Non-equilibrium Dynamics in Complex Networks

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DECLARATION

I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier in whole or in part for a degree / diploma at this or any other Institution / University.

Rishu Kumar Singh

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Contents

List of Figures

Synopsis

Complex systems comprising a large number of strongly interacting components that possess non-trivial dynamics are ubiquitous in nature, examples ranging from the immune system of organisms that consist of an enormous variety of cells performing various specialized functions in a coordinated manner to ecological food webs coupling a large number of species to each other in terms of trophic relations. In the economic domain also we observe a variety of complex systems, such as financial markets and the network of international trade between nations. A common framework often used for describing the structure of such systems is that of networks [1–4], a network being defined as a set of nodes (or vertices) that are connected with one another through links (or edges). For example, in the context of ecology, nodes are the different species and links are the possible predator-prey, competitive or mutualistic interactions between them.

While the topological arrangement of the connections between the constituent parts of a complex system can often reveal fascinating insights, such an exclusively structural approach often cannot explain its dynamical behavior. Complex systems in nature are often subject to various types of environmental stimuli and perturbations that keep them far from equilibrium. To describe the behavior of such non-equilibrium systems we need to take recourse to a dynamical perspective. In this thesis, we look at the relation between the structural properties of a complex system and the features arising from the collective dynamics of its elements. We focus on understanding how robustness of a complex system (either in terms of survival of activity in its components or stability of the statistical

1

properties of its dynamics) can arise as a result of the interaction between its different parts. We show that a set of concepts from statistical physics can provide a common toolbox for explaining features of widely different out-of-equilibrium complex systems, e.g., random walks for explaining movement of prices in markets as well as extinction of species in ecological communities. Similarly the concept of waiting time is useful when considering phenomena involving the intervals between successive extinctions in an ecological system or the duration between two successive transactions in a financial market.

We begin with a short overview of the literature on dynamics of complex networks in **Chapter 1**. We give special emphasis on the long-standing debate as to whether increased complexity in systems is conducive for its robustness, known in the literature as the stability-diversity debate [5].

In **Chapter 2** we consider the role of inter-dependence on the global stability of coupled networks [6, 7]. Many complex systems that occur in biological, technological and socio-economic contexts are seen to be strongly influenced by the behavior of other systems. Such interdependence can result in perturbations in one system propagating to others, potentially resulting in a cascading avalanche through the network of networks. Recent studies of percolation of failure processes in a system of two or more connected networks have suggested that interdependence makes the entire system fragile. However, a proper appraisal of the role of interdependence on the stability of complex systems necessarily needs to take into account the dynamical processes occurring on them. Compared to a purely structural approach (such as percolation, that considers the effect of removing nodes or links), a dynamical systems perspective provides a richer framework for assessing the robustness of systems. Indeed, investigating how fluctuations from equilibrium in a local region of a complex system can propagate to other regions forms the basis for addressing the dynamical stability of systems. Extending this framework to the context of interdependent networks can potentially offer us insights on why such systems are

ubiquitous in the real world in spite of their structural fragility.

Motivated by these observations, in this chapter we describe the results of our investigations on the dynamics of coupled networks. Our principal finding is that an optimally strong interdependence between networks can increase the robustness of the system in terms of its dynamical stability. Unlike percolation-based approaches where failure is often identified exclusively with breakdown of connectivity so that increasing interdependence necessarily enhances fragility, our dynamical perspective leads to a strikingly different conclusion. In particular, we show that the system has a much higher likelihood of survival for an optimal interdependence, with both networks facing almost certain catastrophic collapse in isolation. Our results suggest that interdependence may be essential in several natural systems for maintaining diversity in the presence of fluctuations that are potentially destabilizing. Thus, interdependence need not always have negative repercussions. Instead its impact may depend strongly on the context, e.g., the nature of coupling and the type of dynamics being considered.

In **Chapter 3** we consider the stability-diversity debate, viz., the question of whether higher complexity (e.g., as a result of increasing the number of nodes, connection density or the range of interactions strengths in a network) makes a system more vulnerable to disturbances arising from small perturbations in the state variables. While empirical observations appear to suggest that more diverse (and hence more complex systems) tend to be more resistant to external interventions and also less likely to show catastrophic collapses, theoretical investigations suggest otherwise. We view this question in the perspective of long-term dynamical evolution of many coupled dynamical elements where activity in each of the nodes may cease (corresponding to extinction) as a result of interaction with other modes [9, 145]. A cascading sequence of failures across the network will manifest as a large-scale catastrophic collapse of the system resulting from initially small perturbations in the dynamical variables and we ask under what conditions such phenomena will be possible. An important variation we introduce in our model is the

3

dependence of the interaction strengths with which a node interacts with its neighbors on the total number of neighbors that it has. In particular we focus on the case when the average interaction strength of a node is inversely proportional to its in-degree (i.e., the total number of incoming connections from other nodes of the network). Such a reciprocal relation between strength and degree is motivated by empirical observations over several decades, e.g., "...species that interact feebly with others do so with a great number of other species. Conversely with species with strong interactions are often part of a system with a small number of species..." (R. Margalef, *Perspectives in Ecological Theory*, University of Chicago Press, Chicago, 1968, p. 7).

While the extreme long-term behavior of the model is qualitatively similar to the case when the strengths are independent of degree, viz., the asymptotic fraction of nodes with persistent activity is inversely related to the number of nodes, the connection density and the dispersion of interaction strengths, a critical difference is that scaling strength by degree makes the the system exhibit extremely long transients during which most nodes remain active. Surprisingly, the more connected a system, the longer is the duration of the transient period characterized by the network possessing a large fraction of active nodes. Thus, observations made in short timescales may well conclude that systems having more nodes (i.e., more diversity) and higher link density (i.e., more connected) will be more stable in the sense of having a higher fraction of nodes exhibiting dynamical activity than systems that are less diverse or less connected – in agreement with claims made in the empirical literature. On the other hand, at extremely long times, the results will be more consistent with theoretical studies which shows that more complex systems would tend to have lower number of active nodes surviving. Thus, we provide a novel resolution to the long-standing debate over the relation between complexity and stability by showing that the answer depends on the time-scale of observation.

In **Chapter 4** we consider a different type of complex system than the ones we have considered so far, viz., financial markets. Financial markets constitute a prototypical ex-

ample of a complex system because of the large number of components involved whose dynamics is often unpredictable. However, despite such unpredictability, the existence of statistical regularities that have been reported for many different markets across geographically separated locations, e.g., the observation of "inverse cubic law" in the distribution of price fluctuations (measured in terms of logarithmic returns, i.e., difference in the logarithm of prices at successive time intervals). Many studies reporting these observations have used daily trade data, thereby neglecting the significant intra-day patterns in trading activity. Motivated by these observations, we study the high-frequency (HF) [10, 11] trade data obtained from the National Stock Exchange (NSE), India to characterize the statistical properties of the collective dynamics of such a complex non-equilibrium system. Such HF data, by providing information about market movements at the maximum possible resolution, helps one to understand the microstructure of the dynamics of this complex system. Such an analysis will also provide benchmarks against which existing models of collective behavior in markets can be tested.

Using HF data from different intervals we show that the gross statistical properties of the market as a whole are in general stationary even though those of its constituents, i.e., individual stocks, are not. In particular we see that the distribution of transaction sizes (i.e., units of stock involved in a single transaction) when trades carried out in the entire market are considered does not change its nature over time. However, for individual stocks, this distribution can differ significantly between one period and another. We have also investigated the statistics of waiting times between successive trades. For individual stocks we observe that the distribution of waiting times is non-exponential which we link to the frequent occurrence of successive transactions that involve large returns (price changes). We also find that the cumulative volatility of price movements increases linearly with time within a trading day. However there is deviation from linearity at the ends suggesting that the non-Gaussian character arises because of significant volume of end-of-day trading. The correlation between the intervals separating successive trades and the magnitude of price fluctuations implies that the distributions of the waiting times and that

5

of step lengths of the random walk [12, 13] executed by the price of a financial asset may not be completely independent unlike what is generally assumed in many studies.

We conclude in **Chapter 5** with a summary and general discussion of the implications of our results.

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Chapter 1

Introduction

The concept of equilibrium has been central to physics from the advent of Newtonian mechanics in the seventeenth century. The key idea of competing forces that act on an object of interest balancing each other, such that the dynamical properties of the object become time-invariant, has permeated beyond classical mechanics into statistical physics where it is associated with the principle of detailed balance. It is a crucial simplifying assumption that enables theoretical analysis of a large number of physical phenomena. However, many processes in nature, in particular those that occur in the living world, are far from equilibrium and thus could not be analyzed using the tools of physics for a long time. This is because while techniques for studying equilibrium phenomena have been developed much earlier, it is only in late 19th and early 20th centuries that the first steps were taken to theoretically understand out-of-equilibrium phenomena starting with investigations in kinetic theory. Einstein's development of the theory of Brownian motion [1] is one of the early landmarks in this venture. Around the same time, an alternative approach based on analyzing individual trajectories of random dynamical systems was proposed by Langevin [2]. In this approach, the time-evolution of a variable of interest is described using a stochastic differential equation [3], which is a physically more intuitive approach than the one based on description of probability densities as in the Fokker-Planck equa-

tion [4]. These two complementary approaches for studying non-equilibrium systems, one based on a description of dynamical trajectories and the other on probability densities, are fundamentally connected by the concept of random walks [5].

One of the earliest examples of the theoretical treatment of non-equilibrium dynamics of systems is provided by the seminal work of Kramers [6] in calculating the thermal rate of escape of a particle by jumping across a potential barrier. The escape problem has since been applied to domains ranging from chemical kinetics (e.g., calculating the rate of chemical reactions) to transport theory [7, 8]. The question of how long it takes for a particle to escape from a potential well can also be seen as a specific case of the more general class of first-passage problems [9]. Such problems are fundamentally connected to the question of persistence in a dynamical system (e.g., how long does the system remain in a given region in phase space) [10], and have applications across widely different domains. For instance, the framework of first-passage has been used to study problems as diverse as the inter-spike intervals in the firing of neurons [11] and the time taken by asset prices to cross a given threshold [12]. Non-equilibrium systems also exhibit irreversibility and dissipation, demonstrating the "arrow" of time manifest in all macroscopic phenomena in nature. In addition, far-from-equilibrium processes are often characterized by self-organization leading to patterns in space and/or time [13]. One of the most striking examples of these is the occurrence of temporal oscillations in the molecular concentrations, as well as, propagating target and spiral waves in the Belusov-Zhabotinsky chemical reactions [14–16]. The investigation of such processes were pioneered by among others, Ilya Prigogine [17, 18] whose contributions were recognized with the Nobel Prize in Chemistry in 1977 [19], and Hermann Haken, whose work on lasers provides a classic example of how a system system comprising many degrees of freedom exhibit self-organization when driven with a supra-threshold stimulus [20, 21]. Indeed, most complex systems that are composed of many interacting components (or subsystems) and whose collective behavior cannot be simply expressed in terms of the superposition of properties of individual elements, typically exhibit non-equilibrium behavior.

One of the signatures of complex self-organization in natural systems is the occurrence of $1/f$ noise [22], i.e., the the observable of interest has a frequency spectrum such that the power spectral density for a particular frequency is inversely proportional to the frequency. This type of frequency spectrum is ubiquitous in nature and has been observed in phenomena as different as the statistics of flooding of the river Nile to the luminosity of stars [23]. Occurrence of $1/f$ noise has been identified with self-organized criticality [24, 25] in driven dynamical systems. Such processes exhibit scaling phenomena with algebraic decay of correlations across space and/or time, that have been connected with the occurrence of statistically self-similar structures in nature such as in mountains and coastlines [26] and in fluid turbulence [27].

Thus, beginning from the investigation of simple systems in the early 20th century, such as a particle diffusing in a solution, the study of non-equilibrium phenomena in physics has come a long way, addressing problems associated with complex systems such as the living cell and ecosystems. It is striking that while these developments were taking place in the physical sciences, simultaneously a parallel approach was being constructed in the realm of social sciences for studying similar phenomena. Strikingly, five years before Einstein's seminal paper on Brownian motion, Bachelier had proposed his own theory of the same phenomenon [28] in the context of describing the behavior of fluctuating prices in markets. Although unnoticed at the time, the work of Bachelier was used as a starting point by Itô to develop stochastic calculus and the theory of geometric Brownian motion [29]. This theory has played a fundamental role in modern financial theory and practice, including the development of option pricing formulas pioneered by Black and Scholes [30] and independently by Merton [31]. While most of these theoretical developments have assumed Gaussian behavior for price fluctuations, empirical studies have shown that the distribution of asset price returns exhibit strong deviation from Gaussian nature [32, 33]. In particular, evidence suggests that the second moment of the distribution may diverge in many cases in reality. This problem was sought to be resolved in subsequent work by Mantegna and Stanley [34], wherein they showed that even though

11

price fluctuation distributions exhibit fat tails, the tails are exponentially decaying at the far ends resulting in a finite second moment for the price variations. From the 1990s physicists have used various techniques from statistical physics to understand the properties of such distributions associated with the dynamics of markets. This link between non-equilibrium phenomena in natural (e.g., Brownian motion) and socio-economic (e.g., financial markets) domains have provided a rich cross-fertilization of ideas between these two apparently very different areas [35].

1.1   Nonlinearity

A primary reason that the dynamical properties of complex systems cannot be derived in a straightforward manner from that of their subsystems is the intrinsically nonlinear character of such systems. Nonlinearity leads to the failure of the principle of linear superposition, e.g., in a pendulum undergoing large-amplitude oscillations [36], and is ubiquitous in natural and man-made systems [37,38]. The analysis of such systems is generally problematic as techniques designed for analyzing linear systems are rarely of much efficacy in these cases [39]. However, understanding how the nonlinearity of dynamical systems affects their behavior is of fundamental importance and is also crucial for many applications, such as, understanding climate change [40, 41], biological systems [42], optical systems, electromagnetic systems, solid state devices, etc. [43].

A particular class of nonlinear systems are the discrete dynamical systems, also known as nonlinear maps [44], that can exhibit extremely complicated dynamics [45] ranging between steady state, periodic oscillations and chaos, depending on the value of system parameters [46–48]. Although of interest in their own right, individual maps are of limited usefulness when describing complex systems. This problem can, however, be overcome by considering a system of coupled nonlinear maps, e.g., coupled map lattices [49]. Such models have been used to describe spatiotemporal intermittency [50, 51], spatially

extended chaos [52], synchronization [53, 54], cascading failures [55], etc., in dynamical systems with multiple degrees of freedom. However, complex systems composed of many interacting units are not always embedded on regular lattices in finite spatial dimensions and may acquire arbitrary connection topologies which are described using their statistical properties, viz., average number of neighbors of a given element, distribution of interaction strengths, etc. These structures, which deviate from regular lattice based arrangements, are termed networks [56] and have become a cornerstone of contemporary research in complex systems [57–59]. We shall now briefly discuss the concepts and techniques of such systems that are relevant for the work reported in the thesis.

1.2 Networks

A network is a set of nodes connected by links, which may by directed or undirected, thus classifying the network as directed or undirected. All information about the network topology is contained in its adjacency matrix

$$\mathbf{g} = \{g_{ij}\}_{N \times N}, \tag{1.1}$$

which is a matrix with elements 0 and 1, N being the number of nodes. If two nodes i and j of the network are connected by a link, then $g_{ij} = 1$, else $g_{ij} = 0$. A directed network is characterized by an asymmetric adjacency matrix, whereas for an undirected network $\mathbf{g}$ is symmetric. In addition, the links of a network can also have weights assigned, which we denote by

$$\mathbf{J} = \{J_{ij}\}_{N \times N}, \tag{1.2}$$

where the weights J_{ij} are drawn according to some rule, or some statistical distribution depending on the problem. For example, $J_{ij} = g_{ij} \ \forall \ i, \ j \ = \ 1, ..., N$.

In order to characterize the neighborhood of a node i, we define its degree

$$k_i = \sum_j g_{ij}.$$ (1.3)

It is to be noted that for an undirected network, for which $g_{ij} = g_{ji}$, it does not matter in (1.3) whether the summation is with respect to the first index or the second. However, for directed networks the asymmetry of the adjacency matrix **g** implies that the in-degree of node, $k_i^{in} = \sum_j g_{ij}$, defined as the number of neighbors of i, is in general different from its out-degree, $k_i^{out} = \sum_j g_{ij}$, which is a measure of the number of nodes to which i is connected. An important quantity characterizing networks is its degree distribution $p(k_i)$ and has been used to classify different types of networks. For example, random networks have Poisson degree distributions [60] whereas scale-free networks possess power-law degree distributions [61].

Another concept closely related to the average degree of a node $\langle k \rangle = \sum_i k_i p(k_i)$ is the connection density of the network

$$C = \frac{\langle k \rangle}{N},$$ (1.4)

where N is the number of nodes in the network. Here it is assumed that the network under consideration is undirected. An analogous definition can be written for the case of directed networks. These and other related concepts about networks are described in Refs. [56, 62].

