On the Asymptotic Symmetry Algebra of Classical and Quantum Gravity

By

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DECLARATION

I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree / diploma at this or any other Institution / University.

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List of Publications arising from the thesis[1]

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2. **Double soft graviton theorems and Bondi-Metzner-Sachs symmetries**,

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[1] As it is standard in the High Energy Physics Theory (hep-th) community the names of the authors on any paper appear in their alphabetical order.

Conferences and workshops attended

1. “Chennai Strings Meeting 2022", at “The Institute of Mathematical Sciences", Chennai, India from 21-24 February 2023.

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6. “Indian Strings Meeting 2019” at “Indian Institute of Science Education and Research (IISER)”, Bhopal from 22-27 December 2019.

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9. “Indian Strings Meeting” at “Indian Institute of Science Education and Research (IISER)”, Trivandrum from 16-21 December 2018.

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Dedicated to my friends and family

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Contents

Synopsis

0.1 Introduction

This thesis is concerned with a study of symmetries in classical and quantum gravitational scattering in $d \geq 4$ dimensions. More in detail, the goal of the thesis is to contribute towards an understanding of the symmetry algebra whose corresponding conservation laws constrain the $\mathcal{S}$-matrix of a gravitational theory. In a non-gravitational field theory such as massive scalar field theory, the symmetries of scattering are simply the isometries of underlying spacetime. In a theory of gravity (and in general in any gauge theory) the group of symmetries is typically infinite-dimensional. Several examples are well understood now. In three spacetime dimensions with a negative cosmological constant, the symmetry group that preserves the space of solutions of Einstein's equations is generated by the infinite-dimensional Virasoro algebra whose central charge is $\frac{3l}{2G}$. Since the early 60s, it has been known that the group of symmetries that preserve solutions to Einstein's equations in four dimensions with vanishing cosmological constant is an infinite dimensional group, where the asymptotic spacetime translations are enhanced to so-called angle dependent *Supertranslations*. This symmetry group is the celebrated BMS group. The action of this symmetry group on space of solutions can be represented by its action on the gauge invariant "free data" which are parametrized by fields at the boundary of spacetime known as Null Infinity. As the symmetry is represented by its action on the boundary of a spacetime, such gravitational symmetries are known as *Asymptotic Symmetries*.

In the last decade or so, the understanding of asymptotic symmetries in four dimensions has been significantly enhanced due to its relationship with gravitational observables in classical theory and factorization theorems in perturbative quantum gravity (PQG). These factorization theorems of the PQG $\mathcal{S}$-matrix are known in the literature as *Soft Theorems*. Although the complete "tower" of possible asymptotic symmetries that includes the classical BMS group is still being discovered, we now understand fairly well that in addition to the infinite-dimensional extension of spacetime translations captured by supertranslations, the symmetry group of gravitational scattering in four dimensions also includes an infinite dimensional extension of the Lorentz group, known as *Superrotations*. For real solutions to Einstein's equations, these superrotations are parametrized by diffeomorphisms of the celestial sphere metric. However in the case of quantum scattering, when external states are definite helicity states (and hence correspond to complexified solutions to linearised equations), superrotations can also be parametrized in terms of the so-called loop group. In this thesis, we will focus on the former definition of superrotations as in $d > 4$ dimensions no definition of loop group is known in the literature. This group of symmetries containing both supertranslation and superrotation shall be called Generalised-BMS (GBMS) in $d > 4$.

Inspired by the equivalence between asymptotic symmetries and Soft Theorems in $d = 4$, this thesis aims to contribute towards the understanding of the existence of similar equivalence in higher dimensions focusing on $d = 6$. In particular, we have identified the correct radiative degrees of freedom in $d = 6$ that are compatible with both Supertranslation and Superrotation action. We proposed conserved Superrotation charges, whose correctness is checked by the fact that they reproduce the correct action on "free data". Then elevating this symmetry as the conjectured symmetry of the $\mathcal{S}$-matrix in PQG, we establish their relation to Soft Theorems in $d = 6$.

In the following, in section-0.2, we first discuss some necessary preliminaries for studying asymptotic symmetries in $d \geq 4$ dimensions. Then in section-0.3, we state some universal factorization properties of the PQG $\mathcal{S}$-matrix in generic dimensions and their known equivalence with asymptotic symmetries in $d = 4$. In section-0.4 we discuss how extending the above program in higher even dimensions relation between conservation law corresponding to supertranslation symmetry of PQG $\mathcal{S}$-matrix and one of its factorization was built. In relation to this, we discuss the BMS Symmetries in $d = 6$. This serves as the background for Section-0.5, 0.6, and 0.7, which discuss new results of the thesis. We conclude with section-0.8, where the primary content of the thesis is very briefly summarized.

0.2 Preliminaries

In this thesis, we only consider massless fields. That is, we either consider the vacuum Einstein's equations or where the source is a massless stress-energy tensor. In either of these two cases, the field equations can be recast as an initial value problem with the characteristic "initial data" specified at the future of past null infinity of the manifold. Spacetimes we consider will have two disjoint Null infinities, one at the past and one at the future. Both past and future null infinity in general d-dimension has a topology $\mathbb{S}^{(d-2)} \times \mathbb{R}$.

A particularly suitable coordinate system for studying asymptotic symmetries at future null infinity is the retarded Bondi coordinates (u, r, z^a), where r is the radial distance from the origin, $u = t - r$ is the retarded time, and (z^a) co-orodinatize the celestial sphere $\mathbb{S}^{(d-2)}$. All the solutions to Eintein's equation with normalisable sources and vanishing cosmological constant asymptote to the flat metric, which in these co-ordinates take the form

$$ds^2 = -du^2 - 2dudr + r^2\gamma_{ab}dz^a dz^b. \tag{1}$$

Such a parametrization of the space of solutions can be made explicit in the Bondi gauge as follows. Consider

$$ds^2 = Me^{2\beta}du^2 - 2e^{2\beta}dudr + g_{ab}(dz^a - U^a du)(dz^b - U^b du), \tag{2}$$

where the Bondi gauge condition is given by

$$g_{rr} = 0 \quad g_{ra} = 0 \quad \det\left(\frac{g_{ab}}{r^2}\right) = \det(\gamma_{ab}). \tag{3}$$

Here, $M(u,r,z^a)$, $U^a(u,r,z^a), \beta(u,r,z^a)$ and $g_{ab}(u,r,z^a)$ are hitherto un-determined. Asymptotic flatness is ensured by demanding that the Weyl tensor "peels" off suitably fast at large r. However, these fall-off conditions on the Weyl tensor do not uniquely fix the fall-off conditions on the metric components, and exploring this freedom leads to a sufficiently large class of asymptotically flat solutions. Weakening the fall-off conditions typically leads to the enlargement of asymptotic symmetry algebra.

In this thesis, we explore these subtleties in Six dimensions. In the following, we shall discuss the fall-off conditions and physical considerations that led to the proposal of asymptotic symmetries in $d = 6$ to be BMS. Then we explain our results regarding the attempt to extend this to GBMS in $d = 6$. But, before doing that, we first briefly mention the factorisation theorems of the PQG $\mathcal{S}$-matrix named Soft Theorems and their known connections to the conservation laws corresponding to the asymptotic symmetries in $d = 4$.

0.3 Soft Theorems & Asymptotic Symmetries in $d = 4$

In any spacetime dimension, consider any scattering amplitude ($\mathcal{A}_{n+1}$) containing $i = 1, \cdots, n$ finite energy particles of any mass, spin, and one soft (energy $\omega \to 0$) graviton. In the expansion of the soft energy, the amplitude can be factorized in terms of the amplitude containing solely these other n finite energy particles ($\mathcal{A}_n$) and some factors which depend upon the data of the external states only. We can write this factorization as follows [1]:

$$\mathcal{A}_{n+1} = \left[\frac{1}{\omega}S^{(0)} + S^{(1)} + \omega S^{(2)}\right]\mathcal{A}_n + O(\omega^2) \tag{4}$$

Note that, the above expression is written in a manner that the factors $S^{(0)}$, $S^{(1)}$, and $S^{(2)}$ are independent of soft energy ω. These are called the *Soft Factors*. These factorizations hold for any arbitrary theory of quantum gravity that behaves like PQG on a background spacetime in the low-energy. The first two soft factors $S^{(0)}$ and $S^{(1)}$ are universal in the sense that they don't depend on the details of the particular interaction term of the theory. In $d = 4$ due to the presence of infrared divergence coming from loops, one is forced to make the mathematical statement (4) in tree level only. However, in $d > 4$, this is an all-loop statement.

Let, $|\text{in}\rangle$ and $\langle\text{Out}|$ denote the "in" and "Out" states of a scattering process. Then one can write the amplitude $\mathcal{A}_{n+1}$ and $\mathcal{A}_n$ in terms of the matrix elements of the $\mathcal{S}$-matrix. Let, p^μ and $\epsilon_{\mu\nu}$ be the momentum and polarisation tensor of the soft graviton with polarisation label λ. Let, $\mathfrak{a}_\lambda(\omega, \hat{z})$ be the operator that creates a soft graviton with energy ω in the "Out" state whose direction on the celestial sphere can be denoted using $\hat{z}$. Let, k^i_μ and $\mathcal{J}^i_{\mu\nu}$ be the momentum and angular momentum of the i-th finite energy particle respectively.

Now, using the explicit expression of the *Leading Soft Factor* $S^{(0)}$ and picking $1/\omega$ contribution in $\mathcal{A}_{n+1}$, one can write the *Leading Single Soft Graviton Theorem* [2] as follows:

$$\lim_{\omega\to 0} \omega\, \langle\text{Out}|\, \mathfrak{a}_\lambda(\omega, \hat{z})\mathcal{S}\, |\text{in}\rangle = \sqrt{8\pi G_N}\left(\sum_i \frac{\epsilon^{\mu\nu}_\lambda k^i_\mu k^i_\nu}{(p/\omega)\cdot k^i}\right)\langle\text{out}|\, \mathcal{S}\, |\text{in}\rangle \tag{5}$$

Similarly, using the explicit expression of the *Subleading Soft Factor* $S^{(1)}$ and picking $O(1)$ in ω contribution in $\mathcal{A}_{n+1}$, one can write the *Subleading Single Soft Graviton Theo-*

rem [1] as follows:

$$\lim_{\omega\to 0}(1+\omega\partial_\omega)\,\langle\mathrm{Out}|\,\mathfrak{a}_\lambda(\omega,\hat{z})\mathcal{S}\,|\mathrm{in}\rangle = -i\sqrt{8\pi G_N}\left(\sum_i \frac{\epsilon_\lambda^{\mu\nu}k^i_\nu p^\rho \mathcal{J}^i_{\mu\rho}}{p\cdot k^i}\right)\langle\mathrm{out}|\,\mathcal{S}\,|\mathrm{in}\rangle\,. \qquad (6)$$

Note that, in the equation-(5) and equation-(6) in the LHS the operator $\mathfrak{a}_\lambda(\omega,\hat{z})$ creates an additional graviton in the outgoing state compared to RHS. The energy of the graviton is given by ω and $\hat{z}$ denotes its direction. The specific limits on ω used in (5) and (6) respectively ensure the leading and the subleading limits.

Universal behaviour of the above factorization property of PQG $\mathcal{S}$-matrix prompts one to ask if there is a “symmetry origin" of them. One finds that, indeed in $d = 4$ these factorization theorems can be rederived if the known asymptotic symmetries of the classical gravity are elevated as the symmetries of the PQG $\mathcal{S}$-matrix.

Let us briefly mention the asymptotic symmetry algebras which are of interest here. In the seminal works [3, 4], in $d = 4$, the asymptotic symmetry algebra was obtained to be the celebrated BMS algebra, which is a semidirect product of Supertranslation (ST) and the Six-dimensional conformal algebra on the celestial sphere. The super-translations are parametrized by smooth functions $f(z^a)$ on the celestial sphere at $\mathcal{I}^\pm$. In terms of basis spherical harmonics, linear combinations of $Y_{lm}(z^a)|\,l\ \leq\ 1$ correspond to global translations. In $d = 4$, there are at least two distinct infinite dimensional extensions of this six-dimensional conformal algebra. Since, local conformal transformations on $\mathbb{S}^2$ are infinite dimensional, extending the global conformal algebra to the local conformal algebra, one gets an infinite dimensional extension of BMS algebra known as extended BMS. Instead, if one extends this global conformal algebra to all smooth diffeomorphisms on $\mathbb{S}^2$, the corresponding infinite-dimensional extension is known as Generalised BMS (GBMS) algebra. Thus superrotations are parametrized by smooth vector fields $V^a(z^a)$ on the celestial sphere at $\mathcal{I}^\pm$. This definition of Superrotation in $d = 4$ will be of particular interest to us, as we shall attempt to generalize it to higher even dimensions. It is important to note that, Supertranslation (ST) algebra is a subalgebra of both BMS and GBMS. Superrotation algebra is subalgebra of GBMS.

In $d = 4$ leading (5) and subleading (6) soft graviton theorem can be shown to be equivalent to the Supertranslation and Superrotation symmetry of the PQG $\mathcal{S}$-matrix [5, 6] respectively. A key input from the classical gravity in establishing this connection is as follows. One can obtain conserved Noether charges corresponding to Supertranslation ($Q^{d=4}_{\rm ST}$) and Superrotation ($Q^{d=4}_{\rm SR}$). Then, conjecturing them as the symmetry of the PQG $\mathcal{S}$-matrix, these conservation laws in the quantum theory can be cast as the following Ward identities:

$$\langle \mathrm{Out}|\,[Q^{d=4}_{\rm ST}, \mathcal{S}]\,|\mathrm{in}\rangle = 0 \qquad \langle \mathrm{Out}|\,[Q^{d=4}_{\rm SR}, \mathcal{S}]\,|\mathrm{in}\rangle = 0 \tag{7}$$

Then, (5) and (6) imply (and is also implied by) the first and the second Ward identity in (7) respectively.

Since due to the lack of IR divergence coming from loops soft theorems are even more robust in higher dimensions, a natural question arises as follows. Does there exist an analogous relation between soft theorems and asymptotic symmetries in higher dimensions?

0.4 BMS Symmetries in $d = 6$

Early works regarding the asymptotic symmetries concluded about a trivial asymptotic symmetry (Poincare) in higher dimensions [7]. Since, the Soft Theorems (5) and (6) hold in any dimensions, this prompted recent works to revisit these works. This corresponds to the revision of stricter large-r fall-off chosen in earlier works. The argument behind this revision is as follows. In any general d dimension, radiation contribution comes from the $O(r^{(d/2-3)})$ term of the large-r expansion of the angular part (g_{ab}) of the metric. However, in any dimension in linearized gravity, supertranslation changes $O(r)$ of this angular metric. In $d = 4$, it's a coincidence that these two orders match. Thus, in $d = 4$ disallowing supertranslation, will automatically exclude all radiative solutions from the solution space. This makes the enlargement of the Poincare symmetry to include Supertranslation a physical necessity. Such necessities are not present in $d > 4$. In higher even d it is possible to consistently set $O(r)$ of this angular metric to be zero, and still get radiative solutions. Since setting $O(r)$ term to be zero is not a supertranslation invariant condition, this effectively reduces the asymptotic symmetry group to Poincare. Since in higher dimensions there is no phenomena to dictate the "correct choice" of the large-r fall-off, inspired by asymptotic symmetries and Soft theorems connection in $d = 4$, one wishes to seek whether there is a BMS compatible fall-off by weakening the earlier stricter fall-offs.

In [8], starting from the Leading Soft Theorem (5) a Ward identity of the form

$$\langle \text{Out}|\,[Q_{\text{ST}}^{d=2m+2}, \mathcal{S}]\,|\text{in}\rangle = 0 \tag{8}$$

was derived and the Supertranslation charge ($Q_{\text{ST}}^{d=2m+2}$) was read-off from it. This charge was then derived at the classical level in linearized gravity in [9]. The authors introduced some additional u fall-off conditions for the dynamical mode, which leads to the vanishing of the divergence coming from the boundaries of $\mathcal{I}^+$.

In [10], the above result was generalized for non-linear gravity focusing on $d = 6$. The BMS-compatible fall-off conditions for $d = 6$ can be written as:

$$\begin{aligned}
&M = -1 + \sum_{n=1}^{\infty} \frac{M^{(n)}(u,z)}{r^n}, \quad \beta = \sum_{n=2}^{\infty} \frac{\beta^{(n)}(u,z)}{r^n}, \quad U_a = \sum_{n=0}^{\infty} \frac{U_a^{(n)}(u,z)}{r^n} \\
&g_{ab} = r^2\gamma_{ab}(z) + rC_{ab}(z) + D_{ab}(u,z) + \sum_{n=1}^{\infty} \frac{g_{ab}^{(n)}(u,z)}{r^n},
\end{aligned} \tag{9}$$

where $D_{ab}(u,z)$ is the dynamical mode in $d = 6$. From the physical consideration, it should scale as $1/|u|^{(2+\epsilon)}$. Supertranslation action violates this u fall-off of the dynamical mode due to the presence of C_{ab} (which was absent in the linear analysis of [9]). However, physical news tensor $\partial_u D_{ab}$ remains invariant under any redefinition of dynamical mode by the addition of a pure function of C_{ab}. Exploiting this fact, in [10] it was identified that the following redefinition of the dynamical mode keeps the u fall-off invariant under supertranslation action.

$$\tilde{D}^{\mathrm{ST}}_{ab} = D_{ab} - \frac{1}{4}\gamma^{cd}C_{ac}C_{bd} - \frac{1}{16}\gamma_{ab}C_{cd}C^{cd} \tag{10}$$

The Noether charge for Supertranslation ($Q^{d=6}_{\mathrm{ST}}$) was found to be the same as in [9], except for the replacement $D_{ab} \to \tilde{D}^{\mathrm{ST}}_{ab}$. From the above results, it is natural to claim that, for BMS compatible solution space of the classical theory, the correct classical degree of freedom to quantize for obtaining the graviton in quantum theory is not D_{ab} but $\tilde{D}^{\mathrm{ST}}_{ab}$. Corresponding Ward identity

$$\langle \mathrm{Out}|\,[Q^{d=6}_{\mathrm{ST}}, \mathcal{S}]\,|\mathrm{in}\rangle = 0 \tag{11}$$

correctly reproduces the leading soft graviton theorem (5).

Since in $d = 4$ there is a generalization from BMS to GBMS by including Superrotations, a natural question is, whether the same holds for $d = 6$ at the classical level. And if yes, at the quantum level, can we obtain the Subleading Single Soft graviton theorems (6) from the Ward identities constructed of the Noether charges corresponding to such asymptotic symmetries in $d = 6$? This is the question which we explore in this thesis.

0.5 GBMS Symmetries in $d = 6$

Inspired by the generalization of symmetry algebra from BMS to GBMS in the $d = 4$ case, we start with the generalization of the fall-off conditions chosen for studying the

BMS algebra. Let us start with the following fall-off conditions [11]:

$$M = \sum_{n=0}^{\infty} \frac{M^{(n)}(u,z)}{r^n}, \quad \beta = \sum_{n=2}^{\infty} \frac{\beta^{(n)}(u,z)}{r^n}, \quad U_a = \sum_{n=0}^{\infty} \frac{U_a^{(n)}(u,z)}{r^n}$$
$$g_{ab} = r^2 q_{ab}(z) + rC_{ab}(u,z) + D_{ab}(u,z) + \sum_{n=1}^{\infty} \frac{g_{ab}^{(n)}(u,z)}{r^n} \tag{12}$$

Here, $q_{ab}(z)$ is obtained from any area preserving ($\sqrt{q} = \sqrt{\gamma}$) smooth diffeomorphisms of unit round sphere metric $\gamma_{ab}(z)$. From Einstein's equation, we get $\partial_u C_{ab} = -\bar{\mathcal{R}}^{TF}_{ab}$, where $\bar{\mathcal{R}}^{TF}_{ab}$ is the trace-free part of the Ricci tensor corresponding to q_{ab} metric. This implies, $C_{ab}(u,z) = \bar{C}_{ab}(z) + uT_{ab}(z)$, where, $T_{ab} = -\bar{\mathcal{R}}^{TF}_{ab}$. Given $q_{ab}(z)$, $\bar{C}_{ab}(z)$ and $D_{ab}(u,z)$; metric can be solved at all order in r. $D_{ab}(u,z)$ is the dynamical mode.

