STUDIES IN TWO-BODY REACTIONS OF STRONG AND ELECTROMAGNETIC INTERACTIONS

THESIS

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BY

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This thesis is based upon the work done during the years 1967-1971, on some two body reactions of strong and electromagnetic interactions, under the guidance of Professor Alladi Ramakrishnan, Director, The Institute of Mathematical Sciences, Madras.

It is a pleasure to record gratefully the benefit derived from useful discussions and collaboration with co-research workers, and due acknowledgement has been made at the appropriate places. Six papers which form the subject matter of this thesis have already been published in established journals. The available reprints are attached at the end of this thesis (please see back cover). The Computations have been done on the IBM 1620 Computer at the Fundamental Engineering Research Establishment, Madras and on the IBM 1130 Computer at the Advanced Centre for Bio-Physics, University of Madras, Madras. I am thankful to the concerned authorities for making available the Computer facilities.

I am deeply indebted to Professor Alladi Ramakrishnan for his constant encouragement and guidance throughout the preparation of this work and to the Institute of Mathematical Sciences for excellent facilities offered for research. I am also thankful to the Council of Scientific and Industrial Research, Government of India and the Institute of Mathematical Sciences for financial support.

(A.Sundaram)

C O N T E N T S

Introduction

In this thesis we are primarily concerned with the study of some two body inelastic processes at high energy in the Regge pole model. An attractive feature of the Regge pole model is that there is almost complete functional freedom of the trajectory and residue function for negative values of the momentum transfer. But the same feature also works against the model in the sense that an unlimited number of parameters can be introduced into the theory to patch up the momentum transfer dependence at fixed energy. A noteworthy feature of our study of the two body inelastic processes is the restriction of the number of parameters to a minimum by using the known form of the Regge trajectory, by equating the residues to the field theoretic counterpart at the pole point and/or by invoking some higher symmetry like SU(3) or U(6,6) for the vertices and resorting to the assumption of exchange degeneracy for the trajectories as indicated recently by Duality theory.

To provide a background for our study we present here a brief summary of the various models used to describe the two body inelastic processes:

Quasi two body channels in which one or both of the 'particles' in the final states are resonances, are a prominent feature of collisions in the intermediate energy range (in the

range of incident momenta from 2 to 8 Gev/c). It is observed that the angular distributions for these reactions are generally peaked in the region of small momentum transfers. The predominance of small momentum transfers implies that glancing collisions are most important in these reactions and indicates that these reactions are mediated by a long range force corresponding to the exchange of nearby singularities in the crossed channel. There are various theoretical models that have been used to describe the inelastic peripheral processes. Some of these models are

1) One particle exchange model[1)]

a) with form factors[2)]

b) with absorptive corrections (absorptive peripheral model)

2) Coherent droplet model[3)]

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1) C.J.Goebel, Phys.Rev.Lett. 1, 337 (1958); G.F.Chew and F.E.Low, Phys.Rev. 113, 1640 (1959); S.D.Drell, Phys.Rev. Lett. 5, 342 (1960); Rev.Mod.Phys. 33, 408 (1961).

For historical review of the OPE model and absorption model see: J.D.Jackson, Rev. of Mod. Phys. 37, 484 (1965); A.C.Hearn and S.D.Drell in 'High Energy Physics', (Academic Press, Inc., New York, USA); M.H.Ross and G.L.Shaw, Phys. Rev.Lett. 12, 627 (1964).

2) E.Ferrari and F.Sellari, Nuovo Cim. Suppl. 24, 453 (1962) and references therein.

3) N.Byers and C.N.Yang, Phys.Rev. 142, 976 (1966).

3) Regge pole model[4)]

The essential feature of the unadorned OPE model is the presence in the differential cross section of the meson propagator. The idea that such a factor is responsible for the peaking at small momentum transfers though attractive is not borne out quantitatively by specific calculations. The model gives cross sections with too large a magnitude and often fails to decrease with increasing momentum transfer.

Ferrari and Sellari among others have suggested form factors as the solution to the problem of the momentum transfer dependence. But in many cases, very rapidly varying factors are needed to get agreement with experiment and in certain cases where vector mesons are exchanged, the form factors need to have a structure corresponding to much smaller masses than the mass of the exchange particles. Thus the prescription for choosing the form factors seems to be rather adhoc.

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4) There is of course a great volume of literature on the application of Regge poles to two body processes. A detailed summary of the work done in the field of Regge poles are given in the books:

1) P.D.B.Collins and F.J.Squires, Regge poles in particle physics, Springer Tracts in Modern Physics, 45 Springer-Verlag (1968).

2) Barger and Cline, Phenomenological theories of High Energy Scattering, Benjamin (1969),

and the Reviews:

R.J.N.Phillips in "Proceedings of the International School of Physics", Effora Majora Academic Press, p.268 (1966)

Gerald E.Hite, Reviews of Modern Physics, 41, 669(1969).

L.Betrochi in "Proceedings of the Heidelberg Conference" (1968).

J.D.Jackson in "Proceedings of the Lund International Conference on 'Elementary Particles' ", edited by G.Von Dradel (1969).

See Appendix A for a brief summary of "Regge phenomenology"

In the absorptive peripheral model the assumption is that the presence of competing open channels at high energies cause a reduction of the low partial wave reaction amplitudes below the values given by the simple peripheral model while leaving the higher partial waves essentially unaltered. This brings about a reduction of the reaction cross section and important modifications of the angular distributions. The OPEA seems to be successful in predicting correctly the momentum transfer dependence of the angular distributions and the magnitude of the cross sections in the case of pion exchange though there are cases of disagreement especially for high spins in the final state.

The coherent droplet model has as its corner stone the idea that all exchanges occur with the same momentum transfer dependence at least at high enough energies. At intermediate energies it is difficult to decide whether this first approximation is adequate. For instance in charge exchange scattering this approximation is not enough. Also it should be noted that in production of high spin resonances the model would tend to give broader distributions in t than elastic scattering because of the helicity flip amplitudes which vanish as t 0.

However the main defect of the OPE (one particle exchange) and other related models is the inability to predict correctly the energy dependence of reactions when exchanged particles

with spin one or greater are involved because the OPE model has the well known asymptotic energy dependence where J is the spin of the exchanged particle. Exactly here, the Regge pole model scores over other models. The Regge pole model attempts to describe direct-channel reactions in terms of analytic properties of the crossed channel amplitudes as a function of the cross channel angular momentum. The concept of analyticity in the crossed-channel angular momentum variable j, sets the Regge pole model apart from other peripheral models. The difficulty regarding the energy dependence of the cross section encountered in the OPE models, is avoided in the Regge pole model by associating particles, with the same quantum numbers but different discrete values of angular momentum with the poles in the complex angular momentum plane.

The poles are assumed to move or trace out trajectories as a function of the momentum transfer variable t. The symbol (t) designates the position of the poles as a function of t and one associates a known particle with each trajectory. The trajectory and residue function should also smoothly extrapolate to their value at the pole of physical particles. The differential cross section, in the case of single Regge pole exchange, will be asymptotic to and since (t) is required to be less than or equal to unity for the physical values of t in the s-channel, agreement with experiment could be had. In the non-relativistic potential scattering, for the case of superposition of Yukawa potentials,

Regge clearly showed that the only singularities in the complex angular momentum are simple poles. But this j-plane-meromorphy is not justified in the relativistic case and there are more complex singularities such as cuts besides the poles.

The plan of the thesis is as follows:

In Chapter I, we study tensor meson production in pion-nucleon collisions. The processes $\pi^- p \quad f^0 n$ and $\pi^+ p \quad A_2^+ p$[5] are studied using Reggeized π and exchanges respectively and the results are compared with the predictions of the absorptive peripheral model and the experimental data.

In Chapter II, the process $p\bar{p}$ [6] is studied. Applying U(6,6) symmetry to the vertices in order to uniquely fix the Lagrangian and coupling constants, we Reggeize the dominant K trajectory and our predictions for the angular dependence of the differential cross section and the energy dependence of the total cross sections are compared with available experiment.

In Chapter III, we are concerned with isobar production in πN collisions[7]. Here we study the processes[7] $\pi^+ p \quad \pi^0 \Delta^{++}$, $\pi^+ p \quad \Delta^{++}$ and $K^+ p \quad K^0 \Delta^{++}$ taking into

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5) A.Sundaram and R.Sridhar, Nuovo Cim. 50A, 969 (1967).

6) A.Sundaram and K.Srinivasa Rao, Nuovo Cim.61A, 755 (1969).

7) A.Sundaram, Lettere al Nuovo Cim. 1, 177 (1971).

account poles and cuts due to pomeron plus Regge pole exchange. We also discuss the question whether nonsense wrong signature points should be included or not, in the parametrization of Regge pole amplitude. Exchange degeneracy for trajectories and SU(3) symmetry for the residues are assumed to reduce the number of parameters. The theoretical calculations for the differential cross sections and the density matrix elements are compared with the experimental results.

In Chapter IV, we study the effect of mixing the irreducible representations of SU(3) group[8]. It is known that symmetry is broken in realistic situations. Many methods have been discussed in the literature on the symmetry breaking mechanism. The symmetry breaking we introduce is different from other known methods. We point out that symmetry breaking manifests in mixing various rotation irreducible representations of the symmetry group a fact analogous to the d state admixture to the otherwise symmetrical s-state ground state wave function of the deuteron. This has become particularly useful in the problem of identifying the Roper resonance which has all the quantum numbers same as nucleon. In this chapter we study the consequences of representation mixing in SU(3) and several sum rules are presented and some of them can be tested against experiment.

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8) Alladi Ramakrishnan, T.S.Santhanam and A.Sundaram, preprint (1967).

In Chapter V[9], the implications of the current algebra SU(2) x SU(2) on the electromagnetic form factors are studied.

In Chapter VI[10], we study the process p $\pi^-\Delta^{++}$ using the vector dominance model (VDM), and as a further application of the VDM we calculate the -2π coupling constant[11].

In addition to my papers referred herein, I am also enclosing at the end of the thesis, a reprint of my paper, in collaboration with Professor Alladi Ramakrishnan, T.S.Santhanam and P.S.Chandrasekaran, entitled "Helicity matrices in the generalized Clifford algebra" (Jour. Math. Anal. Appl. Vol.26, 275, 1969) in support of this thesis.

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9) T.S.Santhanam, A.Sundaram and K.Venkatesan, preprint (1968).

10) A.Sundaram, Nuovo Cim. 58A, 905 (1968).

11) A.Sundaram, Prog. Theor. Phys. 43, 843 (1970).

CHAPTER: I

REGGE POLE MODEL AND TENSOR MESON PRODUCTION+

1. Introduction:

Recent experiments on the πN interactions[1] in the Gev region exhibit a strong peaking of the production cross section for the processes:

(1)

(2)

in the region of small momentum transfer indicating a peripheral interaction and suggesting a One Particle Exchange (OPE) mechanism. However it is well known that the Born term model fails to reproduce the experimentally observed peaking and the angular distribution due to the following difficulties:

i) the slope of the production cross section is not steep enough,

ii) the absolute magnitude of the production cross section is too big in some cases,

iii) the unitarity bounds are badly violated in angular momentum,

and iv) the energy dependence is often wrong, in particular for vector meson and other higher spin exchanges.

+ A.Sundaram and R.Sridhar, Nuovo Cim. 50A, 969 (1967).

1) Aachen-Birmingham-Bonn Hamburg-London (I.C.) - Munchen Collaboration, Nuovo Cim. 31, 729 (1964), Phys.Rev.138B,897(1965).

So the Born term model has to be corrected, either by an overall form factor[2,3] or by inclusion of effects due to competition from other open channels, as in the absorption model[4,5] and in the K matrix method[6]. The above processes 1) and 2) have been studied by Hogassen et. al.[7] using absorptive peripheral model and they find that:

i) in the case of meson production, their model reproduces correctly the slope of the angular distribution but the absolute magnitude of the cross section is too big by a factor of 2.

ii) in the case of A_2 meson production, their model is unable to account for either the absolute magnitude of the cross section or the slope of the differential cross section. In an attempt to improve the results of the absorption model, they tried

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2) E.Ferrari and F.Sellari, Suppl. Nuovo Cim. 24, 453 (1962); Nuovo Cim. 27, 1450 (1963).

3) J.D.Jackson and H.Pilkhun, Nuovo Cim. 33, 906 (1964) and errata 34, 1841 (1964).

4) N.J.Sopkovich, Nuovo Cim. 39, 186 (1962); J.D.Jackson in Proceedings of the 1966 Stony Brook Conference on High Energy Two Body Reaction (unpublished); J.D.Jackson, Rev.Mod.Phys. 37, 484 (1965); J.D.Jackson in Proceedings of the 1965 Les Houches Summer School (Gordon and Breach Science Publishers, Inc., New York 1966)

5) A.C.Hearn and S.D.Drell in High Energy Physics (Academic-Press Inc., New York, to be published) (SLAC-PUB-176, ITP-200,1966); U.Maor in Proceedings of the Conference on Nuclear and Particle Physics, University of Glasgow, 1966 (CERN/TC/Physics 66-26) unpublished.

6) Dietz, K and Pilkhun, H; Nuovo Cim. 37, 1561 (1965); 29, 928 (1965); R.C.Arnold, Phys.Rev. 136B, 1388 (1964).

7) H.Hogassen, J.Hogassen, B.Keyser and B.E.Y.Svensson, Nuovo Cim. 42A, 323 (1966).

the introduction of a rather rapidly varying form factor which gave rise only to a steeper slope for the differential cross section. Moreover it is to be observed that their energy dependence of the total cross section is wrong.

They conclude that the absorption model, though successful in reproducing the experimental data for reactions where a pion is exchanged and where the spins of the outgoing particles are low (an excellent example is the reaction πp p[8]), becomes worse as soon as the spins of either the exchanged particles or the particles in the final state, increase.

Here we study the processes (1) and (2) by a simple Regge pole model and see whether it could explain the experimental observations. The reason why we resort to the Regge pole model for these processes is based on the success of the Regge pole model in predicting a correct t and s dependence of cross sections in many inelastic reactions[9]. Moreover, in cases where only a few trajectories are exchanged, the parameters entering in the calculations are minimum and so provide a good test of the model. The process (1) is known to proceed mainly via π exchange while for the process (2) we assume exchange to be dominant. The process (1) has been studied earlier by Regge pole models by Thews[10] and Frautschi and Jones[11]. Frautschi and

8) For some typical results obtained within the absorption model, see e.g. J.D.Jackson, J.D.Donohne, K.Gottfried, R.Keyser and B.E.Y.Svensson, Phys.Rev. 139B, 428 (1965).

9) R.K.Logan, Phys.Rev.Lett. 14, 414 (1965); D.P.Roy, Nuovo Cim. 40A, 513 (1965); Phys.Rev. 146, 1218 (1966).

10) R.L.Thews, Phys.Rev. 155, 1624 (1967).

11) S.Frautschi and Lorella Jones, Phys.Rev.164, 1918 (1967).

Jones have studied the process (1) paying particular attention to the nature of the kinematical constraints[12] on the residue of the pion trajectory and they obtained a good fit to the data. Their observation that the variation of the residue can be explained entirely in terms of "kinematical" effects, is in contrast to the earlier work of Thews who neglected all the threshold constraints.

However the proximity of the pion pole to the physical t region suggests the importance of the Born term. We use a model which consists of an "evasive" pion with kinematical singularities determined by the Born amplitude rather than the factors suggested by the crossing matrix[11,13]. Recently, such a model applied by Abrahams and Maor[14] to π^+p Δ^+ has been found to give the correct experimental features.

The Born term has been calculated by Hogassen et. al.[7] and we follow their notation.

2. <u>The Regge Pole Contribution to πN fN</u>.

The one particle exchange contribution to the process (1) and (2) is illustrated in fig. . The four momenta of particles a,b,c,d are denoted by and their masses by m_a, m_b, m_c and m_d, respectively. We have

and the square of the momentum

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12) L.L.Wang, Phys.Rev. 142, 1187 (1966).

13) J.D.Jackson and G.E.Hite, Phys.Rev. 169, 1248 (1968).

14) G.S.Abrahams and U.Maor, Phys.Rev.Lett. 25, 621 (1970).

transfer $-t =$. Moreover a nucleon of mass m is described by the usual Dirac spinor normalised to $=2m$ on which act the conventional Dirac matrices and . The spin 2 particle is characterized by a symmetric traceless second rank tensor .

Then the Feynman amplitude for the process (1) using an elementary pion exchange is given by[7)]

(3)

where G is the πNN coupling constant and g_2 is the $\pi\pi f$ coupling constant. If the π meson is now treated as a Regge pole, the propagator in ( ) is expected to be replaced by[15)]

(4)

where x_t, p_t and q_t are the cosine of the scattering angle, the initial and final momenta in the centre of mass system (c.m.s.) respectively.

Now

(5)

15) S.C.Frautschi, Gell-Mann, F.Zachariasen, Phys.Rev.126, 2204 (1962); I.J.Muzinich, ibid 130, 1571 (1963); M.M.Islam, Nuovo Cim. 30, 579 (1963); M.M.Islam and R.Pinon, ibid 30, 837 (1963); D.P.Roy, N.C. 40A, 513 (1965); B.Margotis and A.Rottstein, Nuovo Cim. 45A, 1010 (1966).

where are the Mandlestam variables. For large s, this reduces to

(6)

The first two factors in (4) arise from the Sommerfeld-Watson transform of the amplitude in the t channel. The third term brings out the threshold dependence from the residue of the trajectory and b is a slowly varying function of t and is treated as a constant. The asymptotic approximation for the Legendre function gives

(7)

The factor for small value of t can be treated as a constant which is denoted by b'. The pion trajectory has the form

(8)

Now b is determined from the fact that ( ) should coincide with its field-theoretic counterpart at the pole point. Then the modified propagator is given by:

Thus the π-Regge trajectory exchange contribution to the reaction $\pi^- p$ is

(10)

The above expression yields the following differential cross section which is summed over final and averaged over initial spin states[7]:

(11)

where is the magnitude of the three momentum of particle a in the rest frame of particle C, given by

(12)

3. Regge pole contribution to π^+p A_2^+p:

Since the A_2 meson has G parity -1, the possible trajectories that can be exchanged in the t channel are p,p' and . Here we consider only exchange as we are mainly interested in comparing the Regge pole model with the absorption model calculations[7]. Moreover, it is pointed out that for the process (2), the Pomeron might not be contributing with full strength in the forward direction[16]. Also the charge exchange reaction π^+n A_2^0p which proceeds mainly via I=1 exchange is found to have the same cross section as that of process (2), at pion

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16) L.L.Wang, Phys.Rev.153, 1664 (1967).
Also Morrison (Phys.Lett.25B, 230 ( )) has suggested that all diffractive processes satisfy the rule

where $p_i(f)$ and $J_i(f)$ are the initial (final) intrinsic parity and spin of the particle. However this rule is not satisfied for the process π^+p A_2^+p. Morrison has recently suggested that this rule may be valid for the process (2) and conjectured the possible existence of another A_2 meson with $J^P = 1^+$ or 2^-, which corresponds to the energy independent component in the behaviour of the cross-section for the reaction π^+p A_2^+p with beam momentum.

momentum 3.65 Gev/c[17] indicating that diffraction dissociation is not the main contribution at intermediate energies. The matrix element for the process $\pi^+ p$ $A_2^+ p$ with the exchange of elementary meson is given by[7]:

(13)

and

(14)

In the above expression is the coupling constant and and are the vector and tensor couplings of the meson to the nucleons.

If we now treat the exchanged meson as a Regge pole, the modified propagator, using the Reggeization procedure discussed earlier will be given by:

(15)

17) G.Borsen, L.Lovell, E.Marquit, B.Roe, D.Sindair, and J.Van der Valde, Phys.Rev.Lett. 16, 1177 (1966).

Identifying the above expression with its field-theoretic counterpart at the pole point, is determined to be:

(16)

Now substituting (15) and (16) into (13) and summing over final and averaged over initial spin states, the following expression for the differential cross section is obtained[7]:

(17)

where .

In the above expression , the magnitude of the three momentum of particle a in the rest frame of particle C, is given by

(18)

and with , denoting the centre of mass production angle q' the magnitude of the centre of mass three momentum of either particle in the final state

(19)

4. Results and discussion:

i) $\pi^- p \quad f^o n$:

We have taken for the pion trajectory the form

. The value of the slope is not well determined. We find the value = 0.8 yields good agreement with experiment+). We have used the following values for the coupling constants:

= 1.6 and = 27

Now the process $\pi^+ n \quad f^o p$ had earlier been studied by Thews and Frautschi and Jones. Thews[10] required rapidly varying

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+) The experimental situation concerning the pion trajectory is characterized by the following numbers. = 0.34 Gev^{-2} for the reaction (M.Markytan, Nucl.Phys.B10,193(1969), =1.2 Gev^{-2} for the reaction pp $pn\pi^+$ in the double Regge pole model (E.L.Berger, Phys.Rev.Lett.21, 701 (1968)), =1.5 ± .50 Gev^{-2} for the reactions , (B.Haber, U.Maor, G.Yekutieli and E.Gotsman, Phys.Rev.168, 1773 (1968)), M.Markytan and P.Schmid (Lettere al Nuovo Cim. Vol.III, 57 (1970) conclude that for the same processes the slope of the pion trajectory is always less than 1 Gev^{-2}. Thews (ref.10) has obtained a value = 0.69 ± 0.29 Gev^{-2} for the process (1) which agrees with our value.

residue function and he did not take into account the question of kinematic singularities. Frautschi and Jones[11] used the kinematic factors including those omitted by Wang in detail and observed that the rapid variation of the residue is mainly due to these further kinematic effects, rather than dynamics. Here, in our model, we consider an "evasive" pion with kinematical factors determined by the behaviour of the Born amplitude at thresholds and pseudothresholds rather than the factors suggested by the crossing matrix. The reduced residue is smoothly continued from the coupling constants calculated at the pion pole. In Figures are shown the calculated cross sections as compared to the experimental results for the reaction

at 4 Gev/c, 10 Gev/c and 16 Gev/c and

at 6 Gev/c[18] incident laboratory momenta. We find that our model reproduces the results reasonably well.

ii) π^+p A_2^+p:

Here we have used = 1.2 and =3.7. The coupling constant is evaluated from the expression

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18) The experimental data are taken from the following sources:

: 4 Gev/c data: Aachen-Birmingham-Bonn-Hamburg-London (IC), Munchen Collaboration, Nuovo Cim.31, 729(1964)

6 Gev/c data: F.Bruyan et. al. in Proceedings of the International conference on High Energy Physics, Dubna, 1964 (Atomizdat, Moscow, 1965) Vol.1, p.422.

10 Gev/c data: M.Wahlig et. al., Phys.Rev.147, 941 (1966).

16 Gev/c data: J.Ballam et. al., Phys.Lett. 31B, 489 (1970).

(20)

where Q is the magnitude of the three momentum transfer for the vector meson in the 2^+ rest frame+). The trajectory is quite well known from πN charge exchange reactions and other reactions and we take the trajectory as =0.57 + 0.91t. In Fig. we compare the results of our calculation for the cross section of the reaction $\pi^+ p$ $A_2^+ p$ to the experimental one at 4 Gev/c[1] incident momentum . The curve marked ( ) is the result of the unmodified Born approximation; it is much too big and does not show any decrease with increasing momentum transfer. The absorption model result is marked ( ). It also fails to reproduce the experimental data, though the shape of the curve is better than that obtained for the Born approximation. The Regge pole model fit is marked ( ) and it yields a good agreement with experiment. In Table I we give the energy dependence of the total cross section. It decreases rapidly as the energy increases. But the experimental data at higher energies reveal a non-decreasing, essentially energy independent cross section. The present model cannot account for this phenomenon. Probably at higher energies the p and p' trajectories dominate. So, we conclude that the success of our model

+) We have used a width of 80 Mev for the decay A_2 π .

is limited to intermediate energies upto 6 Gev/c incident momentum.

Table I

Incident-pion momentum in Gev/c	Total cross-section in mb		
	Regge-pole model	Experiment	Absorption model
4	0.27	0.25[1)]	2.5
6	0.095	0.075 ± 0.026[19)]	-

In conclusion, we note that the Regge pole model with constant residues and kinematical factors, determined by the behaviour of the Born amplitude at thresholds and pseudothresholds, explains successfully the experimental data for the processes (1)[+] and (2).

19) CERN-Ecole Polytechnique Collaboration; Dubna Conference (1964).

+) However it should be mentioned that Yock and Gordon (P.C.M. Yock and D.Gordon, Phys.Rev.157, 1362 (1967)) have found that inclusion of form factors for the process $\pi^- p \rightarrow f^0 n$ brings theory into better agreement with experiment, but this brings in additional parameter m_c which is the "cut off" mass. Our Regge pole fit seems to be as good as the fit obtained by Yock and Gordon. Also even though at 10 Gev/c the predicted cross section as shown in Fig.(4) exceeds the reported cross section by about a factor 2, this should not be taken as a serious argument against the model, as pointed out by Yock and Gordon.

