MATSCIENCE REPORT 78

# PROCEEDINGS OF THE CONFERENCE ON

# NUCLEAR PHYSICS

MYSORE,

MARCH 1–4, 1973

HE INSTITUTE OF MATHEMATICAL SCIENCES, MADRAS-600020. (India)

Proceedings of the
CONFERENCE on NUCLEAR PHYSICS

held at
The Institution of Engineers, Mysore
from March 1-4, 1973

Convenor
Professor Alladi Ramakrishnan
Director, MATSCIENCE, Madras

Editor
Dr.K. Srinivasa Rao
MATSCIENCE, Madras.

THE INSTITUTE OF MATHEMATICAL SCIENCES
MADRAS-600020, INDIA

## FOREWORD

The conference on Nuclear Physics, conducted by MATSCIENCE, was held at the Institution of Engineers, Mysore, from March 1 to 4, 1973. This conference, in the true spirit of MATSCIENCE, brought together a small group of active nuclear physicists from various parts of the country, mostly theoreticians, who were provided a better opportunity for exchanging their ideas than that possible in the bigger gatherings. It was inaugurated by Professor Alladi Ramakrishnan, Director of Matscience on March 1, 1973, who welcomed the participants.

Participants to the conference from institutions other than MATSCIENCE, who presented their research work, belonged to: Atomic Minerals Division of DAE (Delhi), Bangalore University, Berhampur University, Bhabha Atomic Research Centre, Indian Institute of Technology (Bombay), Madras Engineering College, Madras University, N.G.M.College (Pollachi), Punjabi University, S.R.College (Trichi), Saha Institute of Nuclear Physics, Shivaji University and Vikram University where teaching in physics is going on and the complete list of those who represented various institutions at the conference is given at the end of this report.

The academic programme of the conference is given at the beginning of this report. Of the 29 talks delivered, 22 were related to nuclear physics, 2 were related to many-body theory and the rest (5) were related to elementary particle physics. The arrangement of the proceedings follows the academic programme. The lectures are not verbatim as delivered, but are reproduced

here as supplied after the conference by the authors themselves.

The organizers of the conference wish to thank all those participants who promptly submitted their manuscripts soon after the conference. We wish to take this opportunity to record our grateful thanks to: Smt. Lalitha Ramakrishnan (Vocal) and party for their delightful music concert on the evening of March 1; His Highness the Maharaja of Mysore for his gracious permission to the participants to visit his palace; the Government of Mysore for accommodating the participants in their Guest House; the Institution of Engineers for making their lecture halls available; the Physics Department of Mysore University for making their slide and over-head projectors available; and Professor A.P. Jambulingam of the Technical Teachers' Training Institute for allowing us to get the graphs transferred onto stencils using the electronic stencil cutter.

The editor expresses his grateful thanks to Drs.K.H.Mariwalla V.Radhakrishnan, N.R.Ranganathan, T.S.Santhanam, and R.Sridhar for actively taking part in the organization and the conduct of the conference and to Mr.N.S.Sampath and Mr.S.Krishnan for their help in the organization of the conference.

K.Srinivasa Rao
Editor

CONFERENCE ON NUCLEAR PHYSICS
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ACADEMIC PROGRAMME

March 1

Session 1 — 12.30 p.m. : Inauguration by Professor Alladi Ramakrishnan, Director, MATSCIENCE

12.40 p.m. : Professor Alladi Ramakrishnan*
"Generalized Gell-Mann-Nishijima relations" - p.1

Session 2 — 3.00 p.m. : Dr.K.Srinivasa Rao*
"Photopions from nuclei" - p.4

Dr.T.S.Santhanam*
"Unified descriptions of electromagnetic and weak interactions and leptonic quarks" - p.37

March 2

Session 3 — 9.00 a.m. : Dr.V.Devanathan*
"Some unfamiliar problems in muon capture" - p.40

Mr.Nandakishore Tiwari++
"A semi-classical treatment of collective states in transition nuclei"

Mr.A.R.Tekumalla
"A new spin half wave equation" -p.48

Session 4 — 11.00 a.m. : Dr.R.Shanta*
"Finite life-time effects of hole-states in single particle knock-out reactions" (Abstract) -p.66

Dr.J.Mahalanabis
"A study of pion-scattering by $^{12}C$ in the impulse approximation" -p.67

Dr.K.Srinivasa Rao
"Photoproduction of neagive pions from $^{12}C$" -p.84

Session 5 — 2.30 p.m. : Dr.N.G.Puttaswamy*
"Studies in stripping and pick-up reactions" -p.98

Dr.T.Nagarajan
"Beta spectrum shape and nuclear structure" -p.110

Mr.P.Ramaswamy
"Positron annihilation - A nuclear tool for solid state" p.118

Dr.S.Srinivasa Raghavan
"Deuteron form factors with soft-core potentials" p.129

(5.00 p.m : Excursion to Brindavan gardens)

March 3

Session 6 9.00 a.m. :

Dr.K.H.Mariwalla*
"Nuclear Physics in a larger perspective" p132

Mr.S.S.Ray++
"Calculation of $^{40}Ca(d,p)^{41}Ca$ reaction cross section including deuteron break-up channel"

Dr.S.D.Sharma
"Positive deformation of odd-odd nuclei and the core rotation" -p.145

Mr.M.T.Teli
"Polarization of neutrons in high-energy anti-neutrino scattering" -p.150

Session 7 11.00 a.m. :

Dr.L.Satpathy*
"Four body correlation and quartet states in nuclei" -p.168

Dr.Jyoti K.Parikh
"Self-consistent calculations for p-f shell nuclei" -p.185

Mr.R.Parthasarathy
"On determination of induced tensor coupling constant, in muon capture" -p.195

Mr.P.R.Subramanian
"Trace techniques for angular momentum operators" -p.201

(Afternoon: No session)

March 4

Session 8 9.00 a.m. :

Dr.B.K.Jain*
"A consistent study of the nucleus" -p.210

Dr.K.P.Joshi
"Effective interactions in nuclei and realistic interactions" -p.239

Session 9 11.00 a.m. :

Dr.N.R.Ranganathan
"Generalized Clifford groups" (abstract) -p.257

Dr.V.Radhakrishnan
"A general method to determine particle density in a self-consistant fashion for a many-body system" -p.259

Professor N.Ananthakrishnan
"Prospects of fusion power" -p.261

Session 10 2.00 p.m. : Dr.R.Sridhar
"Equations of motion method"
(abstract) -p.271

Mr.Jagdev Gargi
"Coriolis and a symmetric effects on moment of inertia of odd-odd deformed nuclei"-p.273

Mr.G.Alagar Ramanujan
"On super-luminal inertial frames"
(abstract)-p.281

OPEN DISCUSSION

(3.00 p.m. : Break-up Tea Party)

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* Duration of lecture was 40 minutes. All other lectures were of 20 minutes duration. Time was allowed for discussion at the end of every lecture.

++ Paper not submitted

# GENERALIZED GELL-MANN-NISHIJIMA RELATIONS

Alladi Ramakrishnan
MATSCIENCE, The Institute of Mathematical Sciences,
Madras-600020

Our most recent work at MATSCIENCE involving the Generalized Clifford Algebra (G.C.A.) has led us to a new view point on the internal quantum numbers of the quarks which has been postulated by Gell-Mann to be the fundamental consitments of the nucleons in the nucleus.

We start with the fundamental cyclic n x n matrix:

$$C = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \qquad (1)$$

which has the property $C^n = I$, i.e. C is one of the n-th roots of the unit matrix of dimension n. The eigen values of the matrix C are the n roots of unity:

$$1, \omega, \omega^2, \ldots, \omega^{n-1}, \qquad (2)$$

where $\omega$ is the n-th primitive root of unity. The corresponding eigen vectors can be placed together as columns forming a matrix which we call as the U-matrix. Hence

$$U^{-1} C U = B, \qquad (3)$$

where B and U are matrices given by

$$B = \begin{bmatrix} 1 & & & & \\ & \omega & & & \\ & & \omega^2 & & \\ & & & \ddots & \\ & & & & \omega^{n-1} \end{bmatrix} ; \quad U = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega & \cdots & \omega^{n-1} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & \omega^{n-1} & \cdots & (\omega^{n-1})^{n-1} \end{bmatrix} \qquad (4)$$

B is a diagonal matrix and U is the well-known Sylvester matrix.

We immediately observe that B and C possess an interesting commutation relation:

$$CB = \omega BC \cdot \tag{5}$$

This we shall call the $\omega$-commutation relation which is the basis of G.C.A. It is an inexplicable fact in the history of matrix theory that this commutation relation has not been noticed till very recently.

The matrices $B, B^2, \ldots, B^{n-1}$ commute as they are powers of the matrix B. Let $\eta_1$ take any one of the eigen values (2) of the B-matrix and let

$$\eta_2 = \eta_1^2 \ , \ \eta_3 = \eta_1^3 \ , \ \ldots \ , \eta_{n-1} = \eta_1^{n-1} \tag{6}$$

then we define a set of quantum numbers $s_1, s_2, \ldots, s_n$ as the following linear combinations of the $\eta_i$'s:

$$\begin{aligned}
s_1 &= \tfrac{1}{n}\{\eta_1 + \eta_2 + \cdots + \eta_{n-1}\}, \\
s_2 &= \tfrac{1}{n}\{\omega^{n-1}\eta_1 + (\omega^{n-1})^2\eta_2 + \cdots + (\omega^{n-1})^{n-1}\eta_{n-1}\}, \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
s_n &= \tfrac{1}{n}\{\omega\eta_1 + \omega^2\eta_2 + \cdots + \omega^{n-1}\eta_{n-1}\}.
\end{aligned} \tag{7}$$

These can be written as a vector-matrix equation:

$$\vec{s} = \tfrac{1}{n} S \vec{\eta} \tag{8}$$

where

$$\vec{s} = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix} \quad ; \quad \vec{\eta} = \begin{pmatrix} 0 \\ \eta_1 \\ \eta_2 \\ \vdots \\ \eta_{n-1} \end{pmatrix} \qquad \text{and} \qquad S = U.$$

Since the first value in $\vec{\eta}$ is set equal to zero, the choice of the first column in S is irrelevant. We notice that in the case of the quark, we identify the quantum numbers $s_1$, $s_2$ and $s_3$ as:

$$s_1 = Q \quad , \quad s_2 = Y - Q \quad \text{and} \quad s_3 = -Y \quad , \qquad (9)$$

where Q and Y are the charge and hypercharge, respectively. We assert that the commuting generators of the Lie algebra are the linear combinations of the commutin elements of the G.C.A. (Viz.$B, B^2, \ldots, B^{n-1}$) and that the shift operators of the Lie algebra can be expressed as linear combinations of the products of B and C.

We now proceed to define eigen values $I_{kl}$ which are the differences between $s_k$ and $s_l$ $(k \neq l)$:

$$I_{kl} = s_k - s_l \; ; \quad k, l = 1, 2, \ldots, n. \qquad (10)$$

For $n = 3$, we can identify $I_{12}$, $I_{23}$ and $I_3$, as the Z-components of I-spin, U-spin and V-spin, respectively, in the language of SU(3). We also recognize the equation (10) for n = 3, as the Gell-Mann-Nishijima relation:

$$I_{12} = 2I_z = s_1 - s_2 = 2Q - Y \quad \text{or} \quad Q = I_z + \frac{Y}{2}. \qquad (11)$$

But equation (10) can be applied to the SU(n) quarks, and this consititutes of the eneralized Gell-Mann-Nishijima relations. What we have described now is the correspondence between the

quantum numbers of the quarks in the language of the Lie and Clifford algebras.

If we consider physical particles as composed of quarks with $\eta^i(j)$ denoting the $j$ -th quantum number of the $i$ -th particle, then we can define the quantum number of the physical particle $\eta(j)$ as:

$$\eta(j) = \sum_i \eta^i(j) . \qquad (12)$$

In a similar way, the composite quantum number of a physical particle can be defined. The vector matrix relation given by equation (8) still holds between $s(j)$ and $\eta(j)$ .

## Reference

Alladi Ramakrishnan, "L-matrix theory or the Grammar of Dirac matrices", Tata-McGraw Hill Co., (1972). See Appendix II of this book for a complete list of the author's publications on this subject.

# PHOTOPIONS FROM NUCLEI

K. Srinivasa Rao
MATSCIENCE. Institute of Mathematical Sciences
Madras-20.

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Abstract:

Configuration mixing particle-hole models, besides the simple independent particle model, have been used in a quantitative analysis of photoproduction of positive pions from $^{16}O$. The effect of two-particle-two-hole correlations in the ground state wave function of $^{16}O$, as well as the effect of the Saxon-Woods basis, instead of the harmonic oscillator basis, on the cross sections has been studied. The theoretical cross sections are in good agreement with experimental data. In the case of $^{11}B$, the surface production mechanism accounts for the experimental charged pion photoproduction cross sections when the simple independent particle model is used for the description of nuclear states. A comparison of the results from our theory is made with those obtained by others using the 'elementary' particle treatment for nuclei.

## 1. Introduction:

A quantitative analysis of photoproduction of charged pions from nuclei is used as a probe for studying the structure of nuclei. Our studies[1-5] of charged pion photoproduction from nuclei are based on a model of direct interaction between the incident photon and the individual nucleons, coupled with the

impulse approximation. The nuclear transition operator is expressed in terms of the free single nucleon photoproduction amplitudes obtained from dispersion relations. On the basis of the realization that the positive pion photoproduction process is similar to the muon capture process, since the same initial and final nuclear states are involved in both process, and the fact that nuclear structure effects have been found [6,7] to play a significant role in the muon capture process, we use the available particle-hole models to describe the nuclear states in the case of our study of $^{16}O(\gamma, \pi^+)^{16}N$ for which experimental data are available. A spherical description for the ground state of $^{16}O$ and particle-hole wave functions for the low-lying bound states of $^{16}N(J^P = 2^-, 0^-, 3^-, 1^-; T=1)$ yield cross sections smaller than those obtained with the independent particle model but are still larger, by almost a factor of two, than the experimental results. This discrepancy can be accounted for by invoking the phenomenological surface production mechanism. We also find that a better agreement between theory and experiment can be obtained, without invoking the surface production mechanism, if we use the realistic Kuo[8] wave functions with 'screening' for $^{16}N$ states, together with the ground state wave function of $^{16}O$ which explicitly includes two-particle-two-hole correlations. Use of the Saxon-Woods basis instead of the conventional harmonic oscillator basis is found[4] to improve the agreement between theory and experiment.

In the case of a non-closed shell nuclei, like $^{11}B$ we are constrained to the use of the simple independent particle model for the nuclear states, due to the absence of detailed information about the same. We find that the use of the phenomenological surface production mechanism yields[2] results, for charged pion photoproduction from $^{11}B$, in reasonable agreement with the experimental data.

To avoid the usuage of the impulse approximation and a detailed description of the nuclear states, Kim and Primakoff[9] treat the initial and final nuclei themselves as 'elementary' particles, so that all the nuclear structure complexity is imbedded in certain nuclear from factors. This elementary particle treatment applied by Kim and Primakoff[9] to the study of $\beta$ -decay ft values for various nuclei has been used by Kim[10] to study the partial muon capture rate: $^{16}O(0^+) \rightarrow {}^{16}N(2^-)$. The same treatment has been applied to the theory of photopions from nuclei by Griffiths and Kim[11] and more recently by Pascaul and Sanchez-Gomez[12]. In the elementary particle treatment of pion photoproduction, cross sections for ground to ground state transitions can alone be calculated since nuclear form factors are available only for such transitions. The present available experimental results for pion photoproduction correspond to any of the stable, low-lying states of the final nucleus, and not only to the ground state. Therefore, it is nearly impossible to compare theory with experiment. This is the main limitation for elementary particle treatment of pion photoproduction from nuclei.

## 2. Available Experimental Data:

Early experimental investigation[13] on photoproduction of charged pions from complex nuclei were devoted to the total cross section based on an observation of emitted pions rather than of the product nuclei. These experiments have shown that the sum of the $\pi^+$ and $\pi^-$ cross sections exhibit an exact $A^{2/3}$ dependence, where A is the mass number of the nucleus, and this $A^{2/3}$ dependence has been readily explained by means of a surface production model proposed by Wilson[14] and Butler[15].

The first experimental investigation of reactions of the type $A(\gamma, \pi^{\pm})B$, where the residual nucleus B is identified by its characteristic radio-activity, has been made by Hughes and March[16] in 1958. They studied the reaction $^{11}B(\gamma, \pi^-)^{11}C$ where $^{11}C$ was identified by its positron activity. Subsequently, Hummel and his coworkers[17] at Illinois, U.S.A. have reported cross sections for the following reactions: $^{11}B(\gamma, \pi^-)^{11}C$, $^{11}B(\gamma, \pi^+)^{11}Be$, $^{16}O(\gamma, \pi^+)^{16}N$ and $^{27}Al(\gamma, \pi^+)^{27}Mg$ while Nydhal and Forkman[18] at Lund, Sweden, have measured the cross sections for the reactions: $^{27}Al(\gamma, \pi^+)^{27}Mg$, $^{51}V(\gamma, \pi^+)^{51}Ti$ and $^{60}Ni(\gamma, \pi^-)^{60}Cu$. The last of these reactions has also been investigated by March and Walker[19]. More recently, Blomquist et. al[20] at Lund, have reported the cross sections for the reactions $^{41}K(\gamma, \pi^+)^{41}A$, $^{27}Al(\gamma, \pi^+)^{27}Mg$ and $^{65}Cu(\gamma, \pi^+)^{65}Ni$. Since these experimentalists measure the radioactivities of the final nuclei, they report

total pion photoproduction cross sections which are sums of 'partial' cross sections due to photon induced transitions to specific states of the final nucleus which are stable against nucleon emission.

3. Early Theoretical Studies:

The work at MATSCIENCE on pion photoproduction was initiated by Alladi Ramakrishnan, Devanathan and Ramachandran[21] with the study of elastic photoproduction of neutral pions from deuterium in the impulse approximation using the amplitudes of Chew, Goldberger, Low and Nambu[22] for photoproduction of pions from single nucleons. Photoproduction of charged and neutral pions from the three nucleon targets, $^{3}H$ and $^{3}He$ has been studied by Ramachandran and Ananthanarayanan[23]. In a series of papers Devanathan and Ramachandran[24] have obtained compact general expressions for the cross sections for pion photoproduction from nuclei by performing an explicit summation over all magnetic quantum numbers.

Laing and Moorehouse[25] studied the process $^{11}B(\gamma, \pi^-)^{11}C$ and calculated the cross sections assuming volume production and surface production of pions. Hughes and March made detailed comparison of their experimental results with the theory of Laing and Moorehouse and find good agreement with the theoretical values obtained assuming surface production of pions.

More recently, Engelbrecht[26] and Saunders[27] have studied photoproduction of neutral pions from closed shell nuclei, taking the final state interactions of the outgoing pion with the residual nucleus through an optical potential .

The above-mentioned theoretical studies on pion photoproduction from complex nuclei were based on the impulse approximation and are not concerned with aspects of nuclear structure which depend sensitively on specific details of the nuclear model, while we are concerned precisely with this aspect here.

## 4. Nuclear Structure and charged pion photoproduction

The free nucleon pion photoproduction amplitude has the general structure $\underline{\sigma}\cdot\underline{K} + L$ where $\underline{K}$ and L are the spin-flip and spin non-flip parts of the amplitude. For $\underline{K}$ and L we use the Chew, Goldberger, Low and Nambu[22] (C.G.L.N.) amplitudes * which have been found to be extremely successful upto incident photon energies of 500 MeV. given below:

$$\underline{K} = i\frac{2\sqrt{2}\,\pi e f}{\sqrt{\mu_0 \nu_0}}\left[\frac{1}{1+\frac{\mu_0}{M}}\left\{\underline{\epsilon} + \frac{2(\underline{\mu}\cdot\underline{\epsilon})\,\underline{k}}{(k^2+1)}\right\} + \lambda\hbar^{-}\underline{\mu}\times(\underline{\nu}\times\underline{\epsilon}\right.$$

$$\left. \mp\alpha\left\{\mu_0^2\underline{\epsilon} + \underline{\mu}\times(\underline{\nu}\times\underline{\epsilon}) - H_{-}\frac{(\underline{\mu}\cdot\underline{\epsilon})\,\underline{\mu}}{M\mu_0}\right\}\right], \qquad (1)$$

$$L = \frac{2\sqrt{2}\,\pi e f}{\sqrt{\mu_0 \nu_0}}\,\underline{\mu}\cdot(\underline{\nu}\times\underline{\epsilon})\,\lambda\hbar^{+-}, \qquad (2)$$

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* We use natural units $\hbar = c = 1$ = pion mass.

here

$$e^2 = 1/137 \quad , \quad f^2 = 0.08 ,$$

$$\lambda = \frac{\mu_p - \mu_n}{4Mf^2} \quad , \quad \alpha = \frac{\mu_p + \mu_n}{2M\mu_0} ,$$

$$h^{--} = \frac{1}{3\mu^3} e^{i\delta_{33}} \sin\delta_{33} \quad , \quad h^{+-} = -\frac{2}{3\mu^3} e^{i\delta_{33}} \sin\delta_{33} ,$$

e being the electromagnetic coupling constant,

f the unrationalized, renormalized $\pi$-N coupling constant,

$\mu_p$ , $\mu_n$ the anamolous magnetic moments of proton and neutron,

M the nucleon mass,

$\underline{\mu}$, $\mu_0$ the momentum and energy of the pion,

$\underline{\nu}$ , $\nu_0$ the momentum and energy of the photon,

$\delta_{33}$ the dominant $(J = 3/2, T = 3/2)$ $\pi$-N scattering phase-shift,

$\underline{k}$ the momentum transfer to the nucleon, $(\underline{k} = \underline{\nu} - \underline{\mu})$,

$\underline{\varepsilon}$ the polarization of the incident photon and

$$H_- = \begin{cases} 0 \text{ for } \pi^+ \text{ photoproduction,} \\ 1 \text{ for } \pi^- \text{ photoproduction.} \end{cases}$$

In the expression (1) for $\mathcal{K}$, the upper and lower signs refer to $\pi^+$ and $\pi^-$ photoproduction, respectively.

The single nucleon pion photoproduction transition operator can be written as:

$$t(\gamma N \to N\pi^{\pm}) = (\underline{\sigma}\cdot\underline{K} + L)\,\tau_{\mp}\,\exp(i\underline{k}\cdot\underline{r}), \quad (3)$$

where $\underline{r}$ is the position vector of the nucleon, $\underline{k}$ the momentum transfer to the nucleon, $\tau_{\mp}$ the nucleon isospin operator and $\gamma N \to N\pi^{\pm}$ stands for either $\gamma p \to n\pi^{+}$ or $\gamma n \to p\pi^{-}$. For convenience in the evaluation of matrix elements, we rewrite (3) as:

$$t = \sum_{n=0,1} (\sigma_n\cdot\underline{K}_n)\,\tau_{\mp}\,\exp(i\underline{k}\cdot\underline{r}) = \sum_{\lambda,m_\lambda} t^{\lambda}_{m_\lambda}, \quad (4)$$

where $\sigma_0$ is the unit operator and

$$t^{\lambda}_{m_\lambda} = 4\pi \sum_{n,\ell} i^{\ell}(-1)^{\ell+n-\lambda+m_\lambda}\, j_{\ell}(kr)\cdot$$

$$(Y^{\ell}(\hat{r})\times\sigma^{n})^{\lambda}_{m_\lambda}\,[(Y^{\ell}(\hat{k})\times K^{n})^{\lambda}_{-m_\lambda}]^{*}\,\tau_{\mp}. \quad (5)$$

According to the direct interaction model, the interaction of the photon with the nucleus can be expressed in terms of its interaction with the individual nucleons. Thus, the nuclear transition operator is:

$$T = \sum_{i=1}^{A} t_i = \sum_{i=1}^{A} \Big(\sum_{\lambda,m_\lambda} t^{\lambda}_{m_\lambda}\Big)_i , \quad (6)$$

where $A$ is the mass number of the nucleus.

Among the processes mentioned in section 2, for which experimental results are available[17], $^{16}O(\gamma, \pi^+)^{16}N$ is admirably suited for theoretical study, since $^{16}O$, being a doubly closed magic nucleus, is one of the few nuclei which have been extensively studied and good wave functions are available for its ground state and excited states. In the experimental study of $^{16}O(\gamma, \pi^+)^{16}N$, since the radioactivity of the final nucleus has been measured (6.14 MeV $\gamma$ decay from the $3^-$ state in $^{16}O$ see Fig.1). the final nuclear states are restricted to the four low-lying bound states, with spin-parity $J^P = 2^-, 0^-, 3^-$ and $1^-$, which are well established to be the $T = 1$ isobaric analogs of states in $^{16}O$ and this enables us to take the wave functions of $^{16}N$ bound states from those of $^{16}O$ under the valid assumption of good isobaric spin. Thus, the transitions of interest to us are:

$$\gamma + {}^{16}O(J^T = 0^+, T = 0) \rightarrow \pi^+ + {}^{16}N(J^P = 2^-, 0^-, 3^-, 1^-; T = 1) \quad (7)$$

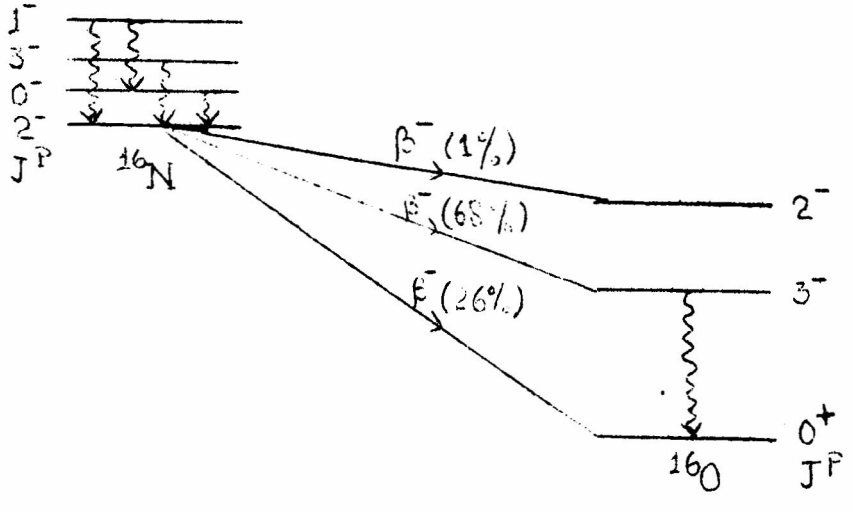


Fig.1 Level scheme for $^{16}O(\gamma, \pi^+)^{16}N$ and subsequent decays. Solid arrows indicate the $\beta$-decays with the branching ratios and wigly arrows the $\gamma$-de-excitations.

In the case of a closed shell nucleus, the basic excitations are of the hole-particle type. So, one is interested in diagonalizing the residual two-body force restricted to the subspace of configurations with one particle-hole pair of energy $\hbar\omega$ to obtain eigenvalues and eigenvectors. This method of calculating the excited state wave functions is called the Tamm-Dancoff Approximation (TDA). The particle-hole wave function of the excited state in TDA is:

$$|J_f M_f\rangle = \sum_{p,h} X^{J_f}_{p,h} (-1)^{j_h - m_h} C(j_p j_h J_f; m_p - m_h M_f) \times a^+_{j_p m_p} a_{j_h m_h} |0\rangle \qquad (8)$$

where h(hole) is used to denote a typical occupied state and p(particle) is used to denote a typical unoccupied state $a^+$ ($a$) is the creation (annihilation) operator of the particle, and $|0\rangle$ is the Hartree-Fock ground state. The $X^{J_f}_{p,h}$ are the mixing coefficients associated with the particle-hole configuration which are normalized as:

$$\sum_{p,h} (X^{J_f}_{p,h})^2 = 1. \qquad (9)$$

It should be noted that in the Independent Particle Model (IPM), there being no configuration mixing, (p,h) take only one value so that $X^{J_f}_{p,h} = 1$.

The nuclear transition operator (3), becomes in the occupation number representation:

$$T = \sum_{\alpha,\beta} \langle \alpha | \sum_{\lambda,m_\lambda} t^{\lambda}_{m_\lambda} | \beta \rangle \, a^{+}_{\alpha} a_{\beta} \,, \qquad (10)$$

where $\alpha$ and $\beta$ are particle states. Treating the ground state of $^{160}$ to be spherical using (3) and the Wigner-Eckart theorem, we obtain the following expression for the transition matrix element in TDA:

$$Q_{TDA} = \langle J_f M_f | T | 0^+, g.s. \rangle$$

$$= \sum_{\substack{p,h \\ \lambda, m_\lambda}} X^{J_f}_{p,h} \frac{[j_p]}{[J_f]} \langle j_p \| t^{\lambda} \| j_h \rangle \delta_{J_f,\lambda} \delta_{M_f,m'_\lambda} \qquad (11)$$

where $[J] = (2J+1)^{1/2}$. Substituting expression (5) for $t^{\lambda}_{m_\lambda}$ in eq.(11); after squaring $Q_{TDA}$ and summing over $M_f$, we get:

$$|Q_{TDA}|^2 = 16\pi^2 \sum_{\substack{n,\ell,n',\ell' \\ p,h,p',h'}} i^{\ell-\ell'} (-1)^{\ell+n+\ell'+n'} X^{J_f}_{p,h} (X^{J_f}_{p',h'})^{*} \times$$

$$\times \frac{[J_p][j_{p'}]}{[J_f]^2} \Big\{ \sum_{M_f} (Y^{\ell}(\hat{k}) \times K^{n})^{J_f}_{-M_f} [(Y^{\ell'}(\hat{k}) \times K^{n'})^{J_f}_{-M_f}]^{*} \times$$

$$\times \langle J_\ell(kr) \rangle_{p,h} \langle j_{\ell'}(kr) \rangle_{p',h'} \times$$

$$\times \langle j_p \| (Y^{\ell}(\hat{r}) \times \sigma^{n})^{J_f} \| j_h \rangle \times$$

$$\times \langle j_{p'} \| (Y^{\ell'}(\hat{r}) \times \sigma^{n'})^{J_f} \| j_{h'} \rangle^{*}, \qquad (12)$$

where

$$\langle j_\ell(kr)\rangle_{p,h} = \int_a^\infty u_p^*(r)\, j_\ell(kr)\, u_h(r)\, dr, \qquad (13)$$

are the radial integrals with a = 0 for volume production and $a = r_0$ . the nuclear radius, for surface production. The quantity in curly brackets in expression (12) can be simplified, following Devanathan and Ramachandran[24], to:

$$\sum_{M_f} \left(Y^{\ell}(\hat{k}) \times K^{n}\right)^{J_f}_{-M_f} \left[\left(Y^{\ell'}(\hat{k}) \times K^{n'}\right)^{J_f}_{-M_f}\right]^* =$$

$$= \frac{1}{4\pi} \sum_N (-1)^{J_f} (-1)^{\ell-\ell'} \frac{[\ell][\ell'][J_f]^2}{[N]}\, C(\ell\ell' N; 000)$$

$$\times W(\ell\ell' n n'; N J_f)\ \left(Y^N(\hat{k}) \cdot \left[K^n \times (K^{n'})^*\right]\right). \qquad (14)$$

Further, the reduced matrix elements in (12) can be easily evaluated using standard techniques to give:

$$\langle j_p \| \left(Y^{\ell}(\hat{r}) \times \sigma^n\right)^{J_f} \| j_h \rangle = \frac{[\ell_h][1/2][j_h]}{(4\pi)^{1/2}[j_p]} \times$$

$$\times C(\ell_h \ell \ell_p; 000) \begin{pmatrix} \ell_p & 1/2 & j_p \\ \ell_h & 1/2 & j_h \\ \ell & n & J_f \end{pmatrix}. \qquad (15)$$

The differential cross section for pion photoproduction is given by

$$\frac{d\sigma}{d\Omega} = (2\pi)^{-2} \mu\mu_0 |Q|^2, \qquad (16)$$

under the assumption that the entire energy of the photon is given to the outgoing pion ($\mu_o = \nu_o$).

It is now well known that two series of positive parity levels in $^{16}O$, obey rotational band systematics. Brown and Green[28] proposed a model in which these two bands are considered as mixtures of 'deformed' two-particle-two-hole (2p-2h) and four-particle-four-hole (4p-4h) states with the spherical shell model ground state. Several theoretical studies have been made to estimate the amount of deformed components in the ground state wave function of $^{16}O$. In 1969, Purser et. al.[29] made a direct measurement of the 2p-2h admixtures in the ground state wave function of $^{16}O$ by investigating the pick-up reactions $^{16}O(d,t)^{15}O$ and $^{16}O(d,{}^3He)^{15}N$ at a deuteron energy of 20 MeV. Their results confirm the existence of such pair excitations with intensities comparable to these predicted by Brown and Green[28]. The ground state wave function of $^{16}O$ can be approximated by:

$$|0^+, g.s.\rangle = \alpha\,|0p\text{-}0h\rangle + \beta\,|(1d_{5/2}^{2})_{J=0,T=1}\,(1p_{1/2}^{-2})_{0,1}\rangle + \gamma\,|(2s_{1/2}^{2})_{0,1}\,(1p_{1/2}^{-2})_{0,1}\rangle \qquad (17)$$

where configurations with particles and holes separately coupled to (J,T) other than (0,1) are neglected, based on the observation of Zamick[30] that they lie much higher in energy

and hence their coupling to the $|0\hbar\omega - 0h\rangle$ state is much weaker. In table 1, we list the values of the parameters, $\alpha$, $\beta$ and $\gamma$ determined by Purser et. al.[29]

Table 1. Ground state wave function of $^{16}O$

| $^{16}O$ model | $\alpha$ | $\beta$ | $\gamma$ |
|---|---|---|---|
| I (PS) | 1.00 | 0.00 | 0.00 |
| II (Expt. $^{16}O(d,t)^{15}O$) | 0.87 | 0.26 | 0.27 |
| III (Expt. $^{16}O(d,^3He)^{15}N$) | 0.82 | 0.54 | 0.20 |

(PS denotes Pure Shell model wave function)

It should be mentioned here that most nuclear reactions calculations to-date involving $^{16}O$ have been made with $\alpha = 1$, $\beta = \gamma = 0$, the exceptional ones being those of Walker[31] and Green and Rho[7], who have shown that several theoretical-experimental discrepancies, for muon capture and photo-disintegration of $^{16}O$, can be resolved by assuming the ground state to have 2p-2h admixtures.

When 2p-2h admixtures are taken into account in the ground state wave function of $^{16}O$, one should consider the ground state and low-lying T = 1 states of $^{16}N$ to have 3p-3h admixtures to the

usual 1p-1h states. Walker[31] claims, using a method of generating the $^{16}N$ states from a deformed ground state, that the 3p-3h mixing can be as large as the 2p-2h mixing of the ground state. On the other hand, Green and Rho[7] argue both on theoretical and empirical grounds that the 3p-3h admixtures are insignificant for the $^{16}N$ quartet states. Following these arguments, we take the quartet T = 1 states of $^{16}N$ to be described by simple 1p-1h combinations.

If we denote by $Q^{J_f}(0p-0h)$ the matrix element for charged pion photoproduction, given by (11), when the ground state of $^{16}O$ is treated as spherical and by $Q^{J_f}(2p-2h)$ the matrix element when the ground state wave function of $^{16}O$ is described by (17), then it can be shown[3] that:

$$Q^{J_f=0^-,1^-}(2p-2h) = \left(\alpha + \frac{(-1)^n\gamma}{\sqrt{12}}\right) Q^{J_f=0^-,1^-}(0p-0h) \,, \tag{18}$$

and

$$Q^{J_f=2^-,3^-}(2p-2h) = \left(\alpha + \frac{(-1)^n\beta}{6}\right) Q^{J_f=2^-,3^-}(0p-0h) \,, \tag{19}$$

where n can take the values 0 and 1 (eq.4). As before, the differential cross section is given by eq.(16) where Q should now be replaced by $Q^{J_f}(2p-2h)$. In IPM, it can be shown[5] that the cross section for the $0^+$ $2^-$ 'partial' ...

cross section is given by:

$$\sigma^{J_f=2^-}(2p-2h) = \left(\alpha - \frac{\beta}{6}\right)^2 \sigma^{J_f=2^-}(0p-0h). \quad (20)$$

Since this transition $0^+ \rightarrow 2^-$ gives rise to the dominant contribution to the total cross section, it is significant to note that the factor $\left(\alpha - \frac{\beta}{6}\right)^2 \approx 0.53$ when $\alpha = 0.82$ and $\beta = 0.54$ as in table 1. Thus, in IPM, there is almost a factor of two reduction in the cross section for the $0^+ \rightarrow 2^-$ transition when we take the 2p-2h correlations in the ground state of $^{16}O$ into account which is a consequence of nuclear structure effect on the cross section.

The radial integral given by eq.(13), involves obviously radial single-particle wave functions $u(r)$, which are solutions of the radial Schrödinger equation

$$\left[-\frac{1}{2M}\frac{d^2}{dr^2} + \frac{l(l+1)}{2Mr^2} + v(r)\right]u(r) = \varepsilon u(r), \quad (21)$$

with the normalization

$$\int_0^\infty u^2(r)\,dr = 1. \quad (22)$$

The conventional harmonic oscillator potential

$$v(r) = \frac{1}{2}M\omega^2 r^2, \quad (23)$$

leads to analytic solutions for $u(r) = r R_{n\ell}(r)$ given by:

$$R_{n\ell}(r) = \left[\frac{2^{n+\ell+1}}{\sqrt{\pi}(2n+2\ell-1)!!}\right]^{1/2} \frac{1}{b^{\ell+3/2}} P(r) r^{\ell} \times \exp(-r^2/2b^2), \qquad (24)$$

where $b = \sqrt{\hbar/M\omega}$ is the oscillator size parameter and

$$P(r) = 1 \quad \text{for } n = 1,$$

$$P(r) = \frac{2\ell+3}{r^2} - \frac{r^2}{b^2} \quad \text{for } n = 2.$$

Results of shell model calculations[32], using single particle Woods-Saxon (WS) wave functions, indicate that their effect on level spacings can be as large as those obtained by adding a core polarization correction to the interaction. Also, results from electron scattering[33] show that the realistic WS single particle states can predict the position of the second diffraction minima in $^{16}O$ and $^{12}C$ in agreement with experimental data while the harmonic oscillator basis cannot. In the light of these results, we consider it interesting to study the effect of WS basis on the cross sections for $^{16}O(\gamma, \pi^+)^{16}N$ since high momentum transfers, as in the case of electron scattering occur in the first pion-nucleon resonance region. So, if instead of the harmonic oscillator potential (23), we were to use the realistic Saxon-Woods potential

$$v(r) = -U f(r) + 2U_S g(r)(\vec{\ell}\cdot\vec{\sigma}) + V_c(r, R_c), \qquad (25)$$

where

$$f(r) = \left[1 + \exp\left(\frac{r - R_o}{a}\right)\right]^{-1},$$

$$g(r) = \frac{1}{r}\frac{df}{dr},$$

$$V_c(r, R_c) = Z_A Z_\mu e^2 h(r),$$

with

$$h(r) = \begin{cases} 1/r & \text{for } r \geq R_c, \\ \frac{1}{2R_c}\left(3 - \frac{r^2}{R_c^2}\right) & \text{for } r < R_c, \end{cases}$$

$$R_o = R_c = r_o A^{1/3},$$

with $r_o = 1.25$ fm and $a = 0.65$ fm, then one should solve the Schrödinger equation numerically. The central (U) and spin-orbit ($U_s$) potential strengths are fitted to the single particle energies ($\varepsilon$) of the spectra of neighbouring (A+1) and (A-1) nuclei. Thus, in the Saxon-Woods basis, the single-particle wave functions are obtained as numerical solutions of the Schrödinger equation.

5. 'Elementary' particle treatment of nuclei.

In the 'elementary' particle treatment of nuclei, proposed by Kim and Primakoff[9], the usage of the impulse approximation and the details of the nuclear wave functions are avoided, by imbedding all the complexity of the nuclear structure into certain nuclear form factors, which can often be found explicitly by using CVC hypothesis and the PCAC hypothesis along with a pion-pole dominance assumption. Griffiths and Kim[11] studied pion photoproduction by treating the initial

and final nuclei as elementary particles, especially the cases $^{3}He(\gamma,\pi^{+})^{3}H$, $^{11}B(\gamma,\pi^{-})^{11}C$ and $^{16}O(\gamma,\pi^{+})^{16}N$, where nuclei with spin less than or equal to two occur for which adequate spinorial wave functions were used. The approach of Griffiths and Kim[11] is to start with the identity of Callan and Treiman[34]:

$$S_{\mu} = i q_{\nu} \int d^{4}x\, e^{-iq\cdot x}\, \theta(x_{0}) \langle N_{f} | [A_{\nu}^{(\pm)}(x), j_{\mu}^{(0)}] | N_{i} \rangle \qquad (26)$$

where $A_{\mu}^{(\pm)}(x)$ is the axial vector current $j_{\mu}(x) = V_{\mu}^{(3)}(x) + \frac{1}{\sqrt{3}} V_{\mu}^{(8)}(x)$ is the electromagnetic current, $V_{\mu}^{(i)}(x)$ being a member of the octet of vector currents and $q$ is the pion four-momentum. Performing the four-dimensional integrations eq.(26), by parts and using

(i) the soft pion limit $(q \to 0)$,

(ii) the PCAC hypothesis and,

(iii) the algebra of currents,

in conjunction with the Lehmann-Symanzik-Zimmerman[35] reduction technique, the photoproduction matrix element is related to the nuclear axial vector matrix element.

$$M^{(\lambda)} = n_{i} n_{f} \sqrt{4\pi}\, e\, \epsilon_{\mu}^{(\lambda)}(k) \langle N_{f} \pi^{\pm} | j_{\mu}(0) | N_{i} \rangle$$
$$\simeq i \frac{n_{i} n_{f}}{a_{\pi} m_{\pi}} \sqrt{4\pi}\, e\, \epsilon_{\mu}^{(\lambda)}(k) \langle N_{f} | A_{\mu}^{(\pm)}(0) | N_{i} \rangle \qquad (27)$$

where $\epsilon_\mu^{(\lambda)}(k)$ is the photon polarization vector, $n_i$ and $n_f$ are the normalization factors, $e$ the electric charge, $a_\pi = 0.95$ the pion decay coupling constant and $m_\pi$ the mass of the pion. Since the most general form of the nuclear matrix element of the axial current satisfying both the Lorentz and gauge invariance requirements involves the axial vector and induced pseudoscalar form factors for the transition $N_i \rightarrow N_f$, the next vital step is the evaluation of these form factors. Using pion-pole-dominance assumption, these form factors are related to electromagnetic form factors which in turn are determined from electron-nuclear scattering experiments. The cross section is related to $\sum_{spins}\sum M^{(\lambda)}(M^{(\lambda)})^\dagger$ and the evaluation of this quantity proceeds on standard lines depending upon the spins of the initial and final nuclei--in the case of $^3H$ and $^3He$, being spin $\frac{1}{2}$ nuclei, Dirac spinors are involved; in the case of $^{11}B$ and $^{11}C$ being spin 3/2 nuclei, Rarita-Schwinger spinors appear; in the case of $^{16}O(0^+)$ and $^{16}N(2^-)$, the former requires no polarization vector while the latter is associated with the symmetric, rank 2 polarization tensor $W_{\mu\nu}^{(M)}$ $(\mu,\nu = 1,2,3,4\,;\ M = \pm 2, \pm 1, 0)$.

More recently, Pascaul and Sanchez-Gomez[12] introduced invariant form factors using the covariant spin formalism in the study of pion photoproduction when the initial and final nuclei are treated as 'elementary' particles. They claim that their new formulation is valid for nuclei of arbitrary spin. Further, they get results which are completely different from those obtained by Griffiths and Kim, which they assert is due to an algebraic error in the paper of Griffths and Kim.

## 6. Results and discussions

### 6.1. Photoproduction of positive pions from $^{16}O$.

In the first instance, numerical calculations for the reaction $^{16}O(\gamma, \pi^+)^{16}N$ were made by Devanathan et. al[1] treating the ground state of $^{16}O$ to be spherical and the low-lying states of $^{16}N$ to be described by either the IPM or the particle-hole wave functions of Gillet and Vink Mau[36] in the Tamm-Dancoff Approximation - GV(IDA). The harmonic oscillator size parameter (b) value was varied between 1.5 fm and 2.0 fm. and its effect on the partial cross sections was found to be negligible except for the $2^-$ final state of $^{16}N$. So, we take the value of b obtained from charge distribution measurements, viz. b = 1.76 fm. In general, we find that the major contribution to the total cross section comes from transitions to $2^-$ and $3^-$ levels, while the contribution from transition to $0^-$ level is almost negligible. Thus, our results clearly indicate the relative importance of the final states of the nucleus as against the expectation of Meyer et. al[17] that the cross section would depend more on the number of states than on their specific details. The energy dependence of the total cross section calculated in the IPM, when compared with the experimental results of Meyer et. al.[17] show that the theoretical results are almost four times larger-than the data.

The cross section can be reduced by introducing a cut off parameter in the radial integral (13). The integral has limits 0 and $\infty$ in the volume production model, whereas in the surface production model the limits are $r_0$ to $\infty$. Physically, the surface production model amounts to assuming all or most of the pions produced in the interior of the nucleus are directly, reabsorbed. In the case of surface production of pions, the radial integrals $\langle j_\ell(kr)\rangle_{p,h}$ given by (13) have to be evaluated numerically. Unfortunately, $r_0$, the lower limit of integration, is not a well defined quantity because of the diffuse surface of the nucleus and furthermore the radial integrals are sensitive to $r_0$. The total cross section is found to be very sensitive to $r_0$.

The particle-hole wave functions yield cross sections which are smaller than those obtained with the IPM, but they are still larger, by almost a factor of two, than the experimental results. We have shown that this discrepancy can be accounted for by invoking the phenomenological surface production mechanism.

The particle-hole wave functions used for the bound states of $^{16}N$ have been calculated by Gillet and Vinh Mau[35] not only in the TDA but also in the Random Phase Approximation (RPA)-the RPA wave functions include long-range correlations in the ground state of $^{16}O$ in an approximate way. We find that the TDA and RPA results do not differ appreciably from each other.

Subsequently, Srinivasa Rao and Devanathan[3] have explicitly taken the two-particle-twohole (2p-2h) correlations in the ground state of $^{16}O$ in our study of $^{16}O(\gamma,\pi^+)^{16}N$. This amounts to treating the ground state to be non-spherical. We have also used the wave functions of Kuo[8] for the bound states of $^{16}N$, calculated with matrix elements derived from the realistic Hamada-Johnson potential which include core-polarization (or 'screening') corrections. Without invoking the phenomenological surface production mechanism, we find that a better agreement between theory and experiment can be obtained when we use the Kuo wave functions with 'screening' for $^{16}N$ states, together with the ground state wave function of $^{16}O$ which explicitly include 2p-2h correlations.

Recently, I redid calculations for $^{16}O(\gamma,\pi^+)^{16}N$ in the Woods-Saxon (WS) basis[4], instead of the harmonic oscillator basis, for reasons given earlier, taking the 2p-2h correlations in the ground state wave function as before and for $^{16}N$ states we take the particle-hole wave functions obtained in TDA by Perez using a WS potential with spin-orbit coupling given by (25). The major difference between WS and harmonic oscillator (h.o.) particle-hole calculations is that less configuration mixing occurs in the WS case. A detailed comparison of the h.o. and WS radial wave functions and radial integrals for $^{16}O(\gamma,\pi^+)^{16}N$ has been made, elsewhere.

In fig.2, a comparison is made between our theoretical cross sections obtained in both the h.o. basis and WS basis with experimental results of Meyer et. al.[17] for $^{16}O(\gamma,\pi^+)^{16}N$.

