FROM PARTONS TO CONSTITUENT QUARKS: A STUDY OF HADRON STRUCTURE FUNCTIONS

by

V. Ravindran

A THESIS IN PHYSICS

Presented to the University of Madras in Partial Fulfillment of
The Requirements of the Degree of Doctor of Philosophy

March 1993

The Institute of Mathematical Sciences,
C.I.T. Campus, Tharamani,
Madras 600 113 INDIA

INSTITUTE OF MATHEMATICAL SCIENCES

DEDICATED TO

PILLAIYAR

INSTITUTE OF MATHEMATICAL SCIENCES LIBRARY
CHENNAI - 600 113

INSTITUTE OF MATHEMATICAL
LIBRARY

ACKNOWLEDGEMENTS

I would like to acknowledge all those who made my stay at Matscience very enjoyable and fruitful.

First of all I thank my advisor Prof. M.V.N. Murthy for his care and guidance which have been the motivating factors behind my progress.

I cannot forget Pari who gave me his loving company and encouragement.

It has been a privilege to work with my friends Indumathi and Prakash with whom I have collaborated at different stages of the work that went into this thesis.

For many useful and informative discussions I thank Profs. G. Date, R. Ramachandran, N. D. Hari Dass, S. Uma Shankar, R. Vasudevan, G. Rajasekaran.

I remember with pleasure many moments of fun with my friends from Matscience—Manu, Shaji, Chaks, Vythee, Muthu, Sundar, Varghese, Mary, Florence and Chitra, Prem and Sakthi Murugesan from Madras University, Settu, Jayavel from Anna University and Ravichandran and Kandasamy from my school days.

I thank my akka Manisha for her encouragement.

I thank the library staff and non-academic staff for their invaluable help at various instances in the past.

The cheerful times that I spent with Shubashree will stay in my mind.

My love, respect and eternal gratitude to my parents and my sisters for making all this possible.

ITUTE OF MATHEMA
LIB

THE INSTITUTE OF MATHEMATICAL SCIENCES
C I T CAMPUS, THARAMANI, MADRAS - 600 113, INDIA

Phone(O): 044-235 1856, 0588, 1049, 1050
Fax: 044-235 0586; Grams: MATSCIENCE
Telex: 041 8960 PCO IN PP WDT 20
E-mail: murthy@imsc.ernet.in

Dr M V N Murthy

CERTIFICATE

This is to certify that the Ph.D thesis submitted by V. Ravindran to the University of Madras, entitled *FROM PARTONS TO CONSTITUENT QUARKS: A STUDY OF HADRON STRUCTURE FUNCTIONS* is a record of bonafide research work done by him under my supervision. The research work presented in this thesis has not formed the basis for the award to the candidate of any Degree, Diploma, Associateship, Fellowship or other similar title. It is further certified that the thesis represents independent work on the part of the candidate and collaboration was necessitated by the nature and scope of the problems dealt with.

Dr. M.V.N. Murthy

March 1993

Thesis Supervisor

Contents

0 Abstract 1

0.1 Weak Spin Dependent Structure Functions and Polarized Drell Yan Process . . 4

0.2 Quark Model Analysis of Proton Spin 7

0.3 Partons to Constituent Quarks 7

0.4 Quark masses and Polarised Electroproduction 8

1 Introduction 10

1.1 Deep Inelastic Scattering 11

1.2 Structure functions 13

1.3 Parton model 16

1.4 Free field theory 19

1.5 Factorisation method 25

1.6 Operator Product Expansion 32

1.7 EMC result and the spin content of the proton 41

2 Weak spin dependent structure functions 49

2.1 Introduction 49

2.2 Basic formalism 51

2.3 QCD Parton model 52

2.3.1 Hard scattering cross sections 53

2.3.2 Conclusion 57

2.4 Factorisation method 58

2.4.1 Hard scattering coefficients 61

2.4.2 Conclusion 68

3 Polarised Drell-Yan process 69
3.1 Introduction . . . 69
3.2 Operator definitions . . . 70
3.3 Hard scattering coefficient . . . 74
3.4 Conclusion . . . 77

4 Phenomenology of proton spin 78
4.1 The Model . . . 78
4.2 Phenomenology . . . 81
4.3 Conclusion . . . 84

5 Moments of structure functions 85
5.1 Introduction . . . 85
5.2 Moments of Structure Functions. . . . 87
5.3 Numerical estimates of m_{eff} . . . 91
5.4 Model Calculation of Matrix Elements. . . . 98
5.5 Conclusion . . . 107

6 Quark Masses and Polarised Electroproduction 109
6.1 Introduction . . . 109
6.2 Massless Theory . . . 110
6.3 Massive theory . . . 113
6.4 Conclusion . . . 118

7 Conclusions 119

A $W^{\pm}g$ fusion process 123

B Compton scattering 125

C Evaluation of Triangle diagram 127

D State Normalisation in the various models 129

Bibliography 131

Chapter 0

Abstract

It is well known that high energy hadronic processes such as Deep Inelastic Scattering (DIS) and Drell-Yan (DY) processes are excellent tools to understand the structure of hadrons. These processes act as testing grounds for the predictions of Quantum Chromo Dynamics (QCD), which is the main candidate for the underlying theory of strong interactions. This thesis is a study of these processes in general.

The deep inelastic electron-proton scattering cross section can be factored in terms of a product of a purely leptonic part specified by a tensor $L_{\mu\nu}$ and a hadronic part parametrized by a tensor $W_{\mu\nu}$. The hadronic tensor $W_{\mu\nu}$ is related by Fourier transformation to the matrix element of the commutator of two electromagnetic currents localized at two different space-time points sandwiched between proton states. Due to lack of knowledge about the structure of these current matrix elements, the hadronic tensor is expressed in terms of unknown structure functions which are, in general, functions of the kinematical variables involved in the process. The number of independent second rank tensors and associated structure functions can be obtained by properly enumerating the independent scattering amplitudes and considering the symmetries of the process such as parity, time reversal, etc. It was Bjorken who found, using current algebra techniques [2], that these functions scale as a function of the dimensionless quantity, $x = -q^2/2p \cdot q$ (later called Bjorken variable), in the limit where q^2 and $p \cdot q$ become large. Here q is the four-momentum transferred to the proton, p, the momentum of the proton. This scaling was verified experimentally and can be understood simply by assuming that the

strong interaction has scale invariance at high energies. Keeping these in mind, Feynman proposed a model called parton model which assumes that the hadrons are made up of noninteracting point like objects called partons. The lepton proton cross section can then be expressed as an incoherent sum of the lepton parton cross sections properly convoluted with parton probability distribution functions.

The product of the currents appearing in the hadronic tensor can also be expressed using Wilson's Operator Product Expansion (OPE) in terms of local operator matrix elements (OME) evaluated between hadron states with perturbatively calculable coefficient functions. The various operators that can enter into this expansion depend on the underlying theory which is Quantum Chromodynamics (QCD). Proper interpretation of these OME sheds light on the structure of the hadron.

Yet another approach called the factorization method, which is the field theoretical version of the parton model, is an elegant method to analyse high energy hadronic processes. This approach carries with it the operator definitions for parton distribution functions. In many respects this approach is analogous to the OPE approach and has many interesting features. One such feature is that it gives results which are free of mass singularities. This approach also serves as a link between conventional OPE and the parton model.

On the experimental front, scaling in DIS is well established. The analysis of DIS in the framework of OPE and the Renormalisation Group (RG) predicts logarithmic scaling violations. These have also been observed in unpolarised electron-proton and muon-proton DIS. Recently European Muon Collaboration (EMC) [1] has done polarised μP scattering experiments and has measured the spin dependent structure function $g_1(x, -q^2)$ at an average $\langle -q^2 \rangle = 10.7(GeV)^2$. In the naive parton model this quantity is related to polarised quark distribution functions. Using the measured first moment $\int_0^1 dx g_1(x, -q^2)$ along with known values of F and D (F and D are the $SU(3)_f$ reduced matrix elements for axial octet current between the octet baryon states) the net spin contribution to proton spin, coming from the quarks, is found to be almost zero. This result is striking because it contradicts the naive quark model prediction. This provided a fresh impetus to studying aspects of both unpolarised and polarised hadronic processes.

There are several theoretical papers on the issue of the proton spin. Altarelli and Ross [7] have shown that, in the parton model, gluons can in principle contribute to the first moment of $g_1(x, Q^2)$ even at large $Q^2(\equiv -q^2)$. Rigorous analysis of these calculations both in parton model as well as using OPE can be found in the paper by Carlitz, Collins and Mueller [9]. The OPE analysis of this function suggests that there does not exist a relevant gluonic operator that contributes to the first moment of the spin dependent structure function. This first moment involves only proton matrix element of the axial vector current. This matrix element being a non-perturbative object has been analysed extensively using current algebra techniques as well as chiral anomalies.

The parton model analysis of the gluonic contribution to this structure function involves the evaluation of the off-shell photon-gluon ($\gamma^* g$) fusion process. This process is both infrared and collinear singular and has to be regularised and renormalised. Hence, though there may be, in principle, a large gluon contribution, there exists, in the computation, a manual factorisation of soft and hard parts of the process. This leads to an ambiguity in defining the polarised parton distribution functions and therefore also the gluon component in such a model.

Using completely different approach, Bodwin and Qiu [10] and A.V. Manohar [11] have shown that gluons do not contribute to the first moment of $g_1(x, Q^2)$. Bodwin and Qiu have suggested a subtraction scheme to remove soft part of $\gamma^* g$ fusion cross section through which gluons contribute to the structure function. Note that gluon interacts with γ^* only through a quark loop. So γ^* can see gluon polarisation only when the quarks are off mass shell. So this is the region of the cross section which is contributing to the structure function. The left-over part can then be interpreted as the probability of finding a real quark inside the polarised gluon. Such a division leads to zero gluon contribution to the first moment of this structure function.

This thesis is concerned with analysing various hadronic processes and the corresponding structure functions using OPE, parton model and factorization method. Because of recent interest in extracting spin dependent structure of hadrons, our main focus will be on polarised processes. However we also consider some aspects of structure functions not

directly related to the polarised processes. To begin with, we briefly introduce the concept of structure functions of hadrons in strong interaction physics. The tools used in this thesis such as factorization, parton model and OPE are then described. We have studied both perturbative and nonperturbative aspects of hadronic processes in this connection. In the first part we discuss various inputs that might be of use in resolving the proton spin puzzle. Since the EMC result is not sufficient to get full information about the spin contributions from the constituents of the proton, we study other processes such as polarised weak (neutrino) DIS and polarised DY experiments. We have calculated perturbatively the quark and gluonic contributions to spin dependent structure functions appearing in weak DIS [12, 13] and to the polarized Drell-Yan function in polarized DY process [14]. The calculations are performed both in the parton model and using the factorization method. The study of the nonperturbative part includes the analysis of operator matrix elements appearing in the OPE of DIS. A possible explanation for the EMC result using a simple model [42] is presented. We then show that using OPE and an effective equation of motion for the constituents, the various moments of structure functions can be related by a mass parameter whose value, evaluated from current experimental data on these structure functions, confirms the validity [40, 41] of the constituent picture of hadrons. We analyse the Operator Matrix elements appearing in the first moment of the structure function $g_1(x, Q^2)$ using current algebra [43] techniques. A summary of the main results covered in this thesis is as follows:

0.1 Weak Spin Dependent Structure Functions and Polarized Drell Yan Process

The electromagnetic spin dependent proton structure function $g_1(x, Q^2)$ is not adequate to get full information about the spin contributions coming from various constituents of the proton. Hence, we attempt to study theoretically other processes such as polarized weak DIS and polarized DY to obtain a complete description of the spin dependent parton densities. Gluons can in principle contribute in the leading order to first moments

of these functions, as pointed out by Altarelli and Ross. We have therefore calculated gluonic contributions in addition to quark contributions to the new structure functions $g_1(x,Q^2), g_3(x,Q^2)$ and $g_4(x,Q^2)$ which appear in weak DIS and to the polarized DY function $W(\tau,Q^2)$. The calculation of gluonic contribution involves mass divergences (both infrared and collinear) and ultraviolet divergences (UV). The parton model calculation of gluonic contribution to these DIS structure functions and polarised DY function involves both infrared divergence which appears when the initial gluon is on shell and collinear divergence when the out going quarks are massless. These can be avoided by keeping masses non zero and gluon off-shell, finally applying a suitable renormalisation scheme to subtract them out. This renormalisation changes the bare parton distribution functions into renormalised ones. This fixes the definition of parton distribution functions in a particular scheme. In the factorization method, the evaluation of gluonic contribution to these DIS structure functions and polarised DY function involves in addition to usual mass singularities, ultraviolet singularity which occurs in the evaluation of the bilocal operators between polarised gluon states. We have used dimensional regularisation to regularise and $\bar{M}S$ scheme to renormalise them. This method, as we know, preserves gauge invariance, Lorentz invariance and analyticity properties.

We first calculate quark and gluonic contributions to the weak structure function in the parton model. The mass singularities occurring in the gluonic contribution are regularised and renormalized in massive gluon scheme. The combination $\int_0^1 dx(g_1^{W^+P}(x,Q^2)+ g_1^{W^-P}(x,Q^2))$ is found to be directly proportional to the singlet part of $\int_0^1 dx g_1^{\mu P}(x,Q^2)$. Superscripts denote the processes in which these functions are measured. These functions can be expressed in terms of the parton spin dependent distribution functions, $\Delta q = q^+(x,Q^2) - q^-(x,Q^2)$ and $\Delta g = g^+(x,Q^2) - g^-(x,Q^2)$, where $q^+(x,Q^2)$ is the probability of finding a quark carrying a fraction x of the momentum of the proton and with helicity parallel with respect to the proton helicity, while $q^-(x,Q^2)$ is the probability of finding a quark with its helicity opposite to that of the parent proton. Similarly, the definition of $\Delta g(x,Q^2)$ is analogous to that of $\Delta q(x,Q^2)$. In terms of the moments $\Delta q(Q^2) = \int_0^1 dx\, \Delta q(x,Q^2)$ and $\Delta g(Q^2) = \int_0^1 dx\, \Delta g(x,Q^2)$, the first moment of the weak

$g_1(x,Q^2)$ is proportional to $\sum_i(\Delta q_i(Q^2)+\Delta\bar{q}(Q^2)-\alpha_s\Delta g/2\pi)$ where i runs over various quark flavours. Therefore a measurement of this combination directly yields the quark singlet contribution to the proton spin unlike the EMC result where one has to resort to low energy data. Much of the controversy regarding proton spin structure arose precisely from the value of this singlet moment. In the gluonic sector $g_3(x,Q^2)$ and $g_4(x,Q^2)$ are both free of mass singularities since they do not receive any gluonic contribution. Hence, we find that the Callan-Gross sum rule $g_4(x,Q^2)=2xg_3(x,Q^2)$ in the weak sector is not affected by gluonic correction. Also $\int_0^1 dx g_3^{(W^++W^-)P}(x,Q^2)$ measures the valence quark contribution to the proton spin, $\Delta u_V+\Delta d_V$, where subscript denotes the valence part of up (u) and down (d) quarks.

Alternatively we use Factorization method to evaluate quark and gluon contribution to the weak spin dependent structure functions. The gluonic contribution to the first moment of $g_1^{(W^++W^-)P}(x,Q^2)$ is found to vanish and $g_{3,4}^{(W^+\pm W^-)P}$ do not receive leading order gluonic contribution. We have done the calculation with all the quarks and gluons massive. The results are shown to be independent of mass. So there is no mass singularity, thus verifying the proposed factorization in the spin dependent weak sector. The parton distributions defined in this formalism should not be confused with those in the parton model as they are defined with respect to the model and the renormalisation scheme adopted in it.

We also analyse polarized Drell-Yan process $P\,P\rightarrow\mu^+\,\mu^-+X$ using the factorization method and derive operator definitions for polarized parton distribution functions which are consistent with those that occur in DIS. We demonstrate that a factorization analogous to that in the unpolarized DY case holds in this process. We have explicitly shown the factorization of mass singularities to order α_s in the gluonic sector. The gluonic contribution to the first moment of the polarized DY function from the 'Compton' subprocess turns out to be zero. On the other hand, QCD parton model analysis of the gluonic contribution to polarised DIS involves only the Compton process. This process suffers from both infrared and collinear divergences. If we apply same subtraction procedure that is adopted in DIS, we get a combination for the first moment of DY function

to order α_s which is analogous to that appearing in the first moment of $g_1^{\mu P}(x)$. Invoking universality, that is parton distributions are process independent, we conclude that the gluonic contribution to the first moment of polarized DY function vanishes. So the leading order gluonic contribution to the first moment of polarized Drell-Yan function is consistent with the results obtained from polarized DIS.

0.2 Quark Model Analysis of Proton Spin

The spin dependent proton structure function $g_1(x, Q^2)$ measured at EMC is analysed. We study the various inputs that go into the first moment of this structure function. The basic inputs are polarised valence, sea quarks and gluon distribution functions. We show that the quark model prediction for polarized valence quark distributions suitably modified to accomodate Bjorken sum rule using Carlitz-Kaur model may be consistent with the measured value of the first moment of $g_1(x, Q^2)$.

0.3 Partons to Constituent Quarks

We analyse the various moments of proton structure functions $F_2(x, Q^2)$ (spin independent) and $g_1(x, Q^2)$ (spin dependent) using the operator product expansion (OPE). Using an effective equation of motion $(i\not{\partial} - m^i_{eff})\psi_i = 0$, where m^i_{eff} is an effective mass scale of the quark inside the proton, we show that allowed successive moments can be related by a mass parameter, ie

$$\begin{aligned} R_F &= \frac{\int dx x^2 F_2(x, Q^2)}{\int dx F_2(x, Q^2)} = \frac{m^2_{eff}}{M^2_P}, \\ R_G &= \frac{\int dx x^2 g_1(x, Q^2)}{\int dx g_1(x, Q^2)} = \frac{m^2_{eff}}{M^2_P}. \end{aligned} \tag{0.1}$$

Here M_P is the mass of the proton. Present experimental data on these structure functions imply that the mass parameter turns out to be around $300 MeV$ for three flavoured proton. For example, using Sloan parametrization [45]of the spin independent distributions based on experimental data, we get $R_F = 0.105$ or $m_{eff} = 303 MeV$ which vindicates our

expectation that m_{eff} is of the order of constituent quark mass. This is not affected even when one considers different flavour masses. Recently measured spin dependent structure function $g_1(x, Q^2)$ supports the result i.e. R_G turns out to be 0.091 or equivalently $m_{eff} = 283MeV$. Quantum Chromodynamics corrections to the relations are considered and we have found that dropping terms of order $O(\alpha_s)$ in the Wilson's coefficients would lead to errors of the order 10-15 per cent for various moments. In deed such a relation is valid even for ratios of higher moments and the result within experimental errors agrees with $m_{eff} \approx 300$. We strengthen our results by evaluating these moments in a bag model with non strange quarks.

0.4 Quark masses and Polarised Electroproduction

It is shown that in a theory with strictly massless quarks, the usual OPE for polarized electroproduction, when combined with U(1) anomaly

$$\partial_\mu j^\mu_{i5} = 2im_i\bar{\psi}_i\gamma_5\psi_i + \frac{\alpha_s}{2\pi}\tilde{F}_{\mu\nu}F^{\mu\nu}, \tag{0.2}$$

predicts a vanishing singlet quark contribution. Non-singlet contribution is ambiguous unless Goldstone modes are introduced explicitly. Introducing a small quark mass changes the whole picture and leads to a conclusion consistent with Altarelli and Ross [7]; Carlitz, Collins and Mueller [9]. Our analysis singles out the operator that directly measures the quark spin contribution.

To summarize, this thesis is concerned with analysis of DIS structure functions and polarised DY function and their moments which are relevant in understanding the spin structure of proton. We have made use of Parton model, OPE and Factorization methods and simple models for this. Also, we have emphasised the importance of obtaining the consistent definition of the parton distribution functions as they depend on the model and the renormalisation scheme. Within each of the above models this exercise is carried out and the comparison with experimental data depends on the adopted scheme. This thesis is based on the following papers:

1. Phenomenology of the Proton spin, Pramana J.Phys 34 (1990) 103, with M.V.N. Murthy.

2. From Partons to Constituent Quarks, Int.Jour.Mod.Phys A6 (1991) 3933, with D. Indumathi and M.V.N. Murthy.

3. Quark masses and Polarised Electroproduction, PRL-TH/91-07, with Debajyoti Choudhury and N.D. Hari Dass.

4. Gluonic Corrections to spin dependent weak structure functions, Phys. Lett B278 (1992) 175, with Prakash Mathews.

5. Gluonic Contributions to spin dependent weak structure functions: Method of Factorisation, Int.Jour.Mod.Phys. A7 (1992) 6371, with Prakash Mathews.

6. Can polarized Drell-Yan shed more light on the Proton spin?, Mod.Phys.Lett A7 (1992) 2695, with Prakash Mathews.

Chapter 1

Introduction

Information on the structure of hadrons can be got using structureless particles as probes which scatter off the hadron. If the scattering is elastic, then there exists a considerable amount of scattering cross section in the large angle region if the target is point particle. But experimentally there is suppression in this region. This is due to the fact that proton is not really point like but it has structure. This structure is described by two elastic form factors, G_E and G_M. The first one is called the electric form factor and the second one is called the magnetic form factor. These are nothing but charge and magnetic distributions inside the proton when they are Fourier transformed. When the energy or the momentum transferred is very high, the scattering is no longer elastic but inelastic. This can be expressed as

$$l + P \rightarrow l' + X,$$

where X is the unobserved final many particle state. At low q^2 ($q \equiv$ momentum transferred from the lepton to hadron vertex), there are prominent resonance peaks in the cross section. As q^2 increases i.e. about $Q^2 \equiv -q^2 = 2\text{GeV}^2$, these peaks disappear and there is a continuum in the cross section. This means that the cross section scales. This was first theoretically predicted by Bjorken using current algebra. This can be understood by assuming that the strong interaction is scale invariant. Feynman later systematised the above scenario through his model called parton model. This model assumes that the proton is made up of point-like, non interacting, spin-half objects called partons. In this model, the lepton-proton scattering cross section can be expressed as an incoherent sum

of elastic lepton-parton scattering cross sections convoluted with the parton probability density inside the proton. Such a picture reproduces the observed scaling behaviour.

In this chapter we will study deep inelastic scattering using a simple model and field theoretical techniques. The model assumes that the proton is made up of point-like, noninteracting, spin-half particles. We know that the deep inelastic cross section can be factored into a leptonic part which is calculable and hadronic part which is not calculable. First we parametrise the hadronic tensor in terms of unknown functions called structure functions using symmetries of the process. Using free field approach, we show that these functions are related to the proton matrix element of some local operators. We then give the physical interpretation of these functions. This leads to the parton model interpretation of these structure functions.

Another approach to this problem is to use the operator product expansion (OPE)[56, 17] since we are interested in the contribution coming from the product of two electromagnetic currents when the square of their separation vanishes. We know that OPE is the expansion of product of two or more operators in terms of well defined local operators with appropriate singular coefficients called Wilson's coefficients. Hence in the OPE method, moments of the structure functions are related to matrix elements of some local operators. All we know about these operators are their symmetry properties and dimensions. Once the simple model is used, these operators can be constructed out of the fields and their derivatives. We have studied the interpretation of these operators in connection with the parton model.

1.1 Deep Inelastic Scattering

The cross section for the process $\mu P \rightarrow \mu X$ can be written as

$$\sigma = \sum_{\sigma'} \int \frac{d^3k'}{(2\pi)^3 2k'_0} \sum_X (2\pi)^4 \delta^{(4)}(p_X + k' - k - p) | < \mu\ (k', \sigma') X \mid T \mid \mu\ (k, \sigma)\ p > |^2.$$

where T is the transition matrix and σ, σ' and s denote the spin indices of incoming muon, outgoing muon and the proton respectively. Various momenta of the particles are specified in the figure.

The differential cross section in the one-photon exchange can be expressed as the product of leptonic part ($L_{\mu\nu}$) and hadronic part ($W_{\mu\nu}$). That is,

$$k'_0 \frac{d\sigma^{(s)}}{d^3k'} = \frac{1}{k.p} \left(\frac{\alpha}{q^2}\right)^2 L^{(\sigma)}_{\mu\nu} W^{\mu\nu}_{(s)} \tag{1.1}$$

where

$$L^{(\sigma)}_{\mu\nu} = \frac{1}{2} \sum_{\sigma'} (\bar{u}_{\sigma'}(k') \gamma_\mu u_\sigma(k))^\dagger (\bar{u}_{\sigma'}(k') \gamma_\nu u_\sigma(k)) \tag{1.2}$$

$$W^{(s)}_{\mu\nu} = \frac{1}{2\pi} \sum_X (2\pi)^4 \delta^{(4)}(p_X - p - q) \langle p\,s | J_\mu(0) | X \rangle \langle X | J_\nu(0) | p\,s \rangle , \tag{1.3}$$

and $\alpha = e^2/4\pi$. Since all the information about the structure of the proton is contained in the hadronic tensor $W_{\mu\nu}$, we concentrate only on $W_{\mu\nu}$. Also this is the only part which cannot be calculabled within the framework of perturbation theory due to the lack of knowledge about the explicit form of the current hence its matrix element between hadronic target states is complicated. We will proceed to extract as much information as possible about the target using reasonable approximations.

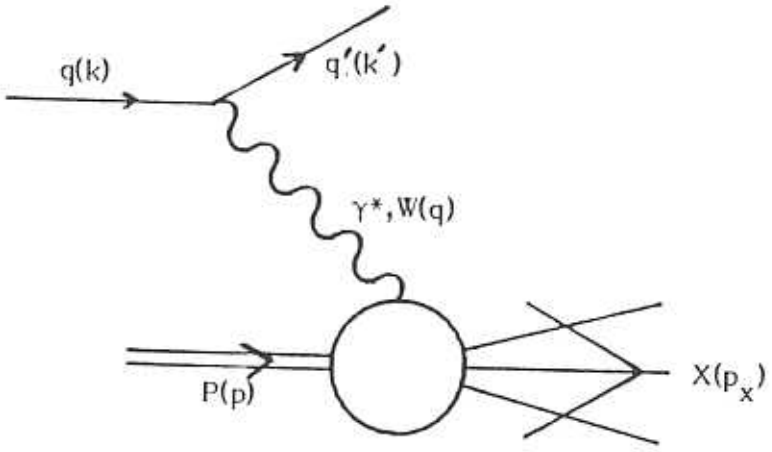

Figure 1.1 Lepton-Hadron DIS process.

Now, using the exponential representation for the Dirac delta function, we can express $W_{\mu\nu}$ as the Fourier transformation of the commutator of the currents:

$$W_{\mu\nu} = \frac{1}{2\pi} \int d^4\xi \; e^{iq.\xi} \langle ps | [J_\mu(\xi), J_\nu(0)] | ps \rangle . \tag{1.4}$$

Note that the second term namely $J_\nu(0)\ J_\mu(\xi)$, is included for convenience, this however, does not contributing due to kinematical reasons.

The hadronic tensor can be related to the absorptive part of the virtual Compton amplitude as

$$W_{\mu\nu} = \frac{1}{\pi}\,\mathrm{Im}\,T_{\mu\nu}\ , \tag{1.5}$$

where

$$T_{\mu\nu} = i\int\ d^4\xi\ e^{iq\cdot\xi}\ \langle\, p\,s\,|\ T J_\mu(\xi) J_\nu(0)\ |\, p\,s\,\rangle\ , \tag{1.6}$$

where T stands for the time ordered product of the currents. This can be proved by using unitarity of S-matrix elements.

1.2 Structure functions

Even though it is not possible to calculate $W_{\mu\nu}$, it can be parametrized in terms of functions called structure functions with appropriate second rank tensor coefficients. The first question we can ask is that how many structure functions (second rank tensors) are needed to express $W_{\mu\nu}$ completely. This can be enumerated using the eqn.(1.6). That is, the number of independent structure functions is equal to the number of independent forward virtual Compton amplitudes. These functions are, in general, functions of Lorentz invariants such as p^2,q^2 and $p\cdot q$ since their coefficients are second rank tensors. Bjorken [2] observed, using current algebra techniques that these functions scale as a function of the single variable, $x = -q^2/2p.q$ at large energy and momentum transfer ($p\cdot q$ and $-q^2 \to \infty$, called the DIS or Bjorken limit). x is called Bjorken variable and this scaling is called Bjorken scaling. This has been successfully tested in the SLAC-MIT DIS experiments [61] and later by the EMC group at CERN [1].

The structure functions can be classified into spin independent and spin dependent types, depending on whether their coefficients are spin independent or dependent. These tensor coefficients are constructed out of vectors and tensors available in the theory such as p^μ, q^μ, $g_{\mu\nu}$ and $\epsilon_{\mu\nu\lambda\sigma}$. There are ten allowed helicity amplitudes for the forward virtual

Compton process. On imposing time reversal invariance symmetry, this reduces to eight independent amplitudes. Out of these eight we find that three of them are spin independent while the rest five are spin dependent. One set of independent amplitudes is given below. The processes are expressed in terms of the angular momentum of the particles involved, i.e., $j_1 j_2 \rightarrow j_3 j_4$:

- $\frac{1}{2}\ ,\ -1 \rightarrow -\frac{1}{2}\ ,0$
- $\frac{1}{2}\ ,\ 0 \rightarrow -\frac{1}{2}\ ,1$
- $(\frac{1}{2}\ ,-1 \rightarrow \frac{1}{2}\ ,\ -1)\ \pm\ (-\frac{1}{2}\ ,\ -1 \rightarrow -\frac{1}{2}\ ,\ -1)$
- $(\frac{1}{2}\ ,\ 0 \rightarrow \frac{1}{2}\ ,\ 0)\ \pm\ (-\frac{1}{2}\ ,\ 0 \rightarrow -\frac{1}{2}\ ,\ 0)$
- $(\frac{1}{2}\ ,\ 1 \rightarrow \frac{1}{2}\ ,\ 1)\ \pm\ (-\frac{1}{2}\ ,\ 1 \rightarrow -\frac{1}{2}\ ,\ 1)$

The spin independent structure functions are denoted by $F_i^{\alpha}(x)$ $(i = 1,2,3)$ and spin dependent structure functions by $g_i^{\alpha}(x)$ $(i = 1-5)$ where α denotes the process in which these functions are measured. Note also that the parity invariance reduces the total number to four. Therefore of the eight there are four parity conserving structure functions and four parity violating structure functions. The parity conserving ones are measured when the probe is photon and parity violating ones are measured when the probe is either $W^{\pm}$ or Z.

As mentioned above, the tensor coefficients are constructed using the following properties.

1. Hermiticity: $W_{\mu\nu} = W^{*}_{\nu\mu}$
2. Time reversal invariance: $A_{\ \mu_1\ M_1 \rightarrow\ \mu_2\ M_2} = A_{\mu_2 M_2 \rightarrow \mu_1 M_1}$
3. Parity invariance: $A_{\ \mu_1\ M_1 \rightarrow\ \mu_2\ M_2} = A_{-\mu_1 - M_1 \rightarrow -\mu_2 - M_2}$
4. Current conservation: $q^{\mu}\ W_{\mu\nu} = q^{\nu}\ W_{\mu\nu} = 0$
5. Lorentz covariance

6. Spin dependent part should be linear in s^μ

Here $A_{\mu_1 M_1 \to \mu_2 M_2}$ is the amplitude, where the subscripts denote the helicities of particles involved. Current conservation requires that the following independent combinations contribute to the spin independent part of $W_{\mu\nu}$:

1. $a_1\ (-g^{\mu\nu} + q^\mu q^\nu/q^2)$,

2. $a_2\ (p^\mu - q^\mu\ p.q/q^2)(p^\nu - q^\nu\ p.q/q^2)$,

3. $a_3\ \epsilon^{\mu\nu\lambda\sigma}\ q_\lambda\ p_\sigma$.

The last contribution is a parity violating one. Time Reversal invariance and hermiticity require that a_1 and a_2 are real while a_3 is purely imaginary .

The spin dependent part of $W_{\mu\nu}$ will depend on s^μ in addition to the usual vectors and tensors. The typical second rank tensors which are linear in s are

1. $b_1\ \epsilon^{\mu\nu\lambda\sigma}\ q_\lambda\ s_\sigma$,

2. $b_2\ \epsilon^{\mu\nu\lambda\sigma}\ q_\lambda(s_\sigma - p_\sigma\ s.q/p.q)$,

3. $b_3\ \frac{q.s}{p.q}\ (-g^{\mu\nu} + q^\mu q^\nu/q^2)$,

4. $b_4\ \left[R^\mu R^\nu\ \frac{q.s}{p.q} + (R^\mu T^\nu + R^\nu T^\mu)\ \right]$,

5. $b_5\ \left[R^\mu R^\nu \frac{q.s}{p.q} - (p^\mu s^\nu + p^\nu s^\mu)\right]$.

where $R_\mu = p_\mu - q_\mu p.q/q^2$, $T_\mu = s_\mu - q_\mu s.q/q^2$ and the last three structures are parity violating. Time reversal invariance and hermiticity require that b_1 and b_2 are purely imaginary while the rest of the b's are real constants. The hadronic tensor can in general be written as

$$W_{\mu\nu} = \overline{W}_{\mu\nu} + \widetilde{W}_{\mu\nu} \tag{1.7}$$

where $\overline{W}_{\mu\nu}$ is the part which contains only spin independent terms and $\widetilde{W}_{\mu\nu}$ contains only spin dependent terms, viz,

$$\begin{aligned}
\overline{W}_{\mu\nu} &= G^3_{\mu\nu}\, F_1(x,Q^2) + \frac{1}{p.q} R_\mu R_\nu\, 2F_2(x,Q^2) + \frac{i}{p.q}\epsilon_{\mu\nu\lambda\rho} p^\lambda q^\rho\, F_3(x,Q^2), \\
\widetilde{W}_{\mu\nu} &= \frac{i}{p.q}\epsilon_{\mu\nu\lambda\rho} q^\lambda \left[s^\rho\, g_1(x,Q^2) + \left(s^\rho - \frac{s.q}{p.q} p^\rho \right) g_2(x,Q^2) \right] + \\
&\quad \frac{q.s}{p.q} g^3_{\mu\nu}\, g_3(x,Q^2) + \frac{1}{p.q}\left[G^4_{\mu\nu}\, g_4(x,Q^2) + G^5_{\mu\nu}\, g_5(x,Q^2) \right]
\end{aligned}$$

where

$$\begin{aligned}
G^3_{\mu\nu} &= -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \\
G^{4,5}_{\mu\nu} &= \frac{1}{2}\left\{ R_\mu R_\nu \frac{q.s}{p.q} \pm \frac{1}{2}\left(R_\mu T_\nu + R_\nu T_\mu \right) \right\}
\end{aligned}$$

1.3 Parton model

In the parton model [62], hadrons which are extended objects with finite size, mass etc. can be thought of as being made up of pointlike, spin half- constituents called partons which are held together by their mutual interaction. The nature of the interaction can be described by a gauge theory-quantum chromodynamics (QCD)[65]. At the level of the parton model, hadrons are described in terms of virtual partonic states. But it is quite difficult to write the wave function for hadrons in terms of the partonic states due to lack of knowledge about the confining mechanism. On the other hand one can calculate scattering of partons among themselves or with other point-like objects such as leptons, photon etc.

Consider inclusive electron(muon)-hadron scattering by a single virtual photon exchange. Let the process be highly inelastic in the sense that large energy and momentum are transferred to the hadron during the interaction. In the centre of momentum frame, due to the large energy and momentum transferred, the hadron is Lorentz contracted and the interaction is time dilated. Due to the first effect, the electron takes less time to traverse the hadron. Since the interaction is time dilated, electron will see only a single virtual partonic state during interaction. This virtual state is nothing but a state of a

definite number of partons each carrying a specific fraction, x, of the hadron's momentum. Under this condition the whole process can be described of as the scattering of the electron by a virtual parton with a momentum given by a fraction x of the proton's momentum in the centre of mass (cm) frame. The cross section can be classically described, i.e., it can be expressed in terms of probabilities instead of amplitudes.

In the parton model, the DIS cross section of a virtual photon coming from the lepton of polarisation λ off a proton of polarisation H can be written as

$$\sigma^{\gamma^*(\lambda)P(H)}(x) = \sum_{a(h)} \int \int dz\, dy\, \hat{\sigma}^{\gamma^*(\lambda)a(h)}(z)\, f_{\frac{a(h)}{P(H)}}(y)\, \delta(x - yz), \tag{1.8}$$

where $\hat{\sigma}$ is the parton level cross section and $f_{a(h)/P(H)}(x)$ is the probability of finding a parton of type a with helicity h and momentum fraction x inside the proton of helicity H. Using the above equation, the unpolarised cross section is found to be

$$\begin{aligned} \sigma^{\gamma^* P}(x) &= \frac{1}{2}\left[\sigma^{\gamma^*(+)P(+)}(x) + \sigma^{\gamma^*(+)P(-)}(x)\right] \\ &= \sum_a \hat{\sigma}^{\gamma^* a} \otimes f_{\frac{a}{P(+)}} \end{aligned} \tag{1.9}$$

where $\hat{\sigma}^{\gamma^* a}(x) = \frac{1}{2}\left[\hat{\sigma}^{\gamma^*(+)a(+)}(x) + \hat{\sigma}^{\gamma^*(+)a(-)}(x)\right]$ and $f_{\frac{a}{P(+)}}(x) = f_{\frac{a(+)}{P(+)}}(x) + f_{\frac{a(-)}{P(+)}}(x)$. Here the following properties are used:

$$f_{\frac{a(+)}{P(+)}}(x) = f_{\frac{a(-)}{P(-)}}(x), \qquad f_{\frac{a(-)}{P(+)}}(x) = f_{\frac{a(+)}{P(-)}}(x)$$

and

$$\sigma^{\gamma^*(+)a(+)}(x) = \sigma^{\gamma^*(-)a(-)}(x), \qquad \sigma^{\gamma^*(+)a(-)}(x) = \sigma^{\gamma^*(-)a(+)}(x).$$

The above properties result from the parity invariance of the distribution functions which strong interaction respects.

Similarly the polarised cross section defined as

$$\Delta\sigma^{\gamma^* P}(x) = \frac{1}{2}(\Delta\sigma^{\gamma^*(+)P(+)}(x) - \Delta\sigma^{\gamma^*(+)P(-)})(x) \tag{1.10}$$

can be written as

$$\Delta\sigma^{\gamma^* a}(x) = \Delta\hat{\sigma}^{\gamma^* a} \otimes f_{\frac{\Delta a}{P(+)}} \tag{1.11}$$

where $\Delta\hat{\sigma}^{\gamma^* P}(x) = \frac{1}{2}(\sigma^{\gamma^*(+)P(+)}(x) - \sigma^{\gamma^*(+)P(-)}(x))$ and $f_{\frac{\Delta a}{P(+)}}(x) = f_{\frac{\Delta a(+)}{P(+)}}(x) - f_{\frac{\Delta a(-)}{P(+)}}(x)$. Note that there is a pertubatively calculable part which is cross section and distribution function which is not calculable. This is a reflection of the fact that the photon-hadron DIS cross section is not completely calculable in perturbation theory.