Connection with the fall-off conditions (9) chosen for studying BMS algebra must be stressed here. If one restricts to the unit round metric γ_{ab} on the $\mathbb{S}^4$ i.e. $q_{ab} = \gamma_{ab}$, then $T_{ab} = 0$, and $M^{(0)} = -1$, i.e. one essentially recovers (9), and the corresponding symmetry algebra is BMS. Demanding preservation of the fall-off conditions (12), one obtains an infinite dimensional extension of the Lorentz subalgebra of original BMS algebra, parametrized by any smooth vector field V^a on $\mathbb{S}^4$. Correspondingly, one gets Generalised-BMS (GBMS) algebra in $d = 6$. GBMS is a semi-direct product of ST and Diff($\mathbb{S}^4$). Henceforth, by Superrotation in $d = 6$, we shall mean this extension of Lorentz algebra. Superrotation vector fields are given by:

$$\xi^u_{\text{SR}} = u\alpha(z)$$
$$\xi^a_{\text{SR}} = V^a(z) - u\mathcal{D}_b\alpha(z)\int_r^{\infty} e^{2\beta(u,r',z)} g^{ab}(u,r',z)dr'$$
$$\xi^r_{\text{SR}} = -\frac{r}{4}[\mathcal{D}_a\xi^a_V(u,r,z) - uU^a(u,r,z)\mathcal{D}_a\alpha(z)]\,. \tag{13}$$

Here, $\alpha = \frac{1}{4}\mathcal{D}_a V^a$. Action of the superrotation on $\bar{C}_{ab}$, T_{ab}, and D_{ab} can be written as:

$$\delta_{\text{SR}}\bar{C}_{ab} = \mathcal{L}_V\bar{C}_{ab} - \alpha\bar{C}_{ab}$$

$$\delta_{\text{SR}} T_{ab} = \mathcal{L}_V T_{ab} - 2(\mathcal{D}_a \mathcal{D}_b \alpha)^{TF}$$

$$\begin{aligned}\delta_{\text{SR}} D_{ab} &= u\alpha\partial_u D_{ab} + \mathcal{L}_V D_{ab} \\ &\qquad + u\Big\{\frac{1}{4}\mathcal{D}^2\alpha C_{ab} - U^{(0)}_{(a}\mathcal{D}_{b)}\alpha + \frac{1}{2}q_{c(a}\mathcal{D}_{b)}(C^{cd}\mathcal{D}_d\alpha) - C_{c(a}\mathcal{D}_{b)}\mathcal{D}^c\alpha \\ &\qquad - \mathcal{D}^c\alpha\mathcal{D}_c C_{ab} + \frac{1}{2}q_{ab}U^{(0)c}\mathcal{D}_c\alpha - \frac{1}{4}q_{ab}\mathcal{D}_c(C^{cd}\mathcal{D}_d\alpha)\Big\} \qquad (14)\end{aligned}$$

Similar to [10], we shall work on the decompactified sphere ($\mathbb{R}^4$). Borrowing from the terminology used in $d = 4$, we call the case of $q_{ab} = \gamma_{ab}$ metric on $\mathbb{S}^4$ (or δ_{ab} metric on $\mathbb{R}^4$) as Bondi frame. In the Bondi frame, $T_{ab} = 0$ and $C_{ab} = \bar{C}_{ab}$; and hence, the superrotation action (14) takes a simpler form. Now, superrotation action takes away from the Bondi frame i.e. $\delta_{\text{SR}} T_{ab} \neq 0$, even starting from Bondi frame where $T_{ab} = 0$.

Due to the generalization r fall-off condition from (9) to (12), there arises a need for further field redefinition of radiative degrees of freedom, such that the u fall-off at the boundaries of the $\mathcal{I}^+$ is maintained. This generalisation should capture the information of non-zero T_{ab}, but should smoothly reproduce the redefinition (2.28) in the Bondi case ($T_{ab} = 0$, $C_{ab} = \bar{C}_{ab}$). We shall look at the effect of going linearly away from the Bondi frame. In this case, a natural generalization of field redefinition becomes:

$$\begin{aligned}\tilde{D}_{ab} &= D_{ab} - \frac{1}{4}q^{mn}\bar{C}_{am}\bar{C}_{bn} - \frac{1}{16}q_{ab}\bar{C}_{mn}\bar{C}^{mn} \\ &\qquad - u\Big[\frac{1}{4}q^{mn}(\bar{C}_{am}T_{bn} + T_{am}\bar{C}_{bn}) + \frac{1}{8}q_{ab}T_{mn}\bar{C}^{mn}\Big] + O(T^2). \qquad (15)\end{aligned}$$

Note that supertranslation and superrotation action on this redefined radiative field can be written as:

$$\delta_{\text{ST}}\tilde{D}_{ab} = f\partial_u\tilde{D}_{ab} \qquad (16)$$

$$\delta_{\text{SR}}\tilde{D}_{ab} = \mathcal{L}_V\tilde{D}_{ab} + u\alpha\partial_u\tilde{D}_{ab} \qquad (17)$$

Thus, the u fall-offs are not violated by supertranslation or superrotation action starting from a Bondi frame.

0.6 Conserved Superrotation Charge

In [11], we obtained the conserved charge corresponding to superrotation symmetry in the Bondi frame. Similar to the $d = 4$ case, the charge can be split into two kinds of terms. Borrowing from the terminology used in $d = 4$, we refer to the terms linear in dynamical mode $\tilde{D}_{ab}$ as the Soft charge and terms quadratic in $\tilde{D}_{ab}$ as the Hard charge. Hard superrotation charge in $d = 6$ can be written as:

$$Q_{\rm SR}^{\rm Hard} = \frac{1}{8\pi G_N}\int_{\mathcal{I}^+}\Big[u\alpha(z)\mathcal{T}^{(4)}_{uu}(u,z) + V^a(z)\mathcal{T}^{(4)}_{ua}(u,z)\Big]. \tag{18}$$

Here, $\mathcal{T}^{(4)}_{uu}(u,z)$ and $\mathcal{T}^{(4)}_{ua}(u,z)$ are uu and ua components of the $O(r^{-4})$ terms in the large r expansion of the stress-energy tensor $\mathcal{T}_{\mu\nu}$ respectively. Using the stress-energy tensor for pure gravity, Hard superrotation charge for pure gravity was obtained to be:

$$Q_{\rm SR}^{\rm Hard} = \frac{1}{32\pi G_N}\int_{\mathcal{I}^+} N^{ab}\Big(\mathcal{L}_V\tilde{D}_{ab} + u\alpha N_{ab}\Big). \tag{19}$$

where, $N_{ab} = \partial_u\tilde{D}_{ab}$ is the news tensor.

An independent derivation of the Hard Superrotation Charge was also given starting from the Symplectic structure defined on the Hard Phase Space. Hard Phase space consists of the canonical pair $(\tilde{D}_{ab},\, N_{ab})$. A symplectic structure can be defined in this Phase space as follows:

$$\Omega_{\rm Hard}(\delta,\delta^{'}) = -\frac{1}{32\pi G_N}\int_{\mathcal{I}^+}\delta\tilde{D}_{ab}\wedge\delta^{'}N_{ab}. \tag{20}$$

Hard Superrotation charge is obtained from this symplectic structure as follows:

$$\Omega_{\text{Hard}}(\delta_0, \delta_{SR}) = \delta_0 Q_{\text{SR}}^{\text{Hard}}. \tag{21}$$

The charge thus obtained matches the Hard superrotation charge for pure gravity(19) obtained from the stress-energy tensor. In [11], the following superrotation soft charge was proposed for any generic Bondi frame ($\bar{C}_{ab} \neq 0$):

$$\begin{aligned} Q_{\text{SR}}^{\text{Soft}} = \frac{1}{128\pi G_N} \int_{\mathcal{I}^+} u V^b(x) \Big[\partial^4 \partial^a \tilde{D}_{ab} - \frac{4}{3} \partial_b \partial^2 \partial^{ef} \tilde{D}_{ef}\Big] \\ + \frac{1}{96\pi G_N} \int_{\mathcal{I}^+} (\mathcal{L}_V \bar{C}_{ab} - \alpha \bar{C}_{ab}) \partial^a \partial^m \tilde{D}^b_m \end{aligned} \tag{22}$$

The correctness of the soft charge is tested by the fact that they produce correct action on the Kinametic data $(\bar{C}_{ab}, T_{ab})$ in the Bondi frame.

$$\begin{aligned} \{Q_{\text{SR}}^{\text{Soft}}, \bar{C}_{ab}\} &= \delta_{\text{SR}} \bar{C}_{ab} \\ \{Q_{\text{SR}}^{\text{Soft}}, T_{ab}\} &= \delta_{\text{SR}} T_{ab}. \end{aligned} \tag{23}$$

Here, the right-hand side of (23) is obtained from the spacetime action (14) after putting $T_{ab} = 0$ for the Bondi case. This soft charge (22) is further justified by the fact that they reproduce the correct subleading soft graviton theorem in the quantum theory, as shall be discussed in the next section.

Finally, we have the total Superrotation charge in pure gravity in $d = 6$ given as:

$$\begin{aligned} Q_{\text{SR}} &= Q_{\text{SR}}^{\text{Soft}} + Q_{\text{SR}}^{\text{Hard}} \\ &= \frac{1}{128\pi G_N} \int_{\mathcal{I}^+} u V^b(x) \Big[\partial^4 \partial^a \tilde{D}_{ab} - \frac{4}{3} \partial_b \partial^2 \partial^{ef} \tilde{D}_{ef}\Big] \\ &\quad + \frac{1}{96\pi G_N} \int_{\mathcal{I}^+} (\mathcal{L}_V \bar{C}_{ab} - \alpha \bar{C}_{ab}) \partial^a \partial^m \tilde{D}^b_m + \frac{1}{32\pi G_N} \int_{\mathcal{I}^+} N^{ab} \Big(\mathcal{L}_V \tilde{D}_{ab} + u\alpha N_{ab}\Big). \end{aligned} \tag{24}$$

This charge produces the correct action on the dynamical data, namely:

$$\{Q_{\mathrm{SR}}, \tilde{D}_{ab}\} = \delta_{\mathrm{SR}}\tilde{D}_{ab}, \tag{25}$$

where $\delta_{\mathrm{SR}}\tilde{D}_{ab}$ is the spacetime action of superrotation given in (16).

0.7 Implication to the $\mathcal{S}$-matrix

As we already mentioned one of the primary motivations for choosing boundary conditions such that one gets a non-trivial asymptotic symmetry algebra is the possible implication of this symmetry to the quantum gravity $\mathcal{S}$-matrix. Specifically, we ask how the GBMS symmetry of the $\mathcal{S}$-matrix is related to the soft graviton theorems mentioned in (5) and (6).

At this point, an important conceptual aspect needs to be mentioned here. apriori there are two independent GBMS algebras: (1) GBMS^{+} acting on $\mathcal{I}^{+}$, labelled by $(f^{+}(z), V_{+}^{a}(z))$ and (2) GBMS^{-} acting on $\mathcal{I}^{-}$, labelled by $(f^{-}(z), V_{-}^{a}(z))$. Inspired from [12], we can identify a diagonal subalgebra GBMS^{0} of GBMS^{+} $\times$ GBMS^{-} can be identified as the symmetry of the gravitational scattering problem. This is done through the following matching conditions:

$$f^{+}(z) = f^{-}(-z) \qquad\qquad V_{+}^{a}(z) = V_{-}^{a}(-z) \tag{26}$$

The vacua are degenerate because the GBMS action on a vacuum state creates a new vacuum. Motivated from [10], a convenient choice for labeling the vacua is to choose them to be the eigenstates of the operators $\bar{C}_{ab}$ and T_{cd}. One studies the soft theorems in the ordinary Fock vacuum and we choose this vacuum as $|0\rangle = |\bar{C}_{ab} = 0, T_{cd} = 0\rangle$. One can construct the finite energy “in" and “out" states from this vacuum.

In our case, we consider the scenario of a massless scalar field coupled to gravity and consider a scattering process in which all finite energy particles except the soft graviton are scalar particles. One has the following Superrotation charge in this case. The Soft charge is given by (22) and the Hard charge is obtained from (18) using the energy-momentum tensor for the scalar field. Finally, for the GBMS symmetry of the $\mathcal{S}$-matrix, we can write a Ward identity of the form:

$$\langle \text{Out}| \, [Q_{\text{SR}}^{d=6}, \mathcal{S}] \, |\text{in}\rangle = 0 \tag{27}$$

In [11], we showed that, starting from the subleading soft graviton theorem (6) in $d = 6$ and specializing to the case where in the external states finite energy particles are only scalars, one can derive the Ward identity (27).

0.8 Conclusion

We now summarise the primary content of the thesis. The primary goal of this thesis is to draw the lessons from the recent discovery of symmetry algebra that constrains gravitational scattering in Four dimensions and generalize it to higher even dimensions. In particular, in $d = 6$ we identified the radiative degrees of freedom in the Bondi frame that preserve correct early and late time behaviour ($|u| \to \infty$) upon Supertranslation and Superrotation action.

The main result of this thesis is a proposal for the Superrotation charge in the Bondi frame beyond linearised gravity. We proved that this charge has correct action on the Dynamical and Kinematic data. In the case of a scalar field coupled to gravity, by promoting the superrotation symmetry to the symmetry of Quantum Gravity $\mathcal{S}$-matrix, we established the connection with the Subleading Soft Graviton Theorem.

Outline of the thesis

In this thesis, we explore the asymptotic symmetries of gravity in higher dimensions and their implications both at the classical as well as quantum level.

- Chapter 1 will introduce the basic background and motivation for studying asymptotic symmetries.
- Chapter 2 will motivate the study of Asymptotic Symmetries in higher dimensions, and survey the works done in the literature that will be relevant for our work.
- Chapter 3 will introduce GBMS symmetries in $d = 6$.
- Chapter 4 will discuss the consequence of these symmetries to classical gravity and the conserved charges they lead to.
- Chapter 5 will discuss the consequence of these symmetries to quantum gravity, in particular to the perturbative quantum gravity $\mathcal{S}$-matrix. Connection of the GBMS symmetries with a particular type of factorization theorems of $\mathcal{S}$-matrix, namely subleading soft graviton theorem is established.
- Chapter 6 summarises the key results, points out the open issues, and tries to chart a pathway to how to solve them.

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Chapter 1

Introduction

The subject matter of study in this thesis is certain symmetries in classical and quantum gravitational scattering in $d \geq 4$ spacetime dimensions. More in detail, the goal of the thesis is to contribute towards an understanding of the symmetry algebra whose corresponding conservation laws constrain the $\mathcal{S}$-matrix of a gravitational theory. The obvious symmetries of any scattering are the isometries of the underlying spacetime *i.e.* Poincare symmetry for the case when underlying spacetime is Minkowski. However, for gauge theories and gravity, there are further non-trivial symmetries of the scattering and typically they are infinite-dimensional. For gauge theories, these symmetries correspond to so-called *large gauge transformation symmetries*, which are infinite-dimensional extensions of the global internal symmetries (see [13–17] and references therein). For gravitational theories, they correspond to infinite-dimensional extensions of the isometries. In this thesis, we shall study the implication of such infinite-dimensional symmetries in gravitational theories, both in classical and quantum setups.

In [12], it was identified that the symmetries of the flat spacetime $\mathcal{S}$-matrix of gravitational theories in $d = 4$ are encoded in the celebrated Bondi-Metzner-Sachs (BMS) algebra [3, 4]. The isometries of the flat spacetimes are given by the Poincare algebra, which is a semi-direct product of the Translation and Lorentz algebra. BMS algebra is an infinite dimensional extension of this Poincare algebra, where the Translation subalgebra of the Poincare algebra gets an infinite dimensional extension in the form of so-called angle-dependent *Supertranslations*. Hence BMS is a semi-direct product of Supertranslation and Lorentz algebra.

Although the application of the BMS symmetries to the understanding of the Quantum Gravity $\mathcal{S}$-matrix is relatively new, at the classical level, the role of these symmetries has been explored since the early works by Bondi, Metzner, Vanderburg, and Sachs [3,4]. All solutions to Einstein's equation with vanishing cosmological constant and normalizable sources asymptotes to Flat Spacetime. This solution space is known in the literature as *Asymptotically Flat Spacetimes*. The role of the BMS symmetries when looked at from the perspective of this asymptotic behaviour of spacetime is as follows. The action of the BMS symmetries preserves this solution space of Einstein's equation. The action of these symmetries on the space of solutions can be represented by their action on the gauge invariant "free data", which are parametrized by fields at the boundary of spacetime known as Null Infinity [18]. As the symmetry is represented by its action on the boundary of a spacetime, such gravitational symmetries are known as *Asymptotic Symmetries*.

In the case of gravitational scattering, one has some incoming data at the past boundary and outgoing data at the future boundary and the scattering process relates them. Corresponding to each of these boundaries one has an independent asymptotic symmetry algebra. In [12], from the product of these two asymptotic symmetry algebras, symmetry of the gravitational scattering was identified through certain prescription called antipodal matching conditions.

The supertranslation symmetries of the gravitational scattering are related to certain observable effects in classical gravity [19]. Consider a pair of test masses placed at a certain initial distance in the "far zone" of gravitational radiation. As a burst gravitational wave passes through them the distance between them oscillates. If one observes them a long time after the burst gravitational wave has passed one sees a small permanent change in the distance between them [20]. This observable effect is called the *Displacement Memory Effect*. This memory effect can be understood as a consequence of infinitely many conservation laws in a gravitational scattering corresponding to energy conservation at each angle.

This displacement memory effect is a position space effect, which is related to the large-time behaviour of the metric fluctuations of the asymptotically flat class. Corresponding low-frequency behaviour of these metric fluctuations obtained via Fourier transform is intricately linked to certain important factorization properties of the Quantum Gravity $\mathcal{S}$-matrix [19]. Consider, a scattering amplitude containing gravitons and other particles with any mass and spin. In the limit when the energy [1] of one of the gravitons goes zero the amplitude factorizes in terms of a universal (process and theory independent) factor depending only on the data of the external states. A graviton of vanishing energy in the limiting sense is called a *Soft Graviton*. The particular factorization of Quantum Gravity $\mathcal{S}$-matrix discussed here is called the *Weinberg's Soft Graviton Theorem*, based on the seminal work [2] where it was studied first.

Since at the classical level, Supertranslation Symmetries are related to the displacement memory effects, and on the other hand displacement memory effects are also related to Weinberg's Soft Graviton Theorem corresponding to scattering in Quantum Gravity, one can ask if the Weinberg's Soft Graviton Theorems are related to Supertranslation symmetries of the quantum scattering. Indeed, in seminal work [5], it was shown that the Weingberg's Soft Graviton Theorem is equivalent to the conservation laws corresponding to Supertranslation Symmetries of the Quantum Gravity $\mathcal{S}$-matrix.

Thus, one has an interesting triality of relations among the following: memory effects, soft graviton theorems, and asymptotic symmetries. Since they correspond to the Infrared or low energy behaviour of the theory, such a triality of relations is called *Infrared Triangle*. In fact, there are similar triality of relations in any gauge theory [21], but since this thesis is

[1] Since we shall be working throughout with the unit $\hbar = c = 1$, we shall use the term energy and frequency interchangeably.

concerned with gravitational scattering, we shall restrict our discussion to the gravitational Infrared Triangle only.

This triality however is not the only triality that exists in the infrared sector of gravity in $d = 4$.

In deriving Weinberg's Soft Theorem one does an expansion in the energy of the soft graviton and Winberg's Soft Theorem corresponds to factorization of the amplitude in the leading order (pole term) of the soft graviton energy. However, there exists universal factorization at the subleading order in the energy of the soft graviton as well. Such a factorization corresponds to the celebrated *Cachazo-Strominger Soft Graviton Theorem* [1], named after the authors who conjectured them first.

Just like the Weinberg's Soft Graviton Theorem is related to an observable effect in classical gravity called displacement memory, the Cachazo-Strominger Soft Graviton Theorem can also be related to a classical observable effect called *Spin Memory Effect* [22]. In the "far zone" of gravitational radiation, this memory effect can be measured from the relative time delays between beams on clockwise and counterclockwise orbits of particles, induced by radiative angular momentum flux. This memory effect can be understood as a consequence of infinitely many conservation laws in a gravitational scattering corresponding to angular momentum conservation at each angle.

Motivated by Ads/CFT correspondence [23], in $d = 4$, the asymptotic symmetries were further extended by introducing infinite dimensional extensions [24, 25] of the Lorentz subalgebra [2] of the BMS algebra. Such infinite dimensional extensions of Lorentz algebra are called *Superrotations*.