The cross sections are plotted as a function of the incident photon energy for the ground state wave functions given in table 1. The markings on the curves are evidently self-explanatory. The dashed line curve marked KUO(III) H.O. is the one obtained with the Kuo[8] wave functions for $^{16}N$ and the ground state wave function III given in table 1, in the h.o. basis with the 'equivalent' size parameter value of b = 1.6 fm, while the solid line curve also marked KUO(III)H.O. is for b = 1.76 fm. The most interesting feature is the flattening of the cross section obtained in the WS basis around the first pion-nucleon region ( $\sim$ 320 Mev) resulting in a theoretical reproduction of the right order of magnitude and shape of the experimental data. It is in this energy region that high momentum transfers (400-600 MeV) occur. It is to be noted that in comparison with results in the h.o. basis the large reduction due to the change of basis is compensated for by the smaller reduction due to small configuration mixing in the particle-hole wave functions obtained by Perez[37] in WS basis.

As far as the theoretical study of $^{16}O(\gamma, \pi^+)^{16}N$ is concerned we find that with the resolution in ambiguity of the nuclear wave functions, we are able to almost reproduce the experimental data both in magnitude and direction without resorting to either surface production of pions or final state interactions (FSI) of the outgoing pion with the residual nucleus. We wish to recall the finding of Saunders[27] that around 250 MeV incident photon energy, the FSI is more significant for neutral than for charged pion photoproduction and that in the case of charged pion photoproduction FSI are of some importance only in the forward angles ($<30\%$) and there too the reduction in the differential

cross section is at most of the order of 20-25 %. We therefore conjecture that an explicit calculation including FSI will enable us to make the correct choice of the ground state wave function of $^{16}O$, given by either II or III of table 1.

In fig.3 we compare the results obtained in our theoretical study with those of Griffiths and Kim[11] and Pascaul and Sanchez-Gomez[12] in their 'elementary' particle treatment of nuclei for the $0^+ \rightarrow 2^-$ 'partial' cross section in $^{16}O(\gamma, \pi^+)\,^{16}N$. The results of Pascaul and Sanchez-Gomez[12] are completely different from those of Griffiths and Kim[11] which is attributed by the former to a wrong equation used by the latter. Pascaul and Sanchez-Gomez[12] obtain two curves, using two different forms for the invariant vector form factors which fit the experimental data for their variation as a function of momentum transfer. One of these (shown in figure 3) is in remarkable agreement with our results obtained for the $0^+ \rightarrow 2^-$ transition in the Saxon-Woods basis, using the Perez wave function for the $2^-$ state of $^{16}N$. The experimental points given are those of Meyers et. al. for the total cross section which is obtained as the sum of the 'partial' cross sections to the four low-lying bound states of $^{16}N$. In fig.3, we have also shown, for the sake of comparison our results obtained in the Saxon-Woods basis with Perez wave functions for $^{16}N$ states, the total cross section which is a sum due to transitions to the four low-lying bound states of $^{16}N$.

6.2. Positive pion photoproduction from $^{11}B$:

Devanathan, Srinivasa Rao and Sridhar[2] have studied $^{11}B(\gamma, \pi^+)^{11}Be$ using the IPM for the nucleus and volume and surface production mechanisms. It is well known that there are only two-low-lying states of $^{11}Be(J^P = \frac{1}{2}^-, \frac{1}{2}^+)$ which are bound and subsequently decay by $\beta$-emission and these are the transitions which are observed by Dyal and Hummel[17] in their experimental results. As in the case of $^{16}O(\gamma, \pi^+)^{16}N$, the process $^{11}B(\gamma, \pi^+)^{11}Be$ is similar to muon capture, in that the same initial and final nuclear states are involved. Rood[38], in his study of muon capture by $^{11}B$ to the two lowest levels of $^{11}Be$, has observed that while in the case of $^{16}O(0^+) \rightarrow {}^{16}N(2^-, 0^-, 3^-, 1^-)$ transitions only one particle is involved, in the case of $^{11}B(3/2^-) \rightarrow {}^{11}Be(1/2^-, 1/2^+)$ each of the three protons in the $1P_{3/2}$ level of the $^{11}B$ ground state may make the transition to a neutron in the $1P_{1/2}$ or $2S_{1/2}$ level in the final state leaving the other two protons in the $1P_{3/2}$ level. Using the IPM, if we follow the analysis of Rood[38], we get a $1/\sqrt{2}$ factor when we evaluate the pion photoproduction matrix element : $Q = \frac{1}{\sqrt{2}} \langle f | T | i \rangle$ for $^{11}B(\gamma, \pi^+)^{11}Be$. We find that a good quantitative agreement with experimental results of Dyal and Hummel[17] can be obtained, when we invoke the surface production mechanism with $r_0 = 2.55$ fm. and use the size parameter value determined by the electron scattering experiments: b = 1.55 fm.

6.3. Negative pion photoproduction from $^{11}B$

Devanathan, Srinivasa Rao and Sridhar[2] have studied the reaction $^{11}B(\gamma, \pi^-)^{11}C$ using the IPM to describe the initial and final nuclear states and in the absence of a detailed description of the same study the cross section in the volume and surface production models. Since the states of $^{11}C$ have not yet been enumerated correctly, following Laing and Moorehouse we assume that the bound states of $^{11}C$ stable against nucleon emission result from the single particle transitions: $1P_{3/2} \rightarrow {}^1P_{3/2}$ $1P_{1/2}$, $1d_{5/2}$, $1d_{3/2}$ and $2s_{1/2}$. Using the values of the parameters: $b$ = 1.55 fm. and $r_0$ = 2.55 fm. which give a good fit with experimental data for $^{11}B(\gamma,\pi^+)^{11}Be$, we find that our results obtained in the surface production model assuming the bound states of $^{11}C$ to result from only the single particle transitions: $1P_{3/2} \rightarrow 1P_{3/2}, 1P_{1/2}$ and $1d_{5/2}$ are in reasonably good agreement with experimental results of Dyal and Hummel[17].

Griffiths and Kim[11] have made a calculation using the 'elementary' particle treatment of nuclei only for the transition to the ground state of $^{11}C$. They write down an Adler-Weisberger type sum rule to argue that the total cross section should be about five times the cross section to the ground state of $^{11}C$. They therefore multiply their result obtained for the ground state transition by a factor five and compare it favourably with experimental results. This suggestion of Griffiths and Kim[11] that the ground state contribution is about one-fifth of the sum over all contributions is not confirmed by the recent results

on $^{11}B(\gamma,\pi^-)^{11}C$ of Janecek[39]. Furthermore, the error in Griffiths and Kim[11], pointed out by Pascaul and Sanchez-Gomez[12] might also adversely alter the results quoted by the former.

Recently, Janecek[39] has theoretically studied $^{11}B(\gamma,\pi^-)^{11}C$ using the multipole amplitudes of Berends, Donnachie and Weaver[40] and the impulse approximation. He has considered the transitions from the ground state of $^{11}B$ to all those states of $^{11}C$ which lie under 7.545 MeV, the threshold energy for $\alpha$-decay of $^{11}C$-viz. the levels $3/2^-$(g.s.), $1/2^-$, $5/2^-$, $3/2^-$, $1/2^+$, $7/2^-$, $5/2^+$ and $3/2^+$. He finds that the major contributions to the cross section come from the transitions to the ground state and the $7/2^-$ state of $^{11}C$. He also finds that among the five multipoles considered: $E_{0+}$, $E_{1+}$, $E_{2^-}$, $M_{1+}$ and $M_{1^-}$, the electric $(E_{0+})$ and magnetic $(M_{1+})$ dipoles give rise to two peaks and these account for the shape of the cross section curve. However, experiment contains only one peak corresponding to the position of the electric dipole peak. Janecek[39] suggests that an experimental measurement of the differential cross section would reveal whether the negative pion photoproduction from $^{11}B$ is in fact dominated by the electric dipole transition. Janecek[39] however finds that neither the form nor the magnitude of the total cross section studied as a function of incident photon energy agrees with experimental results. In fact, his theoretical results with b = 2fm are about ten times too large!

At present, we can only hope that the advent of the meson factories will provide a wealth of experimental data on pion photoproduction from nuclei, including angular distributions of the cross sections, a study of which will enable us to get a better understanding of the process of charged pion photoproduction from nuclei

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## Figure Captions

Fig.2. Total cross section for $\pi^+$ photoproduction from $^{16}O$. The curves are marked X(Y)Z where X stands for the model for $^{16}N$ states, Y denotes the nature of the ground state wave function of $^{16}O$ and Z refers to the basis in which the calculation has been done.

Fig.3. Cross section for $\gamma + {}^{16}O(0^+) \rightarrow \pi^+ + {}^{16}N(2^-)$. The curves marked (A) and (B) refer, respectively, to the theoretical results of Griffiths and Kim (ref.11) and of Pascual and Sánchez-Gómez (ref.12) obtained assuming 'elementary' particle treatment for nuclei. The curves marked (C) and (D) refer, respectively, to our theoretical results obtained in the Saxon-Woods basis assuming IPM and PEREZ wave functions for $^{16}N(2^-)$ state, including 2p-2h correlations in the ground state of $^{16}O$. The dashed line curve is our result corresponding to the case where the final $^{16}N$ nucleus may be in states of $J^P = 2^-, 0^-, 3^-$ and $1^-$, described by PEREZ wave functions in the Saxon-Woods basis treating the ground state of $^{16}O$ to be 'deformed' (ref.4).

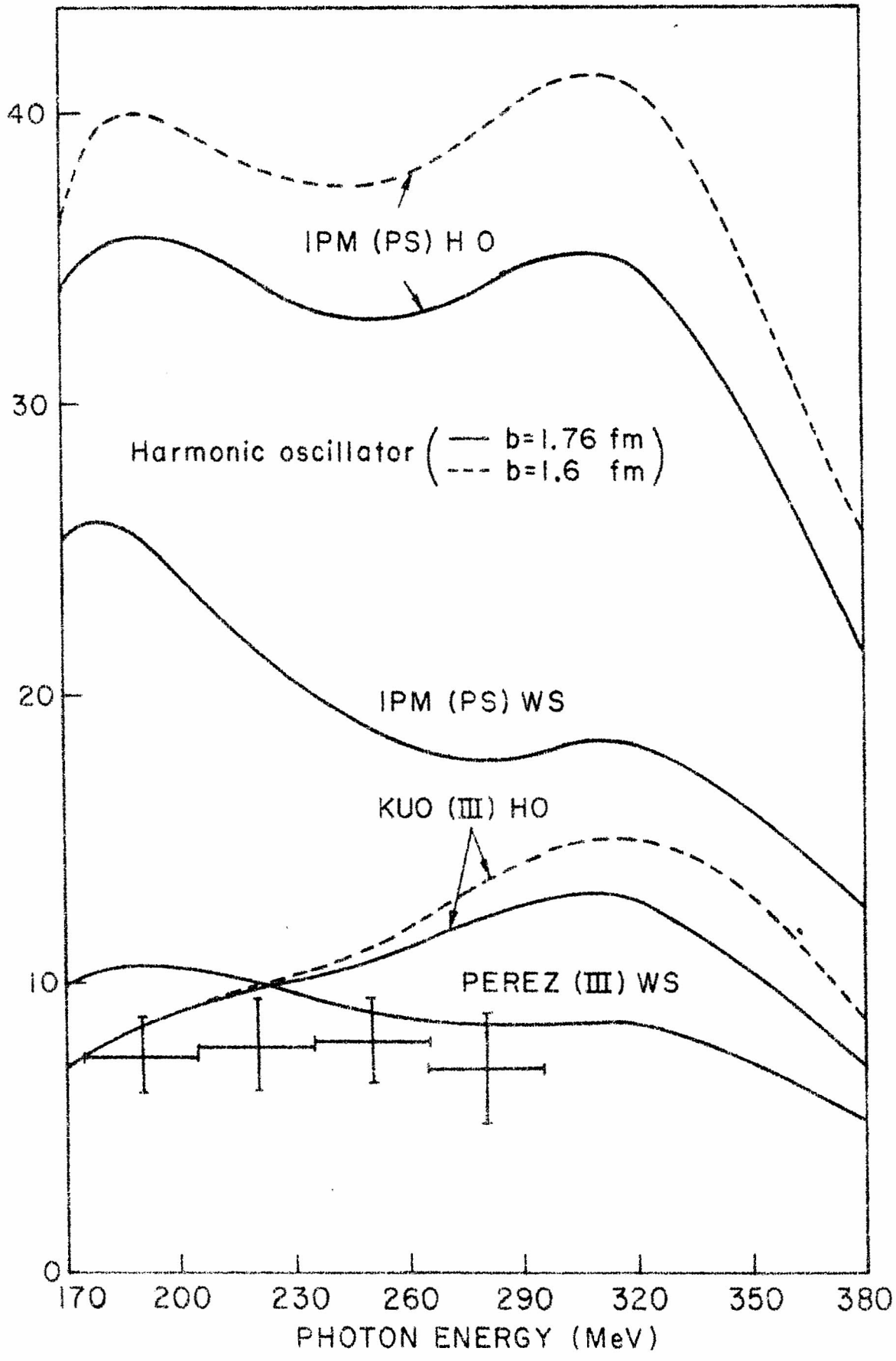
40
30
20
10
0
IPM (PS) H O
Harmonic oscillator (— b=1.76 fm, --- b=1.6 fm)
IPM (PS) WS
KUO (III) HO
PEREZ (III) WS
170
200
230
260
290
320
350
380
PHOTON ENERGY (MeV)

FIGURE 2

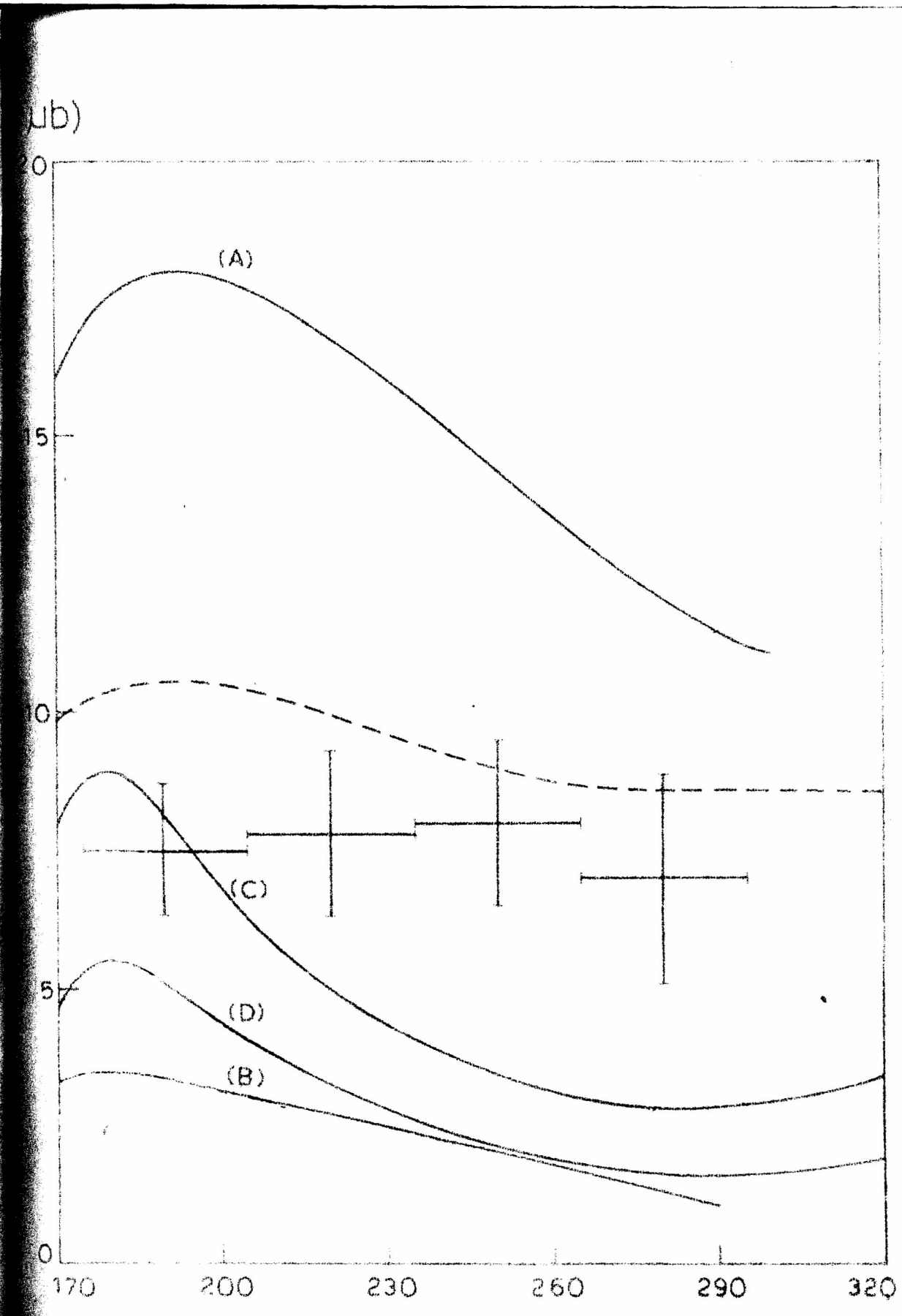
μb)
20
15
10
5
0
(A)
(C)
(D)
(B)
170
200
230
260
290
320
INCIDENT PHOTON ENERGY (MeV)

FIGURE . 3

# unified descriptions of electromagnetic AND WEAK INTERACTIONS AND LEPTONIC QUARKS.

T.S.Santhanam,
MATSCIENCE,MADRAS-600020, INDIA.

Abstract:

An alternative solution to a recently advocated "symmetry clash" in unified theories of electromagnetic and weak interactions is proposed. The model introduces leptonic quarks.

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Of many models proposed recently[1] to put electromagnetism and weak interactions in a unified description, Lipkin[2] has advocated the marriage of these with Han - Nambu [3] model of strong interactions, to avoid "symmetry-clashes" in these domains. We propose here[4], as an alternative, a basic structure in the leptonic domain.

Salam-Weinberg[5] model proposes the scheme $SU(2) \times U(1)$ for electromagnetic and weak interactions. This, as is known, involves a neutral current besides the electromagnetic current. On the other hand, the model of Georgi and Glashow[6] is based on O(3) and has only electromagnetic current as the neutral current. Now, if one tries to unify the scheme with the already successful SU(3) classification of hadrons, the following "symmetry-clash" occurs[2]. While the charge operator in O (3) has only integral or half integral eigenvalues, that of

SU(3) has only integral or third integral eigenvalues. One description of unification involves in enlarging the SU(3) structure to give integral charges. Whether one enlarges this to SU(4) or to Han-Nambu model, an additional degree of freedom is introduced (charm) either directly in the former case or through the permutation symmetry in the latter model. Both these descriptions require the existence of charmed currents.

We propose here that the same "clash" can be overcome by postulating an SU(3) structure to the leptons, and keeping the SU(3) structure for hadrons unchanged. This means introducing "leptonic" quarks. The physical leptons and their antiparticles are constructed from these quarks as a $qqq$ combination. This will mean our assigning four leptons and four new leptons to the eight dimensional representation. As is obvious, there exists a choice to select out four leptons from $(e^-, \mu^-, \nu_\mu, \nu_e)$ and their antiparticles. One can think of cancellation of neutral currents between hadronic and leptonic domains.

The above model would require the existence of new heavy leptons and their couplings to the electrons and muons. Besides, this presupposes a fundamental structure to the leptons which are otherwise known to behave like strictly local objects. Only experiments can decide in favour or against this model. Once having given a structure to the leptons, it is quite

possible that the $\mu - e$ puzzle may have a possible answer in this direction.

The author has benefited from a discussion with Professor E.C.G.Sudarshan.

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# SOME UNFAMILIAR PROBLEMS IN MUON CAPTURE

V. Devanathan
Department of Nuclear Physics, University of Madras,
MADRAS - 25.

## 1. Introduction

The natural source of $\mu^-$ is from $\pi^-$ decay and these muons are found to be polarized in the direction of their flight. When they travel through matter, they are slowed down, caught in Bohr orbits around nuclei by electromagnetic interaction, cascade down to lower orbits by emission of X-rays, known as mu-mesic X-rays and then are finally captured by nuclei by weak interaction. It is this weak interaction capture process that is of interest to us. The investigation has a two-fold purpose: (i) to know precisely the muon capture interaction Hamiltonian and their coupling constants and (ii) to probe the nuclear structure. Experimentally, it is simple to observe the total capture rates and, to a less extent, the partial capture rates in nuclei. It is to a study of these observables, a lot of attention has been devoted all these days. The main purpose of this lecture is to draw the attention to some other observables in muon capture process and stress the importance of investigating these observables.

The asymmetry and polarization of the recoil nucleus, the asymmetry and polarization of the emitted neutron, the cascade process in muon capture and the gamma-neutrino angular correlations in muon capture - these are some of the unfamiliar problems[1-6] in muon capture, to which I have devoted my attention in the last few years. The main motivation for such a study is to look for observables other than scalar. The total and partial capture rates correspond to observables of scalar quantities and the expressions for the capture rates involve the coupling constants and the nuclear matrix elements in some particular combination. By restricting the study to capture rates alone, one will only test this particular combination and it is not possible to infer without any ambiguity the values of the several coupling constants that occur in muon capture interaction Hamiltonian. The asymmetry of recoil nucleus or of emitted neutron in muon capture correspond to a pseudo-scalar observable and the expression for this observable involves the coupling constants and the nuclear matrix elements in a different linear combination. The polarization corresponds to a pseudo-vector quantity and the expression for this observable results in a further different linear combination of the muon capture coupling constants and the nuclear matrix elements. Thus by investigating different observables, one can, in principle, determine in an

unambiguous way the several coupling constants of the muon capture interaction Hamiltonian.

3. Asymmetry and Polarization of the recoil nucleus

Here we are considering a nuclear transition from bound to bound state resulting from muon capture

$$\mu^- + A \rightarrow B + \nu_\mu$$

Due to parity non-conservation in weak interaction, there will be an asymmetry[7-9] in the angular distribution of recoil nuclei about the direction of muon polarization. In general, the recoil nucleus will be polarized and if the nuclear spin $> \frac{1}{2}$, then tensor polarization will also occur besides vector polarization. If the recoil direction is not observed, then it is easy to show that the final nucleus will have only vector polarization in the direction of muon polarization. Since the final nucleus decays by $\beta$-emission, the polarization of the ground state of the final nucleus can be experimentally measured by observing the asymmetry of the $\beta$-decays.

A detailed investigation of the polarization of the recoil nucleus has been made in ref. 7. Here we only discuss the final results. It is found that the recoil nuclear polarization is insensitive to the nuclear wave function. This is indeed a great advantage since one can draw unambiguous information about the coupling constants, uninfluenced by nuclear structure uncertainties. In the particular reaction

$$\mu^- + {}^{12}C \longrightarrow {}^{12}B(1^+) + \nu_\mu$$

it was observed that the capture rate depended very much on the nuclear wave functions used whereas the polarization was independent of them. For instance, the simple j-j coupling shell model yielded a capture rate[10] which is about 5 times the experimental value but the general lp-shell wave function gave a good agreement with experiment. On the other hand, the polarization was found to be almost the same, whichever be the wave function that is used.

3. Asymmetry and Polarization of the emitted neutron in muon capture

Here we are considering muon capture by nuclei resulting in the emission of neutrons

$$\mu^- + X \longrightarrow Y + n + \nu_\mu$$

The final state is a three body system and the use of proper kinematics makes the asymmetry coefficient sensitive to the nuclear structure effects. A plane wave approximation for the emitted neutron greatly simplifies the problem. Bogan[11] has shown that the inclusion of momentum dependent term which contribute less than 10% for the capture rate, greatly disturbs the asymmetry coefficient. Besides, he has given an approximate way of taking into account the final state interaction. The better method of taking into account the final

state interaction of the emitted neutron with the residual nucleus is to represent the nucleus by an optical potential and solve the Schrodinger equation with this optical potential for the neutron. This has been done recently by Boussey and Vinh Mau[12] and calculations are also in progress by our group.

The polarization of the emitted neutron was first studied by Devanathan et. al.[3] The nuclear matrix elements were evaluated and expressed in terms of the density matrix $\rho$ in the spin space of the emitted neutron. Then the longitudinal polarization of the neutron is given by $Tr\,(\vec{\sigma}.\hat{n}\,\rho)/Tr\,\rho$, where $\hat{n}$ is the unit vector in the direction of the emitted neutron. The expression for this involves the neutrino direction and since the neutrino is not observed in the experiment, an integration has to be performed over its direction. The numerical calculations show that the longitudinal polarization of the neutrons is fairly large $\langle \vec{\sigma}.\hat{n} \rangle \approx -0.8$ and so it should not be difficult to observe it experimentally.

4. Cascade process in muon capture and Gamma-Neutrino angular correlations

In many muon capture processes, we come across a nuclear cascade. It is a two-step process resulting in a nuclear transition from an initial spin state $J_i$ to one of the possible intermediate states $J_I$ by muon capture and then to the ground state with spin $J_f$ by gamma deexcitation. The gamma ray yield can be expressed in terms of certain observable correlations[13] involving

the three vectors - the gamma ray momentum $\underline{k}$ , the neutrino momentum $\underline{\nu}$ and the muon polarisation $\underline{P}$ .

The $\gamma$-ray yield $Y$ in a given direction with the neutrino making an angle $\theta_{k\nu}$ in the nuclear cascade process can be written as

$$Y = \frac{(2\pi)^{-5}}{2J_i + 1} k^2 \nu^2 \int |Q|^2 \, d\Phi_\nu$$

where

$$|Q|^2 = \sum_{M_i, M_f, M_I, M_I'} \langle J_f M_f \mid H_a \mid J_I M_I \rangle \langle J_I M_I \mid H_b \mid J_i M_i \rangle \langle J_f M_f \mid H_a \mid J_i M_I' \rangle^* \langle J_I M_I' \mid H_b \mid J_i M_i \rangle^* .$$

An integration over the azimuthal angle $\Phi_\nu$ of neutrino emission, which is not observable, is performed. $H_a$ and $H_b$ are, respectively, the interaction Hamiltonians for gamma emission and muon capture and they have the structure.

$$H_a = \sum_{n, \gamma, m_\gamma} (A_\gamma^{m_\gamma})^* \, U_\gamma^{m_\gamma}(n)$$

$$H_b = \sum_{n, \lambda, m_\lambda} (B_\lambda^{m_\lambda})^* \, V_\lambda^{m_\lambda}(n)$$

In the above expansions, n denotes the nucleon index and $U_\lambda^{m_\lambda}$ , $V_\lambda^{m_\lambda}$ refer to the spherical tensor operators in the nucleon coordinates responsible for nuclear transition. The

spherical tensor components $A_{\lambda}^{m_\lambda}$ and $B_{\lambda}^{m_\lambda}$ refer to the radial field and, in our particular case, they are the spherecal harmonics with arguments denoting, respectively, the gamma and the neutrino directions. Substituting these interaction Hamiltonians, the matrix elements can be evaluated as outlined in Ref. 5 and the gamma ray yield can be expressed in terms of the observable angles $\theta_{\nu k}$ and $\theta_{Pk}$.

In a recent experiment, Miller et al[13] have investigated the gamma-neutrino correlations in muon capture by observing the Doppler-broadened gamma ray, using a method suggested by Grenacs et al[14]. The Doppler shift of $\gamma$-ray energy due to nuclear recoil from neutrino emission is given by

$$E = E_o (1 + \upsilon_o \cos\theta)$$

where E and $E_o$ are the energies of the doppler-shifted gamma ray and of the gamma ray in the rest frame of the emitting nucleus, $\upsilon_o$ is the nuclear recoil velocity and $\theta_{\nu k}$ is the angle between the gamma and the neutrino momenta. By observing the energy distribution of the Doppler-shifted gamma ray, Miller et al. have studied the gamma-neutrino correlations.

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# ON A NEW SPIN HALF WAVE EQUATION*

T.S.Santhanam and A.R.Tekumalla
MATSCIENCE, The Institute of Mathematical Sciences,
Madras-600020

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Dirac's[1] epoch-making paper of 1946 was the starting point of the study of relativistic wave equations. The generalisation of the Dirac matrices and Clifford algebra and their various physical applications have been studied extensively by Professor Alladi Ramakrishnan[2] and his group at Matscience.

Here we shall deal with a different generalisation, an equation for a spin 1/2 massive particle recently advocated by Capri[3] and Santhanam[4], an equation inequivalent to the Dirac equation and obtained by the techniques of higher spin.

## 1. Linear Relativistic Wave Equations for Arbitrary Spin

First we indicate what we mean by techniques of higher spin. In writing linear relativistic equations for particles of arbitrary spin, subsidiary conditions are usually involved which are not compatible when one considers interactions with external fields like, for instance, the electromagnetic field.

Bhabha[5] in 1945 wrote linear relativistic equations of this type without subsidiary conditions but it turned out that his attempts resulted in multi-mass and multi-spin solutions. He investigated relativistic equations of the form

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*Presented by A.R.Tekumalla.

$$(\beta_\mu p_\mu + m)\,\Psi = 0\,, \qquad (1)$$

where $p_\mu$ are the differential operators $-i\frac{\partial}{\partial x_\mu}$, $\beta_\mu$ are four matrices of suitable dimensions describing the spin properties of the particle and $m$ is an arbitrary parameter. The invariance of this equation under the restricted Lorentz group implies the commutator equation

$$[\beta_\mu, I_{\nu\lambda}] = \delta_{\mu\nu}\beta_\lambda - \delta_{\mu\lambda}\beta_\nu\,, \qquad (2)$$

where the $I_{\nu\lambda}$ are the generators of the Lorentz transformation. Bhabha investigated the solution of this commutator equation and gave the most general solution for the matrices $\beta_\mu$ upto some arbitrary constants (see appendix). However, the general philosophy of Bhabha's programme was to take representations of the orthogonal group in five dimensions which branches to a sum of representations of the Lorentz group. Hence he had the additional assumption that the Lorentz generators $I_{\mu\nu}$ are proportional to the commutators $[\beta_\mu, \beta_\nu]$ :

$$I_{\mu\nu} = \lambda\,[\beta_\mu, \beta_\nu]\,. \qquad (3)$$

This helped to fix the arbitrary constants uniquely but resulted in multimass solutions.

Later investigations by Harish Chandra[6] (1947) and Fronsdal and Visconti[7] (1956) gave algebraic conditions on $\beta_\mu$

in order that the equation may describe a particle of unique mass. Harishchandra's condition reads

$$\beta_o^{n+1} = \beta_o^{n-1} \tag{4}$$

and the Umezawa-Visconti condition yields

$$n = 2f \tag{5}$$

where $f$ is the maximum spin contained in the field function.

Recently, Capri (1969) has made an attempt at selecting out a definite spin by imposing further algebraic conditions on the matrices $\beta_\mu$. He constructs four matrices so that the equation satisfies the following conditions:

a) Lorentz Invariance: This is ensured by following Bhabha's prescription for writing the form of $\beta_\mu$.

b) Unique rest mass: To ensure this $\beta_\mu$ must satisfy the Harish-Chandra and Umezawa-Visconti conditions.

c) Hermitianisability: In order that the wave equation be derivable from a Lagrangian and to be able to define a current and energy-momentum tensor, we require the existence of a hermitianising matrix $\eta$ such that

$$\eta^{-1} \beta_\mu \eta = \beta_\mu^\dagger \tag{6}$$

d) Unique Spin: To pick up solutions corresponding to a unique spin, we demand, after Capri :

$$J^2 E = s(s+1) E , \tag{7}$$

where E is the projection operator which projects on to the subspace spanned by the eigenvectors of $\beta_\mu$ corresponding to the non-zero eigenvalues. Explicitly, for Capri:

$$E = \beta_0^{2b-1} . \tag{8}$$

We note here that the additional assumption that the Lorentz generators are proportional to the commutators of the $\beta$ matrices is not used but the other conditions fix $\beta_\mu$ uniquely.

2. Equation for a spin 1/2 particle

The same method has been used by Capri to select out a lower spin by reversing the systematics of the above procedure: in other words, certain algebraic conditions are imposed to eliminate higher spins to get equations for lower spins. Specifically, he has constructed an alternative scheme of equations for a particle of spin 1/2, inequivalent to the Dirac equation. He takes the reducible representation of the Lorentz group

$$D = (1, \tfrac{1}{2}) \oplus (0, \tfrac{1}{2}) \oplus (\tfrac{1}{2}, 0) \oplus (\tfrac{1}{2}, 1) \tag{9}$$

and imposes the following conditions on $\beta_0$, namely $\beta_0^4 = \beta_0^2$ as minimal equation to ensure unique rest mass, $J^2\beta_0^2 = \frac{1}{2}(\frac{1}{2}+1)\beta_0^2$ to make $\Psi$ represent a particle of unique spin. The latter condition is incorporated by demanding that, in the Wild[8] basis ($J^2$ diagonal basis) the diagonal blocks in $\beta_0^2$ corresponding to spin 3/2 be nilpotent and the spin 1/2 block idempotent, and

Finally demanding $\beta_o$ have the minimal equation $\beta_o^4 = \beta_o^2$ .
His matrices are 16 x 16 dimensional and are :

$$\beta_o = \begin{bmatrix} & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ & & & & & & & & 0 & 0 & & & & & & \\ & & & & & & & & \sqrt{2}c & 0 & & & & & & \\ & & & & & & & & -c & 0 & & & & & & \\ & & & & & & & & 0 & c & & & & & & \\ & & & & & & & & 0 & -\sqrt{2}c & & & & & & \\ & & & & & & & & 0 & 0 & & & & & & \\ & & & & & & & & -1 & 0 & & & & & & \\ & & & & & & & & 0 & -1 & & & & & & \\ & & -1 & 0 & & & & & & & & & & & & \\ & & 0 & -1 & & & & & & & & & & & & \\ & & 0 & 0 & & & & & & & & & & & & \\ & & -c & 0 & & & & & & & & & & & & \\ & & 0 & -\sqrt{2}c & & & & & & & & & & & & \\ & & \sqrt{2}c & 0 & & & & & & & & & & & & \\ & & 0 & c & & & & & & & & & & & & \\ & & 0 & 0 & & & & & & & & & & & & \end{bmatrix} ; \quad \beta_1 = \begin{bmatrix} & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ & & & & & & & & \sqrt{2}c & 0 & & & & & & \\ & & & & & & & & 0 & 0 & & & & & & \\ & & & & & & & & 0 & c & & & & & & \\ & & & & & & & & -c & 0 & & & & & & \\ & & & & & & & & 0 & 0 & & & & & & \\ & & & & & & & & 0 & -\sqrt{2}c & & & & & & \\ & & & & & & & & 0 & -1 & & & & & & \\ & & & & & & & & -1 & 0 & & & & & & \\ & & 0 & 1 & & & & & & & & & & & & \\ & & 1 & 0 & & & & & & & & & & & & \\ & & -\sqrt{2}c & 0 & & & & & & & & & & & & \\ & & 0 & -c & & & & & & & & & & & & \\ & & 0 & 0 & & & & & & & & & & & & \\ & & 0 & 0 & & & & & & & & & & & & \\ & & c & 0 & & & & & & & & & & & & \\ & & 0 & \sqrt{2}c & & & & & & & & & & & & \end{bmatrix}$$

$$\beta_2 = \begin{bmatrix} & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ & & & & & & & & \sqrt{2}ic & 0 & & & & & & \\ & & & & & & & & 0 & 0 & & & & & & \\ & & & & & & & & 0 & ic & & & & & & \\ & & & & & & & & ic & 0 & & & & & & \\ & & & & & & & & 0 & 0 & & & & & & \\ & & & & & & & & 0 & \sqrt{2}ic & & & & & & \\ & & & & & & & & 0 & -i & & & & & & \\ & & & & & & & & i & 0 & & & & & & \\ & & 0 & i & & & & & & & & & & & & \\ & & -i & 0 & & & & & & & & & & & & \\ & & -\sqrt{2}ic & 0 & & & & & & & & & & & & \\ & & 0 & -ic & & & & & & & & & & & & \\ & & 0 & 0 & & & & & & & & & & & & \\ & & 0 & 0 & & & & & & & & & & & & \\ & & -ic & 0 & & & & & & & & & & & & \\ & & 0 & -\sqrt{2}ic & & & & & & & & & & & & \end{bmatrix} ; \quad \beta_3 = \begin{bmatrix} & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ & & & & & & & & 0 & 0 & & & & & & \\ & & & & & & & & -\sqrt{2}c & 0 & & & & & & \\ & & & & & & & & -c & 0 & & & & & & \\ & & & & & & & & 0 & -c & & & & & & \\ & & & & & & & & 0 & -\sqrt{2}c & & & & & & \\ & & & & & & & & 0 & 0 & & & & & & \\ & & & & & & & & -1 & 0 & & & & & & \\ & & & & & & & & 0 & 1 & & & & & & \\ & & 1 & 0 & & & & & & & & & & & & \\ & & 0 & -1 & & & & & & & & & & & & \\ & & 0 & 0 & & & & & & & & & & & & \\ & & c & 0 & & & & & & & & & & & & \\ & & 0 & \sqrt{2}c & & & & & & & & & & & & \\ & & \sqrt{2}c & 0 & & & & & & & & & & & & \\ & & 0 & c & & & & & & & & & & & & \\ & & 0 & 0 & & & & & & & & & & & & \end{bmatrix}$$

Note that these matrices have the Dirac matrices at the Centre and some elements above and below on the Dirac 'tails'.

Here we wish to mention that Capri does not achieve what he sets out to. His matrices do not have $\beta_0^4 = \beta_0^2$ as minimal equation and a hermitianising matrix does not exist for his matrices. We shall come to these points later.

A justification for equations of Capri's type has been offered by a group at Matscience led by Santhanam. The problem has been analysed from the algebraic point of view and it is shown that if we project spin s less than f where f is the maximum spin in the field function, the Umezawa-Visconti condition has to be modified to

$$\beta_0^{L+1} = \beta_0^{L-1} \quad \text{where} \quad 2s \leq L \leq 2f \qquad (11)$$

and the condition for unique spin becomes

$$J^2 \beta_0^{L-1} = s(s+1) \beta_0^{L-1} . \qquad (12)$$

For the sequence of representations considered here, the Klein-Gordon divisor $d(\partial)$ which is assumed to be a polynomial in the derivative operators can be written as :

$$d(\partial) = m + \beta_\mu \partial_\mu + a_{\mu\nu} \partial_\mu \partial_\nu + a_{\mu\nu\lambda} \partial_\mu \partial_\nu \partial_\lambda \cdot (13)$$

If the field function can have spin 3/2 also, the coefficient $a_{\mu\nu\lambda}$ must be non-zero. If on the other hand, $a_{\mu\nu\lambda} = 0$ , (the only

other spin in the field function is 1/2) we get, after going through Umezawa-Visconti's procedure,

$$\sum_{\text{Perm}} \beta_\mu \beta_\nu \beta_\lambda = 2\left[\delta_{\mu\nu}\beta_\lambda + \delta_{\nu\lambda}\beta_\mu + \delta_{\mu\lambda}\beta_\nu\right] \quad (14)$$

This equation admits of three distinct algebras of the Duffin-Kemmer-Petaiou type. The first is the DKP algebra describing particles of spin 1 and 0. The second is a new algebra:

$$\beta_\mu \{\beta_\nu, \beta_\lambda\} = 2\,\delta_{\nu\lambda}\beta_\mu , \quad (15)$$

where the bracket stands for the anticommutator. This is the algebra obeyed by the matrices of Capri. We find a third, new, algebra:

$$\{\beta_\lambda, \beta_\nu\}\beta_\mu = 2\,\delta_{\nu\lambda}\beta_\mu . \quad (16)$$

A representation of the last algebra is obtained by taking the hermitian adjoints of the matrices of Capri. These new algebras describe particles of spin 1/2 as can be demonstrated by Capri's method $\left( J^2 E = \frac{1}{2}\left(\frac{1}{2}+1\right) E \right)$.

By choosing representation's with higher spins to begin with and choosing different values of L where $2\lambda \leq L \leq 2\lambda$ and satisfying the condition $J^2 \beta_0^{L-1} = \frac{1}{2}\left(\frac{1}{2}+1\right)\beta_0^{L-1}$, we get a hierarchy of equations describing a spin 1/2 particle.

3. The Hermitianising Matrix:

We shall now show that a hermitianising matrix does not exist for these matrices (and for their hermitian adjoints) and also that these matrices are diagonalisable.

The two algebras given by eqs.(15) and (16) are in general inequivalent. Now if $\beta_\mu$ satisfy either of these algebras, $S^{-1}\beta_\mu S$ also satisfy the same algebra as can be verified by direct substitution. If we now demand the existence of a hermitianising matrix $\eta$ such that $\eta^{-1}\beta_\mu\eta = \beta_\mu^+$, the two algebras become equivalent and $\beta_\mu$ and $\beta_\mu^+$ will have to satisfy both algebras. Then using eq.(15)

$$\beta_0^2(\beta_\mu\beta_\nu + \beta_\nu\beta_\mu) = 2g_{\mu\nu}\beta_0^2$$

and then using (eq.16), we get

$$\beta_\mu\beta_\nu + \beta_\nu\beta_\mu = 2g_{\mu\nu}\beta_0^2 \qquad (17)$$

with $\beta_0^3 = \beta_0$. Thus when $\eta$ exists, the matrices anti-commute but their squares are not unity. Now the matrices of Capri satisfy eq.(15) but do not satisfy eq.(16) nor do they satisfy eq.(17). Hence, from the above argument, a hermitianising matrix does not exist for these matrices. In fact, the matrix $\eta$ given by Capri is incorrect as can be verified directly or inferred from the above arguments.

A parallel argument shows that the hermitian adjoints of the above matrices too are not hermitianisable. It can be

explicitly demonstrated that the moment we insist on a hermitianising matrix, we obtain a trivial generalization of the Dirac matrices.

It is also obvious now that since the Capri matrices satisfy eq.(15), we have $\beta_0^3 = \beta_0$ . Hence, in the canonical form, $\beta_0$ has the form

$$\beta_0 = \begin{bmatrix} 1 & & \\ & -1 & \\ & & 0 \end{bmatrix} \tag{18}$$

and hence is diagonalisable.

4. Representations of the New Algebras

We shall now briefly discuss the realisations of these algebras by matrices. We look for representations which do not satisfy the Dirac Clifford algebra(which trivially satisfy these new algebras). There are 65 linearly independent elements of the first algebra (1, $\beta_\mu$ $\beta_0^{\rho}$ $\beta_1^{\rho_1}$ $\beta_2^{\rho_2}$ $\beta_3^{\rho_3}$ , where $\mu$ =0,1,2,3 and $\rho$ = 0,1) and it can be seen that any set of four matrices of the form

$$\beta_\mu = \begin{bmatrix} 0 & X_\mu \\ 0 & \gamma_\mu \end{bmatrix} \tag{19}$$

where $\gamma_\mu$ are the Dirac matrices and $X$ is an arbitrary matrix of four columns satisfies our first new algebra eq.(15). If some further symmetry properties are required (like, for instance, parity invariance in the choice of the reducible representation) $\beta_\mu$ may be of the form

$$\beta_0 = \begin{bmatrix} 0 & X_0 & 0 \\ & 1 & 0 \\ 0\ 1 & & \\ 0\ Y_0 & 0 & \end{bmatrix}, \quad \beta_i = \begin{bmatrix} & X_i & 0 \\ 0 & \sigma_i & 0 \\ 0\ -\sigma_i & & 0 \\ 0\ Y_i & & \end{bmatrix} \qquad (20)$$

where $X_\mu$ and $Y_\mu$ are now arbitrary matrices of two columns, $\underline{1}$ is a unit matrix in two dimensions and $\sigma_i$ are the Pauli matrices. The matrices of Capri are of this form. The dimensions of X and Y depend on the choice of the reducible representation and are fixed by demanding $\beta_\mu$ satisfy Bhabha's commutator equation for Lorentz invariance. To do this we choose $\beta_0$ in the Wild basis directly, using the above prescription. Thus

$$\beta_0 = \begin{bmatrix} & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ & 1 & 0 & 0 \\ 0\ 0\ 1 & & & \\ 0\ 0\ c & 0 & & \\ 0\ 0\ 0 & & & \end{bmatrix}. \qquad (21)$$

This identifies the non-vanishing coefficients in Bhabha's prescription. This form of $\beta_0$ automatically satisfies the condition for unique spin i.e. $J_0^2 \beta_0 = \lambda(\lambda+\frac{1}{2})\beta_0$ . All the matrices $\beta_\mu$ can now be written down in the original spinor basis since we have identified the non-vanishing arbitrary constants in Bhabha's prescription.

From the form of $\beta_\mu$ displayed, it is clear that a typical characteristic of the matrices is that there are elements only above and below the Dirac part of the matrices. Capri's matrices being of this form, they obey our algebra. Thus it is obvious that the matrices do not have $\beta_0^4 = \beta_0^2$ as minimal equation. Since they obey our algebra, $\beta_0^3 = \beta_0$ .

We now proceed to consider the other algebra defined by eq.(16). By taking the hermitian adjoints of the above matrices, we see $\beta_\mu$ has the form

$$\beta_\mu = \begin{bmatrix} 0 & 0 \\ X_\mu & \gamma_\mu \end{bmatrix} \tag{22}$$

where X has four rows on the horizontal Dirac tail. The number of elements is again 65. And, as before, if further symmetry properties are required,

$$\beta_0 = \begin{bmatrix} \bigcirc & \begin{matrix} 0 & 0 \\ 1 & Y_0 \end{matrix} \\ \begin{matrix} X_0 & 1 \\ 0 & 0 \end{matrix} & \bigcirc \end{bmatrix} \tag{23}$$

and in the Wild basis

$$\beta_0 = \begin{bmatrix} \bigcirc & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & c & 0 \end{matrix} \\ \begin{matrix} 0 & c & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \bigcirc \end{bmatrix} \tag{24}$$

all the $\beta_\mu$ being determined in the original basis as before.

As mentioned earlier, the two algebras discussed here are, in general, inequivalent. If we now require a hermitianising matrix, the two algebras become equivalent and collapse to

$$\{ \beta_\nu , \beta_\lambda \} = 2 g_{\nu\lambda} \beta_0^2 \quad , \quad \beta_0^2 \neq 1 . \tag{25}$$

In this case, the dimension of the algebra is reduced to 17

( $\beta_0^2$ , $\beta_0^{s_0}\,\beta_1^{s_1}\,\beta_2^{s_2}\,\beta_3^{s_3}$ ). Further, the anticommutators commute with all the elements of the algebra

$$\left[\,\beta_\mu\,,\,\{\beta_\nu\,,\beta_\lambda\}\,\right] = 0 \tag{26}$$

for all $\mu, \nu, \lambda$. From eqs.(25) and (26) and Pauli's Fundamental Theorem, it follows that $\beta_\mu$ are either trivial extensions of the Dirac matrices by adding zeros i.e.

$$\beta_\mu = \begin{bmatrix} 0 & 0 \\ 0 & \gamma_\mu \end{bmatrix} \tag{27}$$

or all the $\beta_\mu$ can be reduced to such a form by a single similarity transformation.