Using this model, a physical interpretation [63, 59] of the structure functions can be found. That is, the structure functions relevant to DIS can be expressed in terms of the parton distribution functions as follows:

$$F_2(x) = \sum_i e_i^2 \, x \, \left(f_{\frac{a_i}{P}}(x) + f_{\frac{\bar{a}_i}{P}}(x) \right), \tag{1.12}$$

$$g_1(x) = \frac{1}{2} \sum_i e_i^2 \, \left(f_{\frac{\Delta a_i}{P(+)}}(x) + f_{\frac{\Delta \bar{a}_i}{P(+)}}(x) \right). \tag{1.13}$$

So the measurement of F_2 will tell us the distribution of momentum fraction carried by the partons inside the proton. And the measurement of g_1 will tell us the various spin contributions coming from various partons to the spin of the proton.

It is worth noting that if one introduces interaction among quarks, then there will be contributions coming from this. Specifically, if QCD is the theory for the interaction, gluons will also contribute to these cross sections. These contributions will be in general of higher order.

The beauty of the above model is that we can write the cross section or the structure functions in terms of physically meaningful entities such as parton probability densities inside the proton etc. In particular, these parton distribution functions tell us about the momentum and spin structures of the proton. These are nonpertubative inputs in the theory. Since the cross sections are observables, one can extract these distribution functions experimentally using the relations arrived at in the model. But these functions are assumed to be universal in the sense that these should not change if they are extracted from some other experiments provided the same model is used for these processes. These are universal because they carry information about the target whose structure is not altered by the process.

1.4 Free field theory

In this section, we derive moment sum rules for various structure functions assuming the parton model. As we know, the proton is thought of as a collection of spin 1/2, fractionally charged non interacting point like objects called partons which can be identified with the quarks of Gellmann's quark model. Then, the proton current can be written as sum of quark currents:

$$J_\mu(\xi) = \sum_i e_i \; \bar{\psi}_i(\xi) \; \gamma_\mu \; \psi_i(\xi), \tag{1.14}$$

where e_i's are charges of quarks and ψ's are quark field operators satisfying usual canonical free field commutation relations. Once this form of J_μ is known, the usual field theoretical techniques can be applied to calculate $W_{\mu\nu}$ explicitly.

$$W_{\mu\nu} = \frac{1}{2\pi} \int d^4\xi \; e^{iq.\xi} \langle p\, s | [\; J_\mu(\xi), J_\nu(0)] \; | p\, s \rangle. \tag{1.15}$$

Since the process we have considered is DIS, only a specific region of ξ will be dominant. This can be found as follows: As we know, DIS is specified by the region where $q^2 \to -\infty$, $p.q \to \infty$ with $x = -q^2/2p.q$ fixed. In this region, the above integral will vanish due to highly oscillating nature of the exponential unless $|q.\xi|$ is fixed. The requirement that $q \cdot \xi$ should be fixed leads to $q.\xi = \nu(\xi_0 - q.\xi/|q|) - Mxq.\xi/|q|$ fixed giving $\xi^2 \leq const/-q^2$. This implies that $\xi^2 \approx 0$ is the dominating region. From the causality requirement that the commutator vanishes for space like separation, the allowed dominant region is $\xi^2 \geq 0^+$. This region is called Light cone region.

So our aim is to calculate the commutator of two electromagnetic currents in the limit $\xi^2 \geq 0^+$. Note that the commutator of two operators is the imaginary part of their time ordered product, i.e.,

$$\epsilon(\xi_0) \; [\; J_\mu(\xi), J_\nu(0) \;] = T \; (J_\mu(\xi) J_\nu(0) \;) - (T \; (J_\mu(\xi) J_\nu(0) \;))^\dagger \tag{1.16}$$

where $\epsilon(\xi_0) = \xi_0/|\xi_0|$, is the sign function.

In the following, we shall explicitly calculate the time ordered product of currents using Wick's theorem [67] (since the field operators involved are free field operators). Then we shall pick up only terms which are most singular as $\xi^2 \to 0$. Using Wick's theorem,

$$\begin{aligned} T\,(J_\mu(\xi)J_\nu(0)) \;=\; & :J_\mu(\xi)J_\nu(0): + :\bar{\psi}(\xi)\;\gamma_\mu\; iS(\xi,0)\;\gamma_\nu\;\psi(0): + \\ & :\bar{\psi}(0)\;\gamma_\nu\; iS(0,\xi)\;\gamma_\mu\;\psi(\xi): - Tr\;(iS(0,\xi)\;\gamma_\mu\; iS(\xi,0)\gamma_\nu), \end{aligned} \tag{1.17}$$

where $iS(\xi,\eta) = \langle\, 0 \mid\, T\psi(\xi)\bar{\psi}(\eta)\;\mid 0\,\rangle$. The most singlular part [17] of $iS(\xi,\eta)$ is

$$\frac{i}{2\pi^2}\frac{\gamma_\mu\,(\xi-\eta)^\mu}{[\,(\xi-\eta)^2 - i\epsilon\,]^2}$$

The first and last term of the equation eqn.(1.17) will not contribute to $W_{\mu\nu}$ since they correspond to disconnected diagrams.

Using the identities

$$\frac{1}{(\xi^2 - i\epsilon)^n} - \frac{1}{(\xi^2 + i\epsilon)^n} = 2\pi i\;\frac{(-1)^{n-1}}{(n-1)!}\;\delta^{(n-1)}(\xi^2),$$

$$\gamma_\mu\gamma_\lambda\gamma_\nu = (\sigma_{\mu\lambda\nu\rho} + i\gamma_5\;\epsilon_{\mu\lambda\nu\rho})\;\gamma^\rho,$$

where

$$\sigma_{\mu\lambda\nu\sigma} = g_{\mu\lambda}\;g_{\nu\sigma} - g_{\mu\nu}\;g_{\lambda\sigma} + g_{\mu\sigma}\;g_{\lambda\nu},$$

$$\delta^{(n)}(\xi^2) = \left(\frac{\partial}{\partial\xi^2}\right)^n\;\delta(\xi^2),$$

and $\epsilon_{\mu\nu\lambda\rho}$ is completely antisymmetric tensor with Bjorken-Drell convention, we get

$$\epsilon(\xi_0)\;[\;J_\mu(\xi), J_\nu(0)\;] = \frac{1}{\pi}\xi^\lambda\;\delta^{(1)}(\xi^2)\;\;(\sigma_{\mu\lambda\nu\rho} : \mathcal{O}^\rho_V(\xi,0) : - i\epsilon_{\mu\lambda\nu\rho} : \mathcal{O}^\rho_A(\xi,0) :) \tag{1.18}$$

where

$$\mathcal{O}^\rho_V(\xi,\eta) = \bar{\psi}(\xi)\;\gamma^\rho\;\psi(\eta) - \bar{\psi}(\eta)\;\gamma^\rho\;\psi(\xi)$$

and

$$\mathcal{O}^\rho_A(\xi,\eta) = \bar{\psi}(\xi)\;\gamma^\rho\gamma_5\;\psi(\eta) + \bar{\psi}(\eta)\;\gamma^\rho\gamma_5\;\psi(\xi)$$

Here : : denotes the normal ordering of operators.

It is worth noting that the first and second terms of the eqn.(1.18) contribute to spin averaged and spin weighted cross sections respectively due to the presence of symmetric

and antisymmetric tensors which are ultimately contracted with symmetric and antisymmetric parts of the leptonic tensor. Let us first concentrate on the spin averaged term which is symmetric in $(\mu \leftrightarrow \nu)$:

$$\overline{W}^{(EM)}_{\mu\nu} = \frac{1}{2\pi^2}\int d^4\xi\ e^{iq.\xi}\ \frac{1}{2}\sum_s \langle ps|:\mathcal{O}^{\rho}_V(\xi,0):|ps\rangle \times \sigma_{\mu\lambda\nu\rho}\ \xi^{\lambda}\ \delta^{(1)}(\xi^2)\ \epsilon(\xi_0)\ . \tag{1.19}$$

The bilocal operator $\mathcal{O}^{\rho}_V(\xi,0)$ is made local by using Taylor expansion as

$$\mathcal{O}^{\rho}_V(\xi,0) = \sum_n \frac{1}{n!}\ \xi^{\mu_1}...\xi^{\mu_n}\ \mathcal{O}^{\rho}_{V,\mu_1...\mu_n}(0).$$

This is possible because these operators are well defined as $\xi \to 0$. Even though the matrix element between proton states is not known, one generally expresses this in terms of vectors available in the theory using Lorentz properties of the operator. That is,

$$2p^{\rho}p^{\mu_1}...p^{\mu_n}\begin{pmatrix} A_n(p^2) \\ \bar{A}_n(p^2)\end{pmatrix} = \frac{1}{2}\sum_s \langle ps|\begin{pmatrix} \mathcal{O}^{\rho}_{V,\mu_1...\mu_n}(0) \\ -\mathcal{O}^{\rho}_{\bar{V},\mu_1...\mu_n}(0)\end{pmatrix}|ps\rangle\ , \tag{1.20}$$

where

$$\mathcal{O}^{\rho}_{V,\mu_1...\mu_n}(0) = \partial_{\mu_1}...\partial_{\mu_n}\bar{\psi}(0)\ \gamma^{\rho}\ \psi(0)$$
$$\mathcal{O}^{\rho}_{\bar{V},\mu_1...\mu_n}(0) = \bar{\psi}(0)\ \gamma^{\rho}\ \partial_{\mu_1}...\partial_{\mu_n}\psi(0).$$

Define

$$\begin{pmatrix} f(y) \\ \bar{f}(y)\end{pmatrix} = \sum_n \frac{1}{n!}\begin{pmatrix} A_n(p^2) \\ \bar{A}_n(p^2)\end{pmatrix}(\xi.p)^n = N\int\ dz\ e^{izy}\begin{pmatrix} \tilde{f}(z) \\ \tilde{\bar{f}}(z)\end{pmatrix}, \tag{1.21}$$

where $y = \xi.p$ and N is the normalisation constant. Here, bar and tilde on these functions should not be confused with spin independent and spin dependent parts. Substituting these and performing ξ integral using

$$\int\ d^4\xi\ e^{iq.\xi}\ \epsilon(\xi_0)\ \delta^{(n)}(\xi^2) = \frac{i\pi^2}{4^{n-1}(n-1)!}\ (q^2)^{n-1}\epsilon(q_0)\ \theta(q^2),$$

we get

$$\overline{W}^{(EM)}_{\mu\nu} = N\epsilon(q_0 + xp_0)\left[\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)(\tilde{f}(x) + \bar{\tilde{f}}(x)) + \frac{1}{p.q}\left(p_\mu - \frac{p.q}{q^2}q_\mu\right)\left(p_\nu - \frac{p.q}{q^2}q_\nu\right)2x(\tilde{f}(x) + \bar{\tilde{f}}(x))\right]. \quad (1.22)$$

Note that the hadronic tensor in the free field limit can be expressed in terms of the scaling function $\tilde{f}(x)$ with tensor coefficients which manifestly satisfy current conservation. The above analysis shows that $W_{\mu\nu}$ scales as a function of the Bjorken variable. This scaling behaviour can be understood using the fact that theory is scale invariant on the light cone ($\xi^2 \approx 0$) or large momentum transfers ($Q^2 \to \infty$) and that the hadronic tensor is dimensionless. By carefully taking care of charge, spin, colour etc factors and comparing this with the standard parametrisation of the hadronic tensor, we find that $y\tilde{f}(y) = F_2(y)$ and the Callan-Gross relation [68] that $F_2(x) = 2x\ F_1(x)$ is satisfied. The nth moments of these scaling functions can be found out using the definition of the function $\tilde{f}(y)$:

$$\int_0^1 dx\ x^{n-2}F_2(x) = \frac{1}{2}(-i)^{n-1}\left(A_{n-1}(p^2) + \bar{A}_{n-1}(p^2)\right) \quad (1.23)$$

where nth moment of the function $f(x)$ is defined as $\int dx\ x^{n-1}f(x)$ Note that the moments of scaling function are related to the matrix elements of local operators between proton states.

Let us now consider the antisymmetric part of the hadronic tensor

$$\overline{W}^{(EM)}_{\mu\nu} = \frac{i}{2\pi^2}\,\epsilon_{\mu\nu\lambda\rho}\int d^4\xi\ e^{iq.\xi}\ \xi^\lambda\ \delta^{(1)}(\xi^2)\ \epsilon(\xi_0)\ \langle p\,s\,|:\mathcal{O}^\rho_A(\xi,0):|\,p\,s\,\rangle\ . \quad (1.24)$$

The operator can be made local by using Taylor expansion. $\langle p\,s\,|\ \{\mathcal{O}^\rho_{A\mu_1\ldots\mu_n}(0)\}\ |\,p\,s\,\rangle = B_n(p^2)\{s^\rho p_{\mu_1}\ldots p_{\mu_n}\}$ where $B_n(p^2)$ is a Lorentz scalar carrying all information about the operator in the proton state and $\{\}$ denotes complete symmetrization over enclosed indices. This removes the power suppressed terms (twist three terms).

Note that

$$\xi^{\mu_1}\ldots\xi^{\mu_n}\langle p\,s\,|\ \{\mathcal{O}^\rho_{A\mu_1\ldots\mu_n}\}\ |\,p\,s\,\rangle = \frac{B_n(p^2)}{(n+1)!}\left(\ n!\ s^\rho(\xi.p)^n + n\ n!\ p^\rho(s.\xi)(p.\xi)^{n-1}\ \right).$$

After a little algebra we find

$$\widetilde{W}_{\mu\nu}^{(EM)} = iN\epsilon_{\mu\nu\lambda\rho}\, q^{\lambda}\frac{1}{2p.q}\left[s^{\rho} - \frac{s.q}{p.q}\, p^{\rho} - \frac{s.q}{p.q}\, p^{\rho}x\frac{\partial}{\partial x}\right](\tilde{g}(x) + \tilde{\bar{g}}(x)) \,, \tag{1.25}$$

where

$$\begin{pmatrix} g(y) \\ \bar{g}(y) \end{pmatrix} = \sum_n \frac{1}{n!}\begin{pmatrix} B_n(p^2) \\ \bar{B}_n(p^2) \end{pmatrix}(\xi.p)^n = N\int dz\; e^{izy}\begin{pmatrix} \tilde{g}(z) \\ \tilde{\bar{g}}(z) \end{pmatrix}. \tag{1.26}$$

Here $B_n(p^2)$ and $\bar{B}_n(p^2)$ are defined as

$$\{s^{\rho}p^{\mu_1}...p^{\mu_n}\}\begin{pmatrix} B(p^2) \\ \bar{B}(p^2) \end{pmatrix} = \langle ps|\begin{pmatrix} \mathcal{O}^{\rho}_{A,\mu_1...\mu_n}(0) \\ \mathcal{O}^{\rho}_{\bar{A},\mu_1...\mu_n}(0) \end{pmatrix}|ps\rangle \,,$$

where

$$\begin{aligned} \mathcal{O}^{\rho}_{A,\mu_1...\mu_n}(0) &= \{\partial_{\mu_1}...\partial_{\mu_n}\bar{\psi}(0)\;\gamma^{\rho}\gamma_5\;\psi(0)\} \\ \mathcal{O}^{\rho}_{\bar{A},\mu_1...\mu_n}(0) &= \{\bar{\psi}(0)\;\gamma^{\rho}\gamma_5\;\partial_{\mu_1}...\partial_{\mu_n}\psi(0)\}. \end{aligned} \tag{1.27}$$

Note that $\widetilde{W}_{\mu\nu}^{(EM)}$ also scales. Current conservation is manifest. Comparing this with the standard parametrization of hadronic tensor, we get

$$\begin{aligned} g_1(x) &= -\frac{N}{2}\, x\frac{\partial}{\partial x}(\tilde{g}(x) + \tilde{\bar{g}}(x)), \\ g_2(x) &= \frac{N}{2}\,(\tilde{g}(x) + \tilde{\bar{g}}(x)) - g_1(x). \end{aligned} \tag{1.28}$$

Moments of these scaling functions can be found out as

$$\begin{aligned} \int_0^1 dx\; x^{n-1}g_1(x) &= \frac{1}{4}\,(-i)^{n-1}\left(B_{n-1}(p^2) + \bar{B}_{n-1}(p^2)\right), \\ \int_0^1 dx\; x^{n-1}g_2(x) &= -\frac{n-1}{4n}\,(-i)^{n-1}\left(B_{n-1}(p^2) + \bar{B}_{n-1}(p^2)\right). \end{aligned} \tag{1.29}$$

Also note that $g_2(x)$ is related to $g_1(x)$ as

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dx'}{x'}\, g_1(x'). \tag{1.30}$$

This is called Wilczek-Wandzura sum rule [66].

We now analyse $W_{\mu\nu}^{W}$ in the weak sector (W). The process we consider is $\nu p \to l_{\nu}X$. Since it is a weak interaction, we replace electromagnetic (EM) current by weak current

in the definition. Since we are going to derive sum rules analogous to EM case in the free field theory, the current takes the form

$$\begin{aligned} J_\mu^W(\xi) &= :\frac{1}{2}\,\bar{\psi}(\xi)\,\gamma_\mu\,(1-\gamma_5)\,\psi(\xi): \\ &= \frac{1}{2}\,(J_\mu(\xi)-J_{\mu 5}(\xi)). \end{aligned} \tag{1.31}$$

Here we have not written indices corresponding to weak isospin symmetry since it is not going to affect the derivation of our sum rules. As usual, we evaluate the time ordered product of these currents.

$$\begin{aligned} T(J_\mu^W(\xi)J_\nu^{W\dagger}(0)) &= \frac{1}{4}\Big(TJ_\mu(\xi)J_\nu^\dagger(0)+TJ_{\mu 5}(\xi)J_{\nu 5}^\dagger(0)- \\ &\quad TJ_{\mu 5}(\xi)J_\nu^\dagger(0)-TJ_\mu(\xi)J_{\nu 5}^\dagger(0)\Big). \end{aligned} \tag{1.32}$$

After taking the most singular part of the above expression, we find that the first two terms are identical to those which appear in the electromagnetic case. Hence they reproduce the structure functions $F_1(x), F_2(x), g_1(x)$ and $g_2(x)$. The third and fourth terms are identical in the leading singular limit and the commutator of their sum is found to be

$$\begin{aligned} [J_\mu^W(\xi),j_\nu^{W\dagger}(0)]_{(3+4)} &= -\frac{1}{2\pi}\xi^\lambda\delta^{(1)}(\xi^2)\epsilon(\xi_0)\Big\{\sigma_{\mu\lambda\nu\rho}\Big(:\bar{\psi}(\xi)\gamma^\rho\gamma_5\psi(0)-\bar{\psi}(0)\gamma^\rho\gamma_5\psi(\xi):\Big)+ \\ &\quad i\epsilon_{\mu\nu\lambda\rho}\Big(:\bar{\psi}(\xi)\gamma^\rho\psi(0)+\bar{\psi}(0)\gamma^\rho\psi(\xi)\Big):\Big\} \end{aligned} \tag{1.33}$$

Note the crucial difference between the relative signs of the bilocal operator combinations appearing here and that in the EM case. Also note that coefficient of symmetric tensor(antisymmetric tensor) is axial(vector) tensor contrary to EM case.

Let us consider the second term of the above equation. Its Fourier transform is found to be

$$\frac{iN}{2}\epsilon_{\mu\nu\lambda\rho}\,\frac{p^\rho q^\lambda}{p.q}\,\Big(\tilde{f}_3(x)-\bar{\tilde{f}}_3(x)\Big)\;\epsilon(q_0+xp_0) \tag{1.34}$$

Comparing this with the standard parametrisation, we find that

$$F_3(x)=\frac{N}{2}\Big(\tilde{f}(x)-\bar{\tilde{f}}(x)\Big) \tag{1.35}$$

and its nth moment is given by

$$\int_0^1 dx\ x^{n-2} F_3(x) = \frac{1}{2}(-i)^{n-2}\left(A_{n-2}(p^2) - \bar{A}_{n-2}(p^2)\right) \tag{1.36}$$

Similarly the first term of eqn.(1.33) reduces to

$$\begin{aligned} W^{(2)}_{\mu\nu} &= N\left[\frac{q.s}{p.q}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\frac{x}{4}\frac{\partial}{\partial x} + \right. \\ &\frac{1}{2p.q}\left(R_\mu R_\nu \frac{q.s}{p.q} + \frac{1}{2}(R_\mu T_\nu + R_\nu T_\mu)\right)\frac{x^2}{2}\frac{\partial}{\partial x} + \\ &\left.\frac{1}{2p.q}\left(R_\mu R_\nu \frac{q.s}{p.q} - \frac{1}{2}(R_\nu T_\mu + R_\nu T_\mu)\right)\left(2x + \frac{x^2}{2}\frac{\partial}{\partial x}\right)\right](\tilde{g}(x) - \tilde{\bar{g}}(x)) \end{aligned} \tag{1.37}$$

where R_μ and T_μ are defined already. Comparing with the standard parametrization we find

$$g_3(x) = \frac{N}{4}\, x\frac{\partial}{\partial x}(\tilde{g}(x) - \tilde{\bar{g}}(x)), \tag{1.38}$$

$$g_4(x) = \frac{N}{2}\, x^2\frac{\partial}{\partial x}(\tilde{g}(x) - \tilde{\bar{g}}(x)), \tag{1.39}$$

$$g_5(x) = N\left(2x + \frac{x^2}{2}\frac{\partial}{\partial x}\right)(\tilde{g}(x) - \tilde{\bar{g}}(x)). \tag{1.40}$$

The nth moments of these structure functions are

$$\int_0^1 dx\ x^{n-1} g_3(x) = -\frac{1}{8}(-i)^{n-1}\left(B_{n-1}(p^2) - \bar{B}_{n-1}(p^2)\right), \tag{1.41}$$

$$\int_0^1 dx\ x^{n-1} g_4(x) = -\frac{1}{4}(-i)^{n}\left(B_{n}(p^2) - \bar{B}_{n}(p^2)\right), \tag{1.42}$$

$$\int_0^1 dx\ x^{n-1} g_5(x) = -\frac{1}{4}\frac{n-3}{n+1}(-i)^{n}\left(B_{n}(p^2) - \bar{B}_{n}(p^2)\right). \tag{1.43}$$

Here also one finds both Callan-Gross and Wilczek-Wandzura sum rules among the structure functions:

$$\begin{aligned} g_4(x) &= 2\,x\,g_3(x) \\ g_5(x) &= g_4(x) + 4\,x\int_x^1 \frac{dx'}{x'^2}\, g_4(x) \end{aligned}$$

1.5 Factorisation method

So far, we have discussed various approaches to high energy DIS processes. In the parton model, the q^2 behaviour of the structure functions or the cross sections is completely

controlled by the parton level cross sections while in the OPE, it is controlled by the strength of the singularity of the Wilson coefficients. In both the methods, it does not depend on the distribution functions or the matrix elements. Experimentally observed scaling behaviour of these cross sections implies that partons are in fact free inside the hadron in large q^2 or they weakly interact. Also note that these partons have never been observed experimentally. What is being observed at the final state is a jet of hadrons. So the intuitive picture that partons are free inside a hadron is valid only in the large q^2 limit. Hence in order to write renormalizable local quantum field theory for these particles which behave differently at different energy scales, one should take into account these observed properties of the theory for strong interaction. The quarks and gluons are in fact asymptotic states of perturbative QCD which facilitates asymptotic freedom among these partons at large energies and confinement at small energies via its scale dependent running coupling constant. QCD provides cross sections for parton model and Wilson coefficients for OPE. The successful test of the prediction of scaling violation in the hadronic cross section is the evidence for the existence of such constituents.

If one calculates contribution coming from the parton level cross sections in the parton model to the hadron cross section, one finds divergences of the form $ln\ Q^2/p_i^2$ or $ln\ Q^2/p_i.p_j$ where p_i are momenta of participating partons. These singularities are vulnerable to the QCD perturbation theory relevant to hard scattering processes. In general these mass singularities are of two types of different origins. The first type, which is infrared, occurs due to the massless nature of the gluons and the second type, which is collinear appears when the outgoing particles are collinear to the incoming particles. In some high energy processes such as $\gamma^* g \rightarrow q \bar{q}$ cross section which is relevant to express DIS cross section in terms of gluon distribution function these singularities do occur. In fact, the cross section at large energies goes as $ln\ Q^2/(m^2 - k^2x(1-x))$ and $k^2/(m^2 - k^2x(1-x))$ where m is the mass of the quark and k^2 is the mass of the gluon (if any). Following Politzer, one first factorises the parton level cross section as the product of a part which contains all mass singularities and a part which is mass finite and then absorbs the singular part into the parton distribution when the cross section is plugged into the DIS cross section.

The resultant distribution is then called renormalised one. If this can be done order by order systematically in perturbative QCD, the QCD based parton model is reliable as the perturbation theory for these high energy processes. The procedure by which one factorises mass singularities and absorbs them into parton distribution functions is called factorisation method. So the statement that the cross section factorises into mass singular part and the finite part is called Factorisation theorem. Since the basic parton level cross sections are different for different hard processes, one has to prove the factorisation theorems separately for various processes. There are various all-order proofs for different processes in the literature. Since the mass singularities correspond to long distance physics and finite part corresponds to the short distance physics of the cross section, factorisation is basically factorisation of short and long distance parts of the parton cross sections. In general, the all-order proof of factorisation theorem involves subtractions of mass sensitive parts from the pinch surfaces corresponding to physical picture of the original diagram and reconstruction of these subtraction in the form of hadron matrix element of some operators. The all order proof of factorisation theorem for both unpolarised DIS and unpolarised Drell Yan processes can be found in the literature [16]. These proofs are based on the formal proof of Zimmerman [32]. Until now there is no all order proof for the factorisation theorem in the polarised DIS and DY. Bodwin and Qiu [10] were the first to show the factorisation of mass singularities to order α_s in the gluonic sector of spin dependent DIS cross section. In the same context Manohar [11] has proposed the generalisation of unpolarised factorisation to polarised process by appropriately modifying the operator definitions of the parton distribution functions and has shown the validity to order α_s in the gluonic sector.

In this thesis we have applied the factorisation theorem to Weak DIS. Using this we have calculated gluonic contribution to weak spin dependent structure functions. The independence of the results on mass singularities practically verifies the proposed factorisation theorem in the weak DIS sector. The heuristic derivation (given below) of the factorisation formula in the weak DIS yields the universality of parton distribution functions. We have extended the analysis to polarised DY process also (Chapter 3). We

first derive operator definitions for polarised parton distribution functions from polarised DY cross section. We have also demonstrated that in the free field limit a factorisation analogous to that in the unpolarised DY case holds in this process. By explicit evaluation of gluonic contribution to polarised DY function, we verify the proposed factorisation theorem to order α_s.

In this section we give interpretation of the above free field model. Let us first consider (with out charge, flavour, colour factors etc) the spin independent part of $\overline{W}^{(EM)}_{\mu\nu}$ [37]:

$$W_{\mu\nu} = \frac{1}{2\pi^2}\sigma_{\mu\lambda\nu\rho}\int d^4\xi\ \xi^\lambda\ \epsilon(\xi_0)\ \delta^{(1)}(\xi^2)\ \frac{1}{2}\ \langle p\,s\,|:\mathcal{O}^\rho_V(\xi,0):|\,p\,s\,\rangle, \tag{1.44}$$

where

$$\mathcal{O}^\rho_V(\xi,0) = \bar\psi(\xi)\ \gamma^\rho\ \psi(0) - \bar\psi(0)\ \gamma^\rho\ \psi(\xi). \tag{1.45}$$

The contribution to $F_2(x)$ structure function can be projected from $\overline{W}^{EM}_{\mu\nu}$ using the projector

$$\tilde{\mathcal{P}}^{(2)}_{\mu\nu} = -\frac{x}{2}\left[g_{\mu\nu} + \frac{12x^2 p_\mu p_\nu}{q^2}\right]. \tag{1.46}$$

Noting that $\sigma_{\mu\nu\lambda\sigma}\tilde{\mathcal{P}}^{(2)}_{\mu\nu} \approx \xi\, g_{\sigma\lambda}$ and $\xi^\lambda\ \delta^{(1)}(\xi^2) = \frac{1}{2}\ \partial^\lambda\ \delta(\xi^2)$ we get,

$$F_2(x) = x\frac{1}{4\pi^2}\int d^4\xi\ e^{iq.\xi}\ \ \epsilon(\xi_0)\ \partial_\lambda\ \delta(\xi^2)\ \frac{1}{2}\sum_s\ \langle p\,s\,|:\mathcal{O}^\lambda_V(\xi,0):|\,p\,s\,\rangle. \tag{1.47}$$

Performing integration by parts and neglecting less singular terms, we get,

$$F_2(x)\ \ =\ \ -q_\lambda\frac{i}{4\pi^2}x\int d^4\xi\ e^{iq.\xi}\ \ \epsilon(\xi_0)\ \delta(\xi^2)\ \frac{1}{2}\ \sum_s\ \langle p\,s\,|:\mathcal{O}^\lambda_V(\xi,0):|\,p\,s\,\rangle. \tag{1.48}$$

For convenience, we work in the rest frame of proton in which $p = (M,\vec{0})$ and $q = (\nu,0,0,-\sqrt{\nu^2+Q^2})$ and also use light cone variables since the expression is evaluated on $\xi^2 \approx 0^+$ plane. The light cone coordinates are defined as $a^\pm = (a^0 \pm a^3)/\sqrt{2}$, $\vec{a}_\perp = (a_1,a_2)$. In terms of light cone variables, $p^+ = p^- = M/\sqrt{2}$, $q^+ = -Mx/\sqrt{2}$ and $q^- = \sqrt{2}\nu$. The Bjorken variable becomes $-q^+/p^+$ in the limit q^- and ν going infinity. Now,

$$\begin{aligned} F_2(x)\ \ =\ \ & -x\,\frac{i}{4\pi^2}\,q_\lambda\int d\xi^- d\xi^+ d^2\xi_\perp\ e^{iq^+\xi^- + iq^-\xi^+}\,\epsilon(\xi^+ + \xi^-)\ \delta(\xi_\perp^2 - 2\xi^-\xi^+)\ \times \\ & \frac{1}{2}\ \sum_s\ \langle p\,s\,|:\mathcal{O}^\lambda_V(\xi,0):|\,p\,s\,\rangle, \end{aligned} \tag{1.49}$$

Since $q_\perp$ is zero, $\xi_\perp$ integration can be performed easily using the delta function,

$$\begin{aligned} F_2(x) &= -x\,\frac{i}{4\pi}\,q_\lambda \int d\xi^- d\xi^+ e^{iq^+\xi^- + iq^-\xi^+}\ \tilde{\theta}(\xi^+,\xi^-) \\ &\quad \frac{1}{2}\ \sum_s \langle p\,s\,|:\mathcal{O}^\lambda(\xi,0):|\,p\,s\,\rangle|_{\xi_\perp^2=2\xi^+\xi^-}. \end{aligned} \tag{1.50}$$

where $\tilde{\theta} = [\,\theta(\xi^+)\theta(\xi^-) - \theta(-\xi^+)\theta(-\xi^-)\,]$ Using the following identities in the limit $q^- \to \infty$:

$$\int_{-\infty}^{\infty} d\xi^+ e^{iq^-\xi^+}\,\theta(\pm\xi^+)\ f(\xi^+) = \pm\ f(0)\ \frac{i}{q^- \pm\ i\epsilon}, \tag{1.51}$$

we get

$$F_2(x) = x\frac{1}{4\pi}\int d\xi^-\, e^{-iMx\xi^-/\sqrt{2}}\ \frac{1}{2}\ \sum_s \langle p\,s\,|:\bar{\psi}(\xi^-)\gamma^+\psi(0) - \bar{\psi}(0)\gamma^+\psi(\xi^-):|\,p\,s\,\rangle. \tag{1.52}$$

Inserting complete set of states, doing ξ^- integration using translational invariance and using the projector $P^\pm = \frac{1}{2}\,\gamma^\mp\,\gamma^\pm$ with $\psi^\pm = P^\pm\,\psi$, we get

$$\begin{aligned} \frac{F_2(x)}{x} &= \sqrt{2}\ \sum_n\ \delta(-xp^+ + p^+ - p_n^+)\ \ |\,\langle\,p_n\,|\,\psi_+(0)\,|\,p\,s\,\rangle\,|^2 + \\ &\quad \sqrt{2}\ \sum_n\ \delta(+xp^+ + p^+ - p_n^+)\ \ |\,\langle\,p_n\,|\,\psi_+(0)\,|\,p\,s\,\rangle\,|^2\ . \end{aligned} \tag{1.53}$$

The first term is the probability to annihilate a parton of momentum xp from a proton of momentum p leaving a physical state with momentum $(1-x)p$. This is nothing but the probability of finding a parton with momentum fraction x inside a proton. The second term corresponds to antiparticle. If one writes as

$$\frac{F_2(x)}{x} = f_{\frac{q}{p}}(x) + f_{\frac{\bar{q}}{p}}(x) \tag{1.54}$$

then

$$f_{\frac{q}{p}}(x) = \frac{1}{4\pi}\int\ d\xi^-\, e^{-iMx\xi^-/\sqrt{2}}\ \frac{1}{2}\ \sum_s \langle p\,s\,|:\bar{\psi}(\xi^-)\gamma^+\psi(0):|\,p\,s\,\rangle, \tag{1.55}$$

and

$$f_{\frac{\bar{q}}{p}}(x) = \frac{1}{4\pi}\int\ d\xi^-\, e^{-iMx\xi^-/\sqrt{2}}\ \frac{1}{2}\ \sum_s \langle p\,s\,|:\bar{\psi}(0)\gamma^+\psi(\xi^-):|\,p\,s\,\rangle. \tag{1.56}$$

These functions satisfy the following properties: $f_{\frac{a}{p}}(x) = -\, f_{\frac{a}{p}}(-x)$ and $f_{\frac{a}{p}}(x)$ and $f_{\frac{a}{p}}(x)$ are zero for $x \leq 0$ and $x \geq 1$. Thus $F_2(x)/x$ in the free field theory is proportional to density of quarks and antiquarks with momentum fraction x inside the proton. This coincides with the parton model interpretation of this structure function.

Let us consider the antisymmetric part of $\widetilde{W}^{EM}_{\mu\nu}$[18]:

$$\widetilde{W}^{EM}_{\mu\nu} = -\frac{i}{2\pi^2}\epsilon_{\mu\lambda\nu\rho}\int d^4\xi\ \xi^\lambda\ \epsilon(\xi_0)\ \delta^{(1)}(\xi^2)\langle p\,s\,|:\mathcal{O}^\rho_A(\xi,0):|\,p\,s\,\rangle. \tag{1.57}$$

where

$$\mathcal{O}^\rho_A(\xi,0) = \bar\psi(\xi)\ \gamma^\rho\gamma_5\ \psi(0) + \bar\psi(0)\ \gamma^\rho\gamma_5\ \psi(\xi) \tag{1.58}$$

The contribution to $g_1(x)$ structure function can be projected from $W_{\mu\nu}$ using the projector

$$\tilde{\mathcal{P}}^{(1)}_{\mu\nu} = -\ \frac{i}{2p.q}\ \epsilon_{\mu\nu\lambda\sigma}p^\lambda\ q^\sigma. \tag{1.59}$$

That is,

$$g_1(x) = -\ \frac{i}{4\pi^2}(q_\lambda - 2p_\lambda)\int d^4\xi\ e^{iq.\xi}\ \epsilon(\xi_0)\ \delta(\xi^2)\ \langle p\,s\,|:\mathcal{O}^\lambda_A(\xi,0):|\,p\,s\,\rangle. \tag{1.60}$$

Following the same procedure we get,

$$g_1(x) = \frac{1}{4\pi}\int d\xi^-e^{-iMx\xi^-/\sqrt{2}}\ \langle p\,s\,|:\bar\psi(\xi^-)\ \gamma^+\gamma_5\ \psi(0) + \bar\psi(0)\ \gamma^+\gamma_5\ \psi(\xi^-):|\,p\,s\,\rangle. \tag{1.61}$$

Using translational invariance and the projector $\mathcal{P}^{R,L} = \frac{1}{2}(1 \pm \gamma_5)$ with $\psi^{R,L} = \mathcal{P}^{R,L}\psi$ in addition to the projectors $\mathcal{P}^\pm$, we get

$$\begin{aligned} g_1(x) \;=\; & \sqrt{2}\ \sum_n\ \delta(-xp^+ + p^+ - p^+_n)\left(|\langle\, p_n\,|\psi^R_+(0)|ps\rangle|^2 - |\langle\, p_n\,|\psi^L_+(0)|\,p\,s\,\rangle|^2\right) + \\ & \sqrt{2}\ \sum_n\ \delta(+xp^+ + p^+ - p^+_n)\left(|\langle p_n|\psi^R_+(0)|\,p\,s\,\rangle|^2 - |\langle\, p_n\,|\psi^L_+(0)|ps\rangle|^2\right). \end{aligned} \tag{1.62}$$

The first term is the probability to annihilate a right handed parton of momentum xp from a proton of momentum p leaving a physical state with momentum $(1-x)p$. This is nothing but the probability of finding a right handed parton with momentum fraction x inside a proton with definite polarization. The second term corresponds to left handed particle. Hence, the difference gives the net polarised probability of particle. Similar

interpretation is applied to the rest of the terms which correspond to antiparticle. If one writes $g_1(x)$ as

$$g_1(x) = f_{\frac{\Delta a}{p}}(x) + f_{\frac{\Delta a}{p}}(x) \tag{1.63}$$

then we have

$$f_{\frac{\Delta a}{p}}(x) = \frac{1}{4\pi}\int d\xi^- e^{-ixM\xi^-/\sqrt{2}} \ \langle p\,s\,|:\bar{\psi}(\xi^-)\,\gamma^+\gamma_5\,\psi(0):|\,p\,s\,\rangle \tag{1.64}$$

and

$$f_{\frac{\Delta a}{p}}(x) = \frac{1}{4\pi}\int d\xi^- e^{-ixM\xi^-/\sqrt{2}} \ \langle p\,s\,|:\bar{\psi}(0)\,\gamma^+\gamma_5\,\psi(\xi^-):|\,p\,s\,\rangle. \tag{1.65}$$

These functions satisfy the following relations: $f_{\frac{\Delta a}{p}}(x) = -f_{\frac{\Delta a}{p}}(-x)$ and $f_{\frac{\Delta a}{p}}(x)$ and $f_{\frac{\Delta a}{p}}(x)$ are zero for $x \leq 0$ and $x \geq 1$. Hence $g_1(x)$ is proportional to the density of polarized quarks and antiquarks. This also coincides with the parton model interpretation of this function.