In [26], it was shown that the Cachazo-Strominger Soft Theorem implies the conservation laws corresponding to the Superrotation symmetries of the Quantum Gravity $\mathcal{S}$-matrix. Thus, together with the results of [22], one gets a *Subleading Infrared Triangle* in $d = 4$,

[2] In $d = 4$, there are different definitions of Superrotations corresponding to different infinite dimensional extensions of the Lorentz algebra. We shall discuss these subtleties in detail in Chapter-2.

which encapsulates the triality among the following: Cachazo-Strominger Soft Graviton Theorem, Extended BMS Symmetries, and Spin Memory Effect.

Now, motivated by the development of String theory and other theories with extra dimensions, there has been a flurry of research in understanding the classical gravity in higher dimensions. This motivates one to ask questions regarding the solution space of Einstein's equation with vanishing cosmological constant in $d > 4$ and correspondingly asymptotic symmetries in those spacetime dimensions. Furthermore, due to works by Sen and his collaborators [27–30], it is now well-established that Weinberg's Soft Theorem and Cachazo-Strominger Soft Theorems are true universal statements for any generic theories of quantum gravity in generic dimensions. In $d = 4$ due to Infrared divergence coming from the loops, one is forced to make the Weinberg's and Cachazo-Stromiger's soft theorems as statements at the tree level. In $d > 4$ due to the lack of any IR divergences coming from the loops, the soft theorems are even more robust. Hence, if there exists a connection between symmetries of the scattering and the Soft Theorems in $d > 4$, that is a stronger constraint on the $\mathcal{S}$-matrix. Hence, it is natural to ask if there is a "symmetry origin" of these soft theorems in generic dimensions. Moreover, In $d = 4$, there exists a conjectured duality between the gravity theory in the bulk and a proposed Celestial Conformal Field Theory on the celestial sphere at Infinity. This goes by the name *Celestial Holography* [31,32], and is supposed to be the flat space analogue of the famous Ads/CFT correspondence for spacetimes with a negative cosmological constant. The infrared triangle was a crucial hint for such a conjectured holography. Since Ads/CFT correspondence is conjectured to be true in any spacetime dimensions, it is natural to ask if there is evidence of an Infrared Triangle in $d > 4$ as well. Thus, there is a multitude of motivations to explore the asymptotic symmetries of gravity in higher spacetime dimensions and correspondingly, the symmetries of gravitational scattering in those dimensions.

Early works [7, 33] on asymptotic symmetries in classical gravity in $d > 4$ before the discovery of the Infrared Triangle in $d = 4$ concluded that asymptotic symmetries in

$d > 4$ are trivial, *i.e.* the symmetry algebra is simply the Poincare algebra. This was based on the fact that classical gravity in higher dimensions has no displacement memory in the way it is defined in four dimensions. The same reasoning can be also used to argue that there can be no spin memory effect in higher dimensions. However, with the knowledge of Infrared triangles (both Leading and the Subleading) in $d = 4$, this gives rise to an interesting puzzle in higher dimensions. The universality of soft theorems in generic dimensions suggests that the $\mathcal{S}$-matrix is constrained by an infinite dimensional symmetry, but a lack of the memory effects are seemingly at odd with infinitely many conservation laws that these symmetries will imply.

This motivated a series of recent works [8–10, 34] to critically re-examine these earlier works, and exploiting the room for choosing less restrictive conditions than the earlier works, strong evidence for the Infrered Triangle corresponding to Weinberg's Soft Graviton Theorem was obtained even in higher dimensions.

Since the Cachazo-Strominger Soft Graviton Theorem is also a universal constraint on the $\mathcal{S}$-matrix, it is natural to ask whether there is also a Subleading Infrared Triangle in higher dimensions. This thesis aims to contribute to the understanding of this question. Due to certain conceptual reasons that will be discussed in Chapter-2, it is difficult to make progress in higher odd dimensions and we are forced to restrict ourselves to higher even dimensions. Furthermore, for analytical computability, our concrete calculations are in the lowest $d > 4$ even dimension, *i.e.* d=6.

The rest of this thesis is organized as follows. In Chapter-2 we discuss the basic notion of asymptotic symmetries corresponding to the class of solutions of Einstein's equations called Asymptotically Flat Spacetimes. We discuss early works in this direction in $d \geq 4$. We discuss how to elevate these asymptotic symmetries to the symmetry of the gravitational scattering. We introduce Soft Graviton Theorems and discuss how in $d = 4$ they are equivalent to the conservation laws corresponding to the symmetries of the $\mathcal{S}$-matrix. We discuss how this motivated some recent works to critically re-examine these earlier works related to asymptotic symmetries in $d > 4$ and summarize certain key results in them. This serves as a background for the new contributions in the thesis starting from Chapter-3. In section 3.1, we discuss the boundary conditions that are adopted in our study of asymptotically flat spacetimes in $d = 6$. In section 3.2, we show that the corresponding asymptotic symmetry algebra is the GBMS (which is a semidirect product of supertranslations and superrotations) and evaluate the spacetime action of it. We also identify the correct radiative modes around the so-called Bondi frame, which will be defined in this chapter. In Chapter-4, we propose the conserved superrotation charges in the Bondi frame that generates the corresponding Superrotation symmetries. In Chapter-5, we show that the Ward identities corresponding to the Superrotation charges, which are statements of the conservation laws corresponding to Superrotation symmetries of the $\mathcal{S}$-matrix, follow from the Cachazo-Strominger soft graviton theorem. We summarize our results and address the future directions in Chapter-6.

Chapter 2

Asymptotic Symmetries of Gravity and Soft Graviton Theorems

In Chapter-1 we qualitatively discussed the known triality of relations in $d = 4$ among the following three: Asymptotic Symmetries, Soft Theorems, and Memory effects. Together they form the Infrared Triangles. In this chapter, we shall first explain these notions in detail. Then we shall review the important results in the literature which attempt to extend such a triality in higher dimensions. Our particular focus will be on the relationship between Soft Theorems and Asymptotic Symmetries. As already mentioned, these symmetries corresponding to the solution space of classical gravity can be elevated to the symmetries of classical and quantum gravitational scattering and Soft Graviton Theorems are essentially statements about the conservation laws corresponding to the symmetries of Quantum Gravity $\mathcal{S}$-matrix. In the following, we start with the basic preliminaries regarding the study of Asymptotic Symmetries.

2.1 Preliminaries

We are interested in the *Asymptotically Flat Spacetimes* (AFS) near Null Infinity [35, 36] in d spacetime dimensions. In the geometric framework of Conformal Infinity developed by Penrose, these spacetimes can be described using a manifold $\mathcal{M}$ equipped with a metric $g_{\mu\nu}$ such that they satisfy certain properties: (1) There should exist a conformal embedding from $\mathcal{M}$ to an unphysical manifold $\tilde{\mathcal{M}}$ with boundary $\mathcal{I} = \mathcal{I}^+ \cup \mathcal{I}^-$. Here $\mathcal{I}^\pm$ has a topology $\mathbb{R} \times \mathbb{S}^{(d-2)}$; (2) under the conformal embedding, $g_{\mu\nu} = \Omega^{-2}\tilde{g}_{\mu\nu}$, where Ω is smooth upto and including the boundary $\mathcal{I}$ and on $\mathcal{I}$, $\Omega = 0$, $n_\mu \equiv \nabla_\mu \Omega \neq 0$; (3) metric $g_{\mu\nu}$ satisfies Einstein's equation (with vanishing cosmological constant) with a smooth limit to $\mathcal{I}$. It can be shown that the $\mathcal{I}$ is a null surface.

$\mathcal{I}^-$ is called the Past Null Infinity and $\mathcal{I}^+$ is called the Future Null Infinity. In this thesis, we only consider massless fields. That is, we either consider the vacuum Einstein's equations or the source is a massless stress-energy tensor. In either of these two cases, the field equations can be recast as an initial value problem with the characteristic "initial data" specified at the future of past null infinity of the manifold. In the case of quantum scattering problem, we shall have "in" and "out" data localized at $\mathcal{I}^-$ and $\mathcal{I}^+$ respectively.

Asymptotic symmetries keep the structure of Null infinity preserved. They can be thought of as the symmetries of the solution space of Einstein's Equation with vanishing cosmological constant. Although the asymptotic symmetries in a classical theory can be understood by focusing on one of the Null boundaries, we shall see later that to define them to be the symmetry of the $\mathcal{S}$-matrix one needs to define the symmetry algebra on $\mathcal{I}^+ \cup \mathcal{I}^-$.

A particularly suitable coordinate system for studying asymptotic symmetries at Future Null Infinity $\mathcal{I}^+$ is the retarded Bondi coordinates (u, r, z^a), where r is the radial distance from the origin, $u = t - r$ is the retarded time, and (z^a) co-orodinatize the *celestial sphere* $\mathbb{S}^{(d-2)}$. All the solutions to Eintein's equation with normalizable sources and vanishing cosmological constant asymptote to the flat metric, which in these co-ordinates takes the form

$$ds^2 = -du^2 - 2dudr + r^2\gamma_{ab}dz^a dz^b. \tag{2.1}$$

Here, γ_{ab} is the unit-sphere metric on $\mathbb{S}^{(d-2)}$. Parametrization of the space of solutions can be made explicit in the Bondi gauge. We shall call the metric in this parametrized form as *Bondi Metric*, using which line element can be written as:

$$ds^2 = Me^{2\beta}du^2 - 2e^{2\beta}dudr + g_{ab}(dz^a - U^a du)(dz^b - U^b du). \tag{2.2}$$

Here, the *Bondi gauge condition* is given by

$$g_{rr} = 0 \quad g_{ra} = 0 \quad \det\left(\frac{g_{ab}}{r^2}\right) = \det(\gamma_{ab}). \tag{2.3}$$

Here, $M(u,r,z^a)$, $U^a(u,r,z^a)$, $\beta(u,r,z^a)$, and $g_{ab}(u,r,z^a)$ are hitherto undetermined functions of Bondi coordinates. Asymptotic flatness can be ensured by demanding that the Weyl tensor "peels" off suitably fast at large r. However, these fall-off conditions on the Weyl tensor do not uniquely fix the fall-off conditions on the metric components, and hence, correspondingly on the undermined functions. Exploring this freedom leads to a sufficiently large class of asymptotically flat solutions. Weakening the fall-off conditions typically leads to the enlargement of asymptotic symmetry algebra (ASA).

The discussion in this section implicitly assumes the spacetime dimensions to be even and in [37] it was shown that in odd dimensions there is no useful notion of Null Infinity. Also, in the case when there are massive sources, to define Asymptotically Flat Spacetimes we need to include Timelike Infinities (i^- and i^+). These issues are beyond the scope of this thesis and we shall throughout work in even dimensions with massless sources.

We end this section by illustrating Asymptotically Flat Spacetime using a Penrose dia-

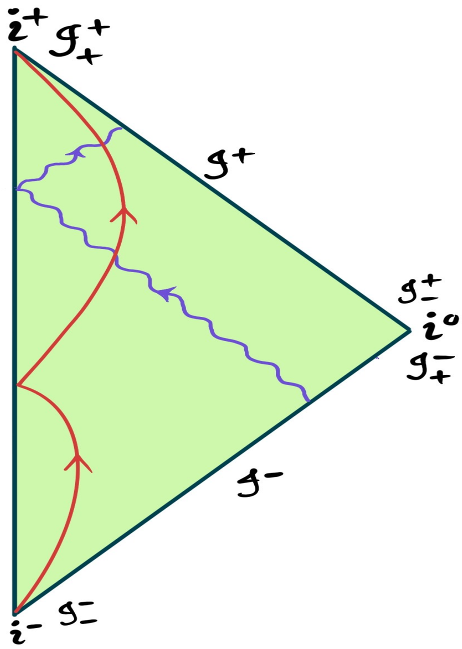

Figure 2.1: *Penrose diagram for Asymptotically Flat Spacetime.* Violet lines represent a Null geodesic and Red lines represent a Timelike geodesic. $\mathcal{I}^{\pm}$ are Future and Past Null boundaries respectively and the subscript $\pm$ denotes their Future and the past end. $i^{\pm}$ denotes the Future and Past Timelike boundaries respectively. i^0 denotes the spacelike boundary.

gram (2.1) and explaining the topology of different boundaries. Null geodesics start at the Past Null boundary $\mathcal{I}^-$ and end at the Future Null Boundary $\mathcal{I}^+$. Since, $\mathcal{I}^{\pm}$ has topology $\mathbb{R} \times \mathbb{S}^{(d-2)}$, each point in the $\mathcal{I}^{\pm}$ lines in the Penrose diagram denotes a $(d-2)$ dimensional sphere. Hence, the past and future ends (denoted by $-$ and $+$ subscripts respectively) of the Null boundaries, represented by points in the Penrose diagram have topology $\mathbb{S}^{(d-2)}$. Timelike geodesics start at the Past Timelike boundary i^- and end at the Future Timelike Boundary i^+. $i^{\pm}$ are points.

2.2 Early Works on Asymptotic Symmetries of Gravity in $d \geq 4$

The study of asymptotic symmetries can be traced back to as early as the sixties. In the seminal works [3, 4], in $d = 4$, the ASA was obtained to be the celebrated BMS algebra, which is a semidirect product of Supertranslation (ST) and Lorentz. ST itself is an infinite dimensional enlargement of the Translation subalgebra of the Poincare algebra (which is the semi-direct product of Translation and Lorentz). ST vector fields are parametrized by a free function $f(z^a)$ on $\mathbb{S}^2$. In $d = 4$, this BMS algebra was further extended in later works. In $d = 4$, there are different proposals for infinite dimensional extension of the BMS algebra, using different infinite-dimensional extensions of the Lorentz subalgebra of the BMS (ST$\rtimes$Lorentz) algebra. In $d = 4$, Lorentz transformation induces the global conformal transformations on $\mathbb{S}^2$. Inspired by an attempt to build a proposed BMS-CFT correspondence (in analogy to Ads/CFT), in the Extended BMS (EBMS) proposal [24, 25], the Lorentz algebra was extended to include local conformal transformations on $\mathbb{S}^2$. Later, inspired by an attempt to build an improved understanding of the Infrared Triangle [1], in the Generalised BMS (GBMS) proposal [38], the Lorentz algebra was extended to include vector fields with generate smooth diffeomorphisms of $\mathbb{S}^2$. Both of these infinite dimensional extensions of the Lorentz algebra are called Superrotation in $d = 4$. In the EBMS case, Superrotation vector fields are parametrized by holomorphic vector fields $V^a(z^b)$ on $\mathbb{S}^2$. On the contrary, in the GBMS case, Superrotation vector fields are parametrized by smooth vector fields $V^a(z^b)$ on $\mathbb{S}^2$. It is important to keep in mind that, ST algebra is a subalgebra of all three proposed ASA in $d = 4$, namely BMS, EBMS, and GBMS.

In [7], the ASA corresponding to AFS of even $d \geq 4$ was studied in their connection to (displacement) memory effect. In [7], it was argued that in $d = 4$ the Supertranslations are tied to the displacement memory effect, and if one uses a strict fall-off such that the ASA

[1] We shall return to this point in a bit more detail in the section-2.4.

is Poincare and thus disallows Supertranslations, generic radiative solutions are automatically excluded. Hence, allowing Supertranslation is essential in $d = 4$. In contrast, in $d > 4$, while the memory effects are seen corresponding to $O(r)$ at the r-expansion of the angular part of the metric (g_{ab}), radiation corresponds to $O(r^{-(d/2-3)})$. Hence, enlargement of the Poincare algebra to include Supertranslation doesn't become a physical necessity. Furthermore, allowing for Supertranslation leads to divergent physical quantities. In this logic, it was argued that the Supertranslation doesn't exist in $d > 4$.

However, new insights from the Infrared Triangle in $d = 4$ have led to revisiting the ASA in higher d in recent works [8–10] and the authors could consistently weaken the fall-off conditions to get a non-trivial asymptotic symmetry bypassing the no-go conditions of [7]. We shall discuss the motivations for this revisit in section-2.4. These new insights in $d = 4$ hinges on the following facts. The asymptotic symmetries in the classical theory can be elevated to the symmetry of the Quantum Gravity $\mathcal{S}$-matrix, and as a consequence of these symmetries of the $\mathcal{S}$-matrix one reproduces the already established results of the Soft Graviton Theorems. So, to proceed further in the next section we review the Soft Graviton Theorems.

2.3 Soft Graviton Theorems

Due to decades of research starting from the early sixties [2] we now understand very robustly that the scattering amplitudes in gravitational theories show the following interesting factorization properties [1, 27–29, 39–44].

In any spacetime dimension, consider a scattering amplitude ($\mathcal{A}_{n+1}$) containing $i = 1, \cdots, n$ finite energy particles of any mass, spin, and one soft (energy $\omega \to 0$) graviton. In the expansion of the soft energy, the amplitude can be written in terms of the amplitude containing solely these other n finite energy particles ($\mathcal{A}_n$) and some factors which depend upon the information of the external states only. We can write this factorization as follows:

$$\mathcal{A}_{n+1} = \left[\frac{1}{\omega}S^{(0)} + S^{(1)} + \omega S^{(2)}\right]\mathcal{A}_n + O(\omega^2). \tag{2.4}$$

Note that, the above expression is written in a manner that the factors $S^{(0)}$, $S^{(1)}$, and $S^{(2)}$ are independent of soft energy ω. These are called the *Soft Factors*.

These factorizations hold for any arbitrary theory of quantum gravity and in any arbitrary dimensions [27–29] and the soft factors are exactly known. We shall write the detailed expressions of the soft factors later. But, before going to the details of the soft factors a few important pieces of information must be stated here. The first two soft factors $S^{(0)}$ and $S^{(1)}$ are universal in the sense that they are not only process-independent but also don't depend on the details of the interaction terms present in theory. The Soft factor $S^{(2)}$ contains a universal piece that is present in any theory of Quantum Gravity and a non-universal piece that depends upon the non-minimal coupling of fields with the Riemann Tensor.

At this point, a few comments about the UV and IR divergence of $\mathcal{A}_{n+1}$ and $\mathcal{A}_n$ must be mentioned. Since the derivation in [27–29] was done using 1-PI effective action of a UV finite theory, the statement (2.4) is well-defined in the UV sense. In $d \geq 5$ due to the absence of infrared divergence coming from the loop momentum, these factorizations are true for all-loop amplitudes. In $d = 4$ infrared divergences force one to restrict these statements at the tree-level, *i.e.* in $d = 4$, $\mathcal{A}_{n+1}$ and $\mathcal{A}_n$ in (2.4) should mean $\mathcal{A}_{n+1}^{\text{Tree},d=4}$ and $\mathcal{A}_n^{\text{Tree},d=4}$. There are additional logarithmic in ω corrections [45] to the soft factors once the loop effects are taken into account.

Let, $|\text{in}\rangle$ and $|\text{Out}\rangle$ denote the "ingoing" and "Outgoing" external states of a scattering process containing the finite energy particles only. Using them, one can write the amplitude $\mathcal{A}_{n+1}$ and $\mathcal{A}_n$ in terms of the matrix elements of the $\mathcal{S}$-matrix. Let, p^μ and $\epsilon_{\mu\nu}$ be the momentum and polarisation tensor of the soft graviton with polarisation label λ. Let,

$\mathfrak{a}_\lambda(\omega, \hat{z})$ be the operator that creates an additional soft graviton with energy ω to the $\langle\text{Out}|$ state, whose direction on the celestial sphere can be denoted using $\hat{z}$. Let, k^i_μ and $\mathcal{J}^i_{\mu\nu}$ be the momentum and angular momentum of the i-th finite energy particle respectively.

Now, using the explicit expression of the *Leading Soft Factor* $S^{(0)}$ and picking $1/\omega$ contribution in $\mathcal{A}_{n+1}$, one can write the *Leading Soft Graviton Theorem* [2] as follows:

$$\lim_{\omega\to 0} \omega \langle\text{Out}| \mathfrak{a}_\lambda(\omega, \hat{z})\mathcal{S}|\text{in}\rangle = \sqrt{8\pi G_N}\left(\sum_i \frac{\epsilon^{\mu\nu}_\lambda k^i_\mu k^i_\nu}{(p/\omega)\cdot k^i}\right)\langle\text{out}|\mathcal{S}|\text{in}\rangle \tag{2.5}$$

This soft theorem is also called *Weinberg's Soft Graviton Theorem*. Here, G_N is the Newton's gravitational constant. Note that, the soft factor is gauge invariant, which can be checked from the fact that when polarisation tensors are taken to be the pure gauge the Soft factor vanishes by momentum conservation.