As an example of such a set of matrices where the equivalence is not immediately obvious, consider

$$\Gamma_\mu = \beta_0^2\,\beta_\mu \tag{28}$$

where $\beta_\mu$ satisfy eq.(15) and may be taken as the Capri matrices. Then $\Gamma_\mu$ anticommute and

$$\Gamma_\mu\Gamma_\nu + \Gamma_\nu\Gamma_\mu = 2g_{\mu\nu}\Gamma_0^2 \,. \tag{29}$$

Further, $\Gamma_\mu$ have the form

$$\Gamma_0 = \begin{bmatrix} & & X_0 & 0 \\ & 0 & -1 & 0 \\ 0 & -1 & & \\ 0 & Y_0 & 0 & \end{bmatrix} \begin{matrix} (6) \\ (2) \\ (2) \\ (6) \end{matrix}$$

$$\Gamma_i = \begin{bmatrix} 0 & X_0\sigma_i & 0 \\ & -\sigma_i & 0 \\ 0 & \sigma_i & 0 \\ 0 & -Y_0\sigma_i & 0 \end{bmatrix} \qquad (30)$$

the figures in parenthesis on the right indicating the number of rows in the various blocks. It is not at once obvious that they can all be brought by the same similarity transformation to the form $\gamma_\mu$ augmented by zeroes. However, consider the transformation

$$\Gamma_\mu' = U^{-1}\Gamma_\mu U \qquad (31)$$

where

$$U = 1 - \begin{bmatrix} 0 & X_0 & 0 \\ 0 & 0 & \\ & & 0 \\ 0 & Y_0 & 0 \end{bmatrix}, \qquad (32)$$

and

$$U^{-1} = 1 - \begin{bmatrix} 0 & X_0 & 0 \\ 0 & 0 & \\ & & 0 \\ 0 & Y_0 & 0 \end{bmatrix}.$$

Then

$$\Gamma_0' = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & 0 \end{bmatrix} \qquad (33)$$

and

$$\Gamma_i' = \begin{bmatrix} 0 & 0 \\ 0 & -\sigma_i \\ \sigma_i & 0 \\ 0 & 0 \end{bmatrix}$$

which is a trivial extension of the Dirac matrices by adding zeroes. However, a hermitianising matrix exists in this case, for,

$$\Gamma_\mu^{\dagger} = \eta^{-1} \Gamma_\mu \eta \,, \tag{34}$$

where $\eta = U \gamma_5 U^{\dagger}$

and $$\gamma_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .$$

The earlier discussion indicates that by adding more of other spins and then requiring $\beta_0^2 \beta_0^{L-1} = s(s+1) \beta_0^{L-1}$ one can have a hierarchy of algebras of the type

$$\beta_{\mu_1}^{(K+2)} \cdots \beta_{\mu_K}^{(K+2)} \{ \beta_\mu, \beta_\nu \} = 2 g_{\nu\lambda} \beta_{\mu_1} \cdots \beta_{\mu_K}, \tag{35}$$

where the K refers to the minimal equation condition

$$\beta^{K+2} = \beta^K$$

The order of the $K^{th}$ algebra is $4^K 2^K \dfrac{4^K - 1}{4 - 1}$ and the realisation of the $K^{th}$ algebra can be obtained from the $(K-1)^{th}$ algebra. For example,

$$\beta_\mu^{(4)} = \begin{bmatrix} 0 & \kappa^{(4)} \\ 0 & \beta_\mu^{(3)} \end{bmatrix} . \tag{36}$$

5. Magnetic Moment

We shall show now that the magnetic moment of a particle satisfying the new equation is identical with that of the Dirac particle. A recent calculation by Capri[10] shows that the magnetic moment in this case is 5/2 times that of a Dirac particle and hence the new equation is a good candidate for representing the proton. However, we assert this result is wrong and we find that the magnetic moment here is the same as for the Dirac particle of mass m. We write the wave equation as

$$(-i\not{D} + m)\,\Psi = 0\,, \tag{30}$$

where

$$D_\mu = \partial_\mu + ie A_\mu\,.$$

We premultiply by $\beta_o$ and, after some algebra, arrive at

$$i\,\partial_t \Psi_1 = e A_o \Psi_1 + \beta_o\,(-i\,\underline{\Gamma}\cdot\underline{D} + m)\,\Psi_1 \tag{31}$$

where $\Psi_1 = \beta_o^2\,\Psi$ and $\Gamma_\mu = \beta_o^2\,\beta_\mu\,\beta_o^2$,

which is identical with the Dirac equation in the space of $\Psi_1$. Hence the magnetic moment of the particle is the same as for a Dirac particle of mass m.

An independent calculation by Malini B.Menon[11] also shows that, for the equation obtained by using the second

algebra, the same value of magnetic moment is obtained.

6. A new violation of the Umezawa-Visconti condition:

Finally we point out another situation in which the Umezawa-Visconti condition is violated and examine the possible consequences. Recently, Hurley[12] (1970) has attempted to construct a linear relativistic wave equation for a particle of unique mass and spin. He constructs a 65+1 component equation which satisfies the Schroedinger equation in the non-relativistic limit and is Galilean invariant and then extends it to obtain a relativistic wave equation. For the relativistic equation, he starts with the reducible representation of the Lorentz group

$$D = (s, o) \oplus (s-\tfrac{1}{2}, \tfrac{1}{2}) . \qquad (32)$$

For Hurley's matrices, we find $\beta_o$ has, as minimal equation $\beta_o^3 = \beta_o$ for any spin s. This is a flagrant violation of the Umezawa-Visconti condition and further, is not covered by the modified condition of Santhanam et. al. as, in this case, we require the maximum spin to be present in the field function and hence we cannot equate the higher terms in the Klein-Gordon divisor to zero. We are investigating this problem more closely and at present it appears to us that a consequence of this violation of the Umezawa-Visconti condition would be that it will not be possible to quantise the field.

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## Appendix

Bhabha has given the most general solution to the commutator equation

$$[\beta_m , I_{rs}] = g_{mr}\beta_s - g_{ms}\beta_r$$

where $I_{rs}$ are the generators of the homogeneous orthochronous Lorentz transformations on the representation $(k,\ell)$. Let

$$\beta^m = \tfrac{1}{2}\sigma_{\dot\mu\lambda}^{\ m} A^{\lambda\dot\mu}. \qquad K_\nu^{\ \mu} = -I_{mn}\sigma^{m\mu\dot\lambda}\sigma^{n}_{\ \dot\lambda\mu}.$$

and

$$L_\mu^{\ \dot\lambda} = I_{mn}\,\sigma^{m}_{\ \dot\mu\nu}\,\sigma^{n\nu\dot\lambda}$$

where the $\sigma$'s are the Pauli matrices and the rows and columns are labelled by upper undotted and dotted indices respectively. Then, the non-vanishing elements of $A^{\lambda\dot\mu}$ (and hence $\beta^m$) are given by

$$\langle k,\ell \mid A^{\mu\dot\lambda} \mid k+\tfrac{1}{2}, \ell+\tfrac{1}{2}\rangle = c\, v^{\beta}(k+\tfrac{1}{2})v^{\dot\lambda}(\ell),$$

$$\langle k+\tfrac{1}{2}, \ell-\tfrac{1}{2} \mid A^{\mu\dot\lambda} \mid k,\ell\rangle = d\, u^{\beta}(k+\tfrac{1}{2})v^{\dot\lambda}(\ell),$$

$$\langle k,\ell \mid A^{\mu\dot\lambda} \mid k+\tfrac{1}{2}, \ell+\tfrac{1}{2}\rangle = c\, v^{\beta}(k+\tfrac{1}{2})v^{\dot\lambda}(\ell+\tfrac{1}{2}),$$

$$\langle k+\tfrac{1}{2}, \ell+\tfrac{1}{2} \mid A^{\mu\dot\lambda} \mid k,\ell\rangle = d\, u^{\beta}(k+\tfrac{1}{2})u^{\dot\lambda}(\ell+\tfrac{1}{2}),$$

where c and d are arbitrary numbers and $u^{\beta}(k)$ and $v^{\beta}(k)$ are matrices of dimensions $(2k+1)\times(2k)$ and $(2k)\times(2k+1)$ res-

pectively satisfying

$$-u_\mu(k+\tfrac{1}{2})\,v^\mu(k+\tfrac{1}{2}) = v_\mu(k)\,u^\mu(k) = 2k+1,$$

$$v_\mu(k)\,v^\mu(k+\tfrac{1}{2}) = u_\mu(k+\tfrac{1}{2})\,u^\mu(k) = 0,$$

$$-v^\mu(k+\tfrac{1}{2})\,u_\nu(k+\tfrac{1}{2}) = K_\nu{}^\mu(k) + (k+1)\,\delta_\nu{}^\mu,$$

$$u^\mu(k)\,v_\nu(k) = K_\nu{}^\mu(k) - k\,\delta_\nu{}^\mu.$$

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# FINITE LIFETIME EFFECTS OF HOLE STATES IN SINGLE PARTICLE KNOCK-OUT REACTIONS*

R. Shyam
Department of Physics
Indian Institute of Technology, Bombay-76

## ABSTRACT

The hole states created in single particle knock-out reactions are physical states but have finite lifetime. The hole is moving in a complex potential, the imaginary part of which is related to the width of the state, and the hole energy and wave functions are complex. The influence of these features on the momentum distributions of the hole is studied for the knock-out of the 1s and 1p protons from $^{12}C$ and is found to be rather insignificant. However, the overlap integral in momentum space shows a significant change. It is also interesting to note that for life times less than the hole transit time, there is a significant effect on the momentum distribution of the knock-out nucleon in the nucleus.

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# A STUDY OF PION-SCATTERING BY $^{12}C$ IN THE IMPULSE APPROXIMATION*

J. Mahalanabis
Saha Institute of Nuclear Physics, CALCUTTA
and
P. K. Banerjee
Burdwan University, Burdwan, WEST BENGAL

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## I. Introduction

Over the last few years there has been a growing interest in pion-nucleus interaction as a probe of nuclear structure over the more conventional nuclear projectiles[1]. So far most of the analysis of pion-nucleus scattering have been done either using Kisslinger optical model[2] or Glauber model[3]. The Kisslinger and Ericson[4] model works well for low incident energies ($\lesssim$ 100 MeV whereas the Glauber multiple scattering theory is expected to be valid only at high energies ( $\gtrsim$ 500MeV) there still is no firmly established theoretical approach to this problem. This is the region which covers the domain of the first pion-nucleon (3-3) resonance, and one may expect that the effective pion-nucleon ( - ) potential near the (3-3) resonance to be dominated by the strong non-local terms.

Recently Binon et al[5] have measured elastic and inelastic cross-sections of 120-280 MeV negative pions scatterd from $^{12}C$. Although the validity of the Kisslinger and Glauber theory is questionable at these energies, these approximations have been used to analyse the scattering data and the agreement obtained

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* Presented by J.Mahalanabis

for the elastic scattering data is quite considerable[6,7]. For the analysis of inelastic scattering of pions by $^{12}C$, the distorted wave impulse approximation and the Glauber theory have been used[8,9] and agreement to the experiment was obtained except at large angles. In the present work we study the scattering of pions using a very simple model for the nucleus and the impulse approximation. The spirit of the impulse model is that we sum the elementary interactions coherently over all the nucleons in the nuclei. Further we use plane wave for the incident pions. We use as an input the elementary pion nucleon phase shifts, the nuclear size as obtained from the elastic electron scattering data.

II. Calculation Method:

The scattering amplitude for the interaction of the pion with the target nucleus may be written in the single scattering approximation as

$$T_{fi} = \langle \Psi_f | T | \Psi_i \rangle = \langle \Psi_f | \sum_{j=1}^{A} t_j | \Psi_i \rangle \qquad (1)$$

where $\Psi_i$ and $\Psi_f$ denote the initial and final nuclear states, A is the number of nucleons in the target nucleus and $t_j$ is the two-body scattering amplitude which denotes the interaction of the pion with the single jth nucleon in the target. The impulse approximation assumes that the two-body scattering amplitude $t_j$ is given by the free pion-nucleon

amplitude i.e. the structure of the target nucleus has no dyna-
mical effect on the pion nucleon scattering process. The scatte
ing amplitude consists of an iso-scalar as well as isovector par
This may be written in the following form[10)]

$$t = e^{i(\underline{k}_f - k_i)\cdot \underline{r}} \left[ \frac{1}{2}(t_p + t_n) + \frac{1}{2}(t_p - t_n)\tau_z \right], \quad (3)$$

where $\underline{k}_i$, $\underline{k}_f$ denote the incident and outgoing pion momenta, $t_p, t_n$ are the interaction amplitudes of the pion with the proton and neutron respectively, $\tau_z$ is the iso-spin matrix i the isospin space of the nucleon. The amplitudes $t_p$ and $t_n$ may be expressed in terms of a spin-independent and spin-indepen dent term as

$$t = g_o + i\,\underline{\sigma}\cdot\underline{g}_1 \quad (3)$$

where $\underline{\sigma}$ is the spin operator of the nucleon. For pion-nucleon scattering at these energies the scattering is dominated by the strong p-wave. Hence if only contributions from the s- and p-waves are considered, $g_o$ and $g_1$ may be expressed as (in eith iso-spin states, T = 1/2 and T = 3/2 for the pion nucleon syste

$$g_o = f_{S_{1/2}} + (f_{P_{1/2}} + 2f_{P_{3/2}})\,\hat{\underline{k}}_f\cdot\hat{\underline{k}}_i \quad (4)$$

$$g_1 = (f_{P_{1/2}} - f_{P_{3/2}})\,\hat{\underline{k}}_f \times \hat{\underline{k}}_i \quad (5)$$

where $f_{\ell_j}$'s are the partial wave amplitudes in the pion nucleon c-m system. The amplitudes are expressed in terms of the phase shifts $\delta_{\ell_j}$ as

$$f_{\ell_j} = \frac{e^{i\delta_{\ell_j}}}{k} \sin\delta_{\ell_j} \qquad (6)$$

and $k$ is the centre-of-mass momentum. Since in the impulse approximation the $\pi$-N scattering amplitude off-energy-shell is assumed to be equal to the free T-matrix on-energy-shell, these are taken from the phase shifts of Roper et al[11].

The ground state of $^{12}C$ and the 4.43 MeV state are assumed to be described by the nuclear shell model in either L-S or j-j coupling extreme. In the limit of L-S coupling the ground and ex-cited states are in $(1s)^4(1p)^8$ configuration coupled to most symmetric state with spin $J = 0^+$ and $2^+$ respectively. In the j-j coupling scheme the ground state is the $(1s_{1/2})^4(1p_{3/2})^8$ configuration and the extied state is a $(1s_{1/2})^4(1p_{3/2})^7(1p_{1/2})$ configuration.

We have used the L-S coupled shell model wave functions for the ground state and the excited state of $^{12}C$ nucleus. The oscillator parameter is taken as $b = 1.64$ fm.

The calculations are done for elastic scattering and inelastic scattering to the 4.43 MeV state and the results are shown in figures 1, 2 and 3.

III. Discussions.

The results of inelastic scattering to the 4.43 MeV show that remarkably good fits to the experimental data are obtained inspite of the crude approximations made in the theory and the simple model used for the nucleus. At large angles there is some discrepancy between theory and experiment. This is due to the fact that the impulse approximation is not reliable at backward angles where the multiple scattering effects and nuclear correlations become important. It is very interesting to see that good agreement is obtained for the inelastic scattering to the 4.43 MeV state which is known to be a deformed state.

We have also used the TDA wave functions to describe the 4.43 MeV state. The work is in progress but preliminary result shows that the cross-section depends considerably on the nuclear wave functions used. For elastic scattering there is a persistant discrepancy between theory and experiment except at lowest energy. Our calculation predicts the first minimum around $Q_{cm} = 65^{o}$-$70^{o}$ for all energies whereas experimentally the position of the minima moves towards small angles for higher energies in terms of the pion, i.e. the theory gives the first minimum consistently at higher momentum transfer which is just the reverse which was obtained by Lee and McManus[8]. One may attribute the discrepancy to be due to the fact that higher partial waves are becoming important at higher energies.

Since inelastic scattering is not as sensitive to the de-

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## Figure Captions

Figs. 1(A) to 1(G) — Comparison between calculated cross-sections for the excitation of 4.43 MeV state in $^{12}C$, and the experimental results of Binon et al.

Figs. 2(A) and 2(B) — Comparison between calculated cross-sections for the excitation of 4.43 MeV state in $^{12}C$ and the experimental results of Binon et al. The continuous line and the dashed curves are the calculations of Glaubers theory[9] using the independent particle and particle-hole wave function, respectively and the dot-dashed curve is due to the impulse approximation used in this work.

Fig. 3. — Differential cross-section for the elastic scattering on $^{12}C$ for $E_\pi$ = 120 MeV is shown together with the experimental data of Binon et al.

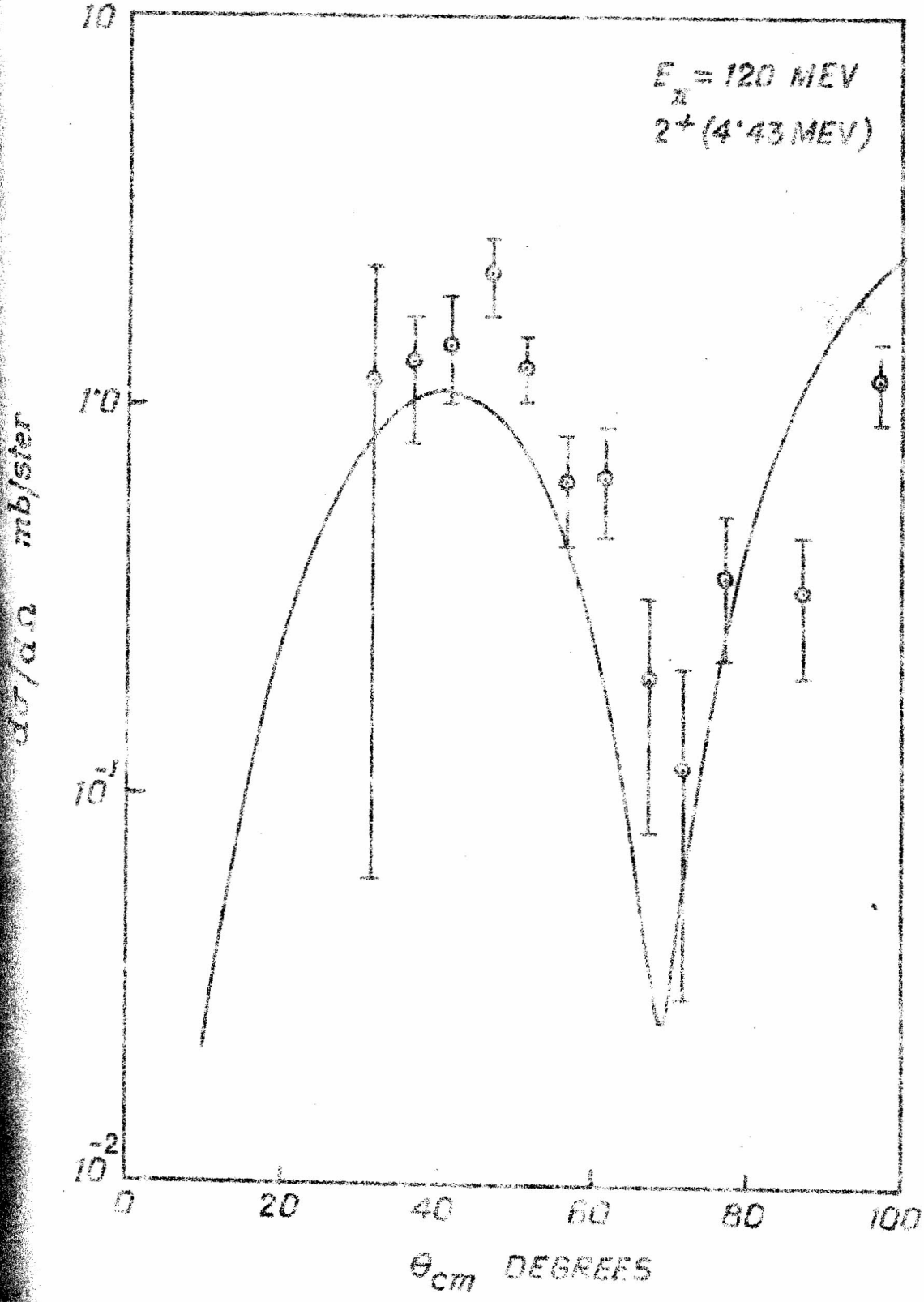

10
1·0
10⁻¹
10⁻²
dσ/dΩ mb/ster
E_π = 120 MEV
2⁺ (4·43 MEV)
0
20
40
60
80
100
θ_cm DEGREES

FIGURE . 1(A)

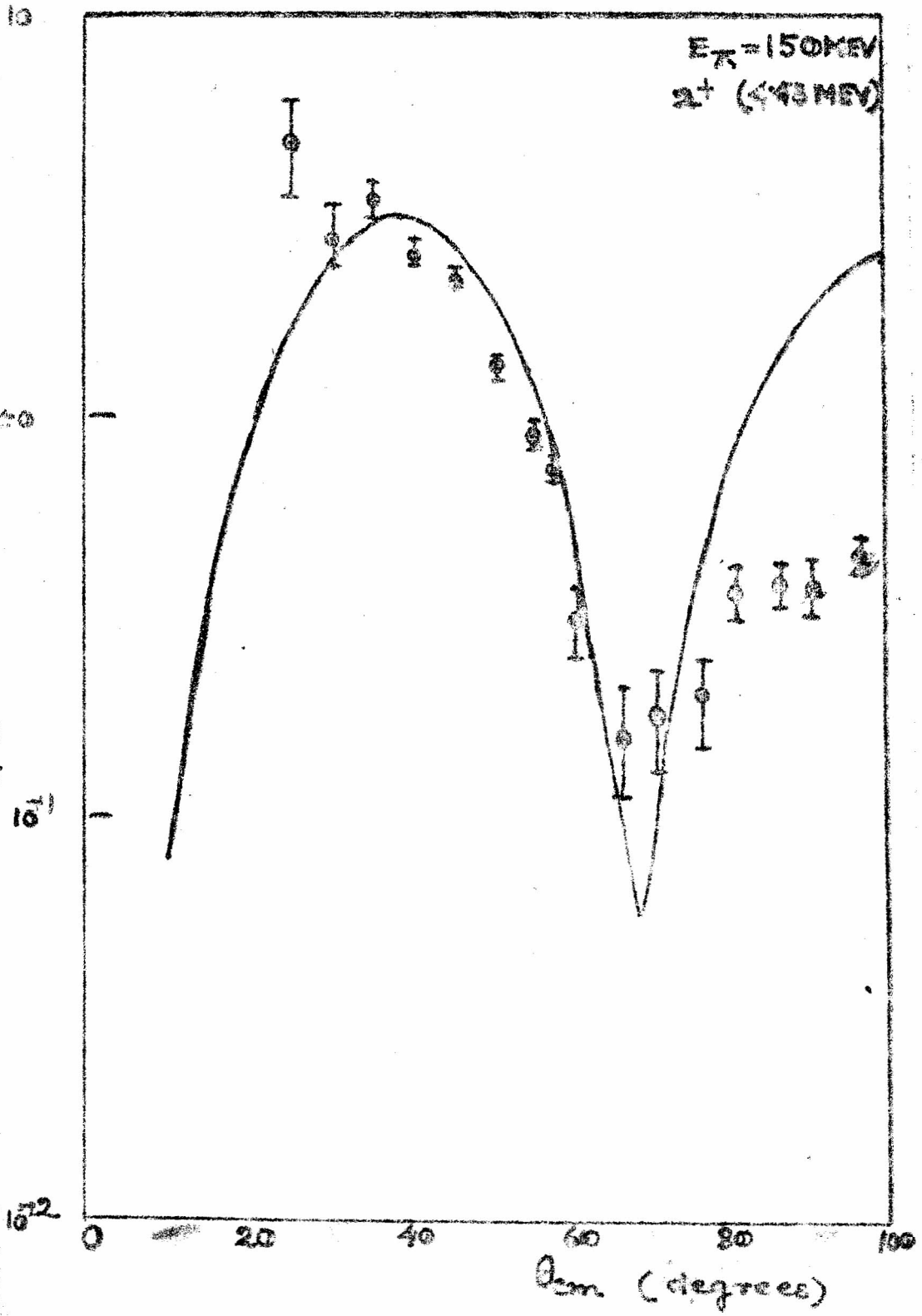

$E_\pi$ = 150 MEV
$2^+$ (4.43 MEV)
10
1
$10^{-1}$
$10^{-2}$
$\frac{d\sigma}{d\Omega}$ (mb/ster)
0
20
40
60
80
100
$\theta_{cm}$ (degrees)

FIGURE . 1(B)

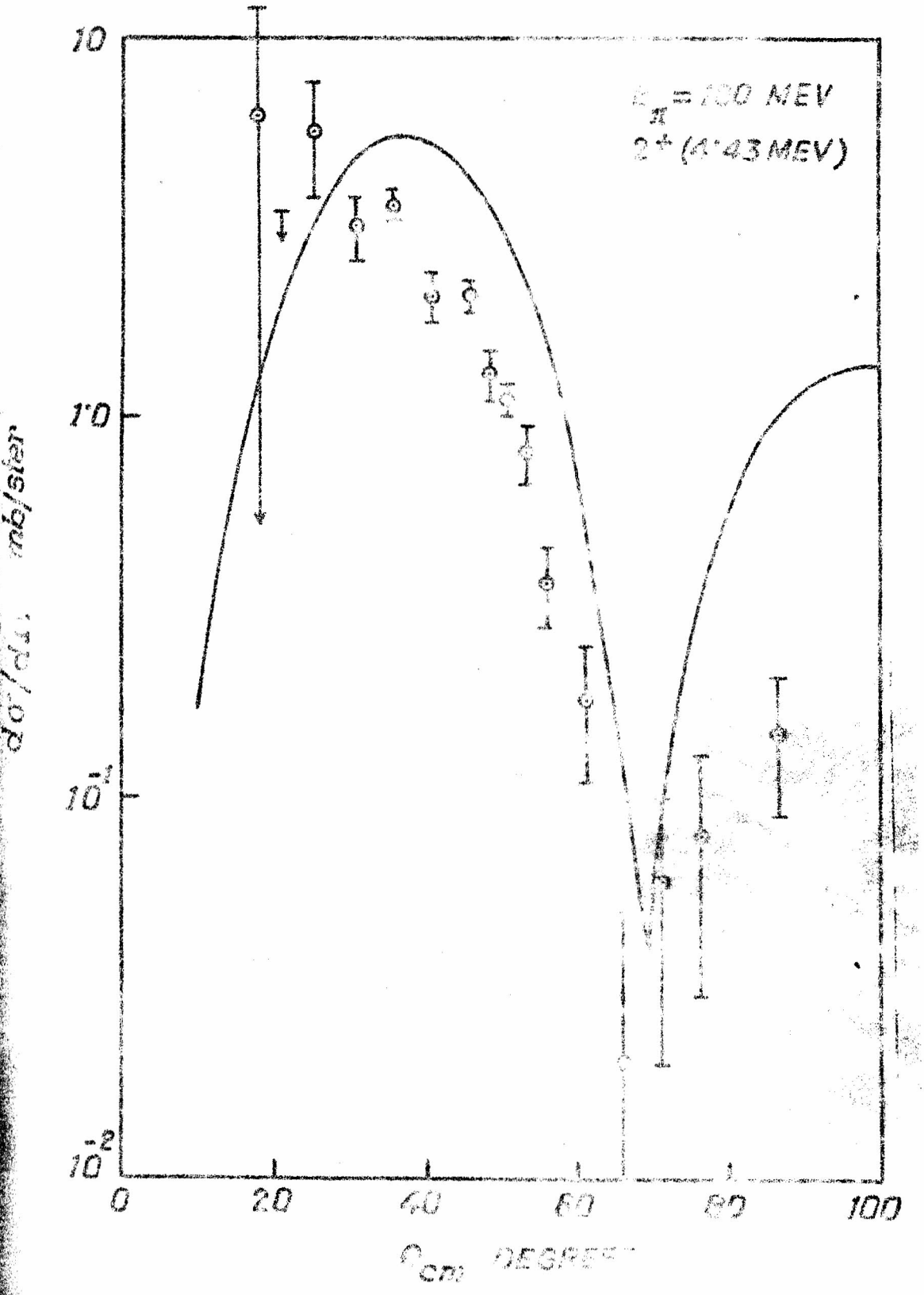

10
1·0
$10^{-1}$
$10^{-2}$
$d\sigma/d\Omega$, mb/ster
$E_\pi$ = 100 MEV
$2^+$ (4·43 MEV)
0
20
40
60
80
100
$\theta_{cm}$ DEGREES

FIGURE . 1(a)

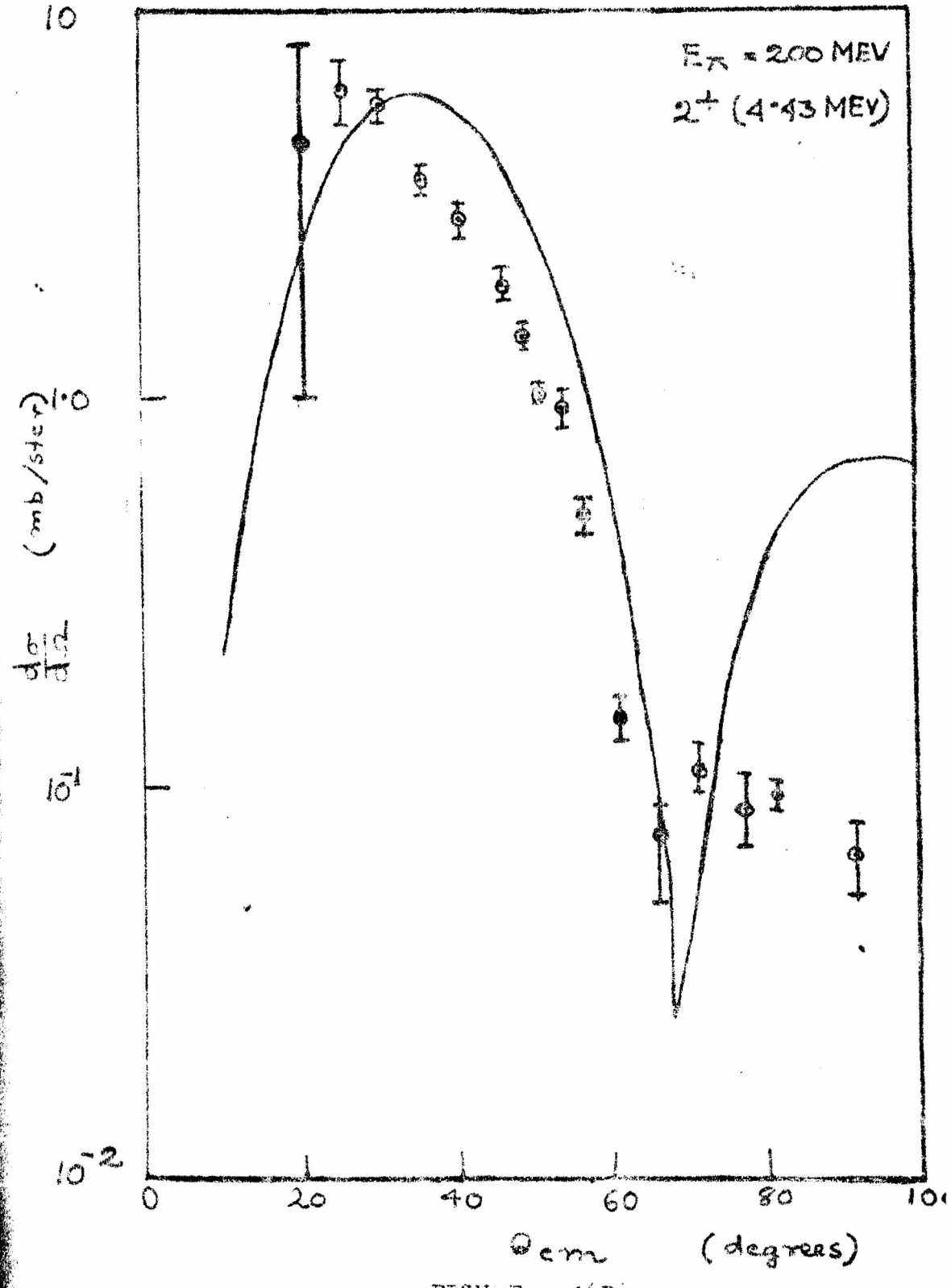

10
1·0
10⁻¹
10⁻²
dσ/dΩ (mb/ster)
E_π = 200 MEV
2⁺ (4·43 MEV)
0
20
40
60
80
100
θ cm (degrees)

FIGURE . 1(D)

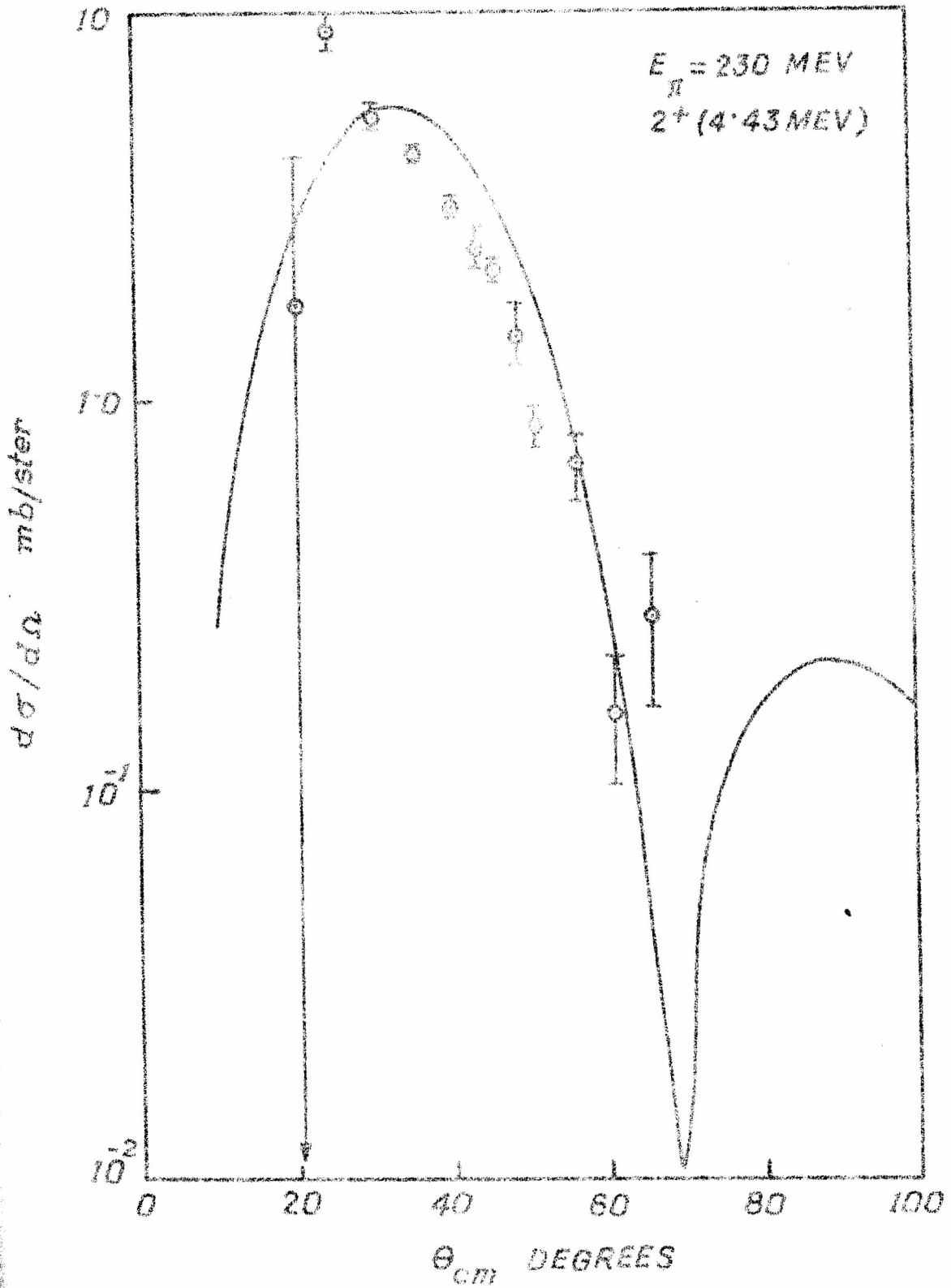
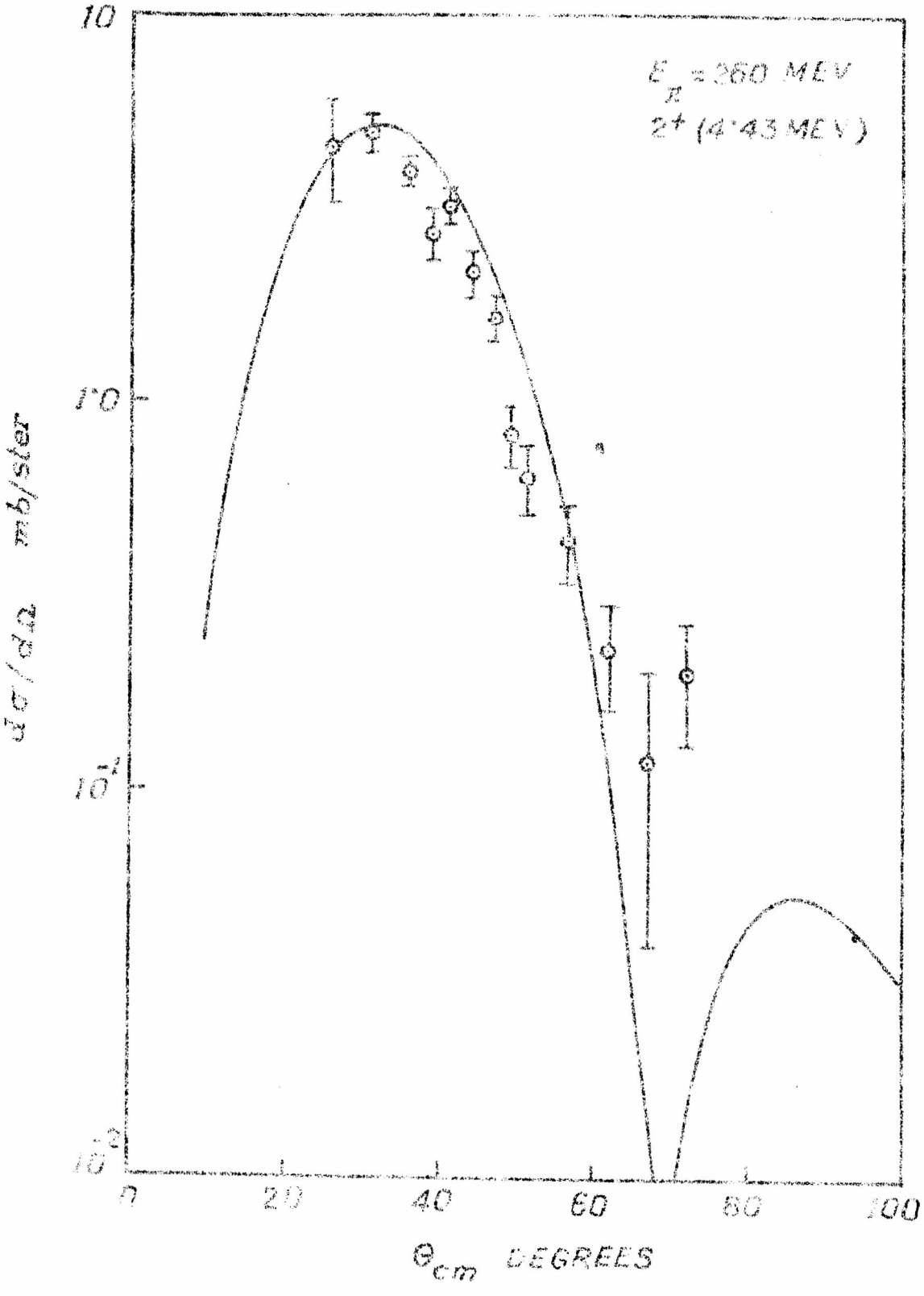
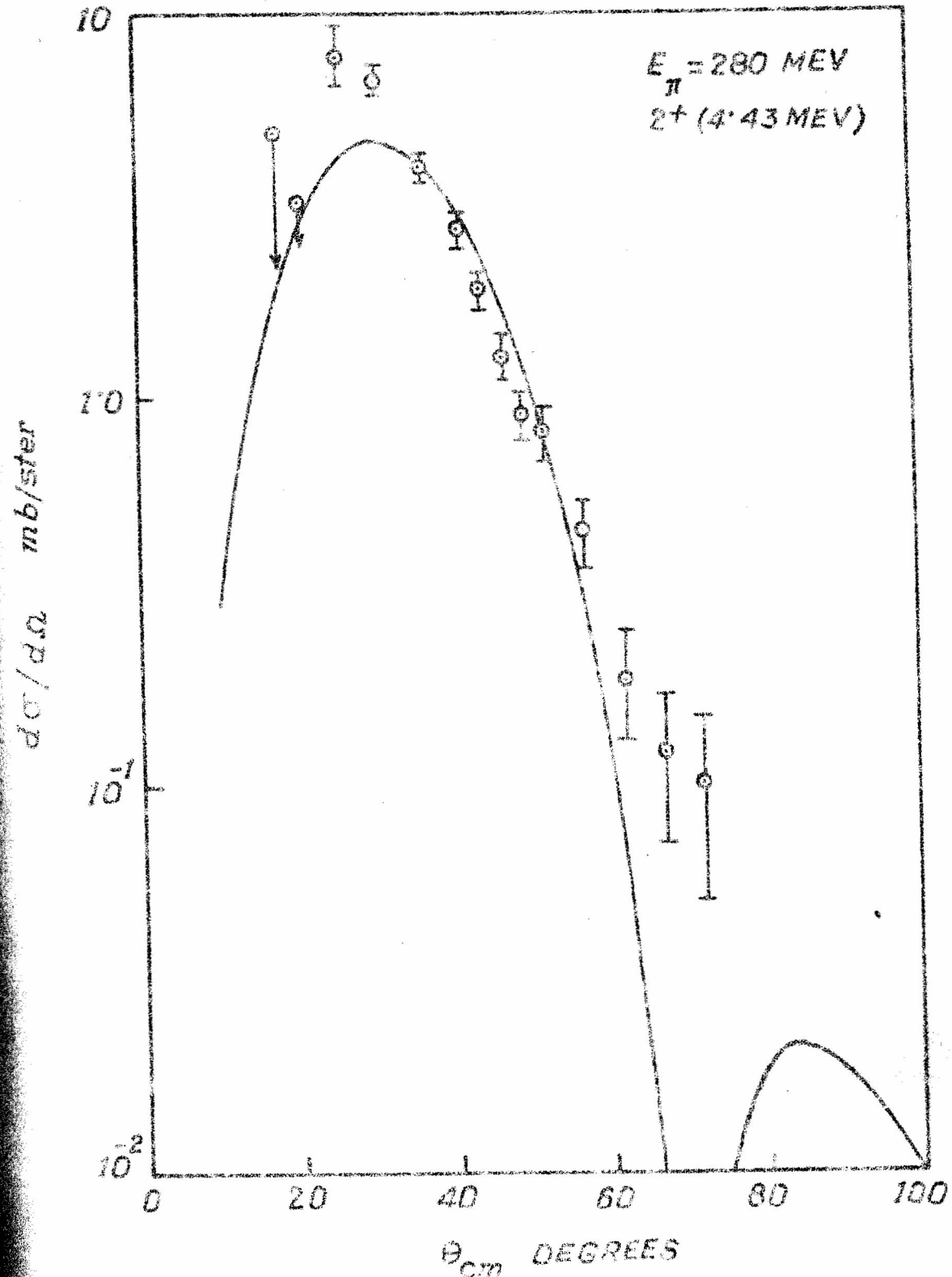

10
1·0
10⁻¹
10⁻²
dσ/dΩ mb/ster
E_π = 230 MEV
2⁺ (4·43 MEV)
0
20
40
60
80
100
θ_cm DEGREES

FIGURE . 1(B)


10
1·0
10⁻¹
10⁻²
dσ/dΩ mb/ster
E_π = 260 MEV
2⁺ (4·43 MEV)
0
20
40
60
80
100
θ_cm DEGREES


10
1·0
$10^{-1}$
$10^{-2}$
$d\sigma/d\Omega$ mb/ster
$E_\pi$ = 280 MEV
$2^+$ (4·43 MEV)
0
20
40
60
80
100
$\theta_{cm}$ DEGREES

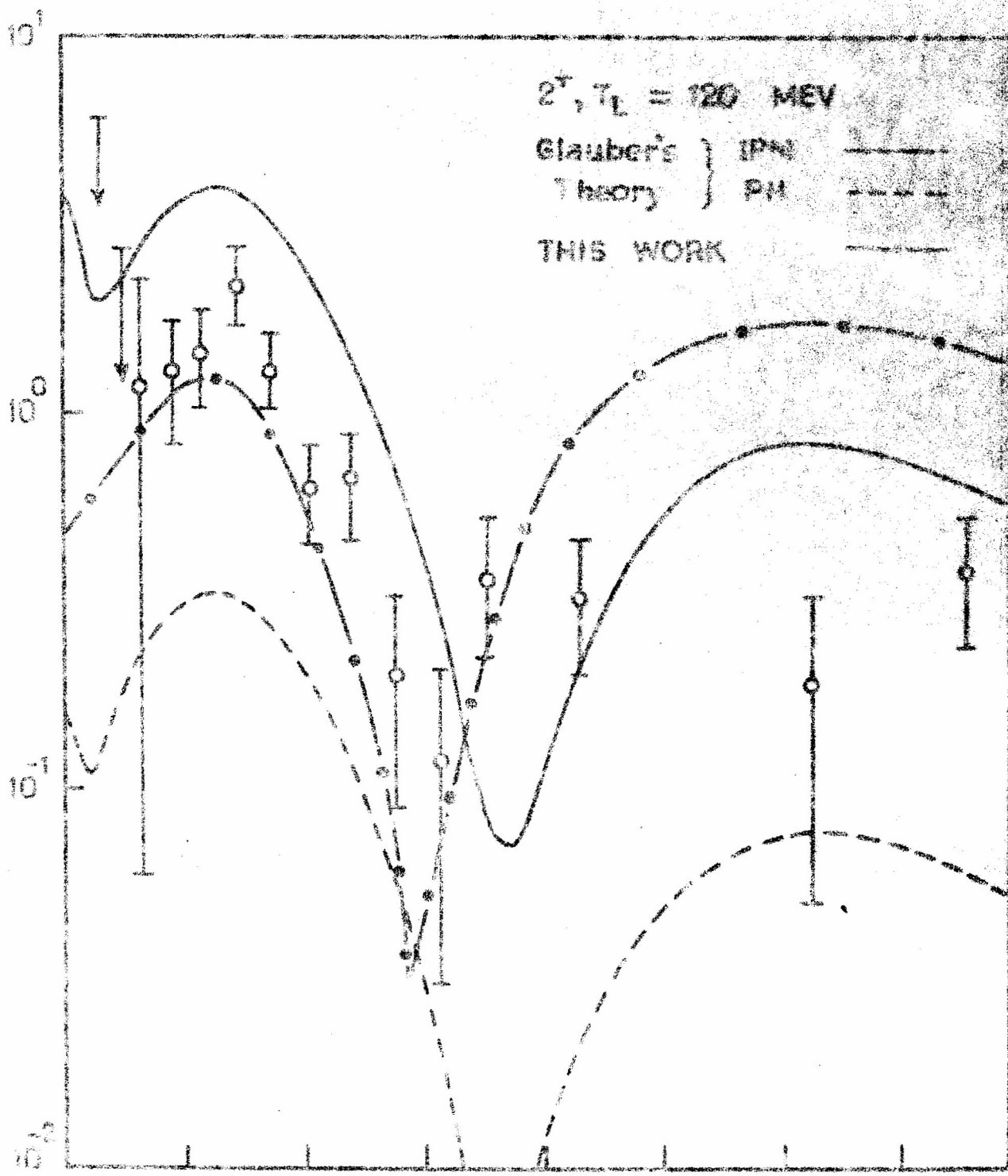
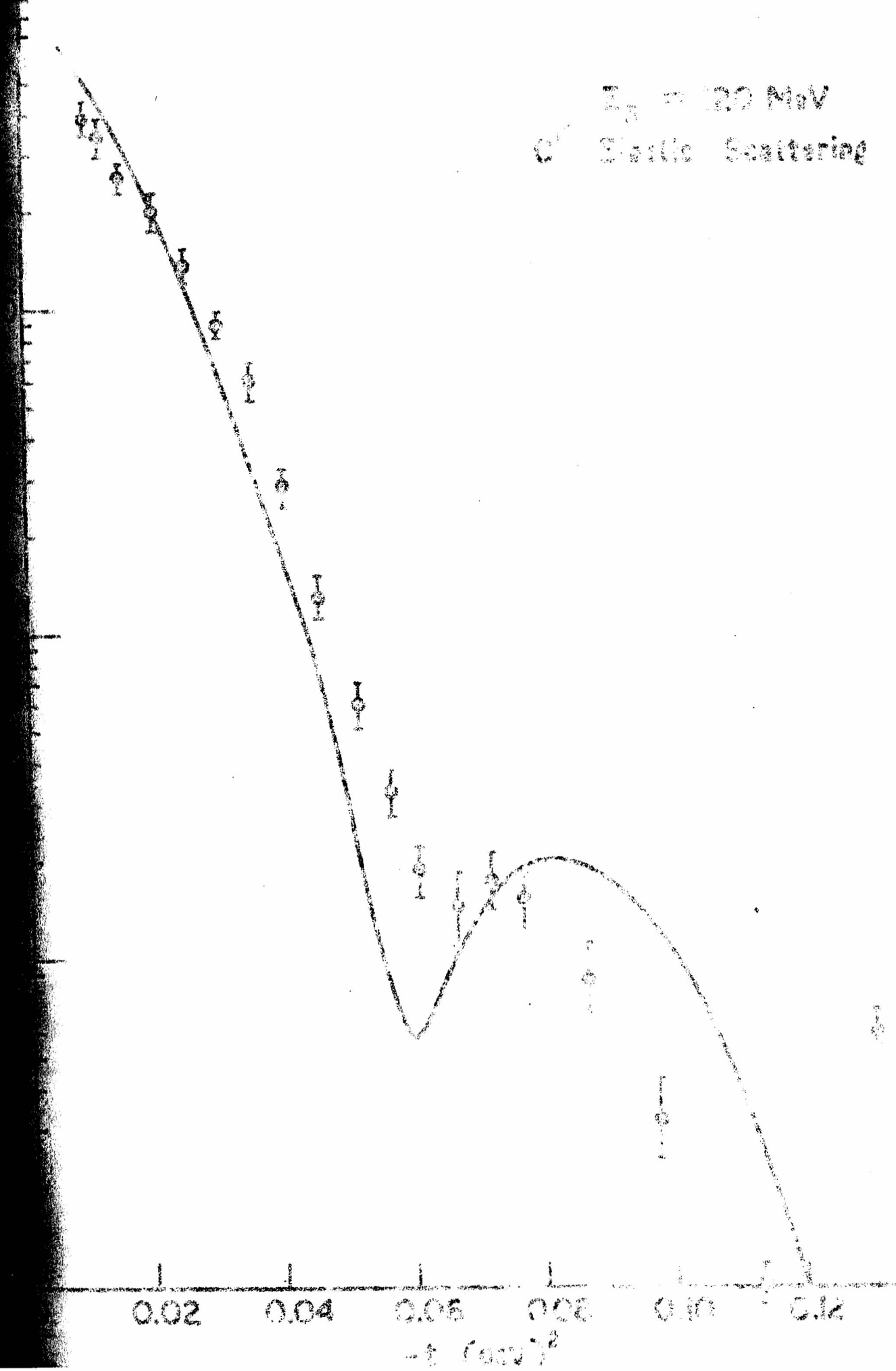

$2^+$, $T_L$ = 720 MEV
Glauber's Theory } IPM
PM
THIS WORK
$10^1$
$10^0$
$10^{-1}$
$10^{-2}$
·02
·04
·06
·08
·10
·12
·14
·16
$-t$ (gev)$^2$

FIGURE 2(A)

$2^+$, $T_L$ = 200 MEV
Glauber's } IPM
Theory } HP
THIS WORK
$10^1$
$10^0$
$10^{-1}$
$10^{-2}$
.02
.04
.06
.08
.10
.12
.14
.16
$-t$ (gev)$^2$


$E_\gamma$ = 18.0 MeV
$C^{12}$ Elastic Scattering
0.02
0.04
0.06
0.08
0.10
0.12
$-t\ (mc)^2$

# PHOTOPRODUCTION OF NEGATIVE PIONS FROM CARBON-12

K.Srinivasa Rao
MATSCIENCE, The Institute of Mathematical Sciences, Madras
INDIA

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Abstract : The cross sections for negative pion photoproduction from $^{12}C$ are calculated, taking into account the two-particle-two-hole correlations in the ground state wave function of $^{12}C$, in the Saxon-Woods basis and the results are compared with those obtained in the harmonic oscillator basis.