We shall now interpret the weak structure function g_3. Since the coefficient of g_3 is symmetric in $(\mu \leftrightarrow \nu)$, consider the symmetric part of $\widetilde{W}^W_{\mu\nu}$. The symmetric part of polarised $\widetilde{W}^W_{\mu\nu}$ is

$$= -\frac{1}{2\pi^2}\sigma_{\mu\lambda\nu\rho}\int d^4\xi\ \xi^\lambda\epsilon(\xi_0)\ \delta^{(1)}(\xi^2)\ \langle p\,s\,|:\widetilde{\mathcal{O}_A}^{\rho}(\xi,0):|ps\rangle. \tag{1.66}$$

where

$$\widetilde{\mathcal{O}_A}^{\rho}(\xi,0) = \bar{\psi}(\xi)\ \gamma^\rho\gamma_5\ \psi(0) - \bar{\psi}(0)\ \gamma^\rho\gamma_5\ \psi(\xi). \tag{1.67}$$

The g_3 structure function can be projected from $W_{\mu\nu}$ using the projector

$$\tilde{\mathcal{P}}^{(3)}_{\mu\nu} = -\frac{1}{2}\left[g_{\mu\nu} + \frac{4x^2p_\mu p_\nu}{q^2}\right]. \tag{1.68}$$

Noting that $\sigma^{\mu\lambda\nu\rho}\ \tilde{\mathcal{P}}^3_{\mu\nu} = g^{\mu\nu}$ and following the same procedure, we get

$$g_3(x) = -\frac{1}{8\pi}\int d\xi^- e^{-iMx\xi^-/\sqrt{2}}\ \langle p\,s\,|:\bar{\psi}(\xi^-)\,\gamma^+\gamma_5\,\psi(0) - \bar{\psi}(0)\,\gamma^+\gamma_5\,\psi(\xi^-):|\,p\,s\,\rangle \tag{1.69}$$

Hence, we have

$$g_3(x) = -\frac{1}{2}\left(f_{\frac{\Delta a}{P}}(x) - f_{\frac{\Delta a}{P}}(x)\right) \tag{1.70}$$

This can be written as

$$g_3 = \int_x^1 \frac{dy}{y}\left(-\frac{1}{2}\delta(1-\frac{x}{y})\right)\left(f_{\frac{\Delta a}{P}}(y) - f_{\frac{\Delta a}{P}}(y)\right). \tag{1.71}$$

Noting that $\delta(1-\frac{x}{y})$ is the quark-W boson Born cross section, one can in general write $\widetilde{W}^W_{\mu\nu}$ as

$$\widetilde{W}^{W(\lambda)P(H)}(x,Q^2) = \sum_{a(h)}\int_x^1 \frac{dy}{y} H^{W(\lambda)a(h)}_{\mu\nu}(x/y,\mu^2,Q^2) f_{\frac{\Delta a(h)}{P(H)}}(y,\mu^2) \tag{1.72}$$

here $H^{Wa}_{\mu\nu}$ is called hard scattering coefficient. This is dependent on the process. Note that we get the same matrix element for both EM and weak currents. The independence of these operators on the process is called universality of parton distribution functions. The properties of these hard scattering coefficients and the parton distribution functions will be discussed later.

1.6 Operator Product Expansion

Gell-Mann proposed famous charge algebra which was extended to the local algebra of charge and current densities to circumvent the problems one faced to find the right theory of strong interaction and to solve it exactly. A number of theoretical techniques have been developed to extract useful informations about strong interaction physics from proposed current algebra. This includes the famous BJL (Bjorken-Johnson-Low) theorem, dispersion technique etc. It is successful in two broad categories of application: low energy theorem and high energy sum rules. Application of current algebra includes Sutherland-Veltman theorem, Cabibbo-Radicati sum rule for Compton scattering, the Adler sum rule for neutrino induced reactions, the Adler-Weisberger relation for pion-nucleon scattering, Callan-Gross sum rule etc. The algebra is basically independent of the specific form of Lagrangian i.e., dynamics, but depends on the canonical commutation relations among basic quantum fields and associated Lagrangian field theory. There are failures in the prediction of current algebra. These can be traced back to difficulties involved in the naive manipulation of quantum fields in local quantum field theories. Importantly one has to

worry about multiplying together two or more operators at the same space-time point. Basically the deviation of the predictions of the current algebra from the observed results is due to the dynamics involved (type of interaction etc). In this direction Wilson proposed famous operator product expansion (OPE) which does not depend on Lagrangian field theory but deals only with fields (basic field operators) and symmetry principles (gauge invariance, local symmetries of the interaction). It is qite successful as we see in this section.

Before we go to proper operator product expansion we study the strong interaction physics using non-Lagrangian models [56] which could help to understand strong interaction physics. This formalism involves no specific Lagrangian and canonical fields. To replace Lagrangian models one proposes two hypothesis: (i) Strong interaction at very high energies or short distances is scale invariant (ii) There exists OPE for the product of two or more currents at short distances.

We know that the equal time commutator of two local field operators $A(\xi)$ and $B(\xi)$ is in general written using current algebra technique [57] as

$$[\ A(\vec{\xi},\xi_0), B(\vec{\eta},\xi_0)\ ] = \sum_n D_n(\vec{\xi}-\vec{\eta})O_n(\xi)\ : \tag{1.73}$$

where $D_n(\vec{\xi}-\vec{\eta})$ are dirac delta function or its derivatives called Schwinger terms and $O_n(\xi)$ are local operators which are not singular. These commutators are used to calculate various other commutators such as $W_{\mu\nu}$ appearing in the strong interaction physics. It is natural to have a more general expression than the commutator of operators. The generalization was first suggested by K.G. Wilson [56]. That is, the generalisation is that an ordinary product of $A(\xi)$ and $B(\eta)$ has an expansion as $\xi \to \eta$ of the form

$$A(\xi)B(\eta) = \sum_n C_n(\xi-\eta)O_n(\xi) \tag{1.74}$$

where O_n are again some local operators and $C_n(\xi-\eta)$ are singular functions as $\xi \to \eta$. The form of these singular functions is $[\ (\xi-\eta)^2 - i\epsilon(\xi_0-\eta_0)\ ]^{-p}$ where p is a real number arrived by using one of the hypotheses that strong interaction is scale invariant. This form can be got from the simple method which will be explained latter. This expansion is called operator product expansion.

We state a few properties of the OPE without proof: OPE exists for free scalar, spinor field theories. It exists for interacting theories. It also exists for product of elementary fields such as $\phi(\xi)\phi(\eta)...$ and composite fields such as product of currents $J(\xi)J(\eta)....$ OPE is useful to calculate Wick product of local free fields and equal time commutator of fields and its time derivatives. OPE is more useful than commutators in describing short distance physics because OPE does not contain any infinite constant since it is just a distribution unlike the Schwinger term which can be infinite.

Now let us derive the form of the singularity stated previously. Consider the matrix element of the product $A(\xi)B(0)$ in the limit $\xi \to 0$

$$\langle \alpha | A(\xi)B(0) | \beta \rangle = \sum_n C_n(\xi) \langle \alpha | O_n(0) | \beta \rangle \tag{1.75}$$

Inserting complete set of states between the operators $A(\xi)$ and $B(0)$ and using translational invariance, the left hand side of the equation can be written as

$$\sum_m \langle \alpha | A(\xi) | m \rangle \langle m | B(0) | \beta \rangle = \sum_m e^{i(p_\alpha - p_m)\xi} \langle \alpha | A(0) | m \rangle \langle m | B(0) | \beta \rangle \tag{1.76}$$

The above series is convergent only when $\xi_0 \to \xi_0 - i\epsilon$ as $E_m - E_\alpha \to \infty$. Also we know from Lorentz invariance that $C_n(\xi) \approx (\xi^2)^{-p}$. These two properties lead to

$$C_n(\xi, \eta) = [\ (\xi - \eta)^2 - i\epsilon(\xi_0 - \eta_0)\ ]^{-p}. \tag{1.77}$$

Note that the above proof requires definite states $|\alpha\rangle$ and $|\beta\rangle$.

The nature of singularity can be got using the hypothesis that strong interaction is scale invariant. We know that the mass and interaction break the scale invariance of the theory. So we assume free massless theory for the time being.

Exact scale invariance of the theory means that the field theory is invariant under the one parameter group of transformation $U(t)$:

$$O_n(\xi) \to U(t)^\dagger O_n(e^t \xi) U(t) \tag{1.78}$$

where $U(t)$ is the scale transformation operator and t is the scale parameter. The scale dimension of the elementary fields in the free field theory is calculated using the canonical

commutation relation and the scale transformation. Under scale transformation, the eqn.(1.75) becomes

$$e^{t(d_A+d_B)}\ A(e^t\xi)B(e^t\eta) = \sum_n C_n(\xi-\eta)\ e^{td_n}\ O_n(e^t\xi) \tag{1.79}$$

where d_A,d_B and d_n are scale dimensions of the operators $A(\xi)$, O_n and $B(\xi)$ respectively. Expanding L.H.S, we get

$$e^{t(d_A+d_B)}\sum_n C_n(e^t\xi - e^t\eta)O(e^t\xi) = \sum_n\ C_n(\xi-\eta)\ e^{td_n}\ O_n(e^t\xi). \tag{1.80}$$

This implies

$$C_n(e^t\xi - e^t\eta) = C_n(\xi-\eta)\ e^{-t(d_A+d_B-d_n)}. \tag{1.81}$$

Since $C_n(\xi-\eta)$ is homogeneous in $(\xi-\eta)^2$,

$$C_n(\xi-\eta) \approx [\ (\xi-\eta)^2 - i\epsilon(\xi_0-\eta_0)\ ]^{(d_n-d_A-d_B)/2} \tag{1.82}$$

From the above relation, it is clear that $d_A + d_B \geq d_n$ leads to singular behaviour of $C_n(\xi-\eta)$.

We calculate the commutator of two operators in the limit $\xi \to \eta$ using OPE and show the presence of Schwinger terms. The commutator can be expressed as

$$[\ A(\xi), B(\eta)\ ] = \sum_n E_n(\xi-\eta)O_n(\xi) \tag{1.83}$$

where

$$E_0(\xi-\eta) = E\left[(-z^2+i\epsilon z_0)^{-p} - (-z^2-i\epsilon z_0)^{-p}\right] \tag{1.84}$$

with $z = \xi - \eta$, here p is determined using the dimensions of A, B and O_n. We express

$$E_0(z) = \left[\int d^3\xi\ \ E_0(\xi)\right]\ \delta^{(3)}(z) + \left[\int d^3\xi\ \left(-\vec{\xi}E_0(\xi)\right)\right]\cdot\vec{\nabla}_z\delta^{(3)}(z) + \ldots \tag{1.85}$$

Hence

$$[\ A(\xi), B(\eta)\ ]|_{\xi_0=\eta_0} = \left(F_0(z_0)\ \delta^{(3)}(z) + \vec{F}_1(z_0).\vec{\nabla}\delta^{(3)}(z) + \ldots\right)O_0 \tag{1.86}$$

where

$$F_0(z_0) = \int d^3\xi \; E_0(\xi),$$
$$\vec{F}_1(z_0) = \int d^3\xi \; \left(-\vec{\xi}E_o(\xi)\right)$$

etc. From the above equation, we can see that the first term corresponds to the usual term in the commutator algebra and the second term corresponds to the Schwinger term. Thus we can reproduce all the commutation relations which appear in strong interaction physics using this present approach.

The other important application of OPE is DIS where one is interested in the calculation of product of two currents when the square of their spatial separation vanishes. In the following we will apply this operator product expansion to DIS and identify these operators with the known operators appearing in the theory which we have already used. Using dimensional analysis, Lorentz invariance and singularity structure of the current commutator on the light cone, one generally expands $J_\mu(\xi)J_\nu(0)$ [17, 60] as

$$\begin{aligned} TJ_\mu(\xi)J_\nu(0) &= (-g_{\mu\nu}\Box - \partial_\mu\partial_\nu)\sum_{i,n}\frac{1}{2}[1+(-1)^n]\frac{1}{\xi^2 - i\epsilon\xi_0}\xi^{\mu_1}...\xi^{\mu_n}C^n_{Li}(\xi^2 - i\epsilon\xi_0) \\ & O_{i,\mu_1...\mu_n} - (g_{\mu\lambda}g_{\nu\sigma}\Box - g_{\mu\lambda}\partial_\nu\partial_\sigma - g_{\nu\sigma}\partial_\mu\partial_\lambda + g_{\mu\nu}\partial_\lambda\partial_\sigma) \\ & \sum_{i,n}\frac{1}{2}[1+(-1)^n]\xi^{\mu_1}...\xi^{\mu_{n-2}}C^n_{2,i}(\xi^2 - i\epsilon\xi_0, g)O^{\lambda\sigma}_{i,\mu_1...\mu_{n-2}} + \cdots. \end{aligned} \quad (1.87)$$

We have shown only symmetric ($\mu \leftrightarrow \nu$) part here. The antisymmetric part will be dealt with separately. It is instructive to compare it with the free field derivation of this product wherein one finds explicitly the same singular behaviour as $\xi^2 \to 0$. The additional functions C^n_{Li} and C^n_{2i} take care of QCD corrections to this formal expansion. The form of the local operators are guided by the theory. Fourier transform of the above expression can be written as

$$\begin{aligned} i\int d^4\xi e^{iq.\xi}TJ_\mu(\xi)J_\nu(0) &= (g_{\mu\nu} - q_\mu q_\nu/q^2)\sum_{i,n}\frac{1}{2}[1+(-1)^n]\left(\frac{2}{Q^2}\right)^n \\ & q^{\mu_1}...q^{\mu_n}\tilde{C}^n_{Li}(q^2,g)O_{i,\mu_1...\mu_n} - (g_{\mu\lambda}g_{\nu\sigma}q^2 - g_{\mu\lambda}q_\nu q_\sigma - \\ & g_{\nu\sigma}q_\mu q_\lambda + g_{\mu\nu}q_\lambda q_\sigma)\sum_{i,n}\frac{1}{2}[1+(-1)^n] \\ & \left(\frac{2}{Q^2}\right)^n q^{\mu_1}...q^{\mu_{n-2}}\tilde{C}^n_{2,i}(q^2,g)O^{\lambda\sigma}_{i,\mu_1...\mu_{n-2}}. \end{aligned} \quad (1.88)$$

By carefully taking care of the dimension, invariance properties, one can construct the local operators with minimum twist (Twist of an operator is defined as : dim$\{O_n\}$- spin $\{O_n\}$.

Since the underlying theory is theory of quarks and gluons (QCD), the minimum twist, local, gauge invariant operators are

$$O_{\psi}^{i,\mu_1\ldots\mu_n} = i^{n-1}\{\ \bar{\psi}\ \gamma^{\mu_1}\ D^{\mu_2}\ldots D^{\mu_n}\psi\} \tag{1.89}$$

$$O^{i}_{A,\mu_1\ldots\mu_n} = 2\ i^{n-2}\{\ F_{\mu_1\lambda}D_{\mu_2}\ldots D_{\mu_{n-1}}F^{\lambda}_{\mu_n}\} \tag{1.90}$$

where $D_\mu = \partial_\mu + iA_\mu$. The factor i^{n-1} is attached to derive the Feynman rules for vertices of composite operators when one calculates QCD corrections to the coefficient functions. The symmetrization $\{\}$ is defined as

$$\{f_{\mu_1\ldots\mu_n}\} = \frac{1}{n!}\ (f_{\mu_1\ldots\mu_n} + \text{permutations}). \tag{1.91}$$

In principle $g_{\mu_i\mu_j}$'s will appear in the formal expansion of the matrix elements, that will make the composite operator traceless. These extra terms actually will not contribute because they produce terms like p^2, $s.q$ etc which give power suppressed contributions to $W_{\mu\nu}$. The allowed operator constructed out of fermion fields is $\bar{\psi}\Gamma_{\mu_1\ldots\mu_n}\psi$ where $\Gamma_{\mu_1\ldots\mu_n}$ can be made up of A_μ, γ_μ, ∂_μ etc. ∂_μ and A_μ will appear together to ensure gauge invariance. There will be only one γ_μ or $\gamma_\mu\gamma_5$ since more than one can always be reduced to one γ matrix. Similarly gluonic composite operators with appropriate quantum numbers can also be constructed out of A_μ's and the covariant derivatives D_μ. These operators are further classified as singlet and nonsinglet under the internal symmetry of the underlying theory. It is not possible to find out the matrix elements of these operators between proton states due to lack of knowledge about the proton state. One way of getting around this is to use Lorentz invariance properties of the operators. The spin averaged matrix element of these operators between single particle (proton) state of momentum p can be expanded in terms of vectors available in the theory

$$\frac{1}{2}\ \sum_s\ \langle p\,s\,|:O^{i,\mu_1\ldots\mu_n}:|\,p\,s\,\rangle = 2A_n(p^2)\ p^{\mu_1}\ldots p^{\mu_n} + \text{trace terms.} \tag{1.92}$$

The constant $A_n(p^2)$ reflects the inability to calculate the matrix element of these operators between proton states. This is the nonperturbative part of the OPE which carries long distance physics of proton. This nice separation of long and short distance physics in $W_{\mu\nu}$ expansion is a consequence of OPE. The measurements of these operators in turn give us a lot of many informations about the target.

Now, finally we write $\overline{T}^{(EM)}_{\mu\nu}$ as

$$\overline{T}^{(EM)}_{\mu\nu} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) t_L + \frac{1}{p.q}\left(p_\mu p_\nu - \frac{p.q}{q^2}(p_\mu q_\nu + p_\nu q_\mu) + \frac{(p.q)^2}{q^2} g_{\mu\nu}\right) 2t_2 \quad (1.93)$$

Using eqn.(1.5), we get

$$F_i = \frac{1}{\pi}\mathrm{Im}\ \mathrm{t_i} \quad (1.94)$$

To avoid confusion we shall consider each term separately. Comparing this with the tensor structures $W_{\mu\nu}$, t_L reads as

$$t_L = \sum_{n,i} \frac{1}{2}\left[1 + (-1)^n\right]\left(\frac{2}{Q^2}\right)^n q_{\mu_1}\cdots q_{\mu_n} C^n_{L,i}(Q^2, g)\ \frac{1}{2}\sum_s \langle p\,s\,|:O^{i,\mu_1\cdots\mu_n}:|\,p\,s\,\rangle. \quad (1.95)$$

Using the eqn.(1.92)

$$t_L = \sum_{n,i} \frac{1}{2}\left[1 + (-1)^n\right]\left(\frac{2p.q}{Q^2}\right)^n C^n_{L,i}(Q^2, g)\ 2A_n(p^2) \quad (1.96)$$

Note the symmetry of $t_L(w)$:

$$t_L(w) = t_L(-w) \quad (1.97)$$

where $w = 2p.q/Q^2$. Since $w \geq 1$ the above expansion is divergent. To get meaningful result out of the above expansion one explores the complex plane spanned by w. Note that $t_L(w)$ is analytic everywhere except along the branch cuts $1 \leq w \leq \infty$ and $-1 \geq w \geq -\infty$. Consider the integral $\frac{1}{2\pi i}\oint_c \frac{T(w)}{w^m} dw$ with c the contour (see ref. [17]). This integral can be explicitly done if it has definite symmetry. This evaluates to

$$\frac{1}{2\pi i}\oint_c dw\ \frac{T^{\pm}(w)}{w^m} = \left[1 \pm (-1)^m\right]\int_0^1 dx\ x^{m-2} ImT^{\pm}(x) \quad (1.98)$$

where $T^{\pm}(w) = \pm T^{\pm}(-w)$. Performing the contour integrals over the contour c shown in the figure ?? on both sides of the eqn.(1.96), we get for L.H.S

$$\frac{1}{2\pi i}\oint_c dw\ \frac{t_L^{\pm}(w)}{w^m} = [1+(-1)^{m-1}]\int_0^1 dx\ x^{m-2}F_L(x), \tag{1.99}$$

Using the formula

$$\frac{1}{2\pi i}\oint_c dw\ w^{m-n} = \delta_{m,n-1}, \tag{1.100}$$

R.H.S evaluates to $\sum_m\ [1+(-1)^{m-1}]\ C_{L,i}^{m-1}(Q^2,g)\ A_{m-1}(p^2)$. These equations imply

$$\int_0^1 dx\ x^{m-1}F_L(x) = \sum_i\ C_{L,i}^m(Q^2,g)\ A_m^i(p^2) \tag{1.101}$$

with $m = odd$. Let us now consider t_2. Note the symmetry of $t_2(w)$

$$t_2(w) = -t_2(-w). \tag{1.102}$$

Following a similar procedure one obtains

$$\int_0^1 dx\ x^{m-1}F_2(x) = \sum_i\ C_{2,i}^m(Q^2,g)\ A_m^i(p^2) \tag{1.103}$$

with $m = even$. The coefficient functions $C_{L,i}, C_{2,i}$ are so chosen that in the free field limit they become unity. Hence they take the form $1 + O(\alpha_s)$ where α_s is the strong coupling constant.

Antisymmetric part [3] of the current-current product can be formally written using the method applied for unpolarised case as

$$\begin{aligned} TJ_\mu(\xi)J_\nu(0) &= -i\epsilon_{\mu\nu\lambda\sigma}\ \partial^\lambda \sum_{n,i}\ \frac{1}{2}\ [1-(-1)^n]\ \frac{1}{\xi^2 - i\epsilon\xi_0}\ \times \\ &\quad E_i^n(\xi^2 - i\epsilon\xi_0, g)\ \xi_{\mu_1}\ldots x_{\mu_n}\ R_i^{\sigma\mu_1\ldots\mu_{n-1}}, \end{aligned} \tag{1.104}$$

where E_i are spin dependent Wilson coefficient functions calculable within the domain of perturbative QCD, and R_i are twist-two, local, gauge invariant operators whose matrix elements are measurable in the experiment. Fourier transform of the above expression reads as

$$\begin{aligned} i\int d^4\xi e^{iq\cdot\xi} TJ_\mu(\xi)J_\nu(0) &= -i\epsilon_{\mu\nu\lambda\sigma}q^\lambda \sum_{n,i}\ \frac{1}{2}\ [1-(-1)^n]\left(\frac{2}{Q^2}\right)^n \times \\ &\quad E_i^n(Q^2,g)\ q_{\mu_1}\ldots q_{\mu_{n-1}}\ R_i^{\sigma\mu_1\ldots\mu_{n-1}}. \end{aligned} \tag{1.105}$$

The matrix elements of these operators between single particle state of definite momentum and spin can be expanded as

$$\langle p\,s\,|:R^{\sigma\mu_1\cdots\mu_{n-1}}:|p\,s\rangle = -a_i^n\{s^\sigma p^{\mu_1}...p^{\mu_{n-1}}\}+... \tag{1.106}$$

where $a_i^n(p^2)$ are unknown functions of Lorentz invariants and s_μ is the spin vector of the state. Hence

$$R_\psi^{\sigma\mu_1\cdots\mu_{n-1}} = i^{n-1}\{\bar\psi\ \gamma_5\ \gamma^\rho D^{\mu_1}D^{\mu_2}...D^{\mu_{n-1}}\psi\} \tag{1.107}$$

$$R_F^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{1}{2}\ i^{n-1}\{\epsilon^{\rho\alpha\beta\gamma}F_{\beta\gamma}D^{\mu_1}...D^{\mu_{n-2}}F_\alpha^{\mu_{n-1}}\}, \tag{1.108}$$

where $\{\}$ denotes complete symmetrization. Observing

$$q_{\mu_1}\cdots q_{\mu_{n-1}}\langle\cdots\rangle_{\mu_1\cdots\mu_{n-1}} = -a_i^n\frac{1}{n!}\left((n-1)!s^\sigma(p.q)^{n-1}+(n-1)(n-1)!p^\sigma(s.q)(p.q)^{n-2}\right)$$

and comparing with the standard parametrization of $W_{\mu\nu}$ we find that

$$h_1(\omega) = \sum_{i,n}\ \frac{1}{2}\left[1-(-1)^n\right]\left(\frac{2p.q}{Q^2}\right)^n E_i^n(Q^2,g)\ a_i^n \tag{1.109}$$

and

$$h_2(\omega) = -\sum_{i,n}\ \frac{1}{2}\left[1-(-1)^n\right]\left(\frac{2p.q}{Q^2}\right)^n E_i^n(Q^2,g)\left(\frac{n-1}{n}\right)\ a_i^n \tag{1.110}$$

Note the symmetry properties of $h_1(x)$ and $h_2(x)$:

$$h_{1,2}(\omega) = -\ h_{1,2}(-\omega). \tag{1.111}$$

Here also we choose similar contour integrals to get meaningful result out of this diverging series. Carrying out the integration on L.H.S for $h_1(x)$ we get

$$\frac{1}{2\pi i}\oint\ dw\ \frac{h_1(w)}{w^{m+1}} = \left[1+(-1)^{m+1}\right]\int_0^1\ dx\ x^{m-1}g_1(x) \tag{1.112}$$

where

$$g_1 = \frac{1}{\pi}Im\ h_1. \tag{1.113}$$

Similarly the R.H.S becomes

$$\frac{1}{2}\sum_i \left[1-(-1)^m\right] E_m^i(Q^2,g)\; a_m^i(p^2), \tag{1.114}$$

Hence

$$\int_0^1 dx\; x^{m-1} g_1(x) \;=\; \frac{1}{2}\sum_i E_m^i(Q^2,g)\; a_m^i(p^2) \tag{1.115}$$

$$\int_0^1 dx\; x^{m-1} g_2(x) \;=\; -\frac{1}{2}\,\frac{m-1}{m}\sum_i E_m^i(Q^2,g)\; a_m^i(p^2) \tag{1.116}$$

where

$$g_2 = \frac{1}{\pi} Im\; h_2 \tag{1.117}$$

with $m = odd$. It is interesting to note that these structure functions satisfy Wilczek Wandzura sum rule.

1.7 EMC result and the spin content of the proton

The European Muon Collaboration EMC has measured the spin dependent structure function, $g_1(x)$, in polarised proton-muon deep inelastic scattering. Let k and k' be the the momenta of initial and final muons and p and s be momentum and spin vectors of the proton. The scattering process is described by two planes: (1) scattering plane (2) polarisation plane. Scattering plane is the plane containing the 3-vectors $\vec{k}$ and $\vec{k'}$. Polarisation plane is the plane containing the 3-vectors $\vec{k}$ and $\vec{s}$. Let ϕ be the angle between them. θ and β are the scattering angle and polarisation angle respectively.

The differential scattering cross section for this process is expressible as the sum of the unpolarised cross section (which does not depend on the polarisation angle) and polarised cross section (which depends on the polarisation angle). Since we are looking at DIS we use standard parametrisation of $W_{\mu\nu}$ expressed [58] in terms of the structure functions F_1,F_2,g_1 and g_2. So the cross section can be written as

$$\frac{d^3\sigma(\beta)}{dxdyd\phi} = \frac{d^3\sigma_0}{dxdyd\phi} - \frac{d^3\Delta\sigma(\beta)}{dxdyd\phi}\,. \tag{1.118}$$

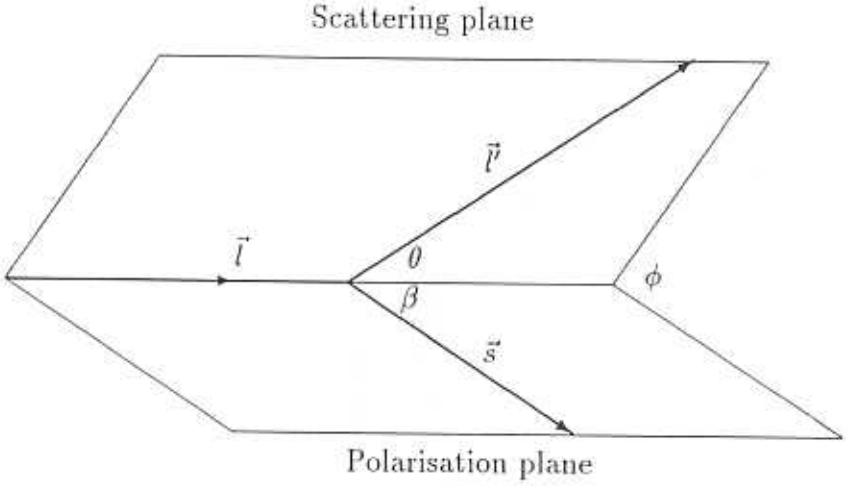

Figure 1.2 Lepton-Hadron DIS process: Production Plane.

The unpolarised and polarised cross sections are given respectively by

$$\frac{d^3\sigma_0}{dxdyd\phi} = \frac{2\alpha^2}{Q^2xy}\left(xy^2F_1(x) + \left(1 - y - \frac{Mxy}{2E}\right)F_2(x)\right) \tag{1.119}$$

$$\begin{aligned}\frac{d^3\Delta\sigma(\beta)}{dxdyd\phi} &= \frac{2\alpha^2}{MExy}\left\{\cos\beta\left[\left(1 - y/2 - \frac{Mxy}{2E}\right)g_1(x) - \frac{Mx}{2E}g_2(x)\right] + \right. \\ &\left. \sin\beta\cos\phi\frac{\sqrt{Q^2}}{\nu}\sqrt{1 - y - \frac{Mxy}{2E}}\left[\frac{y}{2}g_1(x) + g_2(x)\right]\right\}\end{aligned} \tag{1.120}$$

Here $x = -q^2/2p.q$, $y = \nu/E$ and $\nu = E - E'$. The coefficient of g_2, M/E goes to zero in the large energy limit. Furthermore, to disentangle $g_1(x)$ from $g_2(x)$, one generally evaluates the cross section for $\beta = 0$ and π. In both cases, $\sin\beta = 0$. So, by taking the difference in the cross sections in these two cases, it is possible to extract g_1 for various values of x and Q^2 (sufficiently large).

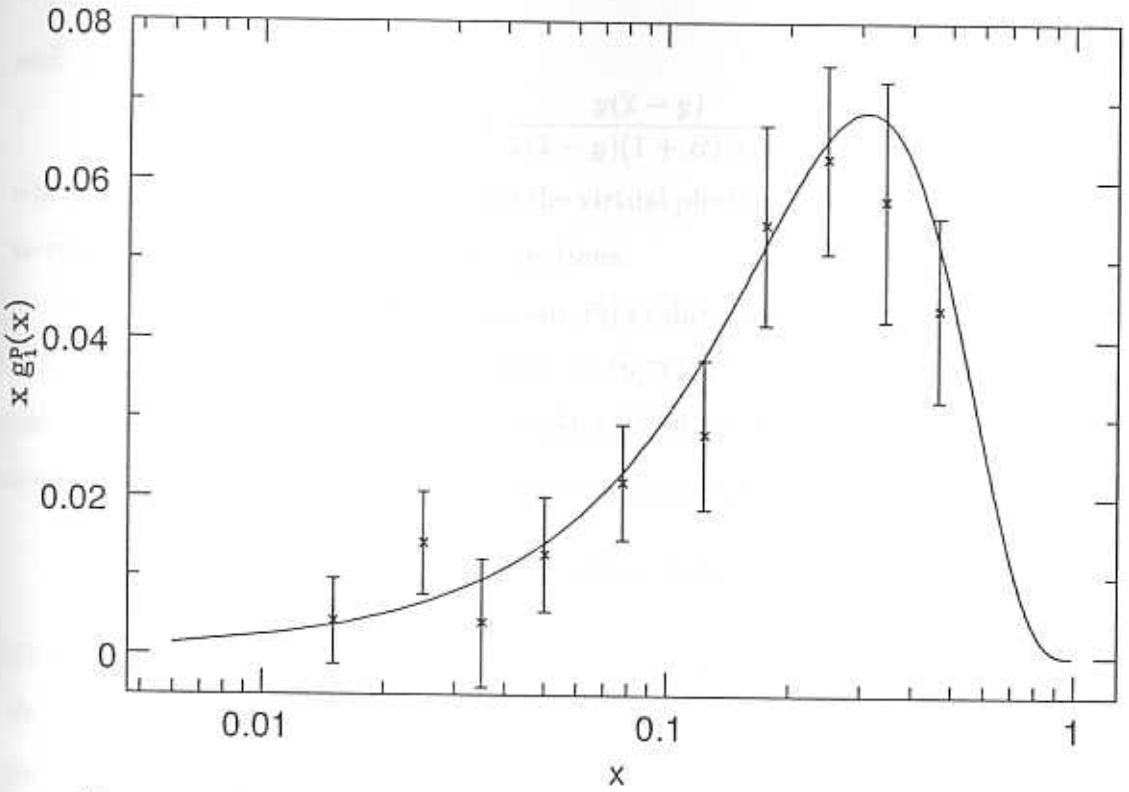

Figure 1.3 The EMC data.

EMC measured the following asymmetry:

$$A_{EMC} = f_t P_t P_b A,$$

where P_t and P_b are target and beam polarisations and f_t the target dilution factor, is the fraction of polarised nucleon in the target. A is given by

$$A = \frac{\sigma^{\downarrow\uparrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\downarrow\uparrow} + \sigma^{\uparrow\uparrow}} \tag{1.121}$$

where $\sigma^{\downarrow\uparrow}(\sigma^{\uparrow\uparrow})$ is the cross section for proton polarised opposite to (along) the beam direction.

In terms of the structure functions defined earlier, the asymmetry A turns out to be at large energies

$$A = D\, A_1$$

where

$$A_1(x) = \frac{g_1(x)}{f_1(x)}$$

and

$$D = \frac{y(2-y)}{2(1-y)(1+R)+y^2}$$

where D is the depolarisation factor of the virtual photon. R is the ratio of the longitudinal to transverse photo-absorption cross sections.

Using the above relations and known $F_2(x)$ data, EMC extracted $g_1(x)$ in the range $0.01 \leq x \leq 0.7$ at average Q^2 i.e $\langle Q^2 \rangle = 10.7\text{GeV}^2$. This set of data is found to be consistent with that of earlier data from SLAC [38] in the region of overlap. Extrapolating to zero and one in the x space, the first moment of $g_1(x)$ is found to be

$$\int_0^1 dx\ g_1(x) = 0.126 \pm 0.010(stat.) \pm 0.015(syst.). \tag{1.122}$$

This value is much smaller than the earlier observation at SLAC and Ellis-Jaffe[48] prediction which is based on OPE and SU(3) symmetry. This discrepancy with SLAC data most probably shows the unreliability in the extrapolation to low x done by the SLAC group. The difference in the Ellis-Jaffe prediction and the observed EMC result can be attributed to various assumptions made in arriving at that prediction. The prime assumptions are (1) assuming free field theory (2) neglecting strange quark contribution (3) assuming that SU(3) is perfect symmetry etc. Apart from the these discrepancies, there are other surprising results which come out of the EMC result.

Let us first study this result in the parton model context. If one assumes the proton is just made up of free quarks and antiquarks then, from eqn.(1.13)

$$g_1(x) = \frac{1}{2} \sum_i e_i^2\ (\Delta q_i(x) + \Delta \bar{q}_i(x)) \tag{1.123}$$

where $i = u, d, s$ and e_i's are their respective charges in units of the electron charge. $\Delta q_i(x) = q^+(x) - q^-(x)$ where $q^{+(-)}(x)$ is the probability of finding a quark of flavour i and momentum faction x with polarisation $+(-)$ inside a proton with polarisation $+(-)$ with respect to the beam direction. So $\Delta q(x)$ is the polarised quark probability density inside the polarised proton. If one assumes SU(3) flavour symmetry i.e., that u,d and s form a triplet in the fundamental representation of SU(3), then the first moment of $g_1(x)$

can be written as

$$\int dx\, g_1(x) = \frac{1}{12}(\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}) + \frac{1}{36}(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2\Delta s - 2\Delta\bar{s}) + \frac{1}{9}(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}) \quad (1.124)$$

where $\Delta q_i = \int_0^1 dx \Delta q_i(x)$ can be defined as the spin contribution to the proton of the ith flavour quark. We have taken first moment because it is proportional to the spin contributions coming from various quarks and antiquarks. To extract the quark and antiquark contribution, $(\Delta u+\Delta\bar{u}+\Delta d+\Delta\bar{d}+\Delta s+\Delta\bar{s})$, to the proton spin, we need to know the non-singlet combination $(\Delta u+\Delta\bar{u}-\Delta d-\Delta\bar{d})$ and $(\Delta u+\Delta\bar{u}+\Delta d+\Delta\bar{d}-2\Delta s-2\Delta\bar{s})$. These combinations are proportional to SU(3) reduced matrix elements for axial octet current between baryon octet states, i.e.,

$$(\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}) = F + D, \quad (1.125)$$

$$(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2\Delta s - 2\Delta\bar{s}) = 3F - D. \quad (1.126)$$

Hence they can be extracted in nucleon and hyperon beta decay experiments respectively. Their experimentally determined values along with EMC result yield the following result for singlet part

$$\frac{1}{2}(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}) = 0.06 \pm 0.047(stat.) \pm 0.069(syst.). \quad (1.127)$$

This is to be compared with the quark model which predicts that most of the proton spin comes from the quarks and antiquarks. This is a surprising result because at very high energies the naive expectation that quark model prediction holds, fails.

We can analyse the $g_1(x)$ structure function in the context of OPE and also justify the use of low energy data to extract singlet part. In OPE formalism the first moment of $g_1(x)$ is found to be

$$\int dx\, g_1(x) = \frac{1}{2} \sum_i e_i^2 \frac{s^\mu}{s^2} \langle p\, s |\ \bar{\psi}_i\, \gamma_\mu\, \gamma_5 \psi_i\ | p\, s \rangle. \quad (1.128)$$

Again if one uses SU(3) symmetry, then ψ's form triplet in the fundamental representation of SU(3). Then the first moment of it becomes

$$= \frac{s^\mu}{s^2}\left[\frac{1}{12}\langle p\,s\,|\bar{\psi}_i\gamma_\mu\gamma_5\lambda_3\psi_i|\,p\,s\,\rangle + \frac{1}{36}\langle p\,s\,|\bar{\psi}_i\gamma_\mu\gamma_5\lambda_8\psi_i|\,p\,s\,\rangle + \frac{1}{9}\langle p\,s\,|\bar{\psi}_i\gamma_\mu\gamma_5\lambda_0\psi_i|\,p\,s\,\rangle\right] \quad (1.129)$$

where $\lambda_{3,8}$ and λ_0 are $SU(3)$ generators and identity respectively. One can identify these operators with moments of quark distribution functions defined in the parton model. This need not be true if one introduces interaction between quarks. Since these operators are conserved we can use the data on them measured in low energy experiments as they are scale independent. In the interacting theory only non-singlet operators are conserved and hence scale independent. So the use of the low energy data for the non-singlet parts is justified. Even this analysis predicts

$$\frac{1}{2}\sum_i \frac{s^\mu}{s^2}\,\langle p\,s\,|\;\bar{\psi}_i\;\gamma_\mu\gamma_5\;\lambda_0\;\psi_i\;|\,p\,s\,\rangle = \;0.06 \pm 0.047(stat.) \pm 0.069(syst.). \quad (1.130)$$

So the quark and antiquark contributions to the proton spin is negligibly small.

The question immediately arises: where is the spin of the proton?. From the angular momentum sum rule i.e.,

$$\frac{1}{2}\sum_i(\Delta q_i + \Delta\bar{q}_i) + \Delta g + \Delta L_z = \frac{1}{2}, \quad (1.131)$$

which states that the spin of quarks(antiquarks), gluons and their orbital angular momenta make up the proton spin. It is clear that $\Delta g(x)$ $(= \int dx(g^1(x) - g^1(x)))$, gluonic contribution to proton spin and ΔL_z, orbital angular momentum contribution should be responsible for the proton spin. Now the question is whether gluons can contribute at large q^2 because this contribution, which starts at order α_s, is damped at large q^2 as $(\log Q^2)^{-1}$. This naive expectation that it will not contribute fails due to the Altarelli-Parisi evolution equation [64] i.e

$$\frac{d}{d\log Q^2/\mu^2}\left(\alpha_s(Q^2)\Delta g(Q^2)\right) = 0 \quad (1.132)$$

to order α_s. This equation along with the fact that α_s damps at large energies tells us that $\Delta g(Q^2)$ grows as $\log Q^2/\mu^2$ at large Q^2. So gluonic contribution to the proton spin may

be substantial. Also because of the eqn.(1.132) gluonic contribution to the first moment of g_1, if any, will be of order α_s^0 and hence will be as important as the quark contribution. So there is no reason to neglect the gluonic contribution to the first moment of $g_1(x)$.