FIGURE CAPTIONS

Fig.1: Feynman diagram for the meson exchange for the processes $\pi^- p \rightarrow f^o n$ and $\pi^+ p \rightarrow A_2 p$.

Fig.2: Differential cross section for the process $\pi^- p \rightarrow f^o n$ at 4 Gev/c incident pion momentum. Experimental results are from Ref.(18). Dotted curve is the result of the absorption model obtained from Ref.(7). Full curve is the Regge pole model result as described in the text.

Fig.3: Differential cross section for the process $\pi^+ n \rightarrow f^o p$ at 6 Gev/c incident pion momentum. Experimental results are from Ref.(18). Dotted curve is the result of the absorption model obtained in ref.(7). Full curve is the Regge pole model result as described in the text.

Fig.4: Differential cross section for the process $\pi^- p \rightarrow f^o n$ at 10 Gev/c. Experimental results are from Ref.(18). Dotted curve is the result of the absorption model obtained in ref.(7). Full curve is the Regge pole model result as described in the text.

Fig.5: Differential cross section for the process $\pi^- p \rightarrow f^o n$ at 16 Gev/c. The experimental data is taken from Ref.(18). The full line describes the result of the Regge pole model as described in the text.

Fig.6: Differential cross section for the reaction $\pi^+ p \rightarrow \Lambda_\Sigma^+ p$ at 4 Gev/c. Experimental results are from ref.(1). Curve marked a) is the Born term model prediction, b) the absorption model result from ref.(7), c) the result of the Regge pole model as described in the text.

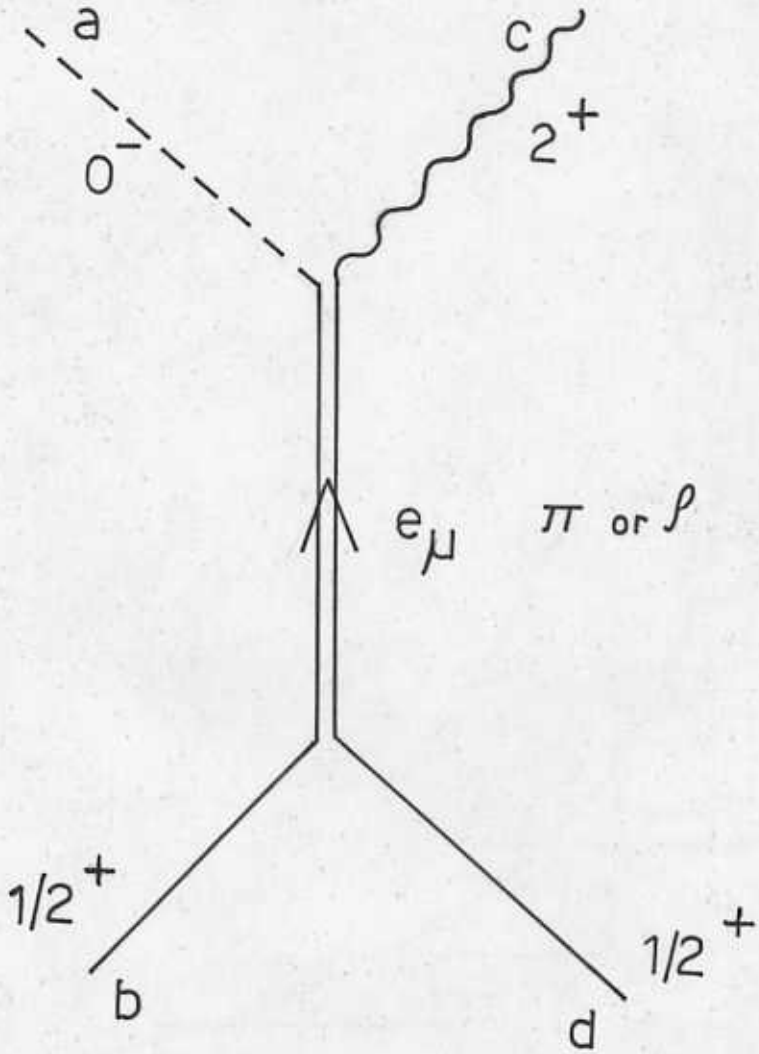

FIG. 1

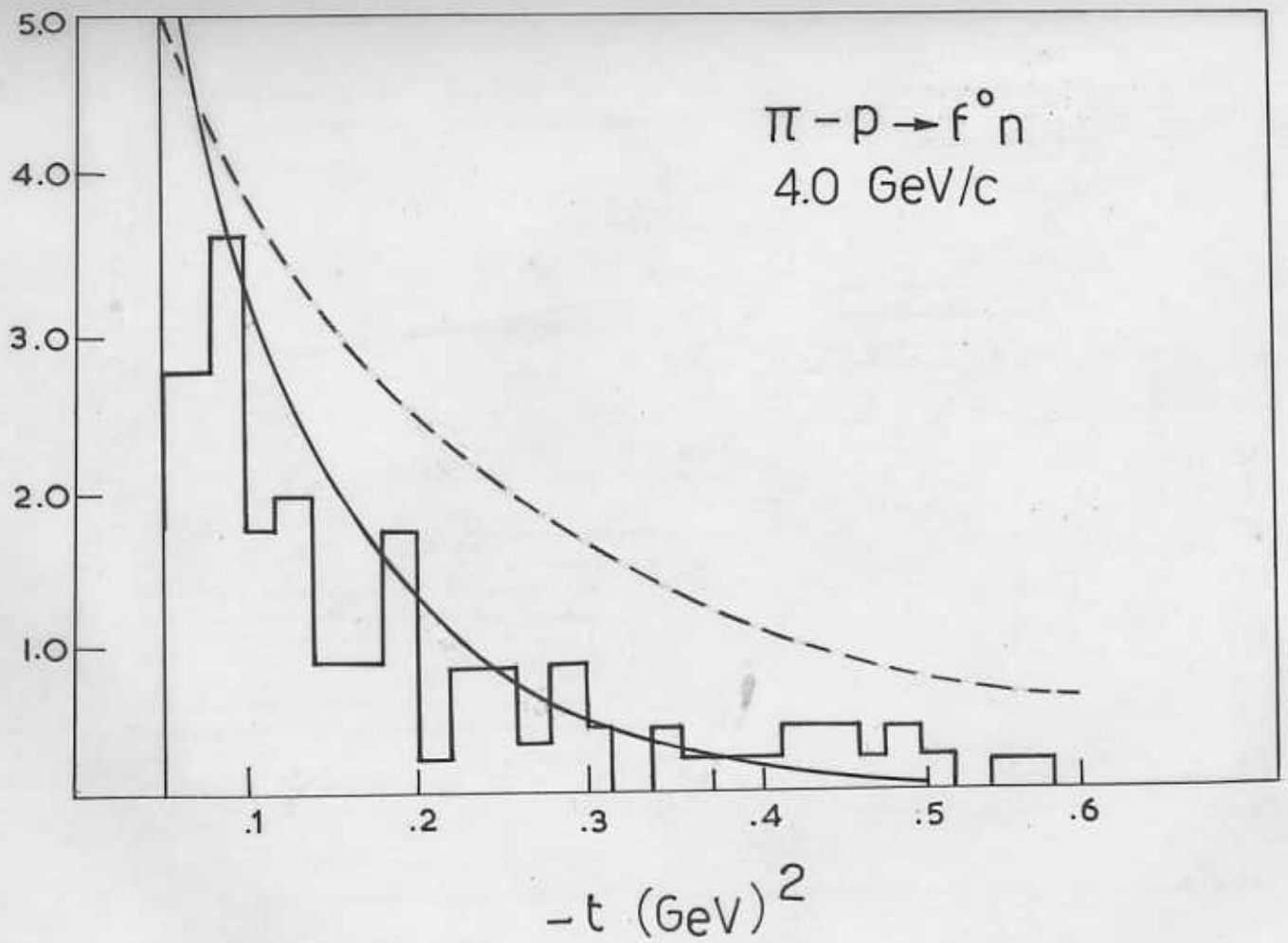

FIG. 2

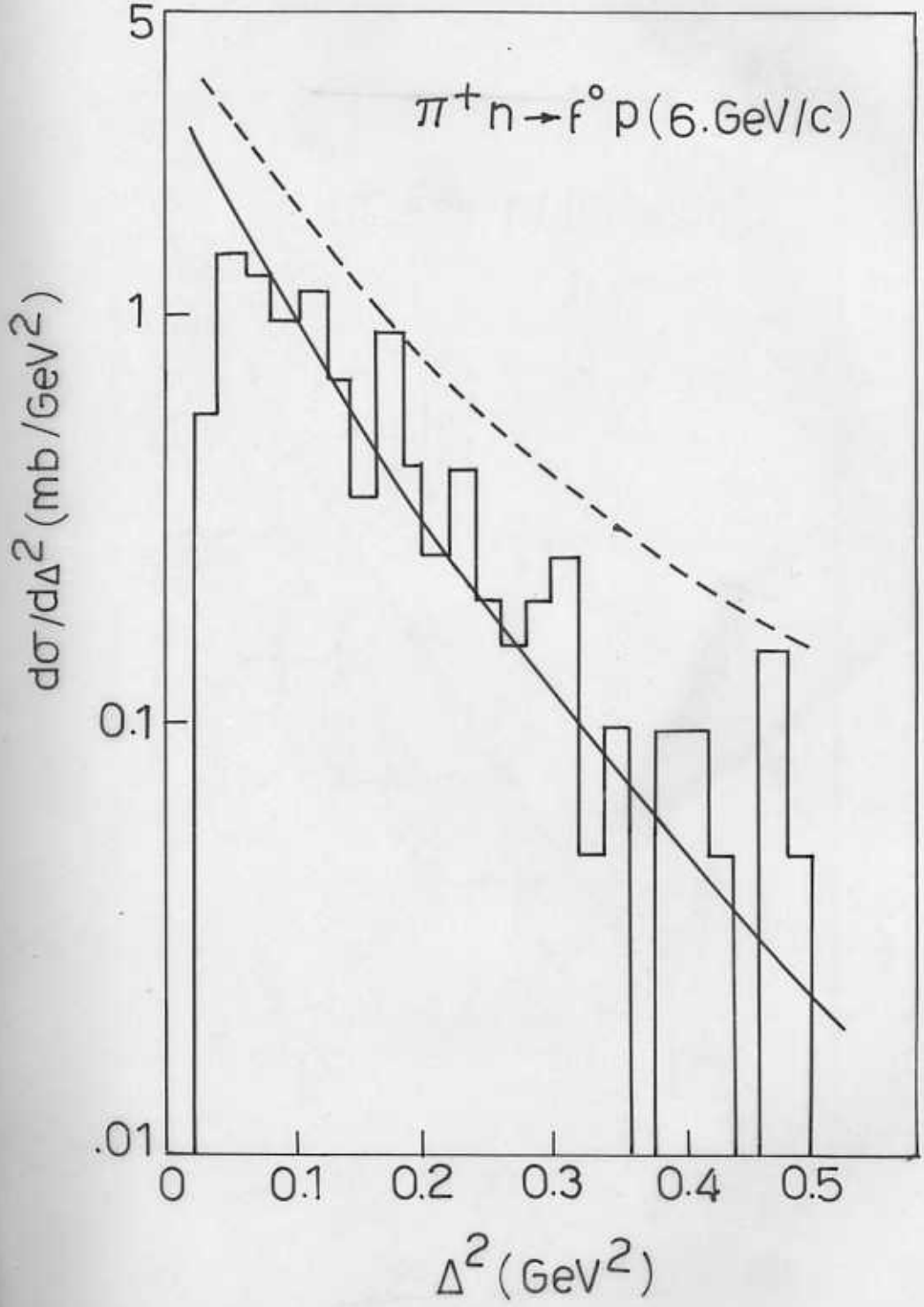
$\pi^+ n \to f^\circ p$ (6. GeV/c)
5
1
0.1
.01
0
0.1
0.2
0.3
0.4
0.5
$d\sigma/d\Delta^2$ (mb/GeV^2)
Δ^2 (GeV^2)

FIG. 3

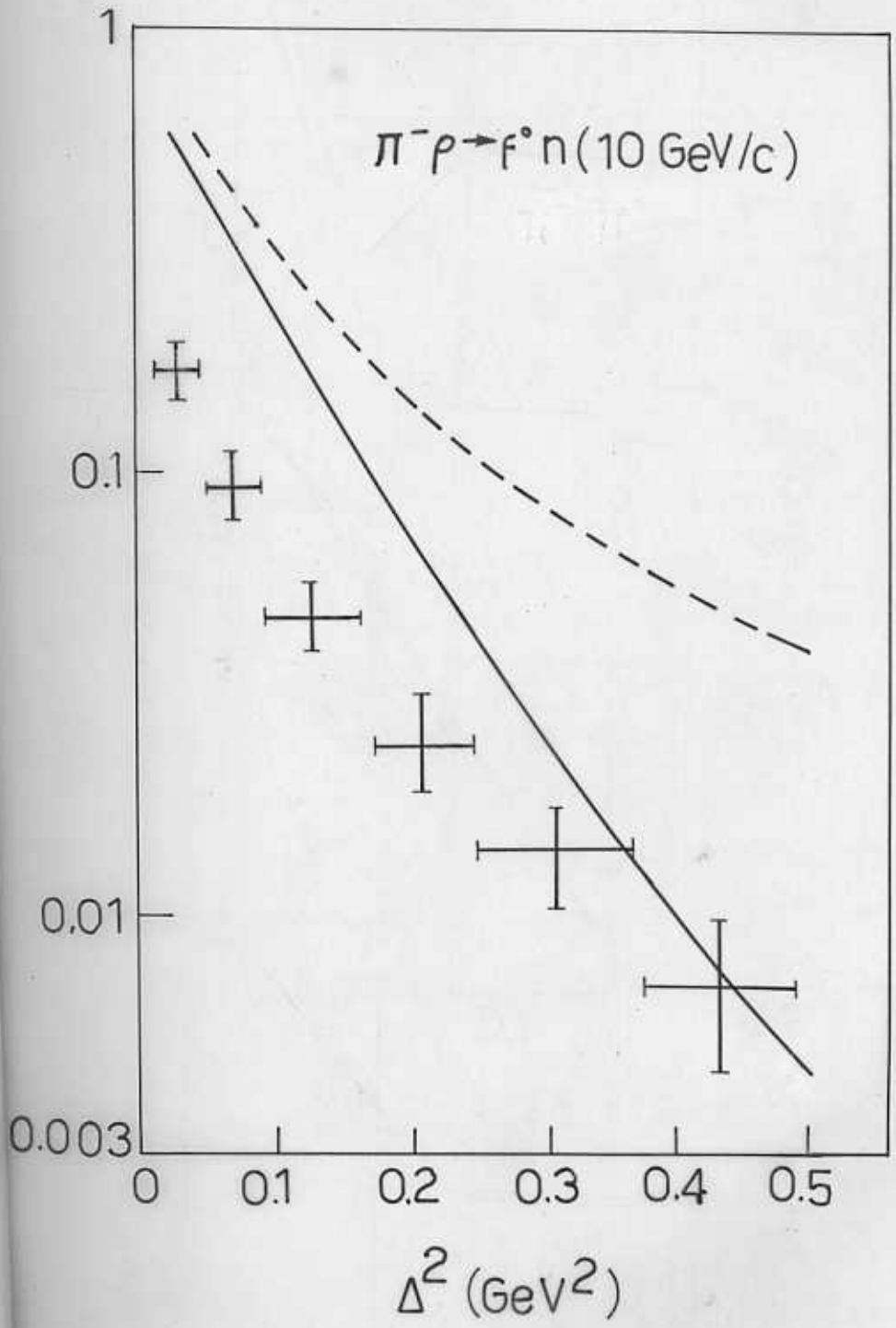

FIG. 4

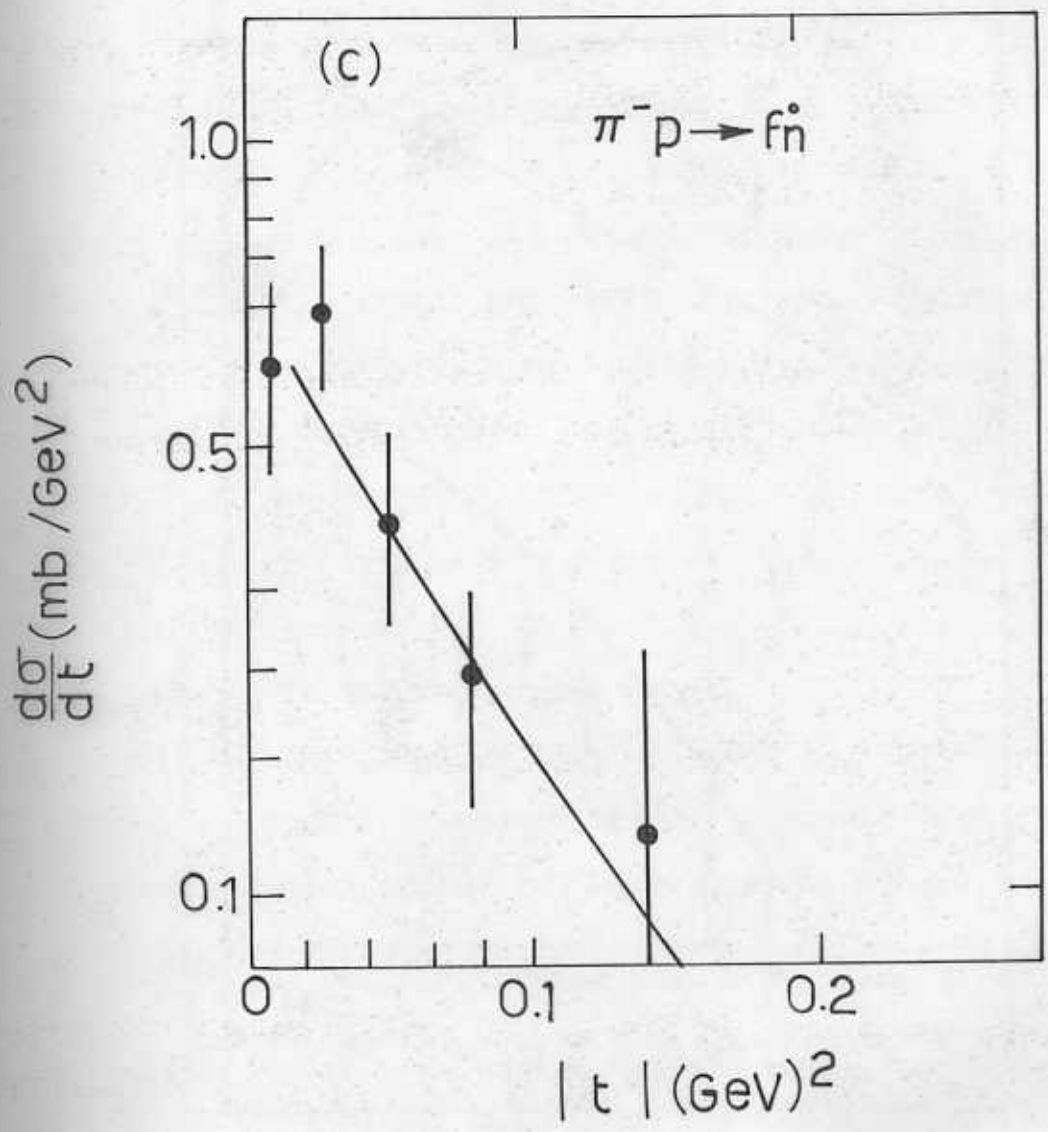
(c)
$\pi^- p \rightarrow f n^0$
1.0
0.5
0.1
$\frac{d\sigma}{dt}$ (mb /GeV^2)
0
0.1
0.2
| t | (GeV)^2

FIG.5

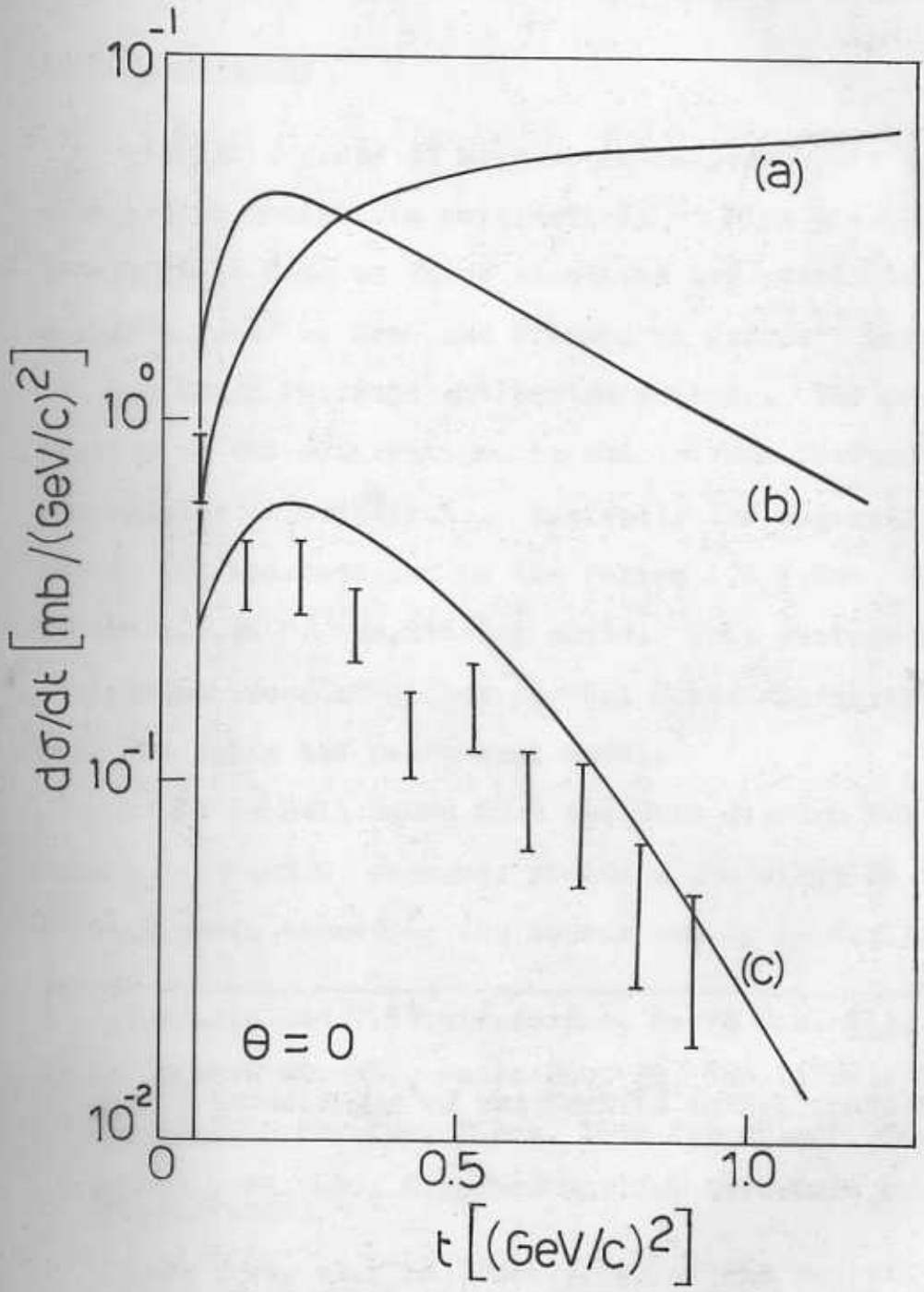

FIG.6

CHAPTER: II

REGGE POLE MODEL AND U(6,6) SYMMETRY FOR $p\bar{p} - \Lambda\bar{\Lambda}$ +

1. Introduction:

Here a study of hyperon-antihyperon pair production from proton antiproton interactions at high energy is made. Experimental data on these reactions are available at various energies from the Cern and Brookhaven groups[1] between 3.0 and 7.0 Gev/c incident antiproton moments. The most striking feature of the observations is the extreme forward peaking of the angular distributions. Typically the majority of the events are concentrated in the region 1.0 > Cos > 0.8 where is the c.m.s. $\bar{p}\bar{\Lambda}$ scattering angle. This feature indicates the predominance of higher partial waves characteristic of what one calls the peripheral model.

It is well known that the Born diagram for the process with a K or/and K exchange yields a contribution from low partial waves exceeding the bounds set by unitarity. The

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+ A.Sundaram and K.Srinivasa Rao, Nuovo Cim. 61A, 755 (1969)

1) B.Musgrave et. al., Nuovo Cim. 35, 735 (1965); R.Bock et. al. in Proceedings of the Twelfth annual conference on High Energy Physics, Dubna, 1964 (Atomizadt, Moscow (1965)).

C.Baltay et. al., Stanford Nuclear Structure meeting, 1963 (unpublished).