Most of the networks encountered in natural or artificial settings, e.g., biological networks [63–66], food webs [67], world wide web [68–70], collaboration networks [71,72], etc. to mention a few, are not frozen in time but continue to grow through addition or deletion of nodes. Such structural evolution of complex networks has been connected to well-known phenomena in statistical physics, such as, self-organized criticality [73–75], percolation [76], etc., making the dynamics of networks (i.e., the process by which the topology

of the network evolves) an important problem in nonequilibrium statistical physics [77]. However, the dynamics of networks by no means exhausts the possibilities of complex dynamical phenomena arising in a network context. For example, even though the class of networks mentioned above are evolving in time, the nodes themselves are essentially static in nature and serve only as structural components of the network. In many cases of interest, however, nodes themselves have intrinsic dynamics (or at least, are associated with a dynamically evolving state), such as in the case of epidemics [78–81]. Other examples include contact processes [82,83] and reaction-diffusion dynamics [84–86] governing the interactions between the nodes in a network. Consideration of such dynamics on networks allow us to address the extremely important problem of stability of a complex system. The question of whether increasing complexity of a system makes it more or less stable was theoretically addressed by Robert May [87]. This early analytical approach involved studying the linear stability in the neighborhood of an equilibrium state of the dynamical system. However, most complex systems exist far from equilibrium. It is thus of interest to ask whether the role of complexity on the stability of a system can be investigated in such scenarios. It also often happens that the failure of a single node can result in a sequence (or chain) of other nodes failing in a cascade. This frequently arises in real world systems such as communication networks [88], electrical power grids [89–93], social and economic systems [94] and complex infrastructural systems [95], which could be subjected to random or targeted attacks [96–101]. These observations have motivated the recent surge of interest in understanding the problem of cascading failures in networks.

1.3   Interdependent Networks

Networks rarely exist in isolation in reality and are generally part of larger systems comprising several other networks. These networks may also have non-trivial dependencies amongst each other [102]. A classic example of such interdependence are found in infrastructure networks, such as that seen between power and communication networks,

that is crucial for their overall functioning [103]. Power grids also depend on the efficiency of transportation networks for their smooth operation, the latter themselves being dependent on the functioning of communication networks [104, 105]. Using a percolation phase transition approach, Buldyrev *et al.* [102] have shown that, unlike in the case of isolated networks, the transition in interdependent networks is discontinuous. This makes the latter more vulnerable to cascading failures in comparison to their isolated counterparts [106]. The percolation based approach has been further extended to the case of more than two interdependent networks [107, 108] and interacting regular lattices [109]. Approaches other than the percolation-based one have been used recently to understand the collective dynamics of interdependent networks [110, 111].

One of the ways to address interdependence amongst two networks is to have qualitatively different types of links connecting members of the different networks, compared to the nature of the intra-network links. Such inter-network links can be either weighted or unweighted. For example, if two ecosystems are connected by migration then the dependency links connecting the two networks can be assigned weights that quantify the rate of migration of different species between their habitats in the two ecosystems. Similar to the weight matrix representing intra-network interactions $\mathbf{J}$, the weights of the dependency links connecting the two networks are given by the matrix

$$\mathbf{D} = \{D_{ij}\}_{N \times N},\tag{1.5}$$

where the first index i represents the i-th node of one network while the second index j denotes the node j in the other network. For the case of dependency links existing only between the corresponding nodes in the two interdependent networks, the weight matrix D is diagonal. Note that this has been assumed to be the case in most studies of interdependent networks reported in literature.

1.4 Financial Markets

Financial markets are one of the paradigmatic examples of complex systems [112–114] whose complexity arise due to a large number of interacting elements. From the 1990s, physicists have been actively involved in studying empirical properties of financial markets [115] that are amenable to tools developed in statistical physics. The contributions made by physicists towards understanding the behavior of finance markets have sometimes been referred to collectively by the term *Econophysics* that was introduced in 1995. The primary aim of this field has been to uncover universal empirical properties that are valid across different markets. Notable amongst such "stylized facts" reported by physicists are power laws describing the nature of the tails in the distribution of fluctuations and that of temporal correlations of volatility [116, 117]. This type of universal behavior has been observed for price & index fluctuations, trading volumes and number of trades. Similar values have been reported for these exponents for different types and sizes of markets [118].

Given the price p_t of some stock at time t, price fluctuations are characterized in terms of log returns, viz.,

$$r_t = \ln p_t - \ln p_{t-\Delta t}, \tag{1.6}$$

where Δt is the time-interval separating the two price values. If q_i denotes the the number of stocks traded in the i-th transaction, also known as trade size of the transaction, then the volume of stocks traded $V_{\Delta t}$ over an interval $[t, t + \Delta t]$ is defined as

$$V_{t,\Delta t} = \sum_{i=1}^{N_{t,\Delta t}} q_i, \tag{1.7}$$

where $N_{t,\Delta t}$ is the number of trades that have occurred during the interval $[t, t + \Delta t]$. The above relation implies that quantities characterizing a trading event, viz., trade sizes, trading volumes and number of trades are not independent of each other.

Although a number of studies have uncovered many of the empirical properties of many different finance markets, most of these observations have been made using daily closing prices that ignores the bulk of information concerning intra-day trading [119]. Empirical analysis using high-frequency data have revealed features which are generally not possible to observe using end of the day trade data. For example, the transactions involved in a stock market never occur at regular intervals implying that the events defining transactions of a stock could themselves be a stochastic process with a given waiting time distribution. This has been demonstrated in a number of studies employing high frequency data of finance markets [120–126]. Motivated by these observations, several studies have employed continuous time random walks [127] to understand financial time series [128–130, 133]. Note that several of the studies show that log returns and waiting times cannot be considered to be independent of each other. This theme has been further explored in the work reported in this thesis.

1.5   Overview of the thesis

The aim of the present thesis is to contribute towards understanding the nonequilibrium dynamics of complex systems composed of a large number of interacting dynamical components which arise in domains varying from natural to artificial settings. Networks provide a common framework to address problems related to complex systems and we use networks of dynamical entities to describe the nonequilibrium dynamics of the systems considered. In this thesis, we look at the properties emerging from an interplay of topology and dynamics by focusing on robustness of the complex system. We use techniques from statistical physics to address the problems considered in the thesis, e.g., random walks, first passage times, persistence probability, waiting times, etc. These concepts are used to understand the diverse problems ranging from the behavior of price fluctuations to the extinction of species in ecosystems.

In **Chapter 2** we consider the problem of two diffusively coupled dynamical networks of nonlinear maps. This problem is directly relevant to the occurrence of interdependent systems in domains ranging from biology to engineering to the society and the economy. Recent studies have shown that interdependence amongst systems makes the overall system of systems more fragile as compared to their individual counterparts. However, these studies focus only on the structural aspects of the system. A dynamical systems perspective is necessary to assess the stability of interdependent systems, which can provide insights into why interdependent systems are ubiquitous despite their structural fragility. Motivated by these observations, we investigate dynamics of coupled networks in this chapter. Our principal finding is that an optimally strong interdependence between networks can increase the robustness of the system in terms of its dynamical stability. The dynamical perspective taken in this chapter produces a view of the role of interdependence that is in stark contrast to percolation based approaches. We show that the system has a very high probability of survival for an optimal range of interdependence, with both networks facing almost certain catastrophic collapse in isolation. This implies that interdependence may be essential in several natural systems for maintaining stability in the presence of fluctuations.

In **Chapter 3** we consider the stability-diversity debate, viz., the question of whether increased complexity makes a system more vulnerable to disturbances arising from small perturbations in its state variables. While empirical observations appear to suggest that more diverse systems tend to be more resistant to external interventions theoretical investigations suggest otherwise. We consider this as a problem of long-term persistence of a system of dynamically interacting elements, whose individual units may cease activity ("go extinct") as a result of interactions with neighboring elements. We ask under what conditions a small perturbation in the dynamical variables can result in a cascade of extinction events propagating through the network. Choosing the interaction weights amongst elements to vary inversely with their number of neighbors, we find that the long-term dynamics is similar to the case when interaction strengths are independent of the

19

degree. However, the scaling of interactions by node degree makes the transient behavior (during which most nodes are active) long-lived. Importantly, the duration of the transient activity increases with the connection density of the network. Hence, when one observes the networks over shorter time scales one will conclude that dense networks are more stable in comparison to their sparse counterparts. These results appear to explain the dominant view in the empirical literature that more complex systems are also more stable. On the other hand, at very long time scales, more densely connected systems will have a much lower number of surviving nodes, which is consistent with earlier theoretical studies suggesting that complexity reduces system stability. Thus our results reported in this thesis provide a novel resolution to the long-standing debate over the relation between complexity and stability. We show that the answer essentially depends on the time-scale at which one is observing the system.

In **Chapter 4** we consider the non-equilibrium behavior of financial markets. As already mentioned earlier these are prototypical examples of complex systems because of the large number of constituent components exhibiting unpredictable dynamics. Despite such unpredictability in their detailed behavior, overall statistically regular properties have been reported for many different markets. Notable amongst them are the inverse cubic law of distribution for price fluctuation (measured in terms of logarithmic returns). Most earlier studies have used daily trade data which neglects the intra-day fluctuations. In contrast, we study the high-frequency(HF) data of price fluctuation for equities listed in the National Stock Exchange(NSE) of India in order to uncover robust statistical features. As the HF data provides information about market movements at the highest possible resolution it should allow one to make inferences about the nature of the underlying dynamics of the system.

Using HF data obtained from different periods in which the market operated we show that the gross statistical properties of the market as a whole are in general stationary even though those of its constituents, i.e., individual stocks, are not. In particular we

see that the distribution of transaction sizes (i.e., number of units of stock exchanged in a single transaction) does not change its nature over time when we are considering the entire market. However, for individual stocks, this distribution can differ significantly from one period to the next. We also find that waiting times between transactions of individual stocks exhibit non-exponential character, and are related to the corresponding log-returns. Specifically, transactions involving larger returns occur closer to each other. This implies that the distribution of waiting times and the step lengths of the random walk executed by a financial asset are not independent of each other, as assumed in many studies. We also report that the cumulative volatility of returns increases linearly with time within a day.

We conclude in **Chapter 5** with a brief summary and discussions of the implications of our results.

Chapter 2

Optimal interdependence enhances the dynamical robustness of complex systems

2.1 Introduction

Many complex systems that occur in biological [134], technological [103] and socio-economic [135] contexts are strongly influenced by the behavior of other systems [136]. Such interdependence can result in perturbations in one system propagating to others, potentially resulting in a cascading avalanche through the network of networks [106, 107]. Recent studies of percolation of failure processes in a system of two [102, 104, 137] or more [138, 139] connected networks have suggested that interdependence makes the entire system fragile. However, a proper appraisal of the role of interdependence on the stability of complex systems necessarily needs to take into account the dynamical processes occurring on them [140, 141]. Compared to a purely structural approach (such as percolation, that considers the effect of removing nodes or links), a dynamical systems perspective provides a richer framework for assessing the robustness of systems [142, 143]. Indeed,

23

investigating how fluctuations from equilibrium in a local region of a complex system can propagate to other regions forms the basis for addressing the dynamical stability of systems [87]. Extending this framework to the context of interdependent networks can potentially offer us insights on why such systems are ubiquitous in the real world in spite of their structural fragility.

In this chapter we show that strong interdependence between networks can *increase* the robustness of the system in terms of its dynamical stability. In particular, we show for a pair of networks that there exists an optimal range of interdependence which substantially enhances the persistence probability of active nodes. By contrast, decreasing the inter-network coupling strength so that the networks are effectively independent results in a catastrophic collapse with extinction of activity in the system almost in its entirety. The increased persistence at optimal coupling is seen to be related to the appearance of attractors of the global dynamics comprising disjoint sets of stable activity. Our results also suggest that the nature of inter-network interactions is a crucial determinant of the role of interdependence on the dynamical robustness of complex systems. For example, increasing the intensity of nonlinear interactions between nodes leads to loss of stability and subsequent transition to a quiescent state, while stronger diffusive coupling between the networks can make a global state corresponding to persistent activity extremely robust.

2.2   Model

Let us consider a model system comprising G interdependent networks. Each network has N dynamical elements connected to each other through a sparse random topology of nonlinear interactions. Interdependence is introduced by diffusively coupling an element i in a network to the corresponding i-th element of other network(s). This framework can be used to represent, for instance, dispersal across G neighboring habitat patches of N interacting species in an ecological system. A continuous dynamical variable z_i^μ

$(i = 1, \ldots, N; \mu = 1, \ldots, G)$ is associated with each node of the coupled networks. In the above-mentioned example, it can be interpreted as the relative mass density of the i-th species in the μ-th patch. We consider generalized Lotka-Volterra interactions between the nodes within a network as this is one of the simplest and ubiquitous types of nonlinear coupling [144, 145]. The dynamical evolution of the system can then be described in terms of the GN coupled equations:

$$z_i^\mu(n+1) = (1 - Dq_i)F(z_i^\mu(n))[1 + \sum_{j=1}^{N} J_{ij}^\mu z_j^\mu(n)]$$

$$+ \frac{Dq_i}{(G-1)} \sum_{\nu \neq \mu}^{G} F(z_i^\nu(n))[1 + \sum_{j=1}^{N} J_{ij}^\nu z_j^\nu(n)]. \quad (2.1)$$

Here $\mathbf{J}^\mu$ is the interaction matrix for the μ-th network, D is a measure of the strength of interdependence via diffusive coupling between networks, and $q_i = 1$ if the i-th species is diffusively coupled across the different networks, while it is zero otherwise ($i = 1, \ldots, N$). The range of the variable z_i^μ is decided by the function $F()$ governing the dynamics of individual elements in the system. Here we consider F to be a smooth unimodal nonlinear map defined over a finite support and having an absorbing state. This class of dynamical systems is quite general and are capable of exhibiting a wide range of behavior including equilibria, periodic oscillations and chaos [146]. For the results shown here we have used the logistic form [147]: $F(z) = rz(1-z)$ if $0 < z < 1$, and $= 0$ otherwise, such that $z = 0$ is the absorbing state, and r is a nonlinearity parameter that determines the nature of the dynamics.

Unlike most studies with logistic map where $r \in [0, 4]$, we specifically choose $r > 4$ such that $F()$ maps a finite subinterval within $[0, 1]$ directly to the absorbing state. Iterative application of $F()$ implies that only a a set of measure zero will remain in the unit interval [44], resulting in a leaky dynamical system [148]. Thus, an isolated node will almost always converge to the absorbing state, corresponding to its extinction. Interaction with other nodes can, however, maintain a node in the active state [$z \in (0, 1)$] indefinitely. We

25

define a measure for the global stability of the system as the asymptotic fraction of nodes in each network that have not reached the absorbing state, viz., $f_{active} = \text{Lt}_{n \to \infty} f_{active}(n)$, where $f_{active}(n) = \sum_{i=1}^{N} \Theta[F(z_i^\mu(n))]/N$ (with $\Theta[x] = 1$ for $x > 0$, and 0 otherwise). Thus, we explicitly investigate conditions under which interdependence between networks can result in persistent activity in at least a subset of the nodes comprising the system. Using an ecological analogy, our focus is on the long-term survival of a finite fraction of the ecosystem as a function of the degree of dispersal between neighboring patches rather than the intrinsic stability of individual species populations.

2.3 Results and Discussion

The degree of interdependence between the networks can be varied by changing the number of pairs of corresponding nodes M ($0 \leq M \leq N$) that are linked via dispersion. The interaction matrix $\mathbf{J}^\mu$ in each network is considered to be sparse, such that only a fraction C of the matrix elements are non-zero with their interaction strengths chosen randomly from a $Normal(0, \sigma^2)$ distribution. For simplicity, we shall focus on a pair of interdependent networks (i.e., $G = 2$) schematically shown in Fig. 2.1 (a), both networks being chosen from the same ensemble so as to have identical parameters r, C and σ. We distinguish between the variables z of the N nodes in the two networks by denoting them as x_i and y_i ($i = 1, \ldots, N$) respectively, their initial values being chosen at random from the uniform distribution [0,1].