In the QCD parton model, gluon can contribute to $g_1(x)$ through $\gamma^* g \rightarrow q\bar{q}$ process. The evaluation of this cross section involves regularisation and renormalisation of mass divergences. These mass divergences are of two types. One is infrared which occurs when the initial gluons are massless and the other is collinear which occurs when the outgoing quarks are massless. These renormalisations leave out ambiguities in the expansion of g_1 in terms of quark and gluon distribution functions. For example [6, 7, 9] in the massive gluon scheme gluons contribute to first moment of $g_1(x)$ but in the massive quark scheme they do not. If one uses the former scheme or equivalently transverse momentum cut off scheme, the first moment of singlet contribution turns out to be proportional to $\sum_i (\Delta q_i + \Delta\bar{q}_i - (\alpha_s/2\pi)\Delta g)$. This result alone can not tell any thing about spin content of the proton as there is one more unknown i.e Δg.

In the OPE formalism [4], gluons do not contribute to the first moment of $g_1(x)$ as there is no twist-two, gauge invariant, local gluon operator in the theory. For higher moments there exist such operators and their coefficients are insensitive to mass divergences.

Despite this, there have been several attempts to include gluonic operator in the first moment of $g_1(x)$ via axial anomaly (anomalous Ward Takahashi identity) (see articles [29, 25, 26, 27, 28]). It is well known that the gluonic axial current k_μ (some times called Chern Simons current) is gauge dependent which forbids its appearance in the OPE. Noting that the forward matrix element of this current is gauge invariant, Lam and Li [5] first included this operator in the first moment of $g_1(x)$. They showed the connection between matrix element of this current and first moment of gluonic distribution function, and emphasised that it would contribute substantially even at large energies to the first moment of $g_1(x)$ using Altarelli-Parisi [64] evaluation equation. This has also been dealt with by Efremov and Teryaev [8]. The gluonic contribution to the spin dependent structure function was first evaluated in the parton model by Gupta et al [6]. Using massive gluon scheme, they have showed that the hard gluonic coefficient is $-\alpha_s/4\pi$ for

each quark(antiquark). Altarelli and Ross [7] and Carlitz, Collins and Mueller [9] have also calculated the gluonic contribution to $g_1(x)$ using different schemes. Both approaches give the same result which is $-\alpha_s/4\pi$ for each quark. Bodwin and Qiu [10] have applied the subtraction procedure usually applied to factorise mass singularities in unpolarised DIS cross section to the polarised DIS to extract gluonic contribution to $g_1(x)$. This involves the subtraction from $\gamma^* g \rightarrow q\bar{q}$ cross section of a part which can be interpreted in terms of the distribution of quarks inside a gluon. This procedure yields the first moment of the hard gluonic contribution to be zero provided that the UV regulators for the spin dependent quark distribution functions respects both gauge invariance and the analyticity structure of the unregulated distributions. Manohar [18, 11] has attempted to generalise the formal factorisation theorem for polarised DIS and has found that gluons do not contribute the first moment of $g_1(x)$. His calculations verify the proposed factorisation theorem to order α_s in the gluonic sector. The singlet matrix element which contributes to the first moment of $g_1(x)$ has also been evaluated using a completely different approach namely QCD sum rule method by Gupta, Murthy and Pasupathy [39].

Chapter 2

Weak spin dependent structure functions

2.1 Introduction

We have seen that the hadronic tensor can be parametrized in terms of unknown scaling functions called structure functions. These structure functions being measurable would tell us about the structure of the hadrons. These are in no way calculable within the domain of perturbative QCD. These structure functions can, in principle be expressed in terms of either hadronic matrix elements of some well defined gauge invariant operators or some probability distribution functions of partons which constitute the proton. The former is the well-known OPE and the latter is the parton model approach. We know from the knowledge of spin independent structure functions [21], measured in electro magnetic (EM) as well as weak (W) DIS, that suitable combinations of $F_2^\alpha(x,Q^2)$ and $F_3^\alpha(x,Q^2)$, where α denotes the process in which these are measured, can disentangle momentum contributions of various constituents to proton's total momentum. In other words $F_2^W(x,Q^2)$ and $F_3^W(x,Q^2)$ are needed in addition to $F_2^{EM}(x,Q^2)$ to get sea and valence contributions to proton since the charge factors of dominant contributions in $F_2^W(x,Q^2)$ and $F_3^W(x,Q^2)$ are approximately equal.

We have seen that in the spin dependent sector, the measurement of $g_1^{EM}(x,Q^2)$ is not sufficient to know the individual spin contributions of quarks and antiquarks to the proton spin. To know this, we had to use low energy data such as values of g_A and g_8. This is again due to the EM charge factors. So the logical step to avoid this problem is to measure the

spin dependent structure function in neutrino-polarised proton DIS experiments [12, 19]. If one uses weak particles namely W and Z bosons as probes, instead of photon three more spin dependent structure functions will be operative in addition to the usual $g_1^{WP}(x,Q^2)$ and $g_2^{WP}(x,Q^2)$ structure functions. They are $g_3^{WP}(x,Q^2)$, $g_4^{WP}(x,Q^2)$ and $g_5^{WP}(x,Q^2)$. These are parity violating structure functions unlike $g_1^{WP}(x,Q^2)$ [19]. If one is interested in only longitudinally polarised proton target, $g_5^{WP}(x,Q^2)$ will not appear in the hadronic tensor.

In this chapter, we analyse these parity violating and parity conserving structure functions in the QCD parton model [15, 12] as well as in the factorisation method [13]. We give a physical interpretation of these structure functions in terms of polarised quark, antiquark and gluon distribution functions. We calculate quark contribution to these to leading order and gluonic contribution to order α_s. The contribution to order α_s is needed due to the scale independence of the combination $\alpha_s(Q^2)\Delta g(Q^2)$, where Δg is the first moment of the polarised gluon distribution function inside the polarised proton. The calculation of the gluonic contribution to these structure functions involves the process $W^{\pm}g \to q_i\bar{q}_j$ where i,j are flavour indices restricted by the weak interaction. This calculation involves regularisation of mass singularities i.e soft divergences. To regularize these singularities we keep both quarks and gluons massive. Also we distinguish the masses of quarks for completeness.

In the parton model we use the massive gluon scheme to renormalise the mass singularities. Ultimately we find that the first moment of $g_1^{WP}(x,Q^2)$ is just proportional to $\Delta q_i + \Delta\bar{q}_i - \frac{\alpha_s}{2\pi}\Delta g$ while the other structure functions $g_3^{WP}(x,Q^2)$ and $g_4^{WP}(x,Q^2)$ do not receive any gluonic contribution from the above mentioned process.

We need not use any soft renormalisation scheme in the factorisation method. In this method we show that the coefficient functions are free of mass singularities. Also the first moment of parity conserving structure function, $g_1^{WP}(x,Q^2)$ does not receive any gluonic contribution unlike in the parton model.

In both the methods, we find Callan-Gross type of sum rule between the parity violating structure functions $g_3^{WP}(x,Q^2)$ adn $g_4^{WP}(x,Q^2)$. We will also show that due to

the weak charge factors, various linear combinations of $g_{1,3,4}^{WP}(x,Q^2)$ give more information about spin content of the proton.

Finally we express the structure functions in terms of matrix elements of axial vector operators derived in the free field limit. We will show that for first moments of these structure functions, to leading order, these quark operators are sufficient even if one introduces interactions because there exists no appropriate gluonic operator in the theory.

2.2 Basic formalism

We consider neutrino-polarized proton Deep inelastic scattering process $\nu,\ \bar{\nu} + P^{\pm}(p) \rightarrow l,\bar{l} + X$, where $+(-)$ denotes the polarization of proton along (opposite to) the beam axis and k,k', p and p_X are momenta of incoming lepton, outgoing lepton, proton and unobserved final hadronic state X. Let us consider the spin dependent part of $W_{\mu\nu}(x,Q^2)$ which can, in principle, be measured in the weak process.

$$\begin{aligned}\widetilde{W}_{\mu\nu}^{W}(x,Q^2) &= \frac{i}{p.q}\epsilon_{\mu\nu\lambda\sigma}q^{\lambda}\left[s^{\sigma}g_1^{WP}(x,Q^2)+\left(s^{\sigma}-\frac{s.q}{p.q}p^{\sigma}\right)g_2^{WP}(x,Q^2)\right] \\ &+\frac{q.s}{p.q}G_{\mu\nu}^{3}g_3^{WP}(x,Q^2)+\frac{1}{p.q}\left[G_{\mu\nu}^{4}g_4^{WP}(x,Q^2)+G_{\mu\nu}^{5}g_5^{WP}(x,Q^2)\right], \quad (2.1)\end{aligned}$$

where $J_W^{\mu}(\xi)$ is the weak hadronic current, s^{μ} the spin vector of the proton, $Q^2(\equiv -q^2)$ the square of momentum transfer and $x \equiv -q^2/2p.q$ is the Bjorken variable. The various second rank tensors in eqn.(2.1) are defined as

$$G^{3}{}_{\mu\nu} = -g_{\mu\nu}+\frac{q_{\mu}q_{\nu}}{q^2}, \quad G^{4,5}{}_{\mu\nu} = \frac{1}{2}\left[R_{\mu}R_{\nu}\frac{q.s}{p.q}\pm\frac{1}{2}\left(R_{\mu}T_{\nu}+R_{\nu}T_{\mu}\right)\right],$$

$$R_{\mu} = p_{\mu}-\frac{p.q}{q^2}q_{\mu}, \quad T_{\mu} = s_{\mu}-\frac{q.s}{q^2}q_{\mu}.$$

It is understood that the model interpretation of g_1^{EM} in EM process is quite different from that measured in the weak process as we will see in this chapter.

To understand the physical interpretation of these structure functions, one makes use of the parton model, OPE or the factorisation theorem. In the parton model, one generally expresses the total cross section in terms of the parton subprocess cross sections

convoluted with appropriate probability distribution functions, that are nonperturbative quantities in the theory, obtained from experiments. In fact, the structure functions are expected in terms of these distribution functions. But in the OPE, the moments of these structure functions are related to hadronic matrix elements of local operators with appropriate quantum numbers, that are again not calculable in the theory for hadron targets. The latter approach lacks the transparency inherent in the physical interpretation of the structure functions via the parton model approach. The factorisation method being the field theoretical version of the parton model is more physical in the sense that it has an operator definition for the probability distribution functions.

2.3 QCD Parton model

In the parton model, lepton-hadron cross section is written in terms of lepton-parton cross sections convoluted with appropriate incoming parton probability density functions. Consider a gauge boson W of polarization λ scattering off a proton with polarization H. The cross section for the process in this parton model can be written as

$$\sigma^{W(\lambda)p(H)}(x) = \sum_{a,h} \int_x^1 \frac{dy}{y}\, \hat{\sigma}^{W(\lambda)a(h)}(x/y)\, f_{\frac{a(h)}{p(H)}}(y)\,, \tag{2.2}$$

where $\hat{\sigma}$ ($\hat{\omega}$)is the gauge boson-parton cross section, y is the fraction of proton momentum carried by parton a, $f_{\frac{a(h)}{p(H)}}(y)$ is the probability of finding a parton of type a with momentum fraction y and polarisation h inside the proton of polarization H.

In general, spin dependent structure functions can be measured by taking the following difference in cross section:

$$\frac{1}{2}\left[\sigma^{W(\lambda)p(+)}(x) - \sigma^{W(\lambda)p(-)}(x)\right]. \tag{2.3}$$

Since $W_{\mu\nu}$ is proportional to the cross section, we can apply convolution. Hence the hadronic tensor in the parton model is found to be

$$\widetilde{W}_{\mu\nu}(x) = \frac{1}{2}\left[W^{+}_{\mu\nu}(x) - W^{-}_{\mu\nu}(x)\right] \tag{2.4}$$

$$= \sum_a \int_x^1 \frac{dy}{y}\, \tilde{\omega}^a_{\mu\nu}(x/y)\, f_{\frac{\Delta a}{p(H)}}(y) \tag{2.5}$$

where $\tilde{\omega}^a_{\mu\nu} = \frac{1}{2}[\omega^{a(+)}_{\mu\nu} - \omega^{a(-)}_{\mu\nu}]$ is the polarised cross section and $f_{\frac{\Delta a}{p(H)}}(y) = f_{\frac{a(+)}{p(H)}}(y) - f_{\frac{a(-)}{p(H)}}(y) = \Delta a(y)$ is the polarized parton probability density.

2.3.1 Hard scattering cross sections

$\widetilde{W}_{\mu\nu}(x, Q^2)$ gets contribution from both quarks as well as gluons. So for notational convenience we write

$$\widetilde{W}^W_{\mu\nu}(x, Q^2) = \widetilde{W}^{Wq+\bar{q}}_{\mu\nu}(x, Q^2) + \widetilde{W}^{Wg}_{\mu\nu}(x, Q^2). \tag{2.6}$$

These cross sections are calculable within the frame work of perturbative quantum chromodynamics (QCD). $\widetilde{W}^{q,g}_{\mu\nu}(x, Q^2)$ involves the evaluation of $W^{\pm}q_i(\bar{q}_i) \to q_j(\bar{q}_j)$ cross sections where the flavours (i, j) are restricted by the electroweak theory. $\widetilde{W}^g_{\mu\nu}(x, Q^2)$ gets contribution from the process $W^{\pm}g \to q_i\bar{q}_j$. Contribution from these processes to the structure functions can be projected out using appropriate projection operators. The parton distribution functions are not calculable in the theory. These are just nonperturbative inputs in the model and can be extracted from experiment. These are the ones which contain full information about the structure of the proton, such as momentum distribution, spin structure etc.

Let us first interpret the structure function $g_1^{WP}(x, Q^2)$ (the superscript is to indicate that it is extracted from weak DIS, in the following we drop this. The superscript is assumed unless otherwise stated) . This can be extracted from the hadronic tensor using the projector:

$$\mathcal{P}^{(1)}_{\mu\nu} = -\frac{i}{2\alpha^2 p.q}\epsilon_{\mu\nu\alpha\beta}q^{\alpha}p^{\beta} \tag{2.7}$$

where $\alpha^2 = 1 - 4x^2p^2/q^2$. Using eqn.(2.6),

$$g_1^{WP}(x, Q^2) = g_1^{(q+\bar{q})}(x, Q^2) + g_1^g(x, Q^2) \tag{2.8}$$

Let us first calculate quark (and antiquark) contribution to $g_1(x, Q^2)$.

$$g_1^{q+\bar{q}}(x, Q^2) = \sum_i \int_x^1 \frac{dy}{y}\, \tilde{\omega}^{Wq_i}(x/y, Q^2)\, f_{\frac{\Delta q_i}{p(H)}}(y, Q^2) + \sum_i \int_x^1 \frac{dy}{y}\, \tilde{\omega}^{W\bar{q}_i}(x/y)\, f_{\frac{\Delta \bar{q}_i}{p(H)}}(y) \tag{2.9}$$

$\tilde{\omega}^{Wq}$ and $\tilde{\omega}^{W\bar{q}}$can be calculated from the Born processes $W^{\pm}(q)q_i(\bar{q}_i(k)) \rightarrow q_j(\bar{q}_j)(k')$ and are found to be at large q^2

$$\tilde{\omega}_1^{Wq,\bar{q}}(z) = \frac{g^2}{8}|U|^2\,\delta(1-z) \tag{2.10}$$

where $z = -q^2/2k.q$ is the Bjorken variable at the parton level, U is the Cabibbo-Kobayashi-Maskawa matrix and g is the weak coupling constant.

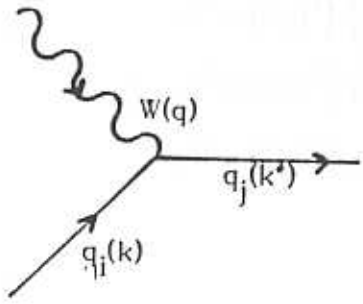

Figure 2.1 Quark Born diagram.

The evaluation of $\tilde{\omega}^{W^{\pm}g}$ involves the process $W^{\pm}g \rightarrow q_i\bar{q}_j$ (See Appendix A for details). This cross section is found to be mass singular. There are two types of singularities: 1. Infrared which arises due to massless nature of the incoming gluon and 2. Collinear which arises when one of the outgoing particle is collinear to the incoming gluon. These singularities are handled by keeping the masses non zero. This is nothing but regularization of soft divergences. In addition we keep flavour masses different. The final expression for this process is too lengthy due to non-zero masses. Since we are interested in large q^2 limit we neglect terms of order $\mathcal{O}(m^2/q^2, k^2/q^2)$. Then we get

$$\begin{aligned}\tilde{\omega}^{Wg}(z,Q^2) \;=\; & \sum_{ij}{}'|U_{ij}|^2\frac{g^2}{8}\frac{\alpha_s}{4\pi}\left[\left((2z-1)ln\frac{Q^2(1-z)}{z(m_i^2-k^2z(1-z))}+3-4z+\right.\right.\\ & \left.\left.\frac{k^2z(1-z)}{m_i^2-k^2z(1-z)}\right)+(m_i\rightarrow m_j)\right],\end{aligned} \tag{2.11}$$

The prime in the summation denotes that flavours are restricted by weak interaction. Note that the above expression diverges as $k^2 \to 0$ and $m^2 \to 0$. Now choose the massive gluon scheme to renormalize the cross sections. This involves keeping quarks massless and gluons massive and absorbing the singular piece into the distribution function. Combining both quark and gluonic contributions and taking first moment we find

$$\begin{aligned}\int_0^1 dx\, g_1^{WP}(x,Q^2) &= \int_0^1 (g_1^{W^+P}(x,Q^2) + g_1^{W^-P}(x,Q^2))dx \\ &= \frac{g^2}{8}\Big[\left(|U_{ud}|^2 + |U_{us}|^2\right)\left(\Delta u + \Delta\bar{u} - \frac{\alpha_s}{2\pi}\Delta g\right) + \\ &\quad \left(|U_{cd}|^2 + |U_{cs}|^2\right)\left(\Delta c + \Delta\bar{c} - \frac{\alpha_s}{2\pi}\Delta g\right) + \\ &\quad \left(|U_{ud}|^2 + |U_{cd}|^2\right)\left(\Delta d + \Delta\bar{d} - \frac{\alpha_s}{2\pi}\Delta g\right) + \\ &\quad \left(|U_{us}|^2 + |U_{cs}|^2\right)\left(\Delta s + \Delta\bar{s} - \frac{\alpha_s}{2\pi}\Delta g\right)\Big], \qquad (2.12)\end{aligned}$$

where $\Delta a = \int dx\, \Delta a(x)$. We can obtain the above relation using transverse momentum cutoff also. Notice that the result is such that the combination $\Delta q + \Delta\bar{q} - \alpha_s \Delta g/2\pi$ appears for each flavour which is reminiscent of the earlier result in polarized electroproduction.

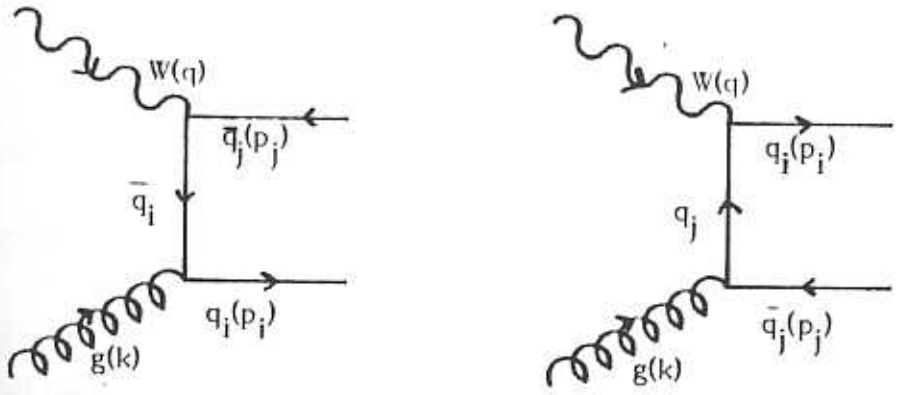

Figure 2.2 Gluon, $W^{\pm}$ fusion diagrams contributing to the process $W^{\pm}P \to W^{\pm}X$.

Since the order of magnitude of CKM matrix elements is

$$U_{ud} \approx U_{cs} \gg U_{cd} \approx U_{us}, \qquad (2.13)$$

the coefficients of $\Delta q_i + \Delta\bar{q}_i - \alpha_s \Delta g/2\pi$ are almost equal for each flavour and approximately equal i.e 1.190 [22]. Hence, the first moment of $g_1^{WP}(x, Q^2)$ becomes,

$$\int dx\, g_1^{WP}(x, Q^2) = 1.190\, \frac{g^2}{8} \sum_i \left(\Delta q_i + \bar{q}_i - \frac{\alpha_s}{2\pi} \Delta g \right) . \tag{2.14}$$

So the measurement of $g_1^{WP}(x, Q^2)$ will directly yield the singlet part of proton spin without resorting to any low energy data unlike in the recent EMC measurement. The low energy quantity g_A, can be measured in the high energy process by measuring the first moment of the combination $g_1^{W^+P}(x, Q^2)$ and $g_1^{W^-P}(x, Q^2)$.

As we mentioned earlier, two more structure functions $g_3^{WP}(x, Q^2)$ and $g_4^{WP}(x, Q^2)$, contribute to longitudinally polarized weak DIS. To extract them in the parton model we use the following projectors:

$$\mathcal{P}_{\mu\nu}^{(3)} = -\frac{1}{2\alpha^2} \left[\alpha^2 g_{\mu\nu} + \frac{4x^2 p_\mu p_\nu}{q^2} \right], \tag{2.15}$$

$$\mathcal{P}_{\mu\nu}^{(4)} = -\frac{x}{\alpha^4} \left[\alpha^2 g_{\mu\nu} + \frac{12x^2 p_\mu p_\nu}{q^2} \right]. \tag{2.16}$$

Since these get contributions both from quarks and gluons one can write as,

$$g_{3,4}^{WP}(x, Q^2) = g_{3,4}^{q+\bar{q}}(x, Q^2) + g_{3,4}^{g}(x, Q^2). \tag{2.17}$$

Following the previous method,

$$g_\alpha^{q+\bar{q}}(x) = \sum_i \int_x^1 \frac{dy}{y}\, \tilde{\omega}_\alpha^{Wq_i}(x/y)\, f_{\frac{\Delta q_i}{p(H)}}(y) + \sum_i \int_x^1 \frac{dy}{y}\, \tilde{\omega}_\alpha^{W\bar{q}_i}(x/y)\, f_{\frac{\Delta \bar{q}_i}{p(H)}}(y) \tag{2.18}$$

where $\alpha = 3, 4$ stand for the structure functions g_3 and g_4 respectively. $\tilde{\omega}_\alpha^{Wq(\bar{q})}$ can be calculated from the Born process $W^\pm(\mathrm{q}) q_i(\bar{q}_i)(k) \to q_j(\bar{q}_j)(k')$ and is found to be at large q^2

$$\tilde{\omega}_\alpha^{Wq(\bar{q})}(z) = \pm f_\alpha \frac{g^2}{8} |U|^2 \delta(1 - z) \tag{2.19}$$

with $f_3 = -1$ and $f_4 = 2z f_3$. This then leads to a Callan-Gross type relation between structure functions $g_3(x, Q^2)$ and $g_4(x, Q^2)$. Note the sign difference between particles and antiparticles. The origin of this sign difference lies in the fact that $g_3^{WP}(x, Q^2)$ and

$g_4^{WP}(x,Q^2)$ corresponds to parity violating structure functions. This would help to extract valence part of the proton spin as we will see later.

Gluonic contribution to $g_{3,4}^g(z,Q^2)$ involves the process $W^{\pm}g \to q_i\bar{q}_j$. To avoid mass singularities we keep all the particles massive. Neglecting terms of order $\mathcal{O}(k^2/q^2, m^2/q^2)$ we get

$$\begin{aligned}\tilde{\omega}_3^{Wg}(z,Q^2) &= \sum_{ij}{}'|U_{ij}|^2\frac{g^2}{8}\frac{\alpha_s}{4\pi}\left[\left((2z-1)ln\frac{Q^2}{m_i^2-k^2z(1-z)}+\right.\right.\\ &\left.\left.\frac{m_i^2}{m_i^2-k^2z(1-z)}\right)-(m_i\to m_j)\right], \qquad (2.20)\\ \tilde{\omega}_4^{Wg}(z,Q^2) &= 2z\tilde{\omega}_3^{Wg}(z,Q^2), \qquad (2.21)\end{aligned}$$

The above expression is a regularized one. Note that when masses of the outgoing particles are equal, the gluonic contribution to $g_{3,4}(x,Q^2)$ is zero. Note also that $g_3^g(z,Q^2)$ and $g_4^g(z,Q^2)$ are related by the factor $2z$. In the scheme used earlier these, terms vanish leaving $g_3^{WP}(x,Q^2)$ and $g_4^{WP}(x,Q^2)$ free of gluonic contribution from Wg fusion diagram. Due to the sign difference between quarks and antiquarks, valence quark contribution to the proton spin can be measured by measuring the following combinations of structure functions i.e.,

$$\int dx\,(g_3^{W+P}(x,Q^2)+g_3^{W-P}(x,Q^2)) = -\frac{1}{8}g^2\,(\Delta u_V+\Delta d_V)\,. \qquad (2.22)$$

This along with the first moment of $g_1^{W+P}-g_1^{W-P}$ would yield Δu_V and Δd_V to proton spin.

2.3.2 Conclusion

We have calculated quark and gluonic contributions to the parity conserving weak structure function ($g_1^{WP}(x,Q^2)$) and parity violating structure functions ($g_3^{WP}(x,Q^2)$ and $g_4^{WP}(x,Q^2)$) in the parton model. We have performed this calculation by keeping all the masses non-zero to avoid mass divergences. We have found that the first moment of $g_1^{WP}(x,Q^2)$ is directly proportional to $\Delta q_i+\Delta\bar{q}_i-\alpha_s\Delta g/2\pi$. So the measurement of $g_1^{WP}(x,Q^2)$ in neutrino-proton polarised scattering would directly yield singlet contribution to proton spin. Here the advantage is that one need not use any low energy

data which one is forced to use in the EMC to extract this combination. The parity violating structure functions do not receive any gluonic contribution. Certain combinations of them can measure valence contribution to the proton spin. So measurement of these structure functions $g_{1,3,4}^{WP}(x, Q^2)$ in neutrino proton polarised DIS would disentangle various contributions coming from constituents to the proton spin.

2.4 Factorisation method

In the analysis of structure functions $g_{1,3,4}^{WP}(x, Q^2)$ using parton model one introduces unknown polarised distribution functions of various constituents. These functions are theoretical inputs in the theory. These are not in principle calculable within perturbative QCD. This is a reflection of the fact that structure functions are not completely calculable within the theory. In the parton model, these functions in some sense form the basis for expressing the structure functions. Their coefficients are determined by the parton cross sections. In a crude sense these distribution functions are verbal definitions and cannot be derived in a rigorous, mathematical manner. But one can write definitions for these distributions in terms of the wave functions of partons in the hadron because these distributions are nothing but probabilities. Any expectation value of the observables such as momentum fraction of a parton or spin contribution of the parton to proton spin can be found as the expectation value of them in the state (wave function) defined. Now the question is whether these wave functions are calculable. As we know, these are not calculable due to the lack of knowledge of the structure of the hadron at various scales.

In summary, in the parton model the total cross section is expressed in terms of parton level cross sections which are calculable, with appropriate probability distribution functions of participating constituent particles which are not calculable. When one goes to higher order contributions, one is bound to get mass singularities. These are regularised and renormalised in a scheme by absorbing these divergences into the parton distribution functions. This fixes the distribution functions in that scheme. So these functions are defined in the parton model only within the scheme adopted.

Now define the soft part of the diagram (cross section) as the part which contains all the long distance physics and hard part as the part which contains all the short distance physics. As we noted, in the parton model only in the lowest order the factorisation of soft part and hard part is maintained. We have seen through $W^{\pm}g \rightarrow q\bar{q}$ cross section that this factorisation fails at next-to-leading order. Hence, one has to do the factorisation manually which leads to different results depending on the choice of factorisation.

In the following, we use a new method called factorisation method [16, 11] to interpret these weak spin dependent structure functions. This method is basically a field theoretical generalization of the parton model. Here, one expands the total lepton-hadron cross section in terms of parton distribution functions. The parton distribution functions are given appropriate operator definitions. The expansion coefficients in this method are no longer just parton level cross sections. Thus, the correct operator definition for parton distribution functions and the freedom that the coefficient functions need not be cross sections ensure the factorization i.e., the separation of hard and soft parts. It should be noted that the formal factorization expansion is true for any target. This helps to determine the coefficient functions order by order. So factorization method differs from the conventional parton model by its following features. 1. Factorization has operator definition for parton distribution functions. 2. The coefficient functions are no longer just parton level cross sections.

Using this method, we calculate both quark and gluonic contributions to these structure functions $g_{1,3,4}^{WP}(x, Q^2)$ to leading order [13]. The quark contributions to the structure functions are identical to that obtained in the parton model. The calculation of gluonic coefficient function involves the evaluation of the matrix element of axial bilocal operators between polarized one gluon state to order α_s in addition to that of usual $W^{\pm}g \rightarrow q\bar{q}$ cross section. It will be shown that the mass singularities appearing in the matrix element cancel that of Wg fusion process leaving the gluonic coefficient function free of mass singularities. This evaluation verifies the proposed factorization to order α_s in the gluonic sector. Thus, in the factorization method, the separation of hard and soft components is automatic.

The factorisation theorem enables us to write the hadronic tensor $\widetilde{W}^{\mu\nu}(x,Q^2)$ as

$$\widetilde{W}^{\mu\nu}_{WA}(x,Q^2) \;=\; \sum_a \int_x^1 \frac{dy}{y} f_{\frac{\Delta a}{A}}(y,\mu^2)\widetilde{H}_{Wa}{}^{\mu\nu}(q,yp,\mu^2,\alpha_s(\mu^2)) + \cdots, \qquad (2.23)$$

where $\widetilde{W}^{\mu\nu}_{WA} = \frac{1}{2}(W^{\mu\nu}_{+} - W^{\mu\nu}_{-})$, $f_{\Delta a/A}$ is the polarised probability distributions of the parton $a(\equiv q_i, \bar{q}_i, g)$ in the polarised hadron target A and is defined as $f_{\Delta a/A_+} = (f_{a_+/A_+} - f_{a_-/A_+})$, $\widetilde{H}^{\mu\nu}_a$ is the polarised hard scattering coefficient defined as $\widetilde{H}^{\mu\nu}_{Wa} = \frac{1}{2}(H^{\mu\nu}_{Wa_+} - H^{\mu\nu}_{Wa_-})$ and the $+(-)$ sign denotes the polarisation of the proton or parton along (opposite to) the beam axis. The ellipses in eqn.(2.23) represent the higher twist contributions. The naive derivation of the above factorisation is given in the introduction using free field approximation. The gauge invariant definitions of polarised quark and gluon distributions in the proton are given in terms of the connected matrix elements of the bilocal operators as [16, 10, 18]

$$\begin{aligned} f_{\frac{\Delta q+\Delta\bar{q}}{A}}(x) \;&=\; \frac{1}{4\pi}\int d\xi^- e^{-ix\xi^- p^+}\left[\langle p,s|\overline{\Psi}_a(0,\xi^-,0_\perp)\gamma^+\gamma_5\mathcal{G}^a_b\Psi^b(0)|p,s\rangle_c + \right. \\ &\left. \langle p,s|\overline{\Psi}_a(0)\gamma^+\gamma_5\mathcal{G}^{a\dagger}_b\Psi^b(0,\xi^-,0_\perp)|p,s\rangle_c\right], \qquad (2.24) \\ f_{\frac{\Delta g}{A}}(x) \;&=\; \frac{i}{4\pi x p^+}\int d\xi^- e^{-ix\xi^- p^+}\left[\langle p,s|F^{+\alpha}_a(0,\xi^-,0_\perp)\mathcal{G}^a_b\tilde{F}^{+b}_\alpha(0)|p,s\rangle_c - \right. \\ &\left. \langle p,s|F^{+\alpha}_a(0)\mathcal{G}^a_b\tilde{F}^{+b}_\alpha(0,\xi^-,0_\perp)|p,s\rangle_c\right], \qquad (2.25) \end{aligned}$$

where $F^{\mu\nu}_a$ is the gluon field strength operator and $\mathcal{G}^a{}_b = \mathcal{P}\exp[ig\int_0^{\xi^-} d\zeta^- A^+(0,\zeta^-,0_\perp)]^a_b$, $\mathcal{P}$ denotes the path ordering of the gluon field operators A^μ. In the above definitions we have used the light cone variables $P^\pm = (P^0 \pm P^3)/\sqrt{2}$, $P_\perp = 0$. For known targets (point like) like quarks and gluons the distribution functions are normalised as

$$f_{\frac{\Delta q+\Delta\bar{q}}{a(h)}}(z) \;=\; h\,\delta(1-z)\,\delta_{a,(q,\bar{q})}, \qquad (2.26)$$

$$f_{\frac{\Delta g}{a(h)}}(z) \;=\; h\,\delta(1-z)\,\delta_{a,g}, \qquad (2.27)$$

where $h = \pm 1$ is the helicity of the incoming parton and $z = x/y$. In general, the above matrix elements cannot be calculated within the domain of perturbative QCD, owing to the complex nature of the proton target. But these are calculable for specific parton targets like quark and gluons. Leading order corrections to these distributions are both

INSTITUTE OF MATHEMATICAL SCIENCES

INSTITUTE OF MATHEMATICAL SCIENCES LIBRARY

ultraviolet (UV) and infrared (IR) sensitive. These can be regularised and renormalised in any scheme. We use dimensional regularisation to regularise the UV divergences and $\overline{MS}$ scheme to renormalise these distributions. This choice is preferred as it ensures gauge invariance and relativistic invariance although it destroys scale invariance. The IR and collinear divergences are avoided by considering on-shell quarks in the cut quark loop and off shell gluons. Unlike the distribution functions $f_{\Delta a/A}$, the hard scattering coefficients are determined in the theory without any reference to the target. These coefficients may also be IR and UV sensitive and should be regularised in the same scheme.

The nth moment of these structure functions can be related to the distribution functions and the hard scattering coefficients as

$$g_i^A(n,Q^2) = \sum_a f_{\frac{\Delta a}{A}}(n,\mu^2)\,\widetilde{H}_a^i\left(n,\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right), \tag{2.28}$$

where $f(n) = \int_0^1 dx\, x^{n-1} f(x)$, i runs over the structure functions. Here we can easily identify $f_{\Delta a/A}(n,\mu^2)$ and $\widetilde{H}_a^i(n,Q^2/\mu^2,\alpha_s)$ with the matrix elements of the twist two local operator and the Wilson coefficient function respectively in the OPE [4].

2.4.1 Hard scattering coefficients

In this section we use the factorisation theorem to calculate the gluonic contribution to the spin dependent weak structure functions of the proton to leading order and the quark contribution to lowest order. The hadronic tensor $\widetilde{W}^{\mu\nu}_{WP}(p,q)$ for the virtual processes $W^{\pm}(q)P(p) \to W^{\pm}X$ can be written as

$$\widetilde{W}^{\mu\nu}_{WP}(x,Q^2) = \sum_{i,j}{}' \int_x^1 \frac{dy}{y} f_{\Delta q_i/P}(y,\mu^2)\,\widetilde{H}^{\mu\nu}_{W^{\pm}ij,q}(q,yp,\mu^2,\alpha_s(\mu^2)) + \sum_{i,j}{}' \int_x^1 \frac{dy}{y} f_{\Delta g/P}(y,\mu^2)\,\widetilde{H}^{\mu\nu}_{W^{\pm}ij,g}(q,yp,\mu^2,\alpha_s(\mu^2)), \tag{2.29}$$

where the prime in the summation denotes summing over quark and antiquark flavours restricted by electroweak theory.

The structure functions $g_i^{WP}(x,Q^2)$ get contribution from quark and gluon sector, and hence can be written as

$$g_i^{WP}(x,Q^2) = g_i^q(x,Q^2) + g_i^g(x,Q^2). \tag{2.30}$$

INSTITUTE OF MATHEMATICAL SCIENCES LIBRARY

36814

As observed in the previous section, for known parton targets we can evaluate $\widetilde{W}^{\mu\nu}_{W,P}$, and hence we compute the lowest order quark contributions (Born diagrams fig. 2.1) to $g_i^{WP}(x,Q^2)$ by appropriately projecting the hadronic tensor. We begin discussing the quark sector (which includes antiquarks as well as quark contributions). Here,

$$g_i^q(x,Q^2) = \sum_i{}' \int_x^1 \frac{dy}{y} f_{\frac{\Delta q_i + \Delta \bar{q}_i}{P}}(y,\mu^2)\, \hat{g}_i^{q,\bar{q}}\left(\frac{x}{y},Q^2\right), \tag{2.31}$$

where $\hat{g}_i^{q\bar{q}}(z,Q^2) = P^i_{\mu\nu}\widetilde{H}^{\mu\nu}_{W\pm q,\bar{q}}$, $x = yz$, $k = yp$, and $z(\equiv -q^2/2k.q)$ is the Björken variable at the parton level and k is the momentum of the incoming quark or antiquark. We find that

$$\begin{aligned}
\hat{g}_1^{q,\bar{q}}(z) &= \frac{g^2}{8}|U_{ij}|^2\,\delta(1-z),\\
\hat{g}_3^{q,(\bar{q})}(z) &= -(+)\frac{g^2}{8}|U_{ij}|^2\,\delta(1-z),\\
\hat{g}_4^{q,(\bar{q})}(z) &= -(+)2z\,\frac{g^2}{8}|U_{ij}|^2\,\delta(1-z),
\end{aligned} \tag{2.32}$$

where g is the weak coupling constant and U_{ij}'s are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. For the processes $W^+P \to W^+X$, using eqn.(2.26) and the $\hat{g}_i(z)$'s evaluated above, we obtain the relations for the structure functions, $g_i^q(x,Q^2)$ to be

$$\begin{aligned}
g_1^q(x,Q^2) &= \frac{g^2}{8}\Big[(|U_{ud}|^2+|U_{us}|^2)\Delta\bar{u}(x,Q^2)+(|U_{cd}|^2+|U_{cs}|^2)\Delta\bar{c}(x,Q^2)+\\
&\quad (|U_{ud}|^2+|U_{cd}|^2)\Delta d(x,Q^2)+(|U_{us}|^2+|U_{cs}|^2)\Delta s(x,Q^2)\Big], \qquad (2.33)\\
g_3^q(x,Q^2) &= \frac{g^2}{8}\Big[(|U_{ud}|^2+|U_{us}|^2)\Delta\bar{u}(x,Q^2)+(|U_{cd}|^2+|U_{cs}|^2)\Delta\bar{c}(x,Q^2)-\\
&\quad (|U_{ud}|^2+|U_{cd}|^2)\Delta d(x,Q^2)-(|U_{us}|^2+|U_{cs}|^2)\Delta s(x,Q^2)\Big], \qquad (2.34)\\
g_4^q(x,Q^2) &= 2x g_3^q(x,Q^2), \qquad (2.35)
\end{aligned}$$

and for the process $W^-P \to W^-X$,

$$\begin{aligned}
g_1^q(x,Q^2) &= \frac{g^2}{8}\Big[(|U_{ud}|^2+|U_{us}|^2)\Delta u(x,Q^2)+(|U_{cd}|^2+|U_{cs}|^2)\Delta c(x,Q^2)+\\
&\quad (|U_{ud}|^2+|U_{cd}|^2)\Delta\bar{d}(x,Q^2)+(|U_{us}|^2+|U_{cs}|^2)\Delta\bar{s}(x,Q^2)\Big], \qquad (2.36)\\
g_3^q(x,Q^2) &= \frac{g^2}{8}\Big[-(|U_{ud}|^2+|U_{us}|^2)\Delta u(x,Q^2)-(|U_{cd}|^2+|U_{cs}|^2)\Delta c(x,Q^2)+
\end{aligned}$$

$$(|U_{ud}|^2 + |U_{cd}|^2)\Delta\bar{d}(x,Q^2) + (|U_{us}|^2 + |U_{cs}|^2)\Delta\bar{s}(x,Q^2)\Big], \qquad (2.37)$$

$$g_4^q(x,Q^2) = 2x g_3^q(x,Q^2), \qquad (2.38)$$

where $\Delta q_i(x,Q^2) \equiv f_{\Delta q_i/P}(x,Q^2)$. Here we have retained even the Cabibbo suppressed terms, the relevance of which will be clear later. Note that eqns.(2.35) and (2.38) are Callan-Gross type relations in the weak spin quark sector.