C.Baltay et. al., in Proceedings of the twelfth annual conference on High Energy Physics, Dubna, 1964 (Atomizadt, Moscow 1965); Phys. Rev. 140B, 1027 (1965).

resulting angular distribution for the produced particle shows too weak an angular dependence. So several attempts[2] have been made to study the process by the absorption model[3] wherein the low partial wave contributions to the cross section are damped by including the elastic interaction in the initial and final states. Cohen Tannoudji and Navelet[2] confirmed the dominance of K exchange. These authors used only γ_μ type of coupling at the KN vertex. Hogassen and Hogassen[2] generalised the model to include an admixture of a Pauli type term $\sigma_{\mu\nu} q_\nu$ and concluded that if the Pauli type term was slightly more than the magnitude of the Dirac type term, the model would no longer fit the data. These considerations reveal some arbitrariness in the choice of the various couplings at the vertices. Moreover the coupling constants are treated as free parameters in these models. Recently Migneron and Watson[4] have studied these reactions using the U(6,6) symmetry for uniquely determining

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2) N.J.Sopkovich, Nuovo Cim. 26, 186 (1962); H.D.D.Watson, Nuovo Cim. 29, 1388 (1963); L.Durand III and Yau Tsi Chiu, Phys.Rev. Lett. 12, 399 (1964); C.Cohen-Tannoudji and H.Navelet, Nuovo Cim. 37, 1511 (1965); H.Hogassen and J.Hogassen, Nuovo Cim. 40A, 560 (1965).

3) N.J.Sopkovich, Nuovo Cim. 39, 186 (1962); J.D.Jackson, in Proceedings of the 1966 Stony Brook Conference on High Energy two body reactions (unpublished); J.D.Jackson in (High Energy Physics) proceedings of the 1965 Les Houches Summer School (Gordon-Breach Science Publishers,Inc., New York 1966); A.C.Hearn and S.D.Drell in 'High Energy Physics' (Academic Press,Inc., New York) SLAC-PUB-176 ITP 200, 1966; U.Maor in Proceedings of the Conference on Nuclear and Particle Physics, University of Glasgow 1966 (CERN/TC/Physics 66-26) unpublished.

4) J.H.R.Migneron and K.Moriarty, Phys.Rev.Lett. 18, 978 (1967); H.D.D.Watson, Phys.Lett. 17, 72 (1966); J.H.R.Migneron and H.D.D.Watson, Phys.Rev. 166, 1654 (1968).

the Lagrangian and coupling constants, while imposing unitarity requirements on the peripheral amplitude with K and K exchange by the inclusion of absorptive corrections. They find that the angular distributions are well reproduced but the energy variations of the cross-section turn out to be far from satisfactory, in that their theoretical cross-sections not only decrease more slowly in the momentum range 3.0 to 5.7 Gev/c but also start rising thereafter, while the experimental results reveal a decrease with increasing energy. This is mainly due to the dominance of K^* exchange at higher energy and the well known defect of the vector exchange in the absorption model.

As the U(6,6) predictions for the three point functions are quite good[5], we use this higher symmetry for the vertices as in ref.4 and use the Regge pole model for the dynamics, assuming K^* -exchange to be dominant. With the known form for the K^* -trajectory and coupling constants and the Lagrangian determined from U(6,6) symmetry our fit is a zero parameter fit and reproduces the experimental data (both the angular dependence and the energy variation) reasonably well. We use the same notation as in ref.4.

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5) A.Salam, R.Delbourgo and J.Strathdee, Proc.Roy.Soc.(London) 284A, 146 (1965); 285A, 312 (1965); M.A.B.Beg and A.Pais, Phys.Rev.Lett. 14, 267 (1965); B.Sakita and K.C.Wali, ibid 14, 404 (1965); Phys.Rev. 139B, 1355 (1965).

It should be noted that U(6,6) symmetric three point functions are not in conflict with physical unitarity.

See R.Delbourgo et. al., Seminar on High Energy Physics and Elementary Particles, Trieste (International Atomic Energy Agency, Vienna, 1965), p.456.

2. U(6,6) interaction and the Regge pole model.

The U(6,6) interaction for the K -exchange is given by

$$L = G J_\mu \varphi_\mu \qquad (1)$$

where is the vector nonet and G is the U(6,6) coupling constant related to the πNN coupling constant by

$$G_{\pi NN} = \frac{5}{3} G \left(1 + \frac{2m}{S}\right) \qquad (2)$$

where S is the pseudoscalar meson mass. The U(6,6) prediction for vector currents relevent to the interaction of the baryon octet with the 1^- meson is given by

$$J_\mu = \frac{P_\mu}{2m}\left(1 + \frac{q^2}{2mV}\right)(\bar{N}N)_F + \left(1 + \frac{2m}{V}\right)\left(\bar{N}\frac{\gamma_\mu}{4m^2}N\right)_{D+\frac{2}{3}F} \qquad (3)$$

where N is the baryon of mass m, q is the momentum transfer, V is the vector meson mass, P_μ and γ_μ are conventional forms for << electric >> and << magnetic >> interactions and the coefficients of $P_\mu/2m$ and $\gamma_\mu/4m^2$ are conventional Sachs form factors F_C and F_M 6).

The basis of the model is an amplitude corresponding to equation (1) which contains the K^* -exchange (Fig.)

---

6) R.G.Sachs, Phys.Rev. 126, 2256 (1962).

$$
\begin{aligned}
T = {} & M_1^{II}(q^2)\, M_1^{I}(q^2)\, \bar{v}(P_1)\, \frac{P_\mu'}{2m}\, v(-P_3)\, \frac{g_{\mu\nu} - q_\mu q_\nu/\mu^2}{\mu^2(K^*) - t}\, \bar{u}(-P_4)\, \frac{P_\nu}{2m}\, u(P_2) \\
& + M_1^{II}(q^2)\, M_2^{I}(q^2)\, \bar{v}(P_1)\, \frac{P_\mu'}{2m}\, v(-P_3)\, \frac{g_{\mu\nu} - q_\mu q_\nu/\mu^2}{\mu^2(K^*) - t}\, \bar{u}(-P_4)\, \gamma_\nu\, u(P_2) \\
& + M_2^{II}(q^2)\, M_1^{I}(q^2)\, \bar{v}(P_1)\, \gamma_\mu\, v(-P_3)\, \frac{g_{\mu\nu} - q_\mu q_\nu/\mu^2}{\mu^2(K^*) - t}\, \bar{u}(-P_4)\, \frac{P_\nu}{2m}\, u(P_2) \qquad (4) \\
& + M_2^{II}(q^2)\, M_2^{I}(q^2)\, \bar{v}(P_1)\, \gamma_\mu\, v(-P_3)\, \frac{g_{\mu\nu} - q_\mu q_\nu/\mu^2}{\mu^2(K^*) - t}\, \bar{u}(-P_4)\, \frac{V_\nu}{2m}\, u(P_2)
\end{aligned}
$$

where p_1, p_2, and $-p_3$ $-p_4$ are the four-momenta of the incoming antiproton and proton and outgoing antibaryon and baryon respectively. P_μ and P_μ' are the sum of fermion momenta at the particle and antiparticle vertices respectively. The superscripts I and II of the form factors refer to the upper and lower vertices of the diagram in Fig.1. Here m is the average mass of the four baryons in each particular channel: $m_1=m_2=m_3=m_4=m$.

In the centre of mass system

$$
\begin{aligned}
s &= (P_1 + P_2)^2 = 4E^2 = W^2 \\
t &= (P_1 + P_3)^2 = -2p^2(1 - \cos\theta) \\
u &= (P_1 + P_4)^2 = -2p^2(1 + \cos\theta)
\end{aligned} \qquad (5)
$$

where E and p are the energy and momenta of each particle. θ is the scattering angle between the incoming proton and outgoing baryon. The helicity amplitudes are defined as

$$
\varphi_i = \langle \lambda_\Lambda \lambda_{\bar{\Lambda}} | T | \lambda_p \lambda_{\bar{p}} \rangle \qquad (6)
$$

where label the helicities of the particles and the index i specifies the helicity dependence. The restrictions imposed by parity and charge conjugation invariance reduce the number of amplitudes from 16 to 6.

Table 1

$\lambda_4\ \lambda_3$ $(\Lambda\ \bar{\Lambda})$ \ $\lambda_1\ \lambda_2$ $(p\ \bar{p})$	$++$	$+-$	$-+$	$--$
$++$	φ_1	φ_5	$-\varphi_5$	φ_2
$+-$	φ_6	φ_3	φ_4	$-\varphi_6$
$-+$	$-\varphi_6$	φ_4	φ_2	φ_6
$--$	φ_2	φ_5	$-\varphi_5$	φ_1

The normalization is such that the differential cross section for the unpolarized particles in the initial state is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{(16\pi E^2)^2}\,\frac{1}{4}\sum_i |\varphi_i|^2 \tag{7}$$

where the summation is over the 16 helicity amplitudes.

If the K^* is now treated as a Regge pole, the factor $1/(\mu^2-t)$ occuring in (1) is expected to be replaced by[7)]

7) S.C.Frautschi, Gell-Mann, F.Zachariasen, Phys.Rev.126, 2204 (1962); I.J.Muzimich, ibid 130, 1571 (1963); M.M.Islam, Nuovo Cim. 30, 579 (1963); M.M.Islam and R.Pinon, ibid 30, 837 (1963); D.P.Roy, N.C. 40A, 513 (1965); Phys.Rev.146, 1218 (1966).

$$\bar{X} = (2\alpha+1)\,\frac{P_\alpha(-x_t) - P_\alpha(x_t)}{2\sin\pi\alpha}\left(\frac{P_t^2}{m_N m_\Lambda}\right)\beta \tag{8}$$

where P_t is the c.m. momentum for the t channel. The first two factors arise from the Sommerfeld-Watson transformation of the amplitude in the t-channel. The third term brings out the threshold dependence of the residue of the K^* trajectory and β is a slowly varying function of t. We shall neglect the t dependence of β. Now for large s we have

$$x_t = \frac{s - (m_N^2 + m_\Lambda^2)}{2P_t^2} \tag{9}$$

Then the asymptotic approximation for the Legendre function gives

$$P_\alpha(x_t) \approx \frac{\Gamma(\alpha+\frac{1}{2})\,2^\alpha}{\pi^{\frac{1}{2}}\Gamma(\alpha+1)}\left[\frac{s - (m_N^2 + m_\Lambda^2)}{2P_t^2}\right]^\alpha \tag{10}$$

The factor $\frac{\Gamma(\alpha+\frac{1}{2})\,2^\alpha}{\Gamma(\alpha+1)}$ varies slowly with t and therefore can be absorbed into β. Thus the propagator is replaced by:

$$X = (2\alpha+1)\left(\frac{e^{-i\pi\alpha} - 1}{2\sin\pi\alpha}\right)\left[\frac{s - (m_N^2 + m_\Lambda^2)}{2m_N m_Y}\right]^\alpha \beta \tag{11}$$

38

β is determined from the fact that X should coincide with its field-theoretic counter part at the pole point, which gives:

$$\beta = -\frac{1}{3}\pi\alpha' \left[\frac{s-(m_N^2+m_\Lambda^2)}{2m_N m_\Lambda}\right]^{-1}$$

$$\alpha' = \left(\frac{\partial\alpha}{\partial t}\right)_{t=m_{K^*}^2} \tag{12}$$

Thus the contribution to each φ_i [4] from the Reggeized K^* exchange is given by:

$$\varphi_1 = X\left\{\left[M_1^2(q^2)(2E^2+P^2(1+x)) + 4M_1(q^2)M_2(q^2)(E^2+P^2)\right]\times (1+x) + M_2^2(q^2)\left[8P^2+2m^2(1+x)\right]\right\},$$

$$\varphi_2 = -X\left\{M_1^2(q^2)\frac{E^2}{m^2}(2E^2+P^2(1+x)) + 4M_1(q^2)M_2(q^2)E^2 + 2M_2^2(q^2)m^2\right\}(1-x),$$

$$\varphi_3 = X\left\{M_1^2(q^2)(2E^2+P^2(1+x)+4M_1(q^2)M_2(q^2)E^2 + 2M_2^2(q^2)m^2\right\}(1-x), \tag{13}$$

$$\varphi_4 = -\varphi_2,$$

$$\varphi_5 = XE\left\{M_1^2(q^2)(2E^2+P^2(1+x)) + \frac{2}{m}M_1(q^2)M_2(q^2)(E^2+P^2) + 2M_2^2(q^2)m\right\}\sin\theta,$$

where $x = \cos\theta$,

$$M_1(q^2) = \sqrt{3}\,\frac{2m}{V}\left(1 - q^2/4m^2\right) G,$$

$$M_2(q^2) = -\sqrt{3}\left(1 + \frac{2m}{V}\right)\left(1 - \frac{q^2}{4m^2}\right) G. \qquad (14)$$

$$M_1 = M_1^{I} = M_1^{II} \quad ; \quad M_2 = M_2^{I} = M_2^{II}$$

Here the K^* -trajectory is assumed to be parallel to the ρ trajectory with $\alpha_{K^*}(0) - \alpha_\rho(0) = m_\rho^2 - m_{K^*}^2$ 8) and is given by

$$\alpha_{K^*} = 0.37 + 0.91\ t\ . \qquad (15)$$

We have made an ad hoc choice of SU_3 masses S = 417 Mev and V = 850 the mean values for 0^- and 1^- nonets respectively, as this prescription has been used elsewhere with success[4,9].

3. Results and discussion.

Figures 2,3,4 show a comparison of the results of the present calculation with those of experiments for incident antiproton momenta of 3.6, 5.7 and 7.0 Gev/c. From the figures it is obvious that the over-all agreement of the angular distribution with experiment is good considering the fact that the present calculation is free of parameters. The total cross sections for the process $p\bar{p} \to \Lambda\bar{\Lambda}$ evaluated at different energies are given in Table 2 along with experimental data of CERN and Yale groups[1]. The observed energy dependence of the total

8) J.P.Ader, M.Capdeville and Ph.Salim, Nucl.Phys.B3407(1967).
9) R.Delbourgo et. al., Seminar on High Energy Physics and Elementary Particles, Trieste (Vienna 1965) p.455.

cross section is also found to be reproduced quite well by the present calculation.

Table.2. Energy dependence of the total cross section $p\bar{p} \rightarrow \Lambda\bar{\Lambda}$.

incident anti-proton momentum (Gev/c)	predicted cross section (μb)	experimental cross section (μb)
3.0	108	117 ± 18
3.6	77	77 ± 20
3.7	74	82 ± 8
5.7	36	40 ± 10
7.0	27	31 ± 13

FIGURE CAPTIONS

Fig.1: The production $p\bar{p} \rightarrow Y\bar{Y}$ in the s-channel or the elastic scattering $p\bar{Y} \rightarrow p\bar{Y}$ in the t-channel.

Fig.2: The one particle exchange diagram for the annihilation process $p\bar{p} \rightarrow \Lambda\bar{\Lambda}$.

Fig.3: Differential cross section for $p\bar{p} \rightarrow \Lambda\bar{\Lambda}$ at 3.6 Gev/c. Data are from ref.(1). The full curve is the prediction of the Regge pole model.

Fig.4: Differential cross section for $p\bar{p} \rightarrow \Lambda\bar{\Lambda}$ at 5.7 Gev/c. The experimental results are taken from ref.(1). The full curve is the prediction of the Regge pole model.

Fig.5: Differential cross section for $p\bar{p} \rightarrow \Lambda\bar{\Lambda}$ at 7.0 Gev/c. The experimental data are from ref.(1). The full curve is the prediction of the Regge pole model.

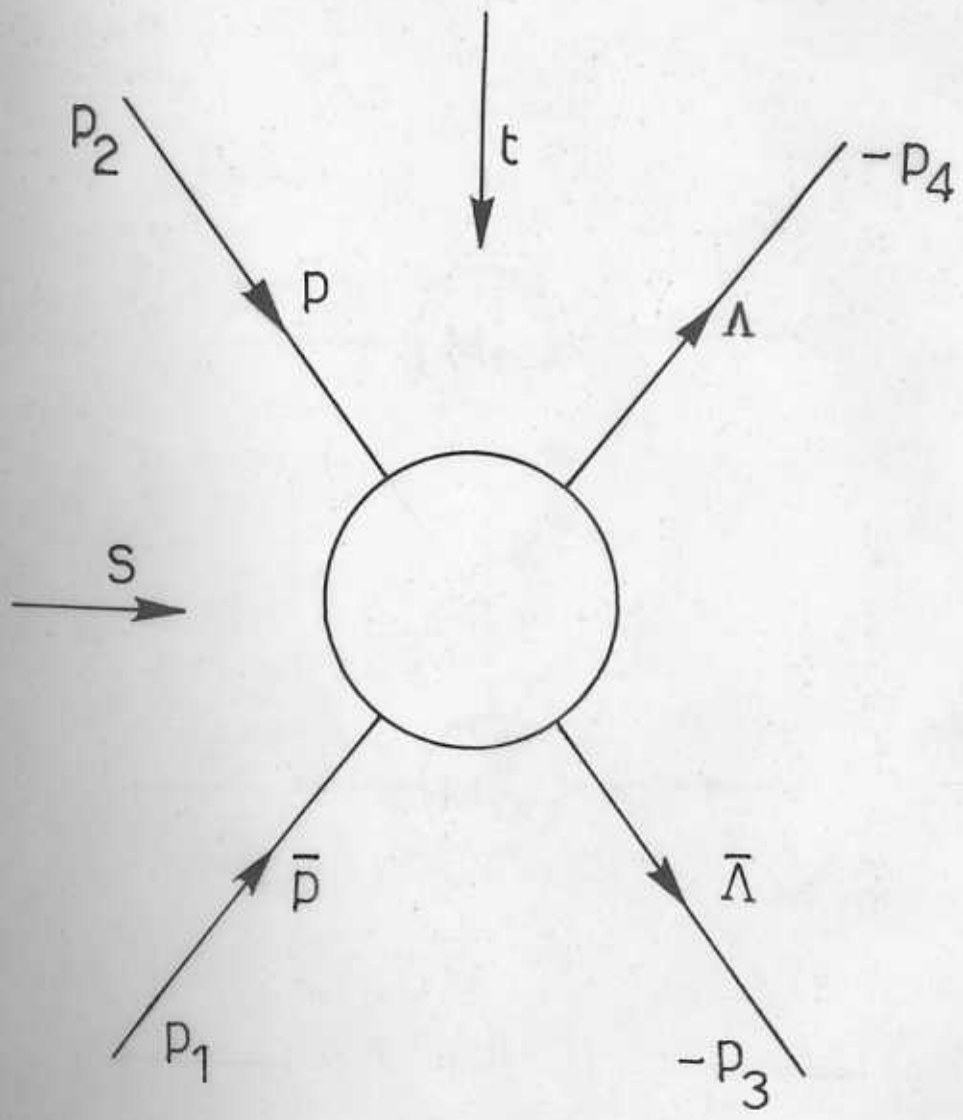

FIG. 1

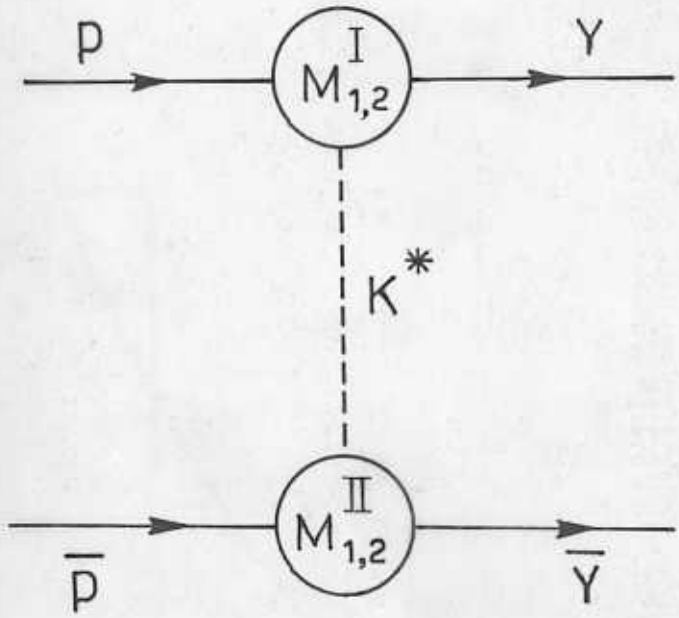

FIG. 2

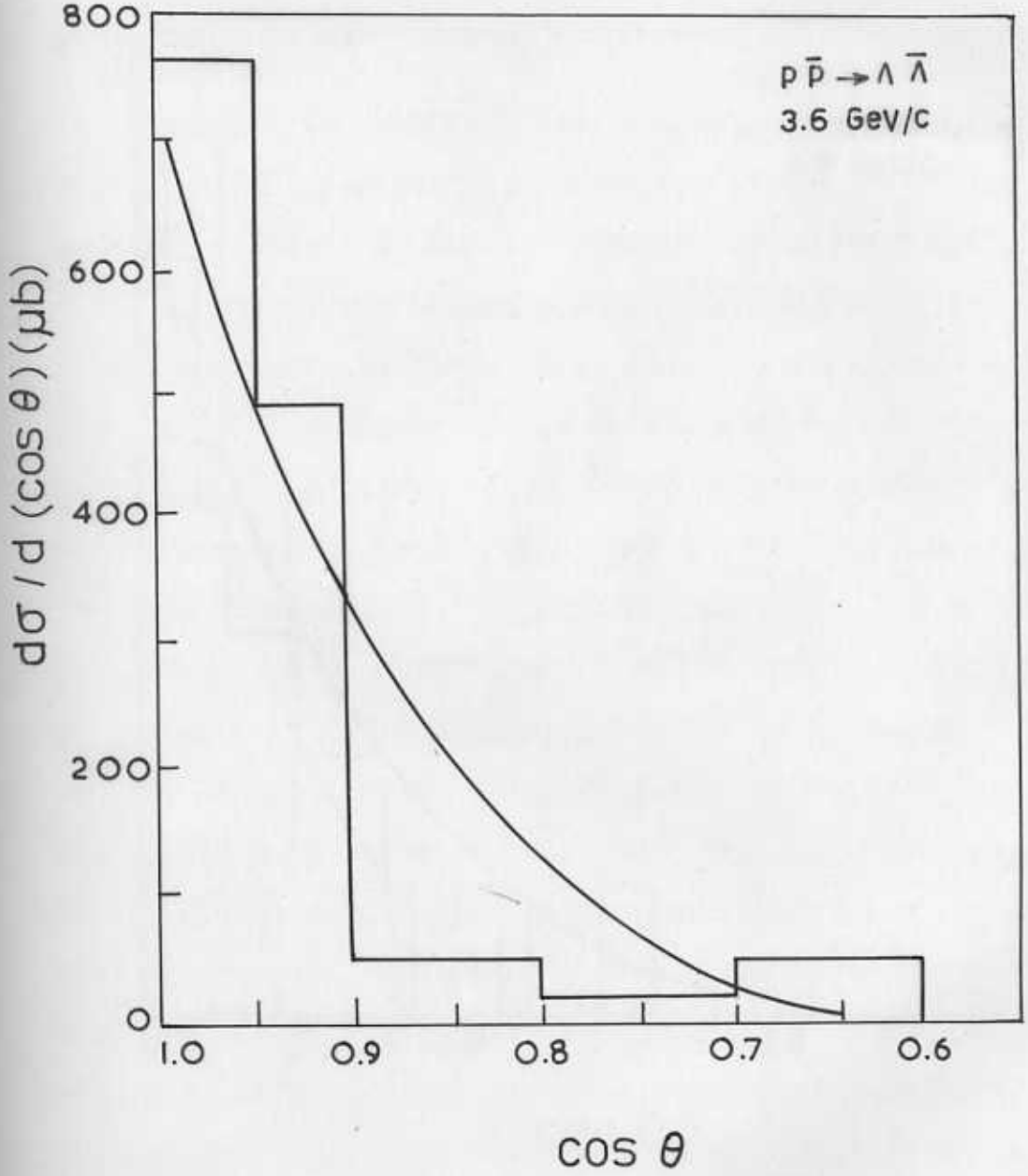
800
600
400
200
0
1.0
0.9
0.8
0.7
0.6
$p\bar{p} \rightarrow \Lambda\bar{\Lambda}$
3.6 GeV/c
$d\sigma / d(\cos\theta)$ (μb)
$\cos\theta$