Fig. 2.1 (b-c) show the time-evolution of the state of the dynamical variables x_i and the global stability measure $f_{active}(n)$ for one of the networks ($N = 256$, $C = 0.1$, $\sigma = 0.01$) where the nonlinearity parameters r_i are distributed uniformly in [4.0,4.1]. As mentioned above, this distribution of r_i implies that the individual node dynamics would almost certainly converge to the absorbing state, and this is indeed what is observed when the networks are isolated, i.e., $D = 0$. However, when the interdependence is increased, e.g., to
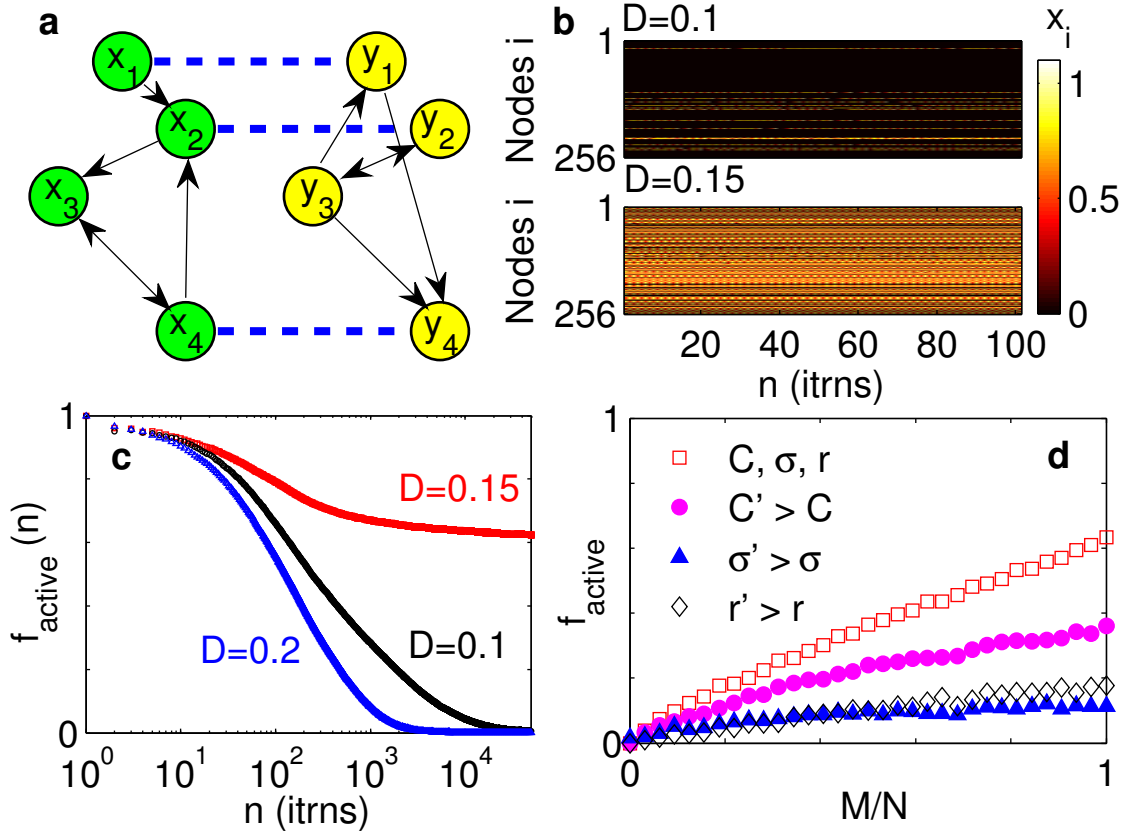
Figure 2.1: (a) Schematic diagram representing two interdependent networks, each comprising N nodes, that have intra-network directed nonlinear interactions (indicated by arrows) and inter-network diffusive coupling between M ($\leq N$) pairs of corresponding nodes (broken lines). (b) Pseudocolor representation of the spatio-temporal evolution of dynamical state x_i for each node i in one of the networks at two different values of the inter-network diffusive coupling strength, viz., (top) $D = 0.1$ and (bottom) $D = 0.15$, with black representing the absorbing state $x_i = 0$, i.e., extinction of activity. Increased interdependence between the networks allows more nodes to maintain persistent activity, i.e., $x \neq 0$. Increasing D further can result in a decrease in the fraction of active nodes f_{active} with time as seen in (c), indicating that long-term persistent activity occurs only within an optimal range of interdependence. (d) Increasing intra-network interactions either in terms of the connection density (C) or their strength (σ) for a given inter-network diffusive coupling strength (e.g., $D = 0.15$), results in a decrease in the fraction f_{active} of nodes with persistent activity. Similar decrease is also observed on increasing the range of the nonlinearity parameter r. However, increasing the number of corresponding node pairs M in the two networks that are coupled diffusively is seen to increase f_{active}, pointing to a fundamental distinction between intra- and inter-network interactions in their contribution to the overall dynamical robustness of the system. Results shown here are obtained for $N = 256$, $C = 0.1$, $C' = 0.3$, $\sigma = 0.01$, $\sigma' = 0.05$, $r \in [4.0, 4.1]$, $r' \in [4.1, 4.2]$ and averaged over 100 realizations.

$D = 0.15$, we observe that a finite fraction of nodes persist in the active state, although for much lower (e.g., $D = 0.1$) and higher (e.g., $D = 0.2$) interdependence the system exhibits complete extinction of activity [Fig. 2.1(b-c)]. Thus an optimal diffusive coupling between corresponding nodes in the two networks enhances the global stability of the system. This suggests, for instance, that ecological niches which in isolation are vulnerable to systemic collapse resulting in mass extinction, can retain species diversity if connected to neighboring habitats through species dispersal. Indeed, for this to happen, it is not even required that all species in the network be capable of moving between the different habitats. As seen from Fig. 2.1 (d), if only a subset of M nodes (out of N) are coupled between the two networks through diffusion, the system exhibits enhanced persistence with f_{active} increasing with M. However, enhancing the intensity of nonlinear interactions within each network by increasing either their connectivity C or range of interaction strengths (measured by the dispersion σ), as well as, amplifying the intrinsic nonlinear dynamics of the nodes by increasing the range of r, decreases the survival probability of active nodes. This is also evident from the variation with C and σ of the probability that a node persists in the active state asymptotically [Fig. 2.2 (a-b)] and is in agreement with earlier studies of global stability of independent networks [145, 149].

Fig. 2.2 (c) shows in detail the contrasting contribution of intra- and inter-network interactions to the robustness of the network in terms of maintaining persistent activity. The probability that a node persists in the active state asymptotically is seen to vary non-monotonically with increasing interdependence D between the networks at different values of the parameters C, σ and r that determine intra-network dynamics. For reference let us focus on the curve for $C = 0.1$, $\sigma = 0.01$ and $r \in [4.0, 4.1]$ [shown using circles in (c)]. We observe that when diffusion is either too low ($D < 0.09$) or high ($D > 0.2$) all activity in the network ceases within the duration of simulation. However, for the intermediate range of values of D, activity continues in at least a part of the network with the persistence probability reaching a peak around $D \simeq 0.16$. Varying the other parameters, such as network connectivity C, intra-network interaction strength σ or the nonlinearity
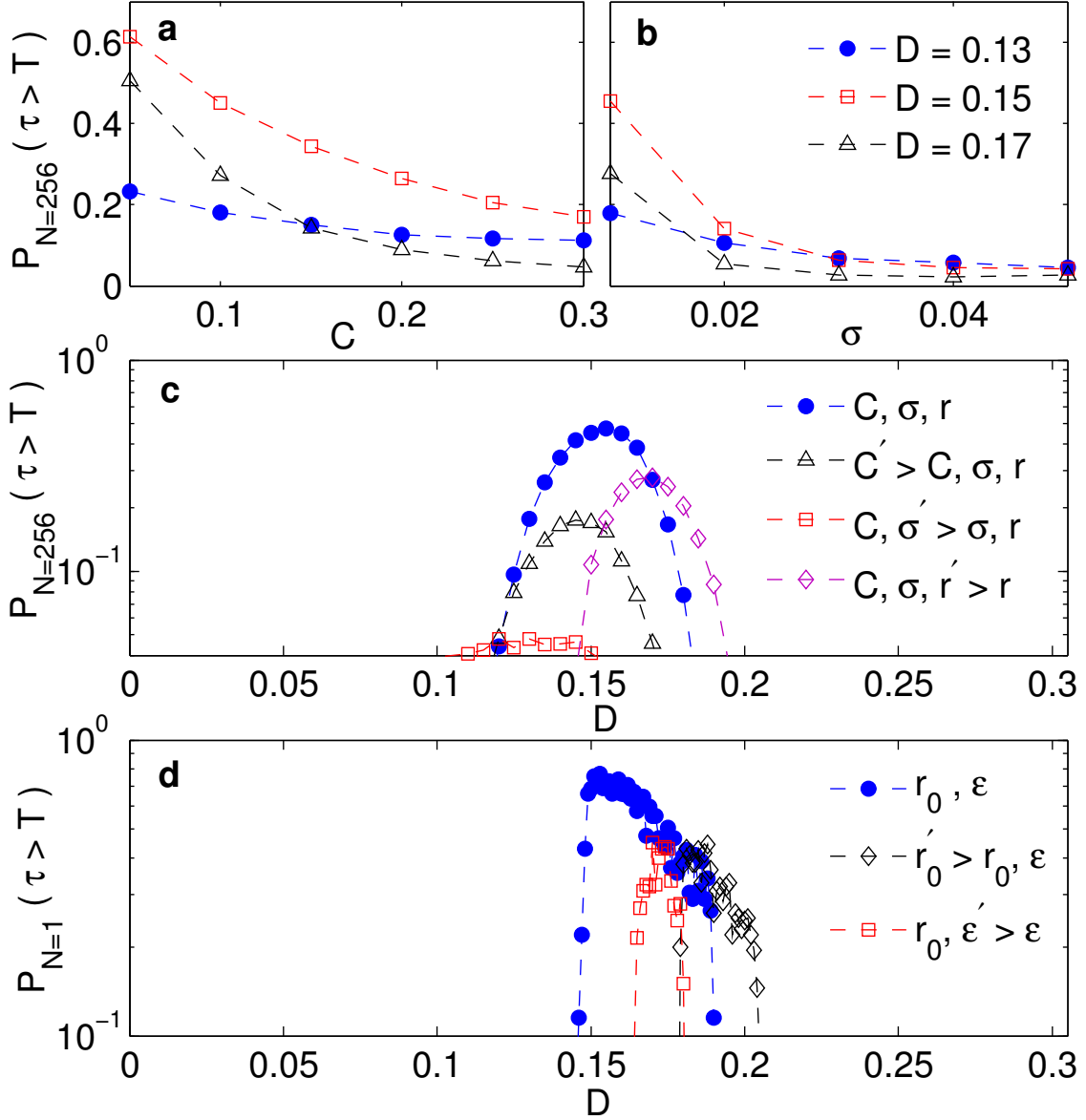
Figure 2.2: (a-c) Probability that nodal activity persists for longer than the duration of simulation $P(\tau > T)$ for an interdependent system of two networks decreases monotonically with increasing connection density C (a) and dispersion of interaction strengths σ (b) as shown for three different values of inter-network coupling strength D [indicated by same symbols in (a) and (b)]. (c) shows that the probability of persistent nodal activity has a non-monotonic dependence on D but decreases with increasing C, σ and r. Each of the networks comprise $N = 256$ nodes. Parameter values used are $C = 0.1$, $\sigma = 0.01$, $r \in [4.0, 4.1]$, $C' = 0.3$, $\sigma' = 0.05$, and $r' \in [4.1, 4.2]$. (d) Probability of persistent activity in a system of two diffusively coupled elements ($N = 1$) whose nonlinearity parameters fluctuate about r_0 due to a noise of strength ϵ. Non-monotonic dependence on coupling strength D is seen, similar to that for the large networks shown in (a). Parameter values are $r_0 = 4.05$, $\epsilon = 0.005$, $r_0' = 4.2$ and $\epsilon' = 0.01$. For all panels, simulation duration is $T = 5 \times 10^4$ itrns and results shown are averaged over 100 realizations.

parameter r, has a simpler outcome, viz., a decrease in the probability that activity will persist in the network at long times. This is shown by the other curves where we increase in turn C (triangles), σ (squares) and r (diamonds). Thus, our results indicate that there exists an attractor corresponding to persistent activity in the network for an optimal range of interdependence (in the neighborhood of $D = 0.15$) which coexists with the attractor corresponding to the extinction of network activity, relatively independent of other parameters.

To understand this in detail, we first note that even when $N = 1$, this much simpler system of two diffusively coupled elements exhibits qualitatively similar features when subjected to noise [Fig. 2.2 (d)]. The multiplicative noise of strength ϵ in the nonlinearity parameter, viz., $r = r_0(1 + \epsilon\eta)$, where η is a Gaussian random process with zero mean and unit variance, is introduced in lieu of the perturbations that each map will feel when connected to a much larger network through nonlinear interactions (Eq. 2.1). The choice of Gaussian noise is motivated by the strength of intra-network interactions, J_{ij}, being drawn from a normal distribution of zero mean and a finite variance. Moreover, if we focus on a single time-evolution step, the perturbations arising from intra-network interactions can be approximated by Gaussian distributed random variables for large networks by using the Central Limit Theorem. Subsequent evolution of the network may introduce correlations among the perturbations arising from the interactions which will not be accounted for in this Gaussian noise approximation. As in the case of the network, we choose $r_0 > 4$ so that an isolated node will almost always converge to the absorbing state, resulting in its extinction. Upon coupling two nodes, however, we observe that the probability of long-term survival of activity in the system becomes finite at an intermediate range of diffusive coupling strength (around $D = 0.15$), similar to that observed for a $N = 256$ network in Fig. 2.2 (c). Thus, understanding the genesis of diffusion-induced persistence for a pair of coupled logistic maps subject to noise [150], may provide an explanation for the same phenomenon observed in the system of interdependent networks described earlier.

The evolution equation for each node in the coupled system comprises two terms, the first representing the intrinsic dynamics of the node with the nonlinearity parameter r effectively reduced by a factor of $(1 - D)$ and the second being the contribution from the other node diffusively coupled to it. Note that the system converges to the absorbing state if the sum of the two terms exceeds 1. A lower bound for the range of D where persistence can occur is obtained by observing that in a persistent system the effective parameter governing the intrinsic dynamics has to necessarily be lower than 4, implying that $D_{c1} = 1 - (4/r)$. The upper bound for persistence is obtained by noting that when $D > D_{c2} = (1 - (1/r))/2$, the dynamics of the two nodes become synchronized asymptotically, effectively making them identical to the uncoupled node that almost surely converges to the absorbing state. In the regime $D_{c1} < D < D_{c2}$, persistence results from out-of-phase oscillations of the two nodes, each alternately visiting disjoint intervals in $(0, 1)$ such that the sum of the terms in their evolution equations never exceeds 1. Thus, regions in the $(0, 1) \times (0, 1)$ domain giving rise to in-phase oscillation converge to the absorbing state (extinction), while the ones mapping to out-of-phase solution persist, resulting in a complex basin of attraction for the persistent activity state of the system as shown in Fig. 2.3. It shows the regions in the phase space $I^2 : (0, 1) \times (0, 1)$ of the system of two diffusively coupled nodes, that correspond to initial states which move out of I^2 and into the absorbing state as a result of the dynamics. When the nodes are isolated ($D = 0$), successive iterations result in these regions increasing in size and eventually taking over the entire domain so that extinction will always happen. Similar behavior is seen for weak coupling (e.g., $D = 0.02$) although the shape of the regions are now modulated as a result of the interaction between the two maps. For high values of coupling also (e.g., $D = 0.3$) we observe the total extinction of activity in the asymptotic limit. However, for an intermediate value of coupling ($D = 0.16$), the complement region defined by trajectories starting from anywhere inside it remaining within I^2, retains a finite measure even at long times, thereby ensuring persistence of activity. Introducing multiplicative noise in the dynamics does not significantly change the structure of the basins shown in
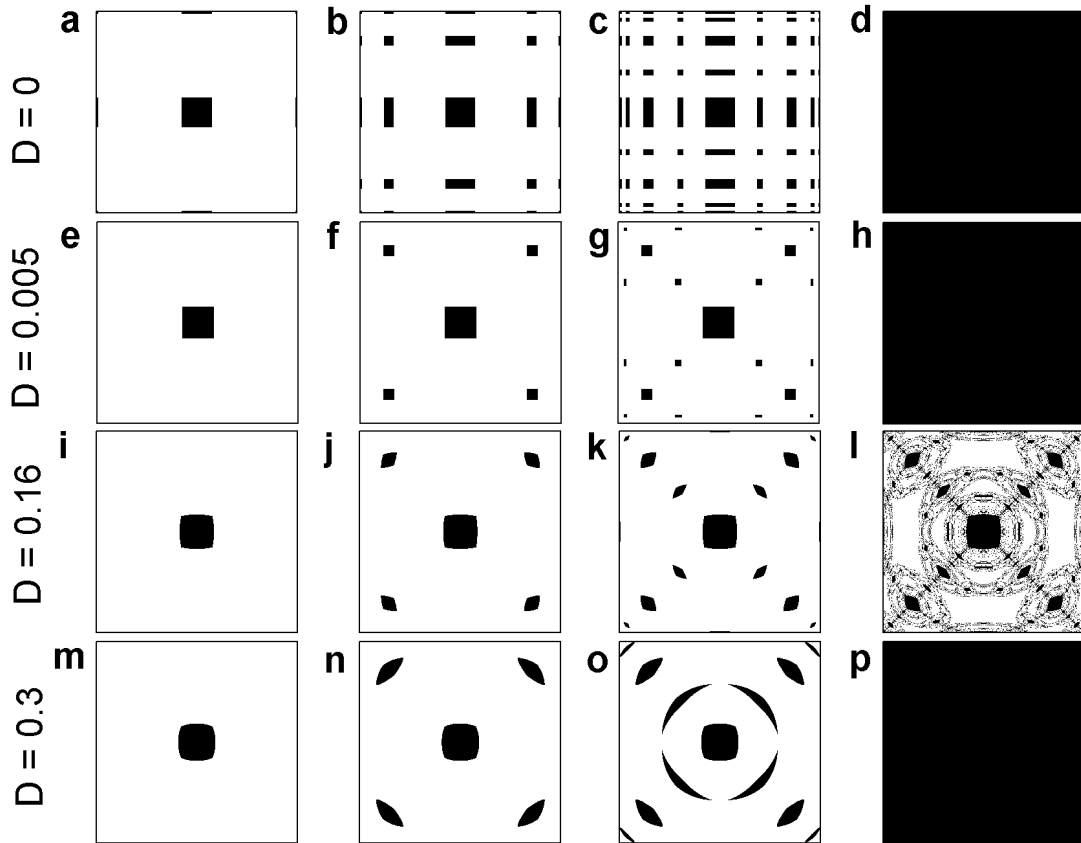
Figure 2.3: Time-evolution of a system of two diffusively coupled logistic maps having $r > 4$, showing (in black) the regions of phase space $I^2 : (0, 1) \times (0, 1)$ that correspond to initial states which lead to trajectories moving out of the I^2 domain resulting in extinction of activity after $n = 1$ (1st column), $= 2$ (2nd column), $= 3$ (3rd column) and $= 2000$ iterations (4th column) in both the maps. As each map has a segment projecting out of I^2, repeated iteration of the system when the maps are isolated ($D = 0$) would eventually drive almost all initial states to extinction (a-d). The same behavior is also observed for a low degree of diffusive coupling ($D = 0.005$, e-h), although the regions are now modulated because of the coupling with the dynamics of the other map. For a stronger diffusive coupling ($D = 0.16$) there is a finite region of phase space that remains within I^2 even after a large number of iterations which corresponds to the basin for the attractor exhibiting persistent activity (i-j). Further increase in the coupling strength (e.g., $D = 0.3$) again results in extinction of activity for almost the entire phase space (m-p).