Similarly, using the explicit expression of the *Subleading Soft Factor* $S^{(1)}$ and picking $\mathcal{O}(1)$ in ω contribution in $\mathcal{A}_{n+1}$, one can write the *Subleading Single Soft Graviton Theorem* [1] as follows:

$$\lim_{\omega\to 0}(1+\omega\partial_\omega) \langle\text{Out}| \mathfrak{a}_\lambda(\omega, \hat{z})\mathcal{S}|\text{in}\rangle = -i\sqrt{8\pi G_N}\left(\sum_i \frac{\epsilon^{\mu\nu}_\lambda k^i_\nu p^\rho \mathcal{J}^i_{\mu\rho}}{p\cdot k^i}\right)\langle\text{out}|\mathcal{S}|\text{in}\rangle\,. \tag{2.6}$$

This soft theorem is also called the *Cachazo-Strominger's Soft Graviton Theorem*. Note that, the soft factor is gauge invariant, which can be checked from the fact that when the polarisation tensor is taken to be the pure gauge the Soft factor vanishes by angular-momentum conservation.

It is important to note here that we now know the existence and the exact formula for the soft factorization for an arbitrary number of soft gravitons in any generic theory of quantum gravity in a generic dimension, where the finite energy particles can have any mass and spin. Such soft graviton theorems are called *Multi-Soft Graviton Theorems* [30, 46]. But, for the purpose of this thesis, it is sufficient to focus on the Single Soft

Graviton Theorems.

Since the factorization at the leading and the subleading order in the energy of the soft gravitons in (2.4) are universal, they serve as a consistency condition for the $\mathcal{S}$-matrix of quantum gravity. One might ask if there are generic symmetries of quantum gravity from which they follow. This is one of the questions the Infrared Triangle program wants to address.

To avoid confusion, it must be stressed here that the Soft Graviton Thorems are theorems in the sense that they are statements about the factorization properties of the quantum gravity $\mathcal{S}$-matrix, which can be derived from the amplitude calculations using Feynman diagrammatic or various other modern techniques. Their derivation apriori doesn't need any reference to the infinite-dimensional asymptotic symmetries. The beauty of the Infrared Triangle program lies in the fact that one gets an independent confirmation of them from the "symmetry origin".

2.4 New Insights from Infrared Triangle

So far we have talked about asymptotic symmetries only in classical theory. One can ask what are the implications of these symmetries at the level of quantum gravity. More specifically, can we say anything about the properties of perturbative quantum gravity $\mathcal{S}$-matrix? Starting with [5], a program was initiated in which certain already known factorization theorems of quantum gravity $\mathcal{S}$-matrix have been found to be a consequence of elevating the asymptotic symmetries of the classical theory as a conjectured symmetry of the $\mathcal{S}$-matrix of the corresponding quantum theory. These factorization theorems are the Soft graviton theorems discussed in the previous section.

In this section, we first discuss how in $d = 4$ Leading and Subleading Soft graviton theorems are related to the asymptotic symmetries. Then, we discuss the early works that hinted similar relations might hold in higher even dimensions as well.

2.4.1 Leading Soft Graviton Theorem & Supertranslation Symmetries in $d = 4$

We want to briefly review how the Leading Soft Graviton Theorem (2.5) is related to the conjectured Supertranslation Symmetry of the quantum gravity $\mathcal{S}$-matrix.

Although, the connection between soft theorem and asymptotic symmetry can be built for finite energy particles with any mass and spins, let us now restrict to perturbative gravity coupled to a massless field for simplicity. In this case, it was shown in [5] that the leading soft theorem (2.5) is a consequence of the conjectured Supertranslation symmetry of the $\mathcal{S}$ matrix.

One can derive the equivalence in two ways, which will be stated below. One can start from the Soft theorem and then derive a Ward identity of Supertranslation for the $\mathcal{S}$-matrix. In this way, one obtains a Ward identity of the form

$$\langle \mathrm{Out}|\, [Q^{d=4}_{\mathrm{ST}}, \mathcal{S}]\, |\mathrm{in}\rangle = 0, \tag{2.7}$$

where, $Q^{d=4}_{\mathrm{ST}}$ is the quantized version of the Supertranslation charge in $d = 4$. Now, since the charge obtained from the Soft theorem matches with the charge obtained from classical gravity this proves that the Soft theorem (2.5) implies Supertranslation Symmetry.

Another way is to start from the classical symmetry and obtain a conserved charge ($Q^{d=4}_{\mathrm{ST}}$). The charges are parametrized by free function $f(z)$ on $\mathbb{S}^2$. Then one can elevate this classical symmetry to the symmetry of the quantum gravity $\mathcal{S}$-matrix by writing a Ward identity of Supertransaltion (2.7). Finally, from this one derives the soft theorem (2.5) as a consequence of the Ward identity (2.7).

A few conceptual points need to be stated here. Apriori there are two independent BMS algebras: (1) BMS^+ acting on $\mathcal{I}^+$, labelled by free function $f^+(z)$ and (2) BMS^- acting on $\mathcal{I}^-$, labelled by free function $f^-(z)$. In [12], a diagonal subalgebra BMS^0 of $\mathrm{BMS}^+ \times$

BMS^- was identified as the symmetry of the gravitational scattering problem. This is done through the antipodal matching $f^+(z) = f^-(-z)$.

It is also important to note that while going from the Ward identity (2.7) to the soft theorem (2.5) one needs to choose the free function $f(z)$ such that it localizes on the particular direction on the celestial sphere corresponding to the direction of the soft graviton. Hence, the Leading Soft Theorem can be thought of as a consequence of Spontaneous Supertranslation Symmetry Breaking from BMS to Poincare.

Another important conceptual point needs to be mentioned here. In $d = 4$, to define the scattering problem certain additional condition called the Christodoulou-Klainerman (CK) condition [12, 47] was required. This condition dictates how the radiative mode should behave at the boundaries of $\mathcal{I}^+$. It is interesting to see how this condition plays a role in the equivalence between the Supertranslation Ward identity (2.7) and the Leading Soft Graviton Theorem (2.5). There are two independent helicity ($\lambda = \pm$) of the soft graviton in $d = 4$. Hence, apriori there are two sets of independent leading soft theorems. But the CK condition relates the positive helicity leading soft theorem with the negative helicity leading soft theorem, and hence there is only one set of independent leading soft theorem. This is consistent with the Ward identities due to the following reasons. While going from Ward identity (2.7) to soft theorem (2.5), there is only one free function to choose from, as the charge is parametrized by a scalar free function $f(z)$ on the celestial sphere. The choice of this function is such that it localizes on a particular direction on the celestial sphere, which gives the direction of the soft graviton.

It is also worth mentioning here that the Supertranslation symmetry in $d = 4$ is related to classical observable effects called gravitational displacement memory [19].

2.4.2 Subleading Soft Graviton Theorem & Superrotation Symmetries in $d = 4$

As before, for simplicity let us restrict to gravity coupled to a massless field. One wants to ask like the leading case, whether in the subleading case also there is an asymptotic symmetry origin of the soft theorem (2.6). In [26], in $d = 4$, starting from the Subleaing Soft Theorem a Ward identity of the form

$$\langle \mathrm{Out}|\,[Q_{\mathrm{SR}}^{d=4}, \mathcal{S}]\,|\mathrm{in}\rangle = 0 \tag{2.8}$$

was derived, where, $Q_{\mathrm{SR}}^{d=4}$ is the quantized version of the Superrotation charge in $d = 4$ corresponding to EBMS algebra. However, the singular nature of the vector fields restricted from proving this equivalence the other way around, namely, Ward identity (2.8) to Soft theorem (2.6). This prompted the authors of [6] to propose a different definition of Superrotation based on Diff($\mathbb{S}^2$) vector field as mentioned in section-2.2. This corresponds to the proposal of GBMS algebra as the ASA for AFS in $d = 4$. In the case of Superrotations corresponding to GBMS, one can go both ways: from Ward identity (2.8) to Soft theorem (2.6) and the reverse. A first principle derivation of the charges corresponding to the Superrorations of this kind was given in [38].

Like the leading case, a few conceptual points need to be stated here. Apriori there are two independent GBMS algebras: (1) GBMS$^+$ acting on $\mathcal{I}^+$, labelled by free functions ($f^+(z)$, $V^a_+(z)$), and (2) GBMS$^-$ acting on $\mathcal{I}^-$, labelled by free function ($f^-(z)$, $V^a_-(z)$). Inspired from [12], a diagonal subalgebra GBMS^0 of GBMS$^+ \times$ GBMS$^-$ can be identified as the symmetry of the gravitational scattering problem. This is done through the following antipodal matching

$$f^+(z) = f^-(-z) \qquad V^a_+(z) = V^a_-(-z). \tag{2.9}$$

It is also worth mentioning that while going from the Ward identity (2.8) to the soft theorem (2.6), one needs to choose the free vector field $V^a(z)$ such that it localizes on the particular direction of the soft graviton. Hence, the Subleading Soft Theorem can be thought of as a consequence of Spontaneous Superrotation Symmetry breaking in the space of degenerate vacua. This corresponds to spontaneous symmetry breaking from GBMS to BMS.

Similar to the leading case a conceptual point about the counting of the independent set of subleading soft theorems needs to be mentioned here. Unlike the leading case, for the subleading case in $d = 4$, CK constraint doesn't relate the negative and positive helicity soft theorems and hence, they are independent sets of soft theorems. This is consistent with the Ward identity because of the following reason. The superrotation charges are parametrized by the two-dimensional vector fields $V^a(z)$ on the celestial sphere. Hence, while going from Ward identity (2.8) to soft theorem (2.6), one has to choose two free functions on the celestial sphere corresponding to two components of the vector field $V^a(z)$. The Choice of these two functions is such that they localize on the particular directions on the celestial sphere corresponding to the directions of the two different helicity soft gravitons.

It is also worth mentioning here that the Superrotation symmetry in $d = 4$ is related to classical observable effects called the gravitational Spin memory [22].

2.4.3 Ward Identities from Soft Theorems in Higher Even Dimensions

In $d = 4$, the Leading Soft Graviton Theorem follows from the supertranslation symmetry of the $\mathcal{S}$-matrix [5]. Since the leading soft graviton theorem (2.5) holds in all dimensions, a natural question is whether supertranslations also exist in all dimensions. Contrary to the classical result of [7], in [8], based on the factorization properties of the perturbative quantum gravity $\mathcal{S}$-matrix, it was argued that the Supertranslation (and correspondingly BMS) holds even in higher even ($d = 2m+2$) dimension and a Supertranslation compatible fall-offs of the Bondi metric (2.2) were proposed. In [8], in all higher even dimensions a Ward identity for the $\mathcal{S}$-matrix of the following form was derived starting from the Leading Soft Graviton Theorem (2.5):

$$\langle \mathrm{Out}|\,[Q_{\mathrm{ST}}^{d=2m+2}, \mathcal{S}]\,|\mathrm{in}\rangle = 0. \tag{2.10}$$

From this Ward identity, the Supertranslation charge ($Q_{\mathrm{ST}}^{d=2m+2}$) can be read-off in generic higher even dimension. This charge was shown to generate the Supertranslation using some proposed commutation relation among the radiative degrees of freedom. However, since there was no first principle derivation of the charge in classical gravity, this created an apparent contradiction with the results of classical gravity [7], the resolution of which will be discussed in the next section.

Inspired from [8], in [48], based on an attempted generalization of Diff($\mathbb{S}^2$) Superrotation in $d = 2m+2$ dimensions (in terms of Diff($\mathbb{S}^{2m}$) vector fields), a Ward identity of Superrotation of the following form was derived in linearized gravity starting from the Subleading Soft Graviton Theorem (2.6):

$$\langle \mathrm{Out}|\,[Q_{\mathrm{SR}}^{d=2m+2}, \mathcal{S}]\,|\mathrm{in}\rangle = 0. \tag{2.11}$$

However, any result on the GBMS symmetries in linearized gravity in higher even dimensions must be consistent with the known results of BMS symmetries in $d = 6$ in non-linear GR [10]. Since in [48] the authors did not allow for Supertranslation modes, it was not clear from their analysis whether one can indeed generalize superrotations in higher dimensions in terms of Diff($\mathbb{S}^{2m}$) vector fields and whether one can properly realize BMS algebra as a subalgebra of this GBMS algebra. This issue was addressed in $d = 6$ in our work [11], which will be the focal point of this thesis. In a very recent work [49], the authors did a much more rigorous analysis which further improved the understanding of GBMS in $d = 6$. But before discussing the Superrotations let us review the resolution of contradictory results regarding BMS in higher dimensions.

2.5 Revisiting the Asymptotic Symmetries in Higher Even Dimensions

As already mentioned, regarding the non-trivial ASA in higher even dimensions, there was a contradiction between the results obtained from classical gravity [7] and from the factorization property of quantum gravity $\mathcal{S}$-matrix [8]. This apparent contradiction was resolved in [9]. The author made a derivation of Supertranslation charge in linearized gravity in higher even dimensions using the Covariant Phase Space Formalism [50]. Despite having the fall-off conditions that allow for Supertranslations, the author was able to get a finite charge by adding certain additional boundary conditions at the boundaries of the $\mathcal{I}^+$ and hence, bypassing the no-go conditions of [7]. Interestingly, these additional conditions also ensure the correct counting for the number of independent soft theorems. Hence, it established the existence of Supertranslation in the higher even dimensions on a stronger footing.

The analysis of [9] was further strengthened in favour of the existence of Supertranslation in higher even dimensions in [10], where the authors did the covariant phase space analysis in non-linear gravity focussing on $d = 6$.

In the following, we first summarize lessons from the above results in a more concrete manner. Then we shall discuss the motivations to extend from BMS to GBMS in $d = 6$, which shall serve as the background for Chapter-3.

2.5.1 Supertranslations in Higher Even Dimensions and Conserved Charges

In [9], the analysis was done in $d = 2m+2$ spacetime dimensions in the linearized gravity coupled to matter. Hence, the author worked with the linearized version of the Bondi metric (2.2), using which line element can be written as:

$$ds^2 = Mdu^2 - 2dudr + g_{ab}dz^a dz^b - 2U_a dz^a du. \tag{2.12}$$

The large-r fall-off conditions chosen for the undetermined parameters of the metric were:

$$\begin{aligned} M &= -1 + \sum_{n=1}^{\infty} \frac{M^{(n)}(u,z)}{r^n}, \quad U_a = \sum_{n=0}^{\infty} \frac{U_a^{(n)}(u,z)}{r^n} \\ g_{ab} &= r^2\gamma_{ab}(z) + \sum_{n=-1}^{\infty} \frac{C_{ab}^{(n)}(u,z)}{r^n} \end{aligned} \tag{2.13}$$

In linear theory, the determinant condition in Bondi-gauge (2.3) ensures that $\gamma^{ab}C_{ab}^{(n)} = 0 \; \forall \; n$, i.e. all $C_{ab}^{(n)}$ are traceless. Solving Einstein's equation one can show that $\partial_u C_{ab}^{(-1)} = 0$ and $C_{ab}^{(m-2)}$ is the free radiative data. Supertranslations are generated by the vector fields:

$$\xi_{\mathrm{ST}} = f(z)\partial_u - \gamma^{ab}(z)\mathcal{D}_a f(z)\partial_b + \frac{1}{2m}\mathcal{D}^2 f(z)\partial_r + \cdots \tag{2.14}$$

Here, $f(z)$ is any smooth function on the celestial sphere $\mathbb{S}^{(d-2)}$, and $\cdots$ denotes the sub-leading (in r) orders of the vector fields. $\mathcal{D}_a$ denotes the covariant derivative compatible with the metric γ_{ab}. The action of Supertranslation preserves the fall-off condition (2.13). This means Supertranslation qualifies as a valid candidate for asymptotic symmetry provided one gets a finite non-zero Noether charge corresponding to it.

Supertranslation does a shift of the $C_{ab}^{(-1)}$ as :

$$\delta_{\mathrm{ST}} C_{ab}^{(-1)} = \Big(-2\mathcal{D}_a\mathcal{D}_b f\Big)^{\mathrm{TF}}. \tag{2.15}$$

Here, the notation $\Big(-2\mathcal{D}_a\mathcal{D}_b f\Big)^{\text{TF}}$ denote the trace-free part of $\Big(-2\mathcal{D}_a\mathcal{D}_b f\Big)$, where the trace has been taken *w.r.t.* the metric γ_{ab}. In the linearized theory, $\delta_{\text{ST}} C^{(n)}_{ab} = 0 \; \forall \; n \geq 0$ (including the radiative order $m-2$). However, later we shall see that this isn't true for non-linear gravity and supertranslation indeed do affect the radiative order as well.

In [9], the Noether charge was calculated for general even dimension $d = 2m+2$ using the covariant-phase space techniques (for review see [50]), . Since the analysis was done in the linearized gravity the hard [2] charge $Q^{\text{Hard,Lin}}_{\text{ST}} = \int_{\mathcal{I}^+} f(z)\mathcal{T}^{\text{Matter}(2m)}_{uu}$ doesn't contain any contribution from the gravitational free data and depends on the matter only. Here, $\mathcal{T}^{\text{Matter}(2m)}_{uu}$ stands for the term at the r^{-2m} order in the large-r expansion of uu component of the matter stress-energy tensor.

The soft charge contained finite as well as the divergent term. The divergence could be cured by putting certain additional $2m-2$ conditions on the behaviour of $C^{(n)}_{ab}$ s at the boundaries of the $\mathcal{I}^+$. These conditions are [9]:

$$\begin{aligned}
&\mathcal{D}^a\mathcal{D}^b C^{(n)}_{ab} = u^{n+1}\Big[\prod_{j=0}^{n}\mathfrak{D}_{j,m}\Big]\mathcal{D}^a\mathcal{D}^b C^{(-1)}_{ab} \quad \forall \;\; 0 \leq n \leq m-3 \\
&\mathcal{D}^a\mathcal{D}^b C^{(m-2)}_{ab}\Big|_{u=\pm\infty,z} \sim O(|u|^{-m+1-\epsilon}) \quad \epsilon > 0 \\
&\mathcal{D}^a\mathcal{D}^b C^{(m+n-2)}_{ab}\Big|_{u=\pm\infty,z} \sim O(|u|^{-m+1+n-\epsilon}) \quad \forall \;\; 1 \leq n \leq m-2, \; \epsilon > 0,
\end{aligned} \tag{2.16}$$

where,

$$\mathfrak{D}_{j,m} = \frac{j(2m-j-3)}{2(j+2)(-2m+j+1)(-m+j+2)}\Big[\mathcal{D}^2 - (j+1)(2m-j-2)\Big]. \tag{2.17}$$

In [9], an aposteriori motivation for putting these conditions was given. It is important to note that, in the d dimension, there are $d(d-3)/2$ leading soft theorems corresponding

[2] similar to the $d = 4$ case, the nomenclature soft and hard is used to denote the part of the charge linear in the gravitational free data and quadratic in the gravity/matter free data respectively.

to the number of polarisations of the graviton. However, all of them are not independent. Supertranslation charges are parametrized by one free function $f(z)$ on the celestial sphere and correspondingly the Ward identity of the form (2.10) has one free function. Using arguments similar to the $d = 4$ case (see section-2.4.1), it can be concluded that this means, there is only one independent soft theorem. This implies one needs $d(d-3)/2 - 1$ extra conditions. In $d = 4$, the Christodoulou-Klainerman (CK) conditions [12, 47] gave the correct counting for the number of independent soft theorems. Hence, in analogy with $d = 4$, these conditions are called the *Generalised CK conditions* in higher dimensions. Among these $d(d-3)/2-1$ generalized CK conditions. $(d-4) = (2m-2)$ conditions are the conditions (2.16) necessary for the finiteness of charge [9]. The remaining $(d-2)(d-3)/2$ other conditions $\mathcal{D}_a U_b^{(0)} = \mathcal{D}_b U_a^{(0)}$ are obtained from the vanishing of magnetic part of the Weyl tensor at $O(r^{-1})$ [8]. Here, $U_a^{(0)}$ is the $O(1)$ term in the large-r expansion of U_a, as defined in (2.13).

The finite part of the soft charge obtained in [9] matches with [8], where it was derived from the soft theorems. This finite Soft charge is given by:

$$\begin{aligned} &Q_{\rm ST}^{\rm Soft,Lin} \\ &= \frac{1}{8\pi G_N}\frac{1}{(2m-1)}\frac{2^{-m}}{\Gamma(m)}\int_{\mathcal{I}^+} f(z)\prod_{l=m+1}^{2m-1}\Big[\mathcal{D}^2-(2m-l)(l-1)\Big]I^{(m-2)}\Big(\mathcal{D}^a\mathcal{D}^b C_{ab}^{(m-2)}\Big), \end{aligned} \tag{2.18}$$

where the operator $I^{(n)}$ stands for n-th antiderivative of the argument with respect to u i.e. $I^{(n)} = [\int_u]^n$. Note that, $\int_{\mathcal{I}^+} = \int d^{2m}z\,\sqrt{\gamma}\int_u$ and the $\int_u I^{(m-2)}\Big(\mathcal{D}^a\mathcal{D}^b C_{ab}^{(m-2)}\Big)$ gives the zero mode.