## 1. INTRODUCTION

In a previous paper[1] (referred to as I) calculations were performed for charged pion photoproduction cross sections from $^{12}C$. There it was pointed out that a study of the reaction:

$$\gamma + {}^{12}C(J^P = 0^+, T=0) \rightarrow \pi^- + {}^{12}N(J^P = 1^+, T = 1) \qquad (1)$$

is interesting from both the theoretical and experimental points of view since only the ground state of $^{12}N$ is stable against nucleon emission and it is a positron emitter. As a first step, in I, we assumed the ground state of $^{12}C$ to be spherical and the cross sections were computed in the harmonic oscillator basis. The purposes of this article are two fold: (i) to treat the ground state of $^{12}C$ more realistically and (ii) to use a realistic Woods-Saxon single particle basis. We are encouraged to consider

these two aspects to be important in the study of pion photoproduction, in view of our recent study[2] of the reaction $^{16}O(\gamma,\pi^+)^{16}N$.

The ground state of $^{12}C$ is known[3] to be better described as a deformed state, as evidenced by the larger transition from the first $2^+$ state. It exhibits the same structure as observed in $^{16}O$. Goswami and Pal[5] introduce two-particle-two-hole (2p-2h) correlations in the ground state of $^{12}C$ through the linearised equation of motion method in their study of the collective states of $^{12}C$, which Goswami and Bur-Touv[6] consider mixtures of spherical and deformed states in $^{12}C$. More recently, Rowe et. al.[7] have calculated the exact ground state of $^{12}C$ by doing a complete matrix diagonalization within the 1p-shell and they have used it in their study of the excitation energies for the first $J^P = 2^+$, $T = 0$ excited state in $^{12}C$ and its quadrupole moment. In the absence of an experimental confirmation of the 2p-2h admixtures in the ground state of $^{12}C$, as has been done for $^{16}O$ by Purser et. al.[8], we use the wave function of Rowe et. al.[7], in our study of reaction (1).

Studies of electron scattering show that realistic Woods-Saxon single particle states can predict the position of the second diffraction minimum in $^{12}C$ in agreement with experimental data while the harmonic oscillator cannot. Since in pion photoproduction near the first pion-nucleon resonance region, high momentum transfers occur, as in the case of electron scattering, we calculate the cross sections for reaction (1) as a function

of the incident photon energy in the Woods-Saxon basis and compare the results with those obtained in the harmonic oscillator basis (ref. 1).

In Section 2, we discuss the ground state wave functions used in the present study, in section 3, we deal with the evaluation of matrix elements and in section 4, we discuss the results obtained. The results obtained for the case of positive pion photoproduction from $^{12}C$ will be reported elsewhere.

2. Ground state wave functions

The ground state wave function of $^{12}C$ ($J^P = 0^+$, $T = 0$) is assumed to contain in addition to the 0p-0h component, 2p-2h components only and from the study of Rowe et. al.[7], we assume that it can be approximated by [7,8]:

$$|0^+, g.s.\rangle = \alpha\ |0p-0h\rangle + \beta |(1p^2_{1/2})_{J=0,T=1}(1p^{-2}_{3/2})_{0,1}\rangle + \gamma\ |(1p^2_{1/2})_{1,0}(1p^{-2}_{3/2})_{1,0}\rangle, \qquad (2)$$

where $\alpha = 0.789$, $\beta = -0.575$ and $\gamma = 0.25$. We denote this deformed Ground State Wave Function for $^{12}C$ by D.G.S. We denote the pure shell model wave function, which corresponds to $\alpha = 1$ and $\beta = \gamma = 0$, for the ground state of $^{12}C$ by PS.

The 2p-2h states in (2) are given by[9]:

$$|2p-2h, (n_1\ell_1 j_1)^2_{J,T}\ (n_2\ell_2 j_2)^{-2}_{J,T}\rangle$$

$$= \frac{1}{2} \sum_{m_1,m_1'} \sum_{m_2,m_2'} \sum_{M,M'} \sum_{\tau_1,\tau_1'} \sum_{\tau_2,\tau_2'} \sum_{T_z,T_z'} (-1)^{M+T_z} \times$$

$$\times C(j_1 j_1 J; m_1 m_1' M) C(j_2 j_2 J; m_2 m_2' M) C(JJ0; MM'0) \times$$

(contd.)

$$\times C(\tfrac{1}{2}\tfrac{1}{2}T;\tau_1\tau_1' T_z)C(\tfrac{1}{2}\tfrac{1}{2}T;\tau_2\tau_2' T_z')C(TTo;T_zT_z'0)$$

$$\times a^{+}_{j_1 m_1 \frac{1}{2}\tau_1} a^{+}_{j_1 m_1' \frac{1}{2}\tau_1'} a_{j_2 m_2 \frac{1}{2}\tau_2} a_{j_2 m_2' \frac{1}{2}\tau_2'} |0\rangle$$

(3)

where the ket $|0\rangle$ is the (Hartree-Fock) closed shell state, $(n_1 \ell_1 j_1)^2$ denotes the two-particle (occupied) state and $(n_2 \ell_2 j_2)^{-2}$ denotes the two-hole (unoccupied) states and $a^+$(a) are the creation (annihilation) operators. We have suppressed the indices $n\ell$ (which accompany j) for $a^+$ and a, without loss of generality.

The ground state of $^{12}N$ ($J^P = 1^+$, T = 1) may be assigned to the configuration where there is a neutron-hole in the $1P_{1/2}$-shell and a proton-particle in the $1P_{3/2}$-shell. This scheme is called as the Independent Particle Model (IPM) here. While the IPM envisages a pure configuration, the particle-hole models allow configuration mixing. The particle-hole wave function, in the Tamm-Dancoff Approximation, in the jj coupling scheme, is of the form

$$|J_f M_f T M_T\rangle = \sum_{p,h} X^{J_f}_{p,h} (-1)^{j_h + m_h} C(j_p j_h J_f; m_p m_h M_f)$$

$$\times (-1)^{\frac{1}{2}+\tau_h} C(\tfrac{1}{2}\tfrac{1}{2}T_f;\tau_p\tau_h M_T)\cdot$$

$$\times a^{+}_{p\, m_p \frac{1}{2}\tau_p} a_{h,-m_h,\frac{1}{2},-\tau_h} |0\rangle \qquad (4)$$

where h(hole) and p(particle) states are characterised by the set of quantum numbers $(n_h \ell_h j_h)$ and $(n_p \ell_p j_p)$, respectively and $X^{J_f}_{p,h}$ are the configuration mixing coefficients with the normalization

$$\sum_{p,h} (X^{J_f}_{p,h})^2 = 1 \quad . \tag{5}$$

A particle-hole configuration mixing calculation for $^{12}C$ has been performed by Gillet and Vinh Mau[11] in the TDA. Since the ground states of $^{12}B(1^+)$ and $^{12}N(1^+)$ have been identified[12] to be T = 1 isobaric analogues of the 15.11 MeV. $1^+$-state in $^{12}C$, we take for the amplitudes of the ground state of $^{12}N$, those corresponding to $J^P = 1^+$, T = 1 (15.11 MeV) state of $^{12}C$, under the assumption of good isobaric spin. This wave function of $^{12}N$ is denoted by GV here.

3. Evaluation of matrix elements.

The transition operator for photoproduction of charged pions from a bound nucleon is given by

$$t = t(\gamma N \to \pi^{\pm} N) = (\underline{\sigma}\cdot\underline{K} + L)\, \tau^{\mp} \exp(i\underline{k}\cdot\underline{r}) \tag{6}$$

where $\tau^{\mp}$ is the isobaric spin operator, K and L are respectively the spin-dependent and spin-independent parts of the amplitude, $\underline{k} = \underline{\nu} - \underline{\mu}$ is the momentum transfer to the nucleon, $\underline{\nu}$ and $\underline{\mu}$ being the incident photon and outgoing pion momenta and $\underline{r}$ is the position vector of the nucleon. The photoproduction amplitudes K and L are functions of momenta and energies of the incident photon $(\underline{\nu}, \nu_o)$ and the out going pion $(\underline{\mu}, \mu_o)$, the

polarization of the photon and the angle of pion emission and they also depend on the magnetic moments of the proton and neutron. We choose the forms of $\underline{K}$ and L given by Chew et. al.[13]. In order to enable us to write the transition operator in spherical tensor notation, we use the unit operator $\sigma_o(=I)$ in spin-space and re-define $\underline{K}$ and L as $K^1$ an $K^o$, respectively, so that:

$$\underline{\sigma}.\underline{K} + L = \sum_{n=0,1} \sigma^n . K^n \qquad (7)$$

Using the Rayleigh expansion for $\exp( i \underline{k} . \underline{r})$, separating the angular and radial parts and forming tensor products of the operators in pairs and using the symmetry and orthogonality properties for the Clebsch-Gordon coefficients, we get:

$$t = \tau^{\mp} \sum_{\lambda, m_\lambda} t^{\lambda}_{m_\lambda} \qquad (8)$$

where

$$t^{\lambda}_{m_\lambda} = 4\pi \sum_{n} \sum_{\ell=o}^{\infty} i^{\ell} (-1)^{\ell+n-\lambda} (-1)^{m_\lambda} j_\ell(kr) \cdot$$
$$\cdot (Y^{\ell}(\hat{r}) \times \sigma^n)^{\lambda}_{m_\lambda} (Y^{\ell}(\hat{k}) \times K^n)^{\lambda}_{-m_\lambda} . \qquad (9)$$

The nuclear transition operator, in the occupation number representation, is given for reaction (1) in the impulse approximation by:

$$\mathcal{T} = \sum_{q=1}^{A} t_q = \sum_{\xi,\eta} \langle \xi | \sum_{\lambda, m_\lambda} t^{\lambda}_{m_\lambda} \tau^{+} | \eta \rangle \, a^{+}_{\xi} a_{\eta} . \qquad (10)$$

where $\xi$ and $\eta$ are the single-particle states.

Using standard techniques, it can be shown[14] that the matrix element of $\mathcal{T}$ between the $|0p-0h\rangle$ component of the initial state[2] and the $|1p-1h\rangle$ final nuclear state[4] is given by:

$$Q(0p-0h) = \langle J_f M_f T M_T | \mathcal{T} | 0p-0h \rangle$$

$$= \delta_{J_f,\lambda}\, \delta_{M_f,m_\lambda}\, \delta_{T_f,1}\, \delta_{M_T,-1} \sum_{p,h} X^{J_f}_{p,h} \frac{[j_p]}{[J_f]} \langle p \| t^{J_f} \| h \rangle, \qquad (11)$$

where we use the notation $[J]$ for $(2J+1)^{1/2}$.

We denote by $Q_{J,T}(2p-2h)$ the matrix element

$$Q_{J,T}(2p-2h) = \langle J_f M_f T M_T | \mathcal{T} | 2p-2h, (n_1 \ell_1 j_1)^2_{J,T} (n_2 \ell_2 j_2)^{-2}_{J,T} \rangle \qquad (12)$$

where the intermediate values $(J,T)$ can be either $(0,1)$ or $(1,0)$ depending upon the $(2p-2h)$ component of the ground state wave function given by (2). The matrix element corresponding to $J = 0$, $T = 1$ is evaluated to be:

$$Q_{0,1}(2p,2h) = \delta_{J_f,\lambda}\, \delta_{M_f,m_\lambda}\, \delta_{T_f,1}\, \delta_{M_T,-1} \times$$

$$\times \frac{1}{\sqrt{3}} \frac{(-1)^{j_1+j_2+J_f}}{[j_1][J_f]} X^{J_f}_{p,h} \langle h \| t^{J_f} \| p \rangle \delta(p,1) \delta(h,2), \qquad (13)$$

where the Kronecker delta product $\delta(p,1) \times \delta(h,2)$ is zero unless the two particles (holes) of the $2p-2h$ initial state component have the same $(n\ell j)$ quantum numbers as the particle (hole) of the $1p-1h$ final state. Expressions (11) and (13) are identical to the ones given by Walker[9]. Similarly, we get for $J = 1$, $T = 0$

case the following expression for (12), after reduction using standard angular momentum techniques :

$$Q_{1,0}^{(2p-2h)} = \delta_{J_f,\lambda}\,\delta_{M_f,m_\lambda}\,\delta_{T_f,1}\,\delta_{M_T,-1}\,3\sqrt{3}\,\frac{[j_h]}{[J_f]} \times W(j_p j_p j_h j_h ; 1 J_f)\, X_{p,h}^{J_f} \times \langle h \| t^{J_f} \| p \rangle\, \delta(p,1)\,\delta(h,2)\,. \tag{14}$$

Hence, the matrix element for reaction (1) is:

$$Q = \langle f \mid \mathcal{T} \mid 0^+, g.s. \rangle = \alpha\, Q(0p\text{-}0h) + \beta\, Q_{0,1}^{(2p-2h)} + \gamma\, Q_{1,0}^{(2p-2h)}, \tag{15}$$

Using the explicit expansion for the reduced matrix elements and the symmetry properties of the Clebsch-Gordon and 9-j symbols, in the IPM, for $J_f^P = 1^+$, we get for (15):

$$Q_{IPM}^{J_f^P=1^+} = \left[\alpha - \frac{(-1)^n}{\sqrt{6}}\beta + (-1)^n\sqrt{\frac{15}{2}}\gamma\right]\frac{[1/2]}{[1]}\,\langle 1p_{1/2} \| t^{J_f^P=1^+} \| 1p_{3/2} \rangle \tag{16}$$

where n can take two values 0 and/or 1 as per eq.(7).

Finally, the differential cross section we are interested in is given by :

$$\frac{d\sigma(0^+ \to J_f^P)}{d\Omega} = (2\pi)^{-2}\,\mu\mu_o \sum_{M_f} |\langle f \mid \mathcal{T} \mid 0^+, g.s. \rangle|^2, \tag{17}$$

here the sum is over the final spins and the bar over the sum denotes the average over photon polarizations.

Numerical Calculations and Discussion.

The single-particle radial wave functions were calculated numerically, using subroutine BDSTS[15], assuming a realistic Woods-Saxon potential of the form

$$v(r) = -Uf(r) + \left(\frac{\hbar}{m_\pi c}\right)^2 U_S\, g(r)(\vec{l}\cdot\vec{\sigma}) + V_c(r, R_c), \tag{18}$$

where

$$f(r) = \left[1 + \exp\left(\frac{r - R_o}{a}\right)\right]^{-1},$$

$$g(r) = \frac{1}{r}\frac{df}{dr},$$

$$V_c(r, R_c) = Z_A Z_\mu e^2 h(r),$$

$Z_A$, $Z_\mu$ = core, particle charge,

$$h(r) = \begin{cases} 1/r & \text{for } r \geq R_c, \\ \frac{1}{2R_c}\left(3 - \frac{r^2}{R_c^2}\right) & \text{for } r < R_c, \end{cases}$$

$$R_o = R_c = r_o A^{1/3},$$

with the following values of the geometrical parameters: $r_o$ = 1.25 fm. and $a$ = 0.65 fm. The central and spin-orbit potential strengths, $U$ and $U_S$ are fitted to the neutron single-particle energies of the spectra given in ref. 7 and these are given in table 1.

Table 1

Values of U and $U_S$ which reproduce the neutron single particle energies* in $^{12}C$.

| n j | (MeV) | U (MeV) | $U_S$ (MeV) |
|---|---|---|---|
| $1P_{3/2}$ | -21.80 | 75.36 | 10.44 |
| $1P_{1/2}$ | -15.65 | | |
| $2s_{1/2}$ | -3.27 | 78.34 | - |
| $1d_{5/2}$ | -4.15 | 78.91 | 5.75 |
| $1d_{3/2}$ | +0.94 | | |
| $1s_{1/2}$ | -50.0 | 87.63 | - |

* 5 MeV was deducted from all the listed single-particle energies, and the potential depths calculated using the corrected energies.

When, for the sake of comparison, we use the harmonic oscillator wave functions, we set the oscillator size parameter value b = 1.64 fm, which is in conformity with elastic and most inelastic experimental electron scattering data[16].

In Fig. 1, we plot the cross section for the reaction(1) as a function of the incident photon energy. The results have been obtained in the harmonic oscillator basis with and without 2h correlations in the ground state wave function of $^{12}C$ and

using the nuclear models IPM and GV to describe the ground state wave function of $^{12}N$. Due to the small configuration mixing the reduction in the cross section is found to be small but we find that the 2p-2h correlations produce a significant reduction in the cross section.

In Fig.2, the cross sections obtained in the Woods-Saxon basis, using the nuclear model IPM for $^{12}N$ and including the 2p-2h ground state correlations in $^{12}C$ are shown. The results obtained in the Saxon-Woods basis are slightly larger than those obtained in the harmonic oscillator basis. This is due to the larger tail of the 1p wave functions in the Saxon-Woods basis. The cross-hatched curve is the one obtained in I, invoking the surface production mechanism. We conclude that use of a realistic ground state wave function and a realistic single-particle basis produces almost the same amount of reduction in the cross sections for $\pi^-$ photoproduction from $^{12}C$, as is obtained by the purely phenomenological surface production model.

It is a pleasure to thank Professor Alladi Ramakrishnan for his interest and encouragement. The numerical computations were made on the CDC 3600 computer at the Tata Institute of Fundamental Research, Bombay. The author is grateful to Dr.J.C.Parikh and Dr.K.H.Bhatt for several interesting discussions.

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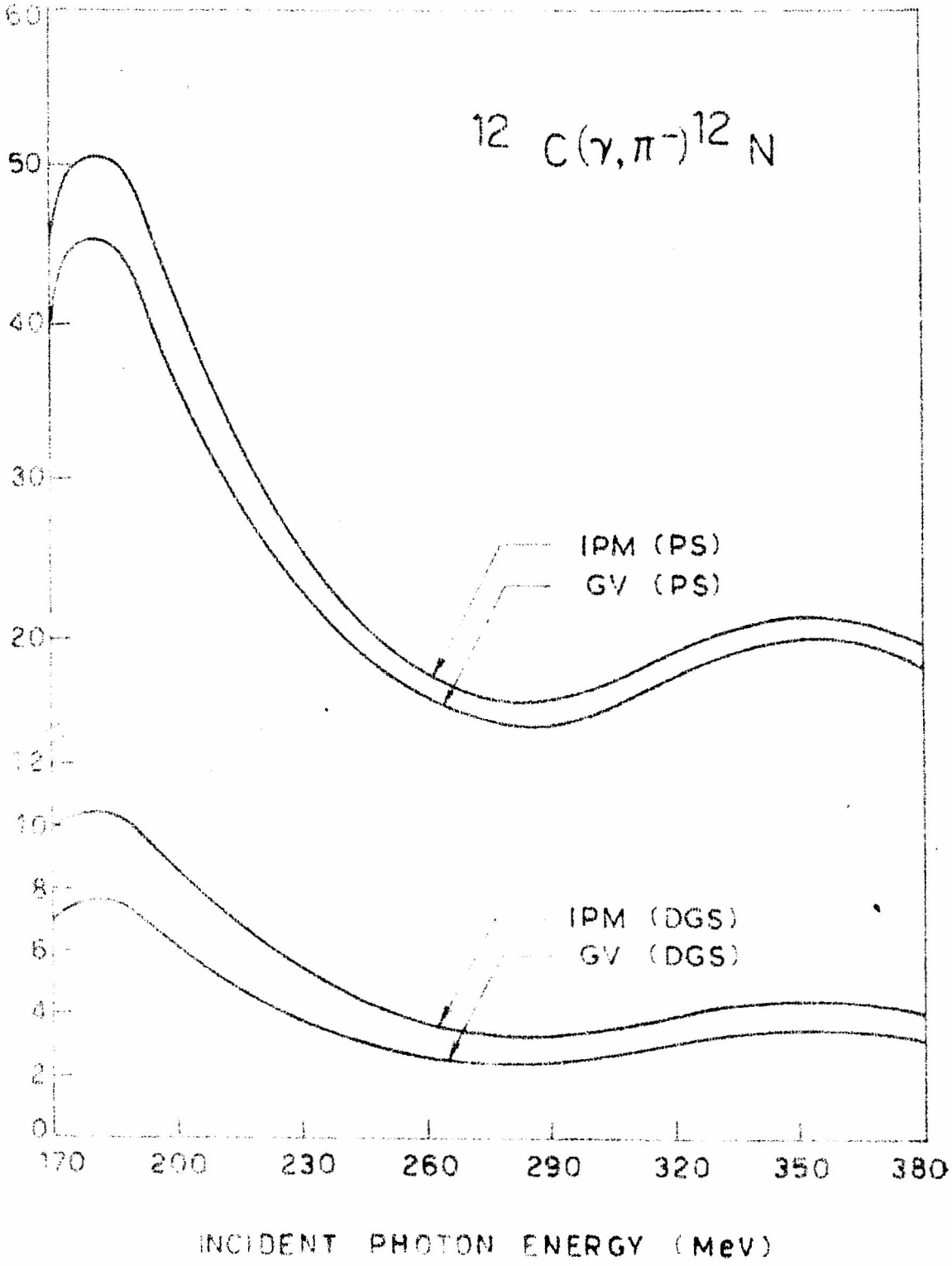
$^{12}C(\gamma,\pi^-)^{12}N$
IPM (PS)
GV (PS)
IPM (DGS)
GV (DGS)
60
50
40
30
20
12
10
8
6
4
2
0
170
200
230
260
290
320
350
380
INCIDENT PHOTON ENERGY (MeV)

FIGURE . 1

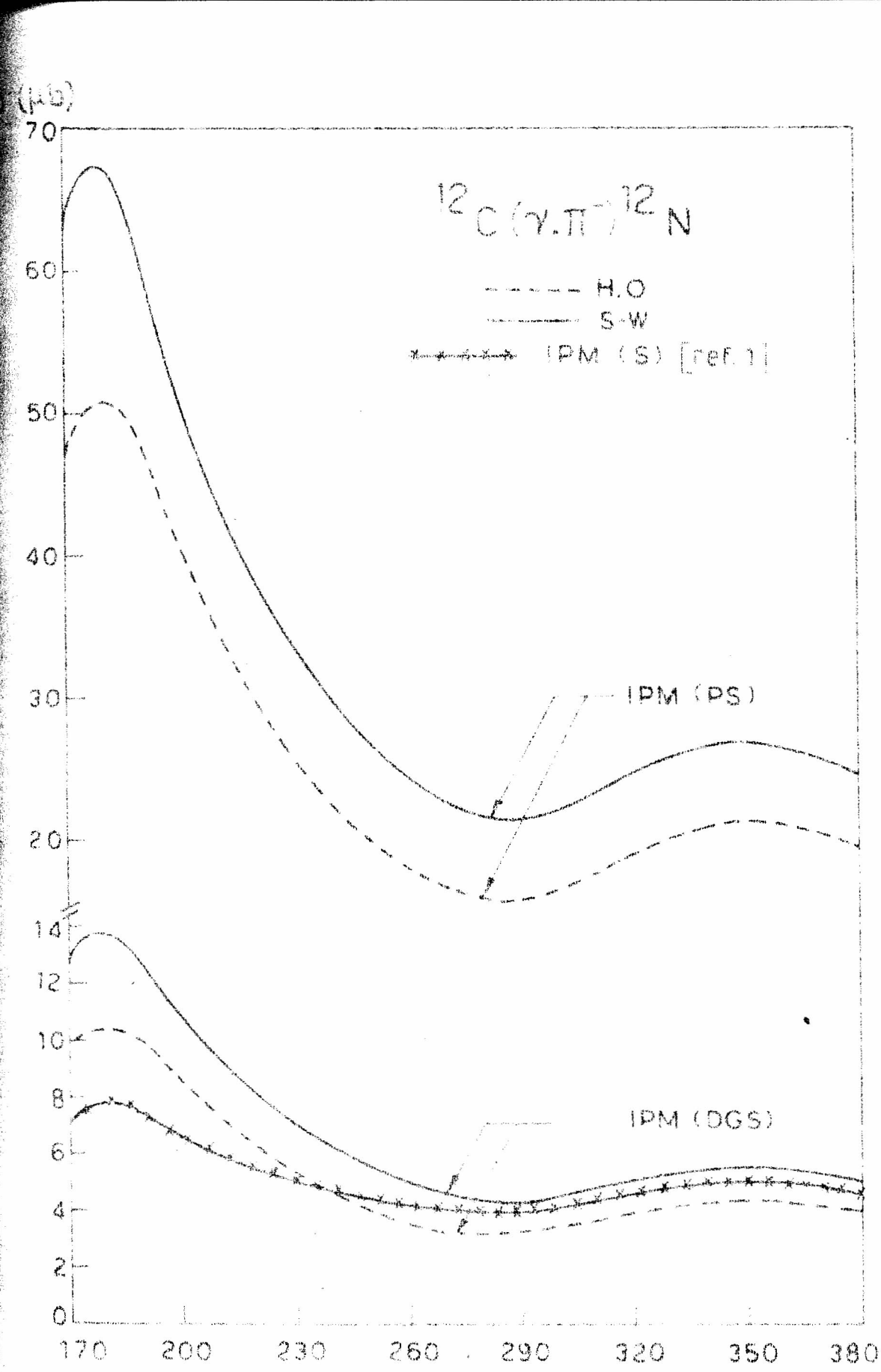
(μb)
70
60
50
40
30
20
14
12
10
8
6
4
2
0
$^{12}C(\gamma,\pi^+)^{12}N$
H.O
S-W
IPM (S) [ref.1]
IPM (PS)
IPM (DGS)
170
200
230
260
290
320
350
380
INCIDENT PHOTON ENERGY (MeV)

FIGURE . 2

# STUDIES IN STRIPPING AND PICKUP REACTIONS

N. G. Puttaswamy
Department of Physics,
Bangalore University, Bangalore.

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## Introduction

The aim of an experimental nuclear physicist has been to find out as much information about a nucleus as possible. In the early days, one determined the positions of the energy levels, their spins and parities. Theoretically also, one can predict these quantities. Shell-model calculations, for example, can predict additional quantities such as the magnetic dipole moment, quadrupole moment, B(M1) and B(E2). However, agreement between experiment and theory with respect to these quantities alone is not enough to test the approximations made in a given nuclear model. One additional valuable information that can be compared is the 'spectroscopic factor' S, which may be determined by a study of nuclear reactions.

Consider reactions of the following type:

$^{37}Cl(d,n)^{38}Ar$ ... Stripping reaction

$^{37}Cl(d,{}^{3}He)^{36}S$ ... Pickup reaction

Let the incident particle energy be moderately high, say 20 to 50 MeV, so that isolated nuclear resonances of the compound nuclear type are not dominant; we shall call these as 'direct reactions'. In the (d,n) or (d,p) reaction one proton or neutron

is stripped from the incident particle and transferred to the target nucleus. In the $(d,{}^3He)$ or $(d,t)$ reaction a proton or a neutron is picked up from the target nucleus. These reactions are also referred to as single-nucleon-transfer reactions.

A single-nucleon-transfer reaction involves a process where the nucleus in an initial state is taken to a final state (in fact, several final states). One can therefore calculate the probability or the amplitude for such a reaction and this is given in terms of the spectroscopic factor S. If the transferred particle is from a $2s_{1/2}$ shell-model level, we may write,

$$S(2s_{1/2}) = n \left\langle \Psi^{JT}(1,2,\ldots,n) \middle| \left\{ \Psi^{J_0T_0}(1,2,\ldots,n-1) \times \phi_{2s}^{j=\frac{1}{2},t=\frac{1}{2}} \right\}^{JT} \right\rangle^2$$

where $\Psi^{JT}(1,2,\ldots,n)$, $\Psi^{J_0T_0}(1,2,\ldots,n-1)$ and $\phi_{2s}^{j=\frac{1}{2},t=\frac{1}{2}}$. are the wave functions of the residual nucleus, the target nucleus (in a stripping reaction), and the stripped $2s_{1/2}$ particle respectively; n represents the number of active nucleons. If, for example, one is interested in the energy levels of $^{38}Ar$ (as seen in a stripping reaction), and if one considers $^{16}O$ and $^{40}Ca$ as closed shells, then there will be 22 active nucleons. See Fig. 1. If one considers $^{28}Si$ and $^{40}Ca$ as closed shells, there will be 10 active nucleons. So far there

xist no theoretical calculations for the case of ?? active ucleons. Shell-model calculations have however been done[1] or the $S_{1/2} - d_{3/2}$ shell nuclei $(20 \leq A \leq 40)$ y assuming that $^{28}Si$ and $^{40}Ca$ form closed shells.

The spectroscopic factor therefore tells us how well e can look upon the residual nucleus as the target plus a single particle in a shell-model state. When calculations are done using the iso-spin formalism, it is customary to give the values of $C^2S$ and not S itself. Here C is a Clebach-Gordon coefficient and depends upon the isospin quantum numbers. In fact, the experimental cross sections will be proportional to $C^2S$ and not to S.

A study of the energy spectrum and the angular distribution of the outgoing particles can give information about the various levels of the residual nucleus. Further, with the use of the DWBA (distorted wave Born approximation) theories, one can extract information about the angular momentum $\ell$ of the transferred nucleon and the corresponding S factor.

Experimental Procedure

In a study of the $^{37}Cd(d,^3He)^{36}S$ reaction, for example[2], one can bombard a target containing $^{37}Cl$ (Say $PbCl_2$) by deuterons. The energy spectrum of the outgoing $^3He$ particle can be studied either by using a magnetic spectrometer or by using a counter telescope. In the later case, one uses combination of $\Delta E$ and E counters so as to enable the identification of the outgoing particle. Fig.2 shows the energy spectrum of $^3He$

particles from the $^{37}Cl(d,^{3}He)^{36}S$ reaction taken at 21°. By taking such spectra at various angles, one can find the intensity of each particle group (i.e., area under the given peak). Fig.3 shows the angular distribution corresponding to the ground-state and the 3.31-Mev state in $^{36}S$. The solid curves in Fig.3 have been obtained from a DWBA calculation using the JULIE code. The optical-model parameters used for the calculation are described in Ref.2. The experimental spectroscopic factor for the transfer of the given nucleon can then be obtained by a comparison of the differential cross sections obtained in the experiment with that predicted by DWBA theory. Hence one can write

$$\left(\frac{d\sigma}{d\Omega}\right)_{exp} = N C^2 S \left(\frac{d\sigma}{d\Omega}\right)_{DWBA}$$

where N = 2.95 is the normalising factor for the $(d,^{3}He)$ reaction.

Table I gives a comparison between the experimental and theoretical spectroscopic factors. The experimental spectroscopic factor has an uncertainty of about 25%, coming mainly from the uncertainties in the optical-model parameters. The theoretical spectroscopic factors should also be accepted with some caution in view of approximations made in the calculations. Under these circumstances, the agreement in the value of $C^2S$ given in Table I is quite satisfactory.

Theoretical calculations:

The theoretical values of spectroscopic factors quoted in Table I are taken from Glaudemans et al[1], who assume an inert $^{28}Si$ core and permit all configurations in the $2s_{1/2}$ and $1d_{3/2}$ shells. They assume the nuclear interaction to be the sum of two-particle interactions, and then reduce the matrix elements into 15 two-particle matrix elements. These, along with the binding energies of a nucleon in the $2s_{1/2}$ and $1d_{3/2}$ shells, constitute 17 parameters which were determined by Glaudemans et al by making a least-squares fit to the energies of known levels. Engelbertink and Glaudemans[4] have performed a calculation for the A = 38 nuclei by using the MSDI (modified surface delta interaction) and by permitting particles to occupy the $1f_{7/2}$ and $2p_{3/2}$ shells also. In this calculation they have 6 parameters which are determined by a least-squares fit to the experimental energies. We have calculated the spectroscopic factors using their wave functions and the results for the $^{37}Cl(^3He,d)^{38}Ar$ reaction are presented in Table II. The experimental values seem to agree closely with the 1964 calculation[1].

There have been attempts to enlarge the shell-model space by including the $d_{5/2}$ levels also[6]. This has been done only for a limited number of nuclei and no calculationsare as yet available for all nuclei in the region $28 \leq A \leq 40$.

In a very recent and detailed shell-model calculation Halbert *et al*[7] have used many alternative interaction Hamiltonians, including the 'realistic' Hamiltonians proposed by Kuo and by Kuo and Brown. They have done seven different calculations ranging from the no-free-parameter case (this is called "$K + {}^{17}O$", where the 'realistic' effective Hamiltonian derived by Kuo from the Hamada-Johnston potential is used along with the single-particle binding energies taken from the known levels of $^{17}O$) to the 14 free-parameter case (this is called RIP, or the radial integral parametrization). They find that the spectroscopic factors, B(M1), B(E2) and such other quantities obtained from the $K + {}^{17}O$ case agree with the six other calculations. It is also interesting to note that they are able to reproduce some of the rotational levels.

Acknowledgement.

The author wishes to thank Mr.C.R.Ramaswamy for his help in the computation of the theoretical spectroscopic factors. Thanks are also due to Professor K.N.Kuchela for his constant encouragement in this work. The financial help provided by a grant from the Department of Atomic Energy, Government of India, is gratefully acknowledged.

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4. G.A.P. Engelbertink and P.W.M. Glaudemans, Nucl. Phys. A123 (1969) 225.
5. M.A. Moinester and W.P. Alford, Nucl. Phys. A115 (1970) 143.
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FIGURE CAPTIONS:

Fig. 1. Ground state of $^{36}_{20}Ar$ according to simple nuclear shell model.

Fig. 2. Spectrum of $^3He$ particles from the $^{37}Cl(d,{}^3He)^{36}S$ reaction at $E_d = 23.35$ MeV and $\theta_{lab} = 21^\circ$.

Fig. 3. Angular distribution of $^3He$ particles from the $^{37}Cl(d,{}^3He)^{36}S$ reaction for (a) the ground state and (b) the 3.31-MeV excited state. The solid curves are the DWBA predictions.

| $E_x$ (MeV) | $J^\pi$ other expts. | (d, $^3$He) Experiment[a] | | | | Theory[b] | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\ell_p$ | Pickup particle | $J^\pi$ | $C^2S$ | $E_x$ (MeV) | $J^\pi$ | $C^2S(s_{1/2})$ | $C^2S(d_{3/2})$ [c] |
| 0.0 | $0^+$ | 2 | $d_{3/2}$ | $0^+$ | 1.31 | 0.0 | $0^+$ | | 0.9083 |
| 2.000 | $0^+$ | - | - | - | - | 4.28 | $0^+$ | | 0.0457 |
| 2.885 | $2^+$ | - | - | - | - | 6.23 | $2^+$ | 0.0039 | 0.1126 |
| 3.304 | $(2^+,3^-)$ | 0 | $s_{1/2}$ | $(1,2)^+$ | 1.21 | 2.83 | $2^+$ | 1.1761 | 0.0274 |
| 3.360 | $0^+$ | - | - | - | - | - | - | | |
| 4.204 | - | - | - | - | - | - | - | | |
| 4.58 | - | 0 | $s_{1/2}$ | $(1,2)^+$ | 1.24 | 5.75 | $1^+$ | 0.6960 | 0.0300 |
| 6.60 | - | 2 | $d_{5/2}$ | $(\leq 4)^+$ | 0.9 | - | - | | |
| | | $\sum C^2S(d_{3/2}) = 1.31$ | | | | $\sum C^2S(d_{3/2}) = 1.1240$ | | | |
| | | $\sum C^2S(s_{1/2}) = 2.45$ | | | | $\sum C^2S(s_{1/2}) = 1.8760$ | | | |
| | | $\sum C^2S(d_{5/2}) = 0.9$ | | | | | | | |

[a] Reference 2.
[b] Reference 1.
[c] Reference 3.

| $E_x$ (MeV) | J (T=1) | 1964 Calculation[a] $(2J+1)C^2S$ | | 1969 Calculation[b] $(2J+1)C^2S$ | | | ($^3$He,d) Experiment[c] $(2J+1)C^2S$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\ell=0$ | $\ell=2$[d] | $E_x$ (MeV) | $\ell=0$ | $\ell=2$[d] | $E_x$ (MeV) | $\ell=0$ | $\ell=2$ |
| -0.17 | $0^+$ | | 1.4902 | 0.25 | | 2.0578 | 0.0 | -- | 1.88 |
| 2.37 | $2^+$ | 0.2014 | 8.1797 | 1.83 | | 5.4263 | 2.17 | 0.064 | 9.86 |
| 4.21 | $2^+$ | 0.1687 | 0.8472 | 3.77 | 0.3341 | 3.9015 | 3.94 | 0.028 | 0.48 |
| 6.50 | $0^+$ | | 0.5737 | 5.24 | 0.1560 | 0.1200 | | | |
| 6.77 | $1^+$ | 0.1560 | 0.1200 | 6.78 | | 0.0083 | | | |
| Ttotal | | 0.4961 | 11.5638 | | 0.4901 | 11.5139 | | 0.092 | 12.24 |
| | | No. of holes = 3.000 | | | No. of holes = 3.0001 | | | No. of holes = 3.11 | |

[a] Reference 1.

[b] Reference 4.

[c] Reference 5.

[d] Reference 3.

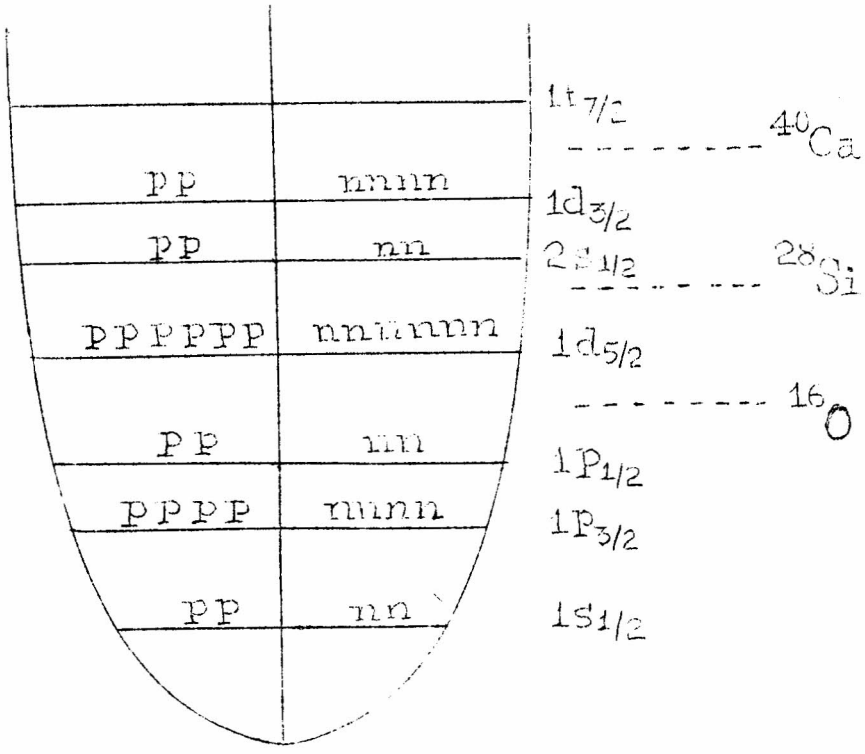
pp
nnnn
pp
nn
pppppp
nnnnnn
pp
nn
pppp
nnnn
pp
nn
1f7/2
40Ca
1d3/2
2s1/2
28Si
1d5/2
16O
1p1/2
1p3/2
1s1/2

Fig.1

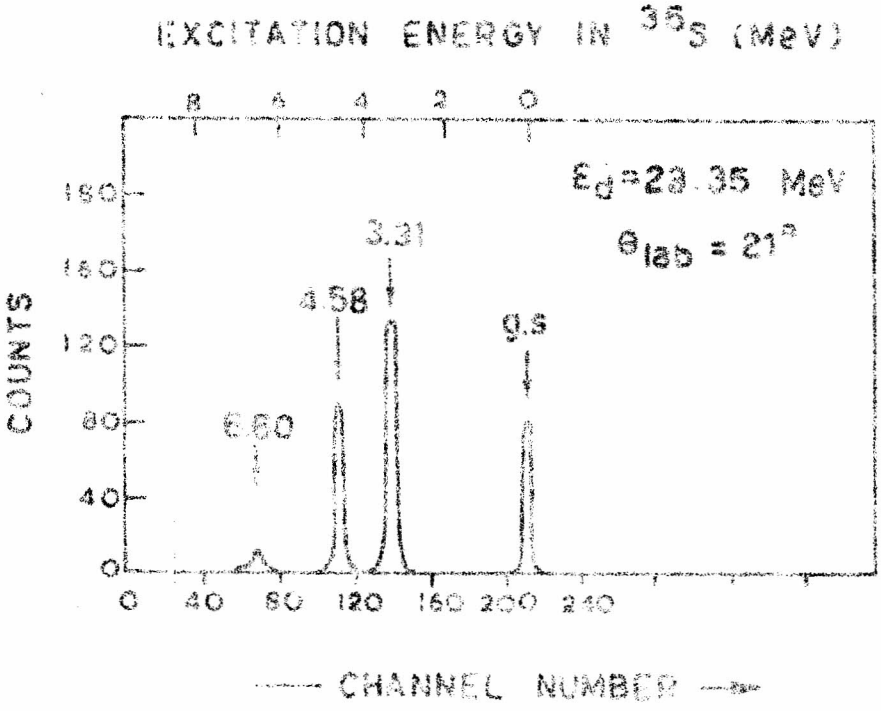
EXCITATION ENERGY IN $^{35}S$ (MeV)
8
6
4
2
0
$E_d$=23.35 MeV
$\theta_{lab}$ = 21°
3.31
4.58
6.60
9.5
COUNTS
160
120
80
40
0
0
40
80
120
160
200
240
CHANNEL NUMBER

FIGURE 2.

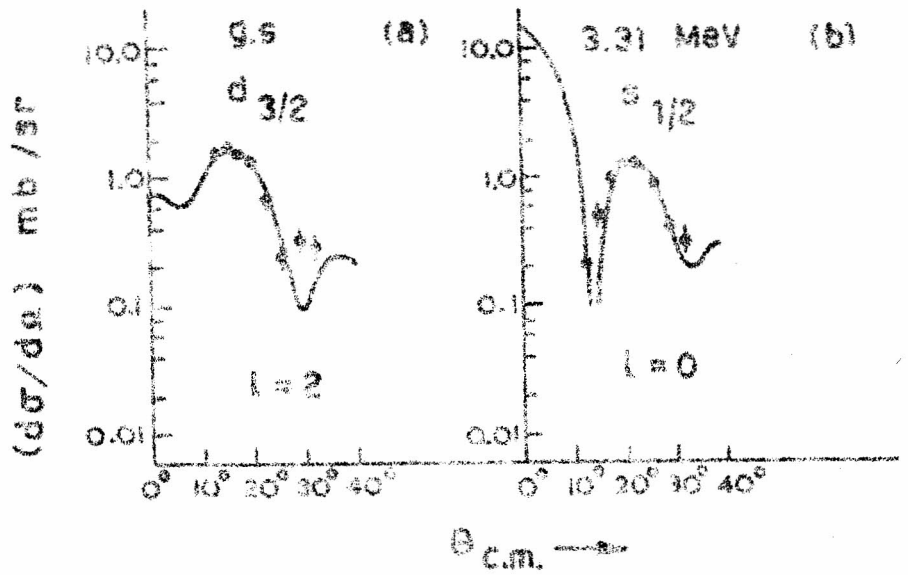
9.5
(a)
3.31 MeV
(b)
$d_{3/2}$
$s_{1/2}$
l = 2
l = 0
10.0
1.0
0.1
0.01
0°
10°
20°
30°
40°
$(d\sigma/d\Omega)$ mb/sr
$\theta_{c.m.}$

FIGURE-3.

# BETA SPECTRUM SHAPE AND NUCLEAR STRUCTURE

T. Nagarajan
Nuclear Physics Department
University of Madras, Madras-600025

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The experimental observables of beta decay are functions of both the properties of weak interaction and the nuclear matrix elements which describe nuclear properties. In order to determine the characteristic features of weak interaction from a study of beta decay, the nuclear matrix elements have to be known or they must enter as a constant factor only. As the latter is the case for allowed decays most of the information about weak interaction has been derived from allowed decays. One piece of information that cannot be obtained from allowed decays is the question of pseudoscalar interaction. Because of parity selection rule this interaction contributes in the lowest order to first-forbidden transitions only.