FIG.3

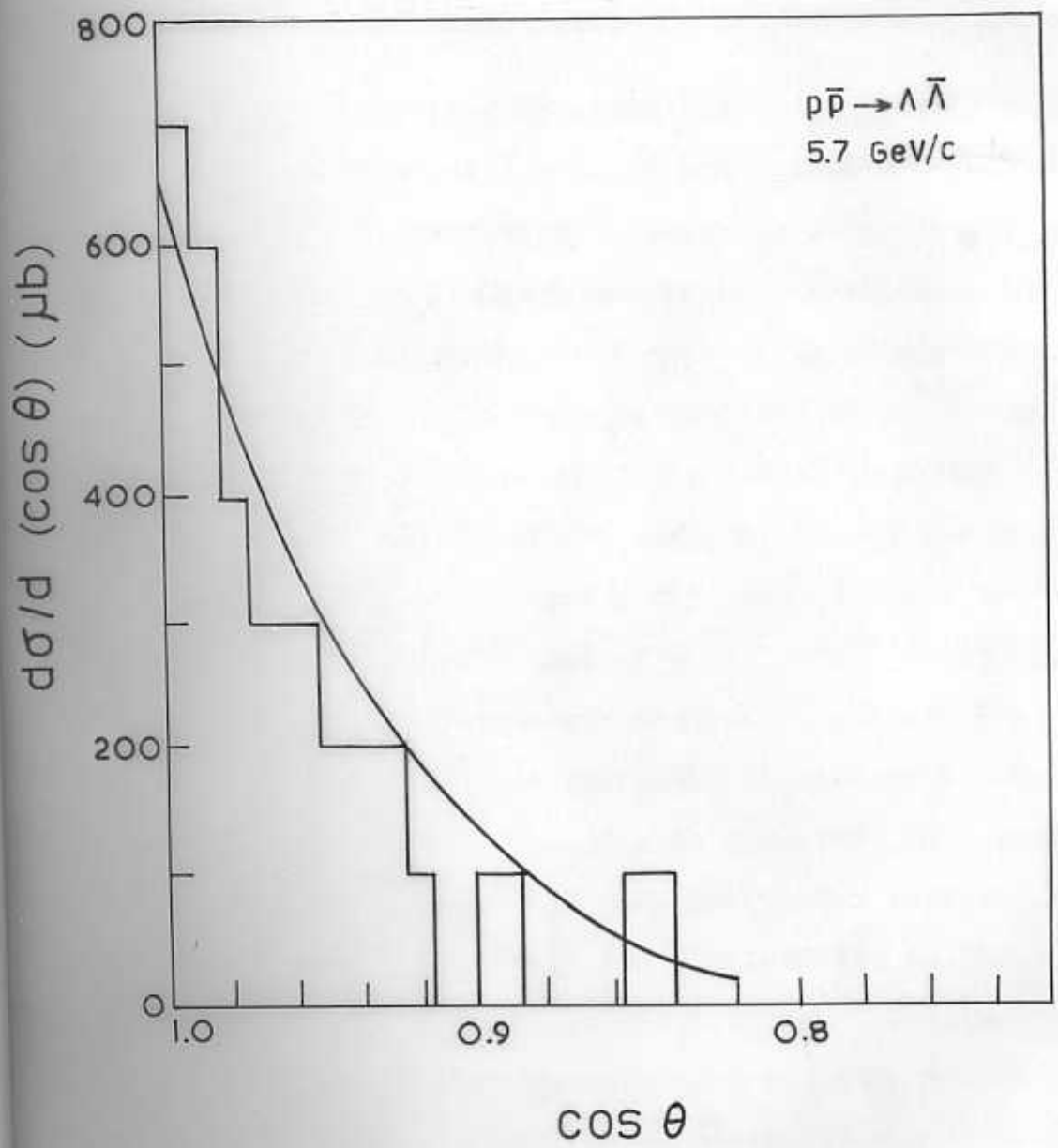
$p\bar{p} \rightarrow \Lambda\bar{\Lambda}$
5.7 GeV/c
800
600
400
200
0
1.0
0.9
0.8
$d\sigma/d(\cos\theta)$ (μb)
$\cos\theta$

FIG.4

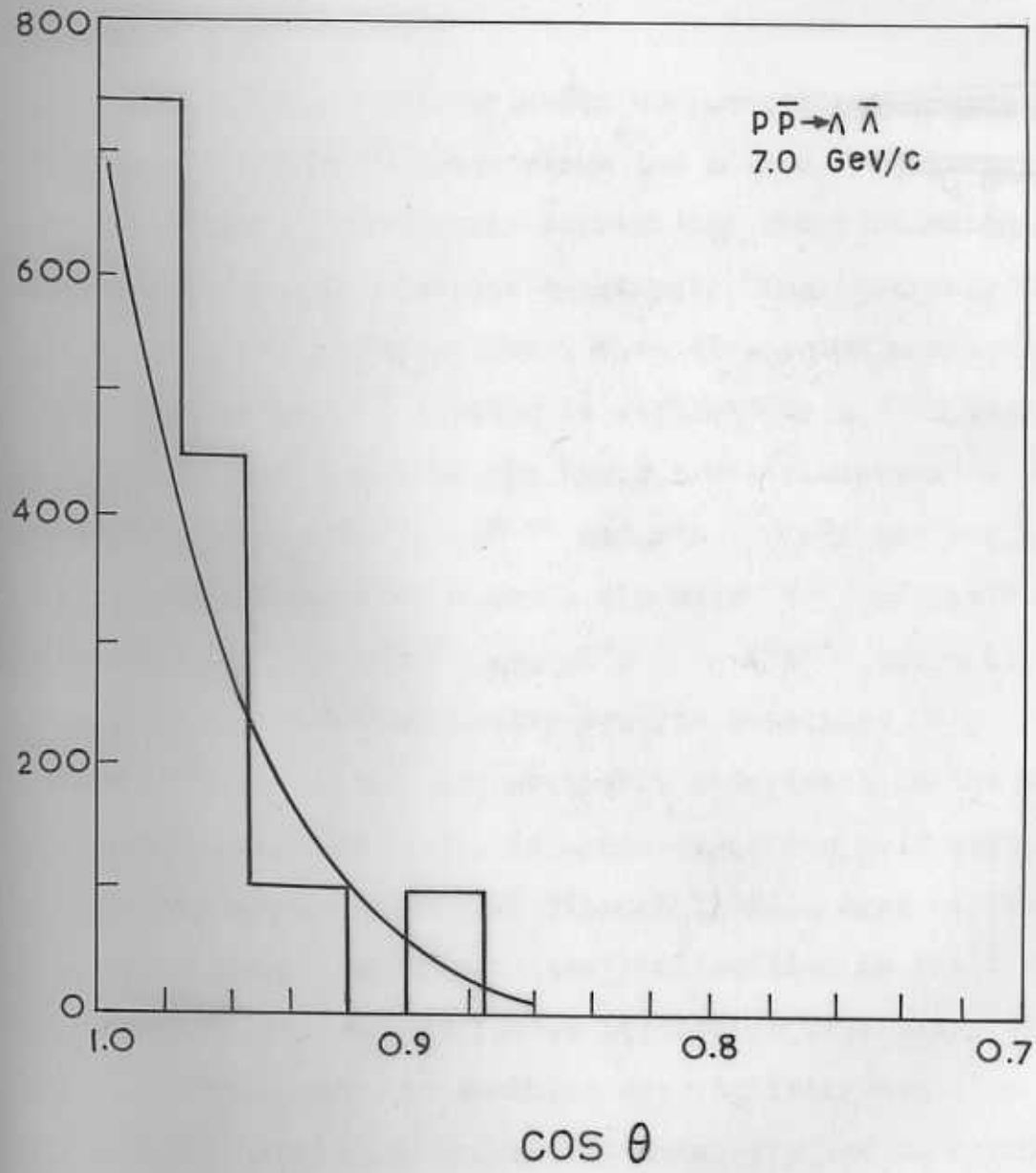
800
600
400
200
0
1.0
0.9
0.8
0.7
$p\bar{p} \to \Lambda\bar{\Lambda}$
7.0 GeV/c
$d\sigma/d(\cos\theta)$ (μb)
$\cos\theta$

FIG.5

CHAPTER: III

REGGE CUTS AND ISOBAR PRODUCTION+

1. Introduction:

We are interested in meson baryon scattering reactions resulting in a pseudoscalar meson and a spin isobar. Indeed there is a lot of similarity between the above reactions and meson baryon charge exchange reactions. Qualitatively one might expect reactions in which the t channel quantum numbers are identical but the isobar is replaced by a baryon to exhibit similar structures in their t distributions[1]. The existing data on $\pi^+p \quad \pi^0 \; ^{++}$ and $\pi^-p \quad \pi^0 n$ are suggestive of a turn over near t 0 and a dip near t -.6 Gev^{-2}. Similarly in $\pi^+p \quad {}^0\Delta^{++}$ and $K^+p \quad K^0\Delta^{++}$ there is absence of dip as in the corresponding protype reactions $\pi^-p \quad n$, and the $K^-p \quad K^0n$. The available structures in the angular distribution in the charge exchange reactions were explained by the dominance of helicity flip amplitudes over helicity non-flip amplitudes. Because of the similarities in the t distribution one expects, in the isobar production case also, the helicity flip amplitudes to dominate over helicity non-flip and double flip amplitudes and one is naturally led to consider the

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+) A.Sundaram, Lettere al Nuovo Cim. Vol.1, 177 (1971).

1) R.D.Mathews, Nucl.Phys. 11B, 339 (1969).

Stodolsky-Sakurai hypothesis[2] that $N\Delta$ couplings are of pure magnetic dipole type. Theoretically an analysis of these reactions is both fruitful and simplified as there are only a limited number of Regge poles (maximum two) exchanged in the t-channel unlike other reactions wherein the theoretical analysis is masked by a host of parameters pertaining to the large numbers of Regge poles exchanged[3]. Therefore a study of the isobar production processes has been made by various authors[1,4]. Krammer and Maor[4] studied the above processes in the pole model, fully incorporating the various kinematical singularities and threshold conditions in their parametrization. Renniger and Sharma avoided the question of the complicated kinematical singularities by having recourse to the use of invariant amplitudes. However there is a strong experimental evidence suggesting the inclusion of more complex singularities (such as cuts) in addition to the simple poles in the complex angular momentum plane

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2) L.Stodolsky and J.J.Sakurai, Phys.Rev.Lett. 11, 90 (1963).

3) For instance in KN elastic scattering there are five Regge pole exchanges viz. p,p', , , A_2.

4) a) H.Capresse and H.Stremnitzer, Nuovo Cim. 44A, 1245 (1966).

b) R.L.Thews, Phys.Rev. 165, 1624 (1967).

c) M.Krammer and U.Maor, Nuovo Cim. 50A, 963 (1967). ibid 52A, 308, 931 (1967); Nucl. Phys.13B, 651 (1969).

d) G.H.Renniger and K.V.L.Sharma, Phys.Rev.178, 2203 (1969).

e) F.Schrempp, Nucl.Phys. 6B, 487 (1968).

as the simple Regge pole model could not satisfactorily explain[5]:

i) the forward peaking in π exchange processes,

and ii) dips and secondary maxima in the momentum transfer distribution in certain inelastic processes.

Because the pion has parity $(-1)^{J+1}$, orbital angular momentum is usually involved in the creation or annihilation of a pion. Thus most reactions via π exchange vanish at zero momentum transfer. Instead of zeros, forward structures (peaks) over a small momentum transfer interval $\Delta t \quad m_\pi^2$ are observed in pn np[6], p π^+n[7], π^+p N^{+++} [8], p $\pi^-\Delta^{++}$ [9] etc. The evidence is very strong that these peaks are associated with π exchange. It has been observed that the mathematics of the Regge representation allows the possibility of nonzero forward amplitudes in certain of these cases, if the π trajectory 'conspires' with a trajectory of opposite parity[10]. This possibility is physically unmotivated as there is no physical particle lying on the conspiration trajectory and does

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5) See for a list of cases wherein the simple Regge pole model finds J.D.Jackson in 'Proceedings of the Lund international conference on Elementary Particles' edited by Von Dradel, p. 63 (1969).

6) G.Manning et.al., Nuovo Cim. 41, 167 (1967)

7) A.M.Boyarski et. al., Phys.Rev.Lett. 20, 300 (1968); P.Heide et. al., Phys.Rev.Lett. 21, 248 (1968).

8) Aachen-Balin, CERN Collaboration, Phys.Lett. 27B, 174 (1968).

9) A.M.Boyarski et. al., Phys.Rev.Lett. 25, 695 (1970)

10) See the review article by L.Bertochi in 'Proceedings of the Heidelberg International Conference on Elementary Particles' edited by Filthuth, p.197 (1967)

not agree with all the known forward π exchange reactions in its basic factorizable form[11]. On the other hand the explanation in terms of cuts is physically compelling. Forward reactions occur as a result of small angle π exchange with compensating elastic or diffractive scattering. The cut contribution should be viewed as a diffractive phenomenon associated with the Regge pole involved in the quantum number exchange. The dips and secondary maxima observed in momentum transfer distribution of a variety of reactions are a natural aspect of this diffractive phenomenon.

So we have studied the processes $\pi^+p \quad \pi^0\Delta^{++}$, $\pi^+p \quad \Delta^{++}$ and $K^+p \quad K^0\Delta^{++}$ invoking Regge poles and Regge cuts[12] due to Regge poles plus n pomerons. Use has been made of the S-channel helicity amplitudes[13] and so there is no problem regarding Kinematic singularity in our approach. It is known that the concept of duality between resonance poles and Regge poles leads to the exchange degeneracy of trajectories due to the absence of resonance poles in channels with exotic quantum numbers. However the empirical , , p', A_2 trajec-

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11) M.Le Bellac, Phys.Lett. 25B, 524 (1967); See J.D.Jackson in 'Proceedings of the Lund International Conference on Elementary Particles' edited by Von Dradel,p.63 (1969).

12) S.Frautschi and B.Margolis, Nuovo Cim.56A, 1155 (1968); A.P.Contogouris and J.Lebrun, Nuovo Cim. 64A, 627 (1969); M.Colocci, Lettere al Nuovo Cim. 4, 53 (1970).

13) G.Cohen-Tannoudji, Ph.Salin and A.Morel, Nuovo Cim.45A, 412 (1968).

tories determined from analyzing πN, KN, πN N (data) are not degenerate[14]. As previous calculations have shown in several other cases we find the inclusion of the Regge cuts in an exchange degenerate scheme for and A_2 trajectories yield good agreement with experiment for the processes π^+p $\pi^0\Delta^{++}$, π^+p Δ^{++} and K^+p $K^0\Delta^{++}$.

2. Regge cuts in elastic and inelastic scattering[15]:

Let us consider the elastic scattering of two spinless particles. At asymptotic energy the impact parameter expansion of the scattering amplitude is given by the Hankel transforms

(1)

with $q = (-t)^{1/2}$ = momentum transfer. Suppose $F(b,s)$ depends on the variable b as follows

(2)

with (3)

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14) G.V.Dass, C.Michel and R.J.N.Phillips, Nucl.Phys.B9,549(1969); V.Barger and R.J.N.Phillips,'Meson exchanges from simultaneous analysis of πN data and dispersion sum rules.(Wisconsin preprint 100-223 April 1969)

15) Here we follow the same procedure as those of A.P.Contogouris and J.P.Lebrun and M.Colocci (ref.12).

where and are real constants and

(4)

Further is assumed to be a real function of x expandable in a Taylor series around x = 0, so that

(5)

where . Substituting the above expression in (1) and interchanging summation and integration we obtain after integrating in b

(6)

where (7)

Suppose now that for n=1 represents the Pomeranchuk Regge trajectory having a slope . Then the term n=1 in (6) can be identified with the contribution of the Pomeranchuk pole to the scattering amplitude:

(8)

If is given, has a residue determined essentially by the real constant ; also the signature factor determines the correct phase of the form .

The argument x of , which is the basis for the construction of the whole series ( ) is easily seen to be related to the inverse Hankel transform of :

(9)

The terms of (6) with $n \geq 2$ can also be associated with singularities in complex angular momentum J for:

i) It is well known[16] that exchange of (n+1) Pomeranchukons with trajectories leads to a moving branch point which at small $|t|$ varies as

(10)

---

16) S.Mandlestam, Nuovo Cim. 30, 1148 (1968); R.Eden, P.Landshoff, D.Olive and J.Polkinghorne, "The Analytic S Matrix" (Cambridge 1966)

From the phenomenological point of view there are several models that take into account both Regge poles and cuts and successfully describe elastic scattering. Some of these models are due to:

a) D.Amati, M.Cini and A.Stanghellini, Nuovo Cim.30, 193 (1963)

b) L.Vanhove, CERN Lectures 65-22 (1965); W.Gottingham and R.F.Peierls, Phys.Rev.137B, 147 (1965).

c) A.P.Contagouris, Phys.Lett. 23, 698 (1966).

d) R.G.Arnold, Phys.Rev.153, 1523 (1967).

e) C.B.Chiu and J.Finkelstein, Nuovo Cim.59A, 92 (1969).

with (t) = 1 + t we immediately obtain (7).

ii) The phase of the leading contributions of the corresponding cut is exp in accord with general theorems on crossing symmetric asymptotic expansion of the scattering amplitude[17].

As the number of exchanged Reggeons increases, the asymptotic cut contributions contain decreasing powers of

.

Thus it is reasonable to associate that $n \geq 2$ terms of (6) with Regge cuts and write

(11)

The specific form of the function represents an ansatz about the cut discontinuity near the branch point; theoretically this is not well known. In ref.(16-a) the form

is used, its derivation is based on the mechanism that produces via elastic unitarity the Amati-Fubini-Stranghellini cuts. Contogouris (16-c) uses

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17) L.VanHove, Phys.Lett.5, 252 (1963); CERN Report 66/842/5-Th676 (June 1966) (Lectures at 1966 Scottish University's Summer School)

A.Logunov, Nguyen Van Hieu and I.Todorov, Annals of Physics 31, 203 (1965).

(12)

which can be derived by a similar mechanism, applied on a somewhat different model. Finally Frautschi and Margolis[12] use the form

(13)

for which some analogy with Glauber's formula for multiple scattering on nuclei might be invoked.

3. <u>Hankel transforms and branch points in inelastic reactions</u>[12]:

Let be the s-channel helicity amplitude for the process a+b c+d with helicities and respectively. Then the contribution of an evasive trajectory (M=0 in the Toller classification) to the scattering amplitude is

(14)

where is the signature and

We assume here that f(s,t) receives also contributions from Regge cuts due to exchange of trajectory R plus n pomeranchuck

trajectories ($n \geq 1$). Thus

(15)

The construction of is in close analogy with the expansions. Then, in our model, the cut contributions are assumed to be defined by the following Hankel transforms:

(16)

where

(17)

and has the form

(18)

with

(19)

It can be shown that the above expression generate a series of cuts with the properties discussed in sec.2.

Here we study the processes

$$\pi^+ p \qquad \pi^o \Delta^{++} \qquad (a)$$

$$\pi^+ p \qquad {}^o\Delta^{++} \qquad (b)$$

$$K^+ p \qquad K^o \Delta^{++} \qquad (c)$$

The process (a) and (b) can proceed only through a single Regge pole exchange with the quantum numbers of the and A_2 trajectory respectively. For the reaction (c) both the and A_2 trajectories contribute. Using the parity and G parity conservations for and A_2 exchanges we get four independent helicity amplitudes viz.

(20)

The differential cross section is then

(21)

Here there are two cases of interest:

i) In the first case zeroes in the nonsense wrong signature points are not present in the parametrization of the Regge pole amplitude[18]. This approach is widely practised by Henyey et. al.[19] who hold the view that the introduction

18) The removal of zeroes is discussed by A.H.Muller and T.L.Trueman, Phys.Rev.160, 1296 (1967); S.Mandlestam and L.L.Wang, ibid 1490 (1967); C.E.Jones and V.L.Teplitz, ibid 159, 1271 (1967).

19) F.Henyey, G.L.Kane, J.Pumplin and M.H.Ross, Phys.Rev.182, 1579 (1969); R.L.Kelly, G.L.Kane and F.Henyey, Phys.Rev. Lett. 24, 1511 (1970).

of zeroes in the nonsense wrong signature points is physically artificial and is incorrect. The absence of such zeroes implies that in conventional Regge pole language important multiplicative fixed poles are found at nonsense wrong signature points in the Regge pole amplitudes. The dips and secondary maxima that are found in the momentum transfer distribution is solely due to pole cut interference or diffraction minimum.

ii) In the other case the Regge pole amplitudes do have the zeroes in the nonsense wrong signature points and the dips and secondary maxima observed in the t distribution is due to the presence of zeroes at these points. This prescription was followed by Argonne group[20] in their investigations.

Case I:

exchange:

(22)

20) R.C.Arnold, Phys.Rev.153, 1523 (1967); Argonne Report No.ANL/HEP 6804, 1968 (unpublished); R.C.Arnold and M.L.Blackman, Phys.Rev.176, 2082 (1968); M.L.Blackman and and G.R.Goldstein, ibid 179, 1480 (1969); M.L.Blackman, ibid 178, 2385 (1969); M.L.Blackman, G.Krammer and K.Schilling, ibid 183, 1452 (1969); A.Capella and J.Tranh Thanh Van, Lett. al Nuovo Cim. 1, 321 (1969).

Similarly

(23)

and

(24)

For A_2 exchange:

(25)

Similarly

(26)

and

(27)

By simple calculations the Regge pole-n pomeron cut is given to be

(28)

where R is the Regge pole , A_2. In the above expressions

(29)

<u>Case II:</u> Here we assume both the vector and tensor trajectories choose nonsense choosing mechanism (Gell-Mann mechanism). Then the Regge pole contributions are given as follows:

(30)

Similarly

(31)

We are interested in the small momentum transfer region ($|t| \leq 1$ Gev). In this region and are slowly varying function of t and so they are absorbed into the . Then using the ( ) cut contribution is evaluated to be

(32)

(33)

(34)

the (A_2-p) cut contribution will be the same as given in ( ). The spin density matrix elements in the rest frame of Δ^{++} are defined by

(35)

where

are the t channel helicity amplitudes related to those in the s-channel by an orthogonal transformation[21)]

21) T.L.Trueman and G.C.Wick, Ann.Phys. 26, 322 (1964).

64

4. Results and discussion:

There are several striking features of the experimental data[22,23] which must be explained by the theoretical effort purporting to give a model of the reactions (a-c). In the reaction (a) the cross section data suggest a 'convexity' near

---

22) The differential cross section data are taken from the following sources:

$\pi^+p \quad \pi^0\Delta^{++}$, 4.0 Gev/c. Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.) - Munchen Collaboration, Nuovo Cim. 32, 688 (1964); Phys. Rev. 138B, 897 (1965).

(36)

where

8.0 Gev/c. Aachen-Berlin-CERN Collaboration, Phys.Lett. 10, 608 (1962); Nucl. Phys. 35, 49 (1962).

$\pi^+p \quad \pi^0\Delta^{++}$, 3 Gev/c. Durham-Mainzer-Saclay Polytechnique-Zurich Collaboration - Paper presented in the "Proceedings of the Lund International Conference on Elementary Particles", edited by Von Gradel, p.158(1969).

(37)

$\pi^+p \quad \Delta^{++}$, 4 Gev/c.data from D.Brown et.al., Phys. Rev.Lett. 12, 602 (1964); D.Brown, University of California Radiation Laboratory Report No.UCRL 10854 (1962) unpublished.

8 Gev/c data from Aachen-Berlin-CERN Collaboration, Phys.Lett.10, 608 (1962).

$\pi^+p \quad \pi^0\Delta^{++}$, 3.5, 5.0, 5.0 Gev/c data are taken from ref.(46); 12.7 Gev/c data are taken from R.G.Solmes et.al., University of Rochester preprint UR-278-515 (1970).

23) The data on the decay density matrix elements come from the following sources:

a) $\pi^+p \quad \pi^0\Delta^{++}$, 2.4 Gev/c points from G.Giacomelli et.al., CERN report No.10102 (1967) unpublished, 4 and 8 Gev/c are from ref.(46).

b) $\pi^+p \quad \pi^+\Delta^{++}$, data are taken from (ref.46).

4. Results and discussion:

There are several striking features of the experimental data[22,23] which must be explained by the theoretical effort purporting to give a model of the reactions (a-c). In the reaction (a) the cross section data suggest a 'convexity' near

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22) The differential cross section data are taken from the following sources:

π^+p $\pi^0\Delta^{++}$: 4.0 Gev/c. Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.) - Munchen Collaboration, Nuovo Cim. 24, 495 (1964); Phys. Rev. 138B, 897 (1965).

8.0 Gev/c. Aachen-Berlin-CERN Collaboration, Phys.Lett. 19, 608 (1965); Nucl. Phys. 8B, 45 (1968).

π^+p $\pi^0\Delta^{++}$: 5 Gev/c. Durham-Nijmejen-Ecole Polytechnique-Turin Collaboration - Paper presented in the "Proceedings of the Lund International Conference on Elementary Particles", edited by Von Dradel, p.158(1969).

π^+p Δ^{++} : 4 Gev/c.data from D.Brown et.al., Phys. Rev.Lett. 19, 608 (1963); D.Brown, University of California Radiation Laboratory Report No.UCRL 18254 (1968) unpublished.

8 Gev/c data from Aachen-Berlin-CERN Collaboration, Phys.Lett.19, 608 (1965).

K^+p $K^0\Delta^{++}$: 3.5, 5.0, 8.0 Gev/c data are taken from ref.(4c); 12.7 Gev/c data are taken from R.S.Holmes et.al., University of Rochester preprint UR-875-313 (1970).