Fig. 2.3 for low values of the noise strength ϵ.

To understand the above results we represent the dynamics of the system as $x_{n+1} = (1 - D)F(x_n) + DF(y_n)$, $y_{n+1} = (1 - D)F(y_n) + DF(x_n)$, where x, y are dynamical variables and $F(x) = rx(1 - x)$. The first term of each evolution equation can be interpreted as a logistic map with growth rate $(1 - D)r$ while the second term represents a contribution from the other map that is diffusively coupled to it. If the sum of the two terms exceed 1 for any of x, y, the corresponding variable goes to the absorbing state. In the weak coupling limit of low D where the first term dominates, it follows that if the effective growth rate $(1 - D)r$ exceeds 4, the system will exit the unit interval almost surely. Thus a lower bound for the range of D in which persistence can be observed is obtained by ensuring that $D > D_{c1} = 1 - (4/r)$. For example, for $r = 4.1$, $D_{c1} \simeq 0.024$. The upper bound of D for persistence is obtained by observing that when the two coupled maps synchronize their activity upon strong coupling, the dynamics reduces to that of an effective 1-dimensional map with $r > 4$ whose trajectories will eventually exit the unit interval with probability 1. Whether synchronization occurs can be investigated by looking at the dynamics of the difference of the two variables, $\delta = y - x$, viz., $\delta_{n+1} = r(1 - 2D)\delta_n[1 - (x_n + y_n)]$. If $D > D_{c2} = (1/2)(1 - (1/r))$, e.g., $D_{c2} \simeq 0.37$ for $r = 4.1$, the difference goes to zero asymptotically resulting in synchronization of the two maps and convergence to the absorbing state. Thus, the system has a possibility of persistence only in the intermediate range $D_{c1} < D < D_{c2}$. In this region, where the individual maps exhibit periodic attractors, persistence can arise through out-of-phase oscillations in the two maps, each alternately visiting two disjoint intervals in (0,1) such that the sum of terms never exceed 1. Thus, regions of $(0, 1) \times (0, 1)$ domain which yield the in-phase solution lead to the absorbing state (extinction), while those giving rise to the out-of-phase solution lead to persistence, resulting in a complex basin of attraction for the persistent activity state [shown in Fig. 2.3 (i-l)]. To show that such stable out-of-phase period-2 solutions exist for an optimal range of D, we can solve the coupled equations $x_{1,2}^* = (1 - D)F(x_{2,1}^*) + DF(x_{1,2}^*)$, and check for stability, thereby obtaining an implicit

33

equation involving the parameters r and D for which $0 < x_1^*, x_2^* < 1$. For specific choices of r and D, we can numerically verify that these solutions are stable, thereby providing confirmatory evidence of the proposed mechanism by which an optimal coupling induces persistence.

The bifurcation diagrams shown in Fig. 2.4 (a-c) indicate how the range of diffusive coupling strengths over which persistent activity is observed changes as we move from the simple case of two coupled maps ($N = 1$) to interdependent networks ($N \gg 1$). As already discussed, diffusively coupling two logistic maps having $r > 4$ allow their states to remain in the unit interval (corresponding to the nodes being active) provided the strength of coupling D remains within an optimal range [Fig. 2.4(a)]. Note that within this range there exists a region, approximately between (0.11,0.18), in which the attractor of the dynamical state of the node occupies a much smaller region of the available phase space $I : (0, 1)$. It is intuitively clear that for such values of D, introducing noise is much less likely to result in the system dynamics going outside the unit interval (thereby making the node inactive). If we now introduce multiplicative noise of low intensity (i.e., small ϵ), the range of D over which persistent activity occurs shrinks [Fig. 2.4(b)]. However, noise does not completely alter the nature of the system dynamics even though the the bifurcation structure is now less crisp. The system appears to be particularly robust in the region referred to earlier where the attractor covers only a small volume of the unit interval. We can compare this case with that of two interdependent networks, each comprising a large number of nodes [Fig. 2.4(c)] where the intra-network interactions are considered effectively to be 'noise'. We observe a reasonable similarity between their bifurcation structures, with the region of persistent activity spanning approximately the same range of D. As in the case of coupled maps with noise, in the case of networks also the system is most robust in the region where the attractor for the unperturbed system of two diffusively coupled maps is confined within a small subinterval inside $I \times I$. The validity of considering the dynamics of coupled nodes embedded within a network as equivalent to the pair being perturbed by an effective noise is further established by the strong resemblance be-
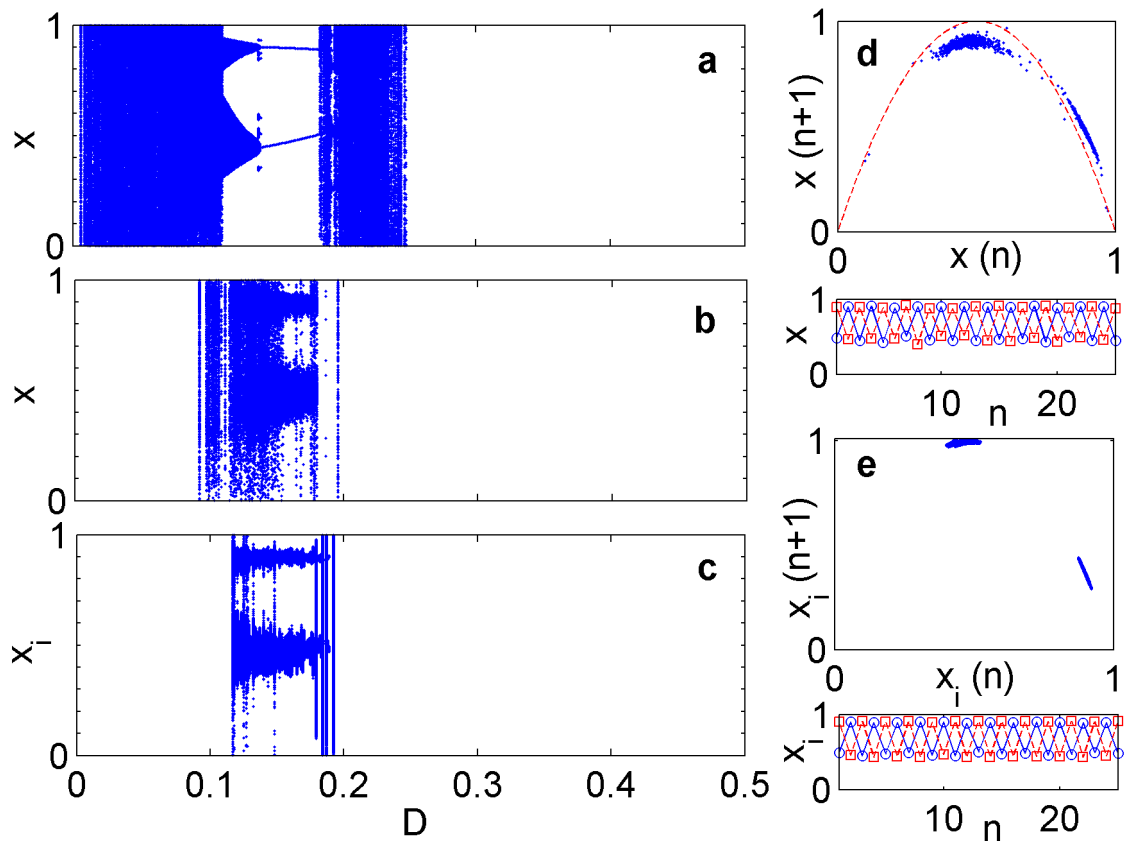
Figure 2.4: Bifurcation diagrams showing the attractor of the dynamical state x of a representative element as a function of the diffusive coupling strength between (a-b) two maps and (c) two networks each comprising $N = 256$ nodes. (a-b) The range of D over which there is long-term persistence of activity in two coupled maps for $r = 4.0025$ (a) is reduced when multiplicative noise of strength $\epsilon = 0.0125$ is introduced (b). The bifurcation structure resembles that of coupled networks shown in (c) for $M = 24$, $C = 0.1, \sigma = 0.01, r \in [4.0, 4.1]$. The contribution of intra-network interactions is qualitatively similar to multiplicative noise, resulting in a similar range of D for which persistence is observed in (b) and (c). (d-e) This similarity is reinforced by comparing the return maps (upper panels) and time-series (lower panels) of the asymptotic dynamical states for (d) two coupled maps with noise [as in (b)] and (e) two networks [as in (c)] for $D = 0.15$. The broken curve in panel (d) represents the return map of an uncoupled logistic map for $r = 4.0025$ shown for comparison.

tween the return maps and time-series for the two cases [Fig. 2.4(d-e)]. As mentioned earlier, to survive indefinitely the dynamical state of each map switches alternately between two disjoint intervals of the unit interval in an out-of-phase arrangement [see the time-series in the lower panels of Fig. 2.4(d-e)], corresponding to a trajectory that jumps between two "islands" of the basin for the attractor corresponding to persistent activity in the coupled system.

The above analysis, apart from explaining why populations that go extinct rapidly in isolation will survive for long times upon being coupled optimally, also helps us understand how the persistence behavior in the system will be affected by increasing the number of interacting components. As can be observed from Eq. (2.1), increasing N keeping C, σ unchanged corresponds to the summation in the interaction term being performed over more components. This suggests that there will be stronger fluctuations, that can be interpreted as a larger effective noise applied to the individual elements resulting in a higher probability of reaching the absorbing state and thereby lowering the survival fraction f_{active}. We have confirmed this through explicit numerical calculations in which N is systematically increased. To ensure that the results reported here are not sensitively dependent on the specific details of the model that we have considered here, we have also carried out simulations with (i) different forms of unimodal nonlinear maps, e.g., $F(x) = (x - l)e^{r(1-x)}$ for $x > l$; 0 otherwise [151], and (ii) different types of connection topologies for the initial network, e.g., those with small-world properties [71, 152] or having scale-free degree distribution [153]. We find in all such cases that the qualitative features reported here are unchanged, with the network connecting the surviving nodes becoming homogeneous asymptotically irrespective of the nature of the initial topology, suggesting that the enhanced persistence of activity in optimally interdependent networks is a generic property.

Finally, Fig. 2.5 shows that when a non-zero mean μ for the distribution of intra-network interaction strengths J_{ij} is used, the optimal range of inter-network coupling D in which
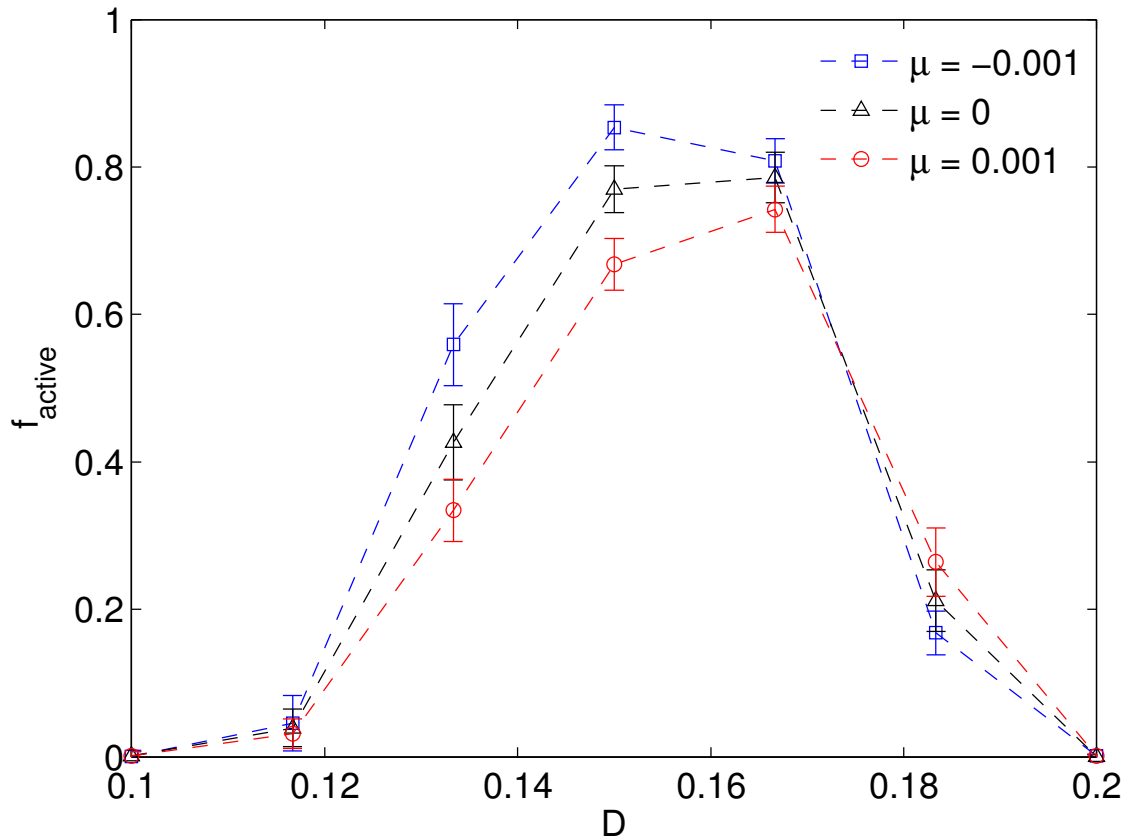
Figure 2.5: The optimal range of inter-network coupling strengths D where the asymptotic fraction of active nodes f_{active} is finite remains almost same when the interaction strengths J_{ij} are chosen from a $Normal(\mu, \sigma^2)$ distribution with different values of the mean μ. Curves representing $\mu = -0.001$, $= 0$ and 0.001 are shown here, which correspond to the number of positive interactions in the network are relatively fewer, equal to and more than the number of negative interactions in the network, respectively. Results shown here are obtained for $N = 128$, $C = 0.1$, $\sigma = 0.01$, $r \in [4.0, 4.1]$ and averaged over 100 realizations.

nodal activity is persistent remains qualitatively unchanged. Note that the absolute value of the mean μ cannot be chosen to be arbitrarily high as that will result in most dynamical trajectories being rapidly ejected outside the unit interval on which the map $F()$ is defined, resulting in extinction of activity in the entire network. We note that as μ is increased from negative to positive values the peak of f_{active} is seen to shift to a relatively higher value of inter-network coupling D.

2.4 Conclusions

To conclude, we have investigated the role of interdependence between constituent networks on the stability of the entire system in a dynamical framework. Unlike percolation-based approaches where failure is often identified exclusively with breakdown of connectivity so that increasing interdependence necessarily enhances fragility [106], our dynamical perspective leads to a strikingly different conclusion. In particular, we show that the system has a much higher likelihood of survival for an optimal interdependence, with both networks facing almost certain catastrophic collapse in isolation. Such enhancement of persistence of activity in a critical range of coupling is analogous to the promotion of synchronization among self-propelled agents for an optimal interaction strength [154]. Our results suggest that interdependence may be essential in several natural systems for maintaining diversity in the presence of fluctuations that are potentially destabilizing. Thus, interdependence need not always have negative repercussions. Instead its impact may depend strongly on the context, e.g., the nature of coupling and the type of dynamics being considered.