Total supertranslation charge in linearized gravity in any general higher even dimension $d = 2m + 2$ is thus given by:

$$\begin{aligned} Q_{\rm ST}^{\rm Lin} =& Q_{\rm ST}^{\rm Soft,Lin} + Q_{\rm ST}^{\rm Hard,Lin} \\ &= \frac{1}{8\pi G_N}\frac{1}{(2m-1)}\frac{2^{-m}}{\Gamma(m)}\int_{\mathcal{I}^+} f(z)\prod_{l=m+1}^{2m-1}\Big[\mathcal{D}^2-(2m-l)(l-1)\Big]I^{(m-2)}\Big(\mathcal{D}^a\mathcal{D}^b C_{ab}^{(m-2)}\Big) \end{aligned}$$

$$+\frac{1}{8\pi G_N}\int_{\mathcal{I}^+} f(z)\mathcal{T}^{\text{Matter}(2m)}_{uu}. \tag{2.19}$$

In $d = 4$, the supertranslation charge obtained from the covariant phase space analysis matches with the "electric charge" obtained from the Weyl tensor [38]. In [9], it was shown that the same is true for higher even dimensions as well, since the charge (2.19) is the same as the "electric charge" ($Q^{\text{Elec}}[\xi_{\text{ST}}]$) obtained from the Weyl tensor:

$$Q^{\text{Lin}}_{\text{ST}} = Q^{\text{Elec}}[\xi_{\text{ST}}] \equiv -\frac{1}{8\pi G_N}\frac{1}{2m-1}\lim_{t\to\infty}\int_{\Sigma_t}\partial_\mu\left[r\sqrt{g}C^{\mu t}{}_{\lambda r}\xi^{\lambda}_{\text{ST}}\right], \tag{2.20}$$

where, ξ_{ST} is the supertranslation vector field and $C_{\mu\nu\rho\sigma}$ is the Weyl tensor.

So far, we have talked about the generic higher even dimensions. Let us now focus on the results of [9] in $d = 6$ in particular, as we will discuss this case in detail for non-linear gravity. For notational ease, in $d = 6$, we shall denote $C^{(0)}_{ab}$ as D_{ab} and $C^{(-1)}_{ab}$ as C_{ab}. Here, $D_{ab}(u, z)$ is the dynamical mode, and $C_{ab}(z)$ is the pure supertranslation mode. Higher $C^{(n)}_{ab}$'s will not be important for discussion in $d = 6$ as they don't contribute at $\mathcal{I}^+$. The supertranslation soft charge in $d = 6$ has a finite and a divergent piece. The divergence is cured by imposing the following u fall-off of the dynamical mode at the boundary of the $\mathcal{I}^+$:

$$\mathcal{D}^a\mathcal{D}^b D_{ab}(u = -\infty, z) = \mathcal{D}^a\mathcal{D}^b D_{ab}(u = +\infty, z) = O(|u|^{-1-\epsilon}), \quad \epsilon > 0. \tag{2.21}$$

Finally, soft supertranslation charge is given by:

$$Q^{\text{Soft,Lin}}_{\text{ST}} = \frac{1}{96\pi G_N}\int_{\mathcal{I}^+} f(z)(\mathcal{D}^2 - 2)\mathcal{D}^a\mathcal{D}^b D_{ab} = \frac{1}{96\pi G_N}\int_{\mathbb{S}^4} f(z)(\mathcal{D}^2 - 2)\mathcal{D}^a\mathcal{D}^b \mathcal{N}^{(0)}_{ab}, \tag{2.22}$$

where $\mathcal{N}^{(0)}_{ab} = \int_u D_{ab}$ is the leading soft mode.

Hard supertranslation charge is given by:

$$Q_{\text{ST}}^{\text{Hard,Lin}} = \frac{1}{8\pi G_N} \int_{\mathcal{I}^+} f(z) \mathcal{T}_{uu}^{\text{Matter(4)}}. \tag{2.23}$$

Finally, one can write the total supertranslation charge in linearized gravity in $d = 6$ as [9]:

$$\begin{aligned} Q_{\text{ST}}^{\text{Lin}} =& Q_{\text{ST}}^{\text{Soft,Lin}} + Q_{\text{ST}}^{\text{Hard,Lin}} \\ =& \frac{1}{96\pi G_N} \int_{\mathcal{I}^+} f(z)(\mathcal{D}^2 - 2)\mathcal{D}^a \mathcal{D}^b D_{ab} + \frac{1}{8\pi G_N} \int_{\mathcal{I}^+} f(z) \mathcal{T}_{uu}^{\text{Matter(4)}} \\ =& \frac{1}{96\pi G_N} \int_{\mathbb{S}^4} f(z)(\mathcal{D}^2 - 2)\mathcal{D}^a \mathcal{D}^b N_{ab}^{(0)} + \frac{1}{8\pi G_N} \int_{\mathcal{I}^+} f(z) \mathcal{T}_{uu}^{\text{Matter(4)}} \end{aligned} \tag{2.24}$$

So far, we have talked about asymptotically flat spacetime in linearized gravity. In [10], the work of [9] was extended to non-linear gravity focusing on $d = 6$. One starts with the general metric (2.2) satisfying the Bondi gauge (2.3) and imposes the following fall-off condition:

$$\begin{aligned} &M = -1 + \sum_{n=1}^{\infty} \frac{M^{(n)}(u,z)}{r^n}, \quad \beta = \sum_{n=2}^{\infty} \frac{\beta^{(n)}(u,z)}{r^n}, \quad U_a = \sum_{n=0}^{\infty} \frac{U_a^{(n)}(u,z)}{r^n} \\ &g_{ab} = r^2 \gamma_{ab}(z) + r C_{ab}(u,z) + D_{ab}(u,z) + \sum_{n=1}^{\infty} \frac{g_{ab}^{(n)}(u,z)}{r^n} \end{aligned} \tag{2.25}$$

Consider the r expansion of the angular part of the metric in $d = 6$ as in (2.25). From the equation of motion, it can be shown that $\partial_u C_{ab}(u,z) = 0$, and given $\gamma_{ab}(z)$, $C_{ab}(z)$ and $D_{ab}(u,z)$ at $\mathcal{I}^+$ the metric can be solved at all r-orders in the bulk. D_{ab} corresponds to the radiative mode.[3] The above r fall-off (2.25) is preserved by the BMS vector fields, where BMS = ST $\rtimes$ Lorentz. The components of Supertranslation (ST) vector fields at all order in r can be written as:

$$\xi_{\text{ST}}^u = f(z)$$

[3] In [10], the authors worked on decompactified sphere, i.e. $\mathbb{S}^4 \to \mathbb{R}^4$, and so $\gamma_{ab} \to \delta_{ab}$. However, upon covariantization of the results obtained at the end, one can recover the $\mathbb{S}^4$ results.

$$\xi^a_{\rm ST} = -\partial_b f\int_r^\infty e^{2\beta}g^{ab}dr'$$
$$\xi^r_{\rm ST} = \frac{r}{4}[U^a\partial_a f - \partial_a\xi^a]. \tag{2.26}$$

The action of supertranslation on C_{ab} and D_{ab} can be written as:

$$\delta_{\rm ST}C_{ab} = -2\Big[\partial_a\partial_b f - \frac{1}{4}\delta_{ab}\partial^2 f\Big]$$
$$\delta_{\rm ST}D_{ab} = f\partial_u D_{ab} + \frac{1}{4}\delta_{ab}\Big[-\frac{4}{3}\partial_c C^{cd}\partial_d f - C^{cd}\partial_c\partial_d f\Big] + \frac{1}{4}C_{ab}\partial^2 f - \partial_c C_{ab}\partial^c f$$
$$-\frac{1}{2}\Big[C_{bc}\partial_a\partial^c f + C_{ac}\partial_b\partial^c f\Big] + \frac{1}{2}\Big[\partial_a C_{bc}\partial^c f + \partial_b C_{ac}\partial^c f\Big] + \frac{1}{6}\Big[\partial^c C_{bc}\partial_a f + \partial^c C_{ac}\partial_b f\Big]. \tag{2.27}$$

It is important to note that, from the saddle-point analysis and the finiteness of the symplectic structure one expects that the radiative degrees of freedom should scale as $|u|^{-(2+\epsilon)}$ ($\epsilon > 0$) at the boundaries of $\mathcal{I}^+$. However, as is evident from (2.27), supertranslation action violates this fall-off.

The News tensor associated with the radiative degrees of freedom is given by $N_{ab} = \partial_u D_{ab}$. Since C_{ab} is independent of u, redefinition $D_{ab} \to D_{ab} + \chi_{ab}$, (where χ_{ab} is any function constructed purely from C_{ab}) doesn't change the physical news tensor.

So, one asks whether there exists a redefinition of the radiative degrees of freedom such that: (1) the redefined field gives same news tensor, (2) u fall-off of this is preserved by supertranslation. It was identified in [10], the correct variable for the radiative degrees of freedom in classical theory and hence, correspondingly, the correct graviton mode in the quantized theory that satisfies the above criteria is not D_{ab}, but a non-linear field redefinition given by:

$$\tilde{D}^{\rm ST}_{ab}(u,z) = D_{ab}(u,z) - \frac{1}{4}\delta^{cd}C_{ac}(z)C_{bd}(z) - \frac{1}{16}\delta_{ab}C_{cd}(z)C^{cd}(z). \tag{2.28}$$

Equipped with this redefinition one finds that:

$$\delta_{\mathrm{ST}}\tilde{D}^{\mathrm{ST}}_{ab}(u,z) = f(z)\partial_u\tilde{D}^{\mathrm{ST}}_{ab}(u,z). \tag{2.29}$$

Using this redefinition one finds a finite supertranslation charge in $d = 6$ for non-linear gravity. The charges can be split into soft and hard parts. Note that the soft and hard parts now depend linearly and quadratically on $\tilde{D}_{ab}$ respectively.

The hard supertranslation charge is given by:

$$\begin{aligned}
Q^{\mathrm{Hard}}_{\mathrm{ST}} &= \frac{1}{8\pi G_N}\int_{\mathcal{I}^+} f(z)\mathcal{T}^{(4)}_{uu}(u,z) \\
&= \frac{1}{8\pi G_N}\int_{\mathcal{I}^+} f(z)\Big[\mathcal{T}^{\mathrm{Matter}(4)}_{uu}(u,z) + \frac{1}{4}N^{ab}(u,z)N_{ab}(u,z)\Big],
\end{aligned} \tag{2.30}$$

where $N_{ab} = \partial_u\tilde{D}^{\mathrm{ST}}_{ab}$ is the News tensor in $d = 6$. Soft Supertranslation Charge is given by:

$$Q^{\mathrm{Soft}}_{\mathrm{ST}} = \frac{1}{96\pi G_N}\int_{\mathcal{I}^+} f(z)\partial^2\partial^{ab}\tilde{D}^{\mathrm{ST}}_{ab}(u,z) = \frac{1}{96\pi G_N}\int_{\mathbb{R}^4} f(z)\partial^2\partial^{ab}\mathcal{N}^{(0)}_{ab}(z), \tag{2.31}$$

where $\mathcal{N}^{(0)}_{ab}$ is the leading soft mode given by:

$$\mathcal{N}^{(0)}_{ab}(z) = \int_u \tilde{D}^{\mathrm{ST}}_{ab}(u,z). \tag{2.32}$$

Hence, we have the following total supertranslation charge:

$$\begin{aligned}
&Q_{\mathrm{ST}} \\
&= Q^{\mathrm{Hard}}_{\mathrm{ST}} + Q^{\mathrm{Soft}}_{\mathrm{ST}} \\
&= \frac{1}{8\pi G_N}\int_{\mathcal{I}^+} f(z)\mathcal{T}^{(4)}_{uu}(u,z) + \frac{1}{96\pi G_N}\int_{\mathbb{R}^4} f(z)\partial^2\partial^{ab}\mathcal{N}^{(0)}_{ab}(z) \\
&= \frac{1}{8\pi G_N}\int_{\mathcal{I}^+} f(z)\Big[\mathcal{T}^{\mathrm{Matter}(4)}_{uu}(u,z) + \frac{1}{4}N^{ab}(u,z)N_{ab}(u,z)\Big] + \frac{1}{96\pi G_N}\int_{\mathcal{I}^+} f(z)\partial^2\partial^{ab}\tilde{D}^{\mathrm{ST}}_{ab}(u,z).
\end{aligned} \tag{2.33}$$

It is important to note how from this charge (2.33) one can obtain the linearized gravity charge (2.24) in $d = 6$. In the case of linearized gravity, the contribution to the stress-energy tensor from the gravitational news is absent. So replacing $\tilde{D}^{\text{ST}}_{ab} \to D_{ab}$ in (2.33) and decompactifying the $\mathbb{S}^4 \to \mathbb{R}^4$ in (2.24), both the charges match.

In [10], the authors worked in non-linear gravity and derived the charge (2.33). Using this charge the connection with the leading single soft graviton theorem can be established in the generic $C_{ab} \neq 0$ case through a Ward identity of the following form:

$$\langle \text{out}| \, [Q_{\text{ST}}, \mathcal{S}] \, |\text{in}\rangle = 0 \Leftrightarrow \langle \text{out}| \, [Q^{\text{Soft}}_{\text{ST}}, \mathcal{S}] \, |\text{in}\rangle = - \langle \text{Out}| \, [Q^{\text{Hard}}_{\text{ST}}, \mathcal{S}] \, |\text{in}\rangle \, . \tag{2.34}$$

As we discussed previously, the correct graviton mode in this case is not D_{ab} but $\tilde{D}^{\text{ST}}_{ab}$ [10]. It is important to note that, C_{ab} can be obtained from a scalar potential ψ, and supertranslated vacua are labeled by this scalar potential.

2.5.2 Motivations to Revisit Superrotations in Higher Even Dimensions

So far we have discussed the Supertranslation symmetries in higher even dimensions, which generalize the Supertranslations in $d = 4$. As already mentioned before, in $d = 4$, there are further infinite dimensional asymptotic symmetries called Superrotations. These symmetries, when elevated as the symmetries of the Quantum Gravity $\mathcal{S}$-matrix in $d = 4$, reproduce the Subleading Soft Graviton Theorems. One might ask if a similar result holds in higher dimensions as well. This requires a generalization of Superrotations to the higher even dimensions. In the works mentioned in Section-2.5.1, earlier no-go conditions disallowing any non-trivial asymptotic symmetry were bypassed and strong evidence for the existence of a fall-off conditions that allow for Supertranslations (and correspondingly BMS) was established. In $d = 4$, by weakening the fall-off conditions that give the asymptotic symmetry algebra to be the BMS, one gets the infinite dimensional extensions of the Lorentz subalgebra of the Poincare algebra, which is called Superrotation. It is natural to ask whether a similar thing can be done in higher dimensions as well and whether a finite conserved charge can be obtained.

To start with, among the two extensions of the BMS algebra in $d = 4$ mentioned in section-2.2, a natural generalization of the EBMS to get Superrotations in higher dimensions is not possible. This is due to the fact that in $d > 4$ corresponding $d - 2$ dimensional local conformal algebra is finite-dimensional. So, one asks if the GBMS algebra of $d = 4$ mentioned in section-2.2 (where Lorentz algebra is extended in terms of smooth vector fields on the celestial sphere) can be generalized to higher dimensions. Henceforth, by Superrotations we shall mean the infinite-dimensional extension of Lorentz algebra in terms of smooth vector fields on the celestial sphere.

Superrotations in higher dimensions have been explored in several earlier works, and our analysis builds upon these results. In [48, 51, 52], the authors studied superrotations in linearized gravity, and the analysis in [53] focussed on understanding the set of boundary conditions that admit an action of superrotations. Analysis of [10] showed that the radiative degrees of freedom require a non-linear field redefinition (2.28) by terms quadratic in C_{ab}. It was not clear whether allowing for Superrotations would require any change in this redefinition. Since $C_{ab} = 0$ is not a supertranslation invariant condition, it can't be put consistently zero once both Supertranslations and Superrotations are allowed. Since the analysis in [48] was done in linearized gravity, the charges obtained there didn't take into account the effect of $C_{ab} \neq 0$ case and it wasn't apriori clear how the charges obtained therein will change once this is taken into account. In this thesis, we shall attempt to answer these puzzles.

In the following chapter, we show that the GBMS admits a natural generalization to six dimensions in which the Poincaré algebra is enhanced to an infinite dimensional algebra composed of supertranslations and diffeomorphisms on the celestial plane $\mathbb{R}^4$ [4] (superrotations). We start with the fall-off conditions of Asymptotically Flat Spacetimes that are

[4] Similar to [10], we shall work on the decompactified version of the celestial sphere $\mathbb{S}^4$.

compatible with these symmetries.

Chapter 3

Generalised BMS Symmetries in Six Dimensions

3.1 Asymptotically Flat Spacetime in $d = 6$

In this section, we review asymptotically flat spacetimes in spacetime dimension $d = 6$. We shall analyse the corresponding asymptotic symmetries at null infinity in $d = 6$ in a modified version of the *Bondi gauge* [3, 4], which shall be described below.

Line element for the above class of spacetimes can be written as follows:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = Me^{2\beta}du^2 - 2e^{2\beta}dudr + g_{ab}(dz^a - U^a du)(dz^b - U^b du)\,, \tag{3.1}$$

where $u = t - r$ is the retarded time, r is the radial distance and z^a coordinatizes the celestial plane[1] $\mathbb{R}^4$. Note that the index of U_a is lowered and raised using the metric g_{ab}.

The parameters M, β, U^a and g_{ab} in (3.1) are functions of the (u, r, z^a) coordinates and they

[1]This is equivalent to the decompactified celestial sphere. The coordinate transformation from the Bondi coordinates to these can be found in [54]. All formulas written in this chapter trivially generalize to the celestial sphere.

have the following large-r expansions near $\mathcal{I}^+$ [8],

$$M = \sum_{n=0}^{\infty} \frac{M^{(n)}(u,z)}{r^n}, \qquad \beta = \sum_{n=2}^{\infty} \frac{\beta^{(n)}(u,z)}{r^n}, \qquad U_a = \sum_{n=0}^{\infty} \frac{U_a^{(n)}(u,z)}{r^n},$$
$$g_{ab} = r^2 q_{ab}(z) + \sum_{n=-1}^{\infty} \frac{g_{ab}^{(n)}(u,z)}{r^n} \equiv r^2 q_{ab}(z) + rC_{ab}(u,z) + D_{ab}(u,z) + \frac{E_{ab}(u,z)}{r} + \frac{F_{ab}(u,z)}{r^2} + \cdots \tag{3.2}$$

We consider the space of asymptotically flat geometries where the metric on the celestial plane, q_{ab}, is chosen to be independent of u. The interested readers can refer to [53] for generalizations to u-dependent q_{ab}. The indices of the components $U_a^{(n)}$, $g_{ab}^{(n)}$ are lowered and raised using q_{ab}. The form of the metric (3.1) ensures that $g_{rr} = g_{ra} = 0$. There is an additional gauge fixing condition, often referred to as the *Bondi determinant condition* which is given as

$$\det\left(\frac{g_{ab}}{r^2}\right) = \det(q_{ab}) = \det(\delta_{ab}) \,, \tag{3.3}$$

where δ_{ab} is the metric on $\mathbb{R}^4$. We would like to point out a key difference between the leading order angular metric chosen in this work (denoted by q_{ab}) with those chosen in earlier literature [8–10]. In previous works [8–10], this metric was either fixed to be the unit sphere metric $\mathbb{S}^4$ (γ_{ab}) or the metric on the plane $\mathbb{R}^4$ (δ_{ab}), and further analysis of asymptotic symmetries was pursued with this choice. This led to the proposal for the asymptotic symmetry group as the BMS group in six spacetime dimensions, which is the semi-direct product of supertranslations and the Lorentz group $SO(5,1)$.