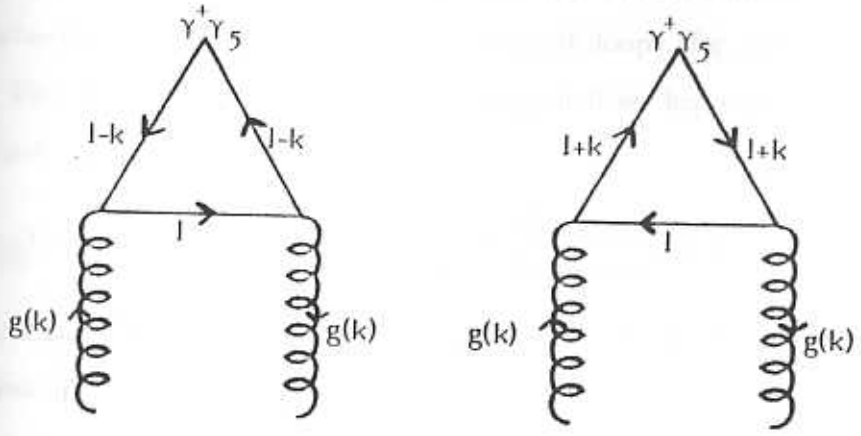

Figure 2.3 Triangle diagrams contributing to polarised distribution function $f_{\Delta q/g(h)}$.

Similarly we evaluate the gluonic contribution to the structure functions $g_i(x,Q^2)^{WP}$ using eqns.(2.24) and (2.25) by replacing the proton target by gluon target as

$$\widetilde{W}^{\mu\nu}_{W^\pm,g}(z,Q^2) = \sum_{ij}{}' \int_z^1 \frac{d\zeta}{\zeta} f_{\Delta q_i/g}(\zeta,\mu^2)\, \widetilde{H}^{\mu\nu}_{W^\pm ij,q}(q,\zeta\, p,\mu^2,\alpha_s(\mu^2)) + \sum_{i,j}{}' \int_z^1 \frac{d\zeta}{\zeta} f_{\Delta g/g}(\zeta,\mu^2)\, \widetilde{H}^{\mu\nu}_{W^\pm ij,g}(q,\zeta\, p,\mu^2,\alpha_s(\mu^2)), \qquad (2.39)$$

where $z(\equiv -q^2/2k.q)$ is the Bjorken variable at the parton level and k is the momentum of the incoming gluon. To evaluate $\widetilde{H}^{\mu\nu}_{W^\pm ij,g}$ to order α_s in the above equation we need to know $f_{\Delta q/g}$ to order α_s, $\widetilde{H}^{\mu\nu}_{W^\pm ij,q}$ to order α_s^0, $f_{\Delta g/g}$ to order α_s^0 and $\widetilde{W}^{Wg}_{\mu\nu}$ to order α_s.

We can evaluate $f_{\Delta q/g}$ and $f_{\Delta\bar{q}/g}$ to order α_s using eqn.(2.24); in the dimensional regularisation scheme, these are given by

$$f_{\Delta q_i/g(h)}(z) = \frac{\alpha_s}{2}\int \frac{d^D l}{(2\pi)^D} 2\pi\delta(l^+ - k^+ + zk^+) 2\pi\delta(l^2 - m_i^2)\theta(l^0)\epsilon^*_\rho(h)\epsilon_\sigma(h)$$

$$Tr\left[\frac{\not{l}-\not{k}+m_i}{(l-k)^2-m_i^2}\gamma^\rho(\not{l}+m_i)\gamma^\sigma\frac{\not{l}-\not{k}+m_i}{(l-k)^2-m_i^2}\gamma^+\gamma^5\right]+\cdots, \quad (2.40)$$

where antisymmetric part $\epsilon_\rho^*(h)\epsilon_\sigma(h) = \frac{ih}{2}\epsilon_{\rho\sigma}$ and $\epsilon_{\rho\sigma}$ is the antisymmetric tensor with $\epsilon_{12} = +1$. Similarly $f_{\Delta\bar{q}/g(h)}$ can be evaluated by reversing the direction of loop momentum which is identical to $f_{\Delta q/g}$. The ellipses in the above equation correspond to certain cut vertex diagrams which do not contribute, the only contribution to $f_{\Delta q/g(h)}$ comes from the cut triangle diagram with quark and antiquark loops (fig. 2.3) (see Appendix C for details). The matrix element is only UV divergent if we keep quarks on shell and gluon off-shell and on regulating we get

$$f_{\frac{\Delta q_i}{g(h)}}(z,\mu_R^2) = h\frac{\alpha_s}{4\pi}\left[(2z-1)ln\frac{\mu_R^2}{m_i^2-k^2z(1-z)}-1+\frac{m_i^2}{m_i^2-k^2z(1-z)}\right], \quad (2.41)$$

where $\mu_R^2 = \mu^2 exp(\gamma_E+\frac{2}{\epsilon})$ and h is the helicity of the gluon target. The above equation suffers from both IR and collinear divergences. The results depend on the way one takes the limit i.e., $k^2 \to 0$ first, finding the moment and then putting $m^2 \to 0$ or vice-versa in eqn.(2.41) [9]. *Here we do not have to choose any of the above prescriptions in our case since these dependences will be shown to disappear at the final stage of our calculation.* It is worth noting that the first moment of $f_{\Delta q/g(h)}$ in the above equation does not have logarithmic divergences consistent with the standard triangle anomaly results.

Next we evaluate the contribution from the box diagrams (see fig. 2.2), by defining $\widetilde{W}_i^g(z,Q^2) \equiv \mathcal{P}^{\mu\nu}{}_i\widetilde{W}_{\mu\nu}(z,Q^2)$. These can be calculated using $W^\pm g$ fusion processes shown in the fig. 2.2, which are cut box diagrams with flavours permitted by electroweak theory. The box diagram is free of UV divergences but suffers from both collinear and IR divergences. To avoid collinear and IR divergences both quarks and gluons are kept massive. In addition we have distinguished the flavour masses. The final expression is more complicated due to higher twist terms $(O(k^4/q^4), O(m^4/q^4)\cdots)$. Since we are only interested in the limit $q^2 \to -\infty$, these terms may be neglected. To this order we obtain

$$\widetilde{W}_1^g(z,Q^2) = \sum_{ij}{}'|U_{ij}|^2\frac{g^2}{8}\frac{\alpha_s}{4\pi}\left[\left((2z-1)ln\frac{Q^2(1-z)}{z(m_i^2-k^2z(1-z))}+3-4z+\right.\right.$$

$$\left.\frac{k^2 z(1-z)}{m_i^2 - k^2 z(1-z)}\right) + (m_{\bar{i}} \to m_j)\Bigg], \tag{2.42}$$

$$\widetilde{W}_3^g(z,Q^2) = \sum_{ij}{}' |U_{ij}|^2 \frac{g^2}{8}\frac{\alpha_s}{4\pi}\left[\left((2z-1)ln\frac{Q^2}{m_i^2 - k^2 z(1-z)} + \right.\right.$$
$$\left.\left.\frac{m_i^2}{m_i^2 - k^2 z(1-z)}\right) - (m_{\bar{i}} \to m_j)\right], \tag{2.43}$$

$$\widetilde{W}_4^g(z,Q^2) = 2z\widetilde{W}_3^g(z,Q^2), \tag{2.44}$$

where for $W^+ g$ the indices $\bar{i} = \bar{u}, \bar{c}$ and $j = d, s$ and for $W^- g$, $\bar{i} = \bar{d}, \bar{s}$ and $j = u, c$. It may be recalled that, in the parton model one only has to calculate the box diagram to evaluate the gluonic contribution to these structure functions. The box diagram is both IR and collinear divergent, and can be regulated in any scheme. This scheme dependency can be suitably absorbed in the parton distribution functions so as to make the measurable structure functions scheme independent. Hence in the parton model, the parton distribution functions can be defined only upto a specific scheme.

From the above results we see that $\widetilde{W}_1^g(z,Q^2)$ is symmetric under $m_{\bar{i}} \to m_j$ but $\widetilde{W}_3^g(z,Q^2)$ and $\widetilde{W}_4^g(z,Q^2)$ are antisymmetric. If we choose all masses equal ($m_{\bar{i}} = m_j$ for all i,j) in eqns.(2.42- 2.44), we get $\widetilde{W}_1^g(z,Q^2)$ non zero, while $\widetilde{W}_3^g(z,Q^2)$ and $\widetilde{W}_4^g(z,Q^2)$ are identically zero. This equal mass result is consistent with that of our parton model calculation [12] wherein we have evaluated the gluonic contributions to weak spin dependent structure functions. It is worth noting that Vogelsang and Weber [15] have also computed the box diagram with different flavour masses in the context of parton model using a different regularisation scheme to regulate collinear divergence. They have also discussed the phenomenological implications. In the limit $q^2 \to -\infty$ the expression for $\widetilde{W}_3^g(z,Q^2)$ and $\widetilde{W}_4^g(z,Q^2)$ are related by $2z$ which is a reflection of a Callan-Gross type relation.

Now we can evaluate the hard gluonic scattering coefficient $\hat{g}_{i,g}^{W^\pm}(z,q^2) \equiv \mathcal{P}_i^{\mu\nu}\widetilde{H}_{\mu\nu}^{W^\pm,g}$ by substituting for $f_{\Delta q + \Delta\bar{q}/g}$, $\widetilde{W}_i^g$, $\hat{g}_{i,g}$ and $f_{\Delta g/g}$ (normalisation condition in eqn.(2.27)),

$$\hat{g}_{1,g}^{W^\pm P}(z,Q^2) = -\sum_{ij}{}' |U_{ij}|^2 \frac{g^2}{8}\frac{\alpha_s}{4\pi}\left[(2z-1)(ln\frac{z}{(1-z)} + ln\frac{\mu_R^2}{Q^2}) + 4z - 3\right], \tag{2.45}$$

$$\hat{g}_{3,g}^{W^\pm P}(z,Q^2) = 0 = \hat{g}_{4,g}^{W^\pm P}(z,Q^2). \tag{2.46}$$

It is interesting to note that in the factorisation method, the coefficient functions (eqns.(2.45) and (2.46)) are free of mass singularities which appeared in the $W^{\pm}g$ fusion and triangle diagrams. The reason is that on evaluating the hard gluonic contribution, the mass singularities of box and triangle diagrams being identical, cancel exactly. As the gluonic scattering coefficient functions in the factorisation method have no mass singularities unlike the naive parton model, there is no IR or collinear prescription dependence. Hence unlike the parton model we need not redefine the parton distribution functions in the factorisation method.

It is now worth understanding the soft independence of the coefficient function by comparing with the gluonic coefficient evaluated in the parton model. The gluonic coefficient function in the factorization method differs from that of parton model by the matrix element of fermionic bilocal operator between polarised gluon states. This comes because we have given operator definitions to the parton distributions and relaxed the assumption that the coefficients are cross sections. The only way to evaluate them is to replace proton by gluon. This leads to the crucial difference between parton model and factorisation method.

The first moment of $\hat{g}_1^{W^{\pm}g}(z,Q^2)$ vanishes identically, leaving the first moment of $\hat{g}_1^{W^{\pm}P}(x,Q^2)$ free of gluonic contribution in this scheme. In the flavour symmetric case (flavour masses equal), the vanishing of the parity violating structure functions $\hat{g}_{3,4}^{W^{\pm}g}(z,Q^2)$ can be understood by constructing two combinations $g_{3,4}^{W^+P}(x,Q^2) - g_{3,4}^{W^-P}(x,Q^2)$ and $g_{3,4}^{W^+P}(x,Q^2) + g_{3,4}^{W^-P}(x,Q^2)$. The first combination is non singlet under flavour symmetry, but the second is a singlet. So no gluonic operator contributes to the first term. Though the second term is singlet, it transforms differently under charge conjugation as compared to the singlet gluonic operator. So this combination too has no gluonic operator. Hence it is apparent that $g_{3,4}^{W^+P}(x,Q^2)$ and $g_{3,4}^{W^-P}(x,Q^2)$ cannot be expressed in terms of matrix elements of local gluon operators. This argument is identical to the parity violating spin independent structure function $F_3^{W^{\pm}}(x,Q^2)$ not receiving gluonic contribution [20, 21]. *This argument breaks down in the flavour non-symmetric case, hence it is surprising that even in the broken flavour symmetric case that we have*

considered $g_{3,4}^{W^\pm P}(x,Q^2)$ *do not receive any gluonic contributions.* On the contrary, in the naive parton model calculation [15] these structure functions receive gluonic contribution due to different flavour masses. The reason is that in the parton model only Wg fusion process is relevant.

Finally we evaluate $g_i^{W^\pm P}(x,Q^2)$ by substituting for quark contribution in eqn.(2.30). The first moment of all these structure functions do not have gluonic contribution. It is interesting to consider the first moment of the combination $g_i^{W^+P}(x,Q^2)+g_i^{W^-P}(x,Q^2)$ which evaluates to

$$\begin{aligned}\int_0^1 dx\, g_1^{WP}(x,Q^2) &= \int_0^1 dx\, \left(g_1^{W^+P}(x,Q^2)+g_1^{W^-P}(x,Q^2)\right), \\ &= \frac{g^2}{8}\left[(|U_{ud}|^2+|U_{us}|^2)(\Delta u+\Delta\bar{u})+(|U_{cd}|^2+|U_{cs}|^2)(\Delta c+\Delta\bar{c})+\right. \\ &\left.(|U_{ud}|^2+|U_{cd}|^2)(\Delta d+\Delta\bar{d})+(|U_{us}|^2+|U_{cs}|^2)(\Delta s+\Delta\bar{s})\right], \quad (2.47)\end{aligned}$$

where Δq_i are the first moments of $\Delta q_i(x)$. Since the order of magnitude of CKM matrix elements are as follows

$$U_{ud}\approx U_{cs}\gg U_{cd}\approx U_{us},$$

the coefficients of $(\Delta q+\Delta\bar{q})$ are almost equal and approximately equal to 1.190 [22]. So the above eqn.(2.47) becomes

$$\int_0^1 dx\, g_1^{WP}(x,Q^2)\approx 1.190\,\frac{g^2}{8}\sum_i(\Delta q_i+\Delta\bar{q}_i). \quad (2.48)$$

Hence the measurement of $g_1^{WP}(x,Q^2)$ would directly yield the quark contribution to the proton spin, unlike the EMC experiment wherein one had to resort to low energy data F and D (D, F are the $SU(3)_f$ reduced matrix elements for axial octet current between the baryon octet states) to extract the singlet piece. The difference $g_1^{W^+P}(x,Q^2)-g_1^{W^-P}(x,Q^2)$ turns out to be the scale independent combination $(\Delta u-\Delta d)\equiv F+D$. The parity violating combination $g_3^{W^+P}(x,Q^2)+g_3^{W^-P}(x,Q^2)$ gives the valence contribution to the proton spin. Here, in contrast to the EM case one can extract the valence and sea part separately [19] similar to the spin independent weak structure functions.

2.4.2 Conclusion

We have calculated quark and gluon coefficient functions to the parity conserving ($g_1^{WP}(x)$) and parity violating ($g_{3,4}^{WP}(x)$) spin dependent structure functions in the weak sector using factorization method. We have done the calculation by keeping all the particles massive. We have distinguished flavour masses for completeness. The non-zero mass avoids mass singularities. Even though we keep these paticles massive, the coefficient functions are found to be free of them verifying factorisation theorem to this order. The reason for this is the exact cancellation of mass singularities coming from the cut box diagram and that coming from the matrix element of fermionic operator between polarised gluon states. The first moment of g_1^g is zero giving no gluon contribution to the first moment of $g_1^{WP}(x, Q^2)$. This fact is used to measure quark singlet part of the proton spin. The parity violating structure functions do not receive any gluonic contribution at all to order α_s. This result along with the sign difference for quark and antiquark contribution enables one to measure valence part of the proton spin using the combination $g_3^{W^+}(x, Q^2) + g_3^{W^-}(x, Q^2)$. Most important is the fact that in the factorization method, one need not do any soft singularity subtraction manually as one does in the parton model. Also one can get more information about quark spin content of the proton as gluons do not contribute to the first moment of $g_1^{WP}(x, Q^2)$ and to all moments of $g_3^{WP}(x, Q^2)$ and $g_4^{WP}(x, Q^2)$.

Chapter 3

Polarised Drell-Yan process

3.1 Introduction

The post European Muon Collaboration (EMC) [1] era has seen a revival of interest in the theoretical study of the polarised structure functions of the proton. The EMC experiment (polarised deep inelastic μP scattering) gave some surprising results regarding the quark spin contribution to the proton spin, namely, that it is negligibly small. The most appealing interpretation of this result is that not only the quarks but the gluons also can contribute to the proton spin, to the lowest order. This is due to the fact that the product of strong coupling constant, α_s, and the first moment of polarised gluon distribution function, $\Delta g(x)$, is of order α_s^0 [5, 7]. Due to this, gluons can in principle contribute to the first moment of polarised structure functions even at large Q^2. This contribution, if non-zero, is as important (magnitude) as the quark contribution to the first moment of structure functions. In the parton model, using the electromagnetic probe to evaluate the gluonic coefficient functions to leading order, one needs a prescription to regulate mass singularities present, and depending on the prescription the gluons may or may not contribute to the first moment of the polarised proton structure function, $g_1^{\mu P}(x)$ [7, 8]. As we have seen in the previous analysis, even if one uses the weak probe to evaluate the gluonic coefficient function, one would draw similar conclusions for the moments of the weak structure functions $g_{1,3,4}^{WP}(x)$ [12, 15]. On the other hand, in the operator product expansion [4] and factorisation method [16, 10, 18, 11], the gluonic coefficient function is free of

mass singularities and its first moment vanishes identically giving no gluonic contribution to the first moment of $g_1^{\mu P}(x)$ to order α_s [10, 11]. Hence, using the factorisation method, in a class of deep inelastic scattering (DIS) processes probed by electromagnetic [10, 11] and weak currents [13], one finds that the gluonic contribution to the first moments of polarised structure functions, $g_1^{\mu P}(x)$ and $g_{1,3,4}^{WP}(x)$, is zero. Under these circumstances, it is therefore instructive to analyse the polarised Drell-Yan (DY) processes as possible sources to measure the gluonic contribution to proton spin. Phenomenologically [34], this process has drawn much attention after the EMC experiment. Further, the experimental possibilities [35] are also bright with the recent technical developments in the acceleration of polarised protons, using the so-called Siberian snakes, which facilitate the polarised proton-proton collision at collider energies.

In this chapter we analyse the polarised DY process using the factorisation method and show [14] that the polarised DY distribution function $\widetilde{W}(\tau, Q^2)$ can be expressed in terms of known operators available in the theory. Further including the appropriate gluonic operators we write a factorisation theorem for polarised DY process and show that the mass singularities factorise leaving the gluonic coefficient functions free of mass singularities to order α_s. In short, we demonstrate factorisation in polarised DY to order α_s.

3.2 Operator definitions

Consider the longitudinally polarised DY process $P_1(p_1)^{\uparrow} + P_2(p_2)^{\uparrow\downarrow} \rightarrow \gamma^*(q) + X(P_X)$, where γ^* is the virtual hard photon that ultimately decays to a lepton pair. The cross section of the process is

$$\tilde{\sigma} = \frac{1}{2K(2\pi)^3}(-g_{\mu\nu}) \int d^4q\ \delta(q^2 - Q^2)\ \theta(q^0)\ \widetilde{W}^{\mu\nu}, \tag{3.1}$$

where K is the flux factor, Q^2 is mass of the virtual hard photon, the sum over photon polarisations yields the factor $-g^{\mu\nu}$ and $\widetilde{W}^{\mu\nu}$ is the Fourier transform of product of hadronic currents sandwiched between two polarised proton states. The polarised DY

function $\widetilde{W}_{P_1P_2}(\tau,Q^2)$ is defined as

$$\widetilde{W}_{P_1P_2}(\tau,Q^2) = -\frac{1}{(2\pi)^3}\int d^4q\ \delta(q^2-Q^2)\ \theta(q^0)\int d^4\xi\ e^{-iq.\xi} \langle p_1s_1;p_2s_2|\ J_\mu(\xi)\ \ J^\mu(0)\ |p_1s_1;p_2s_2\rangle_{sp}. \tag{3.2}$$

We choose the centre of momentum light cone frame where $p_1 = (p_1^+,0,0_\perp)$, $p_2 = (0,p_2^-,0_\perp)$, $q = (q^+,q^-,0_\perp)$, $\xi = (\xi^+,\xi^-,\xi_\perp)$ and the light cone variables are defined as $p^\pm = (p^0 \pm p^3)/\sqrt{2}$. Using the DY limit $p_1^+,p_2^- \to \infty$, $q^+,q^- \to \infty$ with $\tau = q^+q^-/p_1^+p_2^-$ ($\tau = Q^2/s$, where s is the cm energy) fixed, we can express the product of hadron currents in terms of the parton currents which are made up of parton field operators coming from two different protons. In order to proceed, we need to express the current-current product so that the dependences on the two proton momenta separate out. We do this by a Fierz transformation whereby the product of local bilinear (with respect to two different proton states) operators is expressed as sum of bilocal linear operators. Hence the DY function can be neatly separated out in terms of two proton states and we obtain,

$$\widetilde{W}_{P_1P_2}(\tau,Q^2) = \sum_i \frac{e_i^2}{24\pi}\int\frac{dq^+}{q^+}\frac{dq^-}{q^-}\delta(1-\frac{Q^2}{2q^+q^-})\int d\xi^+d\xi^-e^{-iq^+\xi^-}e^{-iq^-\xi^+} \Big[\langle p_1s_1|\ \overline{\psi}_a(\xi^+,\xi^-)\ \gamma_\mu\ \gamma_5\psi_a(0)\ |p_1s_1\rangle\times \langle p_2s_2|\ \overline{\psi}_b(0)\ \gamma^\mu\ \gamma_5\psi_b(\xi^+,\xi^-)\ |p_2s_2\rangle + (a\leftrightarrow b)\Big]\ , \tag{3.3}$$

where i runs over different quark flavours, and the scalar, vector and pseudoscalar terms which appear in the Fierz identity do not contribute for the polarised DY process. From the asymptotic behaviour of Fourier transform of singular functions, it is easy to see that the dominant contribution comes from the region near the singularities [36]. Hence, using $q^+ = x_ap_1^+$ and $q^- = x_bp_2^-$, we have

$$\widetilde{W}_{P_1P_2}(\tau,Q^2) = \sum_i\int\frac{dx_a}{x_a}\frac{dx_b}{x_b}\ \frac{2\pi e_i^2}{3}\ \delta(1-\frac{\tau}{x_ax_b}) \Big[\frac{1}{4\pi}\int d\xi^-e^{-ix_ap_1^+\xi^-}\langle p_1s_1|\ \overline{\psi}_a(\xi^-)\ \gamma^+\ \gamma_5\psi_a(0)\ |p_1s_1\rangle\ \times \frac{1}{4\pi}\int d\xi^+e^{-ix_bp_2^-\xi^+}\langle p_2s_2|\ \overline{\psi}_b(0)\ \gamma^-\ \gamma_5\ \psi_b(\xi^+)|p_2s_2\rangle + (a\leftrightarrow b)\Big], \tag{3.4}$$

where we have also used the fact that $\langle p^\pm s|\ \overline{\psi}\ \gamma^\mp\ \gamma_5\ \psi\ |p^\pm s\rangle = 0$. The above expression provides neat separation of short and long distance physics. By noting that the Dirac

delta function corresponds to the $q, \bar{q} \to \gamma^*$ subprocess, the terms in the bracket can be identified with the polarised quark and antiquark distribution functions of the proton, which are consistent with the DIS polarised distribution functions [10, 11] and hence confirms the universality of the parton distribution functions.

To make the definitions of the parton distribution functions gauge invariant, one has to insert the path ordered exponential given below. With this, one can generalise the above expression by replacing delta function by hard scattering coefficient function $\widetilde{H}_{ab}$ and adding appropriate gluonic operators of the theory. We thus have

$$\widetilde{W}_{P_1P_2}(\tau, Q^2) = \sum_{a,b} \int_\tau^1 \frac{dx_a}{x_a} \int_{\frac{\tau}{x_a}}^1 \frac{dx_b}{x_b} f_{\frac{\Delta a}{P_1}}(x_a, \mu^2)\ \widetilde{H}_{ab}(\frac{\tau}{x_a x_b}, \frac{q^2}{\mu^2})\ f_{\frac{\Delta b}{P_2}}(x_b, \mu^2) + (a \leftrightarrow b) + \cdots, \tag{3.5}$$

where μ^2 is the factorisation scale, $\widetilde{W}_{P_1P_2} = \frac{1}{2}(W_{++} - W_{+-})$, $f_{\Delta a/A}$ is the polarised probability distribution of the parton $a(\equiv q_i, \bar{q}_i, g)$ in the polarised hadron target A, defined as $f_{\Delta a/A_+} = (f_{a_+/A_+} - f_{a_-/A_+})$ and $\widetilde{H}_{ab}$ is the polarised hard scattering coefficient defined as $\widetilde{H}_{ab} = \frac{1}{2}(H_{a_+b_+} - H_{a_+b_-})$ where the $+(-)$ sign denotes the polarisation of the proton or parton along (opposite to) the beam axis. The ellipses in eqn.(3.5) represent the higher twist contributions. The gauge invariant definitions of polarised quark and antiquark distributions in the proton are identified using eqn.(3.4) in terms of the connected matrix elements of the bilocal operators as

$$f_{\frac{\Delta q}{A}}(x) = \frac{1}{4\pi} \int d\xi^- e^{-ix\xi^- p^+} \left[\langle p,s|\ \overline{\Psi}_a(0,\xi^-,0_\perp)\ \gamma^+\ \gamma_5\ \mathcal{G}_b^a\ \Psi^b(0)\ |p,s\rangle_c\right], \tag{3.6}$$

$$f_{\frac{\Delta g}{A}}(x) = \frac{1}{4\pi} \int d\xi^- e^{-ix\xi^- p^+} \left[\langle p,s|\ \overline{\Psi}_a(0)\ \gamma^+\ \gamma_5 \mathcal{G}_b^{a\dagger}\ \Psi^b(0,\xi^-,0_\perp)\ |p,s\rangle_c\right]. \tag{3.7}$$

In terms of the appropriate gluonic operators in the theory the gluonic distribution function is defined as

$$f_{\frac{\Delta g}{A}}(x) = \frac{i}{4\pi x p^+} \int d\xi^- e^{-ix\xi^- p^+} \left[\langle p,s|\ F_a^{+\mu}(0,\xi^-,0_\perp)\ \mathcal{G}_b^a\ \tilde{F}_\mu^{+b}(0)\ |p,s\rangle_c - \langle p,s|\ F_a^{+\mu}(0)\ \mathcal{G}_b^a\ \tilde{F}_\mu^{+b}(0,\xi^-,0_\perp)\ |p,s\rangle_c\right], \tag{3.8}$$

where $F_a^{\mu\nu}$ is the gluon field strength operator, $\mathcal{G}^a{}_b = \mathcal{P}exp\,[ig\int_0^{\xi^-} d\zeta^- A^+(0,\zeta^-,0_\perp)]\,{}^a_b$, $\mathcal{P}$ denoting the path ordering of the gluon field operators A^μ. For known targets like quarks and gluons the distribution functions are normalised as

$$f_{\frac{\Delta q+\Delta\bar{q}}{a(h)}}(z) = h\,\delta(1-z)\,\delta_{a,(q,\bar{q})}, \tag{3.9}$$

$$f_{\frac{\Delta g}{a(h)}}(z) = h\,\delta(1-z)\,\delta_{a,g}\ , \tag{3.10}$$

where $h = \pm 1$ denotes the helicity of the incoming parton. In general, the above matrix elements cannot be calculated within the domain of perturbative QCD, owing to the complex nature of the proton target. But these are calculable for specific parton targets like quarks and gluons. Leading order corrections to these distributions are both ultraviolet (UV) and mass sensitive. These can be regularised and renormalised in any scheme. We use dimensional regularisation to regularise the UV divergences and $\overline{MS}$ scheme to renormalise these distributions. This choice is preferred as it ensures gauge invariance and relativistic invariance although it destroys scale invariance. The mass singularities are avoided by considering on-shell quarks in the cut quark loop and off-shell gluons. Unlike the distribution functions $f_{\Delta a/A}$, the hard scattering coefficients are determined in the theory without any reference to the target. These coefficients may also be mass and UV sensitive and should be regularised in the same scheme.

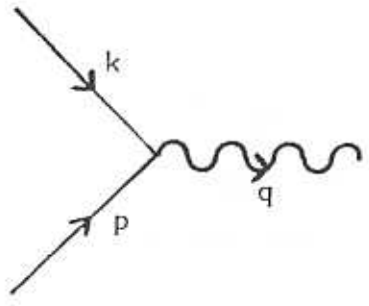

Figure 3.1 Quark Born diagram.

The nth moment of the DY function can be related to the nth moment of the distribution functions and the hard scattering coefficients as

$$\widetilde{W}_{AB}(n,Q^2) = \sum_{ab} f_{\frac{\Delta a}{A}}(n,\mu^2)\ \widetilde{H}_{ab}\left(n,\frac{Q^2}{\mu^2}\right)\ f_{\frac{\Delta b}{B}}(n,\mu^2), \tag{3.11}$$

where $f(n) = \int_0^1 x^{n-1} f(x)dx$. Here we can easily identify $f_{\Delta a/A}(n,\mu^2)$ with the matrix elements of the twist two local operators.

3.3 Hard scattering coefficient

One of the interesting offshoots of the EMC interpretation is that $\alpha_s \Delta g$ is of order α_s^0. Hence the leading order contribution to the DY process comes from the following subprocesses $q\bar{q} \to \gamma^*$, $g(q,\bar{q}) \to \gamma^*(q,\bar{q})$ and $gg \to \gamma^* q\bar{q}$. We therefore use the factorisation theorem to calculate the gluonic contribution to the spin dependent DY function to order α_s and the quark contribution to lowest order. To this order, the quantity of interest is

$$\begin{aligned}
\widetilde{W}_{P_1P_2}(\tau,Q^2) \;=\; & \sum_i \int_\tau^1 \frac{dx_a}{x_a} \int_{\frac{\tau}{x_a}}^1 \frac{dx_b}{x_b} f_{\Delta q_i/P_1}(x_a,\mu^2)\ \widetilde{H}_{q_i\bar{q}_i}\left(\frac{\tau}{x_a x_b},\frac{Q^2}{\mu^2}\right)\ f_{\Delta\bar{q}_i/P_2}(x_b,\mu^2) + \\
& \sum_i \int_\tau^1 \frac{dx_a}{x_a} \int_{\frac{\tau}{x_a}}^1 \frac{dx_b}{x_b} f_{\Delta q_i/P_1}(x_a,\mu^2)\ \widetilde{H}_{q_ig}\left(\frac{\tau}{x_a x_b},\frac{Q^2}{\mu^2}\right)\ f_{\Delta g/P_2}(x_b,\mu^2) + \\
& \sum_i \int_\tau^1 \frac{dx_a}{x_a} \int_{\frac{\tau}{x_a}}^1 \frac{dx_b}{x_b} f_{\Delta\bar{q}_i/P_1}(x_a,\mu^2)\ \widetilde{H}_{\bar{q}_ig}\left(\frac{\tau}{x_a x_b},\frac{Q^2}{\mu^2}\right)\ f_{\Delta g/P_2}(x_b,\mu^2) + \\
& (a \leftrightarrow b).
\end{aligned} \tag{3.12}$$

Using the Born diagram (fig.1), we get

$$\widetilde{H}_{q\bar{q}}(\hat{\tau}) \;=\; \frac{2\pi e_i^2}{3}\,\delta(1-\hat{\tau}). \tag{3.13}$$

To evaluate the gluonic contribution to the DY function $\widetilde{W}_{P_1P_2}(\tau,Q^2)$ we replace the proton targets by quark and gluons and use eqn.(3.6), eqn.(3.7), eqn.(3.8) and eqn.(3.12) and express the resulting 'Compton' subprocess to order α_s (see Appendix B for details) as

$$\begin{aligned}
\widetilde{W}_{qg}(\hat{\tau},Q^2) \;=\; & \sum_i \int_{\hat{\tau}}^1 \frac{dy_a}{y_a} \int_{\frac{\hat{\tau}}{y_a}}^1 \frac{dy_b}{y_b} f_{\Delta\bar{q}_i/g}(y_a,\mu^2)\ \widetilde{H}_{q_i\bar{q}_i}\left(\frac{\hat{\tau}}{y_a y_b},\frac{Q^2}{\mu^2}\right)\ f_{\Delta q_i/q}(y_b,\mu^2) + \\
& \sum_i \int_{\hat{\tau}}^1 \frac{dy_a}{y_a} \int_{\frac{\hat{\tau}}{y_a}}^1 \frac{dy_b}{y_b} f_{\Delta q_i/q}(y_a,\mu^2)\ \widetilde{H}_{q_ig}\left(\frac{\hat{\tau}}{y_a y_b},\frac{Q^2}{\mu^2}\right)\ f_{\Delta g/g}(y_b,\mu^2),
\end{aligned} \tag{3.14}$$

$\hat{\tau}(\equiv \tau/x_a x_b)$ is the DY variable in the parton level. To calculate $\widetilde{H}_{q_i g}(\frac{\hat{\tau}}{y_a y_b}, \frac{Q^2}{\mu^2})$ to order α_s from the eqn.(3.13), we need to know $\widetilde{H}_{qg}(\hat{\tau}, Q^2)$, the Compton process to order α_s, $f_{\frac{\Delta q_i}{q}}$ to order α_s and $f_{\frac{\Delta q_i}{q}}$, $f_{\frac{\Delta g}{g}}$ and $\widetilde{H}_{q\bar{q}}(\frac{\hat{\tau}}{y_a y_b}, \frac{Q^2}{\mu^2})$ to α_s^0. $f_{\frac{\Delta g}{g}}$ to order α_s can be evaluated from definition (eqn.(3.7)) by replacing proton by gluon(polarised) . $f_{\frac{\Delta g}{q}}$ is also similarly evaluated. The only contribution to $f_{\Delta \bar{q}/g(h)}$ comes from the cut triangle diagram (fig. 2.3) with quark and antiquark loops. The matrix element is only UV divergent if we keep quarks on shell and gluon off-shell. The UV divergence is regularised as usual by using dimensional regularisation and renormalised by $\overline{MS}$ scheme. Thus,

$$f_{\frac{\Delta q_i}{g(h)}}(\hat{\tau}, \mu_R^2) = h\frac{\alpha_s}{4\pi}\left[(2\hat{\tau}-1)\, ln\frac{\mu_R^2}{m^2 - k^2\hat{\tau}(1-\hat{\tau})} - 1 + \frac{m^2}{m^2 - k^2\hat{\tau}(1-\hat{\tau})}\right], \quad (3.15)$$

where m is the mass of the quark, k^2 is off-shell mass of the gluon, $\mu_R^2 = \mu^2 exp(\gamma_E + \frac{2}{\epsilon})$ and h is the helicity of the gluon target. Since $\widetilde{H}_{q\bar{q}}$ is given in eqn.(3.13), $\widetilde{H}_{qg}$ can be found if $\widetilde{W}_{qg}$ is calculated.

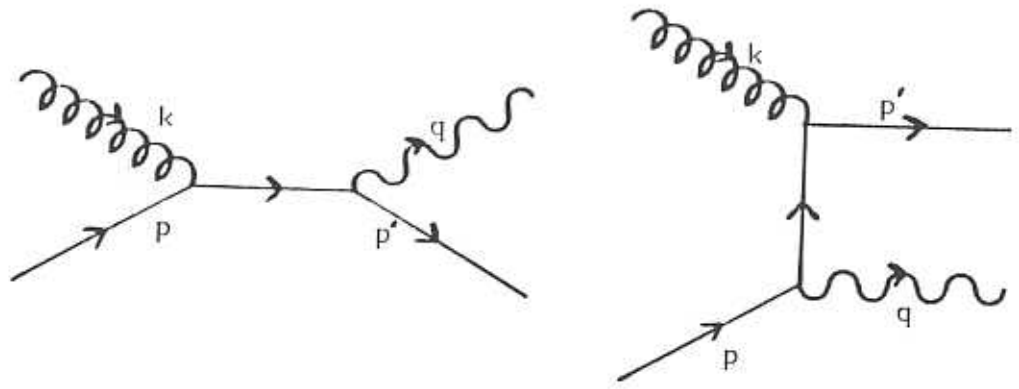

Figure 3.2 Compton subprocess.

As the Compton subprocess diagram (fig.3.2) has only mass singularities, we keep all the masses non zero. Neglecting the power corrections in the large Q^2 and s limit, we get

$$\widetilde{W}_{qg}(\hat{\tau}, Q^2) = \sum_i \alpha_s \frac{e_i^2}{6}\left[(2\hat{\tau}-1)\, ln\frac{Q^2(1-\hat{\tau})^2}{\hat{\tau}(m^2 - k^2\hat{\tau}(1-\hat{\tau}))} - \frac{1}{2}(3\hat{\tau}^2 + 2\hat{\tau} - 3) + \frac{m^2}{m^2 - k^2\hat{\tau}(1-\hat{\tau})}\right]. \quad (3.16)$$

Now we can evaluate the hard scattering coefficient, by using the above results

$$\begin{aligned}\widetilde{H}_{qg}(\hat{\tau},Q^2) &= -\sum_i \alpha_s \frac{e_i^2}{6}\left[(2\hat{\tau}-1)\,(ln\frac{\hat{\tau}}{(1-\hat{\tau})^2}+ln\frac{\mu_R^2}{Q^2})+\frac{1}{2}(3\hat{\tau}^2+2\hat{\tau}-3)-1\right], \\ &= \widetilde{H}_{\bar{q}g}(\hat{\tau},Q^2). \end{aligned} \tag{3.17}$$

We find that a similar analysis of the weak DY processes also yields the same quark and gluon hard scattering coefficient functions but for weak coupling factors. Note that *the above expression is free of any mass singularities, verifying the proposed factorisation to order α_s in the gluonic sector.* In addition, since the first moment of $\widetilde{H}_{qg}(\hat{\tau},Q^2)$ vanishes, *there is no gluonic contribution to the first moment of DY function through the 'Compton' subprocess for both electromagnetic and weak processes.*

Let us now see why this factorisation works and we compare it with that of Bodwin and Qiu [10]. First note that the Compton process is both IR and collinear singular. It is analogous to $\gamma^* g \rightarrow q\bar{q}$ or $W^{\pm} g \rightarrow q\bar{q}$ process. These are basically related by crossing symmetry. If we follow formal subtraction procedure i.e., identifying soft part and subtracting them from the original diagrams then we get the hard part which is mass singular free. Interestingly, the square of the second diagram (Fig.3.2) and contraction of this with the rest, in the limit lead to a set of diagrams which can be interpreted as the probability of finding on-shell quarks and antiquarks inside the polarised gluon. This is what basically one does in showing factorisation property in unpolarised DIS and unpolarised DY cross sections and this is what happens in polarised DIS [10, 11].