23) The data on the decay density matrix elements come from the following sources:

a) π^+p $\pi^0\Delta^{++}$: 3-4 Gev/c points from G.Gidal et.al., UCRL report No.18351 (1968) unpublished; 4 and 8 Gev/c are from ref.(4c).

b) K^+p $K^0\Delta^{++}$: data are taken from (ref.4c).

t 0 followed by a dip near t -0.6 Gev^2. On the other-hand for the reactions (b) and (c) the cross sections show no clear minimum near t 0.6 Gev^2. In our calculation the trajectory is fixed as (t) = 1 + (t-m^2) with m^2 =0.585 Gev^2 and =0.89 Gev^{-2}. The pomeron slope is taken to be 0.4 which is the value needed to fit the new Seprukov data. Treating the free parameters and essentially constants an attempt is made to fit the processes (a) - (c). A reasonable fit is obtained in the case I +)

n=1, =1.323, =0.702, =1

and =10.0

Throughout our calculation we have assumed

and is assumed to be zero. For the process

$\pi^+ p$ $\pi^0 \Delta^{++}$ a good fit is obtained for

(38)

in accordance with the dominance of M1 transition of the isobar and the density matrix elements have the value given by Stodolsky

---

+) We have used $s_0 = m_\pi m_\Delta$ in both the cases I and II.

and Sakurai[+)]

(39)

We correctly reproduce the dip near t = -0.6 Gev^2 which in our model is essentially due to the pole and cut destructive interference. In order to reduce the number of parameters we have assumed exchange degeneracy for the and A_2 trajectories as suggested recently by Duality theory. Also we have demanded SU(3) symmetry and factorization for the pole residues at the vertices. Use has been made of the following SU(3) relations

(40)

for the processes (b) the assumption of the dominant magnetic

---

+) It is known that one of the first models of Δ productions in πN collisions was that of Stodolsky and Sakurai which was based on the exchange of elementary meson. The NΔ vertex was given by assuming the validity of analogy and assuming that the NΔ coupling was of a magnetic dipole type as has been indicated by the experiments on the electroproduction of the Δ. Also see:

L.Stodolsky, Phys.Rev.134,B, 1009 (1964).
J.D.Jackson and H.Pilkhun, Nuovo Cim.33, 906 (1964).
D.P.Roy, Nuovo Cim.40, 513 (1965).

Arguments based on the static model suggest that both the vector and tensor couplings to the baryons are dominantly of the M1 type: See

R.Dashen and S.Frautschi, Phys.Rev.152, 1450 (1966).

dipole coupling at the $A_2N\Delta$ vertex leads to the consideration of amplitudes alone which gives a dip near $t = -.6$ Gev^2. In order to reproduce the experimental data significant contribution from amplitudes have to be taken into account. The cut contribution dominates over the pole contribution at larger momentum transfers. For $|t| > .5$ Gev^2, the dominates over amplitudes and hence the density matrix elements for this process tend to deviate from the values predicted by the fixed pole model. However the recent experimental data[24] on the density matrix elements describing the decay angular distribution of the Δ for the process π^+p Δ are in accord with the predictions of the Stodolsky and Sakurai model. Thus the density matrix elements for larger momentum transfer are not correctly described in this case.

For the case ii) a reasonable fit is obtained for all the processes with the assumption that both and A_2 have M1 type transition at $N\Delta$ and $A_2N\Delta$ vertices. However the slope of the pomeron trajectory is needed to be smaller than the value obtained from Seprukov data. The values of the parameters

---

24) J.V.Major et. al., Bull. Am. Phys. Soc. 15, 493 (1970). The conclusion that production is in favour of the weak cut model has been reached by:
M.Bander and E.Cotsman, Phys.Rev. D2, 1345 (1970).

used are n=1, =3.14, = = 0.12 and =4.0. It should be noted that the same value of is obtained by Frautschi and Margolis[12] in their calculation for the case of elastic-scattering process. We have used for the processes (a-c)

(41)

in accordance with the fixed pole model. The density matrix elements are also in agreement with experiment. The results are compared with the experimental data in Fig. In this case the dip in π^+p $\pi^0\Delta^{++}$ is due to the vanishing of the factor at $t = -.6$ Gev^2. For π^+p $\pi^0\Delta^{++}$ the dip at t .6 Gev^2 is more pronounced but a better fit could be obtained by taking into account a slight contribution for the amplitudes or .

Thus we have studied the isobar production in πN collisions taking into account the Regge poles and the branch points due to the pole plus pomeranchukon. We also looked into the question of whether the nonsense wrong signature points should be included (caseII) or not (case I) in the parametrization of the Regge pole amplitudes. We find that a good fit can be obtained for the process π^+p $\pi^0\Delta^{++}$ in both the cases with the assumption of M1 coupling for the $N\Delta$ vertex. But

the experimental data on the density matrix elements describing the decay angular distributions of the Δ for the processes $\pi^+p \quad \Delta^{++}$ and $K^+p \quad K^0\Delta^{++}$ seem to support the case ii). Also it is seen that the experimental data is compatible with the assumption of exchange degeneracy for the trajectories and SU(3) symmetry for the vertices.

FIGURE CAPTIONS

Fig.1: Differential cross section for the process $\pi^+ p \rightarrow \pi^0 \Delta^{++}$ at 4.0 Gev/c. Experimental data are from ref.22. The full curve is the prediction of the Regge cut model obtained for the case I (without α factors). The dotted curve is the one obtained for case II (with α factors).

Fig.2: Differential cross section for the process $\pi^+ p \rightarrow \pi^0 \Delta^{++}$ at 5.0 Gev/c. Experimental data are from ref.22. The full curve is the prediction of the Regge cut model obtained for case II (with α factors).

Fig.3: Differential cross section for the process $\pi^+ p \rightarrow \pi^0 \Delta^{++}$ at 8.0 Gev/c. Experimental data are from ref.22. The full curve is the prediction of the Regge cut model obtained for case I. The dotted curve is the one obtained for case II.

Fig.4: Differential cross section for the process $\pi^+ p \rightarrow \eta^0 \Delta^{++}$ at 4.0 and 8.0 Gev/c. Experimental data are from ref.22. The full curve is the prediction of the Regge cut model obtained for case I. The dotted curve is the one obtained for case II.

Fig.5: Differential cross section for the process $K^+ p \rightarrow K^0 \Delta^{++}$ at 3.5 and 5.0 Gev/c. Experimental data are from ref.22. The full curve is the prediction of the Regge cut model

obtained for case I. The dotted curve is the one obtained for case II.

Fig.6: Differential cross section for the process $K^+p \rightarrow K^0\Delta^{++}$ at 8.0 Gev/c. Experimental data are from ref.22. The full curve is the prediction of the Regge cut model obtained for case I. The dotted curve is the one obtained for case II.

Fig.7: Differential cross section for the process $K^+p \rightarrow K^0\Delta^{++}$ at 12.7 Gev/c. Experimental data are from ref.22. The full curve is the prediction of the Regge cut model obtained for case I. The dotted curve is the one obtained for case II.

Fig.8: Density matrix elements for the process $\pi^+p \rightarrow \pi^0\Delta^{++}$ Experimental data are from ref.23. The dotted curve is the prediction obtained for both the cases I and II.

Fig.9: Density matrix elements for the process $K^+p \rightarrow K^0\Delta^{++}$. Experimental data are from ref.23. The full curve is the prediction of the Regge cut model obtained for case I. The dotted curve is the one obtained for case II.

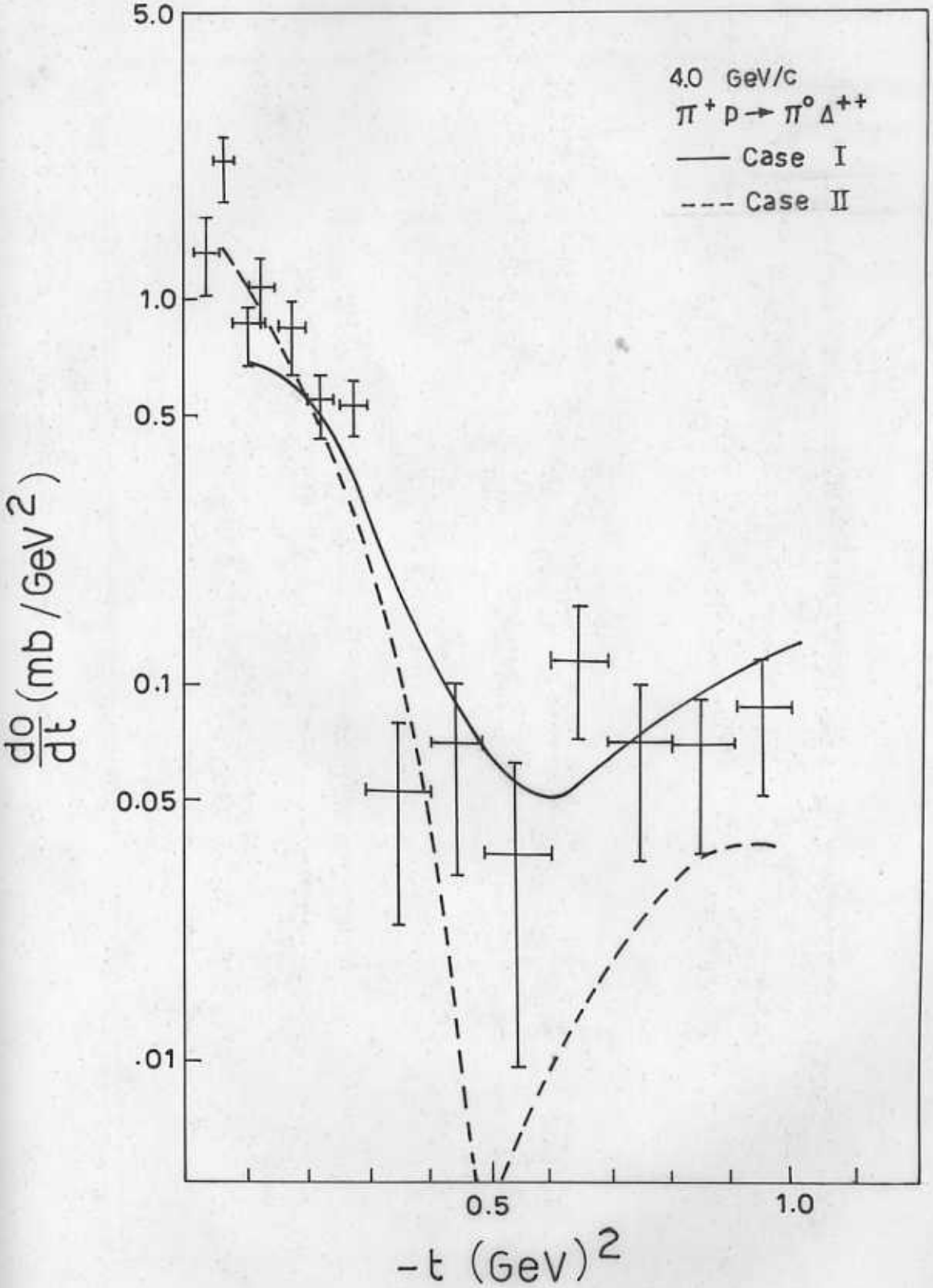

4.0 GeV/c
$\pi^+ p \rightarrow \pi^0 \Delta^{++}$
Case I
Case II
5.0
1.0
0.5
0.1
0.05
.01
$\frac{d\sigma}{dt}$ (mb / GeV^2)
0.5
1.0
$-t$ (GeV)^2

FIG. 1

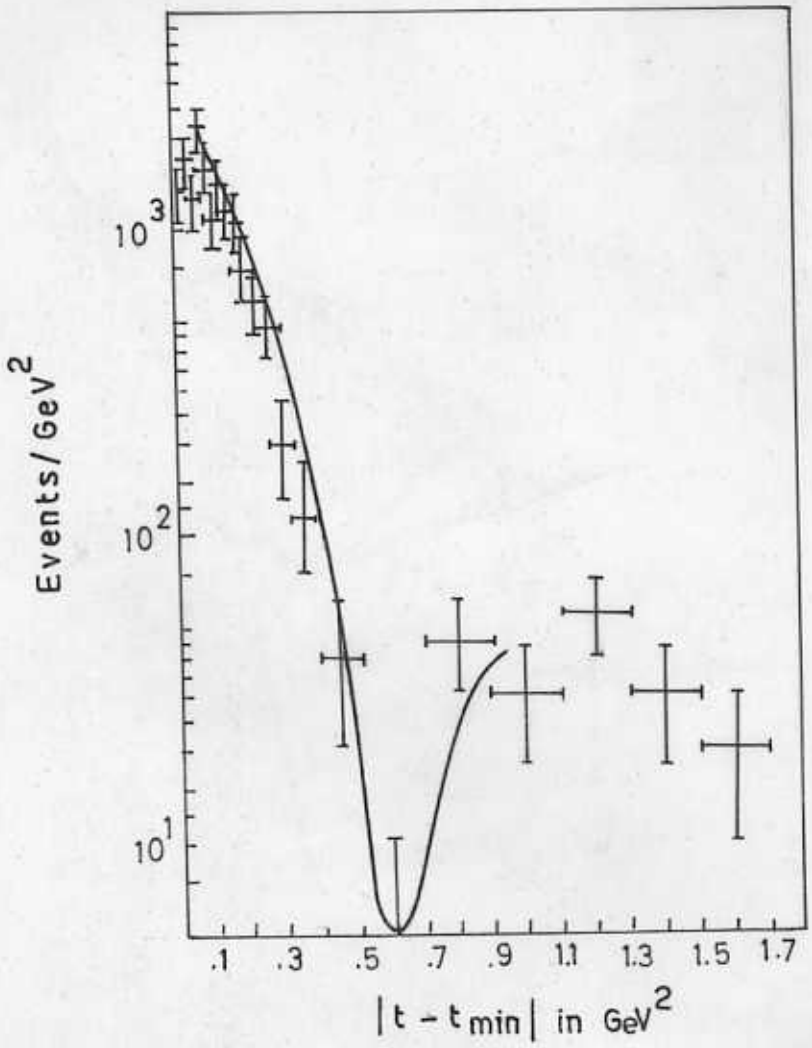

5.0 GeV/c
$\pi^+ p \rightarrow \pi^\circ \Delta^{++}$
Events/GeV^2
10^3
10^2
10^1
.1
.3
.5
.7
.9
1.1
1.3
1.5
1.7
$|t - t_{min}|$ in GeV^2

FIG.2

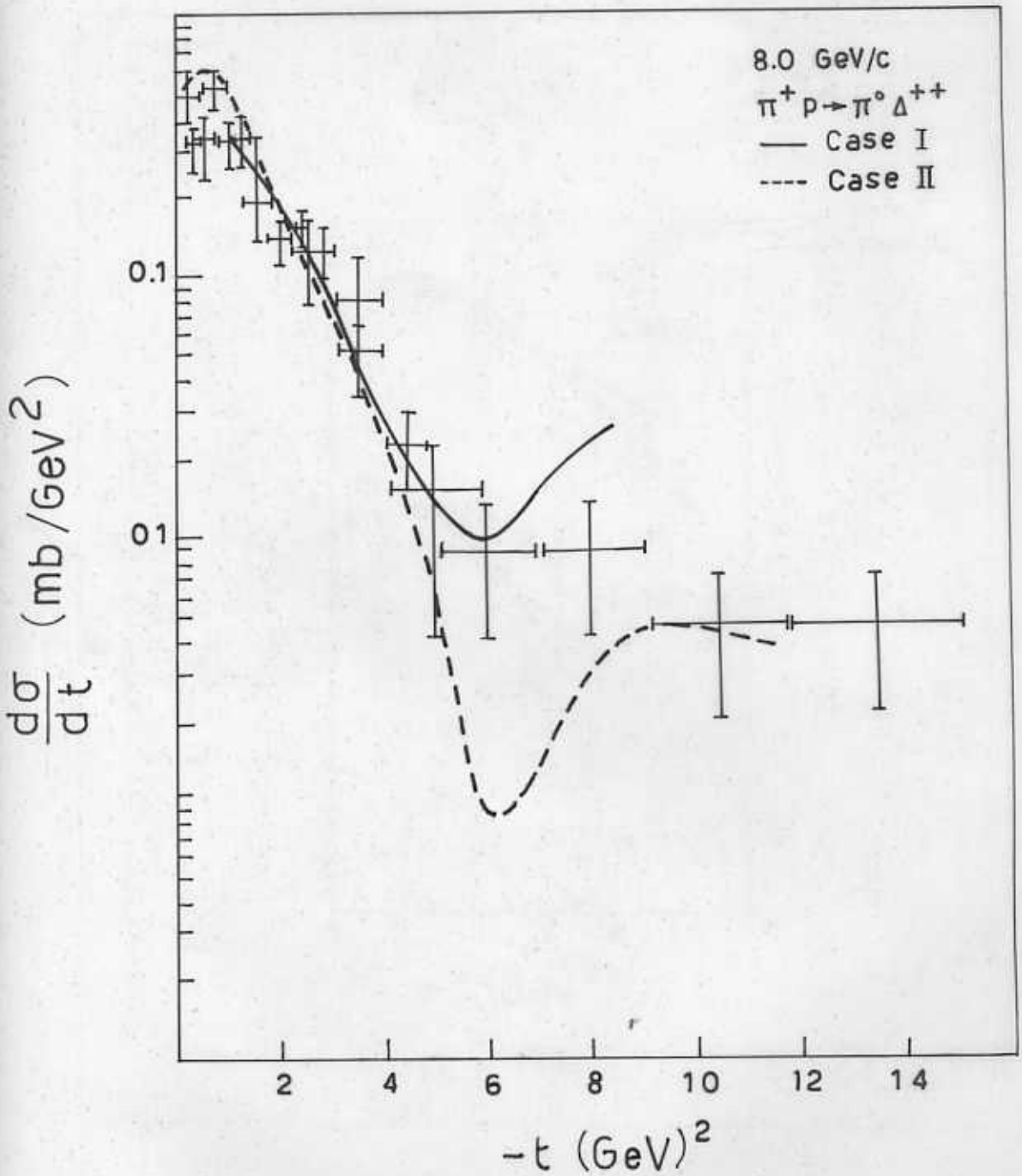

8.0 GeV/c
$\pi^+ p \to \pi^\circ \Delta^{++}$
Case I
Case II
0.1
0.1
$\frac{d\sigma}{dt}$ (mb/GeV^2)
2
4
6
8
10
12
14
$-t$ (GeV)^2

FIG. 3

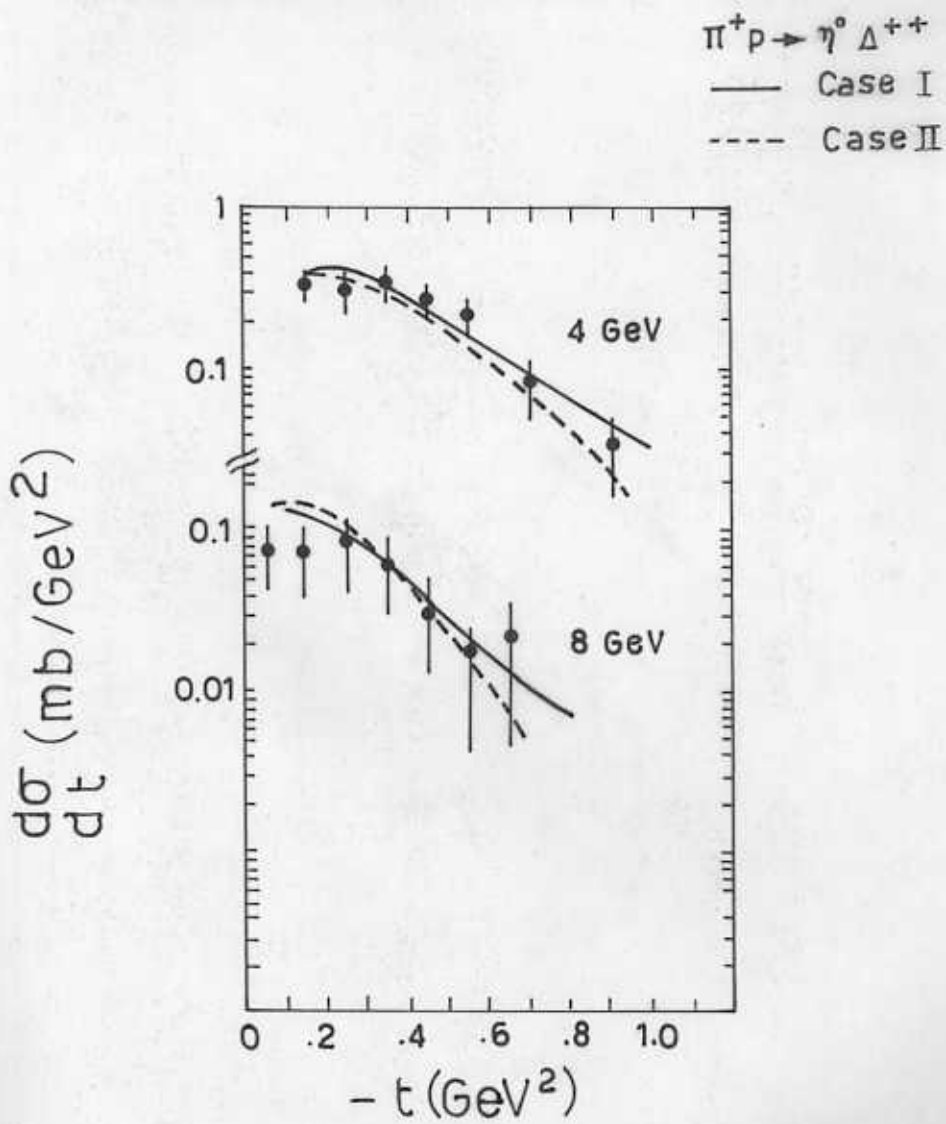

$\pi^+ p \rightarrow \eta^0 \Delta^{++}$
Case I
Case II
4 GeV
8 GeV
1
0.1
0.1
0.01
$\frac{d\sigma}{dt}$ (mb/GeV^2)
0
.2
.4
.6
.8
1.0
$-t$ (GeV^2)