As will be shown, just as in spacetime dimensions $d = 4$, relaxing the metric on the celestial plane to an arbitrary smooth metric (with the determinant condition (3.3)) leads to an extension of the BMS algebra in six dimensions, that we refer to as the generalized BMS algebra. In four spacetime dimensions, the choice $q_{ab} = \delta_{ab}$ is referred to as the *Bondi frame*. However, in six and higher dimensions, the Bondi frame can be understood as the choice for q_{ab} which satisfies the four dimensional Einstein's equation (with or without a cosmological constant)[2]. A metric q_{ab} in the non-Bondi frame does not satisfy the four-dimensional Einstein's equation. However, in spacetime dimensions $d = 4$, a similar definition does not apply to the corresponding two-dimensional angular metric q_{ab} on the celestial plane/sphere. For our purposes, the Bondi frame in six dimensions shall be always referred to as $q_{ab} = \delta_{ab}$.

Using the determinant condition (3.3), it can be shown that the traces of $g^{(n)}_{ab}$ are fixed in terms of $g^{(n-1)}_{ab}$. For example,

$$\begin{aligned}
C^a_a &= 0, \\
D^a_a &= \frac{1}{2} C_{ab} C^{ab}, \\
E^a_a &= C^{ab} D_{ab} - \frac{1}{3} C^{am} C_{mn} C^n_a, \\
F^a_a &= C^{ab} E_{ab} + \frac{1}{2} D^{ab} D_{ab} - C^{am} C_{mn} D^n_a + \frac{1}{4} C^{am} C_{mn} C^{nb} C_{ba} \,.
\end{aligned} \tag{3.4}$$

Having expressed the general form of an asymptotically flat spacetime, we can now solve the Einstein equations for the above family of metrics. This also requires us to impose the following fall-off conditions on the Ricci tensor, which are motivated by demanding the finiteness of energy flux and other physical observables [8],

$$\begin{aligned}
R_{uu} &= O(r^{-4}), \qquad R_{ur} = O(r^{-5}), \qquad R_{ua} = O(r^{-4}), \\
R_{rr} &= O(r^{-6}), \qquad R_{ra} = O(r^{-5}), \qquad R_{ab} = O(r^{-4}) \,.
\end{aligned} \tag{3.5}$$

Using the equations above, we find that all metric components in (3.2) can be expressed in terms of q_{ab}, C_{ab} and D_{ab}. For example, it can be shown that,

$$M^{(0)} = -\frac{\bar{\mathcal{R}}}{12}, \qquad U^{(0)}_a = -\frac{1}{6} \mathcal{D}_b C^b_a \,, \tag{3.6}$$

[2]For example, the cosmological constant is needed when $q_{ab} = \gamma_{ab}$ (the metric of the unit sphere) but is not needed when $q_{ab} = \delta_{ab}$ (the metric of the unit plane).

$$\beta^{(2)} = -\frac{1}{64}C^{ab}C_{ab}, \qquad\qquad \beta^{(3)} = \frac{1}{48}\Big(C^{ab}C_{bm}C^m_a - 2C^{ab}D_{ab}\Big)\,, \tag{3.7}$$

where $\bar{\mathcal{R}}$ is the Ricci scalar for the leading order angular metric q_{ab} and $\mathcal{D}_a$ denotes the covariant derivative w.r.t q_{ab}. The Einstein's equations also imposes the following condition on $C_{ab}(u, z)$,

$$\partial_u C_{ab}(u, z) = -\bar{\mathcal{R}}^{\mathrm{TF}}_{ab} \equiv -\bar{\mathcal{R}}_{ab} + \frac{1}{4}q_{ab}\bar{\mathcal{R}}\,, \tag{3.8}$$

where $\bar{\mathcal{R}}_{ab}$ is the Ricci tensor w.r.t q_{ab}. This implies that the general solution for C_{ab} can be written as

$$C_{ab}(u, z) = \bar{C}_{ab}(z) + uT_{ab}(z)\,, \tag{3.9}$$

where

$$T_{ab} = -\bar{\mathcal{R}}^{\mathrm{TF}}_{ab}\,. \tag{3.10}$$

We conclude this section with a few remarks.

- $D_{ab}(u, z)$ is the unconstrained dynamical data in six dimensions, i.e, it is not determined by the equations of motion.
- In the Bondi frame, $T_{ab} = 0$.
- In $d = 4$, the physical News tensor (which encodes gravitational radiation) is determined by subtracting the *Schouten Tensor* at $\mathcal{I}^+$ from the *Geroch Tensor*, Φ_{ab}. Φ_{ab} is determined by the requirement that in any frame at $\mathcal{I}^+$, the News is gauge invariant under the unphysical Weyl rescaling at $\mathcal{I}^+$. There is a well-defined relationship between Φ_{ab} and the tensor T_{ab} in the spacetime dimension $d = 4$ [55]. However, in spacetime dimension $d = 6$, the analogous relationship between T_{ab} and Φ_{ab} is not clear and is beyond the scope of this work. For an earlier discussion of this issue,

we refer the reader to [53].

3.2 Generalized BMS in six dimensions

In this section, we revisit the asymptotic symmetries at null infinity in six dimensions. The asymptotic symmetry associated to the class of metrics described in the previous section are a set of transformations that preserve the form of the metric at $\mathcal{I}^+$ (3.1) and also satisfies the determinant condition (3.3). Generators of such transformations are vector fields which are divergence free at $\mathcal{I}^+$ (3.17).

3.2.1 Generators of supertranslations and superrotation

Consider a smooth vector field ξ of the following form

$$\xi = \xi^u(u,r,z)\partial_u + \xi^r(u,r,z)\partial_r + \xi^a(u,r,z)\partial_a \ . \tag{3.11}$$

Gauge fixing conditions (3.1) together with (3.3) imply that the vector fields have to satisfy

$$\mathcal{L}_\xi g_{rr} = 0, \qquad \mathcal{L}_\xi g_{ra} = 0, \qquad g^{ab}\mathcal{L}_\xi \det g_{ab} = 0 \ . \tag{3.12}$$

The above conditions fixes the form of the vector fields to be

$$\xi^u(u,r,z) = W(u,z) \ , \tag{3.13}$$

$$\xi^a(u,r,z) = V^a(z) - \mathcal{D}_b W(u,z) \int_r^\infty e^{2\beta(r')} g^{ab}(r') dr' \ , \tag{3.14}$$

$$\xi^r(u,r,z) = -\frac{r}{4}[\mathcal{D}_a \xi^a(u,r,z) - U^a \mathcal{D}_a W(u,z)] \ , \tag{3.15}$$

where $W(u,z)$ is an arbitrary function on the celestial plane, and $V^a(z)$ is any smooth vector field on the celestial plane. The vector field ξ can be determined by the fall off conditions given in (3.2) along with the divergence free condition, which are stated as,

$$\mathcal{L}_\xi g_{uu} = O(1), \qquad \mathcal{L}_\xi g_{ur} = O(r^{-1}), \qquad \mathcal{L}_\xi g_{ab} = O(r^2)\,, \tag{3.16}$$

$$\lim_{r\to\infty} \nabla_\mu \xi^\mu = 0\,. \tag{3.17}$$

Here ∇_μ denotes the covariant derivative compatible with the metric $g_{\mu\nu}$ given in (3.1). Using the above conditions one can fix $W(u,z)$ as:

$$W(u,z) = f(z) + u\alpha(z)\,, \tag{3.18}$$

where $f(z)$ is an arbitrary smooth function on the celestial plane, and $\alpha = \frac{1}{4}\mathcal{D}_a V^a$. Thus, it is evident that the vector field ξ is parameterized by $f(z)$ and $V^a(z)$. The vector fields characterised by $f(z)$ (by setting $V^a(z) = 0$) are called the supertranslation vector fields, while the vector fields characterised by $V^a(z)$ (by setting $f(z) = 0$) are called the superrotation vector fields. Therefore, one can write the supertranslation vector field ξ_f as [10]:

$$\xi^u_f(u,r,z) = f(z)\,, \tag{3.19}$$

$$\xi^a_f(u,r,z) = -\mathcal{D}_b f(z) \int_r^\infty e^{2\beta(u,r',z)} g^{ab}(u,r',z)dr'\,, \tag{3.20}$$

$$\xi^r_f(u,r,z) = -\frac{r}{4}[\mathcal{D}_a \xi^a_f(u,r,z) - U^a(u,r,z)\mathcal{D}_a f(z)]\,. \tag{3.21}$$

Also, one can write the superrotation vector field ξ_V as:

$$\xi^u_V(u,r,z) = u\alpha(z)\,, \tag{3.22}$$

$$\xi^a_V(u,r,z) = V^a(z) - u\mathcal{D}_b\alpha(z) \int_r^\infty e^{2\beta(u,r',z)} g^{ab}(u,r',z)dr'\,, \tag{3.23}$$

$$\xi^r_V(u,r,z) = -\frac{r}{4}[\mathcal{D}_a \xi^a_V(u,r,z) - uU^a(u,r,z)\mathcal{D}_a\alpha(z)]\,. \tag{3.24}$$

Hence, the GBMS algebra is defined as the asymptotic symmetry algebra generated by the supertranslation vector field (ξ_f) and the superrotation vector field (ξ_V). Our primary focus will be on superrotations.

3.2.2 Spacetime action on radiative phase space

Using (3.11), we can derive the action of supertranslations and superrotations on the variables parameterizing the phase space. We note that the background metric q_{ab} remains invariant under supertranslations but transforms under the action of superrotations[3],

$$\begin{aligned} \hat{\delta}_f q_{ab} &= 0 \,, \\ \hat{\delta}_V q_{ab} &= -2\alpha q_{ab} + \mathcal{L}_V q_{ab} = -2\alpha q_{ab} + 2 q_{c(a}\mathcal{D}_{b)} V^c \,, \end{aligned} \tag{3.25}$$

where we have made use of the symmetrization convention $X_{(a}Y_{b)} = \frac{1}{2}(X_a Y_b + X_b Y_a)$. Using (3.25), it is easy to see that under the action of superrotation, a Bondi frame ($T_{ab} = 0$) generically transforms to a non-Bondi frame ($T_{ab} \neq 0$). Upon using a stronger fall off condition, $\hat{\delta}_V g_{ab} = O(r)$, we get a constraint on V^a, which takes the form of a conformal Killing vector (CKV) equation,

$$\mathcal{D}_a V_b + \mathcal{D}_b V_a - \frac{q_{ab}}{2}\mathcal{D}_c V^c = 0 \,. \tag{3.26}$$

The solutions to the CKV equation above are the generators of Lorentz transformations which are finite dimensional. Hence by imposing less restrictive fall offs, we allow an infinite dimensional extension of the Lorentz group in six dimensions[4].

Let us discuss the action of GBMS transformations on the radiative phase space, i.e,

[3] We use the notation $\hat{\delta}_f$ and $\hat{\delta}_V$ to denote the variations computed by setting $V^a = 0$ and $f = 0$ in $\hat{\delta}_\xi$ respectively in any general frame.

[4] This is similar to the four dimensional case. In four spacetime dimensions there are two extensions possible, which is the extended BMS [25] and generalized BMS [38,56]. Extended BMS group is generated by V^a's which are local CKV's in two dimensions. In six dimensions, this extension is not possible as the solution to the CKV is finite dimensional.

$C_{ab}(u,z) = \bar{C}_{ab}(z) + uT_{ab}(z)$ and $D_{ab}(u,z)$. These can be derived by studying the variation $\hat{\delta}_{\xi} g_{ab}$ and expanding it in powers of r. The action of supertranslations gives,

$$
\begin{aligned}
\hat{\delta}_f \bar{C}_{ab} &= \frac{1}{2}\mathcal{D}^2 f q_{ab} - 2\mathcal{D}_a\mathcal{D}_b f + fT_{ab}, \\
\hat{\delta}_f T_{ab} &= 0, \\
\hat{\delta}_f D_{ab} &= f\partial_u D_{ab} + \frac{1}{4}\mathcal{D}^2 f C_{ab} - U^{(0)}_{(a}\mathcal{D}_{b)} f + \frac{1}{2} q_{ab} U^{(0)c}\mathcal{D}_c f - \frac{1}{4} q_{ab}\mathcal{D}_c(C^{cd}\mathcal{D}_d f) \\
&\quad - \frac{1}{2} C_{c(a}\mathcal{D}_{b)}\mathcal{D}^c f - \mathcal{D}^c f \mathcal{D}_c C_{ab} + \frac{1}{2}\mathcal{D}^c f \mathcal{D}_{(a} C_{b)c}\,.
\end{aligned}
\tag{3.27}
$$

These equations generalize the action of supertranslations on the phase space variables in a non-Bondi frame. Upon setting $T_{ab} = 0$ (Bondi frame) we recover the results in [10].

One can see that, T_{ab} is invariant under supertranslations as $\hat{\delta}_f q_{ab} = 0$ (see (3.10) and (3.25)). Similarly, the action of superrotations on the radiative phase space can be derived to be:

$$
\begin{aligned}
\hat{\delta}_V \bar{C}_{ab} &= \mathcal{L}_V \bar{C}_{ab} - \alpha \bar{C}_{ab}\,, \\
\hat{\delta}_V T_{ab} &= \mathcal{L}_V T_{ab} - 2(\mathcal{D}_a\mathcal{D}_b\alpha)^{\mathrm{TF}}\,, \\
\hat{\delta}_V D_{ab} &= u\alpha\partial_u D_{ab} + \mathcal{L}_V D_{ab} \\
&\quad + u\Big\{\frac{1}{4}\mathcal{D}^2\alpha C_{ab} - U^{(0)}_{(a}\mathcal{D}_{b)}\alpha + \frac{1}{2} q_{c(a}\mathcal{D}_{b)}(C^{cd}\mathcal{D}_d\alpha) - C_{c(a}\mathcal{D}_{b)}\mathcal{D}^c\alpha \\
&\quad - \mathcal{D}^c\alpha\mathcal{D}_c C_{ab} + \frac{1}{2} q_{ab} U^{(0)c}\mathcal{D}_c\alpha - \frac{1}{4} q_{ab}\mathcal{D}_c(C^{cd}\mathcal{D}_d\alpha)\Big\}\,.
\end{aligned}
\tag{3.28}
$$

The second equation above can be independently derived by evaluating the variation $\hat{\delta}_V \bar{\mathcal{R}}_{ab}$. Note that the variation of D_{ab} in the equations above are expressed in terms of $C_{ab} = \bar{C}_{ab} + uT_{ab}$ for ease of notation. In the rest of this section and the chapter-4), we analyze the action of GBMS symmetries on the sector of the radiative phase space where $T_{ab} = 0$. It is important to note that, even though this sector is preserved under the action of infinitesimal supertranslations, under infinitesimal superrotations any configuration in the $T_{ab} = 0$ sector is generically mapped to a configuration where $T_{ab} \neq 0$. A general analysis of GBMS symmetries on the full radiative phase space at $\mathcal{I}^+$ is beyond the scope of this thesis.

The variations (3.28) take a simpler form in the Bondi frame, where we have to set $T_{ab} = 0$[5],

$$\begin{aligned}
\delta_V \bar{C}_{ab} &= \mathcal{L}_V \bar{C}_{ab} - \alpha \bar{C}_{ab} \,, \\
\delta_V T_{ab} &= -2(\partial_a \partial_b \alpha)^{\mathrm{TF}} \,, \\
\delta_V D_{ab} &= u\alpha \partial_u D_{ab} + \mathcal{L}_V D_{ab} \\
&\quad + u\bigg\{ \frac{1}{4}\partial^2 \alpha \bar{C}_{ab} - U^{(0)}_{(a}\partial_{b)}\alpha + \frac{1}{2} q_{c(a}\partial_{b)}(\bar{C}^{cd}\partial_d \alpha) - \bar{C}_{c(a}\partial_{b)}\partial^c \alpha \\
&\qquad - \partial^c \alpha \partial_c \bar{C}_{ab} + \frac{1}{2} q_{ab} U^{(0)c}\partial_c \alpha - \frac{1}{4} q_{ab}\partial^c (\bar{C}_{cd}\partial^d \alpha) \bigg\} \,.
\end{aligned} \tag{3.29}$$

From (3.29), it is clear that the dynamical data D_{ab} in the Bondi frame grows as $O(|u|^1)$ as we take $|u| \to \infty$. It is important to note that the expected fall off from the saddle point analysis and computation of the symplectic form [10] where one gets

$$\lim_{|u|\to\infty} \text{Graviton} \sim O\left(\frac{1}{|u|^{2+0^+}}\right) \,. \tag{3.30}$$

Therefore, D_{ab} by itself can not represent the graviton mode (in a frame where $\bar{C}_{ab} \neq 0$) since this is in contradiction with the above expected large u fall-off. In [10], it was already noticed that for the case of supertranslations in the Bondi frame, the supertranslation compatible graviton mode for $\bar{C}_{ab} \neq 0$ is a redefinition of D_{ab}, given by:

$$D_{ab} \to \tilde{D}^{ST}_{ab} = D_{ab} - \frac{1}{4}\bar{C}^m_a \bar{C}_{bm} - \frac{1}{16}\delta_{ab}\bar{C}_{mn}\bar{C}^{mn} \,. \tag{3.31}$$

[5]We would like to draw attention to the notational differences between δ and $\hat{\delta}$. The latter refers to a variation in any general frame, whereas the former is a variation specifically evaluated in the Bondi frame ($T_{ab} = 0$).

Using the following form for $\bar{C}_{ab}$ in the Bondi frame,

$$\bar{C}_{ab} = -2\Big(\partial_a\partial_b\psi\Big)^{\mathrm{TF}} , \tag{3.32}$$

which follows from the vanishing of the Weyl tensor $C_{urab}(u = \pm\infty, z)$ at $O(r^{-1})$ [8,9], the action of supertranslation on $\tilde{D}^{ST}_{ab}$ can be written as:

$$\delta_f \tilde{D}^{ST}_{ab} = f\partial_u \tilde{D}^{ST}_{ab} . \tag{3.33}$$

It is interesting to note that, a similar redefinition but with $\delta_{ab} \to q_{ab}$ and $\bar{C}_{ab} \to \bar{C}_{ab} + uT_{ab}$ ensures that for linear deviations from the Bondi frame, we get the expected u fall-offs for superrotated fields, *i.e*, we find:

$$\begin{aligned} &\delta_f \tilde{D}_{ab} = f\partial_u \tilde{D}_{ab} , \\ &\delta_V \tilde{D}_{ab} = \mathcal{L}_V \tilde{D}_{ab} + u\alpha\partial_u \tilde{D}_{ab} , \end{aligned} \tag{3.34}$$

where

$$\begin{aligned} \tilde{D}_{ab} = D_{ab} &- \frac{1}{4}q^{mn}\bar{C}_{am}\bar{C}_{bn} - \frac{1}{16}q_{ab}\bar{C}_{mn}\bar{C}^{mn} \\ &- u\Big[\frac{1}{4}q^{mn}(\bar{C}_{am}T_{bn} + T_{am}\bar{C}_{bn}) + \frac{1}{8}q_{ab}T_{mn}\bar{C}^{mn}\Big] + O(T^2) , \end{aligned} \tag{3.35}$$

with the fall off condition

$$\lim_{u\to\infty} \tilde{D}_{ab} = O\Big(\frac{1}{u^{2+0^+}}\Big) . \tag{3.36}$$

By using the redefined field (3.35) the News tensor $\partial_u \tilde{D}_{ab} = \partial_u D_{ab}$ is unchanged in the Bondi frame as $T_{ab} = 0$. One might be worried that even though we are finally working in the Bondi frame, it is necessary to include T_{ab} in the definition above (3.35). The explanation for this is as follows. From (3.28) it is clear that T_{ab} transforms non-homogeneously, i.e, even if one starts in the Bondi frame ($T_{ab} = 0$), under superrotations T_{ab} transforms to $-2(\mathcal{D}_a\mathcal{D}_b\alpha)^{\rm TF}$. The terms which are linear in the redefinition above will ensure that the non-homogeneous terms generated from the variation of D_{ab} get appropriately cancelled with the non-homogeneous terms generated by the variation of T_{ab}, which is essential in order to respect the fall off condition. In order to derive the generic form for $\tilde{D}_{ab}$ with appropriate fall off conditions in a general non-Bondi frame, we need to take care of the $O(T^2)$ terms in (3.35) which is beyond the scope of this thesis.

Equations (3.34), (3.35) are among the central results of this chapter as they display the correct phase space variables to use in the Bondi frame in the presence of both supertranslations and superrotations.