Chapter 3

Resolving the complexity-stability debate: A matter of time-scales

3.1 Introduction

Empirical studies of ecological networks suggest that the network has many more weak links corresponding to low-intensity interactions compared to stronger ones [155, 157]. Indeed, theoretical arguments suggest that the preponderance of weak links stabilize the system to perturbations [158]. It has also been argued that systems having many connections can be stable if most of these links are weak [87]. This suggests that a node with many connections would tend to have weaker interactions on average with each of its network neighbors compared to one having fewer links [see schematic in Fig. 3.1 (a)]. Empirical evidence for this is harder to obtain as interaction strengths are often difficult to measure [160]. However, using trophic flow[1] as a measure of the strength of a link, it is possible to see a reciprocal relation between the number of incoming links of a node and the average value of its interaction strengths [Fig. 3.1 (b)]. It suggests that a plausible

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[1]Trophic flow is the transfer of energy between successive levels of a food web as a result of members of a predator species consuming members of its prey species.

mechanism for systems to retain stability while being densely connected is to scale their interaction strength by their degree.

In this chapter we present the striking dynamical consequences when the interaction strengths of nodes in a network are scaled by the number of their connections. For simplicity we have considered the situation where the link weights J_{ij} of a node i are simply reduced proportional to the total number of incoming links k_i, i.e., $J_{ij} \rightarrow J_{ij}/k_i$, although more complex functions can be envisaged. First we show that in the asymptotic limit, the scaling by degree does *not* result in a situation different from that seen in the absence of such scaling, unlike what one may expect from a linear stability argument. This is because as a result of the dynamical evolution, the conditions required for the linear stability argument to be valid are quickly deviated from. Thus, even with degree scaling densely connected networks would tend to have a lower global stability than sparsely connected ones. However, surprisingly, we find that scaling significantly delays the convergence to this asymptotic state for densely connected systems. This transient regime is marked by a much higher number of surviving nodes and thus, when observed at short time scales, one will observe that systems with higher connection density will have more surviving nodes than those having lower connection density (which would have already converged to their asymptotic state). If the system is retained in a non-equilibrium state by being driven through a steady rate of addition of new nodes, one can further extend the transient regime. As in nature, evolving systems are likely to be constantly augmented with newly arriving constituents, we propose this scenario as a plausible mechanism by which the stability-diversity debate for complex systems can be resolved.

3.2   Model

Let us consider a model system comprising N dynamical elements connected to each other through nonlinear interactions having a sparse random topology. The state of the system

at any time n is described by $\mathbf{x}(n)=\{x_1(n), x_2(n), \ldots, x_N(n)\}$ where x_i is a continuous dynamical variable associated with node i ($i = 1, ..., N$). In an ecological context it can be interpreted as the relative mass density of the i-th species in a food-web comprising N interacting species. We consider generalized Lotka-Volterra interactions between the nodes as, apart from its interpretation in terms of predator-prey relations, this is one of the simplest and ubiquitous nonlinear coupling forms between dynamical elements. The time-evolution of the states of the nodes are described in terms of N coupled equations:

$$x_i(n + 1) = f[x_i(n)(1 + \sum_j (J_{ij}/k_i^\alpha(n))x_j(n))], \qquad (3.1)$$

where the sign of the coupling strength J_{ij} determines the nature of, and its magnitude decides the intensity of the effect of interaction with node j on i (in general, $J_{ij} \neq J_{ji}$). Note that the link weights J_{ij} are reduced proportional to the "active" in-degree of a node, i.e., the number of incoming links from neighbors which are active at time n. We have chosen non-zero elements of $\mathbf{J}$ from a $Normal(0, \sigma^2)$ distribution where the standard deviation σ of the interaction strengths is a system parameter that is varied in different simulations. If the topology of the network is sparse, the connection density of the system is specified by the fraction C of non-zero elements of $\mathbf{J}$ (for simplicity we assume that the corresponding adjacency matrix is symmetric, i.e., $J_{ij} = 0 \implies J_{ji} = 0$). The range of values over which x varies depends on the function $f()$ that governs the dynamics of individual nodes. We have chosen the nonlinear map $f(x) = x \exp[r(1 - x)]$, $x > 0$ and 0 otherwise, such that $x = 0$ is the absorbing state and r is the nonlinearity parameter by varying which the map is capable of exhibiting a wide range of behavior including equilibria, periodic oscillations and chaos [151]. In the work reported here the range of r used is such that nodes do not reach the absorbing state in the absence of coupling as we are primarily interested in the instability generated by interactions over the network.

The global stability of the system can be characterized by the asymptotic fraction f_{active}^∞ of nodes in the network that have not reached the absorbing state, viz., $f_{active}^\infty = \text{Lt}_{n\to\infty} f_{active}(n)$,

where $f_{active}(n) = \sum_i \Theta[x_i(n)]/N$ (with $\Theta[x] = 1$ for $x > 0$ and 0 otherwise). The dependence of the strength of interactions J_{ij} ($j = 1, \ldots, N$) of a node i on its degree k_i (i.e., the number of other nodes it is connected to at time n) is quantified by the parameter $\alpha \geq 0$, as k_i^α is the factor by which each of the interaction strengths of node i are scaled in our model. Note that the global stability of networks of nonlinear maps have been investigated earlier (e.g., Ref. [149]) for the special case of $\alpha = 0$ which correspond to the connection strengths of the nodes being independent of their degree. As we show below, if the effective interaction strength of nodes reduces with their degree (i.e., $\alpha > 0$), the convergence time required to reach the steady state may become extremely long and the system properties show novel features in this extended transient period.

3.3 Results and Discussion

Initial values of the dynamical variables x of all N nodes are chosen randomly from a uniform distribution over $[0, 1]$. On beginning the time-evolution of the system, some of the nodes will reach the absorbing state because of interactions with other nodes. As a result the fraction of active nodes f_{active} decreases with time eventually attaining a steady state f_{active}^∞ whose value depends on the parameters N, C, σ and r. As nodes reach the absorbing state, they no longer take part in the collective dynamics, effectively removing them from the network. This results in a reduction in the number of interactions of the remaining nodes which are now therefore more likely to persist in the active state. The simulations are carried out for upto 10^4 iterations beyond which the probability of any more nodes reaching the absorbing state becomes extremely small.

Fig. 3.1 (b-e) shows the dependence of the asymptotic fraction of persistent nodes f_{active}^∞ on the different system parameters. The steady-state results broadly agree with the implications of the May-Wigner stability theorem [2], as has been shown earlier for the case

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[2]The May-Wigner Theorem is a statement about the linear stability of a dynamical system on a random network, expressed as a function of the number of network nodes (N), the connection density (C) and the
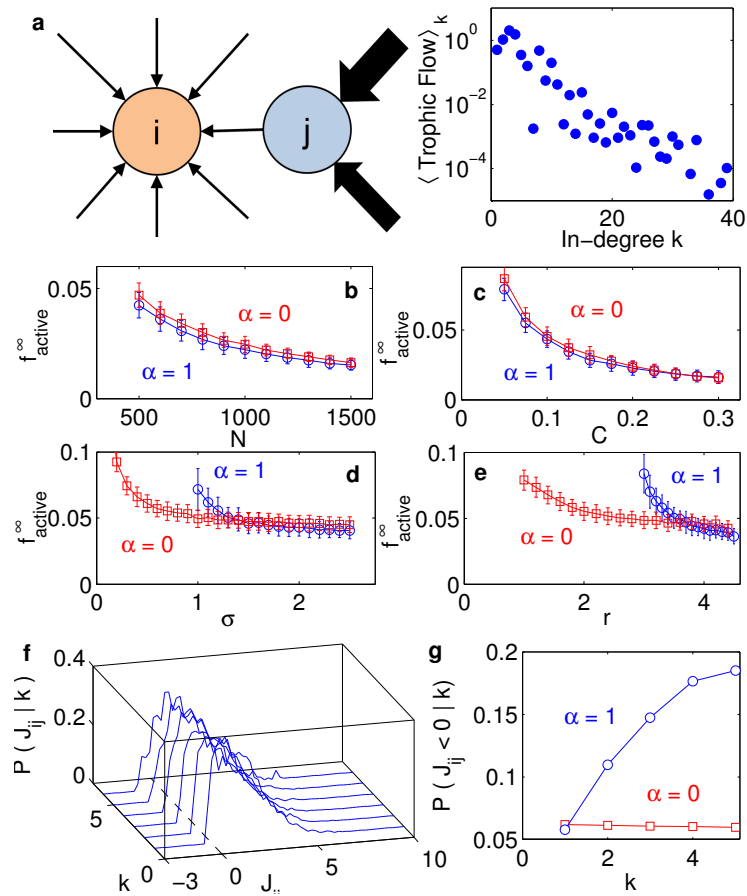
Figure 3.1: (a) Schematic diagram (left) indicating that high in-degree node (i) tends to have weaker interactions (link strength represented by edge thickness) compared to one with low in-degree (j) as suggested by empirical data, e.g., from trophic exchange network reconstructed for South Florida ecosystems during dry season (right) [159]. The interaction strengths of species having k prey species as measured by their mean trophic flow gets progressively weaker with number of interactions (k). Results of incorporating degree-dependent scaling of interaction strengths in a dynamical model of N interacting nodes are shown in (b-e). The variation of the fraction of nodes f_{active} in the network which have persistent activity in the steady state is compared between when the interactions are scaled by their in-degree with scaling index $\alpha = 1$ (circles) and in the absence of such scaling, i.e., $\alpha = 0$ (squares). The global stability of the networks as measured by f_{active} is seen to decrease with increasing number of nodes N (b) and connection density C (c), independent of the scaling index. Increasing the dispersion of interaction strengths σ (d) and the nonlinearity parameter r (e) also result in a similar decreasing trend in f_{active}, even though scaling the interaction strength by degree allows the decline in global stability to occur at larger values of σ and r. (f) The asymptotic distribution of the interaction strengths J_{ij} of a node conditioned on its degree k shows that as it becomes more connected, a node has a higher probability of having negative interaction strengths which makes it more likely to reach the absorbing state (extinction of activity). (g) explicitly shows that scaling the interaction strengths by the degree of each node ($\alpha = 1$) allows nodes with strong negative interactions to continue to be active in contrast to the situation where such scaling is absent ($\alpha = 0$). Results shown here are obtained for $N = 500$, $C = 0.1$, $\sigma = 2$, $r = 4$ and $\alpha = 1$.

43

where the interaction strengths are independent of degree, i.e., $\alpha = 0$ [149]. Although introducing degree dependence in the interaction strengths, viz., by using $\alpha = 1$, does not seem to produce a noticeable difference in the number of active nodes persisting in the long term, we observe that the steady-state network structure in this case is altered. In particular, the distribution of connection weights J_{ij} for nodes having specific degree k_i for the two cases are clearly distinct, with strongly negative weights being more likely to be retained in nodes having many interactions when $\alpha = 1$ (Fig. 3.1, f). By contrast, for $\alpha = 0$, having negative coupling to other nodes is more likely to drive a node to the absorbing states so that the active nodes have a low probability of possessing such links irrespective of degree (Fig. 3.1, g).

Note that the qualitatively similar global stability properties of the system for $\alpha = 0$ and $\alpha = 1$ indicate that the effect of degree dependence of connection strengths cannot be simply approximated as the scaling of the dispersion σ of J_{ij} by the mean degree of the system raised to the power α. A naive application of the Wigner-May stability theorem would then have suggested that for $\alpha > 1/2$, networks having higher average degree will have more nodes persisting in the active state asymptotically than networks having lower average degree. Indeed such a trivial resolution of the complexity-stability debate has been proposed earlier [156]. However, such a simplistic argument does not take into account the time-evolution of the degrees of each of the nodes that radically alters the tenor of the argument. The crucial point to note here is that for $\alpha > 0$, $P(\sum_{j=1}^{N} J_{ij}x_j < -1)$ which determines the probability that a node will evolve to the absorbing state will increase faster for nodes having low degree compared to those having higher degree, which will result in a lower number of extinctions before the asymptotic state is reached (as explained below for $\alpha = 1$).

The most significant effect of introducing degree dependence is seen not in the asymptotic

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variance of the interaction strengths distribution (σ). Describing the population dynamics of the species comprising an ecosystem in terms of a coupled system of differential equations, if we assume that the network of interactions between the species is unstructured, then the theorem says that an equilibrium of the system will be almost certainly stable if the condition $NC\sigma^2 < 1$ holds.
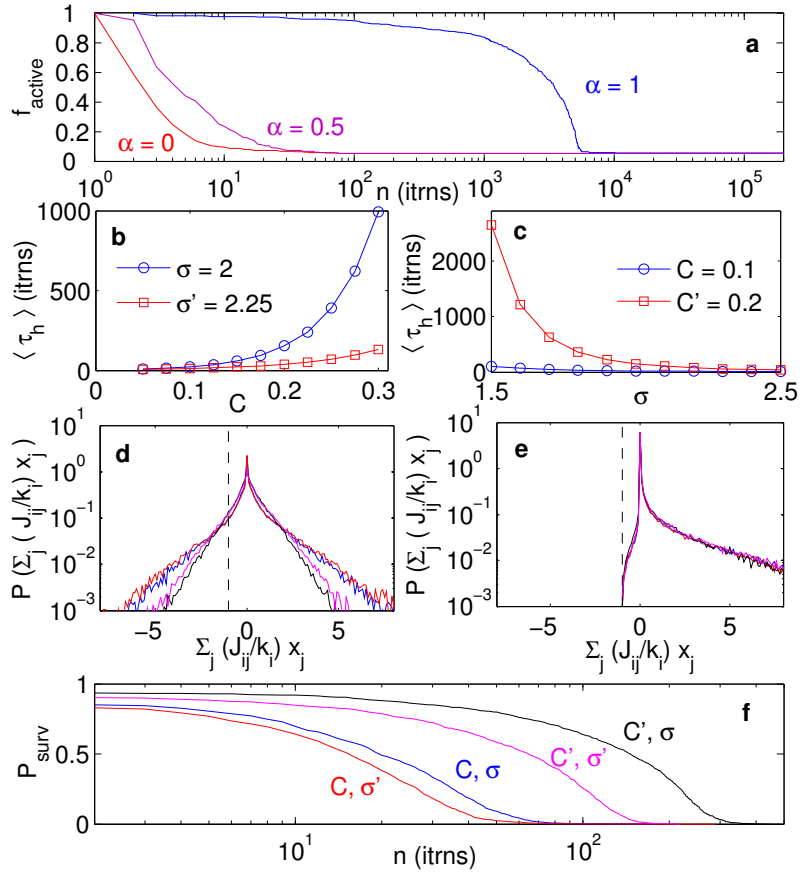
Figure 3.2: (a) Scaling of the interaction strengths of a node i by its degree k_i ($\alpha = 1$) results in a significantly longer transient period during which the fraction of active nodes f_{active} remains much higher than in the asymptotic state. For weaker scaling (e.g., $\sqrt{k_i}$, i.e., $\alpha = 0.5$) or in the absence of any scaling ($\alpha = 0$), f_{active} converges rapidly to its asymptotic value. Results shown for $N = 500$, $C = 0.1$, $\sigma = 1.5$ and $r = 4$. (b-c) The mean duration $\langle \tau_h \rangle$ for which the system is in the transient regime characterized by high f_{active} increases with the connection density C and decreases with the dispersion of the interaction strengths σ ($\alpha = 1$, averaging done over 1000 realizations). (d-e) The distribution of the interaction terms governing the dynamics of individual nodes i, viz., $\Sigma_j J_{ij} x_j / k_i$, is shown for the initial period (d, $n = 1$-10 itrns) and in the asymptotic state (e) for different values of C and σ [color code same as in (f)]. The broken vertical line at -1 represents the critical value below which activity in a node cannot survive. Systems having larger C or a smaller σ exhibit lower dispersion initially with a resulting reduced probability of extinction of activity in a node. In the asymptotic state where there are no further extinction events the distributions for all systems will converge. (f) The probability that a node will survive for a specific duration n, $P_{surv}(n)$ decreases at different rates with time depending on C and σ. The reduced extinction probability for higher C or lower σ [shown in (d)] results in a slower rate of decline in P_{surv} as shown. Results shown for $N = 500$, $C = 0.1 < C' = 0.2$, $\sigma = 2 < \sigma' = 2.25$ and $r = 4$.

45

behavior of the system 3.1 (which as seen from Fig. 3.1 is qualitatively similar to $\alpha = 0$) but in the properties exhibited by it in the transient period before the steady state is reached (Fig. 3.2). The most striking feature we observe is that systems characterized by $\alpha > 0$ will continue in states corresponding to high values of f_{active} for longer periods and with increased α this effect is more pronounced. Thus, when observed at times longer than the scales at which systems with no degree-dependence have reached the asymptotic state corresponding to low values of f_{active} but shorter than the time required for the degree-dependent system to reach its steady state, one may conclude that systems with higher connection density have higher global stability. The dependence of the transient period of f_{active} vs n is clearly evident from Fig. 3.1(a). The duration of the period the system (3.1) is in the transient regime characterized by high f_{active} is measured by the half-life τ_h, i.e., the time upto which $f_{active} > 0.5$. Its mean value for $\alpha = 1$ is seen to increase with higher connection density C and lower dispersion σ^2 of the interaction strengths (Fig. 3.2, b-c). To understand the results, we compare the initial and final distributions of the interaction term $P(\sum_j J_{ij}/k_i)$ in parts (d-e) of Fig. 3.2. A given node i goes extinct when the interaction term $\sum_j J_{ij}/k_i$, executing a random walk depending on the dynamics of neighboring nodes j crosses the boundary at -1. Now, for higher connection density C, the distribution of step lengths of the random walk (which is equal to $J_{ij}x_j/k_i$) has less spread as compared to for a lower value of C((d) of Fig. 3.2). As a result, depending on the connection density, the probability of the random walk of the interaction term to survive on the right of the boundary at -1 is higher for higher C (and a lower σ), thus explaining the observed difference in the length of the transients (Fig. 3.2 (f)). For the same number of surviving nodes, the network for $\alpha = 1$ is more connected on average than for $\alpha = 0$. As a result the number of surviving nodes is the same – the more densely connected network feels the same magnitude of perturbation at higher degree as felt by the relatively sparser network at $\alpha = 0$ which has lower average degree.