FIG. 4

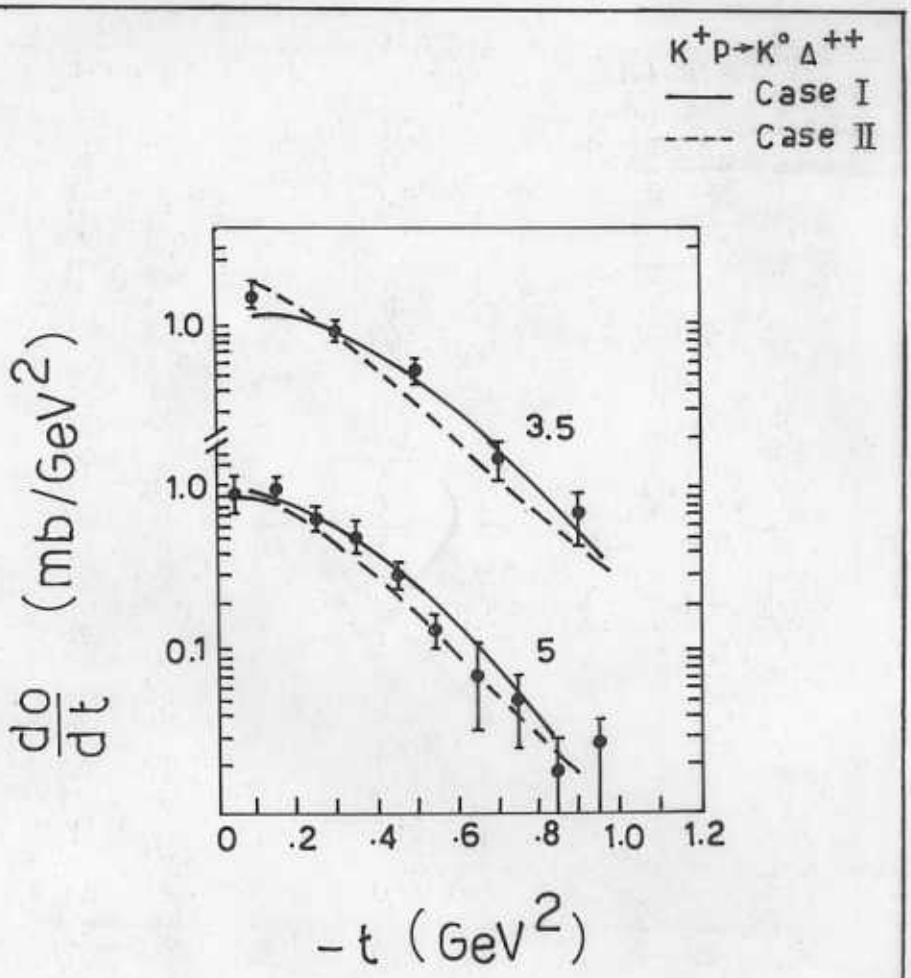


FIG. 5

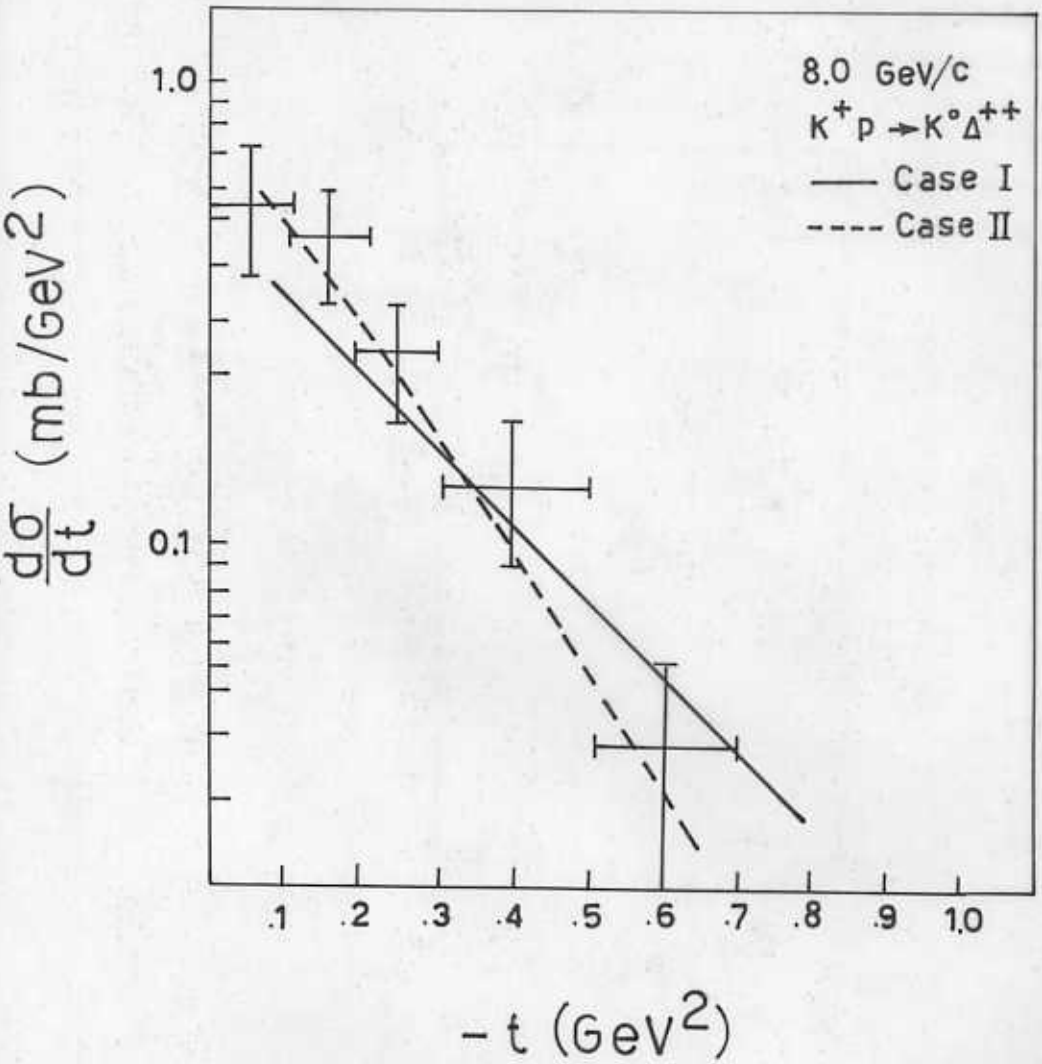

8.0 GeV/c
$K^+p \to K^\circ\Delta^{++}$
Case I
Case II
1.0
0.1
$\frac{d\sigma}{dt}$ (mb/GeV^2)
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$-t$ (GeV^2)

FIG.6

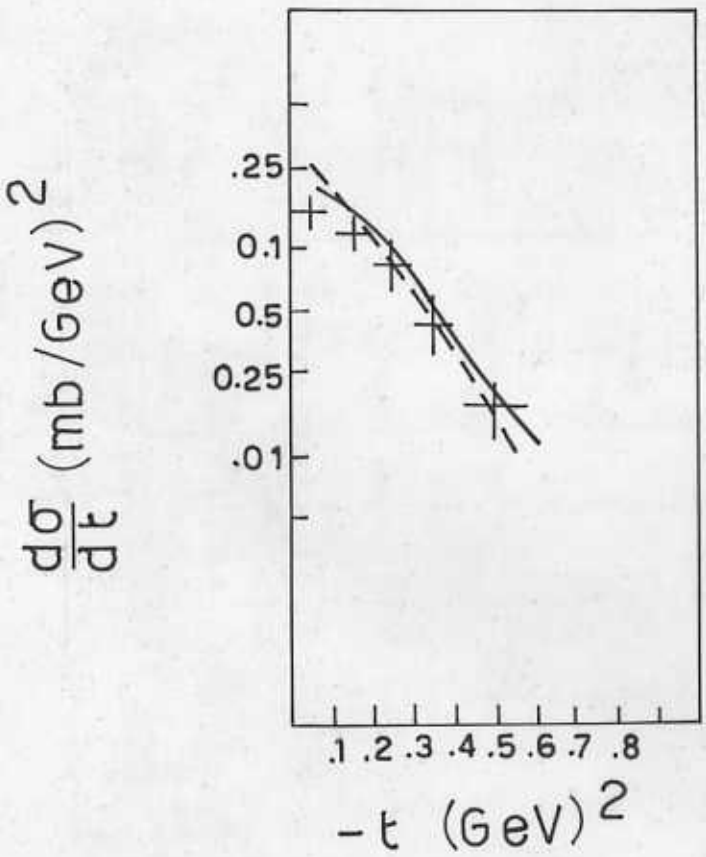
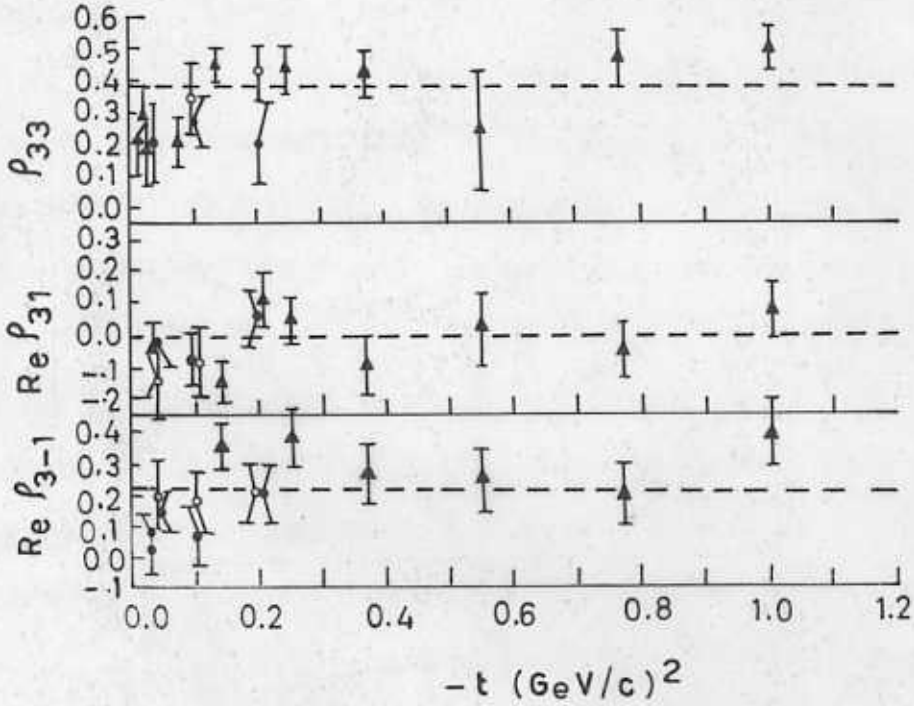

12.7 GeV/c
$K^+ p \to K^\circ \Delta^{++}$
— Case I
--- Case II
$\frac{d\sigma}{dt}$ (mb/GeV)^2
.25
0.1
0.5
0.25
.01
.1 .2 .3 .4 .5 .6 .7 .8
$-t$ (GeV)^2

FIG.7



4 GeV/c

3~4 GeV/c

8 GeV/c

FIG.8

81

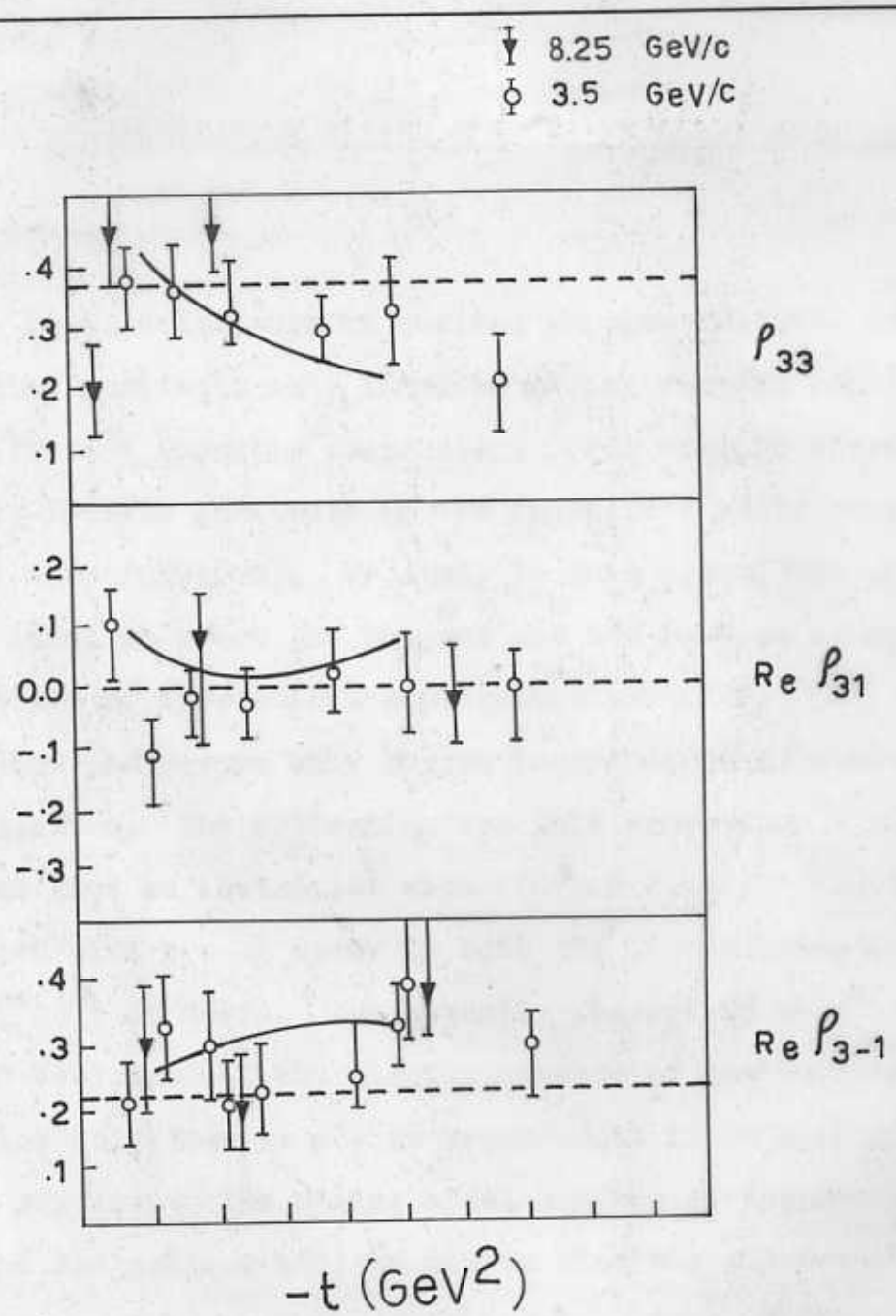

8.25 GeV/c
3.5 GeV/c
.4
.3
.2
.1
ρ_{33}
.2
.1
0.0
-.1
-.2
-.3
Re ρ_{31}
.4
.3
.2
.1
Re ρ_{3-1}
-t (GeV^2)

FIG.9

CHAPTER: V

REPRESENTATION MIXING EFFECTS IN SU(3) SYMMETRY+

1. Introduction:

It is well known in nuclear physics that the symmetry breaking manifests many times in mixing various rotation levels of different symmetry properties. (For example there is a finite D state admixture to the dominant S state deuteron ground state wave function). We study in this paper, the consequences of a model in which the baryons and the isobars belong to an admixture of irreducible representations (IRS) [8] and [10] of SU(3) and assume only charge independence of meson baryon interaction. The motivation for this comes from a simple observation that an isotriplet with Y(hypercharge) = 0 and an isodoublet with Y = -1 occur in both the lowest lying IRS [8] and [10] of SU(3). The recently discovered Roper resonance (1400 Mev) has all the quantum numbers of the nucleon and the problem therefore is now to accommodate it in a SU(3) scheme. This representation mixing model can easily accommodate it. Of course the problem will be now to find the other particles which are its partners. Using this model a study of the baryon meson

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+ Alladi Ramakrishnan, T.S.Santhanam and A.Sundaram, MATSCIENCE preprint (1967).

couplings is made. Relations are found for instance for the strong decays of isobars which have been earlier obtained in the broken SU(3) model where the breaking was introduced as a perturbation in the interaction and these relations are consistent with experiments[1].

2. The Model:

The assumption that has been invoked here is that the baryons and the isobars belong to the reducible representation

(1)

of SU(3). It is immediately apparent that for nucleons and , since they have no counterparts in the representation [10] and for , since it has no counterpart in [8]. The parameter measures the amount of mixing.

3. Strong decays:

Let us consider the strong decays of isobars in this model. The matrix element is denoted as follows:

---

1) J.Leitner, R.Musto and L.O'Raifeartaigh, Nuovo Cim. 45, 169 (1966).
See for a recent clear analysis:
G.Ebel, H.Pilkhun and F.Steiner, Nucl.Phys.B17, 1 (1969).

(2)

Then the contribution to the various relevant decays are given in the tables[2] 1 and 2.

Table 1

Decay

2) The relevant Clebsch-Gordon coefficients have been taken from J.J. de Swart, Rev.Mod.Phys. 35, 916 (1963); P.McNamee and F.Chilton, Rev.Mod.Phys. 36, 1005 (1964).

Table 2

Decay	

Eliminating the parameters, among the observed strong decays the following interesting sum rule is obtained

(3)

This sum rule has been earlier obtained by various authors[3] in broken SU(3), where the symmetry breaking was introduced in a completely different way. This sum rule is well established experimentally[1]. There are many more relations that are predicted among the other strong decays for which sufficient experimental data is not yet available.

---

3) C.Dullamond, A.J.Macfarlane and E.C.G.Sudharshan, Phys.Rev. Lett. 10, 423 (1963); V.Singh and V.Gupta, Phys.Rev. 135B, 1442 (1964); G.Bedi, E.Eberlie and M.Morpurgo, Phys.Rev. 136B, 808 (1964); M.Konuma and K.Tomozawa, Phys.Lett. 10, 347 (1964).

(8)

(9)

These relations also have been predicted in the broken SU(3) model[3]. An analysis of $\bar{K}N$ data may provide information to check many of these sum rules.

4. Baryon-Baryon meson couplings:

The baryon-baryon meson couplings are all expressible in terms of five parameters, after eliminating which the following sum rules are obtained:

(10)

(11)

(12)

(13)

(14)

(15)

(16)

The present knowledge of the coupling constants does not permit a check on these sum rules. However it should be remarked that using forward dispersion relations Lusignoli et. al.[4] have recently estimated NK and N K and found substantial deviation from exact SU(3) predictions.

5. Mass relations and magnetic moments:

The masses of all baryons are expressible in terms of six parameters and therefore no useful prediction is obtained. However in the case of electromagnetic interactions (assuming that the electromagnetic current transforms like the T_1^1

4) M.Lusignoli et. al., Phys.Lett. 21, 210 (1966). Recently Martin and Michael (CERN Preprint Th.1163, 1970) had given the value of 14.5 for the KN coupling constant by comparing the backward differential cross section for the reactions K^-n π^- and πN πN using the value 0.18 ± 0.05 (R.C.Chase et. al., Phys.Lett.30B, 659 (1969)) and = 1.4 (V.Barger, "Coral Cables Conference (1969)) we get good agreement for the sum rules (10) and (12).

component of the octet of SU(3), the following relations are obtained among the magnetic moments of baryons:

(17)

(18)

(19)

(20)

The relations (18)-(20) are those predicted by exact SU(3) of which it is well known that the relation (19) follows from just charge independence.

The same set of relations are also obtained for energy momentum mass differences.

6. Conclusion:

The model so far discussed is essentially different from the models[3] which introduce the symmetry breaking effects through a linear combination of operators. Such types of breaking the symmetry have the following undesirable features[5]:

5) H.R.Rubenstein, Phys.Lett. 22, 210 (1965).
Recently a similar model has been tried to accommodate the Roper resonance by F.Halzen and M.Konuma, preprint RIEP-69, March 1968, Kyoto, Japan. We thank Dr.G.L.Shaw for useful discussion on this point.

The assumption that the mass operator transforms like the IR [8] in the case of SU(3) yields the Gell-Mann-Okubo formula which works well for both the baryons and the mesons. On the other hand in SU(6), the simplest transformation property of the mass operator as the IR [35] or even a simple linear combination of certain representations is certainly inadequate, since for the mesons one has to assume some different linear combination of representations. One might argue that similar uncertainty is there in the parameters characterising the mixing of the representation. However it is hoped that a critical analysis of various experimental informations may be used to fix these parameters approximately. The method is not altogether strange since we are already familiar with mixing[6] and is quite similar to the 'configuration-mixing' in nuclear spectroscopy.

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6) J.J.Sakurai, Phys.Rev. 9, 472 (1962).
S.Okubo, Phys.Lett. 8, 163 (1963).

CHAPTER: VI

INFINITE MOMENTUM LIMIT AND THE ALGEBRA OF CURRENTS+

Recently there have been some attempts[1,2] to find formal solutions of the commutation algebra of the Fourier transform of the current densities using the technique. In particular, Barnes and Kazes[2] have considered the algebra of vector current densities (actually their Fourier transform) with SU(2) as the internal symmetry algebra. By approximating the matrix elements of the commutator with single particle diagonal matrix elements and using the limit, they obtain the isovector form factors at finite momentum. In this chapter, we analyse their results in an arbitrary Lorentz frame and conclude that the Cabibbo-Radicati sum rule[3] is independent of the Lorentz frame[4] and one always gets only a harmonic -dependence for the form factors no matter what momentum limit one employs so long as a finite number of diagonal single particle matrix elements alone are taken into account.

+ T.S.Santhanam, A.Sundaram and K.Venkatesan, preprint MATSCIENCE (1967).

1) R.Dashen and M.Gell-Mann, Phys.Rev.Lett. 17, 340 (1966).

2) K.J.Barnes and Z.Kazes, Phys.Rev.Lett. 17, 978 (1966).

3) N.Cabibbo and L.A.Radicati, Phys.Lett. 19, 697 (1966).

4) C.G.Bollini and J.J.Giambiagi, Nuovo Cim. 21, 107 (1961), I.Saavedra, Nucl.Phys. 74, 677 (1965).

Following Barnes and Kazes, we consider the one dimensional commutator algebra of vector currents

(1)

(2)

where

(3)

The are the vector current densities (fourth component) and i refers to the isospin index. Differentiating eq.(2) with respect to and taking the limit we get the Heisenberg type equation

(4)

with the well known iterated solution

(5)

This is the relation one gets using the algebra.

Consider the following matrix element of between the nucleon states which form the approximate repre-

sentation of the algebra

(6)

By using the Foldy-Wouthysen transformation one can write eq.(6) as

(7)

where

(8)

and

(9)

with

(10)

and

(11)

The is related to the angle appearing in the Lorentz trans-

formation

(12)

(13)

as

(14)

(see ref.(4) for instance). Hence may be represented by[5)]

(15)

so that

(16)

(17)

5) It is clear that is a function of q only when p=0 or p ∞. However if we look at the first few derivatives, it becomes immediately obvious that v's are functions of only q^2 only when we go to the frame p ∞.

Substituting eq.( ) in eq.( ) one easily finds[6)]

(18)

(19)

which as can be verified goes to the Barnes-Kazes form for p ∞. From (10) one can get

(20)

Actually eq.(11) is independent of any momentum limit. From eq.(11) by reexpressing in terms of the Saachs form factors and differentiating with reference to q^2 one can get the Cabibbo-Radicati sum rule without the continuum term.

One can easily see that for small values of p and q, the zero of is pushed up to $q^2 = 30.7\ f^2$ from around $15\ f^{-2}$. However the zero of is brought down. A

6) The presence of p dependent term in the form factors is essentially due to the single particle approximation we use. For example the matrix element $\langle p|V_1 V_1|p'\rangle$ consists of two Feynman diagrams which we have approximated by a single Feynman diagram. When we take the infinite momentum limit, the cross diagram does not contribute so that the p dependent term naturally drops out from the form factors. We thank Professor Zaccariasen for discussions on this point.

detailed comparison is not worth while since in any case one can get only sine or cosine forms or perhaps Bessel functions. The only way, perhaps, to get a more realistic exponential q^2 dependence is to go beyond the single particle matrix elements or to take infinite superposition of single particle matrix elements.

CHAPTER: VII

ON VECTOR DOMINANCE MODEL AND ISOBAR PHOTOPRODUCTION+

The idea that the electromagnetic interactions of hadrons are mediated by vector mesons is very attractive in that it yields a relation between the photoproduction processes and the strong interaction processes. From the knowledge of the strong interaction process we get a prediction for the photoproduction process. The available data for the processes [1] and [2] seem to support the vector-dominance model.

However there are certain ambiguities that are inherent in the vector-dominance model. Since the photons are transversely polarized, one needs to know the density matrix elements for the transverse vector mesons in order to apply the vector-dominance model++ This brings in an ambiguity about the choice of reference frame with respect to which the density matrix elements for the transverse vector mesons are calculated. Frass and Schildnecht[3] have made attempts to

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+) A.Sundaram, Nuovo Cim. 58A, 905 (1968).

1) H.Joos, Phys.Lett. 24B, 103 (1967); K.Kajantic and J.S.Trefil, ibid 24B, 106 (1967); See for a recent survey G.Wolf, DESY Report 70/64 (1970).

2) A.Dar and V.F.Weisskopf, Phys.Rev.Lett. 20, 762 (1968); A.Dar and V.F.Weisskopf, C.A.Levinson and H.J.Lipkin, ibid 20, 1261 (1968).

3) H.Frass and Schildnecht, Nucl. Phys. B6, 395 (1968).

++) herein after referred to as VDM.

justify the view that the values of the density matrix elements of the transverse vector mesons used in the VDM calculations are those measured in the helicity frame. However use of the s-channel helicity frame yields disagreement with experiment in the case of production of charged pions by polarized photons. On the other hand Bialas and Zalewski[4] have succeeded in obtaining VDM predictions which are in accord with the experimental results in the above case, provided they use values of the density matrix elements measured in the Donohue-Hogassen frame.

The extrapolation of the vector meson mass to zero is another questionable procedure that is used in conjunction with VDM. Moreover, the strong interaction data are usually available in the form of differential cross sections. Nothing is known about the relative phases of the amplitudes so that the magnitude of the interference term is not correctly specified. Also in certain cases, analytic continuation of the scattering amplitude from the s-channel to the u-channel has to be considered and so some assumptions have to be made about the effect of crossing from s-channel to u-channel.

In this chapter we study the process in the Regge pole model using the vector dominance assumptions because in this treatment the crossing from the s-channel to u-channel and filtering out the longitudinal components of

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4) A.Bialas and K.Zalewski, Phys.Lett. 28B, 436 (1969).

the vector mesons can be easily accomplished. In section 2 we attempt a simple derivation of the vector dominance model and in section 3 we apply it to the process and discuss the results.

2. The large value of the photoproduction cross section is a significant feature of the study of high energy photoproduction processes. From this fact one gets the idea that the meson is essential for the coupling of electromagnetic field to the hadrons. However it has been stated that vector bosons which are associated with the isospin, baryon and hypercharge currents are universally coupled to all the terms in the corresponding currents. At the present time there is a nonet of vector bosons with spin 1 and negative parity. It is generally assumed that these nine mesons arise from an SU_3 octet and an SU_3 singlet. The hypercharge zero isosinglet of the octet is mixed with the SU_3 singlet to form the physical and mesons. Thus it is the superposition of the and mesons which are assumed to be universally coupled to the hypercharge and baryon currents. Now the following is the simplest form of a relation between the electromagnetic current. of the hadrons and the phenomenological field of the mesons or more general

the field of the vector mesons, , and [5]:

(1)

where, denotes the masses of the vector mesons. The constants are partly determined by the symmetry properties of the e.m. current and the vector meson.