3.2.3 Generalized BMS at $\mathcal{I}^- \cup \mathcal{I}^+$

The GBMS symmetry algebra at $\mathcal{I}^+$ (denoted by $\mathcal{G}^+$) is defined as the symmetry algebra generated by supertranslations and superrotations on the radiative phase space at $\mathcal{I}^+$. Similarly, one can independently define the asymptotic symmetry algebra at $\mathcal{I}^-$ (denoted by $\mathcal{G}^-$). In order to define a gravitational scattering problem that takes the incoming scattering data at $\mathcal{I}^-$ to outgoing scattering data at $\mathcal{I}^+$, one must define a common asymptotic symmetry algebra at $\mathcal{I}^- \cup \mathcal{I}^+$. Motivated from [6, 38], where the analysis was performed in four dimensions, it is natural to propose that in six dimensions, the diagonal subalgebra of GBMS is the symmetry algebra of the quantum gravity $\mathcal{S}$-matrix. The diagonal subalgebra is identified using the antipodal matching conditions on the null generators of $\mathcal{G}^+$ and $\mathcal{G}^-$ which are given as

$$
\begin{aligned}
f_+(z) &= f_-(-z)\,, \\
V_+^a(z) &= V_-^a(-z)\,.
\end{aligned}
\tag{3.37}
$$

Here, $(f_+, V_+^a), (f_-, V_-^a)$ denote the parameterizations used for supertranslations and superrotations at $\mathcal{I}^+$ and $\mathcal{I}^-$ respectively.

Chapter 4

Superrotation Charge in Bondi Frame

In this chapter, we discuss about the superrotation charge in the Bondi frame that generates the right spacetime action on the radiative phase space. Similar to $d = 4$ case, the superrotation charge consists of two independent terms which we refer to as the soft and the hard charge respectively[1]. We remind the readers that the metric at the leading order in large$-r$ in the Bondi frame is

$$ds^2 = -2dudr + r^2 \delta_{ab} dz^a dz^b \ . \tag{4.1}$$

With this choice, we shall proceed onto computing the charges corresponding to the asymptotic symmetries[2] discussed in section-3.2. Computing the charges using the Noether procedure requires a thorough understanding of the symplectic structure in a non-Bondi frame, which is outside the purview of this thesis. However, the same can be used to compute the hard charge even in this case, but obtaining the total superrotation charge is difficult. Therefore, we shall adopt an alternative route to obtain the charge where we exploit the connection between the soft theorem and the Ward identities (corresponding

[1]The nomenclature is motivated by analysis of these charges in four dimensions where the soft superrotation charge is the so-called spin memory [22].

[2]Even though we are working with the metric on the decompactified sphere, none of the physical outcomes will change by considering the metric on the unit sphere.

to the charges).

The charges we obtain by this method can be generalized to gravity coupled to any spin field. Specifically, we shall consider the special case of the gravity coupled to scalars and explicitly demonstrate the equivalence of the Ward identity and the subleading soft theorem in this example. Our work is based on a similar approach by the authors of [48]. We notice certain subtleties associated with their analysis which are delineated and improved upon in the upcoming sections.

4.1 Superrotation Soft Charge

Taking inspiration from the structure of the soft superrotation charge in $d = 4$, we write down the general tensor structure that is covariant and also generates the correct transformation of the radiative data (the necessary Poisson brackets can be read from the symplectic structure given in [10]). Keeping this conditions in mind we propose the Soft Superrotation charge as follows:

$$Q_{\rm SR}^{\rm Soft} = \frac{64\pi^2}{128\pi G_N}\int_{\mathcal{I}^+} uV^b(x)\mathbb{D}^a\tilde{D}_{ab} + \frac{1}{96\pi G_N}\int_{\mathcal{I}^+}(\mathcal{L}_V\bar{C}_{ab} - \alpha\bar{C}_{ab})\partial^a\partial^m\tilde{D}^b_m\,. \tag{4.2}$$

Here, the notation $\int_{\mathcal{I}^+} \equiv \int_{-\infty}^{+\infty} du \int d^4z$ denote integrals over $\mathcal{I}^+$. Explicit expression for the derivative operator $\mathbb{D}^a\tilde{D}_{ab}$ is given as follows [54]:

$$\mathbb{D}^a\tilde{D}_{ab} = \frac{1}{64\pi^2}\Big[\partial^4\partial^a\tilde{D}_{ab} - \frac{4}{3}\partial_b\partial^2\partial^{ef}\tilde{D}_{ef}\Big]. \tag{4.3}$$

in [54], first term in the soft charge (4.2) was derived by relating it to the CFT$_4$ stress tensor on the boundary and also in [57] with arguments relying on the conformal properties of such operators. In Chapter-5, we shall demonstrate how this is consistent with the subleading soft theorem for gravitons coupled to massless scalar particles. Although this has been derived in a specific frame, $\bar{C}_{ab} = 0$ (where $\tilde{D}_{ab} = D_{ab}$), based on the knowledge from Chapter-3, we know that the correct variable to use in a $\bar{C}_{ab} \neq 0$ frame is $\tilde{D}_{ab}$. Thus, the first term is a generalization of the result in [54] from a special Bondi frame to any general Bondi frame obtained via Supertranslation action. The second term in (4.2) is new and this follows by demanding that the superrotation soft charge generates the right spacetime transformations for $\bar{C}_{ab}$ in the Bondi frame. This requires us to make use of the following Poisson bracket derived in [8, 10]:

$$\left\{\int_{-\infty}^{+\infty} du\, \partial^2 \partial^{ab} \tilde{D}_{ab}(u, z),\ \psi(z')\right\} = 96\pi G_N \delta(z, z')\ . \tag{4.4}$$

Note that a derivation of $Q_{\rm SR}^{\rm Soft}$ from a purely asymptotic symmetry perspective requires us to study the symplectic structure carefully. This has been carried out in four dimensions [38], and extending to higher dimensions is beyond the scope of this thesis.

Here, let us to point out that the expression for the Superrotation Soft charge in (4.2), differs from expression of the Superrotation Soft charge given in [48]. For comparing the two expressions we first set $\bar{C}_{ab} = 0$ in (4.2), cause the charge in [48] corresponds to this case. This leaves us with only the first term in (4.2), which after writing the expression for the derivative operator explicitly takes the following form:

$$\frac{1}{128\pi G_N} \int_{\mathcal{I}^+} u V^b(z) \Big[\partial^4 \partial^a D_{ab} - \frac{4}{3} \partial_b \partial^2 \partial^{ef} D_{ef}\Big]. \tag{4.5}$$

After performing an integration by parts, we observe that the second term in (4.5) is proportional to α, and this matches with the soft charge proposed in [48] upto a proportionality factor. However, as we shall show in Chapter-5, the omission of the first term in the soft charge leads to inconsistency from the perspective of Cachazo-Strominger Soft Graviton Theorem[3].

[3] In [48], the commutator of the charge with the radiative data was studied upto a proportionality factor and hence the extra term might have been missed.

4.2 Superrotation Hard Charge

In the previous section we proposed a Superrotation Soft Charge. Now, we shall derive the gravitational superrotation hard charge by two methods. In the first method we make use of the gravitational stress energy tensor derived in [58] and we shall explain it in this section. In Appendix-A we show that the same expression for Gravitational Superrotation Hard charage can also be derived from the symplectic structure defined on the Hard sector of the radiative phase space.

In terms of the stress energy tensor, the supertranslation [10] and superrotation hard charge can be written as:

$$Q^{\text{Hard}}_{\text{ST}}\Big|_{\bar{C}=0} = \frac{1}{8\pi G_N}\int_{\mathcal{I}^+} f\mathcal{T}^{(4)}_{uu}\,, \tag{4.6a}$$

$$Q^{\text{Hard}}_{\text{SR}}\Big|_{\bar{C}=0} = \frac{1}{8\pi G_N}\int_{\mathcal{I}^+} u\alpha\mathcal{T}^{(4)}_{uu} + V^a\mathcal{T}^{(4)}_{ua}\,, \tag{4.6b}$$

Here, $\mathcal{T}^{(4)}_{uu}$ and $\mathcal{T}^{(4)}_{ua}$ are $O(r^{-4})$ terms in the large r-expansion of the stress-energy tensor. Their expressions in pure gravity is given by [58]:

$$\mathcal{T}^{(4)}_{uu} = \frac{1}{4}N_{ab}N^{ab}, \tag{4.7}$$

$$\mathcal{T}^{(4)}_{ua} = \frac{1}{4}\Big[N_{bc}\partial_a D^{bc} - 2N^{bc}\partial_c D_{ab} + 2N_{ca}\partial_b D^{bc}\Big]. \tag{4.8}$$

In the above we have used the notation $Q\big|_{\bar{C}=0}$ to emphasize that the background used for the above computations is the usual flat metric, without turning on supertranslations.

Making use of (4.7) and (4.8) in (4.6) one can write the Hard charge in pure gravity as follows:

$$Q^{\text{Hard}}_{\text{ST}}\Big|_{\bar{C}=0} = \frac{1}{32\pi G_N}\int_{\mathcal{I}^+} f(z)N_{ab}N^{ab}, \tag{4.9}$$

$$Q^{\text{Hard}}_{\text{SR}}\Big|_{\bar{C}=0} = \frac{1}{32\pi G_N}\int_{\mathcal{I}^+} N^{ab}\Big(\mathcal{L}_V D_{ab} + u\alpha N_{ab}\Big)\,. \tag{4.10}$$

As we demonstrated in Chapter-3, the correct phase space variable to use for the radiative data in the Bondi frame (with $\bar{C}_{ab} \neq 0$) is $\tilde{D}_{ab}$ and using the fall off condition (3.36), the form of the hard charge is unchanged,

$$Q^{\text{Hard}}_{\text{ST}} = \frac{1}{32\pi G_N}\int_{\mathcal{I}^+} f(z) N_{ab}N^{ab}, \tag{4.11}$$

$$Q^{\text{Hard}}_{\text{SR}} = \frac{1}{32\pi G_N}\int_{\mathcal{I}^+} N^{ab}\Big(\mathcal{L}_V\tilde{D}_{ab} + u\alpha N_{ab}\Big). \tag{4.12}$$

One can also derive the above charge by analyzing the hard sector of the symplectic structure [59]. The symplectic structure has been derived in [10] by working with $\delta q_{ab} = 0$, which is sufficient for the purpose of deriving the hard charge for both supertranslation and superrotation. In [10], the authors derived the Supertranslation Hard Charge (4.11). In Appendix-A we explain how one can derive superrotation hard charge can from the symplectic structure. The expression obtained in Appendix-A matches with the expression in (4.12).

4.3 Total Superrotation Charge

Since, we already have the expression for both Soft Superrotation Charge (4.2) and Hard Superrotation Charge(4.12), adding them we can write the expression for total Superrotation Charge for pure gravity in Bondi frame as follows:

$$\begin{aligned} Q_{\text{SR}} = &\frac{1}{32\pi G_N}\int_{\mathcal{I}^+} N^{ab}\Big(\mathcal{L}_V\tilde{D}_{ab} + u\alpha N_{ab}\Big) \\ &+ \frac{\pi}{2G_N}\int_{\mathcal{I}^+} uV^b(x)\mathbb{D}^a\tilde{D}_{ab} + \frac{1}{96\pi G_N}\int_{\mathcal{I}^+}(\mathcal{L}_V\bar{C}_{ab} - \alpha\bar{C}_{ab})\partial^a\partial^m\tilde{D}^b_m\,. \end{aligned} \tag{4.13}$$

By making use of the symplectic form (A.1), one can shown that the Poisson bracket between the Hard charge and the radiative data $\tilde{D}_{ab}(u,z)$ reproduces the spacetime action of Superrotation, i.e, $\delta_V\tilde{D}_{ab}(u,z)$.

Chapter 5

Implication to the Quantum Gravity $\mathcal{S}$-matrix

In six (in general in any $d \geq 5$) spacetime dimensions, both the Weinberg's [2] and Cachazo-Strominger's [1] soft graviton theorems are exact constraints on the quantum gravity $\mathcal{S}$-matrix [27]. In four spacetime dimensions, any statement on the $\mathcal{S}$-matrix has to be understood with care as the Dyson $\mathcal{S}$-matrix is infrared divergent. However in higher ($d \geq 5$) dimensions, soft theorems are precise factorisation statements about the $\mathcal{S}$-matrix which is infrared finite. This makes a relationship of the asymptotic symmetries with the soft theorems rather a robust statement on the symmetries of the $\mathcal{S}$-matrix even when loop effects are taken into account. In this chapter, we shall argue that, in $d = 6$ for the case when the external states consists of only massless scalars the Cachazo-Strominger Soft Graviton Theorem imply the Ward identity for Diff($\mathbb{R}^4$) Superrotation Symmetries. The similar analysis with finite energy external gravitons (or any other non-zero spin) requires a careful understanding of quantization of the gravitons in a non-Bondi frame, which we leave for future work.

We begin with the quantization of the soft charge in the Bondi frame. Using the saddle point approximation, the mode expansion of the graviton in the Bondi frame is given as[1] (these formulas are derived in great detail in [54, 60], where in order to match with their conventions we need to replace $u \to \frac{u}{2}$ in our formulas)

$$\tilde{D}_{ab}(u,\hat{z}) = -\frac{\sqrt{8\pi G_N}}{(2\pi)^3}\int_0^{\infty} d\omega\omega\Big[\tilde{\mathfrak{a}}_{ab}(\omega,\hat{z})e^{-i\omega u} + \tilde{\mathfrak{a}}^{\dagger}_{ab}(\omega,\hat{z})e^{i\omega u}\Big]\,, \tag{5.2}$$

where $\tilde{\mathfrak{a}}_{ab}(\omega,\hat{z})$ and $\tilde{\mathfrak{a}}^{\dagger}_{ab}(\omega,\hat{z})$ are the annihilation and creation operators for the graviton in the vacuum labelled by $\bar{C}_{ab}$, respectively (see section 5.1). They satisfy the following commutation relation

$$[\tilde{\mathfrak{a}}_{ab}(\omega_1,z_1), \tilde{\mathfrak{a}}^{\dagger}_{cd}(\omega_2,z_2)] = \frac{2(2\pi)^5}{\omega_1^3}\delta_{ab,cd}\delta(\omega_1,\omega_2)\delta(z_1,z_2)\,, \tag{5.3}$$

with $\delta_{ab,cd} = \frac{1}{2}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) - \frac{1}{4}\delta_{ab}\delta_{cd}$.

One can now substitute the expansion (5.2) in (4.2) to write the quantized soft charge as[2]

$$Q^{\text{Soft}}_{\text{SR}} = \frac{i}{2\sqrt{8\pi G_N}}\int d^4z \lim_{\omega\to 0} V^b \mathbb{D}^a\Big[(1+\omega\partial_\omega)\tilde{\mathfrak{a}}_{ab} - (1+\omega\partial_\omega)\tilde{\mathfrak{a}}^{\dagger}_{ab}\Big] \tag{5.6}$$

$$- \frac{1}{96\pi^2\sqrt{8\pi G_N}}\int d^4z \lim_{\omega\to 0}\partial^a\partial^m(\tilde{\mathfrak{a}}^b_m + \tilde{\mathfrak{a}}^{\dagger b}_m)(\mathcal{L}_V\bar{C}_{ab} - \alpha\bar{C}_{ab})\,. \tag{5.7}$$

Note that in order to promote the classical expression for the soft charge (4.2) to the quantized version above, we have chosen a particular operator ordering for the terms in the second line. This choice will become clear after defining the vacuum state, as

[1] There is an analogous mode expansion for a field with spin-s in six dimensions,

$$\tilde{X}_{m_1\cdots m_s}(u,\hat{x}) \propto \int_{-\infty}^{+\infty} d\omega\omega\Big[\tilde{\mathfrak{a}}_{m_1\cdots m_s}(\hat{x})e^{-i\omega u} + \tilde{\mathfrak{a}}^{\dagger}_{m_1\cdots m_s}(\hat{x})e^{i\omega u}\Big]. \tag{5.1}$$

[2] The leading and the subleading soft mode take the following form in terms of the creation and annihilation operator

$$\int_{-\infty}^{+\infty} du\tilde{D}_{ab}(u,z) = -\frac{\sqrt{8\pi G_N}}{2(2\pi)^2}\lim_{\omega\to 0}\Big[\tilde{\mathfrak{a}}_{ab}(\omega,z) + \tilde{\mathfrak{a}}^{\dagger}_{ab}(\omega,z)\Big]\,, \tag{5.4}$$

$$\int_{-\infty}^{+\infty} duu\tilde{D}_{ab}(u,z) = \frac{i\sqrt{8\pi G_N}}{2(2\pi)^2}\lim_{\omega\to 0}(1+\omega\partial_\omega)\Big[\tilde{\mathfrak{a}}_{ab}(\omega,z) - \tilde{\mathfrak{a}}^{\dagger}_{ab}(\omega,z)\Big]\,, \tag{5.5}$$

where the factor of 2 in the denominator comes from the fact that we only deal with $\omega > 0$.

done in the following section. Subsequently, we will see that the Ward identity of the superrotation charges on the states built from this vacuum follows from the subleading soft graviton theorem.

5.1 Construction of States and Action of Superrotation Soft Charge

Motivated from [10, 59], a convenient choice for labelling the vacua is to choose them to be the eigenstates of the operators $\bar{C}_{ab}$ and T_{cd}[3]. Soft theorems are usually studied in the Fock vacuum, which corresponds to choosing the vacuum state with zero eigenvalue for $\bar{C}_{ab}$ and T_{cd}, i.e, $|\text{vac}, \bar{C}_{ab} = 0, T_{cd} = 0\rangle$. States with finite energy excitations can be obtained from the vacuum state by acting with the creation operator on these states. For example, a generic incoming state $|\text{in}\rangle$ can be expressed as

$$|\text{in}\rangle = \tilde{\mathfrak{a}}^{\dagger}_{h_1\cdots h_{s_1}}(\omega_1, z_1)\cdots\tilde{\mathfrak{a}}^{\dagger}_{h_1\cdots h_{s_n}}(\omega_n, z_n)\,|\text{vac}, \bar{C}_{ab} = 0, T_{cd} = 0\rangle \ , \tag{5.8}$$

where the operator $\tilde{\mathfrak{a}}^{\dagger}_{h_1\cdots h_s}(\omega, z)$ denotes the creation operator for a particle of spin$-s$ with energy ω and momenta along z. One can similarly define the outgoing states.

For this definition, the reason for the choice of operator ordering in (5.6) is now evident. The action of the soft charge on states defined in (5.8) will not receive any contribution from the second term in (5.6),

$$Q^{\text{Soft}}_{\text{SR}}\,|\text{in}\rangle = \frac{i}{2\sqrt{8\pi G_N}}\int_0^{\infty} d\omega V^b \mathbb{D}^a\Big[(1+\omega\partial_\omega)\tilde{\mathfrak{a}}_{ab} - (1+\omega\partial_\omega)\tilde{\mathfrak{a}}^{\dagger}_{ab}\Big]\,|\text{in}\rangle \ . \tag{5.9}$$

There exists a similar decomposition for the hard charge but the exact structure will not be necessary for our purpose.

[3]Note that we use the same notation for the operators and also the classical fields.

5.2 Action of Superrotation Hard Charge

In this section, we consider the special case of scalar field coupled to gravity, and derive the action of the hard charge on the matter phase space. We will later use this to demonstrate how the Ward identities associated to the superrotation charges are consistent with subleading soft graviton theorem when the external states are massless scalars.

By using the saddle point approximation, the quantized scalar field operator in the Bondi frame is given as

$$\phi^{(2)}(u,z) = -\frac{1}{(2\pi)^3}\int_0^\infty d\omega\omega\Big[\tilde{\mathfrak{a}}(\omega,z)e^{-iu\omega} + \tilde{\mathfrak{a}}^\dagger(\omega,z)e^{iu\omega}\Big]\,, \tag{5.10}$$

where $\phi^{(2)}$ denotes the $\frac{1}{r^2}$ term in the large$-r$ expansion of the field $\phi(u,r,z)$, which is the dynamical mode in $d=6$. The superrotation action on $\phi^{(2)}$ can be written as:

$$\delta_V\phi^{(2)} = \lim_{r\to\infty} r^2\mathcal{L}_\xi\phi = \mathcal{L}_V\phi^{(2)} + u\alpha\partial_u\phi^{(2)} + 2\alpha\phi^{(2)}\,. \tag{5.11}$$

From the inverse Fourier transform of the above equation, we obtain the spacetime action on the creation operator

$$\delta_V\tilde{\mathfrak{a}}^\dagger(\omega,z_s) = \mathcal{L}_V\tilde{\mathfrak{a}}^\dagger(\omega,z_s) - \alpha\omega\partial_\omega\tilde{\mathfrak{a}}^\dagger(\omega,z_s) = V^c\partial_c\tilde{\mathfrak{a}}^\dagger(\omega,z_s) - \alpha\omega\partial_\omega\tilde{\mathfrak{a}}^\dagger(\omega,z_s) \equiv iJ_V(\omega,z_s)\,. \tag{5.12}$$

This is equivalent to evaluating the commutator of the hard charge with the creation operator $[Q^{\text{Hard}}_{\text{SR}}, \tilde{\mathfrak{a}}^\dagger(\omega,z_s)]$.