The beta decay observables, which depend on nuclear matrix elements and therefore throw-light on the structure of nuclear states involved, are (i) log ft value (ii) spectrum shape (iii) $\beta$-$\gamma$ angular and circular polarisation correlation (iv) orientation $\beta$-correlation etc.. The first two observables have been used frequently only to fix the parity and spins of nuclear levels and not so much to elucidate the nuclear structure. It is the purpose of this paper to show that an accurate determination of spectrum shape[1] and log ft values

can lead to a correct elucidation of nuclear structure. Accurate determination of all the matrix elements contributing to a transition will involve an indeterminate number of higher order matrix elements. However, one should keep in mind, that evidences for certain effects are very important for nuclear model considerations, even though the matrix elements themselves cannot be determined accurately. An independent determination of the individual matrix elements from spectrum shape alone is not possible. But in suitable cases, it is always possible to infer model dependent effects from the shape factor and log ft.

Denoting the first-forbidden $\beta$ decay matrix elements in the Cartesian as well as spherical tensor notation[2)]

$$\mathcal{Y}_L^{\mu}(\vec{r}) = r^{\ell} Y_L^{\mu}(\hat{r})$$

$$T^{M}_{KLY}(\vec{r},\vec{\sigma}) = \sum_{\mu'} C(1LK;-\mu',\mu+\mu')\, \mathcal{Y}_K^{\mu+\mu'}(\vec{r})\, \mathcal{Y}_1^{-\mu'}(\vec{\sigma}) .$$

We write :

$$W = C_A \int \vec{\sigma}.\vec{r} = -C_A \frac{4\pi}{\sqrt{3}} \left[\frac{2J_f+1}{2J_i+1}\right]^{1/2} \langle f \| T_{011} \| i \rangle$$

$$V = C_A \int i\gamma_5$$

$$u = C_A \int i\vec{\sigma} \times \vec{r} = \frac{4}{3}\pi\sqrt{2} \left[\frac{2J_f+1}{2J_i+1}\right]^{1/2} \langle f \| T_{111} \| i \rangle C_A$$

$$y = -C_V \int i\vec{\alpha}$$

$$x = -C_A 4 \int \vec{r} = -\sqrt{\frac{4\pi}{3}} \left[\frac{2J_f+1}{2J_i+1}\right]^{1/2} \langle f \| \mathcal{Y}_1 \| i \rangle C_V$$

$$z = C_A \int (\vec{\sigma} \times \vec{r})_{ij} = C_A \frac{8\pi}{3} \left[ \frac{2J_f+1}{2J_i+1} \right]^{1/2} \langle f \| T_{211} \| i \rangle$$

Forming a linear combination of rank zero and one matrix elements

$$V = \xi' v + \xi w$$

$$Y = \xi' y - \xi (u+x)$$

where $\xi' = \frac{1}{4R}$ and $\xi = \frac{\alpha Z}{2R}$.

it is possible to express[3] the electron energy dependence C(W) as follows:

$$C(W) = k \left[ 1 + aw + b/W + c W^2 \right]$$

with $$k = \rho_o^2 + \left(\frac{w}{3}\right)^2 + \rho_1^2 + \frac{W_o^2}{18}(2x+u)^2 - \frac{1}{18}(2x^2+7u^2) + z^2 (W_o^2 - \lambda_1)/12$$

$$\rho_o = V + \frac{w W_o}{3} \quad ; \quad \rho_1 = Y + (u-x)\frac{W_o}{3}$$

$$ak = -\frac{4}{3} uY - \frac{W_o}{9}(4x^2 + 5u^2) + \frac{3}{2} z^2$$

$$bk = \frac{2}{3}\left[ -w\rho_o + (u+x)\rho_1 \right]$$

$$ck = \frac{1}{9}\left[ 4x^2 + 5u^2 \right] + \left[ 1 + \lambda_1 \right] \frac{z^2}{12}$$

(ii)

with similar expressions for $\beta-\gamma$ correlations, $\beta-\gamma$ circularly polarized correlation. In the j-j coupling shell model the reduced matrix elements are given by[2)]

$$\langle \ell \tfrac{1}{2} j \| T_{KL\gamma} \| \ell' \tfrac{1}{2} j \rangle = (-1)^{\ell+j'-j} [\ell][\ell'][j'][K][L]$$

$$\sqrt{\frac{3}{8\pi^2}} \begin{pmatrix} \ell & L & \ell' \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \ell & \ell' & L \\ j & j' & K \\ \frac{1}{2} & \frac{1}{2} & 1 \end{Bmatrix} F_\ell$$

where $$F_\ell = \int r^{\ell+2} R(r) R'(r) dr \ .$$

The shape C(W) of the 340 KeV $\beta$ transition of $Mo^{99}$ viz. $Mo^{99}(\frac{1}{2}^+) \xrightarrow{\beta^-} Tc^{99}(\frac{9}{2}^-)$ has been experimentally measured[1)]. Since the experimental shape and the theoretically deduced one can be normalized at some energy, it is enough to calculate the ratios of matrix elements

$$x/u = 0.266; \quad z/u = -0.316$$

The ratio $y/\xi x$ is obtained from Fujita's relation[4)] which uses CVC theory,

$$y/\xi x = 2.42 \ .$$

The theoretical shape is obtained as

$$C_{th} = 1+0.094 - 0.0047/W + 0.03W^2$$

The comparison between the theory and the experimentally measured shape is shown in figure1. Even though the 514 keV $3/2^-$ level in $Tc^{99}$ (Fig 2) can be described within the framework of shell model, the large log fit value 8.5 of the $\beta$ feed to this level can be explained only by invoking other mechanisms. One of these may be the core excitation where the 142 keV $\frac{1}{2}^-$ level is coupled to the first vibrational $2^+$ level of the core i.e. $Mo^{98}$.

Acknowledgements

It is a pleasure to thank Dr.V.Devanathan for fruitful discussions and Professor K.Venkata Reddy and M.Ravindranath for their assistance in the measurements.

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4. J.J.Fujita, Prog. Theor. Phys. 28 (1962) 338.

## CAPTIONS TO FIGURES

Fig.1. Comparison between measured shape and the one calculated from single-particle matrix elements.

Fig.2. Decay scheme of $Mo^{99}$ due to the present author.

Decay scheme of Mo-99

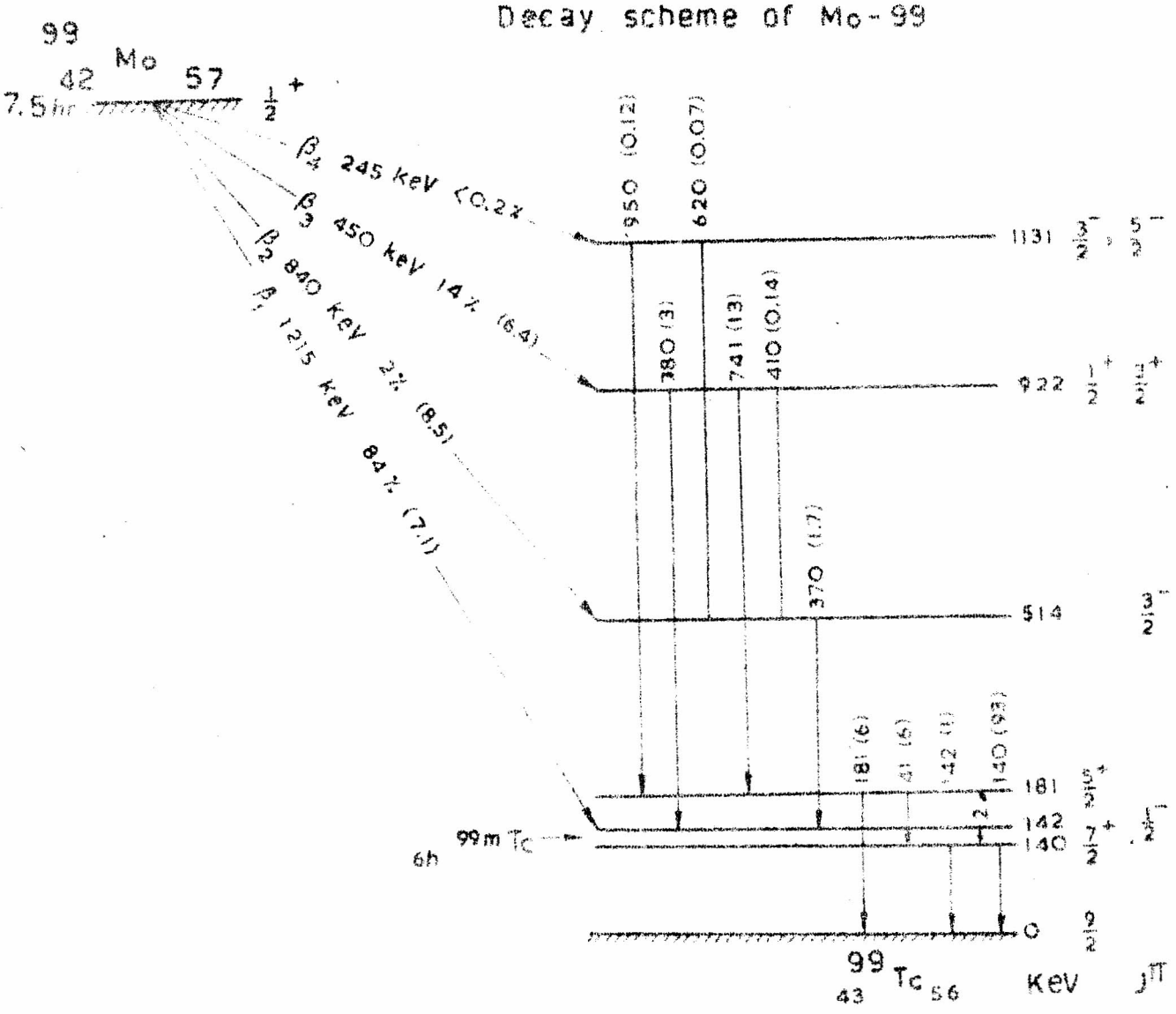


FIGURE . 1

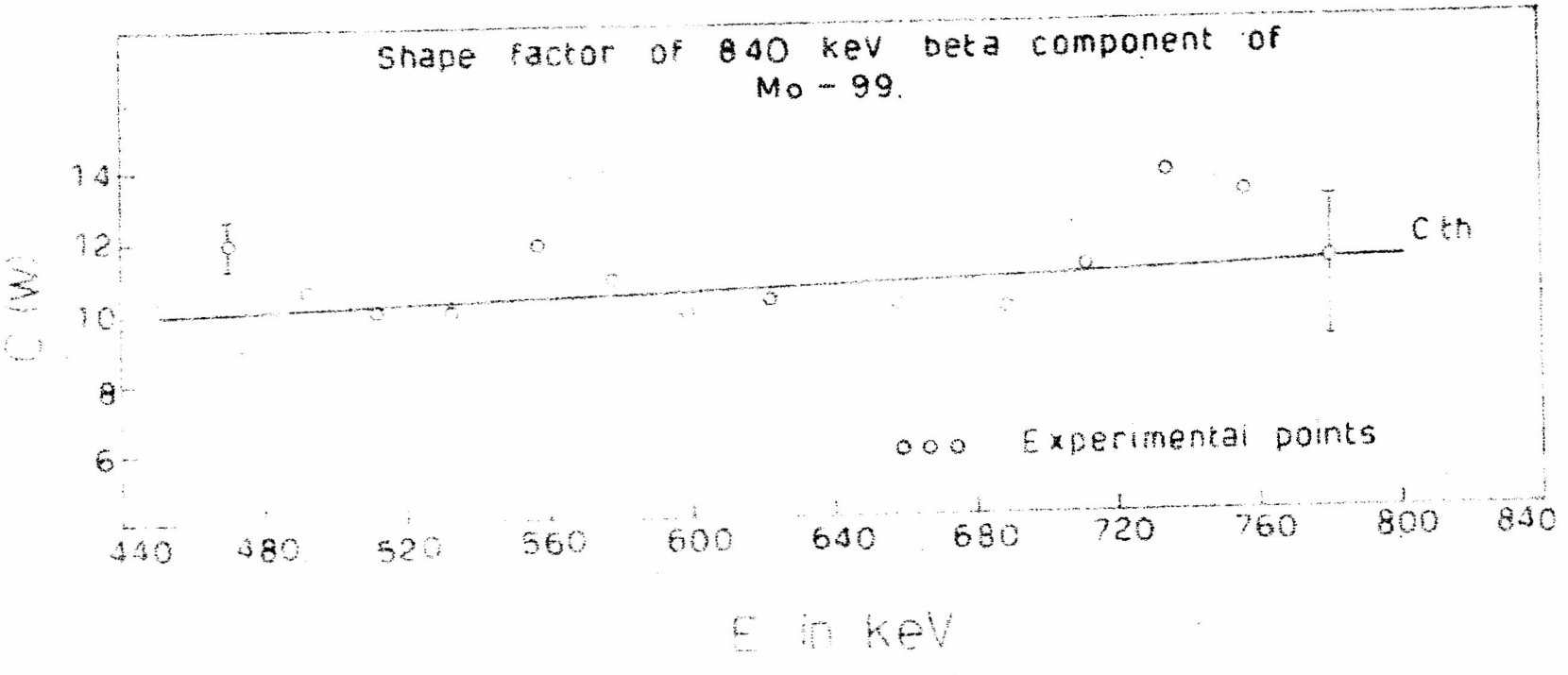
Shape factor of 840 keV beta component of Mo - 99.
C (W)
14
12
10
8
6
C th
o o o Experimental points
440
480
520
560
600
640
680
720
760
800
840
E in keV

FIGURE . 2

# POSITRON ANNIHILATION - A NUCLEAR TOOL FOR SOLID STATE

P. Ramaswamy*, T. Nagarajan, V. Devanathan
and A. Narayanaswamy
Department of Nuclear Physics, University of Madras,
Madras-600025, INDIA

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## 1. Introduction

An electron and positron in bound state called positronium has dimensions 1.06 Å and its triplet and singlet states have half-lives $1.4 \times 10^{-7}$ sec. and $10^{-11}$ sec. respectively. Positronium formation is possible in non-crystalline insulators and amorphous substances where vacant sites of the dimensions of positronium atom exist. This gives rise to long lived component ($10^{-9}$ sec.) in the above substances. It has been established that positronium formation is not possible in metals and crystalline solids.

The annihilation quanta exhibit an angular correlation which is determined by the motion of electrons in solid matter. If P is the centre of mass momentum of the electron-positron system the correlation in the X Y plane is given by

$$P_Z = \theta \quad mc$$

The picture becomes very simple for an ideal metal which can be approximated to 'free electron gas' with Fermi momentum $k_F$. Then the angular correlation is given by

$$N(k) = \text{const.} \left[ k_F^2 - k^2 \right]$$

---

*Presented by P.Ramasamy

The valence electrons of metals such as Na, Al etc. exhibit this behaviour[1]. In these metals the central parabolic distribution due to conduction electrons is superposed on a broader distribution due to core electrons. The picture is not so simple for transition metals where the particle-like states of d-bands show up strongly[1].

2. Momentum distribution of electrons in solids.

If $\Psi_+(\vec{x})$ and $\Psi_-(\vec{x})$ are positron-electron wave-functions, then the momentum distribution of electrons in solid are given by

$$\chi(\vec{k}) = \int d^3\vec{x}\, e^{-i\vec{k}\cdot\vec{x}}\, \Psi_+(\vec{x})\,\Psi_-(\vec{x})$$

The angular correlation of annihilation quanta is given by

$$N(k) = \int_0^\infty\int_0^\infty \chi(\vec{k})\, dk_x\, dk_y .$$

X-ray structure factors provide information on charge distribution of electrons whereas positron annihilation and compton profile techniques provide information on momentum distribution. In fact the latter techniques are especially very sensitive to valence electrons. Calculations of momentum density are more sparese than charge density calculations although at the time of charge density calculations it is not difficult to add a programme to determine momentum density. We can say that momentum density is a more sensitive test of a wave function than charge

density since the momentum depends on the shape of the wave function while the charge depends on its magnitude. The line shape J(q) of Compton Profile has been computed by R.J.Weiss (2) using Hartree-Fock wavefunctions. This integral is identical to N(k) and therefore can be used to obtain contribution to two photon correlation from core electrons.

3. Experimental set-up.

Two $1\frac{1}{4}$'' NaI (TI) crystals mounted on RCA 6199 photo-multipliers are shielded by 5'' thick lead cylinders with collimation slits 0.1 X 3 cms. (Fig. 1) The collimators are at a distance of 2.5 metres resulting in a resolution of 0.8 milliradian and transmission of $0.16 \times 10^{-3}$%. The positron source used is a 75 millicurie $Co^{58}$ evaporated on Mylar foil and area 3.2 X 1 cm. The $Co^{58}$ source was shielded by parabolic lead blocks so that it was not seen by detectors directly. One arm is fixed and the other is movable to a precision of 0.01 mm. by a micrometer screw. Only 511 KeV spectrum was seen in the singles. An ORTEC coincident unit of 0. 1$\mu$ sec. resolving time is used. The gates were set to accept only the photopeaks of 511 KeV.

The movable detector was moved in steps of 1 mm. and the coincidence and singles were accumulated for 2000 seconds. Five to seven runs were taken for each sample. The coincident counts were normalised for both the singles and the normalised counts of different runs were added point-wise. The above data reduction including calculation of error-bars and folding about the C.G. was done by IBM 1130.

4. Investigations of single crystals of alkali halides.

The 2Y angular correlation data in strongly ionic crystals should resemble the momentum distribution of the Wheeler compounds, since positron can be bound so tightly to the negative ions. But atomic states like $C\,l^{-}\,e^{+}$, $ClP_S$ or $H\,P_S$ are not possible as there can be no vacant sites to accommodate Wheeler atoms. But the main features of angular distribution should be derivable from the best wave functions of electrons in the outer shells of free negative ions. Any discrepancy between the angular distribution obtained from H.F, Wavefunctions of negative halide ions and the experiment should come from the crystalline character of the alkali halides. In order to test this hypothesis and also to see whether there is any asymmetry in the momentum distributions along the various crystal orientations detailed work has been done on KCl and NaCl crystals.

Since long exposures to the 810 KeV gamma rays of $Co^{58}$ result in the production of F centres, the Kcl and Nacl are coloured. To avoid this the positron source and sample were put inside the perspex vessel and it was well lit to cause optical bleaching.

Single crystals of KCl, NaCl etc. were grown from the melt by Kyropoulos techniques and the samples were prepared with their planes parallel to 100, 110, 111 etc. faces. The angular distributions corresponding to these plane orientations are shown in Fig. 2. Even though the angular distributions indicate rough

spherical symmetry, there is a slight asymmetry. The momentum and density distributions are given by

$$\rho(k) = \text{const.} \quad \frac{1}{k_z} \quad \frac{dN(k_z)}{dk_z}$$

and

$$N(k) = \text{const.} \quad k_z \quad \frac{dN(k_z)}{dk_z} .$$

These are shown in Figs. 3 and 4. Fig. 5 shows the angular distribution in oriented single crystal of KCl with and without application of an R.F. field. This effect needs further investigation.

5. Results and Discussion

A new mechanism of positron annihilation in single crystals has been suggested by Dekhtyar[3]. According to him there is a very high probability for the positrons to get reflected from internal faces of the crystals. Hence some of the positrons may escape from the crystal. This cannot happen in the case of a polycrystalline aggregate. Poly crystalline samples of KCl and NaCl were prepared with 25 Kg/cm$^2$. We compared the angular distributions of the polycrystalline samples and single crystals. There was no change in the areas under the angular distribution curves. If the mechanism suggested by Dekhtyar were correct the area of the angular distribution curve for the poly crystalline sample must have been more. Our observations raise doubts about the validity of Dekhtyar's hypothesis.

The very surprising question is that an intermediate life time is observed in ionic crystals. Attempts are being made to

use Bloch wave functions for electrons in KCl and calculate the momentum distribution. Further investigations are under way.

## ACKNOWLEDGMENTS

It is a pleasure to thank Dr.J.A.D. Mathew of University of York for fruitful discussions. We thank Professor C.Ramasastry and Dr.S.Radhakrishna, I.I.T., Madras for extending their full facilities.

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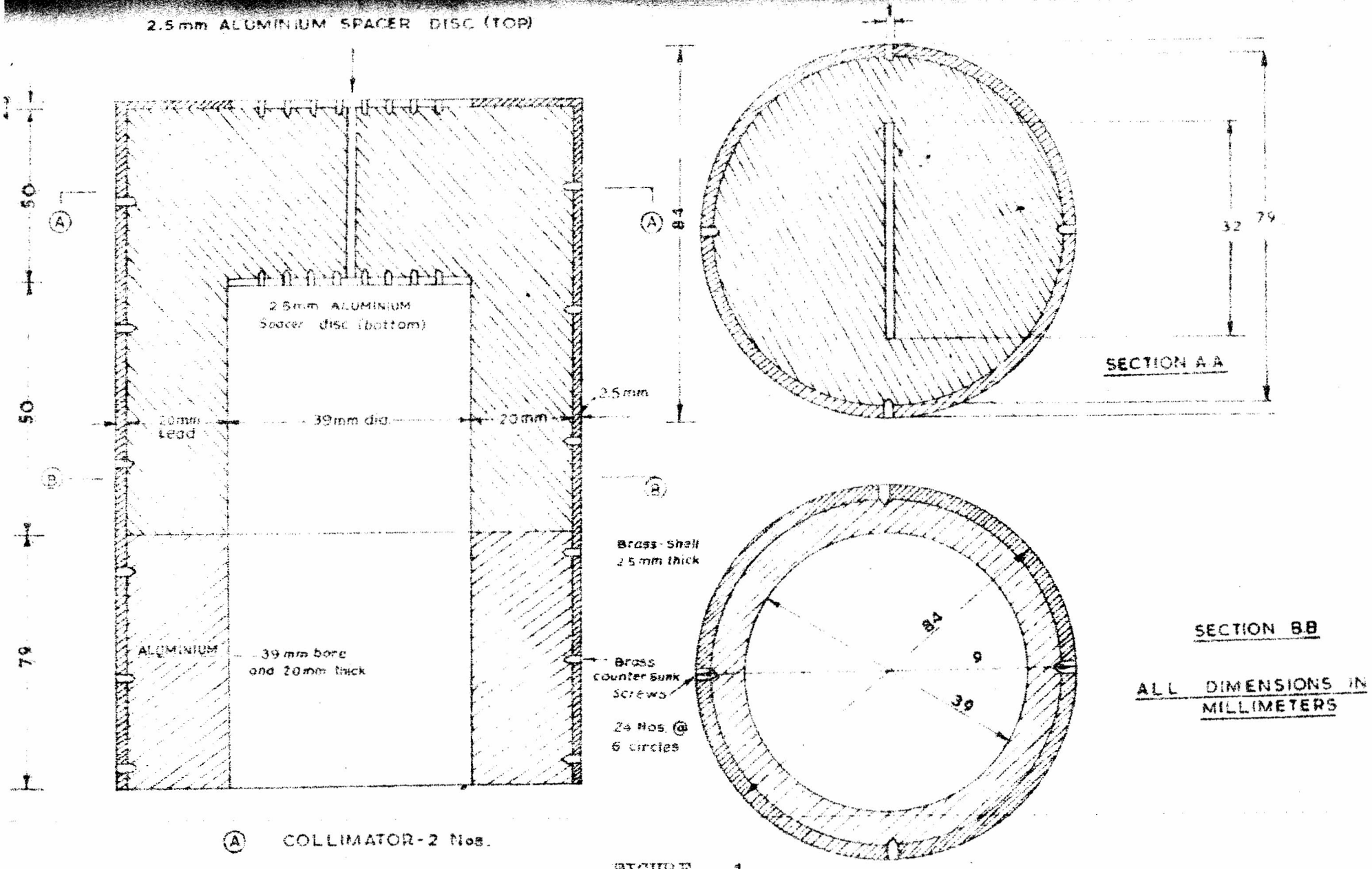


Ⓐ COLLIMATOR-2 Nos.

FIGURE . 1

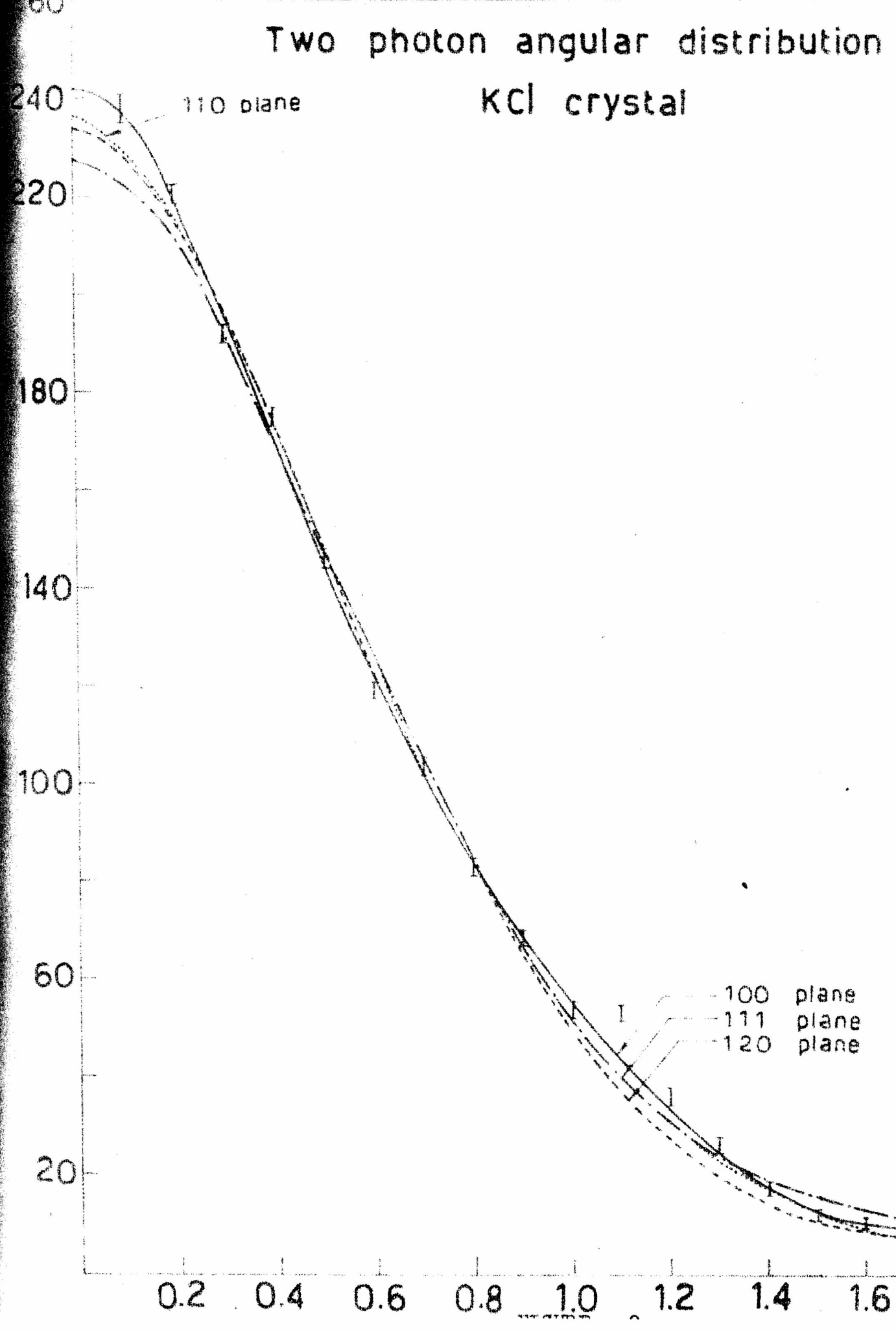
Two photon angular distribution
KCl crystal
110 plane
100 plane
111 plane
120 plane
240
220
180
140
100
60
20
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6

FIG. III

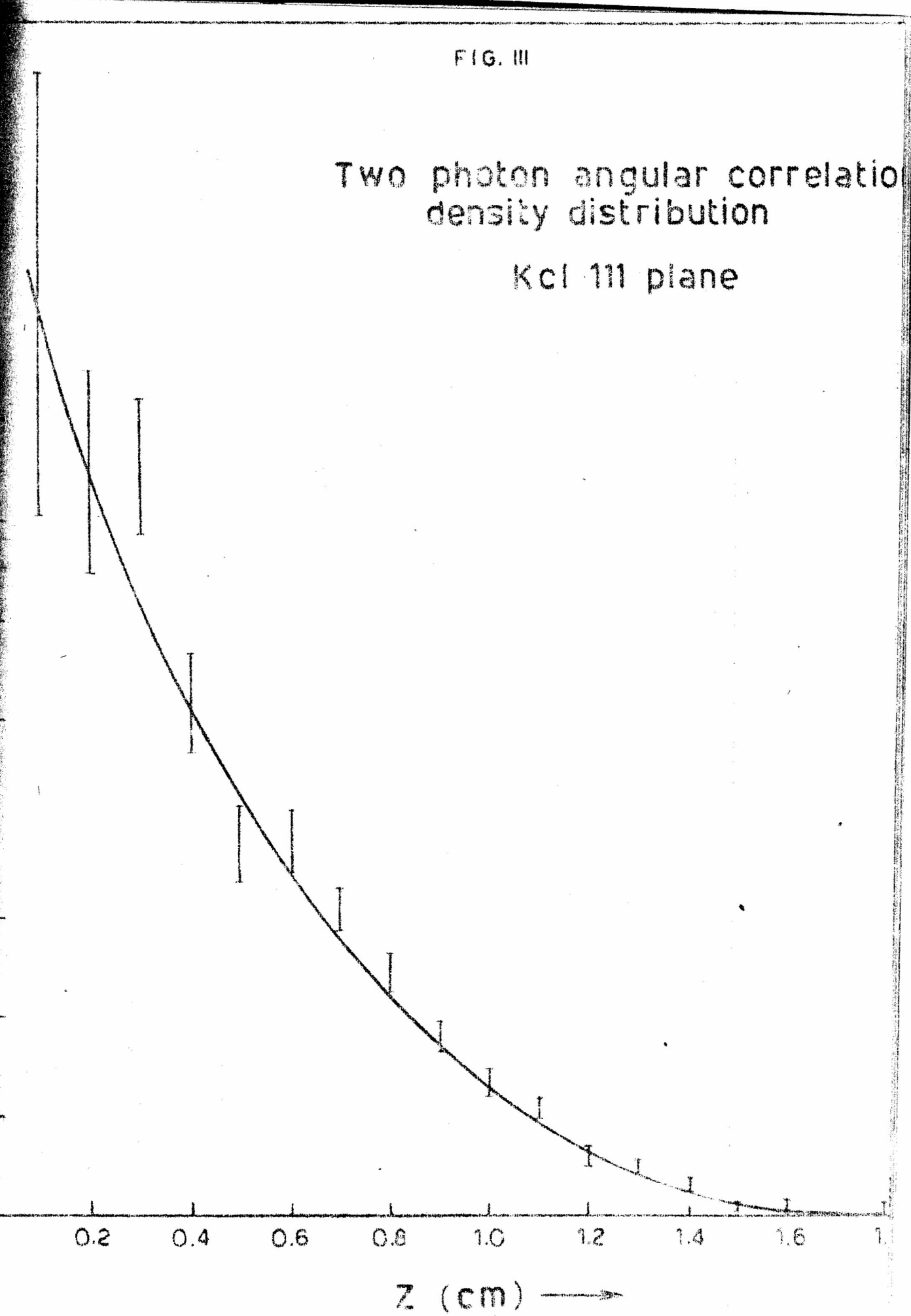


FIGURE . 3

FIG. IV

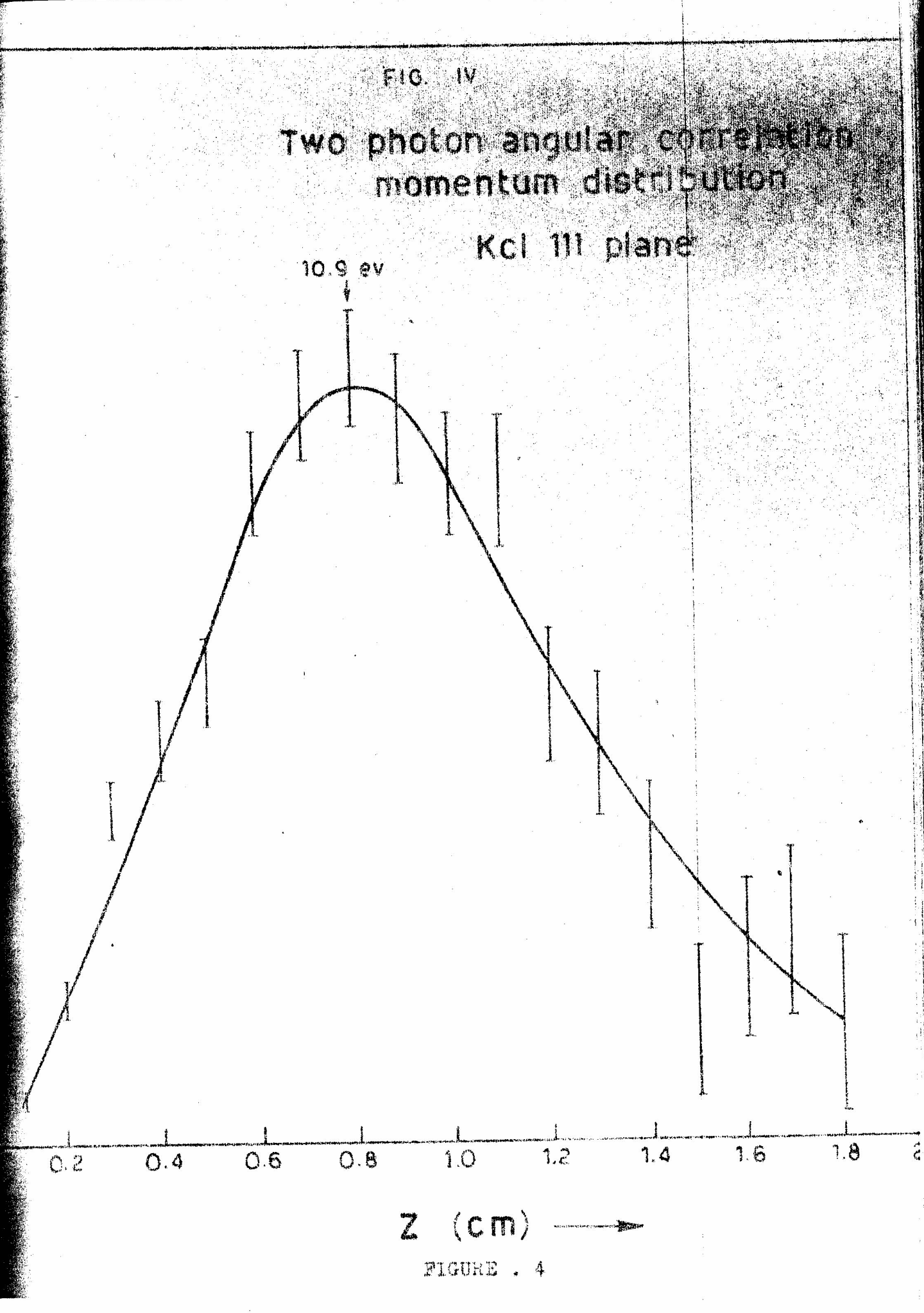


FIGURE . 4

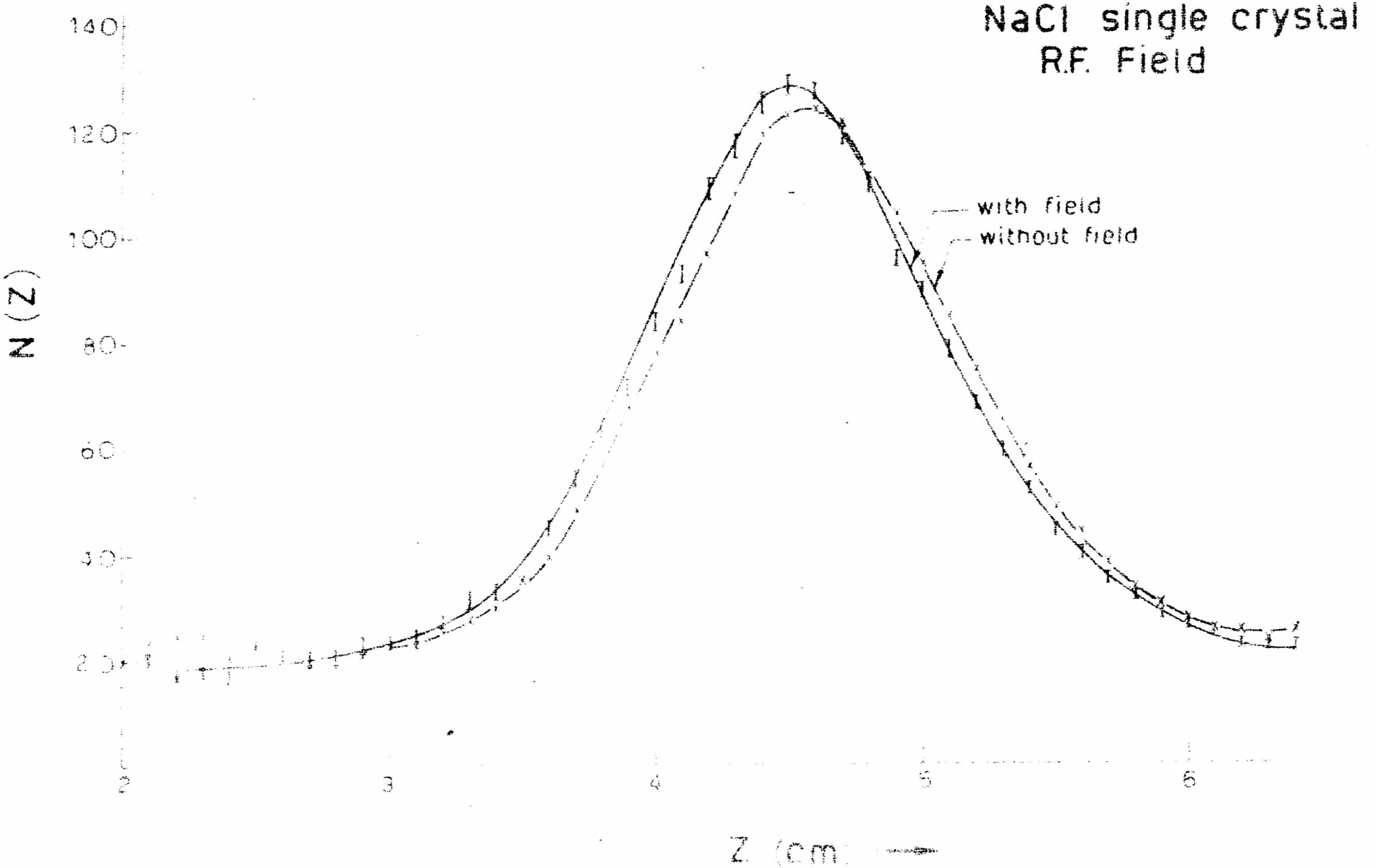
NaCl single crystal
R.F. Field
with field
without field
N (Z)
140
120
100
80
60
40
20
2
3
4
5
6
Z (cm)

FIGURE . 5

# DEUTERON FORM FACTORS WITH SOFT-CORE POTENTIALS*

S.Srinivasa Raghavan,
Physics Department, College of Engineering, Guindy,
MADRAS-600025

Kishi[1] has shown that the values of the deuteron from factor for the Kishi et al[1] potential and the Hamada - Johnston potential (both contain hard-core) agree well with each other for low q's (momentum transfer), but differ significantly for high q's. In this note we have evaluated the deuteron form factors with two different soft-core potentials in order to see whether both the potentials give identical results in the form factor calculations. In our calculations we use the soft-core potential of Reid[2] and the velocity-dependent potential (soft-core) of Nestor et al[3]. The soft-core deuteron wave functions for the Reid potential are given in Table XVI of ref.2, and for the Nestor et al . potential, we obtain the ground state deuteron wave functions by direct numerical integration[4] of the Rarita-Schwinger equation.

The explicit forms for the deuteron form factor are

$$F_o(q) = 2\, G_{ch}^{(s)}(q)\, I_c(q)$$

$$F_2(q) = 2\, G_{ch}^{(s)}(q)\, I_Q(q)$$

$$F_{mag}(q) = \sqrt{\frac{2}{3}}\left(\frac{q}{2M_c}\right)\left\{ 2\, G_{ch}^{(s)}(q)\, I_{m1}(q) + 2\, G_{mag}^{(s)}(q)\, I_{m2}(q)\right\} \qquad (1)$$

*To appear in Prog. Theor. Phys. (1973)

in which

$$I_c(q) = \int (u^2 + w^2)\, j_0(x)\, dr$$

$$I_Q(q) = \int 2ur\,(u - 8^{-1/2} w)\, j_2(x)\, dr$$

$$I_{m1}(q) = \frac{3}{4} \int w^2 [j_0(x) + j_2(x)]\, dr$$

$$I_{m2}(q) = \int [(u^2 - w^2/2)\, j_0(x) + w\,(2^{-1/2} u + 2^{-1} w)\, j_2(x)]\, dr \qquad (2)$$

where

$$x = qr/2 .$$

In the above equations $G_{ch}^{(s)}(q)$ and $G_{mag}^{(s)}(q)$ are the electromagnetic form factors of the nucleon and the subscripts c, Q and m's stand for the charge, quadrupole and magnetic moment, distribution of the deuteron respectively.

We have evaluated the integrals in Eq.(2) numerically for the potential of Reid and Nestor et al. and used Eq.(1) for calculating the form factors. For the purpose of comparison with experiments[1] we have also evaluated the quantity

$$A(q) = F_0^2 + F_2^2 + F_{mag}^2 \qquad (3)$$

Our calculated values for the form factors indicate that the Reid potential agrees well with experiments[1,5] for all q's.

The form factors for small q are essentially determined by the wave function in the outer region where OPEP is dominant[1]. Because of this fact, for small q, Reid potential which has a OPEP tail agrees better with experiments than the potential of Nestor et al. which has no OPEP for its tail. For higher q's our calculations clearly show that the deuteron form factor data favour Reid potential. Perhaps, a velocity-dependent potential with a OPEP tail and a large deuteron D-state probability (Nestor potential has $\sim$ 2% . D-state probability compared to Reid's value $\sim$ 7%) may show a better agreement with experiments than the Reid soft-core potential for the form factors. Calculations with such a potential are in progress.

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# NUCLEAR PHYSICS IN A LARGER PERSPECTIVE

K.H.Mariwalla
MATSCIENCE, The Institute of Mathematical Sciences
Madras-20, India.

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## I. What is nuclear physics?

First of all I shall like to say a few words on what is nuclear physics. In the popular mind the present age is the Nuclear Age. This impression has been to a large extent contributed by the demonstrated destructive power of the so called "Atom Bomb" and the much fanfare of claims regarding the "limitless" source of energy from controlled fission and fusion processes. From a technical viewpoint, Nuclear Physics proper covers only a small part of the developments in modern physics. Considering the state of poor understanding of nuclear phenomena it is difficult to characterise nuclear physics: the only common denominator being that it deals with nuclei.

But a variety of problems in electrodynamics, solid, liquid and gaseous states of matter could involve some considerations of the nucleus. Hence it would be more appropriate to define nuclear physics as dealing with those phenomena which involve active interaction between nucleons and nuclei and which may result in change in the state of the nuclei in interaction. For instance in the case of liquid $He^4$ its' statistics is determined by the nuclear spin; but this is not an active interaction but rather "passive" or "kinematic" effect and hence not part of nuclear physics per se.

Within the above definition nuclear physics was born, so to say, in the discovery by Becquerel of Radioactivity in 1896. May I seek your indulgence to recall that three types of rays were discovered: $\alpha$-rays, $\beta$-rays and $\gamma$-rays. The qualitative explanation of $\alpha$-decay phenomena by Gamow set the tune for the

applicability of Quantum theory to the study of nuclear phenomena. The $\gamma$-rays turned out to be electromagnetic waves. The success of quantum theory in the case of electromagnetic interactions (in molecules, atoms and nuclei) encouraged Fermi to give an anologous formulation for the phenomena involving $\beta$-decay. It was soon realised that $\beta$-decay belongs to a very wide class of phenomena which are collectively characterised as weak interactions. It is a surprising fact that the Fermi type formulation involving the motion of currents is found to be always possible for describing these phenomena. Unfortunately, however, unlike the electromagnetic theory no systematic procedure has yet been discovered to extract finite results for such a field theory. The forces which keep the nucleus together may roughly be characterised as strong interactions. The application of quantum theory has been even much less successful here. Consequently, the understanding of the fundamentals of nuclear theory is non-existant in this larger perspective. As a result, the subject has been practically split into two subjects.

(1) Nuclear Physics. This deals with the structure of a nucleus and with phenomena involving low energy nuclear physics. It does not concern itself with fundamental questions but is essentially confined to determine the effective interaction. It is this which is popularly called nuclear physics.

(2) The High Energy or Elementary Particle Physics: This deals with the study of phenomena involving particles responsible for nuclear forces in their nascent state. All the fundamental questions of nuclear physics form the subject matter here.

II. Problems and scope of Nuclear Theory:

What are the problems and scope of nuclear physics in the narrow sense as defined above. Since there is no basic theory of

nuclear physics, one has to set up models based on classical experience to fit the data, assuming the basic background structure to be quantum theory. There are two principal studies involved. (1) Interaction between "free nucleons" (2) the interaction between nucleons in nuclei. When one talks of interaction one is always looking for an effective interaction. To find it we need experimental data. If we had all the data we want it may be possible to determine empirically the effective interaction completely. Though a large amount of data is available at present, it is still not large enough for a complete determination of the interaction. However considerable progress has been made. The development of shell model of the nucleus is an excellent example of the painstaking collection and study of nuclear data which showed that in spite of strong initial objections it is nevertheless possible to describe the structure of a nucleus in terms of a shell model if one assumes a strong enough spin-orbit coupling. A bye-product of these studies has been the first extensive use of various "internal (as against geometric symmetry groups) symmetry groups" such as $SU_2$, $SU_3$ and $SU_4$ in nuclear physics much before they became fashionable in elementary particle theory. There are of course other nuclear structure models which are, however, used to explain only a particular class of phenomena. The scattering data on free nucleons and between nucleons and nucleus and its theoretical study forms another important branch of study called Nuclear Reactions. In the world of ideas it is relevant to mention here the tremendous impact that the Bohr's idea of the compound nucleus had on the developments of nuclear reaction theory. A close look at all these developments show that there is a striking parallel between (what we call) Reactions on the one hand and Structure (Shell Model) on the other.

However, in spite of all these "successes" the scene in nuclear physics is still far from satisfactory. Firstly, in the absence of any basic theory the various models, however successful they are in a given local setting, are at best only a stop gap arrangement. Secondly, even at the level of what is usually called nuclear theory, the quality of agreement between theory and experiment is unsatisfactory.

At this point it is useful to add that in high energy physics, the situation is not basically different. In the absence of a basic theory the phenomenological theories and models are the usual theoretical approaches. Each of these models however is successful in only a limited area. One may therefore question the very efforts made in developing such models to fit the data. On the other hand we recall that the birth of almost every physical theory has been preceded by a long trial of data fitting. For instance, Newtonian mechanics (experiments of Galileo, Huygens, Newton and others), Newtonian Gravitation (work of Kepler, Galileo and others before them), Maxwell's electromagnetic theory (discovery of Coulomb's law; the laws of Ampere and Faraday), Special relativity (various experiments on electrodynamics of moving bodies and attmpts to fit the data) and Quantum theory (discovered in trying to fit data on spectral lines of atoms and molecules). Apart from this argument based on historical developments in physics let me add that these phenomenalogical and model making studies have also been of immediate application. I have in mind here, the application of models and methods developed in these studies to other situations elsewhere in physics and engineering.

The case of the development of quantum theory is of particular interest to us. As already mentioned quantum mechanics was first formulated in the study of electromagnetic phenomena in atoms and molecules. The understanding of quantum electrodynamics has also enabled one to make considerable progress in the understanding of other nuclear phenomena by using electromagnetic interaction as a probe. A certain superficial anology of weak interaction "theory" with the electromagnetic theory is also perhaps responsible for most of the progress in the weak interaction theory. Any other advance in the understanding of weak and strong interaction physics is mainly phenomenological. It is therefore not possible to rule out that Quantum Theory is strictly applicable only to electromagnetic processes and that the nuclear phenomena are outside the scope of Quantum Mechanics. However in the absence of availability of any other tangible mechanics, quantum mechanics is the best we have. Hence within the scope of quantum theory and Relativity one would like to know if it is possible to predict some general features of a nuclear theory from some qualitative considerations. In the following we attmpt such an amalysis.