In fact the assumptions that

a) the photon is the spin singlet of an SU_3 octet,

b) the physical and are defined by

(2)

where and are respectively the I=0 members of an SU_3 singlet and octet, and that

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5) Some of the papers on the VDM are:
J.J.Sakurai, Ann.Phys.11, 1 (1960); M.Gell-Mann, Phys.Rev. 124, 953 (1961); M.Gell-Mann, Phys.Rev.125, 1067 (1962); J.J.Sakurai, Proceedings of the International School of Physics "Enrico Fermi" Course XXVI (1963); Y.Nambu and J.J.Sakurai, Phys.Rev.Lett.8, 79 (1962); M.Gell-Mann, D.Sharp and W.G.Wasner, Phys.Rev.Lett.8, 261 (1962); R.F.Dashen and D.H.Sharp, Phys.Rev.133B, 1585 (1964); L.Stodolsky, Phys.Rev.134B, 1099 (1964); G.Barton and B.G.Smith, Nuovo Cim.36, 436 (1965); M.Ross and L.Stodolsky, Phys.Rev.149, 1172 (1966); D.S.Beder, Phys.Rev.149, 1023 (1966); N.M.Kroll and T.D.Lee and B.Zumino, Phys.Rev. 157, 1376 (1967).

c) the vertex is invariant under SU(3) lead to a sum rule for the coupling constant

(3)

The mass formula gives the mixing angle

(4)

using the mixing angle we obtain

6)

(5)

equation ( ) implies the following relationship between the T matrix elements for the reaction

(6)

and the T matrix elements for the reaction

(7)

(8)

---

6) Modifications due to symmetry breaking change the ratio : : to 9:0.65:1.33 (R.J.Oakes and Sakurai, Phys.Rev.Lett.19,1266(1967)) or 9:1.21:1.34 (T.Das, V.S.Mathur and S.Okubo, Phys.Rev.Lett.19, 470 (1967))

If one sets $k^2=0$, one obtains the following relationship between both the photoproduction process and the corresponding strong interaction process for the transversely polarized vector bosons

(9)

Taking the absolute square of the above amplitudes we get+)

(10)

3. Application of the vector-dominance model to

p $\pi^-\Delta^{++}$ yields the relation

(11)

+) Because the meson is weakly coupled to nonstrange mesons we ignore the term.

where are the vector mesons , and . We shall evaluate the left hand side in a Regge pole model assuming the dominance of and A_2 trajectory[7].

Now the Regge pole amplitude for the process and the physically observed process is the same except for a change to . Apart from kinematical factors we shall write the proportionality relation

(12)

where and are t-channel helicity amplitudes for reactions N $\pi\Delta$ and πN Δ respectively, (h) denotes the particular helicity configuration. The reason formula (12) can hold is that both and A_2 trajectories have parity $(-1)^j$. Because of this property their coupling to πV is unique and automatically gauge invariant. Moreover they contribute to those amplitudes for which the helicity of is ± 1[8]. Notice this simple justification of relation (12) does not apply in particular to π exchange.

In terms of π and A_2 exchanges, the differential cross section for the process π^+p Δ^{++} [9] reads

7) We essentially adapt the same procedure followed by B.Diu and M.Le Bellac, Phys.Lett.24B, 416 (1967).

8) From the a theorem due to Gottfried and Jackson(Nuovo Cim. 33, 309 (1964) if a meson (resonance) with natural parity is produced from an incident pseudoscalar meson through a t-channel exchange, then a natural parity exchange cannot contribute to those amplitudes making up.

9) L.L.Wang, Phys.Rev.153, (1967).

are reduced residue function obtained by removing all threshold factors, so that they vary smoothly with t. Though the pion trajectory is a low lying trajectory compared to and A_2, it contributes significantly in the forward direction because of the proximity of the pion pole to the physical region. A recent analysis of Ader et. al.[10] shows that for the process $\pi^+ p$ Δ^{++}, the natural parity exchange contribution dominates over the unnatural parity contribution

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10) J.P.Ader, M.Capdeville, G.Cohen Tannoudji and Ph.Salin, CERN Report Th.856 March (1968).

away from the forward direction as the energy increases. For instance the density matrix elements for the decay decreases rapidly away from the forward direction. For incident pion momenta 3-4 Gev[11] = 0.826 in the range $0 < -t < .1$ and is equal to 0.276 slightly above $t = -0.2$ Gev^2. So we assume the pion contribution is dominant upto $t = -.2$ Gev^2 and then A_2 takes over.

Now eqn.(12) can be written in terms of reduced residue functions defined after equation (13). For the case of the isovector photon we get the relation

(14)

using the above relation we get the isovector photon contribution. Similarly from the Regge pole amplitude for taking into consideration only those amplitudes for which the vector meson helicity is ± 1 and multiplying by , the isoscalar photon contribution could be obtained. Adding these two contributions we get

(15)

11) D.G.Brown, Ph.D.Thesis, UCRL report 182564 (May 1968); D.G.Brown, G.Cridal, R.W.Birge, R.Bacastow, S.Fung, W.Jackson and R.Pu, Phys.Rev.Lett. 19, 664 (1967).

In the above expression we have ignored the interference term. The photon-vector coupling constant can be obtained from leptonic decays[12] and and we take the value = 0.45. In the energy range 3 5.8 Gev, the relation was tested and agreement with experiment is found. However recently experimental data for the process p $\pi^-\Delta^{++}$ are available from 2-16 Gev/c[13]. Comparison of the VDM prediction at higher energies suggests that the interference terms should be taken into account.+) Recently Gotsman[14] has studied the isobar photoproduction process in the VDM model. He made a Regge pole fit to the strong interaction reactions π^+p Δ^{++} and π^+p Δ^{++}. Assuming the exchange of π, A_1, and A_2

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12) S.C.C.Ting, Proc. Intern. Symposium on Electron and Photon Interaction at High Energies, Stanford, California (1967). We take the meson decay width to be 130 Mev.

13) A.M.Boyarski et. al., Phys.Rev.Lett. 22, 148 (1969).

14) E.Gotsman, Phys.Rev.186, 1543 (1969).

+) Since the isoscalar contribution is only 10% of the isovector contribution it is neglected. Recently Ader et. al. (J.P.Ader, M.Capdeville, Nucl.Phys. B17, 127 (1970)) have studied the process p $\pi^-\Delta^{++}$ taking only the π^- and A_2 contributions assuming π contribution to be small for $|t| < .3$ Gev^2 and only A_2 trajectory contributes in that region. This in a way justifies our approximation in taking the just term of (15) for comparison with the experimental data. But we find from the recent experimental data, there is a discrepancy between the theoretical prediction and experimental data.

trajectories for π^+p Δ and the exchange of B and trajectories for π^+p Δ, a reasonable fit to the existing data has been obtained. He also considered the continuation of the vector meson mass to zero mass assuming Gottfried-Jackson frame for the transverse helicity amplitudes.

The experimental data on the charge symmetric reaction n $\pi^-\Delta^+$ [15] have recently been made available. By taking the combination the interference term can be annihilated and in this case the following relation is obtained[16]

(16)

This result is in disagreement with experimental data+) if we

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15) A.M.Boyarski et. al., Proc. of the International Symposium on Electron and photon interactions at High Energies, Liverpool, England, September 1969, ed. by D.W.Braben, (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England (1970)).

16) A.Dar and V.F.Weisskopf, CERN Report No.Th.934, 1968(unpublished); A.Dar, Nucl. Phys.B11, 634 (1969); C.Iso and H.Yoshii, Ann.Phys.47, 424 (1968).

+) Also experimentally it is found that the cross section for the process n $\pi^+\Delta^-$ seems to be greater than that of p $\pi^-\Delta^{++}$ much in contrast to the prediction of Dar(ref.16)

use 0.5 and use the density matrix element for transverse vector mesons evaluated in the helicity frame (see fig.).

Thus application of VDM to the process p $\pi^-\Delta^{++}$ suggests that there is a large interference between the trajectories of opposite signature. Also the predictions of the VDM, involving charge symmetric reactions seem to be in disagreement with experiment+). However there are difficulties in the cross section determination for π^+p Δ^{++} since both the and Δ have large widths[17]. Better data for π^+p Δ^{++} would of course throw much light on the model.

+) Recently W.Schmidt and D.R.Yennie, Phys.Rev.Lett.23,623 (1969) showed that several experiments involving photon e interactions with nucleons or nuclei are in disagreement with the predictions of the VDM. They suggested that these discrepancies may be removed either by changing the structure of the electromagnetic current (field current identity) or by allowing for a mass dependence in the vector-meson amplitudes.

17) M.Walter, Deutsche Akademie der Wissenschuften Zu Berlin-Zeuthen Report No.PHE 69-1 (unpublished).

4. As another application of the vector dominance model, we evaluate the coupling constant[+]. This coupling constant appears as a form factor in the process

(17)

various estimates have been given for this coupling constant. Dispersion theoretic calculations[18] yield a value of the order of unity while Donnachi and Shaw[19] obtain a value = 0.04 ± 0.15 from the analysis of photoproduction data. A.Chatterji[20] using "hard pion" techniques has obtained a value of 0.03. Here we evaluate the coupling constant using the Veneziano model[21] and invoking the vector dominance assumption.

+) A.Sundaram, Prog.Theo.Phys. 43, 843 (1970)

18) H.Wong, Phys.Rev.Lett. 5, 70 (1960); S.Okubo and B.Sakita ibid 11, 60 (1963); K.Kawarabayashi, M.Suzuki, ibid 16, 255 (1960); 16, 384 (1960).

19) A.Donnachi and G.Shaw, Ann. of Phys. 37, 333 (1966)

20) A.Chatterji, Phys.Rev. 170, 1578 (1968).

21) G.Veneziano, Nuovo Cim.57A, 190 (1968)

Also see for application of the model:

C.Lovelace, Phys.Lett. 28B, 265 (1968)

K.Kawarabayashi, S.Kitakado and H.Yabuki, Phys.Lett. 28B, 462 (1969)

H.Goldberg and Y.Srivastava, Phys.Rev.Lett. 22, 749,1340 (1969)

K.Igi, Phys.Lett.28B, 330, (1968)

M.Jacob, CERN Report Th.1010. Lectures delivered at the VIII Internationale Universitats Wochen fur Kernphysik der Universitat Graz Schladming 24 Feb - 8 March 1969.

Now the amplitude for the process (17) is of the form

(18)

where is the photon polarization vector and are the four momenta of the three pions. are the isospin indices. The coupling constant is related to F(s,t,u) by

(19)

By using the vector dominance model one can relate the process (17) to the process $\pi\pi$ π , as the isoscalar photon can alone contribute for the process (17). Now to obtain the amplitude for the process (18) we use the Veneziano model since this model possesses Regge behaviour at high energy, crossing symmetry and 'duality'.

Thus the amplitude for the process $\pi\pi$ π in the Veneziano model[21] corresponding to the exchange of meson is given by

(20)

where B is the Euler function

(21)

and is the trajectory function given by = + 0.88
The constant is determined by comparing (20) with the elementary exchange contribution and equating the residues+)

(22)

In the above expression = 0.88 Gev^{-2}. Now the application of the vector dominance assumption to the process (17) yields the relation

(23)

where is the direct photon-omega coupling using (19),

+) The and couplings are defined through

(21) and (23) we obtain the relation

(24)

We take for the value 6.5×10^{-2} e[22] and =

Setting = 115 Mev[23] and = 17 Gev^{-1} [24] we determine

= 0.12

which is in agreement with the value obtained by Donnachi and Shaw viz.,

$$= 0.04 \pm 0.15 \qquad (25)$$

However it should be mentioned that in writing down the Veneziano amplitude for the process (17) the following

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22) S.C.C.Ting in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September 1968 (CERN, Scientific Information Service, Geneva, Switzerland) p.43

23) J.E.Augustin et. al., Physics Letters 28B, 503 (1969). This width corresponds to a value of about 1.8 for

24) This value when used in the Gell-Mann-Sharp-Wasner formula (M.Gell-Mann, D.Sharp and W.G.Wasner, Phys.Rev.Lett. 8, 261 (1962) yields the observed value for

difficulty arises[+]. In order to eliminate undesirable poles with even angular momenta, Veneziano required, for the process $\pi\pi$ π , the condition

(26)

which however becomes in the present case (the process (17))

since (27)

Nevertheless we did the calculation using the Veneziano model just to see what value for the coupling constant one obtains by using the Veneziano model.

Iroshinikov et. al. used[25] Virasaro[26] amplitude for the process $\pi\pi$ π in order to avoid the above difficulty.

+) However, the Veneziano-Lovelace formula (ref.20) for $\pi\pi$ scattering gives without Veneziano's constraint (s) + (t) + (u) = 1, reasonable predictions for S wave I=0 and I=2 scattering lengths and with Veneziano's constraint it fails; more over the Veneziano condition cannot be satisfied in any reaction because it depends strongly on the masses of the external particles involved (see H.Caprasse, Lett. Nuovo Cim. 2, 101 (1969)), Miyato (Prog.Th.Phys.Vol.45, 984 (1971)) with the help of secondary terms has constructed Veneziano-like amplitudes with Veneziano-like constraints (s)+ (t) + (u) = constant (not necessarily integers).

25) G.S.Iroshinikov, Y.P.Nikitau and A.S.Chernov, Soviet Physics JETP LETTERS 10, 95 (1969).

26) M.A.Virasaro, Phys. Rev. 177, 2309 (1969).

The value obtained by them for the coupling constant is of the order of unity in agreement with the authors of ref. But it has been pointed out[27] that this form of the amplitude leads to a contradiction when applied to the process $\pi\pi - \pi N$ which is known to be dominated by the process (17).

Recently Murtaza and Harun-ar-Rashid[28] used the amplitude given by Cooper[29] who has employed an elegant spurion technique to write down the Veneziano-type amplitude for the process (17) by considering the generalized Veneziano representation for the process $K\bar{K}$ 3π given by Barda Kci and Ruegg[30]. In this model the amplitude for the process

(28)

(where is the isoscalar current) is given by $A(s,t,p_1^2)$ + five other cyclic terms in s,t and u width

(29)

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27) I.Raszillier and D.H.Schiller, Academia Republicu Socialiste Romania Report, Institutul De Fizica, Bucharest 1969 (unpublished).

28) G.Murtaza and A.M.Harun-ar-Rashid, Phys.Rev.D2, 236 (1970)

29) F.Cooper, Phys. Rev. D1, 1140 (1970)

30) K.Barda Kci and G.Ruegg, Phys.Lett. 28B, 342 (1968).

The amplitude for the process π $\pi\pi$ is obtained by taking $p_1^2 = m$, whereas the amplitude for the process (17) is obtained if we set $p_1^2 = 0$. The amplitude may be written as an infinite sum of Beta functions and this helps in avoiding the contradiction obtained in the case of Virasaro amplitude. By considering the exchange they also obtain the value for the coupling constant to be of the order of unity.

FIGURE CAPTIONS

Fig.1: Diagram for vector meson dominance model for the process p $\pi^-\Delta^{++}$.

Fig.2: Differential cross section for the process p $\pi^-\Delta^{++}$ at 3 5.8 Gev/c. experimental data are from German Bubble Chamber Collaboration Phys.Lett.23, 707 (1966). o predicted value using relation (3) at 4 Gev/c; the data for π^+p Δ^{++} being taken from A.B.B. H.L.M. Collaboration, Phys. Rev. 138B, 897 (1965); predicted value using relation (3) at 3 5 Gev/c the data for π^+p Δ^{++} being taken from D.G.Brown, Ph.D. Thesis,UCRL Report 182564 (1968).

Fig.3: Vector dominance comparison of Δ production. The data has been taken from ref.15.

FIG.1

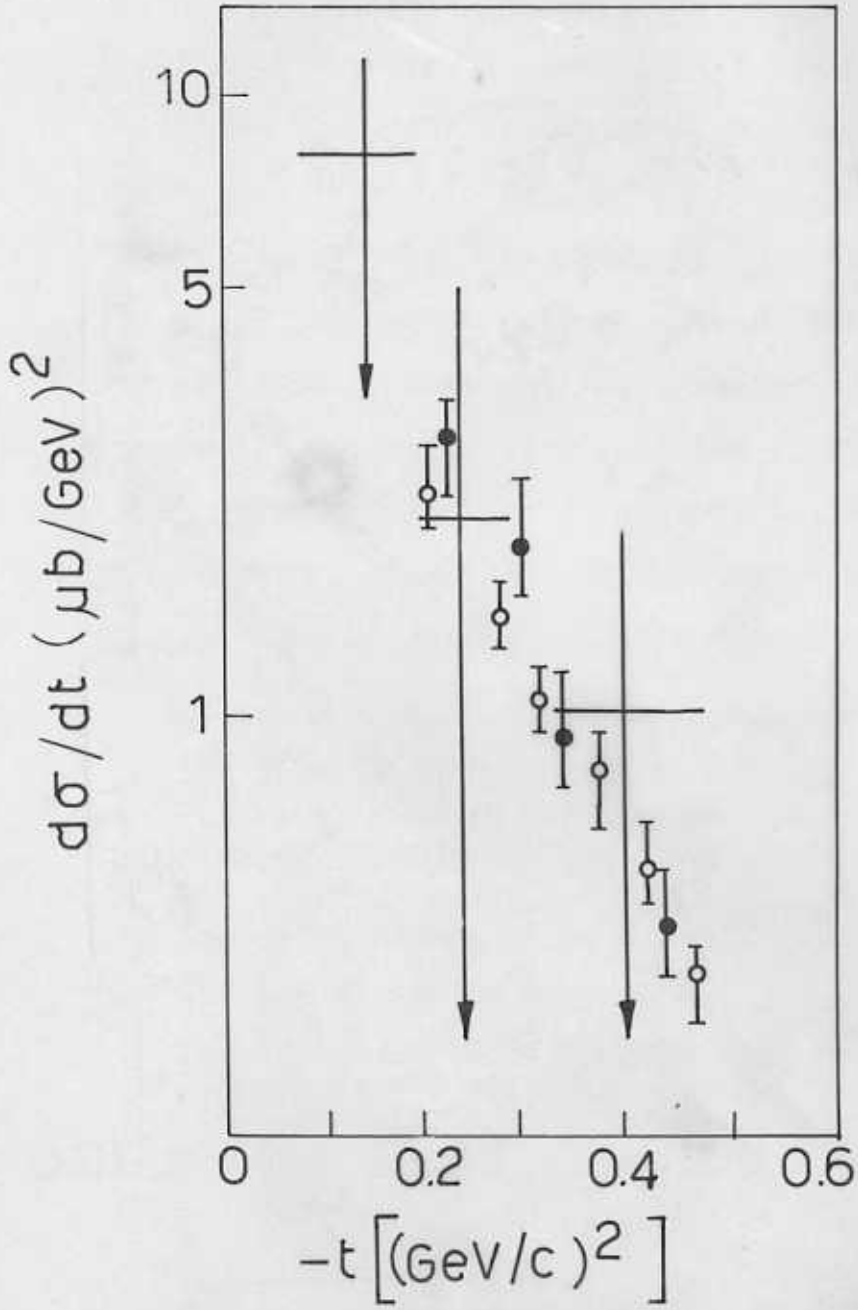

FIG.2

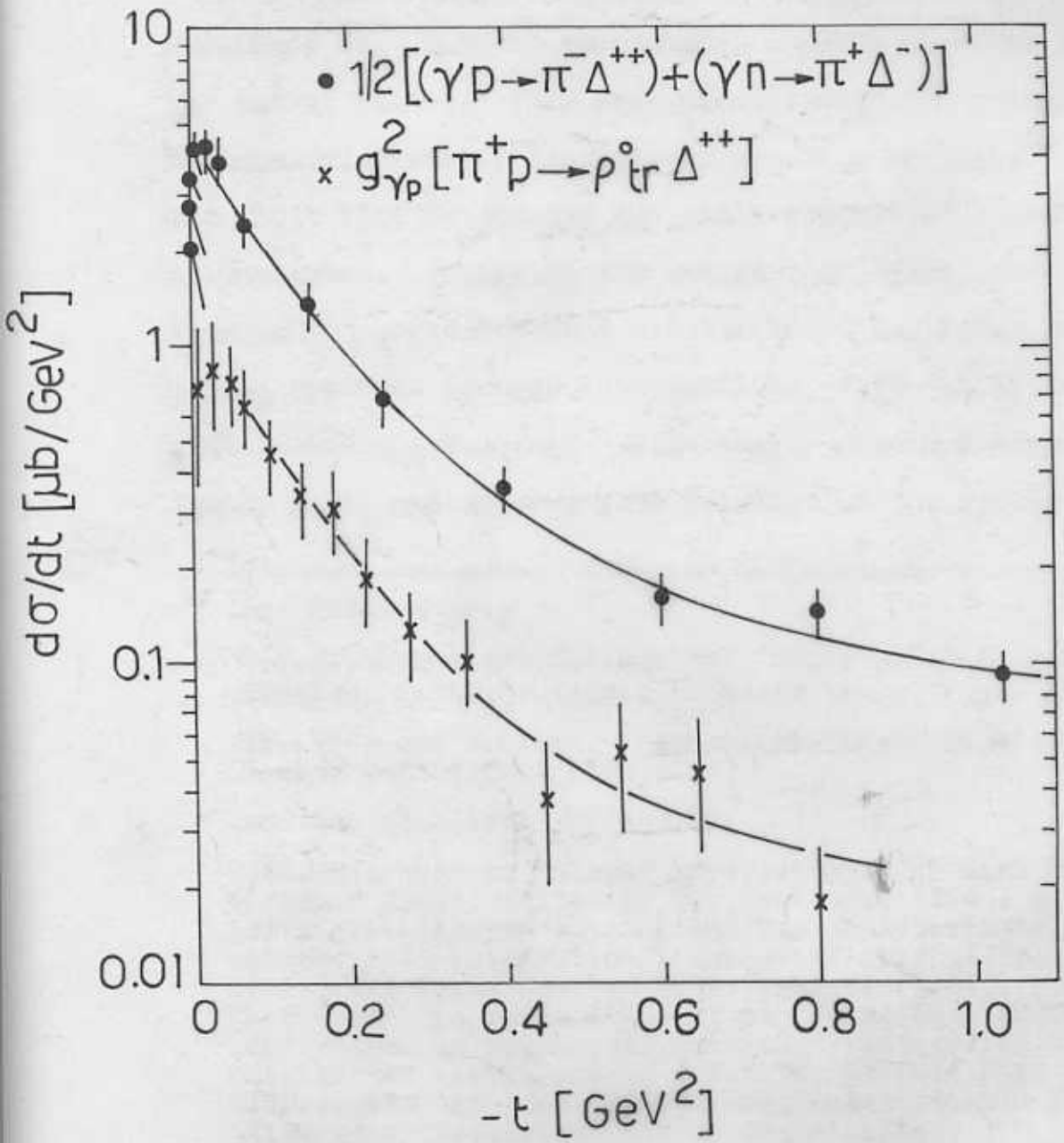
• 1/2[(γp→π⁻Δ⁺⁺)+(γn→π⁺Δ⁻)]
× g²γp[π⁺p→ρ°tr Δ⁺⁺]
dσ/dt [μb/GeV²]
10
1
0.1
0.01
0
0.2
0.4
0.6
0.8
1.0
-t [GeV²]

FIG.3

APPENDIX A:

REGGE PHENOMENOLOGY

Here we give some relevant extracts from the lore of Regge phenomenology+. Regge poles are poles of the scattering amplitude that move in the complex angular momentum plane as the energy varies. They were discovered Mathematically in the non-relativistic Schrodinger equation by Regge[1] who for the first time introduced the whole concept of complex angular momentum. Following the analysis of Regge, Chew and Frautschi[2] suggested that all the known particles and resonances could be grouped into families, each family being associated with a given Regge trajectory. A spin J member of the family would appear as a manifestation of the passage the Regge

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+) For details see:

P.D.B.Collins and E.J.Squires, Regge poles in particle physics, Springer Tracts in Modern Physics 45, 1968.

V.Barger and D.Cline, 'Phenomenological theories of High Energy Scattering' (Benjamin)

and the excellant reviews by:

B.M.Udgaonkar in "Strong interactions and High Energy Physics" (Plenum Press) edited by R.G.Moorhouse (1964) p.223; R.J.N.Phillips in 'Strong and Weak interactions', Proceedings of the 1966 international school of physics 'Ettore Majorana' (Academic Press) edited by A.Zichichi (1966), p.268; L.Betrochi in Proceedings of the Heidelberg International Conference on Elementary Particles (Interscience) ed. H.Filthuth (1968) p.197; G.E.Hite, Rev.Mod.Phys.41, 669(1969); J.D.Jackson in Proc. of the Lund International Conference on Elementary Particles, ed.Von Dradel (1969).

1) T.Regge, Nuovo Cim.14, 951 (1959); ibid 18, 947 (1960).

2) G.F.Chew and S.C.Frautschi, Phys.Rev.Lett. 7, 394 (1961); S.C.Frautschi and M.Gell-Mann and F.Zachariasen, Phys.Rev. 126, 2204 (1962).

trajectory (t) through an integral or half integral value of J. Actually only alternate integer or half integer values of J along a given trajectory would appear as particles or resonances owing to the concept of "J parity". The square of the mass of the particle of spin J would be the value of t for which Re (t) = J.