5.3 Cachazo-Strominger Soft Graviton Theorem and Superrotation Ward Identity

We start by evaluating the Ward identity for the superrotation charges for massless external scalars built from the vacuum state described in section 5.1, that we expect to be implied from the Cachazo-Strominger Soft Graviton Theorem. This can be written as

$$\langle\text{out}|\,[Q_{\text{SR}},\mathcal{S}]\,|\text{in}\rangle = 0 \implies \langle\text{out}|\,[Q^{\text{Soft}}_{\text{SR}},\mathcal{S}]\,|\text{in}\rangle = -\,\langle\text{out}|\,[Q^{\text{Hard}}_{\text{SR}},\mathcal{S}]\,|\text{in}\rangle\ , \tag{5.13}$$

with the incoming and outgoing states being massless scalars. As explained in the previous section, the charge can be written as a sum of soft and hard charge. Using the expression of soft charge (5.9) and the action of the hard charge on the external states (5.12), the Ward identity can be written as[4]

$$\begin{aligned}\frac{1}{\sqrt{8\pi G_N}}\int d^4z_s\, \mathbb{D}^a V^b(z_s) \lim_{\omega\to 0}(1+\omega\partial_\omega)\,\langle\text{Out}|\,\tilde{a}_{ab}(\omega,z_s)\mathcal{S}\,|\text{in}\rangle \\ = -i\left(\sum_{\text{Out}} J^i_V - \sum_{\text{in}} J^i_V\right)\langle\text{Out}|\,\mathcal{S}\,|\text{in}\rangle\ ,\end{aligned} \tag{5.14}$$

where J^i_V is the operator defined in (5.12) acting on the i^{th} external scalar.

The Cachazo-Strominger subleading soft graviton theorem for the external particles being scalars can be written as:

$$\frac{1}{\sqrt{8\pi G_N}}\lim_{\omega\to 0}(1+\omega\partial_\omega)\,\langle\text{out}|\,\tilde{a}_{ab}(\omega,z_s)\mathcal{S}\,|\text{in}\rangle = -i\left(\sum_i \frac{\epsilon^{\mu\nu}_{ab}k^i_\nu p^\rho}{p\cdot k^i}\mathcal{J}^i_{\mu\rho}\right)\langle\text{out}|\,\mathcal{S}\,|\text{in}\rangle\ , \tag{5.15}$$

where $\epsilon^{\mu\nu}_{ab}$ denotes the polarization tensor (with the polarization indices denoted by a, b) of the soft graviton with momenta $p^\mu = \omega\hat{p}^\mu$, where $\hat{p}^\mu$ denotes the unit null momenta parameterized by the flat coordinates z_s, k^i denotes the momenta of the external scalar

[4] Note that in the equation below we have used crossing symmetry to relate the incoming to outgoing subleading soft graviton mode.

particle (which is parameterised by energy ω_{k_i} and z_{k_i}) and $\mathcal{J}^i_{\mu\nu}$ denotes the total angular momenta acting on the i^{th} external particle and the sum runs over all the external particles. In the flat null coordinates The Cachazo-Strominger Soft Graviton Theorem (5.15) can be expressed as [54]:

$$\frac{1}{\sqrt{8\pi G_N}}\lim_{\omega\to 0}(1+\omega\partial_\omega)\langle\text{out}|\,\tilde{a}_{ab}(\omega,z_s)\mathcal{S}\,|\text{in}\rangle$$
$$=\sum_i\Big[P^c_{ab}(z_s-z_{k_i})\partial_{z^c_{k_i}}+\frac{1}{4}\partial_c P^c_{ab}(z_s-z_{k_i})\omega_{k_i}\partial_{\omega_{k_i}}\Big]\langle\text{out}|\,\mathcal{S}\,|\text{in}\rangle\ , \tag{5.16}$$

where

$$P^c_{ab}(x)=\frac{1}{2}\Big(x_a\delta^c_b+x_b\delta^c_a+\frac{1}{2}x^c\delta_{ab}-\frac{4x_ax_bx^c}{x^2}\Big)\,. \tag{5.17}$$

We will now derive the Ward identity (5.14) from the subleading soft theorem (5.15). As shall be seen below, the linear terms in $P^c_{ab}(x)$ will not affect the calculation and therefore these are an artifact of the gauge choice. In order to derive the Ward identity, we smear the LHS of the soft theorem with the function $\int d^4z_s\,\mathbb{D}^aV^b(z_s)$.

This reproduces the LHS of the Ward identity (5.14). Subsequently, by performing the same operation on the RHS of the soft theorem (5.16), we get the following two terms,

$$\sum_i\int d^4z_s\mathbb{D}^aV^b(z_s)P^c_{ab}(z_s-z_{k_i})\partial_{z^c_{k_i}}=\sum_i V^c(z_{k_i})\partial_{z^c_{k_i}}\ , \tag{5.18}$$
$$\sum_i\int d^4z_s\mathbb{D}^aV^b(z_s)\frac{1}{4}\partial_cP^c_{ab}(z_s-z_{k_i})\omega_{k_i}\partial_{\omega_{k_i}}=\sum_i\alpha(z_{k_i})\omega_{k_i}\partial_{\omega_{k_i}}\ . \tag{5.19}$$

In deriving the above equations we have made use of the following identity (for a derivation of this identity we refer the reader to Appendix-B):

$$\frac{1}{64\pi^2}\Big[\partial^4\partial^aP^c_{ab}(x)-\frac{4}{3}\partial^2\partial^{ef}_bP^c_{ef}(x)\Big]=-\delta^c_b\delta^{(4)}(x)\,. \tag{5.20}$$

Now, adding equation (5.18) and equation (5.19), we recover the J_V operator in (5.12) and hence, we get the RHS of the Ward identity (5.14).

This completes the proof of the Superrotation Ward identity (5.14) as a consequence of Cachazo-Strominger Soft Graviton Theorem (5.15) for the particular case where in the external sates there are only massless scalar particles.

There are some subtleties in extending this proof to particles of arbitray spin in $d > 4$, which we hope to address in a future work.

Chapter 6

Conclusions

Summary

Let us now summarize what we have discussed so far. The central theme of the thesis was to study the symmetry algebra that constrains the $\mathcal{S}$-matrix of gravitational scattering in $d \geq 4$ spacetime dimensions. The primary aim was to learn from the existing literature about the interesting structure called the Gravitational Infrared Triangle in $d = 4$ and explore the possibilities to discover similar structures in higher dimensions.

The infrared Triangle in $d = 4$ is a manifestation of the triality of relations: Soft Graviton Theorems, Asymptotic Symmetries, and Memory effects. Hence, we started with a review of it, with a particular emphasis on the duality between Asymptotic Symmetries and Soft Graviton Theorems. We learned that the Asymptotic Symmetries under consideration preserve the solution space of Einstein's equation called Asymptotically Flat Spacetimes and once this asymptotic symmetry is elevated to the symmetry of the Quantum Gravity, corresponding conservation laws are equivalent to the factorization theorems of the $\mathcal{S}$-matrix called Soft Theorems. We learned how in any generic dimensions there are two universal soft graviton theorems corresponding to the leading and the subleading order in the soft graviton and how in $d = 4$ they are related to the conservation laws corresponding to two sets of infinite dimensional symmetries, namely Supertranslations and Superrotations. Furthermore, we learned that they are related to the third corner of the Infrared triangle, which are observable effects namely Gravitational Displacement Memory and Gravitational Spin Memory respectively. Thus together they form Leading and Subleading Infrared Triangle, corresponding to the Leading and Subleading Soft Theorem respectively.

The robustness and universality of Soft Graviton Theorems create a natural curiosity about the existence of Infrared Triangles in higher dimensions. We learned that after an initial period of conflicting claims about the existence of (or lack of) non-trivial asymptotic symmetries in higher dimensions, there is now increasing evidence for the existence of a Leading Infrared Triangle in any generic even dimensions. This drew our curiosity to explore the existence of a Subleading Infrared Triangle. Among the higher even dimensions, in $d = 6$ it is most well understood as there exist rigorous calculations of conserved charges for Supertranslation in nonlinear gravity.

This serves as a motivation to investigate the existence of a Subleading Infrared Trianle in higher even dimensions, particularly in $d = 6$. Our new results in the thesis are specifically in this direction.

In this thesis, we studied the symmetries of the soultion space of Einstein's equation called asymptotically flat spacetimes in $d = 6$. We worked with the special case where we only have massless fields and therefore we perform our analysis focussing on null infinity. We started by analyzing the equations of motion and the gauge conditions which enable us to identify the free data in the theory (section 3.1). In section 3.2, we found the generic set of transformations that keep the asymptotic form of the metric invariant (thereby defining fall-off conditions) and also respect the gauge conditions. Such transformations are generated by two classes of vector fields, namely supertranslations and superrotations, which are the infinite-dimensional extension of the Poincaré generators. While supertranslations leave the leading order angular metric at $\mathcal{I}^+$ invariant, the action of superrotation vector fields is non-trivial.

Having found the generators of the transformations, in Chapter-4, we computed the charges corresponding to the symmetries by demanding that they generate the correct spacetime action of the phase space variables. For simplicity, we made an assumption by restricting ourselves to variations near Bondi frames. The charges could be split up into two pieces, one the hard piece and the other, the soft piece. Further, we found that the soft charge has a term depending on the choice of the vacuum state labeled by $\bar{C}_{ab}$, which is the $O(r)$ term of the metric component g_{ab}. Following the computations in $d = 4$ [59], we demonstrated how the hard charge can be obtained using the covariant phase space analysis on the hard phase space.

Finally in Chapter-5, we demonstrated how the subleading soft theorems can be used to derive the Ward identities corresponding to the Superrotation symmetry of the $\mathcal{S}$-matrix.

Let us summarise our new contributions in this regard. The primary goal of this thesis was to draw the lessons from the recent discovery of symmetry algebra that constrains gravitational scattering in Four dimensions and generalize it to higher even dimensions. In particular, in $d = 6$, we identified the radiative degrees of freedom in the Bondi frame that preserve correct early and late time behaviour ($|u| \to \infty$) upon Supertranslation and Superrotation action. The main result of this thesis is a proposal for the Superrotation charge in the Bondi frame beyond linearised gravity. We proved that this charge has correct action on the Dynamical and Kinametic data. In the case of a scalar field coupled to gravity, by promoting the superrotation symmetry to the symmetry of Quantum Gravity $\mathcal{S}$-matrix, we established the connection with the Subleading Soft Graviton Theorem.

Thus, our work gives stronger evidence for the existence of an Infrared Triangle in $d = 6$, along with improving the understanding of the GBMS algebra as the asymptotic symmetries in $d = 6$. In a recent work [49], the authors did a rigorous analysis of the phase space of the six-dimensional gravity, where many of the issues not addressed in our work were settled.

Future Directions

To conclude, we point out various issues related to the study of the Infrared Triangle in higher dimensions that are yet to have a clear resolution in the literature.

While showing that Cachazo-Strominger Soft Graviton Theorem implies Superrotation Ward identity we have restricted to the special case of a massless scalar field coupled to gravity and considered the case when in the external states all the finite energy particles except for the soft graviton are massless scalars. Conceptually it is expected that there exists a similar relation when the finite energy particles are of any spin-s (including gravitons which are spin 2). This needs to be shown in future works.

Our entire analysis is based on gravity coupled to massless fields in the classical theory and in the quantum theory the corresponding scattering amplitudes consist of massless particles only. However, the soft theorems hold even when the external particles are massive. The relation between soft theorems and asymptotic symmetries for massive external states was established in $d = 4$ in [56]. A similar derivation for higher dimensions is yet to be done.

Understanding the asymptotic symmetries in higher odd dimensions remains a thorny issue. See [61], for a recent work in this direction. It would be interesting to explore whether there exists Infrared Triangle in higher odd dimensions.

Although, throughout the thesis, we have dealt with the Infraed triangle corresponding to Single Soft Graviton Theorems, in any dimension there exist universal factorization theorems for amplitudes containing more than one soft graviton as well [30, 46, 62, 63] . In $d = 4$, there is increasing evidence that they too have a "symmetry origin" [59, 64, 65]. It will be interesting to explore these questions in higher dimensions.

Appendix A

Superrotation Hard Charge from Covariant Phase Space

From the symplectic structure at $\mathcal{I}^+$ for hard phase space in non-linear general relativity [10, 59], we can give a proposal for the derivation of the superrotation hard charge (4.12) in the Bondi frame. A similar derivation for the supertranslation hard charge is already given in section-3 of [10]. A rigorous derivation of the charge in any generic frame requires a more careful analysis of the symplectic structure at $\mathcal{I}^+$ which involves a study of generic variations of the background metric q_{ab}. This is beyond the scope of the present discussion.

The part of the symplectic form in eq.(3.8) of [10] contributing to the hard charge is given as

$$\Omega^{\text{Hard}}(\delta, \delta^{'}) = -\frac{1}{32\pi G_N} \int_{\mathcal{I}^+} \delta\tilde{D}^{ab} \wedge \delta' \partial_u \tilde{D}_{ab} \, . \tag{A.1}$$

The superrotation hard charge is defined as

$$\delta_0 Q^{\text{Hard}}_{\text{SR}} = \Omega^{\text{Hard}}(\delta_0, \delta_V) \, , \tag{A.2}$$

where the variation δ_0 is defined such that $\delta_0\delta_{ab} = 0$. Upon substituting the variations given in (3.34) we obtain[1]

$$
\begin{aligned}
&\delta_0 Q^{\text{Hard}}_{\text{SR}} \\
&= \frac{1}{32\pi G_N}\int_{\mathcal{I}^+}\delta_0 N^{ab}[2u\alpha N_{ab} + 4\alpha\tilde{D}_{ab}] + \frac{1}{32\pi G_N}\int_{\mathcal{I}^+}\left[\delta_0 N^{ab}\mathcal{L}_V\tilde{D}_{ab} + \delta_0 N_{ab}\mathcal{L}_V\tilde{D}^{ab}\right], \\
&\equiv \frac{1}{32\pi G_N}[\eta^{(1)} + \eta^{(2)}] \,.
\end{aligned}
\tag{A.4}
$$

Our goal is to express the expression on RHS as a total variation in δ_0. The first term in this can be simplified to give

$$
\eta^{(1)} = \int_{\mathcal{I}^+} u\alpha\delta_0(N_{ab}N^{ab}) + 4\int_{\mathcal{I}^+}\alpha\delta_0 N^{ab}\tilde{D}_{ab} \,. \tag{A.5}
$$

As we see below, the second term in the expression above gets canceled by a contribution arising from $\eta^{(2)}$,

$$
\eta^{(2)} = \int_{\mathcal{I}^+}\delta_0(N^{ab}V^c\partial_c\tilde{D}_{ab}) + \int_{\mathcal{I}^+}\delta_0[N^{ab}\tilde{D}^c_b\partial_a V_c + (a\leftrightarrow b)] - 4\int_{\mathcal{I}^+}\alpha\delta_0 N^{ab}\tilde{D}_{ab} \,. \tag{A.6}
$$

Hence, by adding (A.5) and (A.6) we get a total variation in δ_0 on the RHS, which then indicates that we can perform an integration in δ_0 to give $Q^{\text{Hard}}_{\text{SR}}$ as:

$$
Q^{\text{Hard}}_{\text{SR}} = \frac{1}{32\pi G_N}\int_{\mathcal{I}^+} N^{ab}[u\alpha N_{ab} + \mathcal{L}_V\tilde{D}_{ab}] \,, \tag{A.7}
$$

which is the same hard charge derived using different methods in Chapter-4.

[1]We also need the variation of the inverse $\delta_V\tilde{D}^{ab}$ which can be evaluated as

$$
\delta_V\tilde{D}^{ab} = \delta_V(q^{ac}q^{ad}\tilde{D}_{cd}) = \mathcal{L}_V\tilde{D}^{ab} + u\alpha\partial_u\tilde{D}^{ab} + 4\alpha\tilde{D}^{ab} \,. \tag{A.3}
$$

Appendix B

Derivation of the Identity (5.20)

In[1] this appendix we prove equation (5.20), i.e,

$$\frac{1}{64\pi^2}\Big[\partial^4\partial^a P^c_{ab}(x) - \frac{4}{3}\partial_b\partial^2\partial^{ef}P^c_{ef}\Big] = -\delta^c_b\delta^{(4)}(x)\ , \tag{B.1}$$

where $P^c_{ab}(x) = \frac{1}{2}[x_a\delta^c_b + x_b\delta^c_a + \frac{1}{2}x^c\delta_{ab} - \frac{4}{x^2}x^c x_a x_b]$. We first note that the only term in $P^c_{ab}(x)$ that contributes to the equation above is the last term, i.e $\frac{1}{x^2}x^c x_a x_b$, as the terms which are linear in x^a are annihilated by the derivative operators. In order to carefully handle such terms and notice the presence of delta functions, it is instructive to deform pole at $x = 0$ to a slight imaginary value by $x^2 \to x^2 + \epsilon^2$ where the limit $\epsilon \to 0$ has to be carefully taken in the last step of the computation, thus giving rise to terms involving delta functions in the calculation. With this in place we can evaluate the derivatives without worrying about the appearance of the delta function in the intermediate steps. Therefore we get the following terms

$$\frac{1}{64\pi^2}\Big[\partial^4\partial^a P^c_{ab}(x) - \frac{4}{3}\partial_b\partial^2\partial^{ef}P^c_{ef}\Big] = \lim_{\epsilon\to 0}\left\{\frac{1}{2\pi^2}\partial^c_b\Big(\frac{\epsilon^6}{(x^2+\epsilon^2)^4}\Big) - \frac{12}{\pi^2}\frac{\delta^c_b\epsilon^6}{(x^2+\epsilon^2)^5}\right\}. \tag{B.2}$$

[1]We thank R. Loganayagam for useful discussions related to this and also for suggesting several references on this topic.

In order to take the limit we need to keep in mind that these are distributions which are integrated against test functions and hence we integrate the LHS against a spherically symmetric test function $\mathcal{F}(|x|)$ (which has a sufficiently fast fall off) and obtain

$$\int d^4x \mathcal{F}(|x|) \frac{1}{64\pi^2}\Big[\partial^4 \partial^a P^c_{ab}(x) - \frac{4}{3}\partial_b \partial^2 \partial^{ef} P^c_{ef}\Big] \tag{B.3}$$

$$= \lim_{\epsilon\to 0} \int d^4x \mathcal{F}(|x|) \left\{ \frac{1}{2\pi^2}\partial^c_b \left[\frac{\epsilon^6}{(x^2+\epsilon^2)^4} \right] - \frac{12}{\pi^2} \frac{\delta^c_b \epsilon^6}{(x^2+\epsilon^2)^5} \right\} , \tag{B.4}$$

$$= \lim_{\epsilon\to 0} \int_0^\infty d|x| |x|^3 |\mathbb{S}_3| \mathcal{F}(|x|) \left\{ \frac{1}{2\pi^2}\partial^c_b \left[\frac{\epsilon^6}{(|x|^2+\epsilon^2)^4} \right] - \frac{12}{\pi^2} \frac{\delta^c_b \epsilon^6}{(|x|^2+\epsilon^2)^5} \right\} . \tag{B.5}$$

where $\mathbb{S}_3 = 2\pi^2$. As shown in [66], such expressions are simplified by expanding $\mathcal{F}(x)$ in a Taylor series and upon performing the integrals and taking the limit $\epsilon \to 0$ we obtain

$$\int d^4x \mathcal{F}(|x|) \frac{1}{64\pi^2}\Big[\partial^4 \partial^a P^c_{ab}(x) - \frac{4}{3}\partial_b \partial^2 \partial^{ef} P^c_{ef}\Big] = -\delta^c_b \mathcal{F}(0) . \tag{B.6}$$

As this is true for any generic test function $\mathcal{F}(x)$ we conclude that

$$\frac{1}{64\pi^2}\Big[\partial^4 \partial^a P^c_{ab}(x) - \frac{4}{3}\partial_b \partial^2 \partial^{ef} P^c_{ef}\Big] = -\delta^c_b \delta^{(4)}(x) , \tag{B.7}$$

thereby proving the identity (5.20).

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