Let us consider how the interaction term $\sum_j J_{ij}x_j/k_i$ is affected by the connection density C by considering two different values of the latter, viz., C_1 and C_2 ($< C_1$). Note that the

numerator contains a sum of terms that are equally likely to be positive and negative, and therefore is close to zero with a very high probability. Thus the term $\sum_j J_{ij} x_j / k_i$ is effectively smaller for C_1 compared to C_2 in the transient period. This makes the transients long lived as the interaction term approaches the absorbing boundary slower for C_1 compared to C_2. In the long-time limit, the number of surviving nodes is much less than N because of extinctions that occur when the state variable x of any of the nodes reaches the corresponding absorbing state $x = 0$. The number of surviving nodes in the asymptotic limit is lower for C_1 than C_2 because for same value of interaction strength dispersion σ, a node in the sparser network will feel less perturbation effectively than a node in the denser network, even when nominally they have the same distribution of interactions, cf., numerical calculations shown in Fig. 3.2 (d and e).

Before we proceed on to the effect of addition of new nodes to the dynamical properties of the system following (3.1), let us discuss the role played by the parameter α on the transient and steady state properties. We have seen from Fig. 3.1 and 3.2 that α plays a decisive role in governing the transient properties, but becomes almost irrelevant in the long-time limit. This is because in the initial periods of evolution, $\alpha = 1$ scales the weights J_{ij} by a factor k_i, significantly reducing the effective perturbation felt by the node i, when compared with $\alpha = 0$. Thus, making the transients long lived for $\alpha = 1$. However, due to the removal of nodes with time, the reducing degree k_i tends to diminish the effect of division by the degree, resulting in the dynamical properties of the system becoming independent of α in the long-time limit.

The transient properties are particularly interesting in the context of ecosystems, which we know never reach a steady state due to constant removal and addition of new species. Such long-lived transients have always been a source of discrepancy between the predictions made by theoretical models and field experiments. Motivated by these observations, we next study the effect of addition of new nodes at constant rates, with particular attention on the transient properties of the system.
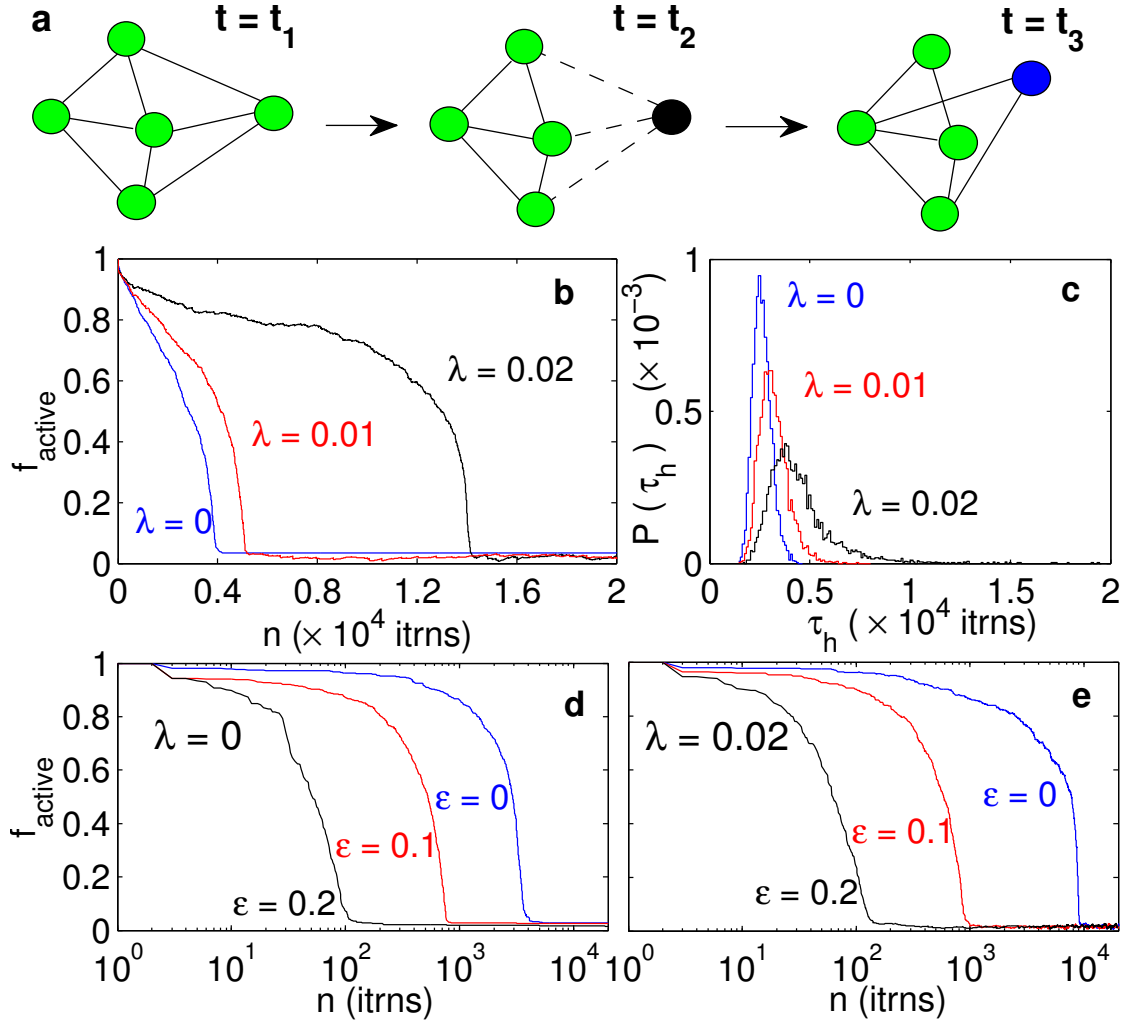
Figure 3.3: (a) Schematic diagram representing network evolution. The initial network of active nodes ($t = t_1$) reduces through extinction of activity in nodes (black node, at $t = t_2$) but also increase through occasional arrival of new nodes that connect to a subset of existing active nodes (blue node, at $t = t_3$). (b-c) Increasing the rate of arrival of new nodes, λ, can increase the duration for which the network remains in the transient regime characterized by high f_{active} (b) as indicated by the distributions of τ_h for different values of λ (c). (d-e) Compared to the deterministic situation ($\epsilon = 0$), the system can converge to the asymptotic state characterized by low f_{active} more quickly when the node dynamics are subject to noise ($\epsilon > 0$) resulting from the nonlinearity parameter fluctuating about a mean value r. Increasing the noise strength ϵ results in reduction of the duration of the transient regime, but as in the deterministic case (b-c), arrival of new nodes at a rate λ [= 0.02, (e)] allows the system to retain a higher fraction of active nodes for longer. Results shown for $N = 500$, $C = 0.2$, $\sigma = 1.5$, $r = 4$ and $\alpha = 1$.

Finally, in Fig. 3.3 we discuss the effect of addition of new nodes to the dynamical system following Eqn. (3.1). Panel (a) of the figure shows the schematic diagram depicting the extinction of a given node followed by the addition of a node connecting to two surviving nodes. We see from the variation of f_{active} with n [Fig. 3.3 (b)] that the transients increase with increasing rate of addition λ, which is the parameter of the exponentially distributed intervals of inter-addition times. This is also confirmed by the distribution of transient times τ_h [see panel (c)], where we see that with increasing λ the distribution shifts towards the right. In Fig. 3.3 (d-e) we study the effect of randomness in the growth parameter, i.e., $r \rightarrow r(1 + \epsilon \eta)$, with $\eta \sim N(0, 1)$ and ϵ the noise strength. Such a randomness arises due to environmental fluctuations in an ecosystem. We see that for a given value of r, increasing magnitude of fluctuations ϵ results in a rapid decay of f_{active} with n. In addition, similar to the case of a deterministic r, we see from (e) that adding new nodes to the system at a rate λ results in an increase in the transient periods, even in the presence of random fluctuations in the nonlinearity parameter r.

The primary implication is about the transient properties of the system. How does the length of the transient depend on structural parameters of the network, viz., connection density and interaction strengths, and the nature of the dynamics of the elements. Empirical investigation of ecosystems suggest that species with many connections tend to have weaker interactions than those having fewer connections [161]. Our model takes this into account by assuming a scaling of interaction strength with the node degree. This apparently simple relation leads to strikingly different behavior in terms of global stability of the system in the short-term. In particular, when observed in the transient state (which can persist for significantly long durations, depending on network parameters) more connected systems would have a much higher fraction of surviving nodes and thus presumably have higher global stability. This appears to run counter to the May-Wigner result that more connected systems would be less stable. It is only in the asymptotic limit, when the system has reached an equilibrium that the results are consistent with that of May. Thus, it appears that the apparent conflict between empirical observations and theoretical

results in the stability vs diversity debate can be simply reconciled as either viewpoint can be validated depending on the time-scale at which the complex system is being observed. When observed over short durations, systems that are far from equilibrium would appear to exhibit a positive relation between connectivity and stability (agreeing with the empirical literature). On the very long-term however as the systems converge to their asymptotic states, more connected systems would collapse to a much smaller set of surviving nodes compared to less connected ones (consistent with the theoretical position). Furthermore, as any real system would be continually perturbed and kept far from equilibrium, it may persist indefinitely in the transient state corresponding to high connectivity being positively related to high stability.

While our model is deterministic, real ecosystems are of course buffeted by noise at all times. The effect of environmental changes has been accounted for to an extent in our model by introducing stochastic fluctuations in the dynamical parameter governing the growth rates of individual nodes. However, even in the presence of such random perturbations, the qualitative aspects of the deterministic model are preserved underlining their robustness.

In the non-equilibrium situation the network is subject to a steady rate of newly arriving nodes, with the intervals between successive arrivals being distributed exponentially. Note that, there is an underlying assumption that new nodes are added to the system independent of the existing ones. However, the assembly of ecological communities do show that which species joins a ecosystem successfully depends on the species that were already existing prior to its joining. Thus, history matters. Taking account of memory in the node addition process may lead to different distribution of inter-arrival time durations whose effect on the stability of the system may be investigated in future studies. One could potentially also make the arrival rate time-varying, e.g., depending on the number of surviving nodes at any given time.

3.4 Conclusions

To conclude, we show that the stability-diversity debate for complex systems can be resolved simply by considering the time-scale in which such systems are being observed if the strengths of interaction scale according to the number of connections of the nodes. While in the asymptotic state the more connected systems will have lower global stability (in terms of fewer number of nodes with surviving activity), consistent with the May-Wigner result for local stability, in the short term dense connectivity will result in extremely long-lived transients compared to sparsely connected systems. Thus, when observed at this time-scale the more connected systems would appear to have a higher number of active nodes and thus possessing higher global stability than systems with lower degree of connectivity. The reason for densely connected systems requiring longer time required to reach the asymptotic state can be explained by considering how the dynamics of nodes is governed by a term representing interactions with other nodes. This term executes a random walk about zero until it strikes the boundary of the absorbing region corresponding to extinction of activity in the node. For nodes having higher number of connections, the steps of the random walk tend to be shorter and correspondingly it takes longer to strike the absorbing boundary, leading to an extended period preceding extinction of activity in nodes. The mechanism is robust, being observable in the presence of random fluctuations in the local dynamics of nodes and moreover is enhanced in the non-equilibrium scenario where networks are subjected to a steady rate of arrival of new nodes balancing the loss of existing nodes through extinction. As in nature complex systems are almost always far-from-equilibrium this may be a possible mechanism by which higher connectivity appears to lead to increased stability even though theoretical results have suggested otherwise.

3.A   Appendix: Empirical data analysis

To investigate whether interaction strengths of nodes depend on their degree we have considered empirical data for ecosystems of Southern Florida compiled by investigators of ATLSS (Network Analysis of Trophic Dynamics in South Florida Ecosystems) from existing data and field-work [159]. The trophic flow data between different components of the ecosystem are available separately for wet (June-November) and dry (December-May) seasons in .paj format from an online database [162]. In the reconstructed network the vertices represent major components of the ecosystem such as trophic species or detritus and the edges represent the exchange of carbon between the components. For our analysis we have considered the predator-prey interactions between all the living compartments, i.e., $N = 122$ trophic species. The panels at the left of Fig. 3.4 show the matrices of estimated trophic flows (in logarithmic scale) between the different species in the dry (top) and wet (bottom) seasons. Note that the flow TF_{ij} is from a prey species j to a predator species i. The number of incoming links k_l for each species l is given by the the number of prey species from which there is a trophic flow into node l. The right panels of Fig. 3.4 shows that the magnitude of flows from prey species of a generalist predator that has many prey tends to be generally smaller than that for more specialist predators (i.e., having fewer prey). The mean interaction strength for a node of degree k (shown in main text) is computed by averaging over the magnitudes of all trophic flows into predator species having k incoming links (i.e., k different prey species).
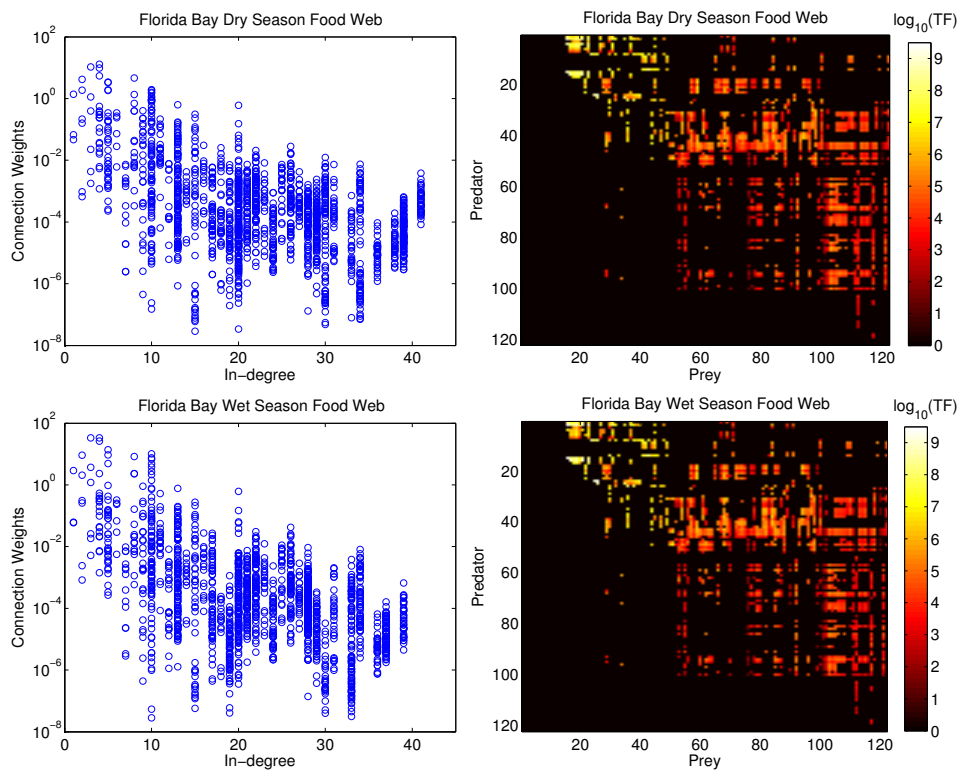
Figure 3.4: Variation of connection weights(on a logarithmic scale) with in-degree of the nodes. Top left shows the empirical data for the dry season and bottom left for the wet season respectively. Corresponding weight matrix is shown on the right, with the color code representing the trophic flow on a logarithmic scale.