In the *parton model* analysis of the above problem, the calculation of gluonic coefficient function in the DY function involves only $\widetilde{W}_{qg}(\hat{\tau},Q^2)$ term. This being mass singular, a particular choice of prescription has to be made to regulate these singularities. Depending on the prescription one gets different results: for $m^2 \neq 0$, $k^2 = 0$ the first moment of $\widetilde{W}_{qg}(\hat{\tau},Q^2)$ is zero, while for $m^2 = 0$, $k^2 \neq 0$ we have,

$$\int d\hat{\tau}\,\widetilde{W}_{qg}(\hat{\tau},Q^2) = -\frac{1}{2}\left(\frac{\alpha_s e_i^2}{3}\right). \tag{3.18}$$

It is interesting to note that *one arrives at the same conclusion even in the polarised DIS case.* Invoking universality of parton distributions we expect the first moment of

$\widetilde{W}_{P_1P_2}(\tau, Q^2)$ in the parton model using the prescription $m^2 = 0$, $k^2 \neq 0$, to be of the form $\Delta q' \ \Delta\bar{q}'$, where $\Delta q' = \Delta q \ - \ (\alpha_s/4\pi)\Delta g$ [7]. Note that eqn.(3.18) verifies this to order α_s. This implies that the first moment of $\widetilde{W}_{gg}(\hat{\tau}, Q^2)$ is $(4\pi e_i^2/3)\ (\alpha_s/4\pi)^2$. Assuming this to be true in the proposed factorisation scheme, one finds that the first moment of $\widetilde{H}_{gg}(\hat{\tau}, Q^2)$ is also zero. Explicit computation of $\widetilde{H}_{gg}$ involves evaluation of eight diagrams and three body phase space integral. Hence, in the factorisation scheme, gluons cannot contribute to the first moment of polarised DY function.

3.4 Conclusion

We have studied the polarised DY process using in the factorisation method and have provided operator definitions for the polarised parton distribution functions which are found to be consistent with that of DIS, and hence the universality is confirmed. We have explicitly shown the factorisation of mass singularities to order α_s in the gluonic sector. The gluonic contribution to the first moment of the polarised DY function from the 'Compton' subprocess turns out to be zero. We have explicitly shown that QCD Parton model analysis of gluonic contribution to polarised DY involves mass singularities and ambiguity in choosing a prescription to avoid them. This status is analogous to that of DIS. Invoking universality, we conclude that the gluonic contribution to the first moment of polarised DY function vanishes.

Chapter 4

Phenomenology of proton spin

The idea of the quark structure of the known hadrons was first due to Gell-Mann and Zweig. They showed that the strongly interacting particles can be built from a triplet of fundamental particles called quarks and corresponding triplet of antiquarks which transform accordingly in $SU(3)$ octet symmetry and to carry fractional charges. As we know, quark model is quite successful in explaining many low energy observations concerning strong, electromagnetic and weak interactions. There are also several successful attempts to accomodate DIS in the list of the applications of quark model with appropriate modifications. The objective of the present chapter is carry out a consistent phenomenological analysis of the polarised structure functions measured by the EMC. This we try to achieve by separately analyzing the various singlet, non-singlet and gluonic contributions to the axial matrix element. The main result is that predictions from the quark model wave functions for the valence quark polarizations can be made consistent [42] with the EMC measurement.

4.1 The Model

We have shown in the earlier chapters that, within the parton model, the singlet distribution functions involve a potentially large gluon term. Separating the valence, sea and gluonic components, we write

$$\Delta u = \Delta u_V + \Delta u_\sigma + \Delta G,$$

$$\begin{aligned} \Delta d &= \Delta d_V + \Delta d_\sigma + \Delta G, \\ \Delta s &= \Delta s_\sigma + \Delta G, \end{aligned} \tag{4.1}$$

where we have separately identified the contributions from valence (v) and sea quarks(σ). Here,

$$\Delta G = -\frac{\alpha_s}{2\pi}\Delta g = -\frac{\alpha_s}{2\pi}\int dx\,(g_+(x) - g_-(x))\,, \tag{4.2}$$

where Δg, the moment of the spin dependent gluon density, arises through the box diagram. However there is some disagreement [24] as to whether this gluonic contribution is calculable using perturbative QCD. For our purposes we treat ΔG as a phenomenological parameter which measures the gluonic contribution to the axial matrix elements.

Our objective is to use the theoretically determined [39] values of Δq_i i.e.,

$$\Delta u = 0.8,\ \Delta d = -0.45,\ \Delta s = -0.165, \tag{4.3}$$

or equivalently the experimentally determined values (from the EMC) and check the consistency of various inputs Δq_V, Δq_σ and ΔG with known physics. To do this we first note that we have no information on ΔG, it is a phenomenological input to the flavour singlet part of Δq. Similarly we have no information of the sea quark spin densities. However proceeding in analogy with results in the unpolarized case, we set,

$$\Delta u_\sigma = \Delta d_\sigma = \frac{1}{k}\Delta s_\sigma = \Delta\sigma. \tag{4.4}$$

The equivalence of u, d quark contributions avoids any isospin violation in the Bjorken sum rule for g_A. For a $SU(3)$ symmetric sea $k = 1$. However it is well known that in the unpolarized case the strange sea distribution k is about $\frac{1}{2}$ of the u, d distributions [69]. The factor k takes care of the breakdown of the $SU(3)$ symmetry or equivalently the quark mass effects.

Next we consider the valence quark contribution. Using the $SU(3)_{flavour} \times SU(2)_{spin}$ wave function of the proton we obtain [70],

$$\begin{aligned} \Delta u_V(x) &= \frac{2}{3}\,u_V(x) \\ \Delta d_V(x) &= -\frac{1}{3}\,d_V(x) \end{aligned} \tag{4.5}$$

where $\Delta q_V(x)(q_V(x))$ is the spin dependent (spin independent) quark density and the weights on the right hand side are obtained by constructing the probabilities for u or d quarks to have spin parallel or anti parallel to the proton spin [71]. With these definitions,

$$\begin{aligned} \Delta u_V &= \int dx\, \Delta u_V(x) = 4/3 \\ \Delta d_V &= \int dx\, \Delta d_V(x) = -1/3 \end{aligned} \tag{4.6}$$

and through Bjorken sum rule we have

$$g_A = \Delta u - \Delta d = \Delta u_V - \Delta d_V = 5/3 \tag{4.7}$$

which is the standard $SU(3)$ value in disagreement with the experimental value($= 5/4$). To avoid this problem we use the Carlitz-Kaur model [72] where the valence quark spin densities are diluted due to their interaction with the gluons and the sea. As a result, we write

$$\begin{aligned} \Delta u_V(x) &= \left(u_V(x) - \frac{2}{3} d_V(x)\right) cos2\theta \\ \Delta d_V(x) &= -\frac{1}{3}\, d_V(x)\, cos2\theta \end{aligned} \tag{4.8}$$

where $\cos 2\theta$, a function of Bjorken x, is the spin dilution factor. A parametrization of $\cos 2\theta$ is then obtained using Regge arguments at small x. The large $x(x \to 1)$ behaviour of $\Delta u_V(x)$ and $\Delta d_V(x)$ should be similar to $u_V(x)$ and $d_V(x)$ due to the constraint

$$|\Delta u_V(x)/u_V(x)|, \quad |\Delta d_V(x)/d_V(x)| \leq 1.$$

The effect of the spin dilution factor will therefore be seen only at small x and $cos2\theta \to 1$ as $x \to 1$. Later we demonstrate that even a simple model in which

$$\begin{aligned} \Delta u_V(x) &= \frac{2}{3} u_V(x)) cos2\theta \\ \Delta d_V(x) &= -\frac{1}{3} d_V(x) cos2\theta \end{aligned} \tag{4.9}$$

yields results identical to the Carlitz-Kaur model. In the next section we consider the phenomenological implications of the various inputs.

4.2 Phenomenology

First we compute the valence quark contribution to Δq. Gupta et al [46] have given parametrizations of $u_V(x)$ and $d_V(x)$ and $cos2\theta$ and various other distributions which fit the $g_1^p(x)$ given by EMC [1]. We concentrate on the valence and $cos2\theta$ distributions given by them. The two sets of distributions given by them are,

Set I	$xu_V(x)$	$1.78x^{.5}(1-x^{1.51})^{3.5}$
$Q_0^2 = 5GeV^2$	$xd_V(x)$	$0.67x^{.4}(1-x^{1.51})^{4.5}$
$\Lambda_{QCD} = 200MeV$	$cos2\theta$	$[1+0.258(1-x)^{1/2}/x^{.1}]^{-1}$
Set II	$xu_V(x)$	$2.75x^{.588}(1-x^{1.51})^{2.69}$
$Q_0^2 = 15GeV^2$	$xd_V(x)$	$8.53x^{1.03}(1-x^{1.51})^{6.87}$
$\Lambda_{QCD} = 90MeV$	$cos2\theta$	$[1+0.2656(1-x)^{1/2}/x^{.1}]^{-1}$
Set III	$xu_V(x)$	$2.26x^{.54}(1-x)^{2.52}[1-1.617(1-x)+$
$Q_0^2 = 15GeV^2$		$3.647(1-x)^2-1.998(1-x)^3]$
$\Lambda_{QCD} = 90MeV$	$xd_V(x)$	$.57(1-x)xu_V(x)$
	$\cos 2\theta$	$[1+.078(1-x)^2/x^{.5}]^{-1}$

Table 4.1 Parametrisations

Third set, with $u_V(x)$ and $d_V(x)$ is given by Diemoz et al [73] (see also Gluck et al [74]), a form suggested by Carlitz and Kaur [72](see also Kaur[75]) based on Regge arguments. The moments Δu_V and Δd_V calculated based on the above parametrizations are given in table (4.2). It is easily seen that the moments are not only independent of the various parametrizations but are insensitive to the details of the model, Carlitz-Kaur or the modified $SU(3)$ model on the other eqn. (4.9). Notice that if $\cos 2\theta = 1$, Δu_V and Δd_V given by either of these models are identical and collapse to the naive $SU(3)$ values. The effect of spin dilution factor is therefore to reduce g_A from 5/3 of $SU(3)$ to approximately 5/4 closer to the experimental value.

Input		Carlitz-Kaur	SU(3) modified
Set I	Δu_V	1.0	1.0
	Δd_V	-0.25	-0.25
Set II	Δu_V	1.013	1.003
	Δd_V	-0.245	-0.245
Set III	Δu_V	1.004	0.99
	Δd_V	-0.24	-0.24

Table 4.2 Valence quark contributions

An important point that emerges from the table (4.2) is that $\Delta u_V/\Delta d_V = -4$ which is exactly the ratio predicted by the naive $SU(3)$. This should not be surprising since we know that the $SU(3)$ predictions for ratios elsewhere (like magnetic moment of baryons, the F/D ratio, etc.,) have been found to be accurate. In fact if the ratios of u and d matrix elements were to be renormalized differently by the effect of sea quarks and gluons such that the ratios differed from $SU(3)$ predictions, it would imply catastrophic isospin violations not found elsewhere. We can therefore safely choose, based on the analysis above,

$$\Delta u_V \approx 1, \ \Delta d_V \approx -0.25. \tag{4.10}$$

Similar numbers have been obtained by Fritzsch [76] where the reduction of g_A from 5/3 to 5/4 has been attributed to the effect of sea quarks and gluons. These, however contribute differently to u and d quarks implying isospin violation, which we have avoided above.

Next we consider the sea quark contribution constrained according to eqn.(4.4). In the simplest case we can take the OZI rule to be exact for axial vector current matrix elements and set $\Delta s_\sigma = 0$. This would imply either $k = 0$ or $\Delta s_\sigma = 0$ $\Delta\sigma = 0$. The former is unlikely since for the unpolarized distributions k is approximately 1/2 from CDHS parametrizations (Abramowicz et al [69]). If the latter is taken to be correct, then

the non-zero value of Δs arises entirely from gluons. Using the theoretical values given in eqn. (4.3), and using eqns.(4.1), we find

$$\Delta G \approx -0.165,\ \Delta u_V \approx 0.965,\ \Delta d_V \approx -0.285. \tag{4.11}$$

The valence quark contribution is not far off from eqn. (4.10). On the other hand if we take $\Delta s_\sigma \neq 0$ and using the valence quark contribution from (4.10) as input in (4.1), we obtain,

$$\begin{aligned} \Delta u &= 1 + \Delta\sigma + \Delta G, \\ \Delta d &= -.25 + \Delta\sigma + \Delta G, \\ \Delta s &= k\Delta\sigma + \Delta G, \end{aligned} \tag{4.12}$$

where $0 \leq k \leq 1$. The case $k = 1$ corresponds to $SU(3)$ symmetric sea which yields $g_A = 1.25$ and $g_8 = \Delta u + \Delta d - 2\Delta s = 0.75$. Experimentally the value of g_8 is close to 0.685 using the measured value of F/D [77]. For $k \leq 1$, we obtain

$$g_8 = 0.75 + 2\Delta\sigma(1 - k) = 0.685. \tag{4.13}$$

For $k \approx 1/2$ as in the unpolarized case $\Delta\sigma = -0.07$ and $\Delta s_\sigma = -0.0335$ which is consistent with the Regge bound [78] on the strange quark polarization.

$$\| \Delta s_\sigma \| \leq -0.072 \pm 0.030. \tag{4.14}$$

A slightly different bound $\| \Delta s_\sigma \| \leq 0.052^{+0.023}_{-0.052}$ has been quoted by Soffer [79]. The value of $\Delta G \approx -0.13$ is slightly different from the $SU(3)$ symmetric case.

In both cases discussed above the sign of the gluon contribution turns out to be negative. Following Altarelli and Ross [7], if $\Delta G = -\alpha_s \Delta g/2\pi$, then the gluon polarization is in turn positive. Cheng and Li [51] have challenged the sign of Δg as obtained. Using the anomalous divergence equations to constrain the size of ΔG, they claim that the resulting sign of ΔG is positive, in fact,

$$\Delta G = 0.16 \pm 0.08. \tag{4.15}$$

They also conclude from this that it spells trouble for naive quark model. We show below that if any, the positive value of ΔG results in a large violation of the bound on the strange quark polarization, eqn.(4.14). Substituting the mean value of ΔG in eqn. (4.12) we find, without affecting the values of Δu_V and Δd_V,

$$\Delta\sigma = -0.36,\ \Delta s_a = -0.325, \tag{4.16}$$

or analogously $k \approx 0.9$ which is very close to the $SU(3)$ symmetric case with $k = 1$. However such large strange sea polarization violates the Regge bound but is still consistent with quark model estimates.

4.3 Conclusion

We have recalculated the valence quark contribution Δu_V and Δd_V to the axial vector matrix elements of u and d quarks Δu and Δd consistent with the Bjorken sum rule. The ratio $\Delta u_V/\Delta d_V = -4$ obeys the $SU(3)$ prediction. Keeping this as the input to the first moment of the proton spin structure function, we have reanalyzed the phenomenology of proton spin. We find the gluonic contribution ΔG to be negative, as claimed by Altarelli and Ross, in order to satisfy the Regge bound on the strange quark polarization. On the other hand Cheng and Li find ΔG to be positive which result they claim is just the wrong sign to accommodate naive quark model predictions. We find, however the quark model predictions for valence Δu_V and Δd_V can be accommodated with positive ΔG while violating the bound on strange quark polarisation.

Chapter 5

Moments of structure functions

5.1 Introduction

In the study of DIS using the method of OPE and renormalisation group (RG), one finds that the moments of structure function of the hadron can be expressed in terms of hadron matrix elements of some local operators constructed out of the fundamental fields which constitute the hadron. The coefficients of these operators called Wilson coefficients are perturbatively calculable unlike the matrix elements. QCD being the theory of strong interaction has been successfully tested experimentally by comparing the ratios of these coefficients at different scales. In other words logarithmic scaling violation predicted by QCD has been successfully observed in DIS experiments. In these cases one generally takes ratios of the same moments of structure functions at different scales which will not involve any non perturbative matrix elements. On the contrary, these matrix elements are not calculable due to lack knowledge about the non perturbative aspects. But these can be extracted from the experiment. These matrix elements in fact carry information about the structure of the hadron. It is now well understood that the operators appearing in $F_2(x)$ carry information about the momentum structure of hadron and that appearing in $g_1(x)$ carry information about the spin structure of the hadron. Thus, ratios of Wilson coefficients at different scales test the theory namely QCD and lowest moments of structure function carry the information about momentum and spin structure of the hadrons. The natural question one would like to ask in this context is:

What will one get if one considers higher moments and ratios of these structure functions at same scale?

In this chapter, we take a look at the non-perturbative matrix elements involved in the computation of the moments of the structure functions of the proton, both polarised as well as unpolarised. In fact, we claim that the systematics associated with these moments implies a definite relation among various non-perturbative matrix elements. These relations reduce the level of computation necessary and enable calculation of ratios of these moments. We support our statements using a simple bag model.

Specifically, consider the two lowest moments of the spin-independent structure function $F_2(x)$, in $e(\mu) - p$ scattering. Their measured values (without including errors) are [38]

$$M_2 \equiv \int dx\, F_2(x) \approx 0.153,$$
$$M_4 \equiv \int dx\, x^2\, F_2(x) \approx 0.016, \quad Q^2 = 15GeV^2.$$

Similarly, for the spin dependent structure function, $g_1(x)$, we find [1]

$$G_1 \equiv \int dx g_1(x) \approx 0.126,$$
$$G_3 \equiv \int dx\, x^2\, g_1(x) \approx 0.011, \quad Q^2 = 10GeV^2.$$

In both the polarised as well as the unpolarised case, the ratio of successive moments is around 0.1. This is true even when we include the next higher moment in the spin-independent case [41]; the moment is down by an order of magnitude compared to the previous one. Is this a coincidence or is there some deeper reason? We show later that the ratios of successive moments can be written as $\frac{m_{eff}^2}{M_P^2}$, where M_P is the proton mass and m_{eff} is some dynamically generated quark mass. Substituting the relevant numbers, we find this mass to be of the order of 300 MeV, which is very close to the constituent quark mass. We also show that these results are far more exact in the spin-independent case than in the spin-dependent case, where the results are model dependent and not so clean.

In section 5.2, we outline the relevant calculations for the moments of structure functions based on the operator product expansion near the light cone. Section 5.3 contains

numerical estimates of various moments and their ratios using parametrisations of experimental data and correcting for finite Q^2 effects. Section 5.4 deals with the calculation of moments in a massive quark bag model. Our conclusions are stated in section 5.5. The normalisation conventions used in the text are given in Appendix D.

5.2 Moments of Structure Functions.

The hadronic tensor, $W_{\mu\nu}$, which appears in the DIS of charged leptons on hadrons, can be formally expressed as the Fourier transform of the commutator of electromagnetic currents sandwiched between hadronic states (see chapter 1 for details), i.e.

$$W_{\mu\nu} \equiv \frac{1}{2\pi} \int d^4x \, e^{iq.x} \, \langle p, s | [J_\mu(x), J_\nu(0)] | p, s \rangle. \tag{5.1}$$

Here p is the proton momentum, s is its covariant spin and q is the momentum transfer at the hadronic vertex. The hadronic tensor contains symmetric and antisymmetric parts in the indices μ and ν:

$$W_{\mu\nu} \equiv W^S_{\mu\nu} + iW^A_{\mu\nu} \tag{5.2}$$

Using Lorentz invariance and current conservation, we get

$$\begin{aligned} W^S_{\mu\nu} &= \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_L + \left(p_\mu p_\nu - (p_\mu p_\nu + q_\mu p_\nu) \frac{p.q}{q^2} + \right. \\ &\left. g_{\mu\nu} \frac{(p.q)^2}{q^2} \right) \frac{2F_2}{p.q}, \end{aligned} \tag{5.3}$$

$$W^A_{\mu\nu} = \epsilon_{\mu\nu\lambda\sigma} \, q^\lambda \left(s^\sigma \frac{1}{p.q} g_1 + \frac{1}{(p.q)^2} (p.q s^\sigma - q.s p^\sigma) \, g_2 \right). \tag{5.4}$$

We shall be interested in the moments of F_2 and g_1 alone. We now apply Wilson's operator product expansion. Near the light cone, the non-local product of two electromagnetic currents can be expanded, leading to a separation of the perturbatively calculable "Wilson coefficient" functions and non-perturbative matrix elements of the theory.

Following Kodaira [4], we can write expressions for the moments of F_2 and g_1 as follows:

$$M_n^{NS}(Q^2) \equiv \int dx\, x^{n-2} F_2^{NS}(x,Q^2) = \sum_i A_n^{i,NS}(\mu^2)\, C_{2,i}^{n,NS}(Q^2,g),$$

$$M_n^{S}(Q^2) \equiv \int dx\, x^{n-2} F_2^{S}(x,Q^2) = \sum_i A_n^{i,S}(\mu^2)\, C_{2,i}^{n,S}(Q^2,g), \tag{5.5}$$

n even, and

$$G_n^{NS}(Q^2) \equiv \int dx\, x^{n-1} g_1^{NS}(x,Q^2) = \frac{1}{2}\sum_i a_n^{i,NS}(\mu^2)\, E_{2,i}^{n,NS}(Q^2,g),$$

$$G_n^{S}(Q^2) \equiv \int dx\, x^{n-1} g_1^{S}(x,Q^2) = \frac{1}{2}\sum_i a_n^{i,S}(\mu^2)\, E_{2,i}^{n,S}(Q^2,g), \tag{5.6}$$

n odd.

Here, C and E are the coefficient functions which are functions of Q^2 and the QCD coupling constant g. The hadronic matrix elements are contained in $A_{n,i}$ and $a_{n,i}$ (These are basically same as A_n and B_n respectively defined in the introduction but for some some numerical factors). The superscripts NS and S refer to non-singlet and singlet contributions while the summation over i runs over all the relevant local operators in the theory of the correct dimension and twist.

In the asymptotic (or scaling) limit, the coefficients corresponding to the gauge fields are zero and we have a simple result with all quark bilinear operators having a coefficient of unity, i.e.,

$$C_{2,i}^{n}(Q^2, g=0) = E_{1,i}^{n}(Q^2, g=0) = 1, \qquad Q^2 \to \infty \tag{5.7}$$

As a consequence, in the leading order, we can combine the singlet and non-singlet contributions to get

$$M_n \equiv \int dx\, x^{n-2} F_2(x) = \sum_i^f e_i^2\, A_n^i,$$

$$G_n \equiv \int dx\, x^{n-1} g_1(x) = \frac{1}{2} \sum_i^f e_i^2\, a_n^i(\mu^2), \tag{5.8}$$

$i = u, d, s$ Here, the matrix elements of the local operators which are renormalized at the scale μ^2 are defined as follows:

$$\frac{1}{2} \sum_i \langle p,s|i^{n-1} \bar{\psi}_i \{\gamma^{\mu_1} D^{\mu_2} ... D^{\mu_n}\} \psi_i |p,s\rangle = 2\, A_n^i\, p^{\mu_1} ... p^{\mu_n}, \tag{5.9}$$

$$\sum_i \langle p,s|i^{n-1} \bar{\psi}_i\, \gamma_5 \{\gamma^{\mu_1} D^{\mu_2} ... D^{\mu_n}\}\, \psi_i |p,s\rangle = -a_n^i \{s^{\mu_1} p^{\mu_2} ... p^{\mu_n}\}, \tag{5.10}$$

where $\{\}$ implies symmetrisation over the enclosed indices. We now proceed to evaluate the moments of F_2 and g_1. We start with the spin averaged function.

$$M_2 = \sum_i \frac{e_i^2}{2M_P^2} \left(\frac{1}{2} \sum_s \langle ps|\bar{\psi}_i\, (i\not{D})\, \psi_i |ps\rangle \right), \tag{5.11}$$

$$M_4 = -\sum_i \frac{e_i^2}{2M_P^4} \left(\frac{1}{2} \sum_s \langle ps|\bar{\psi}_i\, \frac{i}{3}\, (\not{D} D^2 + D_\mu \not{D} D^\mu + D^2 \not{D})\, \psi_i |ps\rangle \right), \tag{5.12}$$

A similar analysis for the moments of the spin dependent structure function gives

$$G_1 = -\frac{1}{2} \left(\frac{s^\mu}{s^2} \right) \sum_i e_i^2\, \langle ps|\bar{\psi}_i\, \gamma_5 \gamma_\mu\, \psi_i |ps\rangle, \tag{5.13}$$

$$G_3 = \frac{s^\mu}{2s^2 M_P^2} \sum_i e_i^2\, \langle ps|\bar{\psi}_i\, \gamma_5\, (\gamma_\mu D^2 + \not{D} D_\mu + D_\mu \not{D})\, \psi_i |ps\rangle. \tag{5.14}$$

In obtaining eqns.(5.11-5.14), we have made use of the fact that $p^2 = M_p^2$ and $s.p = 0$, where M_p is the proton mass. Our aim is to obtain the ratios of moments. We do not, at this stage, know how to compute the hadronic matrix elements in eqns. (5.11) to (5.14). In order to proceed, we introduce the effective equation of motion,

$$[i\partial - m_{eff}^i(\mu^2)]\, \psi_i = 0, \;\; \{\partial^2 + [m_{eff}^i(\mu^2)]^2\}\, \psi_i = 0. \tag{5.15}$$

where $m^i_{eff}(\mu^2)$ is the effective mass of the i-th quark evaluated at the same scale μ^2 at which the operator in the hadronic matrix element is renormalised. We henceforth replace the covariant derivatives by ordinary derivatives and neglect gluonic corrections. In taking this step, we are guided by the spirit of quark model calculations with "constituent quarks". In fact, the m_{eff} appearing in this equation hides our ignorance or inability to calculate the soft gluon contributions which probably dominate the non-perturbative matrix elements. We thus anticipate that this m_{eff} will be of the order of the dynamical quark mass, rather than the current quark mass.

An analogue to m_{eff} can be found when we consider spontaneous chiral symmetry breaking of the QCD vacuum. Even if we begin with a zero quark mass, a dynamical mass is generated due to the non-zero value of the $\langle : q\bar{q} : \rangle$ vacuum condensate [44] in particular, terms like $\langle 0| : \bar{\psi}(0) D\psi(0) : |0\rangle \ \langle 0| : \bar{\psi}(0) D^2\psi(0) : |0\rangle$ show a mass dependent behaviour typified by $m_{dyn}\langle 0| : \psi(0)\bar{\psi}(0) : |0\rangle$ and $m^2_{dyn}\langle 0| : \psi(0)\bar{\psi}(0) : |0\rangle$ respectively. Here, the dynamically generated mass is large, of the order of the constituent quark mass and not the current quark mass.

We now put this equation of motion (5.15) back in the moments (eqns.(5.11) and (5.12)). We immediately observe that

$$M_2 = \sum_i \frac{e_i^2}{2M_P^2} m^i_{eff} \left(\frac{1}{2} \sum_s \langle ps|\psi_i\psi_i|ps\rangle \right), \tag{5.16}$$

$$M_4 = \sum_i \frac{e_i^2}{2M_P^4} (m^i_{eff})^3 \left(\frac{1}{2} \sum_s \langle ps|\psi_i\psi_i|ps\rangle \right), \tag{5.17}$$

i.e., the *same*, unknown, matrix element occurs in both the second and the fourth moment.

Similarly, for the spin-dependent case, the third moment is

$$\begin{aligned} G_3 \ = \ & -\frac{s^\mu}{2s^2 M_P^2} \sum_i e_i^2 \left(m^2_{i,eff} \frac{1}{2} \sum_s \langle ps|\psi_i \gamma_5 \gamma_\mu \psi_i|ps\rangle + \right. \\ & \left. 2m_{eff,i}\langle ps|\psi_i \gamma_5 i\partial_\mu \psi_i|ps\rangle \ \right), \end{aligned} \tag{5.18}$$

Notice that the first term in G_3, apart from mass factors, is very similar to G_1 in eqn.(5.13). Equations (5.13), (5.16)-(5.18) are the basic set of equations. In the next section, we numerically estimate m_{eff} in as model independent a way as possible and later check our calculations in a bag model with non-zero quark mass.

5.3 Numerical estimates of m_{eff}

To get a feeling for the numbers involved, we first consider the spin independent case. To get an order of magnitude estimate of the effective quark mass, we first set all the quark flavours to have equal mass. We see that the ratio of the moments of F_2 is then independent of the matrix elements:

$$\mathcal{R}_F = \frac{M_4}{M_2} = \frac{m_{eff}^2(\mu^2)}{M_P^2}. \tag{5.19}$$

We note here that, by applying our ansatz, $\mathcal{R}_F$ would be the same for ratios of any successive moments of the form $\frac{M_{n+2}}{M_n}$. However, numerically, higher moments are so small that accurate estimation becomes a problem; hence, we concentrate on the case where $n = 2, 4$.

Based on the experimental data [38], Sloan [45] has parametrised the various parton densities. Using these parametrisations, we find

$$M_2 = 0.153, \; M_4 = 0.016 \tag{5.20}$$

(at $Q^2 = 15GeV^2$), so that

$$\mathcal{R}_F = 0.105. \tag{5.21}$$

In other words, $m_{eff} = 303$ MeV, which vindicates our expectation that m_{eff} is of the order of the constituent quark mass.

Next we consider the case when flavour masses are not equal. We therefore re-express the moments in eqns.(5.16) and (5.17) as

$$M_2 = \frac{1}{9}[4\epsilon_u + \epsilon_d + \epsilon_s], \tag{5.22}$$

where

$$\epsilon_q = \int_0^1 dx\, x\, q(x)$$

is the fraction of the proton momentum carried by the quark q. Comparing with eqn. (5.11), we find that

$$\epsilon_q = \frac{1}{2M_P^2}\left(\frac{1}{2}\sum_s \langle ps|\bar{\psi}_q\,(i\not{\partial})\,\psi_q|ps\rangle\right). \tag{5.23}$$

Using eqn.(5.17) we can write the fourth moment as,

$$M_4 = \frac{1}{2M_P^2}[4m_u^2\epsilon_u + m_d^2\epsilon_d + m_s^2\epsilon_s], \tag{5.24}$$

where m_u, m_d and m_s are the effective masses of the u, d and s quarks respectively and cannot be simultaneously determined from a single relation. However, the ϵ's are known. Using the parametrization in ref. [45], these are found to be,

$$\epsilon_u = 0.305,\ \epsilon_d = 0.146,\ \epsilon_s = 0.015 \tag{5.25}$$

(at $Q^2 = 15GeV^2$) In order to estimate M_4, we appeal to the quark model and set $m_u = m_d = 330$ MeV and $m_s = 480$ MeV. This gives a value, $M_4 = 0.019$ which is fairly close to that obtained in eqn.(5.21). On the other hand, using current quark mass values gives an M_4 that is smaller than the observed one by about three orders of magnitude. Hence, a larger value of m_{eff} , of the order of the constituent quark mass, is clearly indicated. It is also obvious from eqn.(5.25) that the effective quark mass one gets is essentially that of the u and d flavours since the momentum fraction carried by the strange quark is nearly negligible.

A similar analysis can be carried out for the spin dependent structure function. We shall just quote the salient features: for details, see Ref. [40]. As pointed out before, the matrix element appearing in the first term of G_3 in eqn.(5.18) is the same as that in eqn.(5.13) for G_1, apart from mass factors. The second term typically gets contributions from the transverse momentum of the quarks and may be expected to be small in comparison with the first term. We shall therefore ignore this term for the moment and justify this step later. Then, we immediately see that the moments of $g_1(x)$ also satisfy a relation similar to that satisfied by the moments of $F_2(x)$ in eqn.(5.19), namely,

$$\mathcal{R}_G = \frac{G_3}{G_1} = \frac{m_{eff}^2(\mu^2)}{M_P^2},$$

again using the assumption that all flavours have equal mass. The measured $g_1(x)$ is positive everywhere for $0.01 \leq x \leq 0.7$. The quoted value of G_1 is

$$G_1 = 0.126 \pm 0.010(stat) \pm 0.015(syst). \tag{5.26}$$

To consistently obtain both G_1 and G_3 from the same set of parametrisations, we use the parametrisations in refs.[1, 45] which yield

$$G_1 = 0.125,\ G_3 = 0.011. \tag{5.27}$$

Hence,

$$\mathcal{R}_G = 0.091 \tag{5.28}$$

(or equivalently),

$$m_{eff} = 283\ MeV, \tag{5.29}$$

again close to the value obtained from the spin independent structure function analysis. However, caution should be exercised in interpreting this result, as the approximation we

have made in the expression for G_3 can only be justified in a model dependent way. We will return to this problem in the next section. It is in this sense that the analysis for the spin independent structure function is cleaner. Thus the use of an effective equation of motion in the OPE throws up a mass scale of the order of the dynamical quark mass. Note that this is a leading order result, as we have dropped $\mathcal{O}(\alpha_s)$ terms in the Wilson coefficients; however, the resulting error in the estimates is not more than 10% to 15% for the various moments.

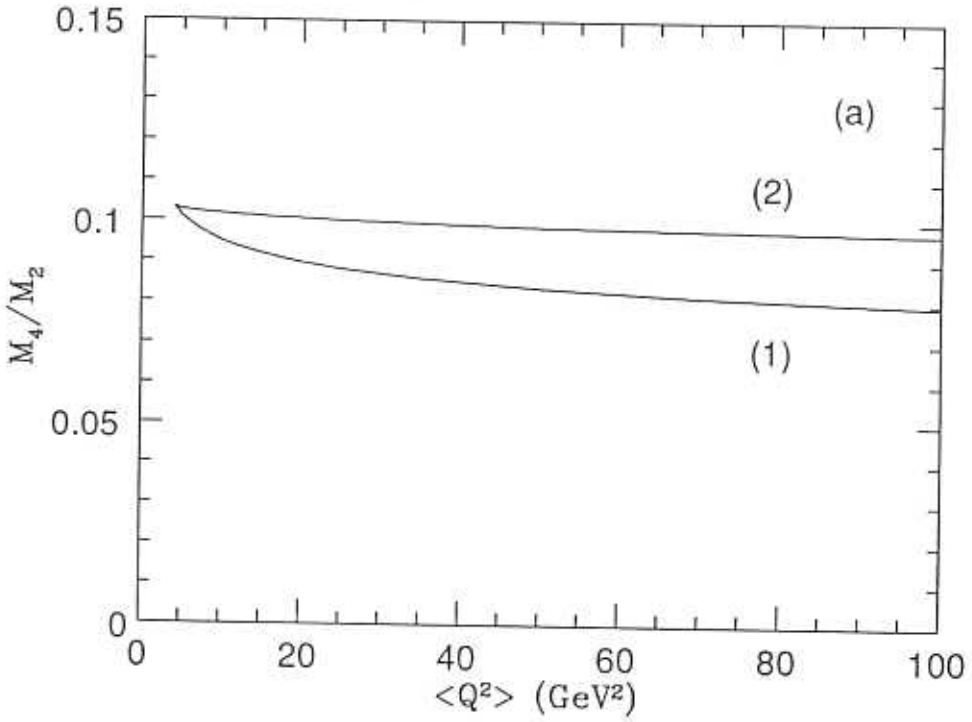

Figure 4.1a The moments ratio M_4/M_2 is plotted as a function of Q^2. The graph labeled (2) includes corrections coming from anomalous dimensions, while graph (1) does not.

Since we know the individual parton densities, it may seem a bit surprising that we do not set m_u, m_d and m_s different from each other right away. We must note here that although the quark model masses fit the results fairly well, eqns.(5.21) and (5.29) are more exact as they are completely parametrisation independent and only depend on the (reasonable) assumption that all the flavours have equal masses.

The results obtained so far are leading order results in the asymptotic limit. However, the data set with which we make comparisons are all parametrised at finite Q^2. To correct

for this, we now include Q^2 dependences in the Wilson coefficients. This is a *perturbative* correction and is expected to be small; in fact, QCD tells us that this is a logarithmic correction, which however is different for the non-singlet and singlet moments. In the process of renormalization, there is operator mixing in the singlet part, while there is no such complication in the non-singlet sector.

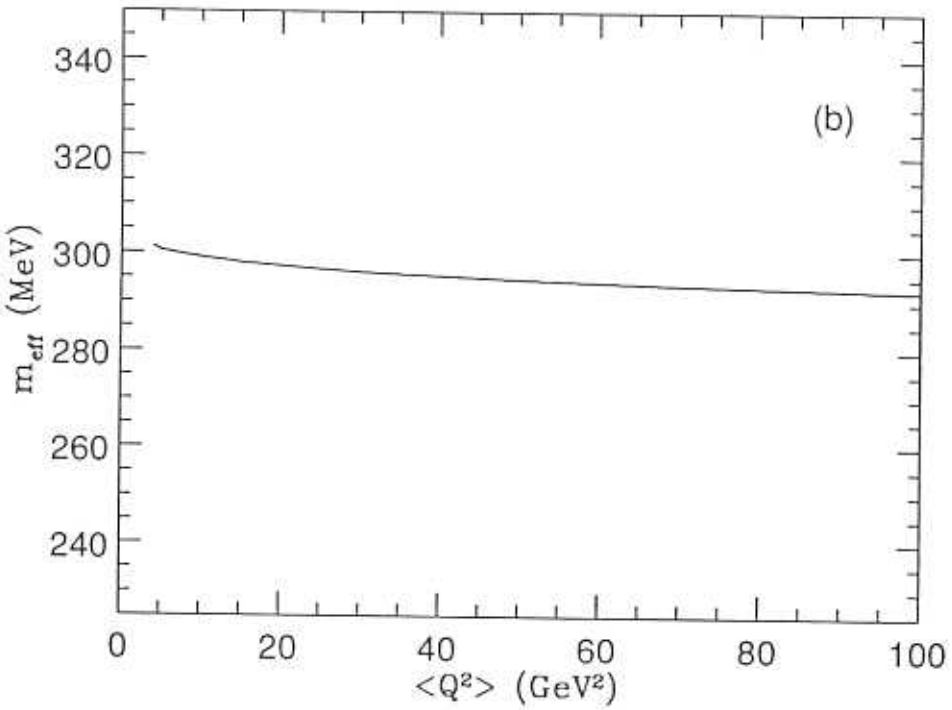

Figure 4.1b The mass, m_{eff}, corresponding to the ratio labeled (2) in fig (4.1a) is plotted as a function of Q^2.