Regge phenomenology, in recent years has acquired sophistication. The Regge pole model has had its ups and downs in explaining the experimental data. But the power law behaviour of the Regge pole amplitude still continues to be attractive. Let us see how the characteristic Regge pole contributions are found.

Let us consider non-relativistic spinless equal mass scattering. The scattering amplitude is written as

(1)

where is the partial wave amplitude, E is the energy, is a Legendre Polynomial and with the c.m. angle. Then if we generalize and to be analytic functions of a complex variable L, the summation can be changed into a contour integral (Sommerfeld-Watson transformation)

(2)

The contour shown in fig.1 surrounds the integer points 0,1,2 ..... along the positive real axis, the vanishing of sin πL gives poles at these points; by Cauchy's theorem the integral equals $-2\pi i$ times the sum of the residues and we recover equation (1). Any other poles on the real axis coming from are avoided as at point A. is closed at .

Suppose we now deform continuously into another contour which runs parallel to the imaginary axis passing through and closed by a semi circle at . The integral stays the same if no new singularities are enclosed, if there are any poles of in the way, as at A and B, we exclude them by extra loops in . The resulting expression for is

(3)

In obtaining (3) we have used the fact that the continuation of can be made in such a way that contribution from infinite semi-circular contour vanishes. The straight section of yields a "back ground" term. From the closed

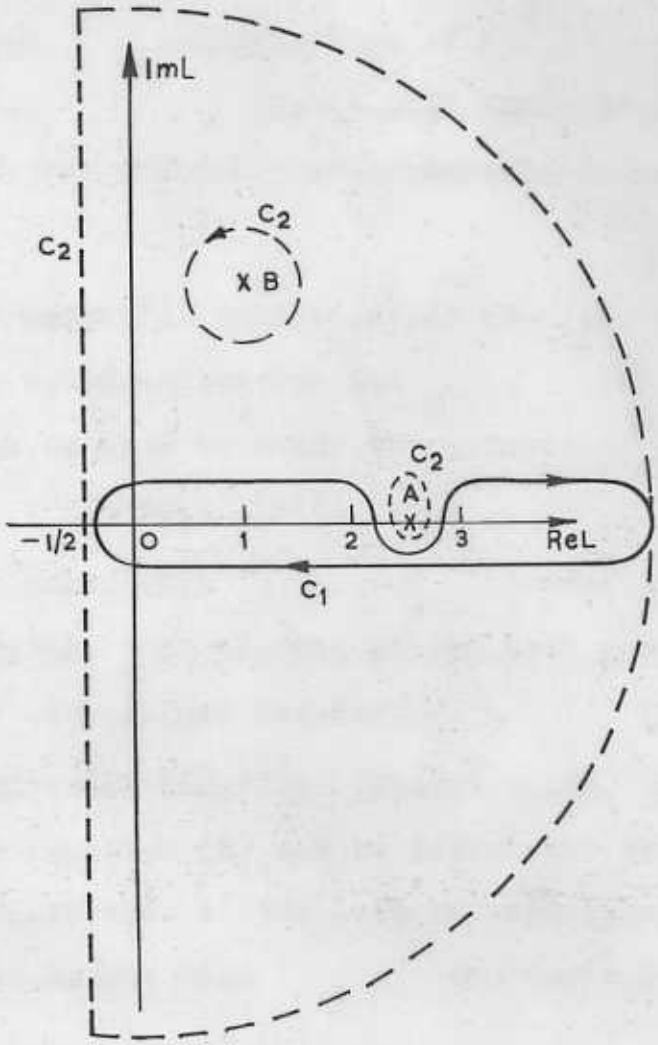

FIG. 1

loops we obtain a sum of characteristic terms, depending on the position and residues of the poles of +). Both and are functions of E. As goes through an integer L, has a pole from the vanishing f sin π , with the correct angular dependence coming from

.

The essential gain in going from (1) to (3) is that (3) gives a valid expression for for all Z. In particular it can be used to study the asymptotic behaviour as . In this limit for . Hence Regge poles with dominate over the back ground integral. The placing of the back ground integral at is optimal because if

and the asymptotic behaviour goes up again. However Mandlestam[3] showed that equation (3) can be recast and the back-ground terms pushed.further to the left by exploiting Legendre functions of the second kind and their relation to viz.,

(4)

+) We have assumed that the only singularities of are simple poles at the points in the upper half plane . These can be proved in potential theory.

3) S.Mandlestam, Annals of Physics 19, 254 (1962).

The resulting expression is

(5)

For it matters little whether one uses the or form. The asymptotic limits are the same.

In the relativistic case, owing to the existence of exchange forces the function to be continued into the complex L plane now contains a factor of the form

(6)

whose behaviour does not allow the Sommerfeld-Watson transformation to be carried out. One therefore defines the amplitudes

(7)

so that

(8)

The partial wave amplitudes are now labelled and

satisfy

for L = even integers

(9)

for L = odd integers

One then continues separately into the complex L plane. The overall result is that Regge pole terms in equation gets replaced by

(10)

The term is called the signature factor. If there are no exchange forces we get degenerate pairs of Regge trajectories with opposite signature and recover the previous formula.

Also in a relativistic process the concepts of crossing plus analyticity allow us to relate the s-channel scattering process A+B A'+B' (s = (Energy)^2, = c.m. scattering angle) to the crossed t channel process $A + \bar{A}'$ $B' + \bar{B}$ with (t = (Energy)^2, = c.m. scattering angle). Since in the s-channel the square of the momentum transfer is just (-t) and since

(11)

we see that <u>high energies and small momentum transfers</u> in the s-channel corresponds to having . Thus in this region we can express the s-channel process in terms of asymptotic form of the t channel process. Thus if is the farthest singularity to the right in the plane+)

(12)

For unequal mass scattering there is always an angular region near = 0 where is of order 1. This range gets smaller as increases but within this range asymptotic formulae that assume are questionable. However it has been pointed out by Freedman and Wang[4] that if the Regge asymptotic form holds nearby in the large region, it may be established also in the questionable region by

---

+) Unlike in potential case, we have no proof that the singularities in the J plane are simple poles. In fact it has been shown that there are branch points as well. In any event a formula like (12) will hold except possibly for logarithmic factors.

4) D.Z.Freedman and J.M. Wang, Phys. Rev. <u>153</u>, 1596 (1967).

analytic continuation. In fact they have shown that Regge trajectories occur in families, the leading parent trajectory occuring with a set of daughter trajectories (t) with zero energy intercepts . These daughter trajectories play a minor role in equal mass situations but for unequal-mass scattering. Their function is to cancel singularities in the asymptotic contribution of the parent trajectory. Van Hove[5] has made an important conceptual advance recently by inventing a simple model which clearly shows how Regge poles could arise in Feynman type field theory. The Van Hove model is based on the Feynman rules and the suggestion that a Regge pole corresponds to the exchange of an infinite number of particles with different angular momenta.

Consider the graph in Fig. corresponding to the exchange of spin J particle. If we assume derivative coupling at the vertices, and that all external particles have spin J zero, the pole term becomes

(13)

where is the relevant coupling constant, and the total amplitude coming from these poles is

(14)

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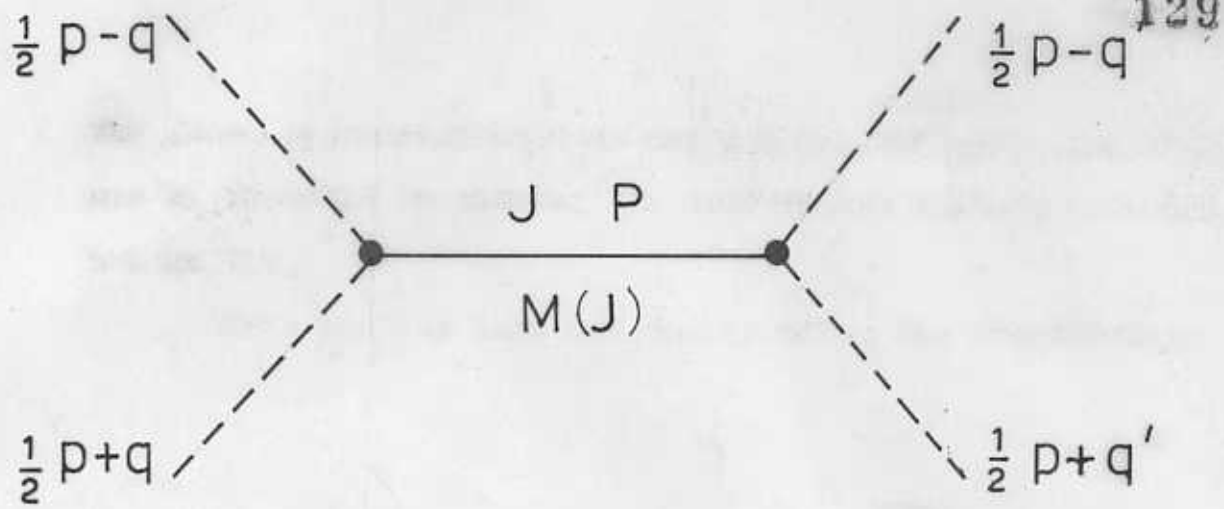
5) L.Van Hove, Phys.Lett. 24B, (1967).
L.Durand, Phys.Rev.Lett. 18, 58 (1967).



Spin J exchange

FIG. 2

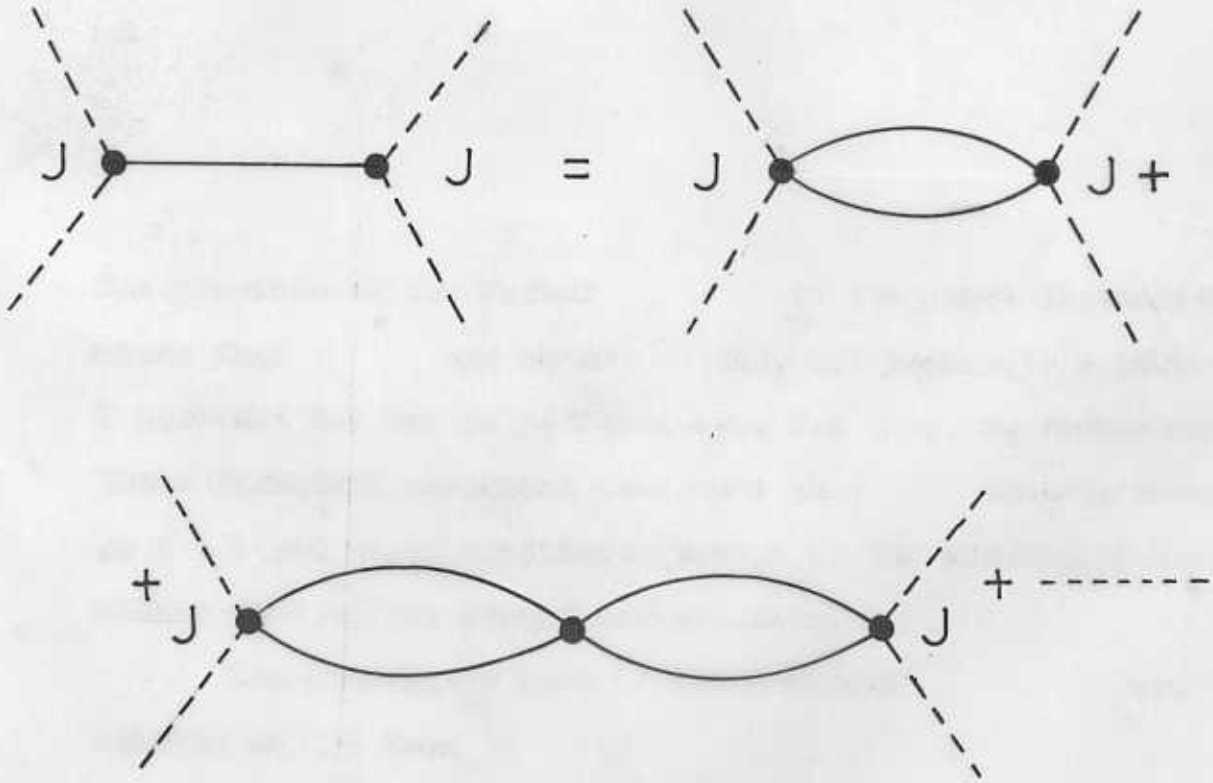


FIG. 3

The above approximation does not satisfy unitarily and so it can be improved by summing the self-energy bubbles illustrated in Fig.

This implies that one really makes the substitution

(15)

where is the contribution of the bubble graph, and we assume that some sort of convergence factors have been put in to make finite.

The Feynman propagator is made up of direct products of the projection operators which could be written as

(16)

The presence of the factor in the above expression means that and hence does not propagate a pure spin J particle but has in addition J-1, J-2 ..... 0, components. These redundent component make sure that behaves decently at t = 0 and these components vanish if the external masses on either side of the propagator are zero.

The propagator part of the numerator can be written in the form

(17)

where and are the centre of mass momenta and scattering angle in the t-channel, except those terms which have behaviour t is set equal to $M^2(J)$. Thus and are perfectly finite at $t = 0$.

The Van Hove model now assumes that there is an infinite set of particles contributing to the amplitude with every other J value

(18)

and that $g^2(J)$, $M^2(J)$ and $\Lambda_J(t)$ can be continued to complex J.

Performing a Sommerfeld-Watson transformation on this sum we can extract the large behaviour. Defining the J= (t) to be the solution of the equation $M_J^2 = t$ the pole due to the vanishing of the propagator leads to

(19)

which is of the Regge form. However we have ignored the poles in J coming from the terms in equation (18) which are there in Feynman calculations. Thus equation (19) is not reasonable at and one must take into account other singularities besides the leading trajectory which was used to derive (19).

APPENDIX B:

REGGEIZATION OF THE S-CHANNEL HELICITY AMPLITUDES

The formulation of high energy exchange models in terms of direct channel amplitudes is dealt with by several authors[1,2]. Here we reproduce some of the results obtained by Cohen Tannoudji et. al. Regge theory, to leading order in s, is easily expressed in the s-channel as in the t channel using the crossing matrix. A simple solution to the problem is possible because the helicity crossing matrix factorizes as noticed by Fox and Leader[3].

First of all let us consider the reaction 1+2 3+4 with spins S_1, S_2, S_3 and S_4 respectively. $s = (p_1+p_2)^2$ and $t = (p_3-p_1)^2$. We consider the exchange of a Regge pole in the t channel ($\bar{4}+2$ $3+\bar{1}$) in order to evaluate the high energy behaviour of physical quantities near the forward direction. All the physical quantities can be expressed for example either in terms of s-channel helicity amplitudes (SHA) M^s or in terms of t channel helicity amplitudes (THA) M^t .
In the case of t channel helicity amplitudes it is easy to incorporate parity and C parity conservations but one is faced with the problem of constraints in t for the THA. The essential

1) Fox: University of Cambridge Thesis (unpublished)
2) G.Cohen Tannoudji, Ph.Salin and A.Morel, 55A, 412 (1968).
3) G.Fox and E.Leader, Phys.Rev.Lett. 18, 628 (1967).

advantage in dealing with the s-channel Helicity amplitudes is that it helps to completely avoid an explicit writing of kinematical singularities and constraints. This is more than a technical improvement because it appears that these kinematical singularities and constraints among t channel helicity amplitudes reveal no continuity at all in terms of the masses of the external particles. The kinematical complications are so sensitive to mass differences that one is lead to the paradoxical situation in which for instance the mass difference between proton and neutron do change the structure of kinematical constraints and as a consequence could change the dynamics. On the other hand in the case of s-channel helicity amplitudes, the solution is continuous in external masses and then completely indifferent to small mass differences. This difficulty does not occur in the s channel if one is interested in the high energy region because for the SHA the constraints always take place far from this region.

Parity conservation at each vertex for the exchange of a particle of spin J and intrinsic parity implies the following relation between the t channel helicity partial wave helicity amplitudes

(1)

The corresponding helicity amplitudes are

(2)

it is easily seen that for large Cos

(3)

so that

(4)

Similarly for the exchange of a particle with parity , G parity g and Isospin I, when particle 4 and 2 are of the same species, G parity then implies

(5)

The S channel helicity amplitudes can be calculated from the t channel amplitudes using the crossing matrix[4)]

(6)

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4) T.L.Trueman and G.C.Wick, Ann.Phys. 26, 322 (1964).

The crossing angles all satisfy $0 \leq \quad \leq \pi$; they are defined by

(7)

where $\Delta = m_3^2 + m_2^2 - m_1^2 - m_4^2$. The phase is +1 unless i and j are both fermions in which case = -1. Using the properties of the function it is found

(8)

and

(9)

due to parity conservation at the vertices. $\overline{4}\,2$ and $\overline{3}\,1$ respectively. Similarly for the G parity conservation the following relations are obtained:

(10)

if particles 4 and 2 are of the same species and

(11)

if particles 3 and 1 are of the same species.

<u>S-channel helicity amplitude:</u>

The S-channel helicity amplitude for an exchange of a Regge trajectory (t) with parity , G parity g isospin I and signature is given as

(12)

Using

(13)

and

(14)

and invoking the parity conservation

gives the following relation for the residues

(15)

The SHA corresponding to the exchange of a given boson Regge trajectory (t) with signature , parity , G parity G and Isospin I is given by

(16)

where

(17)

where the residue factorizes and have the same analyticity

properties as (t). The effect of parity and G parity conservation in terms of these vertex functions:

(18)

if particles i and j are of the same species.

APPENDIX.C

EXCHANGE DEGENERACY

Exchange degeneracy can be understood most easily through potential scattering theory, where one speaks of direct and exchange potentials[1]. The potentials result in angular decomposition of the form

(1)

where and are partial wave amplitudes resulting from the direct and exchange potentials, respectively

In order to avoid the bad behaviour $(-1)^j$ for complex j, one Reggeizes the so called signatured amplitudes , defined by

(2)

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1) R.Omnes and M.Froissart, Mandlestam Theory and Regge poles (W.A.Benjamin Co.,Inc. 1963)

S.C.Frautschi, M.Gell-Mann and F.Zachariasen, Phys.Rev. 126, 2204 (1962).

The full amplitude is given by

(3)

where the final expression results from the assumption that is dominated by a Regge pole at and by the Regge pole at . Exchange degeneracy is the statement that and f are equal. This, of course is true if the exchange potential is zero. In such a case the equality of odd and even signatured amplitudes implies that there is only one trajectory and only one residue function . Approximate degeneracy would result if the exchange potential were small compared to the direct potential or if the exchange potential were not very small, but had a shorter range than the direct potential[2]. In such a situation both the residues and the trajectories would be approximately equal.

In relativistic scattering, one does not work with potentials but with discontinuity and spectral functions. Dispersion relation for the barred amplitudes are written in

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2) A. Ahmadzadeh, Review for Eugene Conference on Regge pole model, Ore meeting (1968).

the form (suppressing the helicity indices)

(4)

where and are discontinuity function accross the and cuts respectively[3]. The functions and play much the same role as direct and exchange potentials in potential theory. A partial wave decomposition of the above expression contains the same undesirable factor $(-1)^j$ as a coefficient of the contribution from . Consequently one again defines even and odd signatured amplitudes and Reggeizes each separately.

Exchange degeneracy implies is zero. Using the Mandlestam representation, the functions and can be written as sums of two dispersion integrals involving the double-spectral functions. While depends on the double spectral functions and , depends on and . Thus, exchange degeneracy is equality to the hypothesis that the contributions from the spectral functions and are negligible compared to that from .

The above argument used unsubtracted dispersion relations for simplicity. Supposely, exchange degeneracy could be formu-

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3) P.D.B.Collins and E.J.Squires, Regge poles in particles physics (Springer, Berlin, 1968) Vol.45, Springer Tracts in Modern Physics.

R.C.Arnold, Phys.Rev.Lett. 14, 657 (1965).

lated in the presence of subtraction terms and conceivably might not depend on the existence of non-zero double spectral functions.

The implications of exact or even approximate exchange degeneracy are very strong. In addition to even and odd trajectories being coincident, the equality of the residues implies that trajectories resulting from exchange degenerate Toller poles do have the same factors.

There has been considerable interest in establishing approximate exchange degeneracy between the pairs[4)]

and . Exchange degenerate pairs of trajectories have the same values of isospin and parity ; the difference in their parities implies that their trajectory and residue functions have different threshold behaviours. Recently two types of exchange degeneracy have been proposed: Strong exchange degeneracy (i.e. equal residues and trajectories) and weak exchange degeneracy (only trajectories are equal)[5)].

Exchange degeneracy also result from Duality Theory.[6)] Application of the finite energy sum rules to the cases where there are no known s-channel resonances (exotic channels), leads the exchange degeneracy of Regge trajectories. Now let us see how exchange degeneracy prediction comes about the finite

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4) G.Abrahams and U.Maor, Phys.Rev.Lett.25, 621 (1970).
5) V.Barger and D.Cline, Phys.Rev.166, 1522 (1967).
6) R.Dolen, D.Horn and C.Schmid, Phys.Rev.166, 1768 (1968).

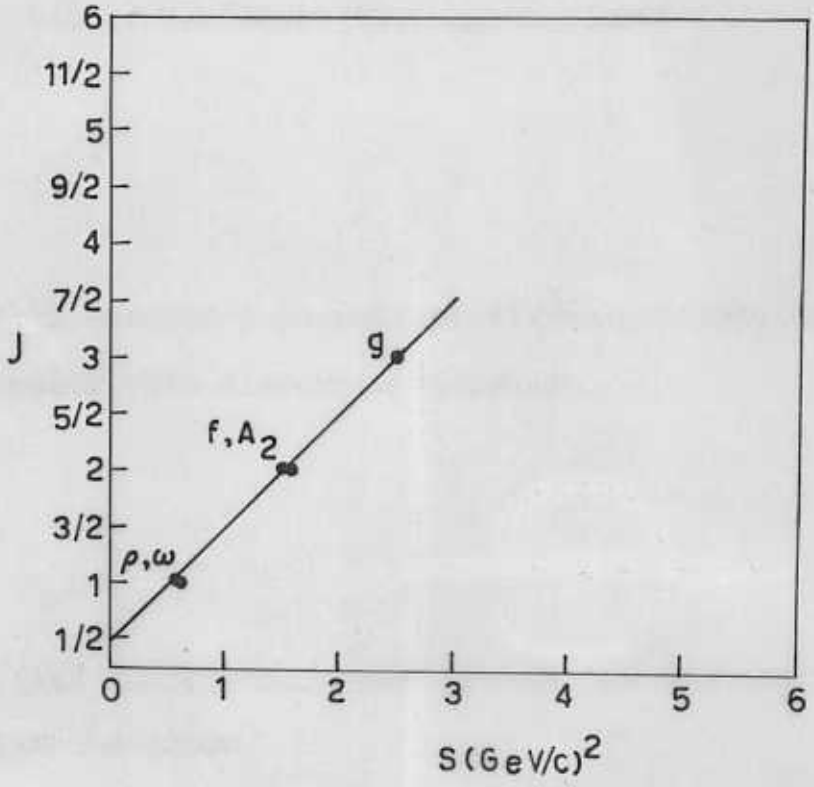

FIG.1

energy sum rules.

Finite energy sum rules are consistency conditions imposed by analyticity on functions that can be expected at high energies as a sum of Regge poles. The contribution of an individual Regge term can be written as

(5)

Let us consider an antisymmetric amplitude that obeys the unsubtracted dispersion relation

(6)

If its leading term has < -1, it will obey the superconvergence relation

(7)

However if the leading Regge term is above -1, we can subtract it from F and the resulting amplitude will obey a superconvergence relation. In order to simplify the calculation we use for the Regge pole the simple power law behaviour

(8)

The amplitude F-R will satisfy the superconvergence relation

(9)

Consider a function F antisymmetric in for fixed t that can be represented by a series of Regge poles for $\geq$ N. The poles can be divided into three classes: stands for all poles which are above -1, for all poles below -1, and for any pole that happens to be at -1. The poles above -1 have to be subtracted from the integrand, the poles below -1, do not appear at all, and the residue of the pole at -1 appears on the right hand side

(10)

Now we cut off the integral at some and express the high energy behaviour by Regge terms whose is below -1:

(11)

We notice that all integrals are now convergent. Performing the integration, we find the following finite energy sum rule

(12)

The generalization of eqn.( ) to arbitrary higher moments is straight forward and we get

(13)

Now a bootstrap equation results if the left hand side of eqn.( ) can be saturated by low energy resonance contribution and if the same resonances give rise to the Regge terms on the right hand side. Freund and Harari[7] conjectured that in this bootstrap scheme the Pomeron enjoys a special status in that it is generated by a non resonating background while all other Regge trajectories are generated by resonances. Applying this to the case of K^+p, and K^+n scattering where there is no s-channel resonances, he got the exchange degeneracy prediction

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7) P.G.O.Freund, Phys.Rev.Lett. 20, 235 (1968)
H.Harari, Phys.Rev.Lett. 20, 1395 (1968)

(14)

as well as other relations among the factorized residue functions etc. equations are acceptable while the residue relations cannot be tested at present.