III. Strength-Range Relationship

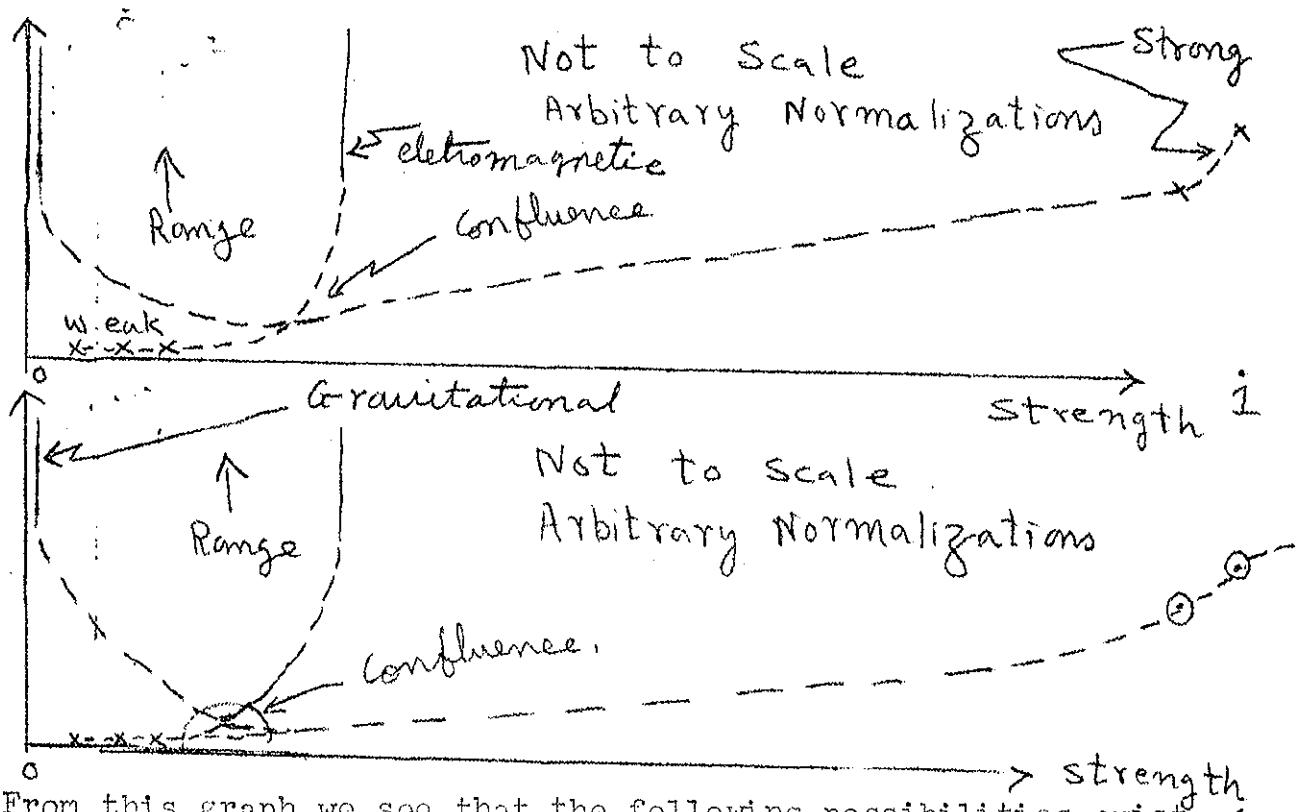
If we consider various interactions in nature, these can be roughly classified in terms of two concepts, viz. Strength and Range of the interaction as indicated in the following table. We must hasten to add that there are several other distinguishing features of these interactions which permit this classification.

| | Strength | Range | Name of the interaction |
|---|---|---|---|
| 1. | Very small (precisely known) | Infinite | Gravitational |
| 2. | Small (values in a certain range) | Very short possibly zero (values in a certain range) | Weak |
| 3. | Large (precisely known) | Infinite | Electromagnetic |
| 4. | Very large (values in a certain range) | Short (values in a certain range) | Strong |

It is clear from this table that if Range is infinite, th strength is a precisely defined number and vice versa. On the other hand if the values of strength of a given type of interaction lie in a certain interval then so do the values of the Range; this is the case for all short range interactions.

A close study of electromagnetic and gravitation theories reveals the role of their static fields which essentially determine their infinite Range and therefore their Strength. We conclude that in the case of short range interactions there is no clear distinction between static and dynamic processes, but the two are interlinked in the sense that a theory based on static interaction even in the most favourable case can atmost only be a rough approximation. The dynami nature of the interaction only implies that there are several fields involved simultaneously in these interactions as is evident from a range of values for Strength in case of short range interactions. For this reason it is all the more important to study the relationship between Strength and Range for short range interactions using the boundary condition, that as range $\longrightarrow \infty$ strength converges

to a well defined number. The following qualitative graphs[1] gives a rough picture of Strength-Range relations.




From this graph we see that the following possibilities exist 1

(1) Weak and strong interaction trajectories have separate asymptotic limits; these being respectively electromagnetic and gravitational coupling constants. In this case we can expect that in an overall theoretical structure there will be substructures of combined "weak and electromagnetic theory" on the one hand and "strong and gravitation theory" on the other; the intersection of the two graphs would provide the point of contact.

(2) Present evidence indicates that the Range of weak interactions is in all probability exactly zero. In this case there is a strong possibility that there is a single trajectory (with branches

on which all the interactions lie. In this connection we note that to-date all attempts to write only electromagnetism and gravitation have failed; hence any link between them has to be via the other interactions viz. short range interactions. Thus for instance heavy spin one particles of the same quantum numbers as the photon couple the electromagnetic field and similarly/heavy spin two particle could couple to the gravitation!

At this point it is necessary to emphasise that for the purpose of our discussion it is not necessary that every point on the strength-Range graph should be realisable in nature; in fact the distinction between various types of interactions is not possible in the first place if that was the case. Moreover it appears that for the case of weak processes there are only discrete points but for strong processes there is in all probability a limit point. Thi would mean that there are infinitely many particles involved in a complete description of strong processes.

It is our hope and conjecture that a more concentrated study will show that gravitational and electromagnetic interaction coupling constants arise from the strength-Range relation for strong and weak interactions roughly in the manner indicated above (i.e. in the infinite Range limit). In further support of this view-point we present the following considerations.

Recall that in analysis of the concept of time and the related concept of simultaneity one is ultimately led to conclude with Einstein that since electromagnetic waves enter in a natural way in prescribing synchronization between clocks, certain properties of these waves must play an active role in the definition of these concepts. This accounts for the leading role assigned to the velocity of light in special relativity. In fact arguments along

these lines led Einstein to the Special and General theories of Relativity as a natural completion of the corresponding ideas of Newtonian mechanics and gravitation. A similar situation exists in the theory of elementary particles. All short range interactions both strong and weak are observable only in terms of electromagnetic and gravitational phenomena, in the sense that the observation and analysis of the experiments involving strong and weak interactions invariably involve electromagnetic and possibly also gravitational phenomenon. Working along this line of thought we conclude that electromagnetic and gravitational phenomena play an important role in the theories of Weak and Strong interactions.

The Strength-Range relations for short range interactions have a further significance independent of gravitation and electromagnetism. For a given coupling constant for strong interactions we can define a certain 'Range' number, say, $\ell_1$; by this we mean that the interaction takes place in the range 0 to $\ell_1$ and effectively vanishes for distances greater than $\ell_1$. It is now possible to do Fourier analysis on $[0, \ell_1]$; discrete numbers which thus arise we call $n_{\ell_1}$. In the same fashion other coupling constants give rise to further numbers $n_{\ell_2}, n_{\ell_3}, \cdots$ In a basic theory of fundamental particles if we thus start with say 3 (this is a necessary minimum) basic coupling constants and their corresponding Ranges it is possible to build up a discrete set of secondary coupling constants (and the corresponding Ranges) by a linear superposition; such a set would clearly have a limit point. It is quite possible that quark and parton models arise just in this manner. Detailed study is here called for.

IV. Space-time dimension

> It is probable that the 3-fold dimensionality of space is a consequence of the law according to which the forces of substances act on each other
>
> Emanuel Kant[2]

A puzzling aspect of any physical theory much taken for granted is the space-time dimensionality. We shall presently show that the dimensionality actually found in nature[3] has much to do with certain discrete and internal symmetries of elementary particle theory. Let the space-time be of d dimensions and pseudo-Euclidean of signature $\pm$ (d-2); the spatial dimension is then (d-1). If d is odd, (d-1) is even; hence spatial inversion in this case is a continuous symmetry and not a discrete operation. Now it is a fact of life that right polarized and left polarized light waves which are mirror images of each other are physically distinguishable. We note parenthetically that the distinguishability between left and right is in a measure responsible for the very existence of life since this distinction plays no inconsiderable role in biological, biophysical and chemico-physical phenomena[4]. If it were possible to convert right polarized and left polarized waves into each other by a mere rotation (as would be the case if space was even dimensional) then there would exist no enantiomorphs and no enzymatic action. From this viewpoint we are led to conclude that asymmetry in nature, as observed in the biological world is a necessary condition for life as is the odd dimensionality of space with which it is interlinked.

Returning to the role of mirror symmetry in particle theory, we note that even though right and left handed waves are distinct,

it is possible to take their linear superpositions to obtain plane polarized and elliptically polarized waves which again exist as "single" entities. On the other hand if we take the corresponding spin group (covering group of Euclidean motions) of the space-time symmetry, we find that its representations for the zero mass case do not even permit a linear superposition if the space is odd dimensional. The existence of neutrinos and antineutrinos is thus an inevitable consequence of the odd dimensionality of space (and even dimension of space-time) which in turn is the sine qua non for the very concept of parity and hence also of its violation.

We saw in the above that two states of opposite parity for light waves are superpasable, but this is not the case for neutrinos. One distinguishes these two cases by saying that right and left polarized waves are related by ordinary, or $\mathcal{P}$ parity, whereas neutrinos and antineutrinos are related by generalized or $\mathcal{CP}$-parity. In the case of light waves $\mathcal{C}$ has the interpretation that there are two types of charges, positive and negative and the electromagnetic waves (whether real or virtual) mediate interactions between them; of this we already know from the electromagnetic theory.

If the space is odd dimensional, clearly the space-time is even dimensional. Since space-time is pseudo-Euclidean, of signature $\pm$ (d-2), the inversion of the sign of all the d coordinates is not possible as was the case for Euclidean spaces. However the operation of changing the sign of all the d(=even) coordinates has determinant +1, as in the case of rotations ! This presents interesting possibilities. To start with we generalize the $\mathcal{CP}$ operation by defining a space-time inversion operator called "Strong Reflection" [5,6] which has the obvious form $\Theta = \mathcal{C}\mathcal{P}T$ where T stands for "Time Reversal".

Since $\theta$ has determinant $+1$, we expect that there is an extension of the isotropy group of space-time (viz. the homogeneous Lorentz group), such that $\theta$ is continuously connected to the identity in the extended group (It is clear that this is not possible if d is odd, for then the determinant of $\theta$ is -1). The complex Lorentz group gives just this extension. We have shown elsewhere[7] that the maximal compact subgroup of the complex Lorentz group gives precisely the group in which $\theta$ is continuously connected to the identity only if d=4. In this case the group in which $\theta$ is continuously connected to the identity is $SU_2 \times SU_2$. This may be considered as an argument in favour of space-time dimensionality of four[3]. On the other hand one could also conclude that given a pseudo-Euclidean space-time with d=4, any internal symmetry group must contain $SU_2 \otimes SU_2$ as a subgroup; the simplest possibilities thus are $SU_3 \otimes SU_3$ or $SU_4$.

In the context of this conclusion we note the following. For the Dirac equation the operation $\theta$ involves a linear transformation by $\gamma_5$; for the case of vanishing mass one obtains the so-called $\gamma_5$-symmetry which gives rise to the characteristic doubling of leptons. In our analysis this doubling is of more general nature and would also extend to strongly interacting particles. We note, that similar conclusion also obtains from considerations of an entirely different nature in the recent work on broken space-time dependent internal symmetries[8].

Summary: In the first two sections we discuss the place of Nuclear Physics vis-a-vis the developments in physics of the last half century. It is concluded that the basis of Nuclear theory can be clarified only in the larger context of physics of all matter and energy. In this connection two general lines of thought are develop using the (1) concept of strength-Range relationship for various interactions; and (2) space-time dimensionality and discrete symmetri

## References


1. The following references contain useful data for drawing such graphs. The present graph however is purely qualitative.

   a. G.Ebel, H.Pilkuhn, Nuclear Physics B.17 (1970), 1
   (compilation of coupling constants and low energy parameters)

   b. Mary E.M.Hogan, Thesis Florida State University, Tallahassee (Uses $SU_3 \times SU_3$ Chiral Lagrangian model and determines several "scattering lengths" and coupling constants)

   c. Gribov, et. al. Zh. Eksper. teor. Fiz (USSR) 41 (1961) 619; Phys. Rev. Letts. 11 (1963) 55-58 (Limits on coupling constants)

It is desirable to collect more such data and make a qualitative study to compare various models.

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3. For other arguments in favour of 4-dimensional space-time, see K.H.Mariwalla, J.Math.Phys. 12 (1971) 96.

4. For a discussion of assymmetry in nature referred to here see for example, K.H.Mariwalla, Ph.D. Thesis (1963) University of Georgia (U.S.A.), Introduction. Glucose, Sucrose and Racemic Compounds (known since Louis Pasteur-1848) are well known enantiomorphic compounds. The connection with biophysical phenomena arises via enzymatic action. We note that to seperate two enantioforms one either requires a physical process (seperation by hand (!) or by use of circularly polarized light) or enzymatic action.

# POSITIVE DEFORMATION OF ODD-ODD NUCLEI AND THE CORE-ROTATION

S.D. Sharma
Department of Physics, Panjabi University, Patiala, INDIA.

## INTRODUCTION

In case of single particle shell model description, the quadrupole moment of the nuclei near magic numbers is predicted to be negative. Away from the closed shells the nuclear core gets deformed due to the incoming odd nucleons. In that case, the quadrupole moments have dominent contribution from the core. The shell model can not explain these enhanced orders of the static quadrupole moments in these regions of deformations. The model is to be modified to include the core rotation due to the developed moment of inertia of the system. Away from magic numbers the quadrupole moments are mostly from core and this contribution is positive. The tendency for positive deformation in case of odd-A nuclei has been discussed by S.A. Moszkowsky[1]. The ground state of Ga, Ca, As, Br, Pb, etc. were found to provide a sticking evidence for this tendency of the nuclear core. The same conclusions were found to hold good in case of some odd-odd nuclei with N=Z also. ($H^2$, $Be^{10}$, $N^{14}$). A similar conclusion was pointed out for all the defomed odd-odd nuclei in the periodic table (not necessarily with N = Z) in his scanning on static and dynamic properties of these nuclei on the basis of symmetric and asymmetric core collection models[2),3)]. The investigation led to the conclusion that the elongation of the nuclear core along the symmetry axis is much favoured in case of these nuclei also. The experimental reports for these nuclei are meager even these days as

regard the studies of their static moments, although the experimental expositions for the energy spectra are quite adequately reported for some of them. The present attempt deals with the theoretical aspect of the problem under the framework of the selected model.

THEORETICAL

Assuming the nuclear core as inert to vibrations, but capable of rotation with odd nucleons attached to its surface, one can generate the eigenvectors while diagonalising the state matrices for the Hamiltonian consisting of the rotational part and the particle parts, including the l.s., $l^2$ and the residual interactions of either type or the Gaussian type. The asymmetry effects enter just rhrough the additional nucleonic waves attached to the coral surface. If we consider only the ground state configurations of odd nucleons in order to study the static moments, the problem gets quite simplified for the symmetric core. The a bove mentioned model thus yields simple expression for the quadrupole moment.

$$\langle Q \rangle = \frac{I(2I-1)}{(I+1)(I+3)} \left\{ Q_0 + \right.$$

$$\left. + \frac{0.415}{41\sqrt{\pi}} A^{1/3}(N_p+\tfrac{3}{2})\left[\tfrac{3}{4} - j_p(j_p+1)\right]\left[\frac{j_p^2(2j_p-1)}{(j_p+1)^2(2j_p+3)}\right]^{1/2} \right\}$$

where

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z_c R_0^2 \beta.$$

Here in the value for Q the first term is the contribution from the core, while the second term is the contribution of the odd proton. It is clear that the contribution of the neutron does not enter into picture but it is only through the modification of the nucleonic wave-pattern on the surface due to the additional dragged neutronic currents that the quadrupole moment gets affected a little. Thus the quadrupole moments of isotopes are expected to differ to quite a small extent only.

For positive defomations the core contribution is positive. Thus the Q is expected to be positive for this case. The opposite contribution due to the odd proton is quite small in magnitude. Note that the deforming Hamiltonian is:-

$$H_{def.} = H_{def.(p)} + H_{def(n)},$$

$$H_{def(i)} = -\beta m_i r_i^2 \omega_i^2 \left[ Y_{20}(\theta_i \varphi_i) \cos\gamma + \frac{1}{\sqrt{2}} \sin\gamma \left[ Y_{22}(\theta_i \varphi_i) + Y_{2,-2}(\theta_i \varphi_i) \right] \right].$$

$i = p, n$

From the expectation values of this part of the Hamiltonian it is quite clear that only the positive deformation can yield the maximum energy of the system. This is also expected from our classical notions about the rotational energies ellipsoids and spherioids of revolutions.

RESULTS.

We shall quote here the cases of only two odd-odd nuclei beyond the lp-shell, whose quadrupole moments are experimentally reported. $Cl^{36}$ has Q = -.017 barns a small negative value, which is quite expected near the magic number 20. In case of $_{65}Tb^{158}$ the experimental Q = +2.5 barns, while the computed value for the best fit parameters of reference 2), is 2.7 barns.

Also the results of scanning for the best fit to the experimental energy-spectra yield strikingly good results for positive deformations only[2]. The speculations of Dr., C.J. Gallagher for positive deformations of almost all the odd-odd nuclei away from magic numbers are found to hold good in these investigations[4),5),6)]. The conclusions are found to hold for about 32 nuclei starting from $Na^{22}$ upto the nucleus $BK^{250}$.

CONCLUSIONS.

The state of positive deformation is the one having the lesser energy, and thus the deformed nuclei are expected to acquire positive deformation while adjusting to the incoming nucleonic currents. The previous analysis by Moszkowsky for N = Z odd-odd nuclei holds in general. The asymmetry effects and the effect of an additive neutron on the core, are quite small. The theoretical approach supports the conclusions of C,J.Gallagher. It is quite quite evident that the odd proton behaves as if it has formed a part of the nuclear core. Thus we expect that at higher excitations the core will go on rotating with the odd proton sticking to its surface and without changing the sign of its quadrupole moment. The relative contributions of core and the odd proton will be held in tact. The cossal of the protonic contribution to the Q-value is not expected in the course of excitations. This result is quite contrary to the one for magnetic moments of odd-odd nuclei[5]. It is to be pointed out that the $\beta^2$-errection to the Q-value will yield results which are even more positive.

From the case of Tensor-corrections to the Q-value in case of Deutron, we expect that the correction on the basis of noncentral force will yield better results. Tendency for positive deformation may be arising due to the distortion of the nuclear core by the attractive tensor forces. As discussed in reference 1), the positive Q-value of the simplest neutron proton system in $3s_1$ state (in case of $H^2$) is accounted for by the attractive tensor forces, but the nature of p-p tensor forces is not known. These matters need lot of investigations in case of heavier odd-odd nuclei.

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# POLARISATION OF NEUTRONS IN HIGH ENERGY ANTINEUTRINO SCATTERING

M.T. Teli
Department of Physics, Shivaji University, KOLHAPUR (India)

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ABSTRACT

The stokes parameters and degree of polarisation of the recoil neutrons in the $\bar{\nu} + p \rightarrow \bar{l} + n$ high energy scattering are discussed for various cases of proton polarisation under both the T-invariance and non-invariance conditions. The polarisation analysis is also made through the relations between any pair of degrees of polarisation which are characteristic of the target polarisation and its energy and are independent of the scattering angle and form factors.

## 1. INTRODUCTION

The high energy neutrino induced reactions are of great importance in testing T-non-invariance and determining weak form factors. The study of polarisation of the particles is necessary in order to determine the second class form factors. Adler[1] calculated the expression for scattering cross section in covariant form by keeping the target neucleon unpolarised and both the final particles polarised. Berman and Veltman[2] studied the transverse polarisation of lepton while Fujii and Yamaguchi[3] that of the proton in the $\nu$ -scattering in order to test the T-non-invariance when the neutron target was unpolarised. They found appreciable transverse polarisation of the particles they studied on the Cabbibo model[4]. Kettley[5] suggested the weak form factors analogous to the electromagnetic form factors for the vector and axial vector currents at the strong vertex in the

$\nu(\bar{\nu})$ - scattering processes and used them to compute the cross section and the polarisation of any one of the final particles by keeping the initial nucleon unpolarised in order to test the T-non-invariance and determine the form factors. Sarkar[6] calculated the expression for the differential cross section in covariant form by keeping the target and the final particles polarised but he neglected the lepton mass, while Lyulka[7] did similar work in laboratory system with either of the final particles unpolarised. Similar literature can be also be found in the work of Sokolov et. al.[8], Teli[9] has calculated a covariant expression for the different cross section of the $\nu(\bar{\nu})$-scattering by keeping all the particles polarised and including the lepton mass.

In this paper we consider the polarisation of the recoil neutron in the high Energy $\bar{\nu} + p \longrightarrow \bar{\ell} + n$ process with proton as the polarised target in the c.m. system of the particles. The polarisation is studied through the stokes parameters and also though the relations between any pair of the degrees of polarisation[10] which either represent an ellipse, a circle or a straight lines characteristic of the target polarisation under both the T-invariance and non-invariance conditions. Important limits are obtained for the $\bar{\nu}$ -energy and scattering angles for definite polarisations of the neutrons.

## 2. NOTATIONS AND CONVENTIONS

We use the following notations,

$$u = \sin \varphi_1, \quad u' = \cos \varphi_1 , \tag{2.1}$$

$$v = \sin \varphi_2, \quad v' = \cos \varphi_2 , \tag{2.2}$$

where ,

$$\varphi_{1,2} = \varphi_4 \pm \varphi_3 , \tag{2.3}$$

with

$$\tan \varphi_3 = at, \quad \tan \varphi_4 = (2b - a)t , \tag{2.4}$$

and

$$a = M/W, \quad b = \nu f_V/g_V, \quad t = \tan \theta/2 . \tag{2.5}$$

$\nu$ , W and $\theta$ are respectively the antineutrino energy, total energy and the scattering angle in the c.m. system. $g_V$ and $f_V$ are respectively the vector and the tensor weak form factors which are functions of $q^2$.

The conventions for the polarisation vectors of the proton ~~of~~ and the recoil neutron are respectively $\vec{s} = (s,s',s'')$ and $\vec{P} = (P,P',P'')$ the components of which describe the degrees of polarisation respectively the Longitudional, the Perpendicular in the reaction plane and the Tranverse to the reaction plane in such a sense that they form a right handed coordinate system.

## 3. STOKES PARAMETERS AND DEGREES OF POLARISATION

### 3.1. T-non-invariance.

We make the following assumptions,

i) Lepton mass neglected. Then the contribution due to the induced scalar and induced pseudoscalar form factor vanishes.

ii) C?C and T-non-invariance. Then $g_V$, $g_A$, and $f_V$ are real while $h_A$ is imaginary.

iii) Neglect the phase difference of $g_V$ and $g_A$ (i.e. they are equal).

iv) $h_A = if_V$ and

v) Neglect the small mass difference of neucleons ($M_n = M_p = M$).

Under the above assumptions we get the following expressions for the stokes parameters of the recoil neutron from the expression of the difference scattering cross section given by Teli[9],

$$(I, P_1, P_2, P_3) = d\sigma_1 [x u' + u^2, x v' - s u^2, -(x v - s' u^2), -(x u - s'' u^2)]. \tag{3.1.1}$$

where

$$x = u' + s v' + s' v + s'' u \tag{3.1.2}$$

and

$d\sigma_1 = d\sigma_0 (1 + s_\ell)$, $s_\ell$-helicity of the anti-lepton, and $d\sigma_0$ the scattering cross section when all the particles are unpolarised.

The polarisation vector is defined as

$$(P, P', P'') = (P_1/I, P_2/I, P_3/I). \qquad (3.1.3)$$

The stokes vector or the polarisation vector satisfy the symmetry property,

$$\vec{P}(\vec{s},\varphi_1) = -\vec{P}(-\vec{s}, \pi-\varphi_1) \text{ and } I(\vec{s},\varphi_1) = I(-\vec{s},\pi-\varphi_1). \qquad (3.1.4)$$

We observe that if the target proton is totally polarised i.e.

$$s^2 + s'^2 + s''^2 = 1, \qquad (3.1.5)$$

then from the equations (3.1.1) to (3.1.3) we obtain

$$P^2 + P'^2 + P''^2 = 1 \qquad (3.1.6)$$

and the recil neutron is also totally polarised. If, however, the target is unpolarised, the neutrons are partially polarised since

$$P^2 + P'^2 + P''^2 < 1. \qquad (3.1.7)$$

In the case of forward $(\theta = 0)$ and backward $(\theta = \pi)$ scattering we have

$$(I, \vec{P}) = d\sigma_1(0 \text{ or } \pi)[(1\pm s), \pm(1\pm s), 0, 0] \qquad (3.1.8)$$

and the recoil neutrons are right (left) handed. However, for $s = \pm 1$, $(I, \vec{P}) = 0$ and antineutrions are not scattered in the forward (backward) direction by the (left) right handed proton targets.

We now discuss the neutron polarisation for various cases of proton polarisation.

We see that

$$X = 0 \quad \text{or} \quad u' + sv' + s'v + s''u = 0 , \qquad (3.1.9)$$

provides various conditions for definite polarisation of the neutron. The stokes parameters then become

$$(I, \vec{P}) = d\sigma_1 ( 1, -s, s', s'') u^2 \qquad (3.1.10)$$

and the neutrons have the same polarisation of the target except that there is a spin flip in the case of the longitudinally polarised target.

I. Target Unpolarised ($\vec{s} = 0$):-

From the eqns. (3.1.9) and (3.1.10) we observe that the recoil beam is completely unpolarised at $u' = 0$ or $\varphi_1 = \pi/2$. The corresponding scattering angles are obtained from the eqn. (2.1) which gives

$$\tan^2 \theta/2 = 1/(2b-a)a . \qquad (3.1.11)$$

At other angles the beam is partially polarised.

II. Target Longitudinally Polarised ($s = \pm 1$):-

Eqn. (3.1.9) gives

$$u' + sv' = 0 \quad \text{or} \quad u' = \mp v' . \qquad (3.1.12)$$

This is possibleonly a t $\theta = \pi$ for $S = \pm 1$ and $\theta = 0$ for $S = -1$. But according to the eqn. (3.1.8) no scattering oocures

in these cases. From the eqn. (3.1.1) we also note that $P_1 = 0$ if

$$v'(u' + sv') = su^2. \qquad (3.1.13)$$

Solving this eqn. we get

$$\cot^2\theta/2 = a^2 - 2ab + 2b^2 \quad \text{for } s = +1$$

and

$$\tan^2\theta/2 = (a^2 - 2ab + 2b^2)/2a^2(2b-a)^2 \quad \text{for } s = -1. \qquad (3.1.14)$$

The angle $\theta$ in these eqns. is the scattering angle at which the polarisation is in the plane perpendicular to the scattering plane.

III. Target Perpendicularly Polarised ( $s' = \pm 1$):-

From the eqn. (31.1.9) we have

$$u' + s'v = 0 \qquad (3.1.15)$$

and the recoil neutrons have 100% perpendicular polarisation without spin flip. The condition for this becomes

$$\varphi_1 \pm \varphi_2 = \pi/2 \quad \text{for } s' = \pm 1, \qquad (3.1.16)$$

i.e. $$\varphi_{3,4} = \pi/4.$$

Using the eqn. (2.4) we get

$$\tan\theta/2 = W/M, \quad s' = +1,$$
$$= 1/(2b-a), \quad s' = -1. \qquad (3.1.17)$$

IV. Target Transversally Polarised ($S'' = \pm 1$):-

From the eqn. (3.1.9) we have

$$u' + s'' u = 0$$

giving

$$\varphi_1 = 3\pi/4 \quad \text{for } s'' = +1, \qquad = \pi/4 \quad \text{for } s'' = -1. \tag{3.1.18}$$

The corresponding scattering angles are given by the eqns. (2.1) to (2.4) in the form

$$\pm a(2b-a)t^2 - 2bt \mp 1 = 0, \quad t = \tan\theta/2. \tag{3.1.19}$$

If $b = a$, then

$$t = (\sqrt{2} \pm 1) W/M, \quad s'' = \pm 1. \tag{3.1.20}$$

If we take $f_v = (\mu_p - \mu_n) g_v/2M$, then using the eqn. (2.5) we get for $b = a$, the antineutrino energy nearly equal to 0.35 M which is nearly 650 times the rest mass energy of the electron. At this energy of the antineutrino, from the eqn. (3.1.1) we see that $P_2 = 0$ and the stokes parameters becomes

$$(I, \vec{P}) = d\sigma_1 (1 + s'' u u'', \; u' + s'' u, \; 0, \; -uu'). \tag{3.1.21}$$

In addtion, at $u' = 0$ or $\varphi_1 = \pi/2$, $P_3 = 0$ and the neutrons have 100% longitudinal polarisation. The scattering angle is given by

$$\tan\theta/2 = W/M = 0.35 + \sqrt{(1 + (0.35)^2)}, \tag{3.1.22}$$

and

$$\theta = 109^\circ 24'.$$

3.2. T-Invariance:-

If in addition to the assumptions made in (3.1) we assume that T-invariance holds, then $g_V$, $g_A$, $f_V$ are real and $h_A = 0$. Under this additional condition, the stokes parameters reduce to the following form[9]

$$[xu' + U,\ xv' - sU,\ -(xv - s'U),\ s''U] \qquad (3.2.1)$$

where

$$x = u' + sv' + s'v \quad \text{and} \quad U = u^2/2 . \qquad (3.2.2)$$

Here also the same conclusion is drawn for $x = 0$ as in the eqns. (3.1.9 and 10). If $s'' = 0$ and $x \neq 0$, the recoil beam has polarisation in the scattering plane.

I. Target Unpolarised ( $\vec{s} = 0$ ) :-

If $x = 0$, then the recoil beam is completely unpolarised and the scattering angles are given by the eqn. (3.1.11).

II. Target Longitudinally Polarised ($s = \pm 1$):-

Here also the same conclusion is true as in the case (3.1.II) for $x = 0$

Neutrons have 100% perpendicular polarisation if

$$xv' = sU . \qquad (3.2.3)$$

Solving this eqn. we get

$$\begin{aligned} \cot \theta/2 &= a - b \quad , \quad s = +1 , \\ &= a(2b - a)/(a - b) , \quad s = -1 . \end{aligned} \qquad (3.2.4)$$

III. Target Perpendicular Polarised ($s' = \pm 1$):-

In this case the stokes parameters are given by

$$[x u' + U,\ x v',\ -(xv - s'U),\ 0] \qquad (3.2.5)$$

with

$$x = u' + s'v .$$

If $x = 0$, then $\varphi_1 \mp \varphi_2 = \pi/2$, $s' = \pm 1$, and the recoil beam has 100% perpendicular polarisation. The scattering angles for this polarisation are given by the eqn. (3.1.17).

IV. Target Tranversally Polarised ($s'' = \pm 1$):-

In this case also the neutrons have 100% tranverse polarisation at the scattering angles given by the eqn. (3.1.17).

4. RELATIONS BETWEEN ANY TWO DEGREES OF POLARISATION:-

We see that the polarisation analysis is not so explicit as is expected from the considerations of the stokes parameters. Takwale and Teli[10] have recently developed a theory for the relations between any two degrees of polarisation of a scattered particle in any process. The relations are either ellipse, circle or straight line whose parameters depend only on the polarisation of a initial particle whose polarisation we wish to use and on its energy in some cases. We now study the polarisation of the recoil neutrons using this theory.

4.1. T-Non-invariance:-

From the eqns. (3.1.1. and 3) we can find a general relation between any pair of P, P' and P''.

From the expressions of P' and P" we have

$$u^2/R(s' - ks'') = (P' - kP'') \qquad (4.1.1)$$

where

$$k = v/u = 1 - a/b\ , \quad R = I/d\sigma_1\ , \qquad (4.1.2)$$

$k = 0$ for $a = b$ or $\nu \simeq 0.35\ M$ and $k \simeq 1$ for $\nu \geq 2M$. Thus the region $0 \leq k \leq 1$ represents the high energy region i.e. $0.35\ M \leq \nu \leq \infty$ .

From the expression of P we have

$$(P + su^2/R)^2 + (P' - s'u^2/R)^2 = (P'' - s''u^2/R)^2 + (1 - u^2/R)^2, \qquad (4.1.3)$$

where the expressions for P' and P" have been used.

Eliminating $u^2/R$ from the eqns. (4.1.1 and 3) we obtain

$$[P(s' - ks'') + s(P' - kP'')]^2 = (1 - k^2)(s'P'' - s''P')^2 + [(s' - s''k) - (P' - kP'')]^2, \text{ and} \qquad (4.1.4)$$

we also have

$$P^2 + P'^2 + P''^2 = 1\ , \quad \vec{s} \neq 0\ . \qquad (4.1.5)$$

From the eqns. (4.1.4 and 5) we can find the locus of any pair of the degrees of polarisation by eliminating the third one. Since eqns. (4.1.4 and 5) are independent of $\varphi_1$, $\varphi_2$, they are independent of $\theta$. They however depend upon the proton polarisation and the value of k or alternatively on the $\not\vec{\nu}$

energy if $f_V/g_V$ is constant. Thus taking k as a parameter, we can get various ellipses or circles depending on the polarisation of the target. In other words, the locii have the characteristics of the target polarisation.

I. Target Unpolarised:-

In this case, from the eqn. (4.1.4) we get

$$P' = k P'', \qquad (4.1.6)$$

which is a straight line passing through (0,0) with slope equal to k.

From the eqn. (3.1.1) we can get the relation between P and p' as

$$(P^2 + P'^2)^2 - (P^2 + P'^2) + P'^2/k^2 = 0. \qquad (4.1.7)$$

Eqn. (4.1.7) represents two loops symmetric about P and P' axes passing through the origin, (1,0) and (-1,0). As k $\longrightarrow$ 1, the loops degenerate into circles of radii 1/2 and centres at ($\pm$ 1/2. 0) belonging respectively to the regions $0 \leq \varphi_1 \leq \pi/2$ and $\pi/2 \leq \varphi_1 \leq \pi$, and are described by the equations.

$$(P \mp 1/2)^2 + P'^2 = 1/4. \qquad (4.1.8)$$

We thus see from the eqn. (4.1.8) that P' and hence P'' do not exceed 50% at any value of scattering angle.

II. Target Longitudinally polarised ( s = $\pm$ 1):-

In this case, eqn. (4.1.4) identically vanishes. Hence from the eqn. (4.1.1) and (4.1.5) we have

$$P' = k P''$$

and

$$P^2 + (1+k^2)P''^2 = 1. \qquad (4.1.9)$$

Thus P' and P'' are linearly related and P and P'' form an ellipse with centre at (0,0), semi kanor axis 1 and semiminor axis $1/\sqrt{(1+k^2)}$. Therefore at any value of scattering angle P' and P'' do not exceed 71% for

III. Target Perpendicularly Polarised ($s' = \pm 1$):-

From the eqns. (4.1.4 and 5) we have

$$P'^2 - kP'P'' + P''^2 - s'P' + s'kP'' = 0 \qquad (4.1.10)$$

and

$$P^4 + k^2P^2P'^2 + k^2P'^4 + 2s'(1-k^2)P^2P' - 2s'k^2P'^3 - (2-k^2)P^2 + P'^2 - 2(1-k^2)s'P' + 1 - k^2 = 0. \qquad (4.1.11)$$

Eqn. (4.1.10) represents an ellipse with centre at the point $[s'(2-k^2)/(4-k^2), -s'k\sqrt{(4-k^2)}]$ and the axes rotated through an angle $\gamma = 3\pi/4$. The ellipse passes through (0,0) (s',0) and (0,-s'k). Thus as $k \to 1$, $P'' \to -s''$ at some scattering angles. The eqn. (4.1.11) represents a pair of ellipses passing through $(\pm 1,0)$, $(\pm\sqrt{(1-k^2)},0)$, $(0,s')$ and $(0,\pm\sqrt{(1-k^2)}/k)$.

For k = 0, eqn. (4.1.10) reduces to a circle given by the eqn. (4.1.8) and has centre at (1/2,0) and hence $P'' \leq 50\%$.

For k = 1, the eqns. (4.1.10 and 11) degenerate respectively into the following ellipses, of the same major

and minor axes,

$$P'^2 - P'P'' + P''^2 - s'P' + s'P'' = 0, \qquad (4.1.12)$$

with centre at (s'/3, -s'/3), and

$$P^4 + P^2P'^2 + P'^4 - 2s'P'^3 - P^2 + P'^2 = 0, \qquad (4.1.13)$$

with centres at ( $\pm$ s'/3, s'/3).

Thus eqns. (4.1.12 and 13) can provide important predictions about any polarisation of the recoil beam.

IV. Target Transversally Polarised (s" = $\pm$ 1):-

From the eqns. (4.1.4) and (4.1.5) we have

$$P'^2 - kP'P'' + k^2P''^2 + s''kP' - s''k^2P'' = 0, \qquad (4.1.14)$$

which represents an ellipse with centre at (-s''/k3, s''/3) and the axes inclinded at an angle $\tan^{-1}(-k/1-k)$.

The locus (P,P') is given by

$$k^4P^4 + k^2(2k^2-1)P^2P'^2 + (1-k^2+k^4)P'^4 + \\ + 2s''kP'^3 - k^4P^2 - k^2(k^2-2)P'^2 = 0, \qquad (4.1.15)$$

representing a pair of ellipses passing through (0,0), ($\pm$ 1,0) and $[0, (-s' \pm k(k^2-1)^{3/2})/(1-k^2+k^4)]$.

For k = 0, eqn. (4.1.15) gives P' = 0 and from the eqn. (4.1.5) we have

$$P^2 + P''^2 = 1 \qquad (4.1.16)$$

which obviously is a circle.

For k = 1, eqns. (4.1.14) reduces to an ellipse.

$$P'^2 - P'P'' + P''^2 + s''P' - s''P'' = 0, \qquad (4.1.17)$$

while eqn. (4.1.15) reduces to the pair of ellipses

$$P^4 + P^2P'^2 + P'^4 + 2s''P'^3 - P^2 + P'^2 = 0 \qquad (4.1.18)$$

with centres at ( $\pm s^2/3$, $-s''/3$).

4.2. T-Invariance:-

From the eqn. (3.2.1) we see that if the target has polarisation in the scattering plane (s'' = 0) then P'' = 0 and we have

$$P^2 + P'^2 = 1. \qquad (4.1.16)$$

We, therefore, consider only two cases of target polarisation I) targer unpolarised and II) tranversally polarised. Following the same method as in (4.1) we arrive at the general relation from the eqn. (3.2.1)

$$(s''P + sP'')^2 + (1 - 1/k^2)(s''P' - s'P'')^2 = (s'' - P'')^2. \qquad (4.2.1)$$

In addition to the eqn. (4.1.5).

I. Target Unpolarised ($\vec{S} = 0$):-

In this case eqn. (4.2.1) gives from the eqn. (3.2.1) we have ,

$$P = u'v'/R \ , \quad P' = -u'v/R \ , \quad R = 1 - U. \qquad (4.2.2)$$

Thus we have

$$P^2 + P'^2 = u'^2/(1-U)^2, \qquad (4.2.3)$$

and

$$v^2 = k^2 u^2 = P'^2/(P^2+P'^2). \qquad (4.2.4)$$

Eliminating $u^2$ from the eqns. (4.2.3 and 4) we obtain the following locus of (P,P'),

$$P^4 + 2(1 - 1/2k^2)P^2P'^2 + (1 - 1/2k^2)^2 P'^4 - P^2 - (1 - 1/k^2)P'^2 = 0. \qquad (4.2.5)$$

Eqn. (4.2.5) represents two ellipses passing through (0,0), $(\pm 1, 0)$ and $(0, \sqrt{(1 - 1/k^2)/(2 - 1/k^2)})$.

For k = 0, P' = 0 and the recoil beam has only longitudinal component of polarisation.

As $k \to 1$, the locus degenerates into the ellipses,

$$4(P \pm 1/2)^2 + 2P'^2 = 1 \qquad (4.2.5')$$

and hence $P' \leq 71\%$.

II. Target Transversally polarised ($s'' = \pm 1$):-

In this case, using the eqns. (4.2.1 and 4.1.5) we obtain various locii.

i) Locus of (P,P').

The locus (P,P') is given by the eqn. (4.2.5) and the same discussion holds here also.

ii) Locus of (P,P").

The locus is found to be an ellipse,

$$P^2/\alpha^2 + (P'' + s'' k^2/(1-2k^2))^2/\beta^2 = 1 \qquad (4.2.6)$$

with

$$\beta = (1-k^2)/(1-2k^2), \quad \alpha = \beta\sqrt{(1-2k^2)}, \qquad (4.2.7)$$

and the centre at $[0, -s'' k^2/(1-2k^2)]$.

For k = 0, on the eqn. (4.2.6) reduces to a circle given by the eqn. (4.1.16) while for k = 1, it reduces to two straight lines,

$$\pm P = s'' - P'' \qquad (4.2.8)$$

+ sign for $0 \leq \varphi_1 \leq \pi/2$ and -ve sign for $\pi/2 \leq \varphi_1 \leq \pi$.

iii) Locus of (P', P'').

In this case, we obtain the following ellipse,

$$2P'^2/k^2 + 4(P'' - s''/2)^2 = 1. \qquad (4.2.9)$$

Thus we conclude that the relations between any pair of the degrees of polarisation are either ellipse, circle or straight line characteristic of the target polarisation and its energy since the neutrino energy is related to the target energy in the c.m. system in a simple way. The scattering angle and form factors can be related to the ecentric angle of the ellipse by the relation

$$P'(\varphi_1, \varphi_2)/P(\varphi_1, \varphi_2) = (\beta/\alpha)\tan\psi, \qquad (4.2.10)$$

where $\alpha$ and $\beta$ are respectively the semi-major and semi-minor axes of the ellipse and $\psi$ is its ecentric angle.

We have not discussed these relations since they are obvious. Thus from the eqn. (4.2.10) $\psi$ is known for a given value of $\theta$ and hence the degrees of Polarisation are directly read from the locus (P,P'). Similar discussion holds for any other locus.

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# FOUR BODY CORRELATION AND QUARTET STATES IN NUCLEI

L.Satpathy
Berhampur University
Orissa

Abstract:

With special reference to "quartet", the nature of different types of four body correlations in nuclei are discussed. A phenomenological quartet shell model is described to predict the intra-shell quartet states which are supposed to be due to a different mode of excitation in contrast to the inter-shell quartet excitation of Arima, Gillet and Ginocchio. A microscopic calculation in the Multiconfiguration Hartree-Fock model provides support to this hypothesis.

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At the outset I wish to say that the term "quartet" is not so widely known or even accepted universally as very genuine. In recent years this subject has aroused wide interest. I will first discuss about what is generally meant by "quartet" and then present our model on intra-shell quartet states.

The idea of quartet[1] has originated in Saclay about a couple of years back by Gillet and Danos. In the simplest term in means that nuclei contain substructures consisting of two protons and two neutrons which are quasibound and quasi independent. There is strong internal interaction between the particles in a quartet, but the interaction between different quartets are weak. This is the result of strong 4-body correlation in nuclei. This feature was quite well-known long back. Wigner's supermultiplet model and the $\alpha$-particle model are the oldest models of nuclear physics which are based on the hypothesis of strong 4-body correlation. Then what is new in the quartet

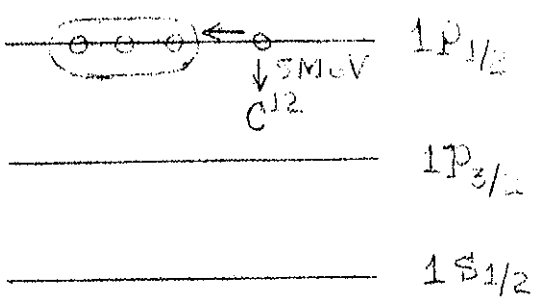
hypothesis? Before answering this question, I would like to point out a few experimental indications of the existence of such four-body substructure in nuclei.

1. The first non-trivial experimental evidence is that the nuclei emit $\alpha$-particles.

2. The binding energies of N=Z even-even 4n nuclei are close to the binding energies of n free $\alpha$-particles (see table 1).

3. In light nuclei the $\alpha$-decay thresholds are quite low compared to the neutron or proton decay thresholds. For example in $O^{16}$, the neutron decay and $\alpha$-decay thresholds are 15.7 MeV and 7.2 Mev respectively and the corresponding values in case of $Mg^{24}$ are 16.2 and 9.3 MeV. This means that a nucleon interacts strongly with the other three nucleons which make up the extracted $\alpha$-particle and much more weakly with the remaining nucleons.

4. In light nuclei, the interaction of a nucleon with the three other members of the quartet and its interaction energy with the core can be estimated. The nucleus $O^{16}$ can be represented in the shell model as



The interaction energy of the last neutron with the core (i.e.$C^{12}$) is known from the separation energy of neutron in $C^{13}$. This is about 5 MeV. The interaction energy of the last neutron with the other three nucleons in the $1p_{1/2}$ orbit which constitute the quartet is $Bn(O^{16}) - Bn(N^{13})$. where Bn denotes the binding energy of neutron. This is about 10 Mev. So about 70 ·/. of the binding energy of the last nucleon comes from its interaction with only 3 nucleons. This quantity gradually decreases for higher nuclei and in case of $Ti^{44}$ which is the last shell-model nucleus, the value is still as large as 50 ·/. Thus even for a nucleus for which j-j coupling is good, the interaction of a nucleon with three nucleons is still equal to its total interaction with 40 other nucleons of the $Ca^{40}$ core.

All these were known since the early days of nuclear physics. In fact these observations have led to $\alpha$-cluster model. Then two question arise. (a) What is new in the quartet hypothesis? (b) How does a quartet differ from $\alpha$-cluster? The newness in quartet hypothesis is that the four nucleon substructure exists in nuclei upto quite high excitation in energy, and this feature is not confined only to light nuclei, but persists in medium and heavy nuclei as well. Using the quartet hypothesis Gillet and Danos have predicted a saw tooth behaviour[3] for the binding energies of the heavy mass nuclei which is really verified experimentally. Recently many heavy ion reaction data obtained ($O^{16}$, $C^{12}$) reaction[3] performed on p-f shell nuclei could be qualitatively understood with the help of quartet hypothesis. The second question about the difference of quartet and $\alpha$-cluster can be answered in the following way.

In a more specific manner, one has to answer how the 4-body correlation envisaged in quartet differs from those of the $\alpha$-cluster or supermultiplet wave function. In $\alpha$-cluster model, 2 neutrons and 2 protons are put in a relative s-wave state and thus represent a close space correlation. This picture is valid and useful only upto the middle of the s-d shell, since l-s coupling breaks down beyond that. The supermultiplet theory is based on the assumption that the nuclear Hamiltonian (a) does not depend upon the spin or isospin coordinates and (b) does not contain a one-body spin-orbit potential. Since for four nucleons the wave function can be fully symmetric in the spatial part and antisymmetric in the spin and isospin part, two protons and two neutrons are put in anlorbit in full spatial symmetry[4] state. So the 4-body correlation in supermultiplet model is completely spatial in nature. Since one-body spin-orbit potential is extremely important for nuclei, beyond the middle of s-d shell, and the two-body interaction is more complicated than assumed in supermultiplet model, the usefulness of this model was confined to few light nuclei.