The evolution of the non-singlet moments can be written in the form

$$M_n^{NS}(Q^2) = \frac{1}{9} A_n^{NS}(\mu^2) \left[\frac{log(\frac{Q^2}{\Lambda^2})}{log(\frac{\mu^2}{\Lambda^2})}\right]^{-d_{NS}^n} \tag{5.30}$$

where Λ is the QCD scale parameter and d_{NS}^n is related to the anomalous dimension of the relevant operator eqn.(5.9). In the case of three flavours [21],

$$d_{NS}^2 = 0.395, \quad d_{NS}^4 = 0.775 \tag{5.31}$$

Similarly, for the singlet part, we have

$$M_n^S(Q^2) = \frac{2}{9}\left(A_n^\psi(\mu^2)\left\{\alpha_n\left[\frac{log\left(\frac{Q^2}{\Lambda^2}\right)}{log\left(\frac{\mu^2}{\Lambda^2}\right)}\right]^{-d_n^-} + (1-\alpha_n)\left[\frac{log\left(\frac{Q^2}{\Lambda^2}\right)}{log\left(\frac{\mu^2}{\Lambda^2}\right)}\right]^{-d_n^+}\right\} + \right. \quad (5.32)$$
$$\left. A_n^G(\mu^2)\left\{\tilde{\alpha_N}\left[\frac{log\left(\frac{Q^2}{\Lambda^2}\right)}{log\left(\frac{\mu^2}{\Lambda^2}\right)}\right]^{-d_n^-} - \tilde{\alpha_n}\left[\frac{log\left(\frac{Q^2}{\Lambda^2}\right)}{log\left(\frac{\mu^2}{\Lambda^2}\right)}\right]^{-d_n^+}\right\}\right),$$

where α and $\tilde{\alpha}$ are parameters which enter into the Q^2 evolution of moments and A^ψ and

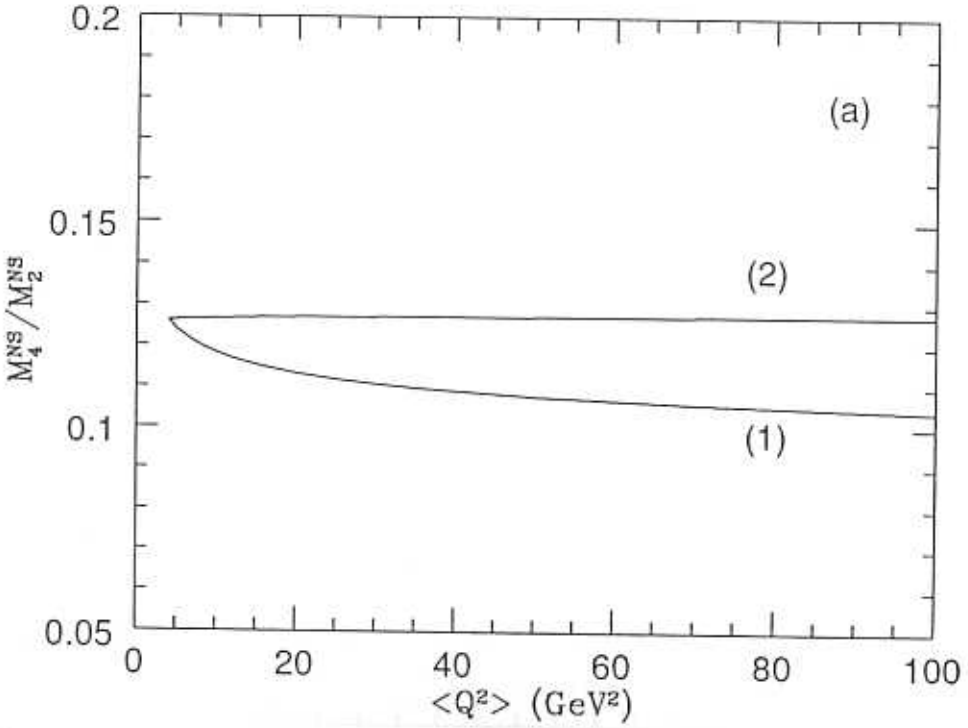

Figure 4.2a The ratio corresponding to the nonsinglet piece in M_4/M_2 is shown as a function of Q^2. Labels (1) and (2) are the same as in Fig. (4.1a).

A^G are the quark and gluon operator matrix elements [4]. For the d's, we have [21],

$$d_2^+ = 0.617, \quad d_2^- = 0.0, \quad d_4^+ = 1.638, \quad d_4^- = 0.76\,. \quad (5.33)$$

On substituting these numbers, we find that the contribution of the gauge fields to the singlet moment is still negligible as the multiplicative logarithmic correction (the coefficient

of A^G in eqn. (5.33)) is less than 0.05 over a large Q^2 range from $4 \leq Q^2(\mathrm{GeV}^2) \leq 100$. Surprisingly, it also turns out that the corresponding coefficients of the nonsinglet and singlet quark terms in eqns. (5.30) and (5.33) are equal in this range, within the same degree of accuracy. Thus, we can simply combine the singlet and non-singlet parts to get

$$\frac{M_4(Q^2)}{M_2(Q^2)} \left[\frac{log\left(\frac{Q^2}{\Lambda^2}\right)}{log\left(\frac{\mu^2}{\Lambda^2}\right)}\right]^{d^4_{NS}-d^2_{NS}} = \frac{\sum_i e_i^2 A_{4,i}(\mu^2)}{\sum_i^2 e_i^2 A_{2,i}(\mu^2)} = \frac{m^2_{eff}(\mu^2)}{M_P^2}, \tag{5.34}$$

where the A's are given by eqn. (5.9).

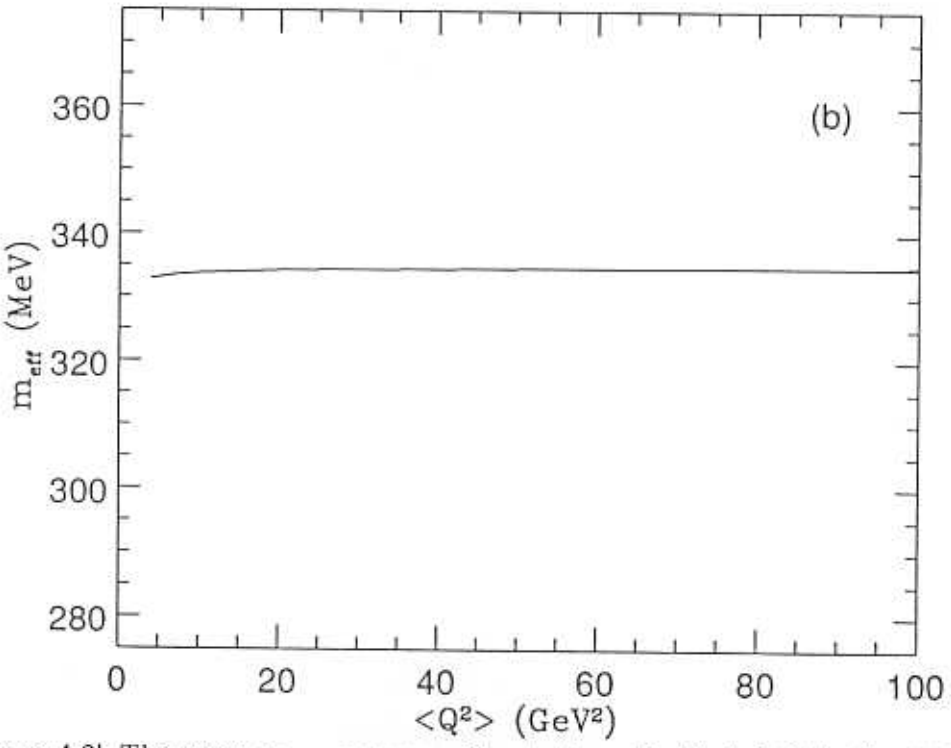

Figure 4.2b The mass, m_{eff}, corresponding to the ratio labeled (2) in fig. (4.2a) is plotted as a function of Q^2.

Using the Q^2-dependent parametrisation for the structure function $F_2(x)$ as given by ref [46], and the anomalous dimension contributions as given above, we calculate the ratios of the moments using eqn. (5.34) over a range of Q^2. The result is plotted in fig (4.1*a*). With leading order Q^2 corrections incorporated, we see that the ratio is almost flat, as it should be, and corresponds to a mass, $m_{eff} \simeq 300$ MeV (fig (4.1*b*)).

On the other hand, since ref. [46] parametrises the individual parton densities as functions of Q^2, we can also plot just the ratio of nonsinglet moments. In this case,

$$\frac{M_4^{NS}(Q^2)}{M_2^{NS}(Q^2)} \left[\frac{log\left(\frac{Q^2}{\Lambda^2}\right)}{log\left(\frac{\mu^2}{\Lambda^2}\right)}\right]^{d^4_{NS}-d^2_{NS}} = \frac{m^2_{eff}(\mu^2)}{M_P^2}, \tag{5.35}$$

which does not involve any approximation unlike in eqn.(5.34). The corresponding ratio is plotted in fig (4.2*a*); again, the ratio is practically Q^2 - independent and yields a slightly higher mass of $m_{eff} \simeq 330$ MeV (fig (4.2*b*)).

We can also correct the ratio $\mathcal{R}_G$ for finite Q^2 analogously; unfortunately, here individual parton densities are not parametrised with Q^2 dependence. The nonsinglet sector is very similar to the corresponding spin independent case; however, the anomalous dimensions in the singlet sector are very different owing to the axial anomaly [4]. In fact, the singlet anomalous dimension is zero for G_1. Retaining an equation analogous to eqn.(5.34), we have

$$\frac{G_3(Q^2)}{G_1(Q^2)} \left[\frac{log\left(\frac{Q^2}{\Lambda^2}\right)}{log\left(\frac{\mu^2}{\Lambda^2}\right)}\right]^{d^3-d^1} = \frac{m^2_{eff}(\mu^2)}{M_P^2}, \tag{5.36}$$

The data [1] is parametrised at $Q^2 \simeq 10 - 15$ GeV^2. For this value, the corresponding mass, corrected for finite Q^2, is $m_{eff} \simeq 318$ MeV (up from 283 MeV without Q^2 corrections). It is, in general, true that finite Q^2 corrections tend to increase the effective mass. Nevertheless, analysis of both spin independent as well as spin dependent structure function moments yields an almost identical effective mass for the quarks. Further, this mass lies in the range of the constituent quark mass.

5.4 Model Calculation of Matrix Elements.

In the previous section, we have seen that the relation between the hadronic matrix elements obtained using a simple assumption regarding the effective mass of quarks can

be made consistent with deep inelastic scattering data on moments. However, we were unable to calculate the moments themselves. We may now ask whether these matrix elements can be calculated in a model which admits an equation of motion similar to eqn. (5.15). The simplest model we can use is the MIT bag model with massive, non-strange quarks which is a modification of the original model and is due to Golowich [47].

In order to keep the chapter self contained, we quickly review the main results of the model. The quarks of mass m satisfy the equation

$$(i\partial\!\!\!/ - m)\,\psi_\alpha(x) = 0 \tag{5.37}$$

inside the bag and the following boundary conditions on the surface of the bag :

$$in.\gamma\psi_\alpha(x) = \psi_\alpha(x), \tag{5.38}$$

$$\sum_\alpha n.\partial[\bar{\psi}_\alpha(x)\psi_\alpha(x)] = 2B. \tag{5.39}$$

The normal $n_\mu = (0, -\hat{r})$ points into the bag at the surface and B is the bag pressure. .

We are interested in solutions corresponding to the proton (ground state) which is completely symmetric in spin, isospin and spatial indices, while being completely antisymmetric in the colour index. These solutions are functions of the three model parameters, namely, the quark mass, m, the bag radius, R, and ω, the mode frequency (here that of the quark ground state). These parameters are constrained by the linear boundary condition, eqn. (5.38). The model thus requires two input parameters to consistently determine m, R and ω. We choose these to be the (known) nucleon axial vector coupling constant, g_A and the charge square radius of the proton, $< r^2 >_p$ (See ref [47] for details of the calculation of g_A and $< r^2 >_p$ within the model).

It suffices for us to state that, given the input g_A and using the linear boundary condition, eqn. (5.38), the parameter ω and the product mR are fixed. The input $< r^2 >_p$

then separately determines m and R, so that the three parameters are uniquely fixed for a given g_A and $< r^2 >_p$. In fig (4.3), we show the variation of the mass (the parameter of interest) with the input parameters g_A and $< r^2 >_p$.

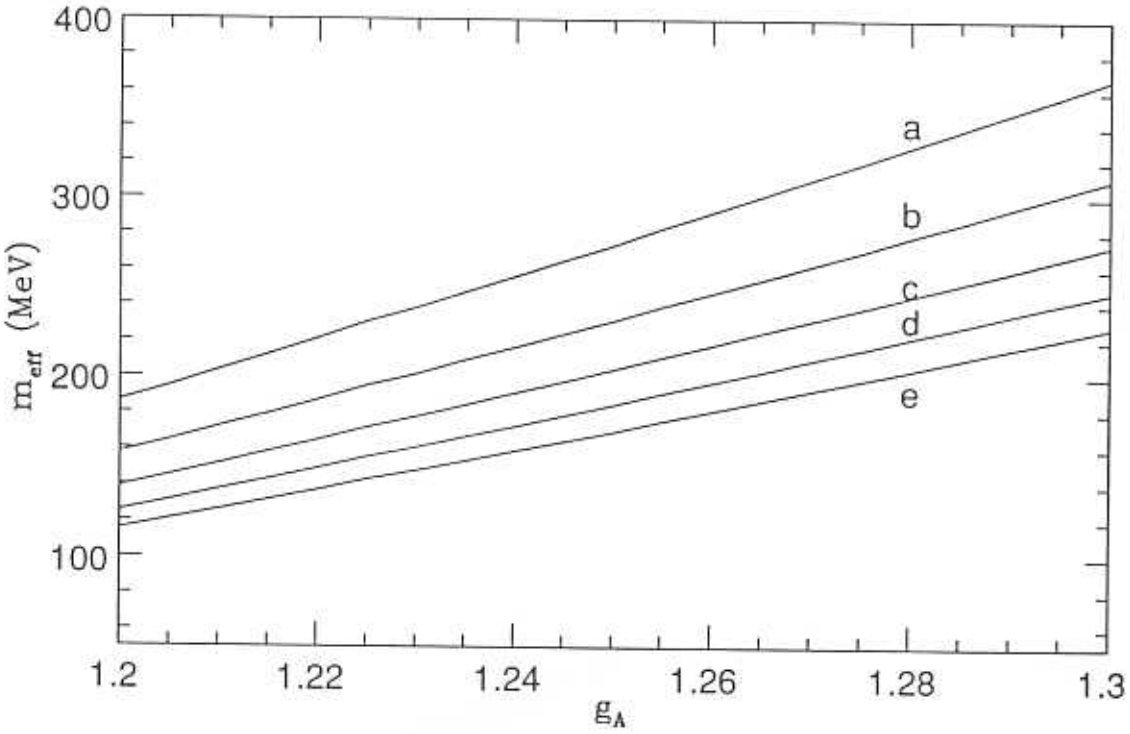

Figure 4.3 Variation of quark mass with the input parameters in the bag model. g_A is plotted along the x-axis, while labels (a) to (e) correspond to $(r^2)_p$ values from 0.25 to 0.65 in steps of 0.1.

We now proceed to evaluate the matrix elements occurring in the OPE which are required in the model calculation. Notice that the spin independent moments involve the evaluation of just one matrix element, namely, $\langle ps|\bar{\psi}\psi|ps\rangle$. Notice also that the normalisation convention of states is different in the bag model and the operator product formalism. (See Appendix D for details of normalisation conventions.) Then

$$\langle ps|\bar{\psi}_q\psi_q|ps\rangle_{OPE} = 2M_P f_q \left[\int d^3x\, \bar{\psi}(x)\psi(x)\right]_{bag}, \tag{5.40}$$

where the left hand side is the relevant matrix element and f_q is the flavour factor,

$$f_u = 2, \quad f_d = 1. \tag{5.41}$$

This matrix element evaluates to

$$\langle ps|\bar{\psi}_q(x)\psi_q(x)|ps\rangle_{OPE} = 2M_P f_q \left(\frac{\omega + 2mR\omega - 2mR}{2\omega^2 - 2\omega + mR}\right) \tag{5.42}$$

in the bag model. Therefore, substituting for the moments in eqns. (5.16) and (5.17), we

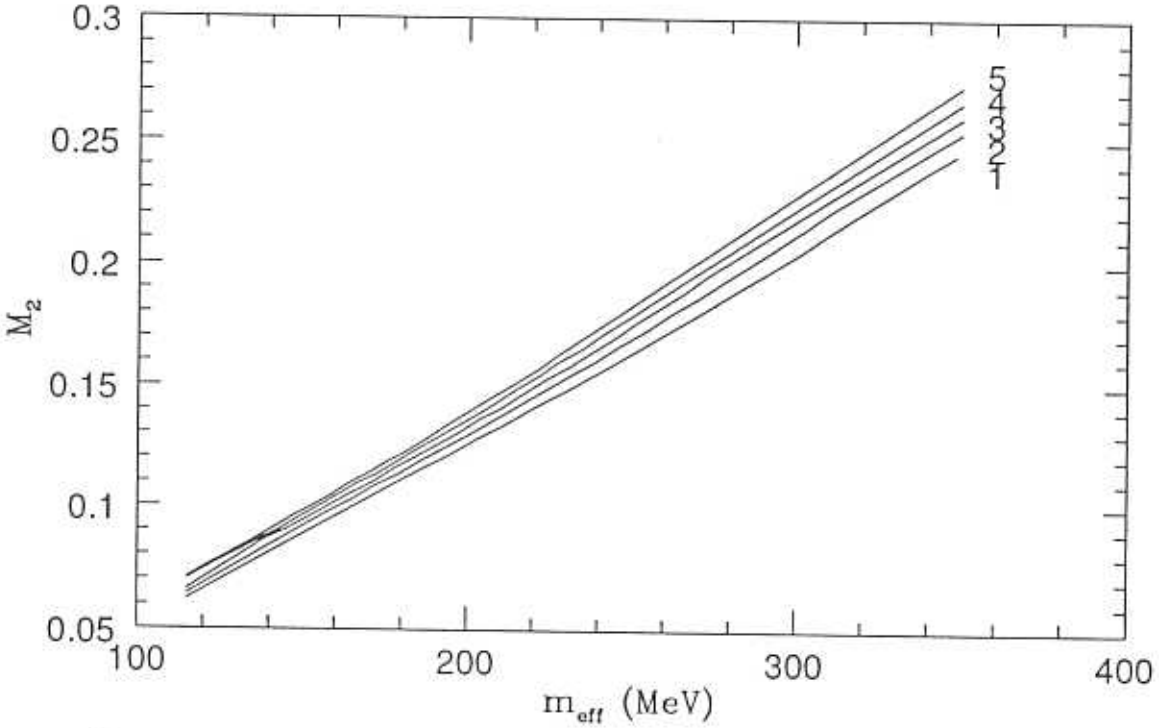

Figure 4.4 Variation of the moment M_2 with quark mass in the bag model. Labels (1) through (5) correspond to the same $\langle r^2\rangle_p$ values as in fig.(4.3).

get,

$$M_2 = \int dx\, F_2^P(x) = \frac{m}{M_P}\left(\frac{\omega + 2mR\omega - 2mR}{2\omega^2 - 2\omega + mR}\right),$$

and

$$M_4 = \int dx\, x^2\, F_2^P = \frac{m^2}{M_P^2} M_2. \tag{5.43}$$

Though M_2 and M_4 have complicated dependences on the bag model parameters, their ratio is model independent to the extent that it depends on the quark mass alone.

In the case of the moments of the spin dependent structure function, we need to evaluate two different matrix elements, namely $s^\mu \langle ps|\bar{\psi}\gamma_5\gamma_\mu\psi|ps\rangle$ which occurs in G_1 as well as in G_3 and $s^\mu\langle ps|\bar{\psi}\gamma_5\partial_\mu\psi|ps\rangle$ which occurs in G_3 alone. The first of these is the standard operator needed for the calculation of g_A. This is given by

$$\begin{aligned} \Delta q &= -\frac{s^\mu}{s^2}\langle ps|\bar{\psi}_q\,\gamma_5\gamma_\mu\,\psi_q|ps\rangle_{OPE} \\ &= g_q\left[\int d^3x\,\bar{\psi}(x)\,\gamma_3\gamma_5\,\psi(x)\right]_{bag}, \end{aligned} \tag{5.44}$$

where $s^\mu = (0,0,0,2M_p)$ and g_q for u and d are

$$g_u = \frac{4}{3},\quad g_d = -\frac{1}{3} \tag{5.45}$$

Substituting for G_1 in eqn.(5.13), we find

$$G_1 = \frac{1}{18}(4\Delta u + \Delta d) = \frac{5}{54}\left(\frac{2\omega^2 + 4mR\omega - 3mR}{2\omega^2 - 2\omega + mR}\right). \tag{5.46}$$

Then, using eqn. (5.18), the first term in G_3 is

$$G_3^{(1)} = \frac{m^2}{M_P^2}\,G_1. \tag{5.47}$$

The second term of G_3 is somewhat more nontrivial. We can write this as

$$\begin{aligned} G_3^{(2)} &= -\frac{m}{s^2 M_P^2}\sum_{q=u,d} e_q^2\, s^\mu\,\langle ps|\bar{\psi}\,\gamma_5\,i\partial_\mu\,\psi_q|ps\rangle_{OPE} \\ &= \frac{m}{M_P^2}\sum_q e_q^2\, g_q\left[\int d^3x\,\bar{\psi}(x)\,\gamma_5\,i\partial_3\,\psi(x)\right]_{bag} \\ &= -\frac{10m}{54M_P^2R}\,\frac{\omega^2 - m^2R^2}{2\omega^2 - 2\omega + mR}. \end{aligned} \tag{5.48}$$

The ratio of the second and first terms in G_3 is then given by

$$\mathcal{R} = \frac{G_3^{(2)}}{G_3^{(1)}} = -\frac{2}{mR}\frac{\omega^2 - m^2R^2}{2\omega^2 + 4mR\omega - 3mR}, \tag{5.49}$$

which is entirely independent of $< r^2 >_p$ and is fixed for a given g_A.

To obtain numerical estimates for the moments $M_{2,4}$ and $G_{1,3}$ in the bag model, we have to first fix the model parameters. From fig (4.3) we see that for a given g_A, the effective quark mass decreases with increasing $< r^2 >_p$. In fig (4.4), we have plotted M_2 as a function of the quark mass for various values of $< r^2 >_p$ from 0.25 to 0.65. Within

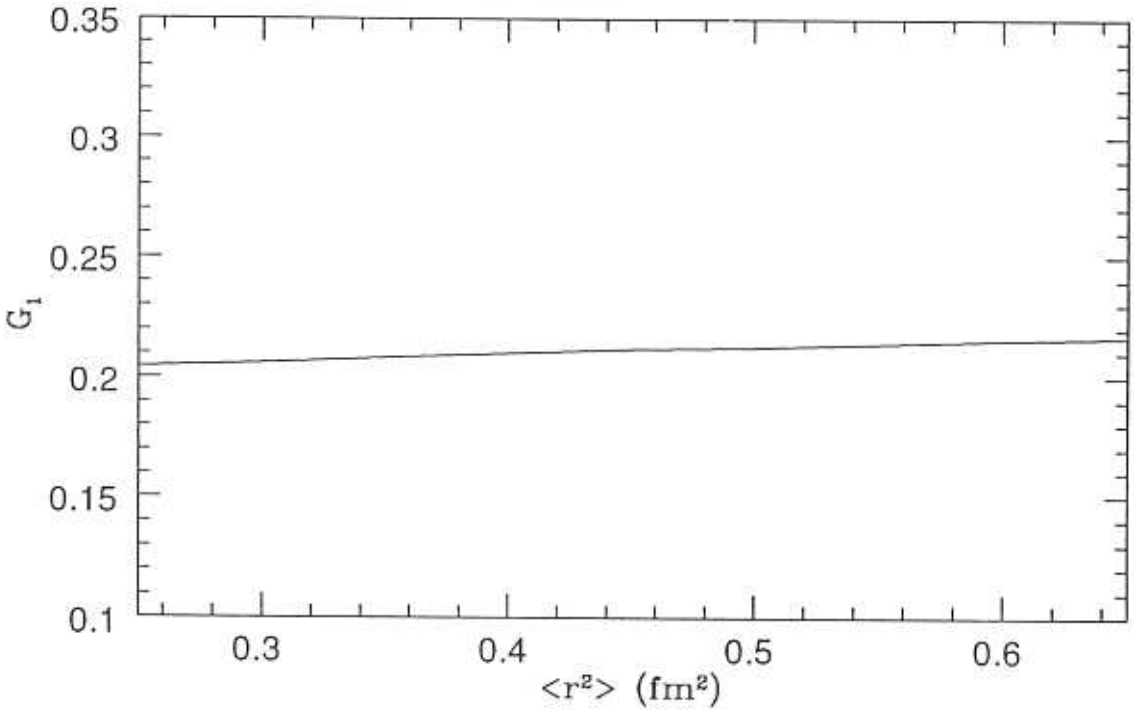

Figure 4.5 G_1 lotted against $(r^2)_p$ for a fixed quark mass of 230 MeV in the bag model.

this range, we see that M_2 is very sensitive to the mass but not to $< r^2 >_p$. We now fix the mass parameter in the bag model using the well measured value of M_2:

$$M_2 = 0.152 \pm 0.011.$$

It is immediately apparent from fig (4.4) that, taking into account the errors on M_2, the effective mass lies in the fairly narrow range

$$210 \leq m_{eff}(MeV) \leq 240. \tag{5.50}$$

This is lower than the $m \cong 300$ MeV obtained in the previous section in a model independent manner. This difference is most probably due to the simplistic nature of our model which considers only two flavours, u and d. Obviously our estimate for M_4 will be lower; M_4 typically varies between 0.008 and 0.01, which is lower than the observed value of 0.014.

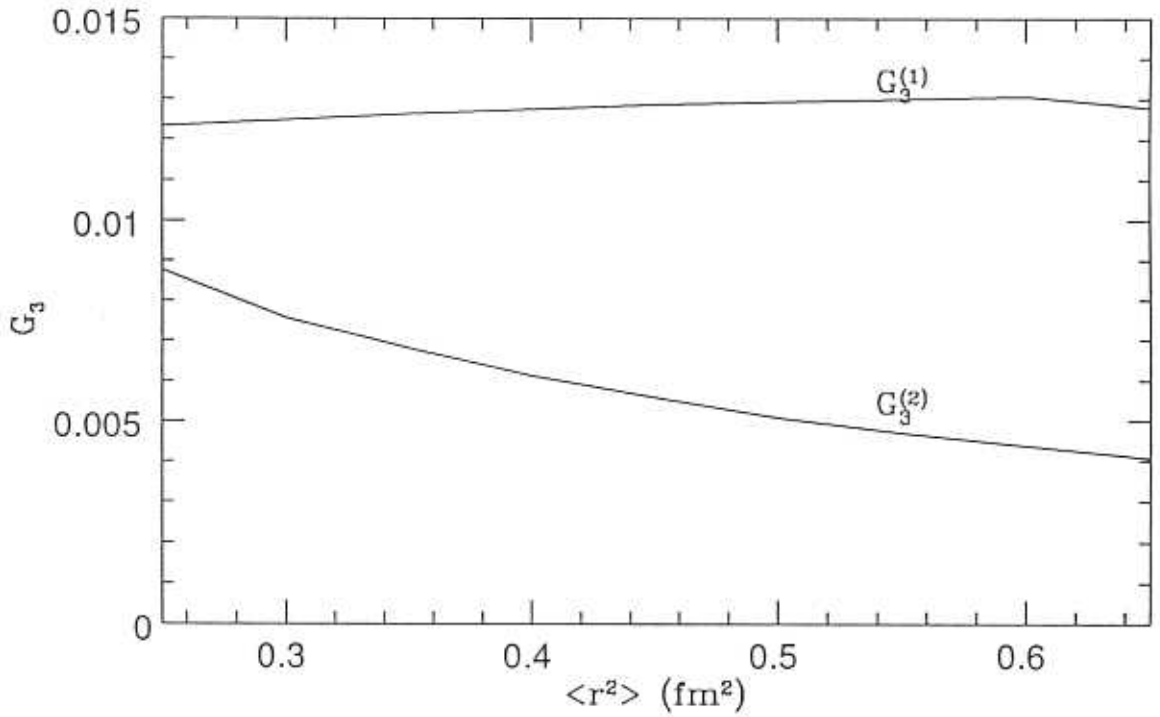

Figure 4.6 The two terms of G_3 given by eqn. (4.10,4.11) plotted against $< r^2 >_P$ for the same quark mass as in fig 4.5 for G_1.

The point we wish to emphasise here is that one can, in fact, calculate the moments of $F_2(x)$ in the bag model, provided the quark mass is taken to be large, somewhat closer to the constituent quark mass.

Using the information from M_2, we therefore fix the effective quark mass to be around 230 MeV, which is the mean value of the range given in eqn.(5.50). For $g_A = 1.254$, the

measured value, this corresponds to $< r^2 >_p^{0.5} = 0.6 fm$, which is slightly smaller than the observed value of 0.83 fm..(See fig (4.3).) With this mass value kept fixed at 230 MeV, we calculate G_1 as a function of $< r^2 >_p$, or, equivalently, g_A apart from a multiplicative factor. The result is shown in fig (4.5). We see that G_1 is a constant, independent of $< r^2 >_p$, to a very good approximation,

$$G_1 = 0.21 \pm 0.007,$$

where the error bars include the variation in the mass range in eqn.(5.50). The first two terms of $G_3(G_3^{(1),(2)})$ are shown separately in fig (4.6), again, as a function of $< r^2 >_p$.

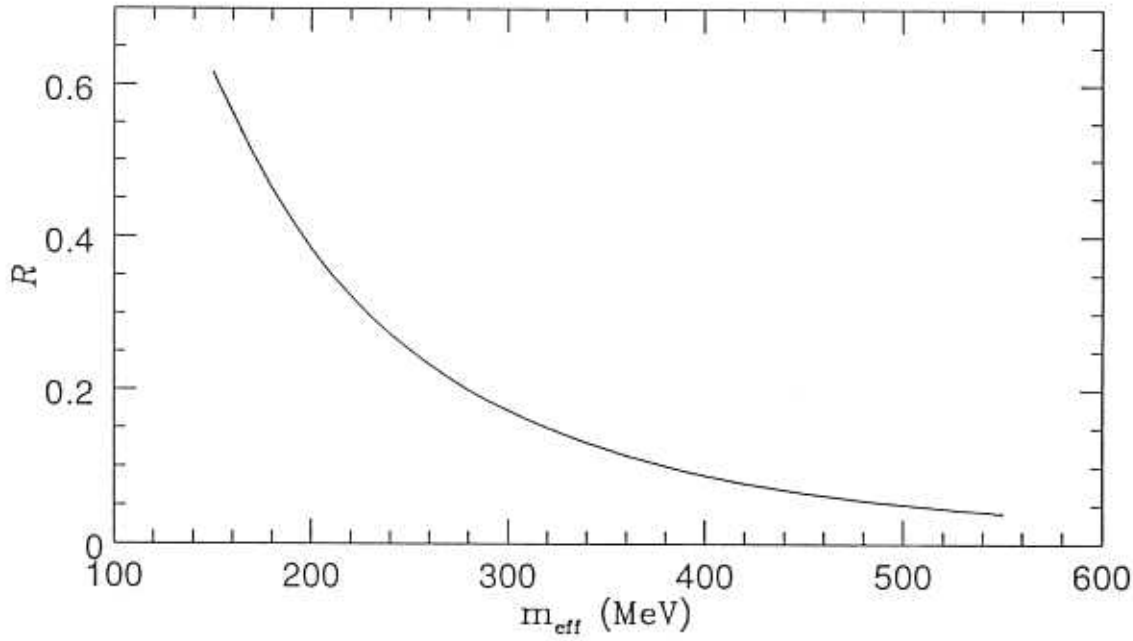

Figure 4.7 Modulus of the ratio $\mathcal{R} = G_3^{(2)}/G_3^{(1)}$ plotted as a function of quark mass in the bag model.

Like G_1, the first term of G_3 is constant and equals 0.012 ± 0.002. The second term, while being smaller than the first term of G_3, is however, highly model dependent. It is therefore possible to choose inputs so that this term can be ignored. Close to the experimental value of $< r^2 >_p$ $(= 0.6889 fm^2)$ and $g_A = 1.31$, the second term of G_3 is about 30% of the first

for a mass of about 230 MeV; moreover, $G_3 \simeq 0.01$, quite close to that obtained from parametrisation of the data. Fig(4.7) shows the ratio of the two terms in G_3 for different quark masses. For larger masses, the second term is indeed small and can be ignored. Hence, the spin dependent moments require a slightly different mass range than the spin independent moments to fit the data. However, the effective mass one requires is still in the range 230 to 300 MeV, and not smaller. This justifies our neglecting the second term in the operator product expansion for the third moment of $g_1(x)$. However, a unique estimate of G_3 cannot be obtained by fixing the mass range through M_2; it depends on the choice of one more input parameter, either g_A or $< r^2 >_p$. However, once the second term is dropped, G_3 depends only on the mass range fixed by M_2.

We now return to a discussion of the bag model value obtained for G_1,

$$G_1 = 0.21,$$

which is very different from that quoted in Section 3 where

$$G_1 = 0.125$$

The reason for the discrepancy in G_1, and the lack of any such discrepancy in the case of the spin-independent structure function is not hard to find, especially when we notice that the Ellis-Jaffe Sum Rule [48] gave a strikingly similar result:

$$G_1 = 0.19.$$

Recall that this value was predicated upon the assumption that the strange quark contribution was zero and using $SU(3)$ Current Algebra. There is the added complication of the gluon contribution to this moment through the ABJ anomaly, which has been widely discussed . This contribution adds to the singlet piece in the first moment of $g_1(x)$, which includes the sea quark contribution and there is no way in which this can be handled

within the bag model. Hence, the bag model reproduces the Ellis Jaffe result rather than the observed value. It is claimed that it is this anomaly contribution which causes the experimental value of G_1 to deviate so sharply from the Ellis Jaffe result. If this is true, and, indeed, such is expected to be the solution to the "proton-spin-puzzle", then the contribution of the anomaly - induced gluon term to the higher moments of $g_1(x)$ must be small. This is because the anomaly contribution is a small$-x$ effect, as it affects the singlet parton densities which are non-negligible only at small x-values, typically $x \leq 0.2$. On the other hand, by taking higher moments of $g_1(x)$, the small$-x$ contributions naturally get suppressed; hence, we would expect the value of the third moment of $g_1(x)$ obtained in the bag model to be closer to the experimental value than the first moment is to its corresponding measured value. We find this to be indeed the case, with G_3 approaching $\simeq 0.008 - 0.010$, as mentioned before.

Hence, our simple model with massive non-strange quarks already reflects (albeit somewhat crudely) the complicated nature of the contribution to the small$-x$ region of $g_1(x)$. It would seem that the failure of this model to predict correctly the first moment of the spin - dependent structure function $g_1(x)$, while providing reasonable estimates in the spin - averaged case, combined with the knowledge that gluonic effects are not included in this model, points to the inclusion of the anomaly contribution as being a prime candidate to the resolution of the discrepancy. It also seems reasonable to expect a generic quark model, or, in fact, *any* non perturbative model capable of evaluating the moments of $g_1(x)$, to give fairly accurate predictions for the third moment and to reproduce the Ellis Jaffe result for the first moment.

5.5 Conclusion

We have analysed the moments of the (experimentally) known proton structure functions, $F_2(x)$ and $g_1(x)$, via the operator product expansion. An effective quark mass, $m_{eff}(\mu^2)$, is introduced through an equation of motion, where μ^2 is the scale at which the operators whose matrix elements occur in the operator product expansion are renormalised. Being

non -perturbative matrix elements, we expect that the corresponding mass occurring in these is of the order of the constituent quark mass rather than the current quark mass. We set up ratios of the various moments and find that these ratios are independent of the unknown matrix elements and depend only on the mass parameter introduced through the equation of motion. Hence, we are able to evaluate this mass, which turns out to be around 300 MeV, thus reinforcing our expectation. This statement is almost model independent in the spin independent case and valid, in the spin dependent case, only in some reasonable, but model dependent, limit.

Having gained confidence in our ansatz, we predict the absolute values of the moments themselves in a bag model with massive quarks. The model we have chosen is not necessarily unique, but is simple enough to allow these calculations to be made. It turns out that the bag model predictions can be made consistent with the measured values of the moments, provided that a mass of around 230 to 300 MeV is chosen, which is approximately consistent with the model independent analysis. While the spin independent case is somewhat straight forward, the spin dependent analysis in the bag model is not. Here, the first moment of the spin- dependent function, $g_1(x)$, reproduces the value calculated by Ellis and Jaffe, while the third moment is consistent (within errors) with the measured ones. We can understand the discrepancy in the first moment as due to the absence of the anomaly term in the bag model. Since this is basically a small$-x$ contribution, this conclusion is consistent with the fact that the third moment comes out all right, as higher moments suppress small$-x$ contributions.

Turning the problem around, it is well known that hadron spectroscopy of the low-lying baryon states is somewhat insensitive to the quark mass parameter in the bag model. Our analysis gives credence to the view that a bag model with massive quarks is probably closer to reality.

Chapter 6

Quark Masses and Polarised Electroproduction

6.1 Introduction

There has been considerable amount of investigation of the spin content of proton by studying the proton matrix element of axial vector currents (both singlet as well as non-singlet). This includes explicit evaluation of these matrix elements in some models. In this context, Cheng and Li [51] have studied the singlet term using the anomalous Ward-Takahashi identity and π^0 dominance for the divergence of the flavour-triplet axial current. Their result on gluonic contribution to the proton spin using EMC data has a different sign than one would naively expect. Brodsky, Ellis and Karliner [50] argued that the spin contribution coming from quarks to the proton spin is negligibly small to leading order in the $1/N$ expansion (where N is the number of colours in large-N QCD) in the Skyrme model (with axial anomaly). Contrary to this result Cohen and Banerjee [55] have showed that the spin content is of order unity in the $1/N$ expansion if one incorporates the effects of coupling of η' to other mesons in a manner suggested by the $U(1)$ anomaly. The singlet matrix element has also been evaluated by Hatsuda [54] in large-N chiral dynamics satisfying the anomalous Ward-Takahashi identity. They have showed that weak η' can reproduce the recent EMC data. In a paper by Jaffe and Manohar [24], various comments are made in this context.

In this chapter it is shown that in a theory with strictly massless quarks, the usual operator product expansions for polarised electroproduction, when combined with U(1)

anomaly, predict a vanishing singlet "quark contribution" [43]. The nonsinglet contribution is ambiguous in such a case unless Goldstone modes are introduced explicitly. In this note we show that finite quark masses, however small, change the picture completely and lead to a result essentially in agreement with Altarelli & Ross [7] and Carlitz, Collins & Mueller [9]. Some important criticisms made by Jaffe & Manohar [24] are shown not to arise in the case of non–zero quark masses. Our analysis singles out the operator that directly measures the "quark content" of proton spin.

6.2 Massless Theory

The central object of interest in the study of polarised electroproduction is the amplitude for the forward Compton scattering of polarised virtual photons off polarised protons [1]. The technique of Operator Product Expansions (OPE) [17] relates the well–known quantity $\int g_1^{ep}(x,Q^2)\,dx$, measured via the polarisation asymmetries, to

$$\sum_n E_n(Q^2)\,\langle p,s|O_n|p,s\rangle_{Q^2} \tag{6.1}$$

where $E_n(Q^2)$ are the so–called coefficient functions and O_n are the operators occurring in the OPE. Hey and Mandula [3], and recently Altarelli and Ross [7] on the one hand, and Carlitz, Collins and Mueller [9] on the other, have convincingly demonstrated that the only relevant operators for eqn. (6.1) are

$$\frac{1}{2}\sum_i e_i^2\,\overline{q_i}\,\gamma^\mu\gamma^5 q_i \tag{6.2}$$

where q_i are various quark fields (ψ_q) and e_i their respective electric charges.