Chapter 4

Non-equilibrium dynamics in a financial market

4.1 Introduction

Financial markets are one of the best known examples of complex systems which are characterized by a large number of interacting components and exhibiting nonlinear dynamics that is inherently unpredictable. Instead of a deterministic description of the time evolution of the components however, the presence of a large number of constituents makes it possible to analyze the statistical properties of the system as a whole, i.e., the market. Indeed, robust statistical features have been reported for many different markets, notable amongst them being the "inverse cubic law" describing the nature of stock price fluctuations in markets of developed [163] and developing economies [164, 165]. Many earlier studies of statistical properties of markets have used daily (or end of day) trade data which does not take into consideration the dynamical behavior of intra-day trading. In recent times, the availability of high-frequency(HF) data containing information about every transaction taking place in the market has made it possible to uncover the properties of stocks markets at the highest possible resolution [166]. Motivated by these studies,
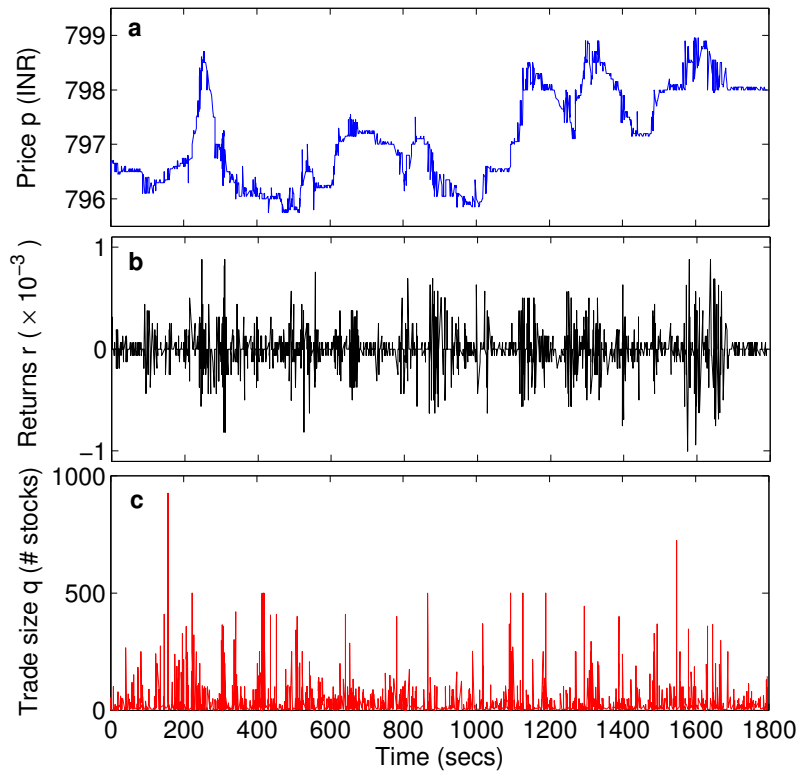
Figure 4.1: Representative time-series of *RELIANCE* stock for 30 minutes beginning at 0950 hrs on December 1, 2012 showing (a) price $p(t)$ of the stock, (b) log-returns $r(t)$, and (c) trade size $q(t)$ as a function of time. Time is measured in seconds, with the origin (0) set at 0950 hrs.

we have used the HF data of the National Stock Exchange(NSE) [167, 168] of India to uncover its principal statistical features. We focus on properties of the market as a whole, as well as, that of individual stocks, e.g., the distribution of trade sizes, the distribution of waiting times between two successive trades, relation between price fluctuations and waiting times, etc. Such a study has implications for our current understanding of the empirical properties of the dynamics of an emerging market, in particular the intra-day behavior of trades and price movements, as well as in modeling such behavior in terms of statistical physics.

56

4.2 Description of the data

In order to study the statistical properties of NSE, we use HF data for the months of December for each year from 1999 to 2012. Note that HF data comes with its own set of challenges, e.g., overwhelming data sizes, unevenly spaced time series, etc. This is easily seen from a sample of the data set for December 2003 as shown below:

```
20031201|MTNL|09:56:29|122.20|10
20031201|MTNL|09:56:29|122.25|50
20031201|MTNL|09:56:29|122.30|40
20031201|SATYAMCOMP|09:56:30|335.25|1000
20031201|SAIL|09:56:30|42.70|700
20031201|M&M|09:56:30|355.95|100
20031201|SATYAMCOMP|09:56:30|335.25|500
20031201|SATYAMCOMP|09:56:30|335.25|100
20031201|RAINCALCIN|09:56:30|25.40|500
20031201|VDOCONINTL|09:56:30|78.85|70
```

where the columns separated by "|" represent respectively the date, name of the stock, time of transaction, price per stock of the stock traded, and the number of stocks traded during the transaction (also termed as the trade size q of the stock). It can be observed from the above data set that many transactions share the same time-stamp. This is because the temporal resolution of recording the transactions is 1 second, so that if two transactions occur within a duration of less than a second of each other, they will have the same time-stamp. However, the ordering of transactions is reported in the correct time-sorted order [169]. For the purpose of the present analysis, we assume that the transactions sharing the same time-stamp take place at the same instant. In addition, the market timings for the trading of common stocks are from 0950 hours to 1530 hours [170–172], so the results reported here are obtained using only trades that occur between 0950 hours and
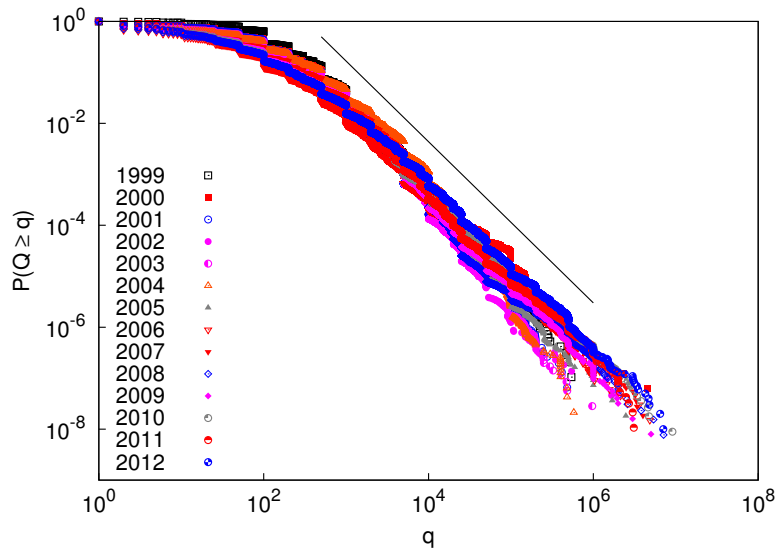
Figure 4.2: Cumulative probability distribution $P(Q \geq q)$ of the trade sizes q for NSE for the month of December for the years 1999 to 2012. Note the doubly logarithmic axis of the graphs, which means that a linearly decaying tail implies the existence of power-law decay of the distribution (the line represents a power-law fit with exponent 1.6).

1530 hours. Also, in order to study the properties of individual stocks we choose four representative stocks, viz., *HDFCBANK* (Finance sector), *INFOSYS* (Infotech sector), *RELIANCE* (Energy sector) and *SUNPHARMA* (Pharmaceutical sector) as these are some of the most important industrial sectors in NSE in terms of market capitalization. We show in Fig. 4.1 the variation of price $p(t)$, returns $r(t)$ and trade size $q(t)$ as a function of time for *RELIANCE* for the first 30 minutes of trading on December 1, 2005 as a typical example of the financial time series.

4.3 Results and Discussion

4.3.1 Distribution of trade sizes

We report the cumulative probability distribution $P(Q \geq q)$ of the trade sizes q for the entire market for the month of December for years from 1999 to 2012 in Fig. 4.2. We see that the distribution for all the months has a linear portion on a log-log graph, thus
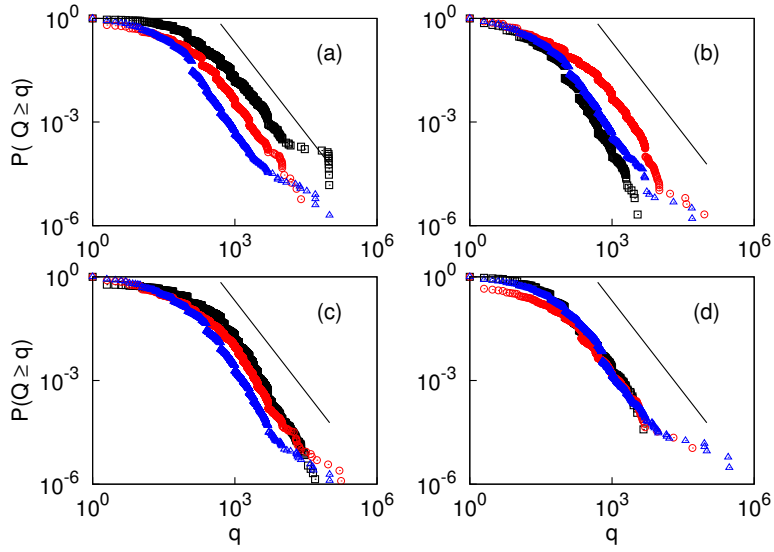
Figure 4.3: Cumulative probability distribution of trade sizes q for the stocks of (a) *HDFCBANK*, (b) *INFOSYS*, (c) *RELIANCE*, and (d) *SUNPHARMA*. The distributions are shown for the December months for the years 2003 (black squares), 2007 (red circles) and 2010 (blue triangles). Similar to the case of the markets, the distribution of trade sizes for individual stocks also exhibits a power law form.

suggesting a power law nature of the tails for the distribution of trade sizes for the market. Fig. 4.3 shows the distribution of trade sizes q for the four individual stocks for the December months of years 2003, 2007 and 2010 respectively. It is evident from the figure that the tails of the distributions of each of the stocks also exhibit power law decay. Hence, the tails of the distribution of the trade sizes q are of the form:

$$P(Q \geq q) \sim q^{1-\alpha}, \tag{4.1}$$

where α is the exponent characterizing the power law distribution. The maximum likelihood estimates of the exponents [173] are shown in Fig. 4.4. It is seen from Fig. 4.4(a) that $\alpha \leq 3$ for almost all periods for the entire market (except 2003). This implies that the trade size distributions for the market belong to the class for which higher than second moments onwards do not exist. As this property holds true for almost the entire duration of December 1999-December 2012, we say that the distribution of trade sizes for the market are stationary, in the sense that it is Lévy stable with second and higher
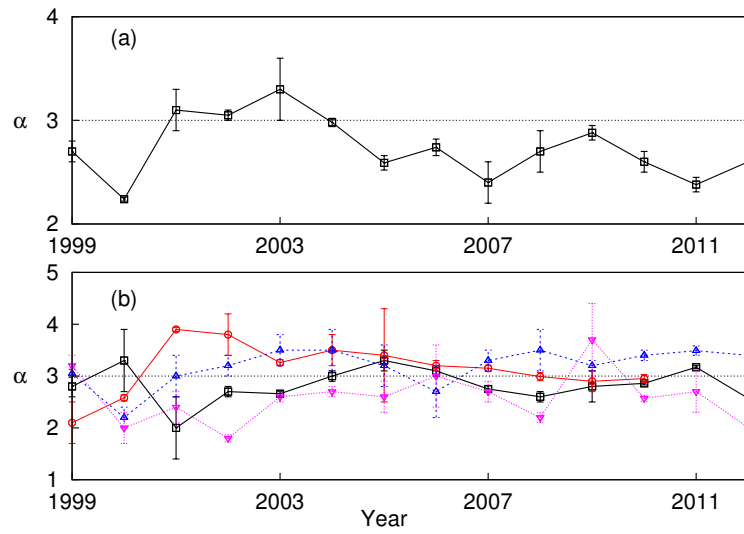
Figure 4.4: Maximum likelihood estimates of the power law distribution of trade sizes for (a) entire market and (b) for the representative stocks: *HDFCBANK* (black squares), *INFOSYS* (red circles), *RELIANCE* (blue triangles), and *SUNPHARMA* (maroon inverted triangles), for the months of December from 1999 to 2012. Please note that the data for *INFOSYS* is not available for the years 2011 and 2012. Bars represent the error in estimating the exponents, obtained using bootstrap technique. The horizontal broken line indicates $\alpha = 3$ which demarcates distributions with Levy-stable nature from those that will eventually converge to Gaussian.
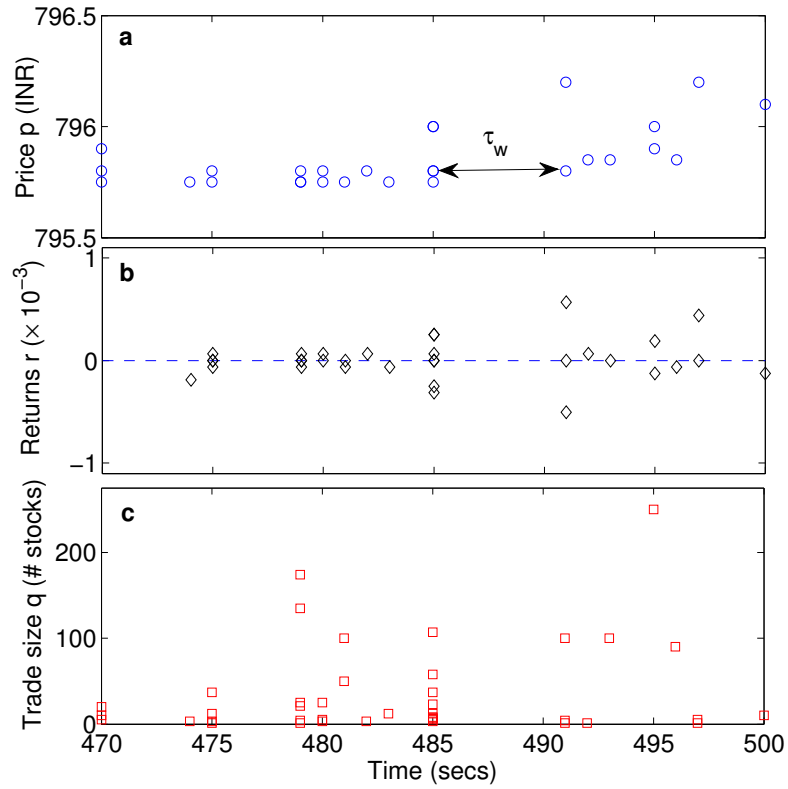
Figure 4.5: A magnified view of the time series of Fig. 4.1 showing that multiple transactions share the same time stamp because the temporal resolution of the recording is limited to 1 second. Also shown is the waiting time τ_w between two successive trades, specifying an interval during which no transaction takes place.

moments diverging. However, this is not true for the case of individual stocks, as seen from Fig. 4.4(b), where we see that the values taken by α for different stocks vary from 2 to 4. In addition, values taken by α for a particular stock exhibit widely different values depending on the period which is being observed, thus suggesting that the dynamics of individual stocks is non-stationary.

4.3.2 Distribution of waiting times and log-returns

An important quantity associated with a given stock is its price $p(t)$ at some time t, that fluctuates from one value to the other. A generic time-series representing price variations of a given stock is shown in Fig. 4.5. We see in (a) that the price $p(t)$ at time t and at $t + \tau_w$ can be the same or different, where τ_w is the waiting time between the two transactions.

61

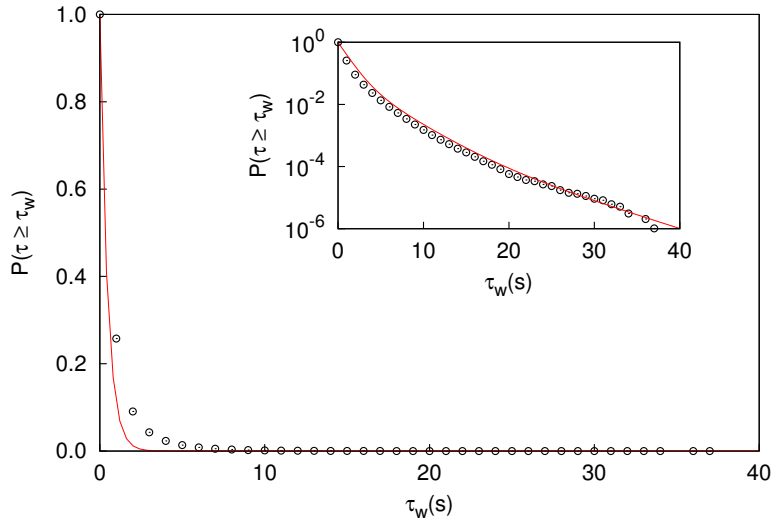Figure 4.6: Cumulative probability distribution of waiting times for successive trades in *RELIANCE* stock for the month of December in 2005. The empirically obtained distribution (circles) has been fitted with an exponential distribution (solid curve) having the same mean waiting time $\langle \tau_w \rangle \approx 0.453$. The inset shows a fit with a theoretical distribution that is a sum of three exponentially decaying curves $\sum_i a_i \exp(-(\tau_w/b_i))$ having different characteristic times τ_w/b_i ($i = 1, 2, 3$) [see text for details].