The four-body correlation envisaged in quartet is not necessarily a spatial correlation. For medium and heavy nuclei where l-s coupling breaks down and j-j coupling is good, 2 protons and 2 neutrons still interact strongly to form a stable structure even though the spatial symmetry is broken badly. Such a structure is called[4] 'quartet' following Gillet and Danos. Thus the four-body correlation is not restricted to spatial part of the wave function as in $\alpha$-cluster or supermultiplet model, but may manifest in the coupled space generated by spin

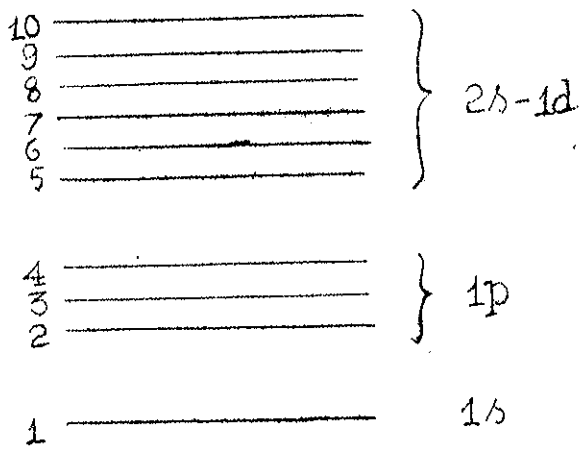
and orbital spaces. The Hartree-Fock intrinsic state, stretched state can be considered to have quartet structure.

It is worth discussing how nuclear force gives rise to such four-body correlation in a general case. Two nucleons in same shell j has maximum binding energy when the angular overlap of their wave function is maximum. For two identical nucleons, the angular overlap is 100 %. when they are antialigned i.e coupled to J=0. This is the usual ideal two body pairing correlation, For a neutron and a proton the angular overlap is 100 %. when (a) they are aligned i.e. J=2j and also (b) anti-aligned J=0. Since the T=0 component of the nuclear force exceeds the T=1 component, it is natural to expect a proton-neutron decoupling over the proton-proton, or neutron-neutron coupling in an assembly of nucleons. A[5] two neutron and two proton would form a substructure in which first each proton-neutron pair will couple to the largest possible angularmomentum J = j+j=n and then these two stretched pair would couple to J=0. It is difficult to prove the goodness of this picture in a many particle system by comparing this wave function with the exact shell model wave function due to the impossibility of obtaining the latter. However, in simple case like $Ti^{44}$, the quartet wave function having the above correlation constructed with only $f_{7/2}$ single particle state is found[6] very close to the true wave function (more than 90 %. overlap with true shell-model wave function). It has been shown by Gillet and Ranyal[7] that in a single j shell the quartet state having the above aligned structure has a lower ground state energy than seniority 0 state if all the even state interaction are attractive and the T = 0 even state interaction is larger than the T = 1 odd one.

Arima, Gillet and Ginnochio have predicted[8] the energies of the quartet states in light 4n nuclei starting from $Be^8$ upto $Fe^{56}$. It is wellknown that the $0^+$ states at 6.06 MeV and 3.35 Mev respectively in $O^{16}$ and $Ca^{40}$ are 4p-4h states. They have taken these states to be one-quartet one-quartet-hole states in analogy to the 1p-1h states. The states predicted by them for the 4n nuclei are of the many-quartet-many-quartet-hole type and are assumed to result from the correlated excitation of 2p and 2n from one major shell to another. These states are hereafter called as inter-shell quartet states. The position of the states were predicted empirically by using mass relationships. We feel that in addition to the existence of inter-shell quartet states there could be another mode of excitation, in which four correlated nucleons can get excited to the unoccupied states in the same major shell. This would give rise to another class of states called as Intra-shell quartet states.

To predict the position of the intra-shell quartet states we devised a model[9] called $^{1}$quartet shell model. We assume the existence of a one-body quartet potential well, the eigenstates of which are used to describe the various quartet-states in 4n nuclei. In this model the 1s, 1p, 2s, 1d,.... oscillators shells are replaced by 1s, 1p, 2s, 1d,.... quartet shells having 1,3,6,.....single quartet states respectively. These single-quartet states are assumed to be nondegenerate and have no other quantum number, except that they have angular momentum zero. The single

quartet levels are represented below in Fig.1.




The ground state of $Ne^{20}$ will be represented as the quartet levels upto no.5 being occupied. The ground-state energy for a 4n nucleus is then given by the expression

$$E_{4n} = \sum_{i \leq n} t_i + \sum_{i<k\leq n} V_{ik} \qquad (1)$$

where t and V are respectively the kinetic energy and the two-body quartet interaction and 'n' designates the last occupied single-particle state. The excitation energy for moving a quartet from the last occupied into the first unoccupied single-quartet state is given by:

$$\Delta E_{n+1,(n)^{-1}} = \sum_{i \leq n-1} t_i + \sum_{i<k\leq n-1} V_{ik} + t_{n+1} + \sum_{i \leq n-1} V_{i,n+1} - E_{4n}$$

$$= E_{4n+4} - 2E_{4n} + E_{4n-4} - V_{n,n+1} \qquad (2)$$

y exciting quartet to other unoccupied levels one derives he following quartet excitation energy is a 4n nucleus:

$$\Delta E_{n+2,(n)^{-1}} = E_{4n+8} - E_{4n-4} - E_{4n} + E_{4n-4} - V_{n,n+2} - V_{n+1,n+2} \quad (3)$$

$$\Delta E_{n+1,(n-1)^{-1}} = E_{4n+4} - E_{4n} - E_{4n-4} + E_{4n-8} - V_{n-1,n+1} - V_{n-1,n} \quad (4)$$

$$\Delta E_{n+1,(n-2)^{-1}} = E_{4n+4} - E_{4n} - E_{4n-8} + E_{4n-12} - V_{n-2,n-1} - V_{n-2,n} - V_{n-2,n+1} \quad (5)$$

$$\Delta E_{n+1,n+2,(n,n-1)^{-1}} = E_{4n+8} - 2E_{4n} + E_{4n-8} - V_{n,n+1} - V_{n,n+2} - V_{n-1,n+1} - V_{n-1,n+2} \quad (6)$$

Eq.(6) describes a two-quartet, two-quartet hole state. The binding energies - E4n are taken from the mass table. The matrix elements of the type Vn,n+1 refer to the interaction between the last occupied and first unoccupied quartet level and are determined by assuming that the first excited $0^+$ state correspond to the quartet excitation of Eq.(2). The analysis of Danos and Spicer[10] on first $0^+$ state in light nuclei provides some support of this assumption. The matrix elements calculated by this procedure are presented in the first row of table 2. In $A^{36}$, there are two $0^+$ states close to one another. We have used both in the alternative calculation. In $Ne^{20}$, the $0^+$ state at 6.72 Mev has been used to fit $V_{5,6}$, since the other $0^+$ at 7.2 Mev is established as an inter-shell quartet state.

If we now consider the quartet excitation (Eqs. 3 - 6) in a particular 4n nucleus, all necessary matrix elements can be obtained from this set under the following assumption.

(a) inter shell matrix element

$$V_{i\in p,\, j\in sd} = V_{n\in p,\, n+1\in sd} = V_{4,5} ,$$

$$V_{i\in sd,\, j\in pf} = V_{n\in sd,\, n+1\in pf} = V_{10,11} .$$

(b) intra-shell matrix element

$$V_{i\leq n,\, j>n} = V_{n,n+1} , \; V_{i<n,\, j=n} = V_{n-1,n} .$$

It is extremely gratifying to note that in case of intershell quartet excitation our Eq.(2) and Eq.(6) reduce to the formula of Arima, Gillet and Ginnochio[8].

It is evident from the table 2 that the $0^+$ quartet states predicted by our model have experimental analogues in those region. We find that for the nuclei in the beginning and end of the shell [for $Ne^{20}$ and $Ar^{36}$], the intershell and intrashell quartet states have comparable energies. For the nuclei in the middle of the shell, i.e. in the case of $Mg^{24}$, $Si^{28}$ and $S^{32}$, these states lie lower than the intershell states calculated by Arima, Gillet and Ginnochio[8]. The general trend of our results is in confirmity with expectation.

From a microscopic calculation, which employs Multi-configuration Hatree-Fock (MCHF) theory[11], we get the evidence in support of the existence of intrashell model of quartet excitation. In this scheme one takes as a trial wave function $|\Psi\rangle$ a superposition of Slater determinant $|I\rangle$ i.e.

$$|\Psi\rangle = \sum_I C_I |I\rangle .$$

The configuration $|I\rangle$ are built from single particle states $|k\rangle$, expanded in an oscillator basis

$$|k\rangle = \sum_a |a\rangle A_{ak} .$$

The nuclear Hamiltonian is given as

$$H = \sum_{\alpha\beta} \langle\alpha|t|\beta\rangle a^+_\alpha a_\beta + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|V|\gamma\delta\rangle a^+_\alpha a^+_\beta a_\delta a_\gamma$$

Then one minimizes $\langle\Psi|H|\Psi\rangle$ treating $C$ and $A$ as parameters with the constraints :

$$\langle i|k\rangle = \delta_{ik}$$

and

$$\langle\Psi|\Psi\rangle = \sum_I C^*_I C_I = 1 .$$

Hence one gets

$$\delta_{C^*_I, A^*} \{\langle\Psi|H|\Psi\rangle - E\langle\Psi|\Psi\rangle - \sum_{ik} \epsilon_{ik}\langle i|k\rangle\} = 0 .$$

This yields the following two equations which are solved simultaneously:

$$\sum_J \langle I|H|J\rangle C_J = E_I C_I ,$$

$$f(A, C, \epsilon) = 0 .$$

In our calculation we have assumed $O^{16}$ as the core and used only 2s-1d shell basis states. Axial symmetry and charge conjugation symmetry are imposed. The configuration $|I\rangle$ are constructed by making particle-hole excitations with respect to that Slater determinant which has the largest overlap with the Hatree-Fock ground-state. The configurations $|I\rangle$ are chosen so that they differ by at least a 2p-2h excitation. Such configurations which have 4p-4h quartet structure have been included. It is ensured that $|\Psi\rangle$ has good isospin $T = 0$ and the total angular momentum projection $K = 0$. In the calculation, Yale potential is used with oscillator length $b = 1.76$ fm, and an average starting energy of 34 Mev. The one-body spectrum has been extracted from the experimental spectrum of $O^{17}$. From our MCHF calculation we found, that out of all the states of a nucleus, a few have nearly pure 4p-4h structure. In table 3 we have presented those states along with the ground-states. The 4p-4h strength of these states in most cases are above 90%. Such high purity states point out the correlated excitation of two protons and two neutrons from one deformed single-particle state to another. In all cases, we find two such states which can be termed as intra-shell quartet states within 12 Mev. In case of $Si^{28}$,

the prolate solution can be understood in the limit of $SU_3$ to be a 4p-4h quartet state with respect to oblate solution. Hence the prolate solution has been presented as an excited quartet state with respect to the oblate ground-state.

Thus our phenomenological quartet shell model and also the microscopic MCHF calculation show clearly that in addition to the intershell excitation, there is another class of excitation which can be termed as intrashell quartet excitation. The latter class of states lie as low and in the middle of the s-d shell even lower than the former class of states. The spectroscopic properties of these states could be ascertained by $\alpha$-transfer reactions.

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## TABLE CAPTIONS

Table 1. Comparison of the binding energies of the 4n nuclei with n times the binding energy of $\alpha$-particle..

Table 2. The matrix elements $V_{n,n+1} = \langle n, n+1 | V | n, n+1 \rangle$ (row 3) are obtained by assuming the first excited experimental $0^+$ states (row 4) to be the one-quartet, one-quartet-hole state of Eq.(2). The excitation energies for the quartet states of the types described by Eqs.(3)-(6) are listed in the rows 5-8. The lower number gives, if available, the experimental value. The underlined quantities represent intershell quartet excitations. The energies of the two-quartet, two-quartet-hole states involving one intershell as well as one intrashell quartet excitation are shown in parentheses. The last two columns (with asterisks) have been calculated by fitting the matrix element $V_{n,n+1}$ assuming the second $0^+$ state as the quartet excitation of Eq.(2).

Table 3. The first row of data gives the strengths of the reference states $C^2_{op-oh}$ defined in Eq.(7). The following rows describe three low-lying excited 4p-4h quartet states. The strength $C^2_{4p-4h}$ and the theoretical and experimental excitation energies $\Delta E$ of these states are given, respectively The subscripts o and p refer to the solutions with oblate and prolate deformations, respectively. In the case of $^{28}Si$, the second state marked with an asterisk is the lowest prolate solution.

TABLE 1

| Nucleus | n × $\alpha$-particle B.E. | Experimental B.E. |
|---|---|---|
| $He^4$ | 28.3 | 28.3 |
| $Be^8$ | 56.6 | 56.5 |
| $C^{12}$ | 84.9 | 92.2 |
| $O^{16}$ | 113.2 | 127.6 |
| $Ne^{20}$ | 141.5 | 160.6 |
| $Mg^{24}$ | 169.8 | 198.3 |
| $Si^{28}$ | 198.1 | 236.5 |

| Nucleus | $^{12}C$ | $^{16}O$ | $^{20}Ne$ | $^{24}Mg$ | $^{28}Si$ | $^{32}S$ | $^{36}Ar$ | $^{40}Ca$ | $^{32}S^*$ | $^{36}Ar^*$ |
|---|---|---|---|---|---|---|---|---|---|---|
| No. of quartets | n = 3 | n = 4 | n = 5 | n = 6 | n = 7 | n = 8 | n = 9 | n = 10 | n = 8 | n = 9 |
| $V_{n,n+1}$ (Mev) | -7.45 | -3.63 | -11.30 | -7.11 | -1.93 | -3.48 | -4.73 | -1.53 | -3.48 | -5.57 |
| $\Delta E^{n+1}_{(n)^{-1}}$ (MeV) | fitted | fitted | fitted | fitted | fitted | fitted | fitted | fitted | fitted | fitted |
| $\Delta E_{exp}$ (MeV) | 7.65 | 6.06 | 6.72 | 6.44 | 4.97 | 3.78 | 4.33 | 3.35 | 3.78 | 5.17 |
| $\Delta E^{n+2}_{(n)^{-1}}$ (MeV) | 9.89 | **12.78** | 13.16 | 11.41 | 8.75 | 12.11 | 4.49 | - | 8.95 | 4.49 |
| $\Delta E_{exp}$ (MeV) | 10.1 | 11.26 | | 10.68 | 6.69 | | 5.17 | | | 4.33 |
| $\Delta E^{n+1}_{(n-1)^{-1}}$ (Mev) | - | 13.71 | 5.11 | 13.16 | 11.41 | 8.75 | 8.11 | | 8.75 | 8.95 |
| $\Delta E_{exp}$ (MeV) | | 14.07 | 7.2 | | | | | | | |
| $\Delta E^{n+1}_{(n-2)^{-1}}$ (MeV) | | | 12.76 | 19.22 | 18.12 | 15.15 | 12.08 | | **15.19** | 13.92 |
| $\Delta E_{exp}$ (MeV) | | | | | | | | | | |
| $\Delta E^{n+1,n+2}_{[n,n-1)^{-1}}$ (MeV) | (15.57) | 15.00 | (37.80) | 25.56 | 13.42 | 17.16 | (13.85) | | 17.16 | 15.53 |
| $\Delta E_{exp}$ (MeV) | | 16.75 | | | | | | | | |

TABLE 3

| NUCLEUS | $^{20}Ne_p$ | $^{24}Mg_p$ | $^{28}Si_o$ | $^{32}S_p$ | $^{36}Ar_o$ |
|---|---|---|---|---|---|
| $C^2_{op-oh}$ (%) | 99.40 | 84.16 | 98.72 | 88.94 | 99.76 |
| $C^2_{4p-4h}$ (%) | 99.80 | 99.80 | 98.40 | 84.46 | 99.80 |
| $\Delta E_{th}$ (Mev) | 5.0 | 5.8 | $2.8^*_p$ | 4.83 | 6.3 |
| $\Delta E_{exp}$ (Mev) | 6.72 | 6.44 | 4.97 | 3.73 | 4.33 |
| $C^2_{4p-4h}$ (%) | 99.60 | 98.01 | 99.20 | 95.06 | 99.60 |
| $\Delta E_{th}$ (Mev) | 7.0 | 9.6 | 12.40 | 11.89 | 7.78 |
| $\Delta E_{exp}$ (Mev) | - | 10.63 | - | - | - |
| $C^2_{4p-4h}$ (%) | 99.20 | 64.96 | 99.20 | 70.73 | 97.81 |
| $\Delta E_{th}$ (Mev) | 8.8 | 13.5 | $15.1_p$ | 13.45 | 17.88 |
| $\Delta E_{exp}$ (Mev) | - | - | - | - | - |

# SELF CONSISTENT CALCULATIONS FOR p-f SHELL NUCLEI

Jyoti K. Parikh

Atomic Minerals Division, NPL Bldgs., Hillside Rd., New Delhi

I shall try to summarize the work that I have been doing in the last year on this subject considering p-f shell nuclei. In nuclear physics, one encounters the problem of solving Schrödinger equation in a many-body framework. Although shell model splits this many-nucleon problem into core + extra-core nucleons where the core is assumed to be inert, the extra-core nucleons for which the two-body interactions should be considered are quite often more than three. In the conventional shell model one solves the Schrödinger equation $H\Psi = E\Psi$ by diagonalizing the Hamiltonian H where $\Psi$ is a j-j coupled (or L-s coupled) many-body wave function. The advantage of this method is that one can get a very large number of states of the nuclear spectra. However, to handle more than 4 extra-core nucleons is an extremely difficult numerical problem.

To treat many nucleons outside the closed core, variational methods would have to be used. There are Hartree-Fock(HF) or Hartree-Fock-Bogoliubov (HFB) methods, where the latter takes pairing correlations into account.

In the variational methods, however, one minimized the Hamiltonian instead of diagonalizing it. Here, the wave function is a Slater determinant and one could treat as many nucleons as the size of the Slater determinant allows. Thus, the advantage of this method is than one can treat very large number of extra-core

nucleons. But the variational wavefunction does not have good angular momentum and one has to apply projection technique to obtain the nuclear spectra.

Formalism.

The formalism for the even nuclei can be found in the literature. The formalism for the odd nuclei is discussed here briefly when the blocking effect due to the odd nucleon has been incorporated. The wave function for the odd nucleon is taken as

$$\psi = \frac{1}{\sqrt{2}}\left(b_i^\dagger + b_{\bar{i}}^\dagger\right) \prod_{\ell \neq i,\bar{i}} \left(u_\ell + v_\ell\, b_\ell^\dagger\, b_{\bar{\ell}}^\dagger\right) |0\rangle$$

where $b^\dagger$ are the creation operators in the deformed basis. $v^2$ are the occupation probabilities and $|0\rangle$ is the vacuum state. Odd proton is put in the $i$ and $\bar{i}$ states with 1/2 probability.

We use the symbol $\ell$ to denote the states other than the $i$ ( $\bar{i}$ ) state and K to denote the $\ell$ as well as the $i$ states. The operators $b_K^\dagger$ can be expanded in spherical basis

$$b_K^\dagger = \sum C_\alpha^K\, a_\alpha^\dagger \quad , \quad b_{\bar{K}}^\dagger = \sum (-1)^{j-m} C_\alpha^K\, a_{\bar{\alpha}}^\dagger \, .$$

Where for a given charge $\alpha$ denotes the quantum numbers ( $j$, $m$ ).

Using the variational method we would like to minimize the shell model Hamiltonian H which is given by

$$H = \sum_{\alpha} \epsilon_{\alpha} a^{+}_{\alpha} a_{\alpha} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} a^{+}_{\alpha} a^{+}_{\beta} a_{\delta} a_{\gamma}$$

In order to solve for $\psi$ , quasi-particle transformation would have to be made.

$$\alpha^{+}_{\ell} = u_{\ell} b^{+}_{\ell} + v_{\ell} b_{\ell} \quad , \quad \alpha_{\ell} = u_{\ell} b_{\ell} + v_{\ell} b^{+}_{\ell}$$

This transformation is such that

$$\alpha_{\ell} | \psi \rangle = 0.$$

Taking the expectation value of H using Wick's theorem and using the above transformation $\langle H \rangle$ could be written as :

$$\langle H \rangle = \sum_{k} ( T_k - \lambda + \Gamma_k ) \rho_k + \sum_{\ell} \Delta_{\ell} t_{\ell} ,$$

where $\rho$ and $t$ are the density matrix and pairing tensor respectively. T, $\lambda$, $\Gamma$ and $\Delta$ are the kinetic energy operator, chemical potential, Hartree-Fock potential and pairing potential respectively. All the symbols are discussed in ref. (3).

Minimizing $\langle H \rangle$ with respect to u, v and $c^{k}_{\alpha}$ we get nonlinear simultaneous equations which would have to be solved numerically using an interactive procedure. Finally we get the variational

energy

$$E_{HFB} = \frac{1}{2} \sum_{\ell} (T_\ell + \lambda - E_\ell) v_\ell^2 + \sum_{i,i} (T_i + \epsilon_i) v_i^2,$$

where $E_\ell$ are the quasiparticle energies.

Having obtained the variational wave-function one has to use projection technique to obtain nuclear spectra. This is discussed in ref.(1,2).

Having extended the HFB formalism of the even nuclei to the odd nuclei we have carried out the calculations for 34 p-f shell nuclei including the odd and even isotopes - using the same model and the same set of parameters. The parameters are so chosen as to reproduce a reasonable fit with the experimental binding energies, separation energies, pair separation energies, spectroscopic factors, quadrupole moments and BE2 values. Having done the calculations for a large number of nuclei one can study various aspects of nuclear structure physics.

(i) The Nuclear Spectra:-

These are obtained for the even nuclei by using the projection technique sup. 1,2 On comparing the spectra obtained from the HF and HFB wave functions, one finds that the includion of the pairing correlations in the HFB method improves the spectra. The pairing correlations decreases the moment of intertia and the spectra are no longer compressed as the PHF spectra.

(ii) The effects of the addition of a neutron on nuclear properties:-

On studying the sets of isotopes which include the even as well as the odd isotopes, one can study the effects of the addition of a neutron on various nuclear properties. The calculations for the isotopes $^{46 \text{ to } 51}$Ti, $^{50 \text{ to } 55}$Cr, $^{54 \text{ to } 58}$Fe, and $^{64 \text{ to } 68}$Zn, are carried out and following observations are made.

a) The present method explains the fluctuations which is observed in the separation energies as one adds a neutron. However, the saturation in the binding energies shown by the calculated results is less rapid in comparison with the experimental results with the addition of a neutron.

b) The single particle levels compress (spread out) as one goes towards (away from) the shell closure which occurs as the f 7/2 shell closes for $^{50}$Ti $^{52}$Cr and $^{54}$Fe or at $^{70}$Zn where p-f shell closes, e.g. The neutron levels of the Ti isotopes compress with the addition of a neutron and spread out for the Fe isotopes.

c) The spectroscopic strengths are measured experimentally by one - nucleon transfer reactions. They give a measure of the configuration mixing. The calculated results are compared with the experimental results in Table 1. The quadrupole moments and BE2 values are calculated using the rotational model approximation. The other aspects of this study such as the variation of deformation, superfluidity, nuclear shapes etc. could be found in ref.(3).

(iii) Variations of nuclear shape and deformations:-

In order to study the variation of the nuclear shape and deformation in a given shell, it is required that one calculates the quadrupole moments of the first excited states, Q2 +, and $BE_2$ values using the same model and the same parameters through out the shell and compares them with the experimental values.

The Q2 + and BE2 values of the even nuclei are calculated using the projection technique. The Q2 + if +Ve (-ve) implies that the nuclear shape is oblate (prolate). It was found that the nuclear shapes are prolate for the Ti, Cr and Fe isotopes, there is degeneracy in the Ni isotopes and the Zn isotopes are oblate shaped. However, the shapes of the Zn isotopes are parameter dependent.

The variation of deformation could be seen by studying the magnitude of the $BE_2$ values given in table II. One sees that there is a sudden drop in the $BE_2$ values for N or Z = 28 nuclei which is due to the f 7/2 shell closure. The deformation decreases with the neutron excess in the Ti, Cr and Zn isotopes and increases in the Fe isotopes.

(Iv) A study of the sets of isotopes with N=26, 28, 30. To study the sets of isotones one has to be the calculations for the odd proton nuclei such as the V, Mn and Co isotopes. Thus one has for example. $^{50}_{Ti}$, $^{51}_{V}$, $^{52}_{Cr}$, $^{53}_{Mn}$, $^{54}_{Fe}$, $^{55}_{Co}$ which is a set of isotones with N=28. The study of these sets of isotones shows that the addition of a proton does not change the nature or

the nuclear shape but it increases its deformation. The quadrupole moments, $BE_2$ values and the spectroscopic strength are calculated and studied in ref.(5).

Finally, it should be pointed out that the versatility of this method is demonstrated by calculating a wide variety of the experimental quantities ranging from the binding energies, separation energies, the nuclear spectra, quadrupole moments, $BE_2$ values and the spectroscopic strengths for 34 nuclei which are isotropes of Ti, V, Cr, Mn, Fe, Co and Zn. This study demonstrates the effects of the addition of a proton (or neutron) on deformation, superfluidity, fluctuations in the separation energies and the configuration mixing.

The calculations to incorporate these wavefunctions in nuclear reaction theory as well as to apply the projection technique to odd nuclei are in progress.

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TABLE - I

The calculated and experimental neutron pick up strengths

| | Calc. | | | | | Exp. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $P_{1/2}$ | $P_{3/2}$ | $f_{5/2}$ | $f_{7/2}$ | | $P_{1/2}$ | $P_{3/2}$ | $f_{5/2}$ | $f_{7/2}$ | Ref. | Exp. |
| 46 | .09 | .36 | .19 | 3.35 | | | .52 | | 3.4 | a | ($He^3$, ) |
| 47 | .07 | .32 | .16 | 4.45 | | | | | | | |
| 48 | .07 | .30 | .20 | 5.43 | | | .50 | | 5.5 | a,b | ($He^3$, ),(p,d) |
| 49 | .04 | .19 | .14 | 6.62 | | | | | | | |
| 50 | .02 | .11 | .10 | 7.76 | | | .48 | | 5.5 | a,b | ($He^3$, ),(p,d) |
| 51 | .13 | .85 | .21 | 7.81 | | | | | | | |
| 50 | .07 | .39 | .19 | 5.34 | | | | | 3.4 | c | (p,d) |
| 51 | .01 | .19 | .05 | 6.74 | | | | | | | |
| 52 | .00 | .03 | .02 | 7.96 | | .8 | .1 | - | 7.52 | d | (d,p) |
| 53 | .20 | .58 | .53 | 7.68 | | | | | | | |
| 54 | .42 | .95 | .91 | 7.72 | | | | | 6.1 | d | (d,p) |
| 55 | .51 | 1.49 | 1.23 | 7.77 | | | | | | | |
| 54 | .00 | .01 | .00 | 7.99 | | | .16 | | 7.88 | e | ($He^3$, ) |
| 55 | .16 | .47 | .53 | 7.83 | | | | | | | |
| 56 | .43 | .95 | .85 | 7.76 | | | 1.43 | .93 | - | f | (p,d) |
| 57 | .54 | 1.48 | 1.19 | 7.79 | | | | | | | |
| 58 | .59 | 1.81 | 1.85 | 7.75 | | | 1.61 | 2.05 | - | f | (p,d) |
| 64 | 1.29 | 2.66 | 2.74 | 7.30 | | .9 | 2.0 | 3.2 | - | g | (d,p) |
| 65 | 1.43 | 3.24 | 2.77 | 7.56 | | | | | | | |
| 66 | 1.54 | 3.62 | 3.04 | 7.80 | | .9 | 2.71 | 4.3 | - | g | (d,p) |
| 67 | 1.58 | 3.76 | 3.74 | 7.92 | | .8 | 2.8 | 3.8 | - | h | (d,p) |
| 68 | 2.00 | 4.00 | 4.00 | 7.97 | | 1.2 | 3.4 | 5.0 | - | g | (d,p) |

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## TABLE II

The experimental $Q_2$ + and BE2 values are compared with the calculated values where the effective charge e = 0.7 has been used. If N or Z = 28, then e is 0.4. For the Ni isotopes both the solutions prolate and oblate are given and those which give better agreements are underlined. The experimental BE2 values are obtained from ref.(13). If the number is in the bracket, it means that the spin of the level is guessed. The harmonic oscillator parameters $b^2$ are given in $fm^2$.

| Nuc. | | $Q_2$+in barns | | BE2(0-2)in $e^2 \times 10^{-50}$ $cm^4$ | | $b^2$ |
|---|---|---|---|---|---|---|
| | | Calc. | Exp. | Calc. | Exp. | |
| $^{44}$Ti | | -0.22 | | 5.6 | | 4.048 |
| $^{46}$Ti | | -0.23 | -0.19±0.10 [a] | 6.5 | 8.0±1.7 | '' |
| $^{48}$Ti | | -0.21 | -0.22±0.08 [a] | 6.2 | 7.0±1.4 | '' |
| $^{50}$Ti | | -0.12 | -0.02±0.09 [a] | 3.2 | 3.2±0.8 | |
| $^{50}$Cr | | -0.38 | | 17.1 | 12.0±0.8 | '' |
| $^{52}$Cr | | -0.27 | | 9.4 | 6.7±0.7 | 4.113 |
| $^{54}$Cr | | -0.28 | | 10.4 | 10.0±0.7 | '' |
| $^{54}$Fe | | -0.29 | | 10,1 | 5.1±0.5 | 4.183 |
| $^{56}$Fe | | -0.32 | -0.34±0.03 [b] | 12.8 | 9.0±1.0 | '' |
| $^{58}$Fe | | -0.34 | | 13.8 | 13.0±3.0 | '' |
| $^{58}$Ni | P | -0.23 | -0.12±0.13 [c] | 6.2 | 7.3±0.2 [c] | 4.288 |
| | 0 | +0.24 | -0.14±0.10 [d] | 7.0 | | |
| $^{60}$Ni | P | -0.21 | 0 ±0.13 [c] | 5.4 | 9.3±0.3 [c] | '' |
| | 0 | 0.24 | -0.01±0.10 [d] | 7.1 | | |
| $^{62}$Ni | P | -0.19 | 0.08±0.12 [c] | 4.5 | 8.8±0.3 [c] | '' |
| | 0 | 0.27 | 0.37±0.20 [d] | 8.6 | | |
| $^{64}$Ni | 0 | 0.27 | 0.35±0.20 [d] | 8.5 | 8.4±0.5 [c] | 4.283 |
| $^{64}$Zn | | 0.33 | | 13.2 | 17.0±1.5 | 4.44 |
| $^{66}$Zn | | 0.31 | | 11.5 | 14.5±1.3 | '' |
| $^{68}$Zn | | 0.31 | | 6.6 | 12.5±1.1 | '' |

a)See ref.(8); b)See ref.(9); c)See ref.(6); d(See ref.(7)

# ON THE DETERMINATION OF INDUCED TENSOR COUPLING CONSTANT IN MUON CAPTURE*

R.Parthasarathy & V.Devanathan,
Department of Nuclear Physics,
University of Madras,
MADRAS

Abstract:

The plausible value of the induced tensor coupling Constant in Muon capture in determined from the analysis of the experimental capture rates in $^{12}C$ and $^{16}O$. This value can be used in the estimation of other observables such as recoil nuclear polarization in muon capture.

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Recent analysis [1] of 'ft' values in mirror $\beta$ decays revived the interest on the study of second class currents. Whether or not such a current is present in weak interaction is still a controversial problem [2] and we make here no attempt to comment upon the same. As a consequence of the observed systematic deviation of the asymmetry parameter :

$\delta = (ft)^+/(ft)^- - 1$ from its zero value, Wilkinson [3] predicts $g_T \approx -4\, g_A$, if second class currents exist. It is to be noted here that the mesonic exchange effects as introduced by Rho [2] modify the expression for $\delta$ and so we cannot rely upon the predictions of Wilkinson using a simple

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* Presented by R.Parthasarathy

formula for $\delta$ given by Huffaker and Greuling [4] on the basis of Impulse Approximation. Also Huffaker and Greuling [4] have predicted $g_T/g_A \approx -3.7 \pm 20\%$ by analysing the 'ft' values of A = 12 system. Besides, Ohtsubo and Fujii [5] have considered muon capture in Hydrogen and reported $g_T/g_A \approx -2$.

It is the purpose of this talk to discuss the plausible value of $g_T/g_A$ and determine its possible ranges by studying the following processes, viz,

$$\mu^- + {}^{12}C(0^+) \longrightarrow {}^{12}B(1^+) + \nu_\mu \qquad (1)$$

$$\mu^- + {}^{16}O(0^+) \longrightarrow {}^{16}N(2^-, 0^-, 1^-, 3^-) + \nu_\mu \qquad (2)$$

In process (2) we confine ourselves to the final states ${}^{16}N\ (2^-, 0^-)$ only as the other two final states, viz, ${}^{16}N(1^-, 3^-)$ do not depend upon either $g_p$ or $g_T$. We obtain the plausible value for $g_T/g_A$ as - 4.5.

We give below in some detail the choice, made for first class current coupling constants. Following the notation used in reference [6] we have

$$g_V = 0.983\ G$$

$$G = 1.02 \times 10^{-5}/M^2$$

where M is mass of nucleon. Recent analysis on quasi elastic neutrino scattering [7] predicts the following $q^2$ dependance (q being the momentum transfer) for $g_A$.

$$g_A(q^2) = -1.23\, g_V \left(1 + \frac{q^2}{m_A^2}\right)^{-2}$$

where $m_A^2 = 0.99\ \text{GeV}^2$. Using CVC and recent analysis [8] on process (1) we write the Weak Magnetism Coupling Constant as

$$g_M(q^2) = 3.7\, g_V \left(1 + \frac{q^2}{m_V^2}\right)^{-2}$$

where $m_V^2 = 0.71\ \text{GeV}^2$. The induced pseudoscalar coupling constant is determined using PCAC and one pion pole dominance. Its value is given by

$$\frac{g_P}{g_A} = \frac{2M m_\mu}{q^2 + m_\pi^2} \qquad (3)$$

where $m_\mu$ and $m_\pi$ are masses of muon and pion respectively. This value is obtained under the assumption that the exchanged virtual pion between 'neutron-proton' vertex and 'neutrino-muon' vertex propagates as a free particle. This being not the real situation Wycech [9] has taken into account the scattering of the virtual pion by other nucleons and he obtains a value of 70% for nuclear matter and 60% for $^{16}O$ using statistical model,

for the correction. Also the exchanged pion can have Coulomb interactions with the other nucleons. This was studied by Baba [10] and the correction for $^{12}C$ and $^{16}O$ are found to be nearly the same, viz, - 3.6%. Including both, the effective correction to $g_p/g_A$ is denoted by $\Delta$ . For the Second Class Current coupling constants we make the choice, viz, $g_S = 0$ from CVC and $g_T = \chi\, g_A$.

We note that in our formalism (equation (5) of reference [6]), $g_p$ and $g_T$ always occur together as a sum and so it is desirable to define another effective coupling constant $\eta\, g_A$ such that

$$\eta\, g_A = g_p + g_T \qquad (4)$$

From equations (3) and (4) and including $\Delta$ we have

$$\eta = \frac{2M m_\mu}{q^2 + m_\pi^2} + \Delta + \chi . \qquad (5)$$

We study the dependance of the capture rates for the processes (1) and (2) on $\eta$ . The nuclear structure uncertainties are eliminated as much as possible by choosing the best available wave functions for $^{12}C$ and $^{16}O$ , viz Kurath's [11] wave functions for $^{12}C$ and wave functions calculated from Migdal's theory [12] for $^{16}O$. From the experimental partial

capture rates for processes (1) and (2) we extract a value for $\eta$ . Using the analysis of Wycech [9] for $\Delta$, we obtain a value for $\chi$ from equation (5). The errors present in the experimental measurements are minimised by doing the chisquare fit for $\eta$ and $\eta \approx 7$ is obtained.

Using the value of $\chi$ so obtained we have also calculated the polarization of $^{12}B$ $(1^+)$ nucleus in process (1) following fef. [13] . It is observed that the values for polarization of the recoil nucleus $^{12}B$ $(1^+)$, calculated (i) neglecting both $\Delta$ and $\chi$ and (ii) including both $\Delta$ and $\chi$ are 0.4884 $\vec{P}_\mu$ and 0.4812 $\vec{P}_\mu$ where $\vec{P}_\mu$ is polarization of muon. This insensitivity is due to the fact that $\Delta$ as given by [9] and $\chi$ as obtained from the analysis, although large when considered individually, mutually cancel away when taken together. If the value of $\Delta$ is set as 70%, then the effect of Second Class Currents is found to increase the polarization of $^{12}B$ $(1^+)$ by 20%.

Till now, no consensus has been reached on the value of $\Delta$. Rho and Green [14] gave a value of 14% for $\Delta$ where as ref.[9] gives 60%. Since $\chi$ depends upon $\Delta$ through equation (5) we can obtain a range for $\chi$ corresponding to $\Delta$ = 14% and 60% and this turns out to be -0.757 and -4.734 respectively, including Coulomb corrections as given by [10] .

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# TRACE TECHNIQUES FOR ANGULAR MOMENTUM OPERATORS *

P.R. Subramaniam and V. Devanathan
Department of Nuclear Physics
University of Madras
Madras-600025.

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ABSTRACT:

Traces of angular momentum matrices for arbitrary spin are derived using their well-established properties. An interesting result, that the angular momentum matrices $J_\lambda$ , $J_\mu$ $(\lambda \neq \mu)$ cannot anticommute when j is neither zero nor half, will simply follow from the formulae obtained.

## 1. Introduction

The purpose of this article is to obtain the traces of angular momentum matrices for systems of arbitrary spin. The development of the techniques is based upon the following facts:

$$1) \quad \vec{J} \times \vec{J} = i\vec{J} \tag{1.1}$$

where $\vec{J}$ is the angular momentum operator.

$$2) \quad J^2 = j(j+1)I \tag{1.2}$$

where I is a unit matrix of order $(2j+1)$.

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*Presented by P.R. Subramaniam.

3) The components of $\vec{J}$ viz. $J_x, J_y, J_z$ are Hermitian. (1.3)

4) $J^2$ and $J_\lambda$ ($\lambda$ = x OR y OR z) can be diagonalized simultaneously in which case the eigen-values of $J_\lambda$ will be $-j, -(j-1), \ldots, (j-1), j$ in steps of unity[1].

Let us now give two important relations which will be used often:

When $\lambda, \mu, \nu$ are different,

1) $$J_\lambda J_\mu - J_\mu J_\lambda = \pm i J_\nu \qquad (1.4)$$

2) $$J_\lambda J_\mu^2 - J_\mu^2 J_\lambda = \pm i (J_\nu J_\mu + J_\mu J_\nu) \qquad (1.5)$$

$\pm$ according as $\lambda, \mu, \nu$ are cyclic OR anticyclic in x,y,z. In the course of the development we will be able to show that $J_\lambda$, $J_\mu$ ($\lambda \neq \mu$) cannot anticommute when j is neither zero nor half[2].

## 2. General Properties of Traces:

The following properties of traces which are very well known will be used often.

a) $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$ (2.1)

b) $\mathrm{Tr}(PQRST) = \mathrm{Tr}(TPQRS) = \mathrm{Tr}(STPQR)$, etc. (2.2)

i.e. the trace of a product of the matrices is not changed by cyclic permutation of the factors.

c) $\mathrm{Tr}(\alpha M) = \alpha\, \mathrm{Tr}(M)$ (2.3)

where $\alpha$ is a scalar and M, a matrix.

d) Trace is invariant under a similarity transformation.

$$\text{Tr}\,(P^{-1}AP) = \text{Tr}(PP^{-1}A), \text{ using (2.2)}$$
$$= \text{Tr}\,(IA)$$
$$= \text{Tr}\,(A)$$

$$\text{Tr}\,(A) = \text{Tr}\,(P^{-1}AP). \qquad (2.4)$$

## 3. List of Formulae

Here is a list of the traces of several frequently encountered matrices.

In what follows i) $\lambda, \mu, \nu$ can be any one of x,y,z

ii) $\rho$, a positive integer, $\geq$ zero

iii) $\epsilon_{\lambda\mu\nu}$ is the Levi-Civita symbol in three dimensions,

iv) $\eta = j(j+1)$, (3a)

v) $\Omega = \text{Tr}\,(J^2) = j(j+1)\,(2j+1)$,

vi) There is no summation over repeated indices.

Let us have an understanding that $J_\lambda^0 = I$, the unit matrix.

1) $\text{Tr}\,(J_\lambda^{2p+1}) = 0$ (3.1)

2) $\text{Tr}\,(J_\lambda^{\rho} J_\mu) = 0, \quad \lambda \neq \mu$ (3.2)

3) $\text{Tr}\,(J_\lambda^{2p} J_\mu J_\nu) = 0$, ( $\lambda, \mu, \nu$ different) (3.3)

4) $\text{Tr}\,(J_\lambda^{2p+1} J J) = \frac{i}{2} \text{Tr}\,(J_\lambda^{2p+2}) \epsilon_{\lambda\mu\nu}$ (3.4)

5) $\text{Tr}\,(J_\lambda^{\rho} J^2) = \text{Tr}\,(J_\lambda^{\rho} J_\nu^2)$

$$= \frac{1}{2}\left[j(j+1)\,\text{Tr}\,(J_\lambda^{\rho}) - \text{Tr}\,(J_\lambda^{\rho+2})\right] \qquad (3.5)$$

( $\lambda, \mu, \nu$ different)

6) $\mathrm{Tr}\ (J_\lambda^2) = \Omega/3$ (3.6)

7) $\mathrm{Tr}\ (J_\lambda^4) = \frac{\Omega}{15}[3\eta - 1]$ (3.7)

8) $\mathrm{Tr}\ (J_\lambda^6) = \frac{\Omega}{21}[3\eta^2 - 3\eta + 1]$ (3.8)

9) $\mathrm{Tr}\ (J_\lambda^8) = \frac{\Omega}{45}[5\eta^3 - 10\eta^2 + 9\eta - 3]$ (3.9)

10) $\mathrm{Tr}\ (J_\lambda^2 J_\mu^2) = \frac{\Omega}{30}[2\eta + 1]$ (3.10)

11) $\mathrm{Tr}\ (J_\lambda J_\mu J_\lambda J_\mu) = \frac{\Omega}{15}[\eta - 2]$ $\lambda \neq \mu$ (3.11)

12) $\mathrm{Tr}\ (J_\lambda^3 J_\mu J_\nu) = \frac{i}{30}\Omega\ (3\eta - 1)\epsilon_{\lambda\mu\nu}$ (3.12)

13) $\mathrm{Tr}\ (J_\lambda^2 J_\mu J_\lambda J_\nu) = \frac{i}{30}\Omega\ (\eta - 2)\epsilon_{\lambda\mu\nu}$ (3.13)

14) $\mathrm{Tr}\ (J_\lambda^2 J_\mu^2 J_\lambda J_\mu) = \mathrm{Tr}\ (J_\lambda^3 J_\mu^3) = \text{zero}, \quad \lambda \neq \mu$ (3.14)

15) $\mathrm{Tr}\ (J_\lambda^2 J_\mu^2 J_\nu^2) = \frac{\Omega}{210}[2\eta^2 - 9\eta + 10]$, ( $\lambda,\mu,\nu$ different) (3.15)

16) $\mathrm{Tr}\ (J_\lambda^3 J_\mu^2 J_\nu) = \mathrm{Tr}\ (J_\lambda^3 J_\nu J_\mu^2) = \text{zero}$, ( $\lambda,\mu,\nu$ different) (3.16)

17) $\mathrm{Tr}\ (J_\lambda^3 J_\mu J_\lambda J_\mu) = \frac{\Omega}{210}[6\eta^2 - 13\eta + 2]$, $\lambda \neq \mu$ (3.17)

18) $\mathrm{Tr}\ (J_\lambda^2 J_\mu J_\lambda^2 J_\mu) = \frac{\Omega}{105}[3\eta^2 - 10\eta + 8]$, ( $\lambda \neq \mu$ ) (3.18)

19) $\mathrm{Tr}\ (J_\lambda^2 J_\nu J_\mu^2 J_\nu) = \frac{\Omega}{210}[2\eta^2 + 19\eta - 11]$, ( $\lambda,\mu,\nu$ different) (3.19)

20) $\mathrm{Tr}\ (J_\lambda^2 J_\mu J_\nu J_\mu J_\nu) = \frac{\Omega}{105}[\eta^2 - \eta - 2]$, ( $\lambda,\mu,\nu$ different) (3.20)

21) $\mathrm{Tr}\ (J_\lambda J_\mu J_\nu J_\alpha J_\beta J_\gamma) = \frac{\Omega}{210}[2\eta^2 - A\eta + B]$ (3.21)

where the successive indices are NOT equal even under the cyclic permutation of the matrices.

$A = 30$, $B = 17$ if the matrices of the three successive pairs

... (... alike) ...

A = 9, B = 10 if the matrices of the three successive pairs are of mixed (cyclic and anticyclic) type.

For the sake of illustration let us evaluate

$$\mathrm{Tr}\,(J_\lambda J_\mu J_\lambda J_\mu), \qquad (\lambda \neq \mu)$$

Let $\lambda, \mu, \nu$ be different

$$J_\lambda J_\mu J_\lambda J_\mu = J_\lambda J_\mu (J_\mu J_\lambda \pm i J_\nu), \text{ using (1.4)}$$

$$= J_\lambda J_\mu^2 J_\lambda \pm i J_\lambda J_\mu J_\nu$$

$$\therefore \mathrm{Tr}[J_\lambda J_\mu J_\lambda J_\mu] = \mathrm{Tr}(J_\lambda J_\mu^2 J_\lambda) \pm i\,\mathrm{Tr}(J_\lambda J_\mu J_\nu)$$

$$= \mathrm{T}(J_\lambda^2 J_\mu^2) \pm i\,\mathrm{Tr}(J_\lambda J_\mu J_\nu), \text{ using (2.2)}$$

$$= \frac{\Omega}{30}[2\eta+1] \pm i\left\{ i\frac{\Omega}{6}\epsilon_{\lambda\mu\nu} \right\},$$

using (3.10), (3.4) and (3.6)

Now $\pm \epsilon_{\lambda\mu\nu} = 1$, as $\pm$ must be used according as $\lambda, \mu, \nu$ are cyclic <u>OR</u> anticyclic in x, y, z.

$$\therefore \quad \mathrm{Tr}(J_\lambda J_\mu J_\lambda J_\mu) = \frac{\Omega}{15}(\eta - 2). \qquad (3.11)$$

## 4. Application of Euler-Maclaurin Sum Formula

Let p be an integer greater than zero. Then using Euler-Maclaurin sum formula, we have[3]

$$1^p + 2^p + \cdots + n^p = \frac{1}{p+1}\left\{\sum_{\nu=0}^{p}\binom{p+1}{\nu} B_\nu n^{p+1-\nu}\right\} + n^p \quad (4.1)$$

Here $B_\nu$ are the Bernoulli's numbers given by

$$B_0 = 1, \quad B_1 = -\frac{1}{2}, \quad B_{2n+1} = 0 \ (n = 1, 2, 3, \ldots\ldots)$$

$$B_{2n} = (-1)^{n+1}\,(2n)!\,\frac{\zeta(2n)}{(2\pi)^{2n}} \times 2, \ (n = 1, 2, 3, \ldots\ldots) \quad (4.2)$$

with

$$\zeta(n) = \sum_{\nu=1}^{\infty}\frac{1}{\nu^{2n}} \qquad (n = 1, 2, 3 \ldots\ldots\ldots)$$

The Bernoulli's numbers can be calculated from the recurrence relation

$$\binom{n}{0} B_0 + \binom{n}{1} B_1 + \binom{n}{2} B_2 + \cdots\cdots + \binom{n}{n-1} B_{n-1} = 0$$

$$(n = 2, 3, \cdots\cdots) \quad (4.3)$$

The first few $B_\nu$ are

$$B_0 = 1, \quad B_1 = -\frac{1}{2}, \quad B_2 = \frac{1}{6}, \quad B_3 = 0, \quad B_4 = -\frac{1}{30},$$

$$B_5 = 0, \quad B_6 = \frac{1}{42}, \quad B_7 = 0, \quad B_8 = -\frac{1}{30} \ \ldots\ldots\ldots \quad (4.4)$$

The first few $\zeta(2n)$ are

$$\zeta(2) = \frac{\pi^2}{6}, \quad \zeta(4) = \frac{\pi^4}{90}, \quad \zeta(6) = \frac{\pi^6}{945}, \ \ldots\ldots \quad (4.5)$$

Equations (3.6), (3.7), (3.8) and (3.9) are obtained
ng (4.1) - (4.5) and (2.4) by considering a representation
which $J_\lambda$ is diagonal.