Let us first consider the case of exactly massless quarks. In this case the octet axial currents are conserved and the singlet axial current has a $U(1)$ anomaly as expressed by

$$\partial_\mu A^{\mu,\alpha} = 0 \qquad \alpha = 1,\cdots,8 \tag{6.3}$$

$$\partial_\mu A^{\mu,0} = 3\,\frac{\alpha_s}{2\pi} tr G^{\mu\nu}\tilde{G}_{\mu\nu} \tag{6.4}$$

with A^α_μ is the axial vector, α_S the (scale dependent) strong coupling constant and $G^{\mu\nu}$ the gluon field strengths. Now consider the matrix element defined by

$$\langle p' |\, A^{\mu,\alpha}(x)\, | p \rangle = e^{iq\cdot x}\, a^{\mu,\alpha}(q) \tag{6.5}$$

where $|p\rangle, |p'\rangle$ are single particle states and $q = p' - p$ is the momentum transfer. The conservation law (6.3) then implies

$$q_\mu a^{\mu,\alpha}(q) = 0\,, \tag{6.6}$$

for all α. Now there are two possibilities for the forward matrix element $a^{\mu,\alpha}(0)$: either it exists or it does not (by the latter we mean that it is either singular at $q = 0$ or the latter is ambiguous). In the first case, we can perform a small-q expansion of (6.6) to yield

$$q_\mu a^{\mu,\alpha}(0) + q_\mu q^\nu \left.\frac{\partial a^{\mu,\alpha}}{\partial q^\nu}\right|_{q=0} + \cdots = 0 \tag{6.7}$$

But

$$a^{\mu,\alpha}(0) = \langle p |\, A^{\mu,\alpha}\, | p \rangle = S^\mu X^\alpha \tag{6.8}$$

as follows from the fact that $a^{\mu,\alpha}$ is an axial vector; S^μ is the spin four-vector of the single particle state $|p\rangle$. Combining (6.8) with (6.7) yields

$$a^{\mu,\alpha}(0) = 0 \tag{6.9}$$

implying the vanishing of $g_8(g_A)$. A similar reasoning for conserved vector currents, however, does not require $v^{\mu,\alpha}(0) = 0$, as $v^{\mu,\alpha}(0)$ are proportional to p^μ and $p\cdot q \sim O(q^2)$. But as is well known, there is another realization of (6.6) which avoids (6.9), namely, the Goldstone mode. According to this, there is a massless singularity in $a^{\mu,\alpha}(q)$ as expressed by

$$a^{\mu,\alpha}(q) = g_8\, \overline{U}(p')\, \gamma^\mu \gamma^5 \lambda^\alpha U(p) - f_{gNN8} \frac{q^\mu}{q^2} \overline{U}(p') \gamma^5 \lambda^\alpha U(p) \tag{6.10}$$

Now (6.6) implies

$$2\, M_N\, g_8 = f_{gNN8} \tag{6.11}$$

which is the well–known Goldberger–Treiman relation. But now $a^{\mu,a}(0)$ is ambiguous due to $\overline{U}(p')\gamma^5\lambda^a U(p)$ vanishing as $S\cdot q$ *i.e.*, one encounters

$$\lim_{q\to 0}\frac{q^\mu S\cdot q}{q^2} \tag{6.12}$$

For the singlet axial current, one has, in place of (6.6),

$$q_\mu a^{\mu,0}(q) = \langle p'|3\frac{\alpha_s}{2\pi}trG^{\mu\nu}\tilde{G}_{\mu\nu}|p\rangle. \tag{6.13}$$

In this case three options present themselves

(i) only the Wigner mode is realised (no Goldstone pole),

(ii) Goldstone pole is realised but without the resolution of the $U(1)$ problem in *QCD*,

(iii) Goldstone pole in the flavour singlet channel is massive due to the $U(1)$ anomaly.

It is easy to verify that for both (i) and (iii) $a^{\mu,0}(0)$ exists and is given by

$$a^{\mu,0}(0) = 3S^\mu G(0) \tag{6.14}$$

where $G(q^2)$ is defined by

$$\langle p'|\frac{\alpha_s}{2\pi}trG^{\mu\nu}\tilde{G}_{\mu\nu}|p\rangle = S\cdot qG(q^2) \tag{6.15}$$

For case (ii), one obtains

$$a^{\mu,0}(0) = S^\mu\left[3G(0) - f_{\eta'}\right] \tag{6.16}$$

where $f_{\eta'}$ is the residue of the Goldstone pole in the singlet axial current. These results agree with the ones given by Jaffe and Manohar [24], but we have not used any singular limiting procedures in obtaining these results.

One of the main criticisms of Jaffe and Manohar regarding the ideas of refs.[7, 9] is based on (6.14) and (6.16); they argue that (6.14) actually implies that $\sum_i \Delta q_i$ of ref.[9] and equivalently $\Delta\Sigma$ of ref.[7] vanishes. Further, if the resolution of the $U(1)$ puzzle in *QCD* is not assumed, Jaffe and Manohar [24] argue that (6.16) gives the misleading impression of a nonvanishing 'quark component' of the proton spin. Thus one sees that

strictly massless quarks yield, upon assuming that the $U(1)$ puzzle in QCD is solved, a vanishing 'quark component' in the singlet part of (6.2) while at the same time yielding an ambiguous result for the nonsinglet part of (6.2). Because of this the OPE in its standard form cannot be used to analyse the polarised electroproduction data. It may be possible to do so by explicitly introducing the octet of Goldstone particles into the picture and by interpreting $\overline{q}\gamma^\mu\gamma^5\lambda^\alpha q$ as $A^{\mu,\alpha} - f\partial^\mu\phi^\alpha$ where $A^{\mu,\alpha}$ and ϕ^α are the octet axial current and the Goldstone fields respectively (the ideas of ref.[50] may also be viewed in this light).

6.3 Massive theory

We shall now show that finite quark masses, however small, alter these conditions totally and automatically yield a nonvanishing 'quark component' to the proton spin. With nonvanishing quark masses, the divergence equations for various axial currents read

$$\partial_\mu A^{\mu,3} = 2im(\overline{u}\,\gamma^5\,u - \overline{d}\,\gamma^5\,d) \tag{6.17}$$

$$\partial_\mu A^{\mu,8} = 2im(\overline{u}\,\gamma^5\,u + \overline{d}\,\gamma^5\,d) - 4im_s\overline{s}\gamma^5 s \tag{6.18}$$

$$\partial_\mu A^{\mu,0} = 2im(\overline{u}\,\gamma^5\,u + \overline{d}\,\gamma^5\,d) + 2im_s\overline{s}\gamma^5 s + 3\frac{\alpha_s}{2\pi}trG^{\mu\nu}\tilde{G}_{\mu\nu}, \tag{6.19}$$

where we have used $m_u = m_d = m$. To highlight the nature of our conclusions let us first assume that all quark masses (including the strange quark) are small and that the $U(1)$ problem of QCD is resolved. Now the Goldstone poles are all displaced from zero by small amounts *i.e.*, $m^2_{Goldstone} \sim m_q$ except the flavour singlet pole. Because of the proximity of the nonsinglet Goldstone pole to $q^2 = 0$, pole dominance is reasonable. However, in our later arguments we shall actually never use pole dominance. Then,

$$a^{\mu,3}(q) = g_A\overline{U}(p')\,\gamma^\mu\gamma^5\,U(p) - fg_{NN8}\frac{q^\mu}{q^2+m_\pi^2}\overline{U}(p')\,\gamma^5\,U(p), \tag{6.20}$$

such that

$$a^{\mu,3}(0) = g_A\,S^\mu \tag{6.21}$$

unambiguously. The divergence relation (6.17) yields

$$2m_N g_A(q^2) - fg_{NN\pi}\frac{q^2}{q^2+m_\pi^2} = g_\pi^5(q^2), \tag{6.22}$$

where

$$\langle p'|\, 2im(\overline{u}\,\gamma^5\, u - \overline{d}\,\gamma^5\, d)|p\rangle = g_\pi^5(q^2)\overline{U}(p')\,\gamma^5\, U(p) \tag{6.23}$$

Evaluating (6.22) at $q^2 = 0$, one gets

$$2m_N g_A(0) = g_\pi^5(0) \tag{6.24}$$

Defining

$$\langle p'|\,\overline{q}\,i\,\gamma^5\, q|p\rangle = \frac{1}{2m_N} S\cdot q Y_q, \tag{6.25}$$

(6.24) is rewritten as

$$2m_N g_A(0) = 2m\,(Y_u - Y_d) \tag{6.26}$$

This is essentially the Bjorken sum rule and clearly indicates that even as $m \to 0$, it is not correct to drop operators like $2m\overline{q}\gamma^5 q$. The $PCAC$ condition is reproduced by substituting the Goldberger–Treiman relation (6.11) in (6.23) to obtain

$$g_\pi^5(q^2) = f g_{NN8}\frac{m_\pi^2}{q^2 + m_\pi^2} \tag{6.27}$$

which is equivalent to

$$\partial_\mu A^{\mu,3} = m_\pi^2 f \Pi^+ \tag{6.28}$$

where Π^+ is the pion field. Likewise, one derives

$$a^{\mu,8}(0) = g_8\, S^\mu \quad \text{and} \quad a^{\mu,0}(0) = g_0\, S^\mu \tag{6.29}$$

where

$$\begin{aligned} 2m_N g_8(0) &= 2m\,(Y_u + Y_d) - 4m_s\, Y_s \quad \text{and} \\ 2m_N g_0(0) &= 2m\,(Y_u + Y_d) + 2m_s\, Y_s + 3(2m_N)\, G(0). \end{aligned} \tag{6.30}$$

The object of interest in polarised electroproduction experiment [1], are the proton matrix elements of $\frac{4}{9}\overline{u}\gamma^\mu\gamma^5 u + \frac{1}{9}\overline{d}\gamma^\mu\gamma^5 d + \frac{1}{9}\overline{s}\gamma^\mu\gamma^5 s$, or equivalently, that of $\frac{2}{9}a^{\mu,0} + \frac{1}{6}a^{\mu,3} + \frac{1}{18}a^{\mu,8}$. The forward matrix element of this operator is given by

$$\int g_1^{ep}(x)dx = \frac{1}{2}\left[\frac{2}{9}a^{\mu,0}(0) + \frac{1}{6}a^{\mu,3}(0) + \frac{1}{18}a^{\mu,8}(0)\right]\frac{S_u}{S^2} \tag{6.31}$$

which, on combining with (6.21) and (6.29), yields

$$\frac{1}{2}\left[\frac{1}{6}g_A + \frac{1}{18}g_8 + \frac{1}{9m_N}\left\{2m(Y_u + Y_d) + 2m_s Y_s + 6m_N G(0)\right\}\right] \tag{6.32}$$

Since $2mY_u$, $2mY_d$, $2m_sY_s$ all approach finite quantities, we see that the isosinglet term $\{\ldots\}$ derives contributions from these in addition to the "gluonic" contribution $3G(0)$. In the treatment of ref.[24] these terms have been ignored (as they do not exist in a theory with massless quarks!).

It should be stressed that neither the Goldberger–Treiman relation nor pole dominance is really required for deriving (6.32). As long as the matrix elements of the axial current are of the type

$$\overline{U}(p')\,\gamma^\mu\gamma^5\,\lambda^a U(p) + \text{terms vanishing as } q \to 0,$$

(6.32) still follows.

Hence the results are easily extended to the realistic situation where m_s is large compared to m. Phenomenologically m_s is seen to be comparable to Λ_{QCD} (say in the $\overline{MS}$ scheme).

We now show that Y_i have the same significance as Δq_i of [9]. To see this, note that the QCD anomaly is flavor independent *i.e.*

$$\partial_\mu \bar{q}_i\,\gamma^\mu\gamma^5\,q_i = 2im_i\bar{q}_i\,\gamma^5\,q_i + \frac{\alpha_s}{2\pi}trG^{\mu\nu}\tilde{G}_{\mu\nu} \tag{6.33}$$

Hence

$$2m_N\,q_\mu a_i^{\mu,0}(q) = S\cdot q\left[2m_iY_i + 2m_NG(q^2)\right]. \tag{6.34}$$

Small-q expansion yields

$$2m_NX_i = 2m_iY_i + 2m_NG(0). \tag{6.35}$$

Our $G(0)$ coincides exactly with $-\Delta g$ of [7, 9] as can be realised from

$$\frac{\alpha_s}{2\pi}trG^{\mu\nu}\tilde{G}_{\mu\nu} = \partial_\mu k^\mu \tag{6.36}$$

and hence

$$\langle p|k^\mu|p\rangle = S^\mu G(0). \tag{6.37}$$

Thus we see that the Y_i introduced here are precisely the same as Δq_i of [9] and related quantities in [7].

Cheng and Li [51] have also made an analysis similar to ours *i.e.*, keeping quark masses finite to obtain equations similar to (6.35). But they have not carefully defined the $q \to 0$ limit and also their motivations are different from ours which is to highlight the importance of nonzero quark masses for a consistent analysis of the OPE for polarised electroproduction.

Hence we see that treating quark masses consistently yields a picture in disagreement with what is proposed in ref[24]. Further our analysis indicates that the 'quark content' of the proton spin is directly measured by the slightly nonforward matrix elements of the operator $2m_i\bar{q}_i\gamma^5 q_i$. It is proposed here that Lattice studies should measure this quantity. It should be pointed out that for very heavy quarks, one expects

$$Y_i \to -G(0) \tag{6.38}$$

as the results of [52] have shown that

$$X_i \to O(m_i^{-3}). \tag{6.39}$$

One may wonder how this is compatible with the interpretation of Y_i as the 'quark content' of the proton spin as the implication of (6.38) is that even very heavy flavors contribute. But the 'gluonic contribution' per new flavor being exactly $G(0)$, it is but a matter of semantics to call Y_i the heavy quark contribution to proton spin.

We believe that a recent lattice simulation [53] of the singlet contribution may support this cancellation mechanism. In this calculation it was found that Δg was zero within the quenched approximation. But the quenched approximation is equivalent to quark masses being very high. Thus this null result may not have a direct bearing on the issue of the proton spin. According to us, the more relevant operator to be measured is $2m\bar{q}\gamma^5 q$ and this should be done outside the quenched approximation. Of course, non–forward matrix elements of this operator have to be evaluated and we do not know how difficult it may be to carry out this measurement on the lattice.

We emphasise that the smallness of the quark masses was initially assumed to justify Goldstone pole dominance. Subsequently we saw that our results hold even in the absence of pole dominance and of the validity of the Goldberger–Treiman relation. Thus (6.32) should be valid even when the quark masses are not small. In particular, there is no reason to believe that the strange quark mass is negligible. In fact, the *EMC* data directly points to nonnegligible effects of the strange quark mass. To see this, note that the experimentally observed $\int g_1^{ep}(x)\,dx$ along with g_A and g_8 implies

$$\begin{aligned} \frac{m_s}{m_N} Y_s + G(0) &= -0.16 \qquad \text{or} \\ \Delta s - (\alpha_s/2\pi)\Delta g &= -0.16. \end{aligned} \tag{6.40}$$

(Here we have used the numbers as given in [24].) If the strange quark were to be treated as massive, this number would be close to zero [24]. It should further be noted that bilinears made out of heavy quark operators cannot all be set to zero as that would contradict the anomaly equation. But there is a good cancellation between the Δq_i and the $\frac{\alpha_s}{2\pi}\Delta g$ terms for heavy quarks making the bilinear $\bar{q}_h \gamma^\mu \gamma^5 q_h$ nearly vanish in the proton state. This cancellation can be understood on recalling that in certain regularisation schemes the entire anomaly is reproduced by the heavy fermion regulators.

The Ellis–Jaffe analysis [48, 52] was made on the assumption that $m_s Y_s/m_N + G(0)$ is zero or equivalently that the strange quarks are extremely heavy. The *EMC* data rules out such a possibility. However, it does not straight away rule out either Y_s or $G(0)$ being zero. But a vanishing $G(0)$ poses the 'problem' of the quarks not contributing to the proton spin. On the other hand $Y_s = 0$ yields $\Delta u + \Delta d$ to be close to the expectation based on the naive quark model (Ellis–Jaffe result). In ref[9] the expectation was raised that after screening out the 'gluonic contributions', $\sum_i \Delta q_i$ should come out close to the quark model result. It is clear from the remarks made above that this can happen only if $Y_s = 0$ or equivalently if

$$G(0) = 3\left[\int dx\, g_1(x) - \int_{EJ} dx\, g_1(x)\right] \tag{6.41}$$

where $\int_{EJ}$ is the value given by the Ellis–Jaffe sum rule. Thus the test for the issues raised by the *EMC* data is an independent experimental verification of either $Y_s = 0$ or of $G(0)$ being given by (6.41). While vanishing of $m_s Y_s/m_N + G(0)$ can be understood in terms of the strange quarks being very heavy, there appears to be no natural principle guaranteeing $Y_s = 0$.

6.4 Conclusion

It has been demonstrated that quark contribution to proton spin is found to be zero in a strictly massless theory. This is achieved by using U(1) anomaly. Also, the nonsinglet contributions are ambiguous unless Goldstone modes are introduced. Non-zero quark masses yield the result consistent with the known results. Thus, this analysis emphasises the importance of mass in studying DIS processes in the OPE formalism.

Chapter 7

Conclusions

In this thesis, we have studied hadronic processes in order to understand the structure of hadrons. Specifically , we have studied aspects of the proton structure both through perturbative as well as nonperturbative techniques. Recent results on the polarised proton structure function measured using electromagnetic probe in deep inelastic scattering experiments (DIS), gave the surprising result that quarks and antiquarks are not responsible for the proton spin to lowest order (i.e., in the very high energy limit). This was obtained using naive parton model and/or operator product expansion. This result relies on the validity of the extrapolation of the data in the Bjorken x space and low energy data on the axial vector coupling constants g_A and g_8. This result implies the existence of a significant gluonic contribution to the spin of the proton, which grows logarithmically as $\log(Q^2/\mu^2)$ at large Q^2. But the measured structure function $g_1(x, Q^2)$ does not tell anything about individual valence and sea quark contributions to the proton spin.

In this thesis, we have shown that informations regarding singlet, as well as valence spin structure of the proton can be got by measuring both parity conserving ($g_1(x, Q^2)$) as well as parity violating spin dependent structure functions ($g_3(x, Q^2)$ and $g_4(x, Q^2)$) in polarised weak DIS. The calculations have been performed both in QCD parton model and factorisation method.

In the parton model, all these spin dependent structure functions are expressed in terms of spin dependent parton distribution functions. We have calculated the coefficient of the quark distribution function to lowest order in α_s and gluonic coefficient function

to order α_s as the first moment of the spin dependent gluon distribution function goes as inverse power of α_s at large Q^2. The singlet part $\sum_i(\Delta q_i + \Delta\bar{q}_i - \alpha_s\Delta g/2\pi)$, which is a combination of the moments (spin contributions) of quarks, antiquarks and gluons, can be extracted from the first moment of the combination $(g_1^{W^+P}(x,Q^2) + g_1^{W^-P}(x,Q^2))$ to leading order, in the massive gluon scheme (k_T cut off also). On the other hand, the parity violating structure function $g_3(x,Q^2)$ yields the pure valence contribution, $\Delta u_V + \Delta d_V$ to the proton spin.

The new factorisation method, which is analogous to that used in spin independent as well as electromagnetic polarised DIS, is also applied to the weak spin dependent structure functions. We have calculated quark as well as gluon coefficient functions by proposing factorising theorem in spin dependent weak DIS. This method gives first moment of $(g_1^{W^+P}(x,Q^2)+g_1^{W^-P}(x,Q^2))$ to be proportional to quark (antiquark) contribution to proton spin alone, unlike in the parton model, where gluons also contribute to this moment. $g_3(x,Q^2)$ and $g_4(x,Q^2)$ are found to be proportional to spin dependent valence quark distribution functions as they do not depend on gluons. Here the result is the same as in the parton model approach. Further, the results we get verify the proposed factorisation theorem in the weak DIS sector to order α_s. It is interesting to note that the structure of mass singularities occurring in the process of evaluation of coefficient functions is identical to that occurring in electromagnetic DIS, confirming the universality of the form of mass singularities in the DIS processes.

In the factorisation method, we find that electromagnetic and weak structure functions do not yield any information about gluonic contribution to proton spin. A logical step in this process of extraction is to study the polarised Drell-Yan process. The motivation is in fact twofold. The first is to extend the factorisation method analogous to unpolarised DY process to polarised DY. The second is to see whether gluonic contribution to proton spin can be extracted from polarised DY process. However, gluons do not contribute to the first moment of the DY function either. In this thesis we have proposed a factorisation theorem for polarised DY process also and have derived operator definitions for spin dependent quark distribution functions. These are found to be consistent with that of

polarised electromagnetic as well as weak DIS, confirming universality. Generalising this by including appropriate operator definitions for the gluonic distribution function, we have calculated both quark and gluonic coefficient functions to the polarised DY function. These results verify the proposed factorisation theorem to order α_s in the gluonic sector.

The analysis of nonperturbative part of hadronic processes involves a model study of the recent EMC result on electromagnetic structure function, study of various moments of both spin independent and spin dependent structure functions and study of the singlet part of the first moment of the spin dependent structure function $g_1(x, Q^2)$ using anomalous Ward-Takahashi identity.

In the model study, we have analysed phenomenologically the first moment of the electromagnetic structure function $g_1(x, Q^2)$. By phenomenologically writing modified quark contributions as sum of valence, sea and gluonic parts and identifying them with theoretically evaluated or equivalently experimentally measured values, we have checked the consistency of various model dependent inputs such as Δq_V (valence), $\Delta\sigma$ (sea) and Δg (gluon) spin contributions with the known physics. In other words, the predictions of quark model wave functions for valence quarks are found to be consistent with the measured EMC result.

We have studied various operator matrix elements coming from moments of spin independent and spin dependent structure functions in the operator product expansion method. In particular, the ratios of various moments are found to be independent of the nonperturbative matrix elements of these local operators but depend on a mass of order 300 MeV which is introduced through an effective equation of motion. This result is independent of structure functions. It is also possible predict the absolute values of various moments of structure functions in a bag model. Interestingly, the prediction in the spin independent sector coincides with the experimental number but only reproduces the approximate Ellis-Jaffe sum rule result in the spin dependent sector. It is to be considered seriously that the matrix elements of these local operators between proton states possess a universal mass scale which is independent of model and process (structure functions). This is to be compared with the universality of mass singularities discussed in the

perturbative analysis of structure functions.

Finally, we have studied the importance of non-zero mass of the quarks using chiral Ward-Takahashi identity. We have showed that zero quark mass results yield results which are inconsistent with the known physics. It is also shown that the first moment of $g_1(x, Q^2)$ in electromagnetic DIS is ambiguous if OPE is used. On the other hand non-zero mass, however small changes the whole picture and yields results consistent with the earlier analysis on the first moment of spin dependent structure function. Hence, the nonperturbative analysis of structure functions advocates the universality of mass scale in the hadron targets and its importance in unambiguously defining and interpreting the structure functions.

APPENDIX A

$W^{\pm} g$ fusion process

In this appendix, we present relevant details to calculate $W^{\pm} g \to q_i \bar{q}_j$ cross section. We calculate this cross section in centre of momentum frame of initial particles $W^{\pm}$ boson and g. Since the diagram suffers both infrared as well as collinear divergences, we keep the masses of the particles non-zero. Let q,k be momenta of initial $W^{\pm}$ boson and gluon respectively of masses q^2 and k^2 respectively. They are parametrised in cm as

$$q = (q^0, 0, 0, -q^3), \qquad k = (k^0, 0, 0, q^3)$$

where

$$q^0 = \frac{\sqrt{\hat{s}}}{2}\left(1 + \frac{q^2 - k^2}{\hat{s}}\right) \qquad k^0 = \frac{\sqrt{\hat{s}}}{2}\left(1 - \frac{q^2 - k^2}{\hat{s}}\right)$$

and

$$q^3 = \frac{q^2}{2\sqrt{\hat{s}}}\frac{\alpha}{z}$$

where

$$\alpha^2 = 1 - \frac{4k^2 z^2}{q^2} \qquad z = -\frac{q^2}{2k.q}$$

and the final state momenta are given by

$$p = (p^0, 0, p^3 \sin\theta, p^3 \cos\theta), \qquad p'^0 = (p'^0, 0, -p^3 \sin\theta, -p^3 \cos\theta)$$

where

$$p^0 = \frac{\sqrt{\hat{s}}}{2}\left(1 + \frac{m^2 - m'^2}{\hat{s}}\right) \qquad p'^0 = \frac{\sqrt{\hat{s}}}{2}\left(1 - \frac{m^2 - m'^2}{\hat{s}}\right)$$

and

$$q^3 = \frac{\sqrt{\hat{s}}}{2}\beta\beta'$$

where

$$\beta^2 = 1 - \frac{(m_i - m_j)^2}{\hat{s}} \quad \beta'^2 = 1 - \frac{(m_i + m_j)^2}{\hat{s}}$$

and θ is the centre of momentum angle.

The two body phase space is quite different due to difference in masses of the outgoing particles.

$$PS_2 = \frac{d^3p}{2p^0(2\pi)^3}\frac{d^3p'}{2p'^0(2\pi)^3}(2\pi)^4\delta^{(4)}(p + p' - q - k)$$

$$= -\frac{\beta\beta'}{16\pi}\int d\cos\theta.$$

APPENDIX B

Compton scattering

In this appendix we give the parametrisations of momenta of the particles involved in $g\,q \rightarrow \gamma^* q$ process and the corresponding phase space. Since the diagram is sensitive to masses of the particles involved one keeps them non zero. Let k and p be the momenta of initial gluon and quark respectively and p' and q be momenta of out going quark and photon respectively. They are parametrised in cm as

$$k = (k^0, 0, 0, k^3), \qquad p = (p^0, 0, 0, -k^3)$$

where

$$k^0 = \frac{\sqrt{\hat{s}}}{2}\left(1 - \frac{m^2 - k^2}{\hat{s}}\right) \qquad p^0 = \frac{\sqrt{\hat{s}}}{2}\left(1 + \frac{m^2 - k^2}{\hat{s}}\right)$$

and

$$k^3 = \frac{\sqrt{\hat{s}}}{2}\left[\left(1 + \frac{m^2 - k^2}{\hat{s}}\right)^2 - \frac{4m^2}{\hat{s}}\right]^{\frac{1}{2}}$$

and the final state momenta are

$$p' = (p'^0, 0, -|q|sin\theta, -|q|cos\theta), \qquad q = (q^0, 0, |q|sin\theta, |q|cos\theta)$$

where

$$p'^0 = \frac{\sqrt{\hat{s}}}{2}\left(1 + \frac{m^2 - q^2}{\hat{s}}\right) \qquad q^0 = \frac{\sqrt{\hat{s}}}{2}\left(1 - \frac{m^2 - q^2}{\hat{s}}\right)$$

$$|q| = \frac{\sqrt{\hat{s}}}{2}\left[1 + \frac{(m^2 - q^2)^2}{\hat{s}^2} - 2\frac{(m^2 + q^2)}{\hat{s}}\right]^{\frac{1}{2}} = \frac{\sqrt{\hat{s}}}{2}\rho$$

and θ is the centre of momentum angle.

The two body phase space is quite different due to non zero mass of the out going particles, that is,

$$PS_2 = \frac{d^3p'}{2p'^0(2\pi)^3}\frac{d^3q'}{2q'^0(2\pi)^3}(2\pi)^4\delta^{(4)}(p'+q'-p-k) = -\frac{1}{16\pi}\rho\int d\cos\theta.$$

APPENDIX C

Evaluation of Triangle diagram

In this appendix we sketch the details of the evaluation of the function $f_{\frac{\Delta g}{g(\lambda)}}$. This involves the evaluation of four diagrams. It is found that only the triangle diagram is non-zero. This diagram is both UV and mass sensitive. To avoid mass singularities we keep both gluon and quarks in the loop massive. To handle UV divergence, we use dimensional regularisation to regularise the diagram and $\overline{MS}$ scheme to renormalise it. Since the diagram involves $\gamma^+\gamma_5$ vertex, one has to be careful in using Dirac algebra of γ matrices. We use t'Hooft-Veltman procedure to calculate trace with γ_5 which in fact reproduces chiral anomaly in QED. The Dirac algebra of gamma matrices in d-dimension is defined as

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} = 2\bar{g}_{\mu\nu} + 2\hat{g}_{\mu\nu},$$

$$\{\gamma_5, \bar{\gamma}_\mu\} = 0 = [\gamma_5, \hat{\gamma}_\mu]$$

where 4 dimensional and $d-4$ dimensional metric tensors are given as

$$\begin{aligned}
\bar{g} &= g_{\mu\nu} \quad \mu = 0,1,2,3 \\
&= 0 \quad \mu \geq 4 \\
\hat{g} &= 0 \quad \mu = 0,1,2,3 \\
&= -\delta_{\mu\nu} \quad \mu \geq 4
\end{aligned}$$

where bar on the γ's denotes that they belong to four dimensional space and hat on them denotes they belong to $d-4$ dimensional space. In the light cone frame, a vector A is

parametrised as $(A^+, A^-, A_\perp)$ or $(A^+, A^-, \bar{A}_\perp, \hat{A}_\perp)$ where

$$A^\pm = \frac{p^0 \pm p^3}{\sqrt{2}},$$

$\bar{A}_\perp$ is 2 dimensional transverse vector and $\hat{A}_\perp$ is $d-4$ dimensional transverse vector. l^+ and l^- integrals in the eqn.(2.40) can be easily done by using two delta functions. These delta functions nead following substitutions for l^+ and l^- terms appearing in the trace.

$$l^+ = k^+(1-x) \quad l^- = \frac{l_\perp^2 + m^2}{2k^+(1-x)}$$

Thus, we have

$$f_{\frac{\Delta q}{q(h)}}(x) = -\alpha_s \int \frac{d^{d-2}l_\perp}{(2\pi)^{d-2}} \frac{l_\perp^2(1-2x) - m^2 - 2(1-x)\, l_\perp^2 \frac{d-4}{d-2}}{(l_\perp^2 + m^2 - k^2x(1-x))^2}$$

This integral is UV divergent. This is renormalised in $\overline{MS}$ scheme.

APPENDIX D

State Normalisation in the various models

In the operator product formalism, the hadron states are normalised according to

$$\langle ps|p's'\rangle = \delta_{ss'} 2E_p (2\pi)^3 \delta^{(3)}(p-p').$$

The spin averaged states are therefore normalised as

$$\langle p|p\rangle = \frac{1}{2}\sum_{s,s'} \langle ps|p's'\rangle = 2E_p\,(2\pi)^3\,\delta^{(3)}(p-p')$$

The spinors satisfy

$$\bar{u}(p,s)u(p,s) = 2M.$$

The proton spin vector is defined as

$$s^\mu = \bar{u}(p,s)\gamma^\mu\gamma_5 u(p,s),$$

$$s.p = 0,\ s^2 = -4M_p^2.$$

The state normalisation in the bag model is as follows:

$$\langle ps|ps\rangle_{bag} = 1.$$

Then, the bag and operator product states are related according to

$$\langle p|\mathcal{O}(0)|p\rangle = 2M_P \int d^3x \, \langle p|\, \mathcal{O}(x)\, |p\rangle_{bag}.$$

We have used the configuration,

$$p^\mu = (M_P, \underline{0}), s^\mu = (0, 0, 0, 2M_P)$$

in the explicit calculation of spin dependent moments.

Bibliography

[1] J. Ashman *et al.* The EMC, Nucl.Phys. B328 (1989) 1.

[2] J.D. Björken, Phys. Rev. D1(1970) 1376.

[3] A.J.G. Hey and J. Mandula, Phys. Rev. D5(1972) 2610.

[4] J. Kodaira *et al.*, Nucl. Phys. B159(1979) 99; B165(1980) 129, K. Hagiwara, Suppl. Prog. Theor. Phys. 77(1983) 100.

[5] C.S. Lam and Bing-An Li, Phys. Rev. D25 (1982) 683.

[6] V. Gupta, S.M. Paranjape and H.S. Mani Pramana 14(1980) 119.

[7] G. Altarelli and G.G. Ross, Phys. Lett B212 (1988) 391.

[8] A.V. Efremov and O.V. Teryaev, In Proceedings of the International Hadron Symposium, Bechyne, Czechoslovakia, 1988, ed. Fischer *etal.* (Czechoslovakian Academy of Science, Prague, 1989), p. 302.

[9] R.D. Carlitz, J.C. Collins and A.H. Mueller, Phys. Lett. B214 (1988) 229.

[10] G.T. Bodwin and J. Qiu Phys Rev D41 (1990) 2755.

[11] A.V. Manohar, Phys. Rev. Lett, 66 (1991) 289.

[12] Prakash Mathews and V. Ravindran, Phys. Lett. B 278 (1992) 175.

[13] Prakash Mathews and V. Ravindran, Int. Jour. Mod. Phys. A7 (1992) 6371.

[14] Prakash Mathews and V. Ravindran, Mod. Phys. Lett. A7 (1992) 2695.

[15] W. Vogelsang and A. Weber, Nucl. Phys B362(1991) 3.

[16] J.C. Collins, D.E. Soper and G. Sterman, Advanced series on Directions in High Energy Physics -Vol 5 (Perturbative QCD) ed. A.H. Mueller.

[17] T. Muta, Foundations of Quatum Chromodynamics (World Scientific 1987).

[18] A.V. Manohar Phys. Rev. Lett. 65(1990) 2511.

[19] V. Ravishankar, Nucl.Phys. B374(1992) 309.

[20] W.A. Bardeen *et al.*, Phys. Rev. D18(1978) 3998.

[21] A.J. Buras Rev. Mod. Phys 52(1980) 199.

[22] Particle data group, Rev of particle properties, Phys. Lett. B239(1990) III 61.

[23] R.D. Carlitz and A.V. Manohar University of California preprint UCSD/PTH 90-32(1990).

[24] R.L. Jaffe and A.V. Manohar, Nucl. Phys. B337(1990) 509.

[25] G. Altarelli and B. Lampe, Z. Phys. C47(1990) 315.

[26] G. Veneziano, Mod. Phys. Lett. A4(1989), 1605.

[27] H.Y. Cheng, Chinese Journal of Physics 29(1991) 67.

[28] A.V. Manohar, University of California preprint UCSD/PTH 90-28(1990).

[29] G. Altarelli, CERN preprint CERN-TH-5675/90(1990).

[30] George Sterman, Introduction to PQCD ITP-SB-91-55.

[31] D. Amati, R. Petronzio and G. Veneziano, Nucl. Phys B140 (1978)54;B146 (1978)29; S. Libby and G. Sterman, Phys. Rev. D18 (1978)3252, *ibid* 4737; R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer and G.G. Ross, Nucl. Phys. B152 (1979) 285; A.V. Efremov and A.V. Radyushkin, Theor. Math. Phys. 44 (1980) 327, 44(1981) 664.

[32] Zimmerman, Ann.Phys. 77 (1973) 536,570.

[33] G.T. Bodwin, Phys. Rev. D31 (1985) 2616; J.C. Collins, D.E. Soper and G. Sterman, Nucl. Phys B261 (1985) 104; B308 (1988) 833.

[34] D. Indumathi, S. Gupta and M.V.N. Murthy, Z. Phys. C42 (1989) 493; *ibid* C47 (1990) 227; H.Y. Cheng, S.N. Lai, Phys. Rev. D41 (1990) 91; Prakash Mathews, R. Ramachandran, Z. Phys. 53 (1992) 305.

[35] G. Bunce et.al. Particle World 3 (1992) 1.

[36] M.J. Lighthill, Introduction to Fourier Analysis and Generalised Functions (Cambridge University Press, Cambridge, 1958); R.L Jaffe, Relativistic Dynamics and Quark-Nuclear Physics (John Wiley and sons 1986) ed. M.B. Johnson, A. Picklesimer.

[37] R.L Jaffe, Relativistic Dynamics and Quark-Nuclear Physics (John Wiley and sons 1986) ed. M.B. Johnson, A. Picklesimer.

[38] SLAC, G. Baum et el., Phys. Rev. Lett. 51 (1983) 1135.

[39] S. Gupta, M.V.N. Murthy and J. Pasupathy, Phys. Rev. D39 (1988) 2547.

[40] D. Indumathi, M.V.N. Murthy and V. Ravindran, Mod. Phys. Lett. A14 (1990) 1125.

[41] D. Indumathi, M.V.N. Murthy and V. Ravindran, Int. Jour. Mod. Phys. A6 (1991) 3933.

[42] M.V.N. Murthy, V. Ravindran, Pramana J.Phys 34 (1990) 103.

[43] Debajyoti Choudhury, N.D. Hari Dass and V. Ravindran, preprint PRL-TH/91-07 (1991).

[44] V.E. Elias, Phys. Rev. D34(1986) 3537.

[45] T. Sloan, Phys. Rep. 162(1988) 45.

[46] S. Gupta, private communication.

[47] E. Golowich and references therein, Phys.Rev. D12 (1975) 2108.

[48] J. Ellis and R.L. Jaffe, Phys.Rev. D9(1974) 1444, Phys.Rev. D10(1974) 1669.

[49] R. L. Jaffe and A. Manohar, Nucl. Phys. **B 337** (1990) 509.

[50] S. Brodsky, J. Ellis and M. Karliner, Phys. Lett. **B 206** (1988) 300.

[51] T. P. Cheng and L. F. Li, Phys. Rev. Lett. **62** (1989) 527.

[52] D. Kaplan and A. Manohar, Nucl. Phys. **B 310** 1988) 527.

[53] J. E. Mandula, Phys. Rev. Lett. **65** (1990) 1403.

[54] T. Hatsuda, Nucl.Phys. B329(1990) 376.

[55] Thomas D. Cohen and Manoj k. Banerjee, Phys. Lett. B230 (1989) 129.

[56] K.G. Wilson, Phys. Rev. 179(1969) 1499.

[57] R. Jackiw, Lectures on Current Algebra and its applications, Princeton Series in physics, Princeton University Press.

[58] Rudiger Voss, CERN-PPE/92-45.

[59] G. Altarelli, Phys. Reports 81(1982) 1.

[60] M.A. Ahmed and G.G. Ross, Phys. Lett. 59B(1975) 369; Nucl. Phys. B111(1976) 441.

[61] SLAC-G. Baum et al., Phys. Rev. Lett. 51(1983)1135.

[62] R.P. Feynman, High Energy Collisions, Gordon and Breach (1970).

[63] E. Reya, Phys. Reports 69(1981) 195.

[64] G. Altarelli and G. Parisi, Nucl. Phys. B126(1977) 298.

[65] D.J. Gross and F. Wilczek, Phys. Rev. Lett 30(1973) 1343, Phys. Rev. D8(1973) 3633, Phys. Rev. D9(1974) 980.

[66] Wandzura and F. Wilczek, Phys. Lett. B172 (1977) 195

[67] J.D. Bjorken and S. Drell, Relativistic Quantum Fields, Mc Graw-Hill, (1965).

[68] Callan and D.J. Gross, Phys. Rev. Lett. 22(1969) 156.

[69] H. Abramowicz et al Z.Phys. C17(1983) 283.

[70] H.S. Mani and M. Noman, Phys. Rev. D24(1981) 1223.

[71] F.E. Close, An introduction to quarks and partons,(London: Academic Press)p.294.

[72] R.D. Carlitz and J. Kaur, Phys. Rev. Lett. 38(1977) 673.

[73] M. Diemoz et al Z. Phys. C39(1988) 21.

[74] M. Gluck, R. Godbole, and E. Reya Preprint DO-Th-88/18.

[75] J. Kaur, Nucl. Phys. B128(1977) 219.

[76] H. Fritzsch, Preprint MPI-PAE/pTh-45/88.

[77] J.M. Gaillard and G. Sauvage, Ann. Rev. Nucl. Part. Sci. 34(1984) 351.

[78] M. Anselmino et al, Santa Barbara Preprint NSF-ITP-88-94.

[79] J. Soffer, Preprint CPT-89/P.2220