The logarithmic return (log-return) associated with price change is

$$r(t) = \ln p(t) - \ln p(t - \tau_w) \tag{4.2}$$

and is shown in Fig. 4.5(b). The random walk nature of price changes is easily seen from the above figure, and the presence of irregular waiting times τ_w makes it an effectively continuous time process. Characterizing the distribution of waiting times is of fundamental importance in order to understand the price dynamics of a given stock. For that purpose, we show the distribution of waiting times for *RELIANCE* for the month of December 2005 in Fig. 4.6. We observe from the figure that the distribution of waiting times $P(\tau > \tau_w)$ cannot be fit by an exponential distribution having the same $\langle \tau_w \rangle$. This implies that the system has inherent long-range memory and that the occurrence of successive transactions are not independent events. This becomes clear upon fitting the empirical distribution of waiting times with a theoretical curve having the form of a sum

62
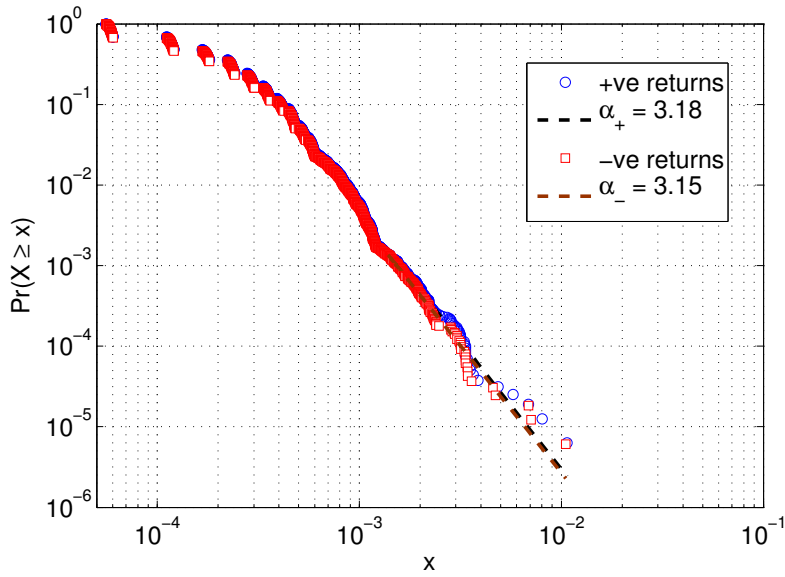
Figure 4.7: Distribution of log-returns of *RELIANCE* (each return being measured over an interval of 1 tick) for the month of December 2005.

of exponentials, viz.,

$$P(\tau \geq \tau_w) = \sum_i a_i \exp(-(\tau_w/b_i)). \tag{4.3}$$

Such theoretical distributions have been used earlier to describe systems having memory [174]. The values of the coefficients used to fit the data shown in Fig. 4.6 are: $a_1 = 0.897, a_2 = 0.1, a_3 = 0.003; b_1 = 1, b_2 = 0.4, b_3 = 0.2$ (we computed the error estimates to be less than 10% in all cases). We also see from Fig. 4.7 that the distribution of returns $Pr(X \geq x)$ has power law decaying tails with exponent close to 3. This is in accordance with the well-known inverse cubic law of price fluctuations reported for financial markets [163, 164, 178].

4.3.3 Properties of log-returns and waiting times

We see in Fig. 4.8 the scatter plot of log-returns r measured for successive transactions against the corresponding time-interval (waiting time τ_w) between them involving *RELIANCE* stock in a particular month. It is observed from the diagram that transactions

63

Figure 4.8: Scatter plot of waiting times τ_w between successive trades and the corresponding log-return r of *RELIANCE* for all transactions that took place in the month of December 2005.

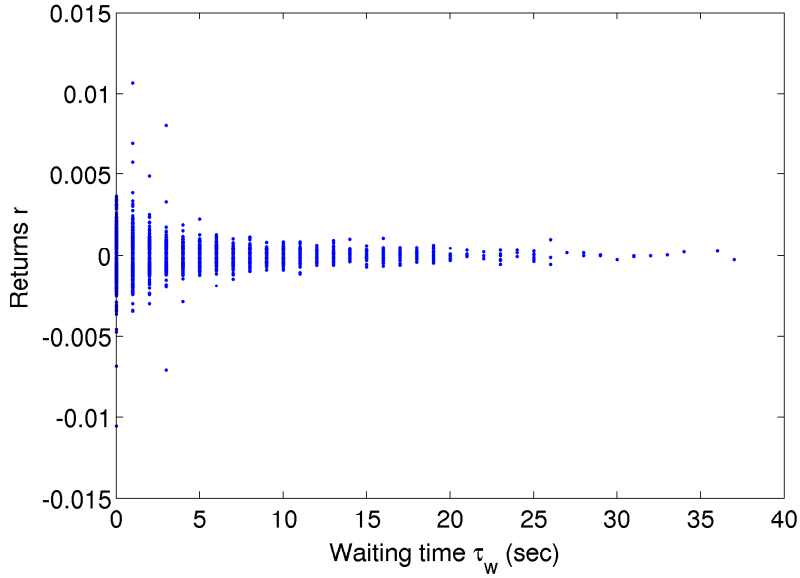resulting in large price changes generally occur closer to each other. To further quantify the observed behavior we look at the distribution of returns conditioned on waiting times, i.e., $P(r|\tau_w)$ in Fig. 4.9. It is evident from the distribution $P(r|\tau_w)$ that larger returns generally occur close to each other in time [130], thus suggesting that the waiting-times and returns may not be independent of each other. This property is also observed for different stocks and for different years. This has implications towards the modeling of price dynamics by continuous time random walks, as market transactions may be better modeled by walks whose step lengths are not chosen independently of the waiting time between successive steps, contrary to what is generally assumed [131, 132].

In order to characterize the dynamics of intra-day trading we define the variance of log-returns over an interval Δt as:

$$\sigma^2(t) = \frac{1}{N_{\Delta t}(t) - 1} \sum_{i=1}^{N_{\Delta t}(t)} (r_i - \langle r_i \rangle)^2 \tag{4.4}$$

where $N_{\Delta t}(t)$ is the number of trades occurring in the interval of length Δt and $\langle r_i \rangle$ is the average return over the interval. We observe from Fig. 4.10 that the variance over the

64

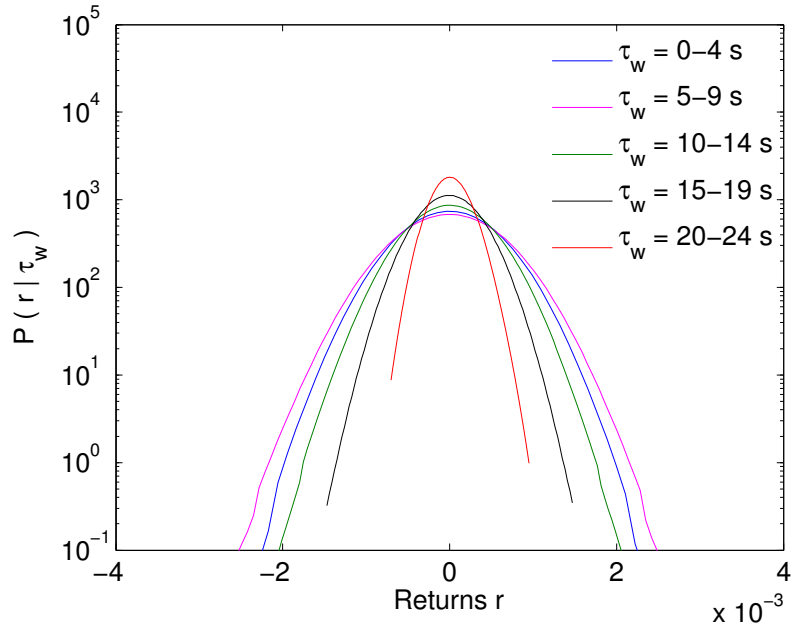Figure 4.9: Conditional distribution $P(r|\tau_w)$ of log-returns $r(t)$ conditioned on the waiting times τ_w for successive trades involving *RELIANCE* stock in the month of December 2005.
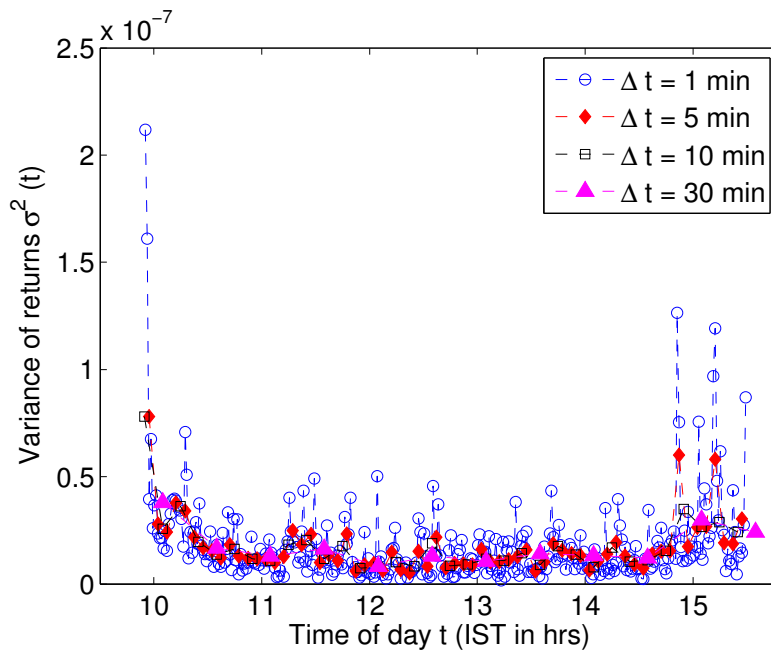
Figure 4.10: Intra-day volatility $\sigma^2(t)$ (measured by variance of the log-returns) during a period Δt shown as a function of time of day t for *RELIANCE* on December 1, 2005, for different intervals Δt.
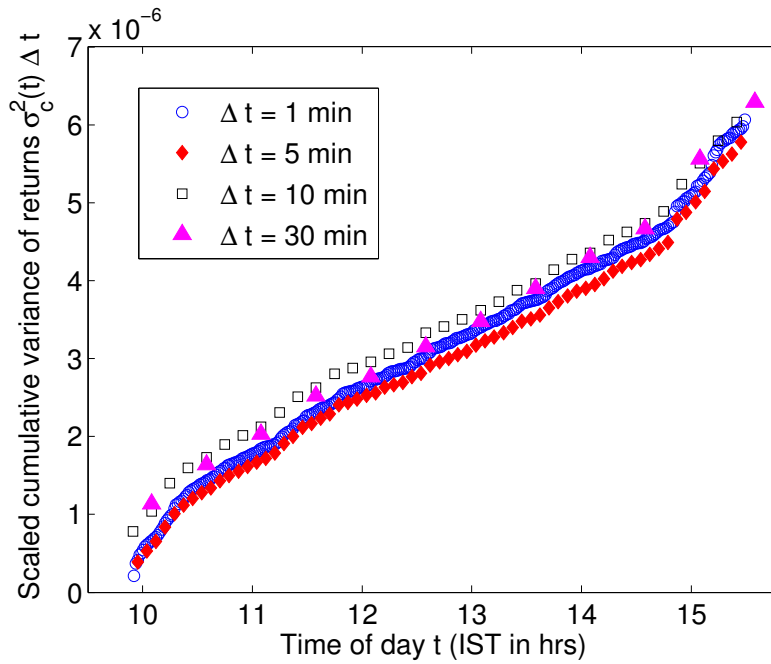
Figure 4.11: Scaled cumulative variance of the log-returns for intra-day trading $\sigma_c^2(t)\Delta t$ as a function of time of day t for *RELIANCE* on December 1, 2005. It is observed that scaling by interval Δt over which the variance is calculated, the curves for cumulative variance $\sigma_c^2(t)$ for different choices of Δt overlap.

intervals is nearly independent of the length of the interval Δt, and find that the scaled cumulative variance $\sigma_c^2(t)\Delta t$ is independent of Δt, as shown in Fig. 4.11. In addition, we also find that the cumulative variance grows linearly with time, thus implying that for the major part of the day price fluctuations of individual stocks are inherently Gaussian.

4.4   Conclusions

To conclude, in this chapter we have looked at the empirical properties of the national stock exchange (NSE) of India, by looking at the high-frequency trade data for the December months from 1999 to 2012. We find that the distributions of the trade sizes exhibit power law decaying tails for the entire market as well as individual stocks. The decay exponent α generally takes values in the interval $(2,3)$ for the market, for almost all the months from 1999 to 2012, a property which is not satisfied for individual stocks.

We also look at the distribution of waiting times between successive transactions for the stocks and find that the distributions deviates from exponential, implying the existence of long-range memory in the trading process. We also find that transactions involving large returns generally occur close to each other. In order to characterize the nature of price fluctuations during a day, we focus on the variance of the returns for intra-day trading, and find that the variance grows linearly with time. This implies that for a major part of the day, price fluctuations are Gaussian in nature.

Chapter 5

Conclusions

The research questions addressed in this thesis belong to the broad area of nonequilibrium dynamics of complex systems, and in particular, that of complex networks. A complex system can be defined as one whose systems-level properties are "emergent" [177] – which cannot always be explained in terms of simple linear superposition of the properties of its constituent elements. This typically arises because of the nonlinearities inherent to a complex system. The investigation of nonlinear properties of systems have played a critical role in understanding a wide variety of natural phenomena, e.g., mass extinctions in ecosystems, fluid turbulence, etc. In many complex systems, even the dynamics of its individual constituents are not known with any degree of certainty. For instance, this is the case for most socio-economic systems, where the behavior of the individual members cannot be described using precise models. Notwithstanding such a limitation, many of the overall properties of such a system can be described in terms of statistically regular features. In the work presented in the thesis, we have used such a framework to analyze different complex systems. In the following subsections we summarize the important results and conclusions reported in the thesis. We conclude with a brief discussion of possible future extensions of the present work.

5.1   Summary of main results

Cascading failures in interdependent networks

Interdependent networks are ubiquitous in nature from natural to artificial settings. In the work presented in the thesis it has been shown that an optimal level of interdependence is necessary for the continued functioning of the network of networks. This is demonstrated using two coupled networks of dynamical elements which, in isolation, are certain to cease activity (become extinct) at some point of time. The approach taken is different from one based on percolation theory, which has generally shown that interdependence amongst systems makes them more vulnerable in comparison to their isolated counterparts. The results have implications for understanding the importance of interdependence of networks of networks that occur in reality, such as a collection of ecological habitats that are connected by migration of species residing in them.

The critical role of observation time-scale in resolving the stability-diversity debate in complex systems

Cascading failures are seen in networks as varied as the power grid and the internet. As many of the networks have non-trivial dynamics associated with their nodes, one can ask what is the influence of connection topology on the overall dynamical stability of such systems. By considering networks of dynamical entities that are capable of exhibiting fixed-point, oscillatory or chaotic dynamics, we have investigated the dependence of the stability of the system to small perturbations on the density of connections between elements. Note that, instability will imply that cessation of activity (extinction) in one node may trigger a cascading process by which a sequence of other nodes will also cease activity – resulting in a significant fraction of the system going extinct. We have shown that when the interaction strengths amongst the nodes are scaled by their degree (number of

connections to other nodes), the transient activity regime becomes extremely long-lived as compared to the situation where the interaction strengths are independent of degree. In particular, when the systems are observed at relatively short time-scales, it will appear that the more complex systems have more active elements – suggesting that complexity promotes stability. However, if they are observed at longer time-scales, more complex systems would have much fewer surviving modes – indicating that complexity results in less stable systems. These results have implications for the complexity vs stability debate in ecosystems, wherein the two opposing camps base their arguments on observations that are made at very different time scales.

Empirical properties of NSE stock market

Financial markets represent a prototypical example of complex systems in which many agents are involved in trading with a common goal, viz., to make a profit. Using high-frequency(HF) trade data from the National Stock Exchange of India, we study the statistical properties of the market as well as that of individual stocks traded in the market. We find that the distribution of trade sizes exhibit power law decaying tails. In addition, the waiting times for arrival of orders of individual stocks exhibit tails decaying slower than exponential. We also find that for most of the time during the day, the variance of returns increases linearly with time. The present study reveals some of the statistical properties of a developing market using HF data which would be need to be taken into account when creating models for such systems.

5.2 Outlook

The problems addressed in the thesis contribute towards a general understanding of the nonequilibrium dynamics of complex systems which arise ubiquitously in natural and artificial settings. Large networks with many interacting components constitute a typical

example of a complex system, examples of which include foodwebs comprising many species and the internet. These systems often change continuously through the addition of new components and/or removal of existing ones, Simple models of such evolving systems may provide significant insights towards understanding how such networks function, as is demonstrated in this thesis. However, one of the assumptions made in the models presented here is that perturbations at any node in the network is felt immediately by the other nodes interacting with it. In other words, they propagate instantaneously across the network. In real systems, the effect of a localized perturbation is often felt elsewhere after a delay because of finite propagation speed of the disturbance. Thus, a natural extension of the work on complex networks presented here will be the inclusion of delay in the interactions between elements within individual networks, as well as, between different networks in the system of coupled networks. It is well known that delay introduces additional time-scales in the system in addition to existing ones and their interplay can lead to non-trivial behavior [175, 176]. In addition, we have looked primarily at discrete-time dynamical systems in this thesis. Considering the collective dynamics of networks of continuous-time dynamical systems could provide another obvious extension of the work reported here.

Not all complex systems are composed of components having predictable dynamical behavior. A classic example is a financial market in which many agents are involved in trading with the common goal of making profit. Lack of knowledge of the dynamics of individual elements implies that the overall properties of a complex system like financial markets can be known only statistically. We have taken this approach to uncover some of the empirical properties of the largest financial market in India. The behavior reported in this thesis will provide crucial pointers towards creating accurate models describing the dynamics of financial markets. In addition, the present study also provides motivation to look at other markets by employing high frequency data of intra-day trades. Such a study will be important in establishing the universality of the observed features at the highest possible temporal resolution.

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