Comments

Let A be a product of n 'J' matrices where $1 \leq n \leq 6$
have

1) Tr (A) $\equiv 0$, if Tr (A) = 0 for $j = \frac{1}{2}$.

2) If Tr (A) $\neq 0$, for $j = \frac{1}{2}$, it will not be zero,
in general, for other j values.

$$\sigma_\lambda = 2J_\lambda \qquad \text{when } j = \frac{1}{2}$$

r example,

a) $Tr(\sigma_\lambda) = 0$ $\qquad$ $Tr(J_\lambda) = 0$ for any j.

b) $Tr(\sigma_\lambda \sigma_\mu \sigma_\lambda \sigma_\nu) = 0$ $\qquad$ $Tr(J_\lambda J_\mu J_\lambda J_\nu) = 0$ for any j.

(No summation ; $\lambda, \mu, \nu$ different)

c) $Tr(\sigma_\lambda \sigma_\mu \sigma_\lambda \sigma_\mu) \neq 0$ $\qquad$ $Tr(J_\lambda J_\mu J_\lambda J_\mu) = 0$ for $j = 1$,

but (No Summation; $\lambda \neq \mu$ ) non zero for other js.

In general to see whether Tr (A) vanishes identically, replace each $J_N$ by $\sigma_N$ and see whether Tr (A) is zero for this particular case. If it is zero for $\sigma_\lambda$ s (i.e. $j = \frac{1}{2}$), it will be true for **ANY** j. We have ONLY GENERALIZED (i.e. we have not proved) the result after observing the behaviour of Tr(A) when A is product of n $J_\lambda$'s with $n \leq 6$.

6. $J_\lambda \cdot J_\mu$ $(\lambda \neq \mu)$ Don't anticommute when j is neither zero nor half.

From (3.10) and (3.11) an interesting result follows:

$$\mathrm{Tr}\,(J_\lambda^2 J_\mu^2) + \mathrm{Tr}\,(J_\lambda J_\mu J_\lambda J_\mu) = \frac{j(j+1)(2j+1)[2\eta+1]}{30}$$

[No summation; $\lambda \neq \mu$]

$$+ \frac{1}{15} j(j+1)(2j+1)[\eta - 2]$$

$$= \frac{1}{30} j(j+1)(2j+1)[4\eta - 3]$$

$$= \frac{1}{30} j(j+1)(2j+1) \times$$

$$\times [4j^2 + 4j + 3] \qquad (6.1)$$

$$(\because \eta = j(j+1))$$

Now the right hand side (R.H.S.) of (3.1) is zero when

$$j = 0, -1, -\tfrac{1}{2} \text{ OR when } 4j^2 + 4j - 3 = 0.$$

$\therefore$ R.H.S. of (6.1) is zero when

$$j = \ldots -1, -\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{3}{2}.$$

Since negative values of j are not permitted $j = 0, \frac{1}{2}$ when R.H.S. of (6.1) is zero.

This then implies that $J_\lambda , J_\mu \ (\lambda \neq \mu)$ cannot anticommute as long as j is neither zero nor half.

For, if they do so,

$$\text{Left hand side of (6.1)} = Tr\left(J_\lambda^2 J_\mu^2\right) + Tr\left(J_\lambda J_\mu J_\lambda J_\mu\right)$$

$$= Tr\left(J_\lambda^2 J_\mu^2\right) - Tr\left(J_\lambda J_\mu J_\lambda J_\mu\right)$$

$$= \text{zero},$$

whereas R.H.S. of (6.1) is not zero when j is neither zero nor half. $\therefore$ $J_\lambda , J_\mu \ (\lambda \neq \mu)$ CANNOT ANTICOMMUTE when j is neither zero nor half. This interesting result[2] follows simply from the trace techniques.


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# A CONSISTENT STUDY OF THE NUCLEUS

B.K.Jain
Nuclear Physics Division, Bhabha Atomic Research Centre,
Trombay, Bombay-400085.

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## 1. Introduction

The mathematically simplest and mentally easily conceivable description of the nucleus is provided by the single particle model. This model assumes that the total interaction of a nucleon with all the other nucleons in the nucleus can be approximated by a single particle potential. This potential, in general, is state dependent and can have any shape. It describes the long range part of the N-N interaction in an average way, and thus is similar, in spirit, to the Hartree-Fock (HF) potential. Phenomenologically, guided by some general considerations, one assumes a certain form for this potential, the parameters of which are fixed by reproducing the observed single particle binding energies of the nucleus. The detailed shape of this potential, and its literal truth can be checked directly by analysing knock-out reactions, such as (p,2p), (e,e'p), the capture reaction ( $\pi^-$,NN) and the elastic electron scattering data. However, in order to have confidence in the potential deduced this way the above reactions should be analysed at various energies. This would reduce the uncertainities due to the assumed reaction mechanism.

Once the shell model[+] potential, and thus the single particle wave function of the nucleus is known reliably, the effect

[+] The word "shell model" in the present text is used to describe, in general, the individual particle description of the nucleus.

due to the strong short range repulsion in the N-N interaction can be introduced following, for example, the Jastrow prescription[1]. The parameters of the Jastrow correlation factor can be deduced by analysing the high energy elastic electron scattering and the pion absorption data. Thus, by analysing the various knock-out reactions, electron scattering and the pion absorption together, it should be possible to determine a consistent and a complete single particle description of the nucleus. In the present talk I illustrate this point by discussing the (p,2p), (e,e'p), (e,e) and ($\pi^-$,NN) reactions on $^6$Li, $^{12}$C and $^{40}$Ca.

2. The (p,2p) and the (e,e'p) Reactions:

In the (p,2p) reaction an incoming proton with energy and momentum $(T_0, p_0)$ is considered to make a quasi-free collision with a bound proton of the target nucleus. Therefore, the momentum of the knocked-out proton in the nucleus is equal and opposite to that of recoiling nucleus. The conservation law obeyed by the two outgoing protons of energy and momenta $(T_1, q_1)$ and $(T_2, q_2)$ give,

$$T_0 = T_1 + T_2 + T_R + E_s , \tag{2.1}$$

$$\vec{p}_0 = \vec{q}_1 + \vec{q}_2 + \vec{Q} , \tag{2.2}$$

where $E_s$ is the proton separation energy, and $T_R$ and Q are the kinetic energy and the momentum of the residual nucleus.

We consider a symmetric coplanar experiment so that the outgoing protons emerge making equal angles with the incident direction and hence $|\vec{q}_1| = |\vec{q}_2| = q$.

In the distorted-wave impulse approximation (DWIA), the cross section for this reaction is given by[2)]

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dT} = F_{kin}(\theta) \left(\frac{d\sigma}{d\Omega}\right)_{pp} P_{pp}(Q), \qquad (2.3)$$

where T is the summed energy of the outgoing protons, $F_{kin}(\theta)$ is a kinetimatic factor, $(d\sigma/d\Omega)_{pp}$ is the free proton-proton cross section in their c.m. system and $P_{pp}(Q)$ is the distorted momentum distribution of the knocked-out proton; is given by;

$$P_{pp}(Q) = \sum_{L,J} S_{J_i J_f}(LJ) (T_f \tfrac{1}{2} \nu_f \tfrac{1}{2} | T_i \nu_i)(2L+1)^{-1} \cdot \sum_M (2\pi)^{-3} |g^M_{LJ}|^2. \qquad (2.4)$$

The quantity $S_{J_i J_f}(LJ)$ is the spectroscopic factor for the angular momenta L, J for the transition $J_i, J_f, T_f$ and $T_i$ are the isospin quantum numbers for the final and initial quantum numbers; $(a\,b\,\alpha\,\beta\,|c\,\gamma)$ is the Clebsch-Gordon coefficients. $g^M_{LJ}$ is evaluated according to the di-proton model[3)] which gives rise to the expression:

$$g^M_{LJ} = \langle \chi^-_{Bc}(\vec{K}_{Bc}, \vec{r}) | \psi^{LJ}_P(\vec{r})\, \chi^+_{oA}(\vec{K}_{oA}, a\vec{r}), \qquad (2.5)$$

where $a = (A-1)/A$, $\chi^{+}_{0A}$ is the distorted wave for the incoming proton, $\chi^{-}_{BC}$ is the distorted wave describing the motion of the c.m. of the two outgoing protons, and the momentum $\vec{K}_{0A}$ and $\vec{K}_{BC}$ are the c.m. momenta and are given by

$$\vec{K}_{0A} = \frac{A}{A+1}\vec{p}_0 , \tag{2.6}$$

$$\vec{K}_{BC} = \vec{q}_1 + \vec{q}_2 \frac{2}{A+1}\vec{p}_0 , \tag{2.7}$$

$\chi^{+}_{0A}$ and $\chi^{-}_{BC}$ have the following forms in WKB approximation

$$\chi^{+}_{0A}(\vec{K}_{0A}, a\vec{r}) = e^{i\vec{K}_{0A}\cdot a\vec{r}}\,\Phi_{0A}(a\vec{r}), \tag{2.8}$$

$$\chi^{-}_{BC}(\vec{K}_{BC}, \vec{r}) = e^{i\vec{K}_{BC}\cdot \vec{r}}\,\Phi_{BC}(\vec{r}), \tag{2.9}$$

where $\Phi_{0A}$ and $\Phi_{BC}$ are slowly varying modulating functions and are given by[4]

$$\Phi_{0A}(a\vec{r}) = \exp\left[-\frac{iE_{0A}}{\hbar^2c^2K_{0A}}\int_0^{\infty} V_{0A}(|a\vec{r} - \hat{K}_{0A}s|)\,ds\right], \tag{2.10}$$

$$\Phi_{BC}(\vec{r}) = \exp\left[\frac{iE_{BC}}{\hbar^2c^2K_{BC}}\int_0^{\infty} V_{BC}(|\vec{r} + \hat{K}_{BC}s|)\,ds\right], \tag{2.11}$$

where $V_{0A}$ and $V_{BC}$ are the complex optical potentials including nuclear and Coulomb potentials of the incident proton and the outgoing di-proton respectively; $E_{0A}$ and $E_{BC}$ are the total

energies of the incoming proton and the outgoing di-proton respectively.

In an (e,e'p) reaction an incoming electron with energy and momentum $(T_0, p_0)$ knocks a bound proton out of the target nucleus so that the electron and the proton emerge with energy and momentum $(T_1, q_1)$ and $(T_2, q_2)$ respectively. We have similar equations for the conservation of energy and momentum as given in eqs. (2.1) and (2.2).

The four-momentum transfer for the electron is given by

$$q_\mu^2 = (T_0 - T_1)^2 - (\vec{p}_0 - q_1)^2 = -4T_0T_1\sin^2(\theta/2), \quad (2.12)$$

where $\theta$ is the angle of scattering of the electron. The momentum transfer of the electron is $\vec{q} = \vec{p}_0 - \vec{q}_1$.

Following McVoy and Van Hove[5], the effective Hamiltonian for the interaction of relativistic electrons with a non-relativistic Pauli particle (correct to second order in inverse nucleon mass) can be written as

$$H_{eff} = -\frac{4\pi e^2}{q_\mu^2}\langle u_f | F_1 e^{-iq_\mu x_\mu} - \frac{F_1}{2M_p}[(\vec{p}\cdot\vec{\alpha})e^{-iq_\mu x_\mu} + e^{-iq_\mu x_\mu}(\vec{p}\cdot\vec{\alpha})]$$
$$-i\frac{F_1+kF_2}{2M_p}\vec{\sigma}\cdot(\vec{q}\times\vec{\alpha})e^{-iq_\mu x_\mu} + \frac{q_\mu^2}{8M_p^2}(F_1+2kF_2)e^{-iq_\mu x_\mu}|u_i\rangle \quad (2.13)$$

where $u_i$ and $u_f$ are the plane-wave Dirac spinors for the incident and scattered electron, $\vec{\alpha}$ is the Dirac operator which

operates on the free electron, $\vec{p}$ and $\vec{\sigma}$ are the Pauli momentum and spin operaters, K is the nucleon anomalous magnetic moment in nuclear magnetons and is taken to be equal to 1.85 n.m. $F_1$ and $F_2$ are the charge and magnetic nucleon form factors depending on $q_\mu^2$. The first term in eq.(2.13) is the Coulomb term, the second, the convection current, the third the spin current term and the last term is the Darwin-Foldy term. F1 and F2 are almost the same inside a nucleus. Following Devanathan[6] we choose an exponential model with mean square radius $\langle a^2 \rangle = 0.64$ fm$^2$; then

$$F_1 = F_2 = \left(1 - \frac{1}{12}\langle a^2\rangle q^2\right). \qquad (2.14)$$

Using the effective Hamiltonian eq.(2.13) one obtains for the (e,e'p) cross section[7], ($c = \hbar = 1$):

$$\frac{d^4\sigma}{dT_1 dT_2 d\Omega_1 d\Omega_2} = \frac{4e^4 \cos^2(\theta/2)}{q_\mu^4} \frac{E_p q_2 q_1^2}{1-(E_p/E_R)[(\vec{q}_2\cdot\vec{q} - q_2^2)/q_2^2]}$$
$$\times \left\{ F_1^2 - \frac{q_\mu^2}{4M_p^2}\left[2(F_1+KF_2)\tan^2(\theta/2) + K^2F_2^2\right] \right.$$
$$+ \frac{F_1^2}{4M_p^2}\sec^2(\theta/2)\left[\frac{(\vec{t}\cdot\vec{p}_0)(\vec{t}\cdot\vec{q}_1)}{p_0 q_1} + t^2\sin^2(\theta/2)\right]$$
$$\left. + \frac{F_2^2}{2M_p}\sec^2(\theta/2)\left[\frac{\vec{t}\cdot\vec{q}_1}{q_1} + \frac{\vec{t}\cdot\vec{p}_0}{p_0}\right]\right\} P_{ep}(Q), \qquad (2.15)$$

where $E_p$ and $E_R$ are the total energies of the outgoing proton and the recoiling nucleus respectively, $\vec{t} = \vec{q} - 2\vec{q}_2$ and $P_{ep}(Q)$ is the distorted momentum distribution of the bound proton; $P_{ep}(Q)$ has the same form as $P_{pp}(Q)$ given by eq.(2.4) except that now $g^M_{LJ}$ is given by

$$g^M_{LJ} = \int e^{i\vec{q}\cdot\vec{r}}\, \chi^{-*}(\vec{q}_2, \vec{r})\, \psi_p^{LJ}(\vec{r})\, d\vec{r}. \quad (2.16)$$

$\chi^-(\vec{q}_2, \vec{r})$, the distorted-wave function of the outgoing proton is given by

$$\chi^-(\vec{q}_2, \vec{r}) = \exp\left[ i\vec{q}_2\cdot\vec{r} + \frac{iE_p}{\hbar^2 c^2 K_p} \int V(|\vec{r} + \hat{K}_p s|)\, ds \right]. \quad (2.17)$$

The c.m. momentum of the outgoing proton, $\vec{K}_p$, is given by

$$\vec{K}_p = \frac{A-1}{A}\, \vec{q}_2. \quad (2.18)$$

From this formalism we observe that the nuclear structure information in these reactions enters through the factor P(Q), which after being corrected for the scattering of continuum particles by the nucleus, is just the Fourier transform of the bound proton wave function. Thus knowledge of P(Q) over the entire momentum range (i.e. $0 \rightarrow \infty$) provides complete information about the single particle nuclear wave function. However, experimentally the range of Q over which meaningful measurements are possible

is restricted by the accuracy of the experiments. In fact, as we see from the experimental cross-section shown in figs 1-5, this range is from zero to about 300 MeV/c. Incidentally this is also the order of the range of the momentum of a bound nucleon which is described by a shell model potential. Hence it is hoped that the analysis of these reactions should provide the shell model character of the nuclear wave function. However, in order to have confidence in the information extracted from these reactions the experiments on them should be done at high energy ($>$ 400 MeV)[8]. At these energies the uncertainities normally encountered in the theoretical analysis due to the multiple scattering of incoming and outgoing particles, the off-shell nature of the two-body t-matrix, three-body nature of the final state and the considerable width of the final state, become unimportant[9].

In figs 1-5 we have plotted the experimentally deduced values of P(Q) from the (p,2p) reaction at incident energies 460 MeV and 1 GeV and for the (e,e'p) reaction at an incident electron energy of 600 MeV compared with that calculated theoretically. The optical potentials for the incoming and outgoing protons are interpolated from the available elastic scattering analyses in the literature. The bound state proton wave functions for $^{12}C$ are generated in a Saxon-Woods potential,

$$V(r) = -V_0 \left[ 1 + \exp\left( \frac{r - r_0 A^{1/3}}{a} \right) \right]^{-1}.$$

Following Elton and Swift[10], the parameters for this potential are taken as,

$V_0$ = 59.01 MeV, $r_0$ = 1.31 fm, a=0.65 fm for the 1p state,

$V_0$ = 63.22 MeV, $r_0$ = 1.31 fm, a=0.65 fm for the 1s state.

These potentials reproduce the single particle binding energies of 1p and 1s shell. We see from these figures that the shape of the distribution in all the curves is well reproduced except for a shift in the (e,e'p) curve for a 1p proton. In the (e,e'p) reaction for a 1p proton, though the peak position does not agree with theory, the full widths at half maximum agree. This may suggest that the exptl. P(Q) has shifted as a whole. From this it seems that before any serious thought could be given to this agreement new experimental data on this reaction should be collected.

The spectroscopic factors derived by normalizing the theoretical peak cross-sections with the corresponding experimental ones are as follows:

| State | | Energy(MeV) | From Sum rule |
|---|---|---|---|
| | 460 | 1000 | |
| 1p | 3.71 | 7.62 | 8.0 |
| 1s | | 5.10 | 4.0 |

As we see the experimental S at 1 GeV agree well with those predicted by the sum rule. The small value of the spectroscopic factor at 460 MeV is partly due to the experimental uncertainty. Since Tyren et al[11], who conducted the experiment at 460 MeV,

have not measured the angular distribution for the 1s knock-out, it is possible that some of the 1p strength is contained in the broad peak which is presumed to correspond purely to 1s knock-out. Taking this uncertainty into account it appears that the assumed 1p wave function satisfactorily accounts for the measured momentum distribution from (p,2p) reaction at different energies and from the (e,e'p) reaction at 600 MeV. For the 1s state there is a good shape agreement in the (p,2p) reaction and (e,e'p) reaction.

In fig 6, we have also plotted the results for $^{40}Ca(p,2p)^{39}K$ corresponding to the knock-out of $1d_{3/2}$ proton. The experimental points are the Elton-Sundberg[12] reanalyzed data of Tyren et al at 460 MeV. The bound state we function is generated in a non-local Woods-Saxon potential,

$$V(\vec{r},\vec{r}') = \pi^{-3/2}\beta^{-3}\, U\left(|(\vec{r}+\vec{r}')/2|\right) \exp\left|-[(\vec{r}-\vec{r}')/\beta]^2\right|,$$

where $$U(s) = V_0 f_0(s) + V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{s}\frac{df_{so}}{ds}\,\vec{l}\cdot\vec{\sigma},$$

and f has Woods-Saxon form. Following Elton and Webb[13], the parameters of this potential are taken as

$V_0 = -78$ MeV, $r_0 = 1.18$ fm, $a=0.5$ fm,

$V_{so} = -10.5$ MeV, $r_{so}=0.80$ fm, $a_{so}=0.45$ fm

$\beta = 0.9$ fm.

As we see from the curve the agreement in shape between the computed and observed results is good. The extracted spectroscopic factor is 2.4. As compared with the value 4.60, obtained from $^{40}Ca(p,d)$ reaction[14], the present value is somewhat low.

From these analyses of proton knock-out reactions on $^{12}C$ and $^{40}Ca$ we may conclude that the relative probability of proton momenta in $^{12}C$ and $^{40}Ca$ in the range $0 \rightarrow 300$ MeV/c can be well reproduced by the state dependent Woods-Saxon potential with a particular choice of parameters. In some cases, however, the absolute value of P(Q) is not reproduced. This may partly be due to the uncertainity in the experimental data and partly in the optical potential parameters used to account for the multiple scattering of the continuum particles.

The legitimacy of these proton wave functions can be further established by checking them against the elastic electron scattering in the low momentum transfer region and against the $(\pi^-, N')$ reaction.

3. Elastic Electron Scattering:

In the Born approximation the cross-section for elastic electron scattering is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Point} |F_{ch}(q)|^2, \qquad (3.1)$$

where the first factor is the cross-section for the scattering from a point charge, $F_{ch}(q)$ is the charge form factor of the

nucleus,

$$F_{ch}(q) = \int e^{i\vec{q}\cdot\vec{r}} \rho_{ch}(\vec{r})\, d\vec{r}, \qquad (3.2)$$

where $\vec{q}$ the momentum transfer, is given by

$$\vec{q} = 2\vec{k}\,\mathrm{Sin}\theta/2 \qquad (3.3)$$

with $\vec{k}$ as the wave vector of the impinging electron and $\theta$ its angle of scattering. $\rho_{ch}(\vec{r})$ is the charge density of the nucleus. In terms of proton density, $\rho_p(\vec{r})$, it is written as,

$$\rho_{ch}(\vec{r}) = \int \rho_p(\vec{r}')\, F_p(|\vec{r}-\vec{r}'|)\, d\vec{r}', \qquad (3.4)$$

where $F_p$ is the proton form factor. $\rho_p(\vec{r})$ itself is written as

$$\rho_p(\vec{r}) = \frac{1}{A} \langle \Phi_o | e \sum_i \delta(\vec{r}-\vec{r}_i)\tfrac{1}{2}(1+\tau_i) | \Phi_o \rangle, \qquad (3.5)$$

where $\Phi_o$ is the nuclear ground state wave function normalized to unity. In terms of single particle description of the nucleus $\Phi_o$ is written as a Slater determinant,

$$\Phi_o = (A!)^{-1/2} \det \phi_{\alpha_1}(1) \cdots \phi_{\alpha_A}(A). \qquad (3.6)$$

Eqns. 3.1-2 suggest that the measurement on the angular distribution of elastically scattered electrons directly give information about the charge distribution in the nucleus. Since $\rho_{ch}(\vec{r})$ is related to nuclear wave function through eqns.3.4-5 these observations should also determine the nature of $\Phi_0$. In fact, the scattering data in the low momentum region should determine the shell model character of the wave function while those above it should be sensitive to the correlation function. Thus by using the single particle wave functions obtained from the proton knock-out reactions to account for the elastic electron scattering data in the low momentum region one should be able to establish further the reliability of these wave functions.

Historically, in fact, the $^{12}C$ and $^{40}Ca$ wave functions used in sec.2 to give the good description of the (p,2p) and (e,e'p) reaction on these nuclei were first obtained by Elton and his collaborators[12,13] by reproducing the elastic electron scattering data.

4. (π⁻,NN) Reaction:

Due to kinematic considerations, negative pions in nuclei are preferentially absorbed by a correlated pair of nucleons. That is why reactions like (π⁻,NN) are used to study the short range correlations (SRC) in nuclei. There is, however, another aspect of this process. Similar to creation of one-hole states in the (p,2p) and (e,e'p) reactions, the (π⁻,NN) reaction creates a two hole state in the nucleus, If, as suggested earlier by us[15],

the pion absorption is described semi-phenomenologically in terms of the elementary reaction

$$\pi + NN \rightarrow N + N ,$$

the $(\pi^-, NN)$ reaction can be used successfully to study the shell model aspect of the nucleus. This would be a logical extension of the single proton knock-out reactions.

In the "two nucleon" model approximation[16], the absorption probability for the back to back emission of nucleons on a nucleus with zero, either initial or final isospin, is given by[17],

$$d\omega = \frac{\rho}{(2\pi)^5} \frac{A(A-1)}{2M_\pi} (2\ell_\pi+1)^{-1} |g_0|^2 \sum_{J, m_\pi, M_S'=\pm 1} (2J+1)^{-1} \times$$

for $T=0$, and $\times \Big| \sum_L \mathcal{J}_{fi}(L S=1 J T=0)(L\,1-m_\pi M_S | J M) F^{\ell_\pi m_\pi}_{L M_L=-m_\pi} \Big|^2 ,$

$$d\omega = \frac{\rho}{(2\pi)^5} \frac{A(A-1)}{2M_\pi} (2\ell_\pi+1)^{-1} |g_1|^2 k^2 (T_f 1 \nu_f 0 | T_i \nu_i)^2 \times$$

for $T=1$, $\times \sum_{m_\pi} \sum_{L=J} (2J+1)^{-1} \Big| \mathcal{J}_{fi}(L S=0 J T=1) F^{\ell_\pi m_\pi}_{L M_L} \Big|^2 , \quad (4.1)$

where $\rho$ is the phase space density and $\ell_\pi$, $m_\pi$ the quantum numbers of the absorbed pion. L,S,J,T are the quantum numbers of the absorbing nucleon pair, and $\mathcal{J}_{fi}$ their coefficient of fractional parentage. $\vec{k} = (\vec{k}_1 - \vec{k}_2)/2$ where $\vec{k}_1$ and $\vec{k}_2$ are the wave vectors of the outgoing nucleons. The integral $F^{\ell_\pi m_\pi}_{L M_L}$ is defined as

$$F^{\ell_\pi m_\pi}_{L M_L} = \int d\vec{R}\, \chi^{-*}(\vec{k}_1, \vec{R}) \chi^{-*}(\vec{k}_2, \vec{R}) \Phi_{L M_L}(\vec{R}, \vec{R}) \phi_{\ell_\pi m_\pi}(\vec{R}) \quad (4.2)$$

where $\chi$ are the distorted waves for the outgoing nucleons, $\Phi_{L M_L}$ is the space part of the two-nucleon wave function in nucleus,

and $\phi_{l_\pi m_\pi}$ is the pion wave function. $g_0$ and $g_1$ are the parameters of the interaction Hamiltonian; they respectively determine the $^{13}S_1 \rightarrow {}^{33}P_1$ and $^{31}S_0 \rightarrow {}^{33}P_0$ transitions of two nucleons. The modulii $|g_0|$ and $|g_1|$ are determined by analysing the reactions:

$$\pi^+ + d \rightarrow p + p ,$$
$$p + p \rightarrow \pi^0 + p + p . \qquad (4.3)$$

The values of $|g_0|$ and $|g_1|$ recently obtained by Figureau and Ericson[18] by analysing the data of Rose[19] are

$$|g_0|^2 = 0.64 \pm 0.05 \text{ fm}^8 ,$$
$$|g_1|^2 = 0.155 \pm 0.08 \text{ fm}^8 . \qquad (4.4)$$

In this formalism the behaviour of two nucleons in nucleus at short relative distance is completely contained in the parameters $g_0$ and $g_1$ and, as we see from eqn.(4.3), it is measured in terms of those present in the free nucleon system. After $g_0$ and $g_1$ are known, the behaviour of the transition strength, is determined by the shell model character of the nucleus.

In figs. 7-8 we have plotted the neutron energy spectrum for $^{12}C$ and recoil momentum distribution for $^6Li$ along with the experimental data. The wave function for $^{12}C$ is the same as that used in sections 2 and 3 to account for proton knock-out and elastic electron scattering data. For $^6Li$ we have used the $\alpha$-d cluster model wave function which fits the $^6Li(p,pd)$ data, elastic

electron scattering[20] and the energy spectrum[21]. The computation of the theoretical curves assumes the absorption of pion from 1S orbit only. As we see from these figures the agreement between experimental data and the theory is good, thus further establishing the reliability of the nuclear wave functions.

The study of pi-mesic X-rays[22] indicates pions in $^{12}C$ are also absorbed significantly from the 2P orbit. Hence, in fig.7, we have also plotted the weighted sum of the absorption from 1S and 2P pionic orbits (dashed curve). The weightage factors are obtained from the X-ray data on $^{12}C$. The agreement with the experimental data does not seem good. This reflects probably either on the values of the weight factors or on the mechanism of absorption.

Now putting the observations of sections 2-4 together it seems that a consistent, hence reliable, single particle aspect of the nucleus can be obtained by analysing the high energy single particle knock-out reactions, elastic electron scattering and the $(\pi^-, NN)$ reactions.

5. Study of SRC:

Due to Pauli exclusion principle the strong repulsive character of the N-N potential in the nucleus affects the relative wave functions of two nucleons only at short distances ($< 1.0$ fm). Beyond this distance the wave function retains the shell model character. Because of this fact, the nuclear wave function which contains the effect of the repulsive nature of

N-N potential at short distance is written in the Jastrow form,

$$\Psi_A = \mathcal{N} \prod_{i \neq j}^{A} f(|\vec{r}_i - \vec{r}_j|)\, \Phi(A), \qquad (5.1)$$

where $\Phi$ is the shell model type wave function, f is the correlation factor and $\mathcal{N}$ is the normalization constant. f, for an infinite repulsive core, is written as

$$\begin{aligned} f(r) &= 0, \quad \text{for } r < r_c, \\ &\to 1, \quad \text{as } r \to r_h \end{aligned} \qquad (5.2)$$

where $r_c$ is the hard core radius and $r_h$ is the healing distance.

The value of $r_h$ and the exact shape of $f(r)$ depends upon the nuclear density and the nature of N-N potential at short distances inside the nucleus. Therefore, the information obtained about f(r) is of great importance in basic nuclear structure calculations. The effect of f(r) in momentum space is to introduce momentum components in the single particle wave function which are much higher than those present in the shell model type wave function. Hence, information about f(r) can be obtained either indirectly by analysing the high momentum transfer part of elastic electron and proton scattering or directly by analysing the pion and photo absorption data. However, as we see from eqn.(5.1), the information obtained about f(r) would be model dependent, and hence uncertain if $\Phi(A)$ is not determined properly before hand. We stress this point as many analyses,

barring a few, of above reactions have been done without determining $\phi(A)$ independently and reliably. On the other hand if one uses for $\phi(A)$ the wave function obtained by the analyses of the reactions type presented in sections 2-4 it should be possible to get the physically meaningful estimates of f(r).

The pion absorption analyses, which are reported in literature, normally describe the pion-nucleus Hamiltonian by a linear sum of basic pion-nucleon interactions. In this procedure the formalism becomes very complicated and, for various reasons, the information obtained about f(r) is not reliable. However, if one is satisfied with the estimate of SRC in nuclei in terms of that present in a free two nucleon system, it is possible to get them in a much simpler way by using the "two nucleon" model.

In the "two nucleon" model description of pion absorption the information about f(r) is contained in the parameters $g_0$ and $g_1$. The best fit values of these parameters can be obtained for a nucleus by comparing the computed and the experimental pion absorption strengths. Then the comparison of the so obtained parameters with these obtained from the elementary reactions (4.3) measures the SRC in the nucleus in terms of those present in the free N-N system. As mentioned in section 4, our work on the $(\pi^-, NN)$ reaction so far has shown that the shape of the neutron energy spectrum in this reaction can be reproduced provided we use the nucleon wave function which is obtained from the analysis of the proton knock-out and the elastic scattering data. Unfortunately, the absolute values of transition strength for $(\pi^-, NN)$ reaction

are not available to determine $g_0$ and $g_1$.

Thanks are due to Dr.N.Sarma for careful reading of the manuscript.

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Figure Captions:

Fig.1. Distorted momentum distribution of 1p proton for the (p,2p) reactions on $^{12}C$ at 460 MeV. Exptl. points are due to Tyren et al.[11]

Fig.2. Distorted momentum distribution of 1p proton for the (p,2p) reaction on $^{12}C$ at 1 GeV Exptl. points are due to Simpson et al[23].

Fig.3. Distorted momentum distribution of 1p proton for the (e,e'p) reactions on $^{12}C$ at 600 MeV. Exptl. points are due to Amaldi et al[24].

Fig.4. Distorted momentum distribution of 1s proton for the (p,2p) reaction on $^{12}C$ at 1 GeV. Exptl. points are due to Simpson et al[23].

Fig.5. Distorted momentum distribution of 1s proton for the (e,e'p) reaction on $^{12}C$ at 600 MeV. Exptl. points are due to Amaldi et al[24].

Fig.6. Angular correlation distribution of 1 $d_{3/2}$ proton for the (p,2p) reaction on $^{40}Ca$ at 460 MeV.

Fig.7. One neutron energy spectrum for absorption of pions on $(1p)^2$ nuclcus of $^{12}C$. Exptl. points are due to Cheshire et al[25].

Fig.8. Recoil momentum distribution for the absorption of pions on $^{6}Li$. Curves (------), (_____) and (__.__) correspond respectively to refs. 21, 26 and 27 respectively.

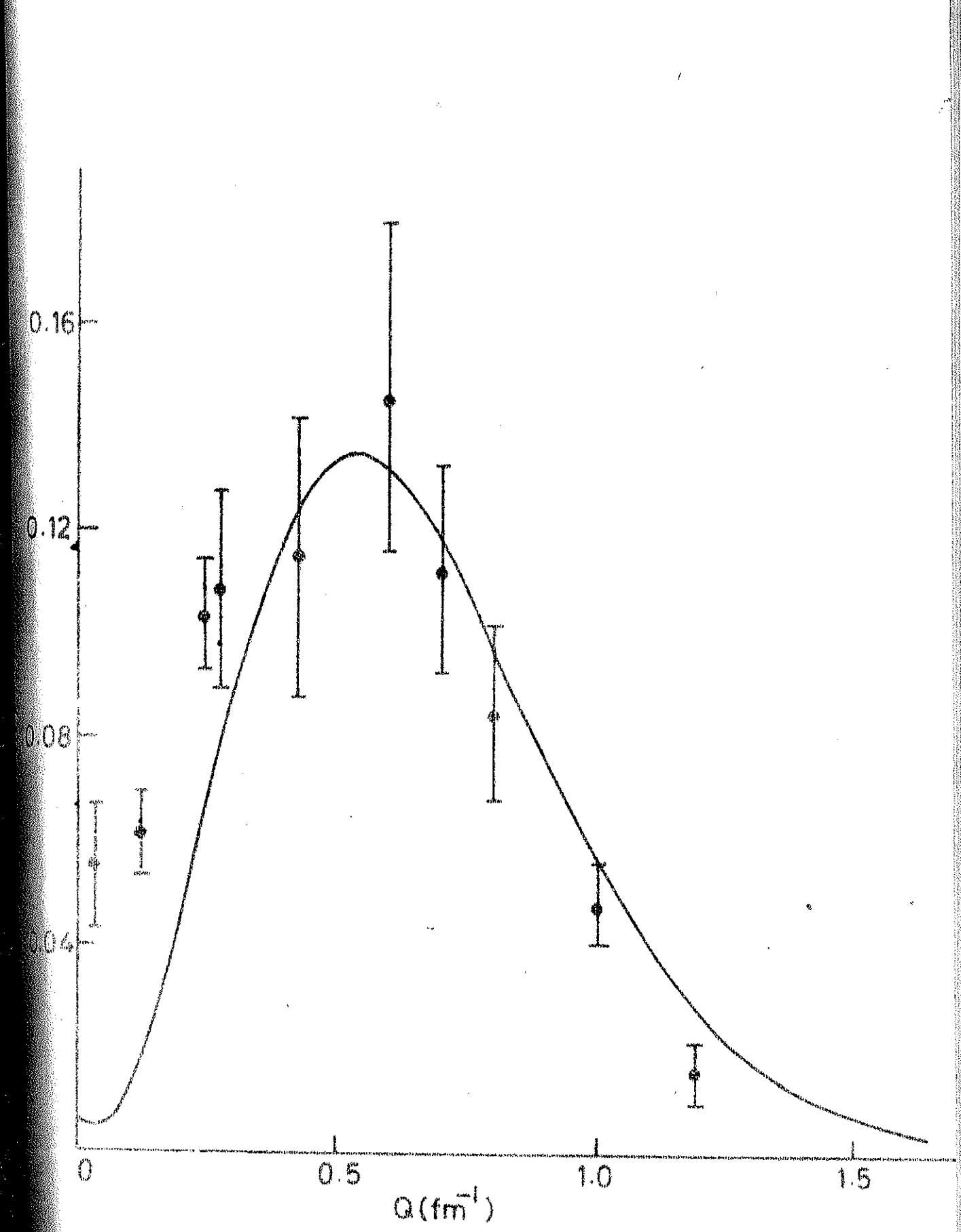


FIGURE . 1

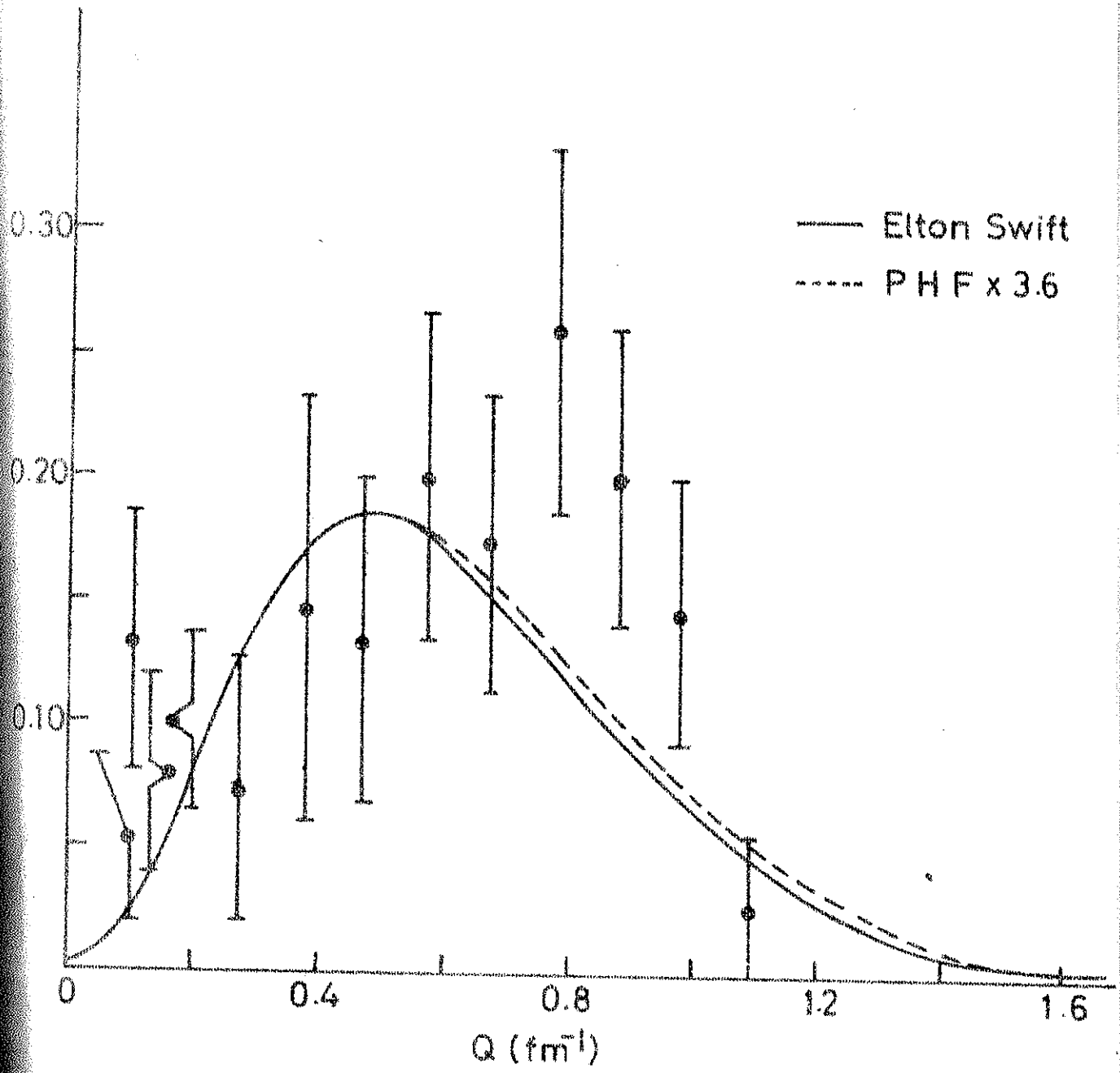


FIGURE . 2

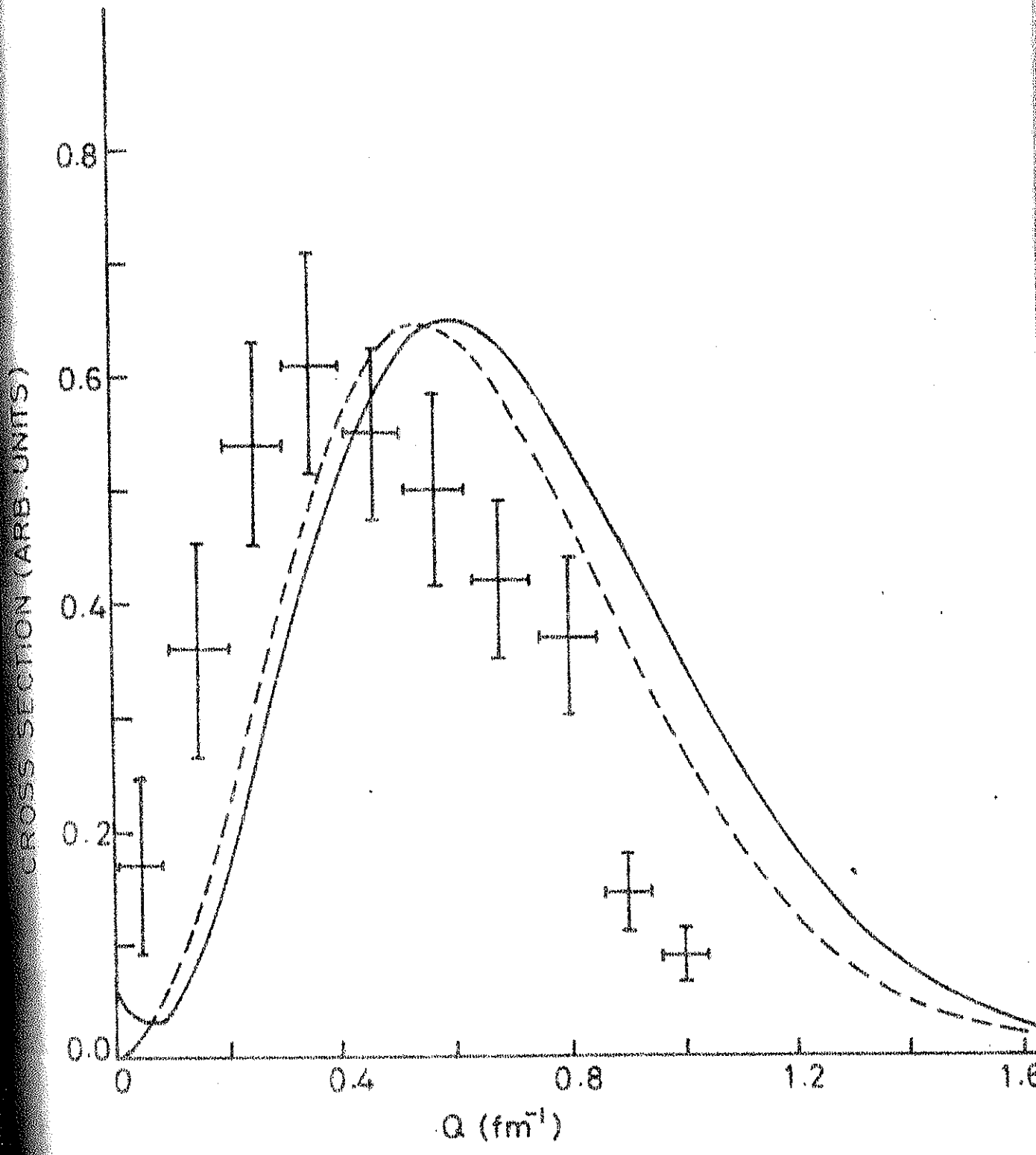


FIGURE . 3

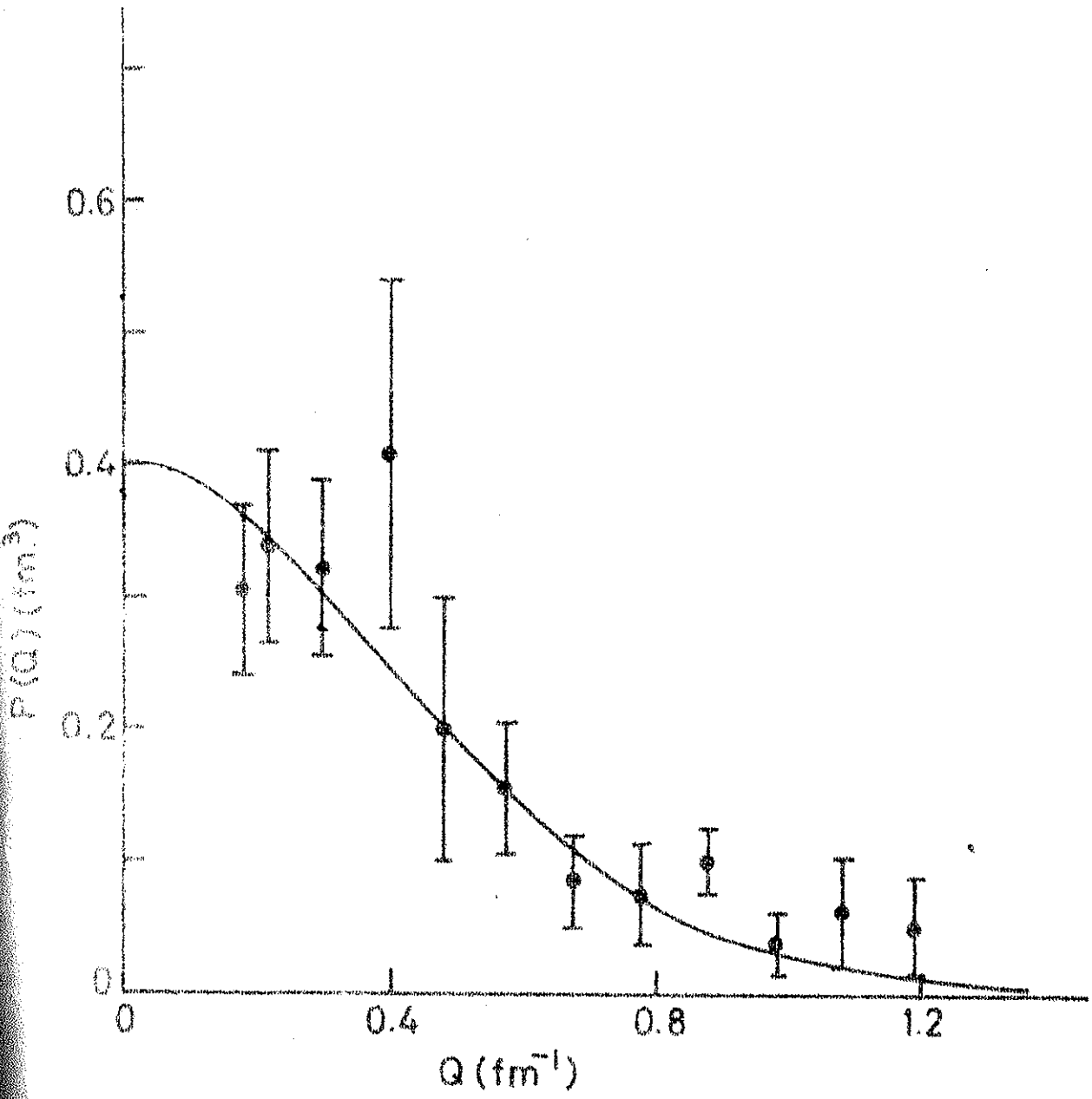


FIGURE . 4

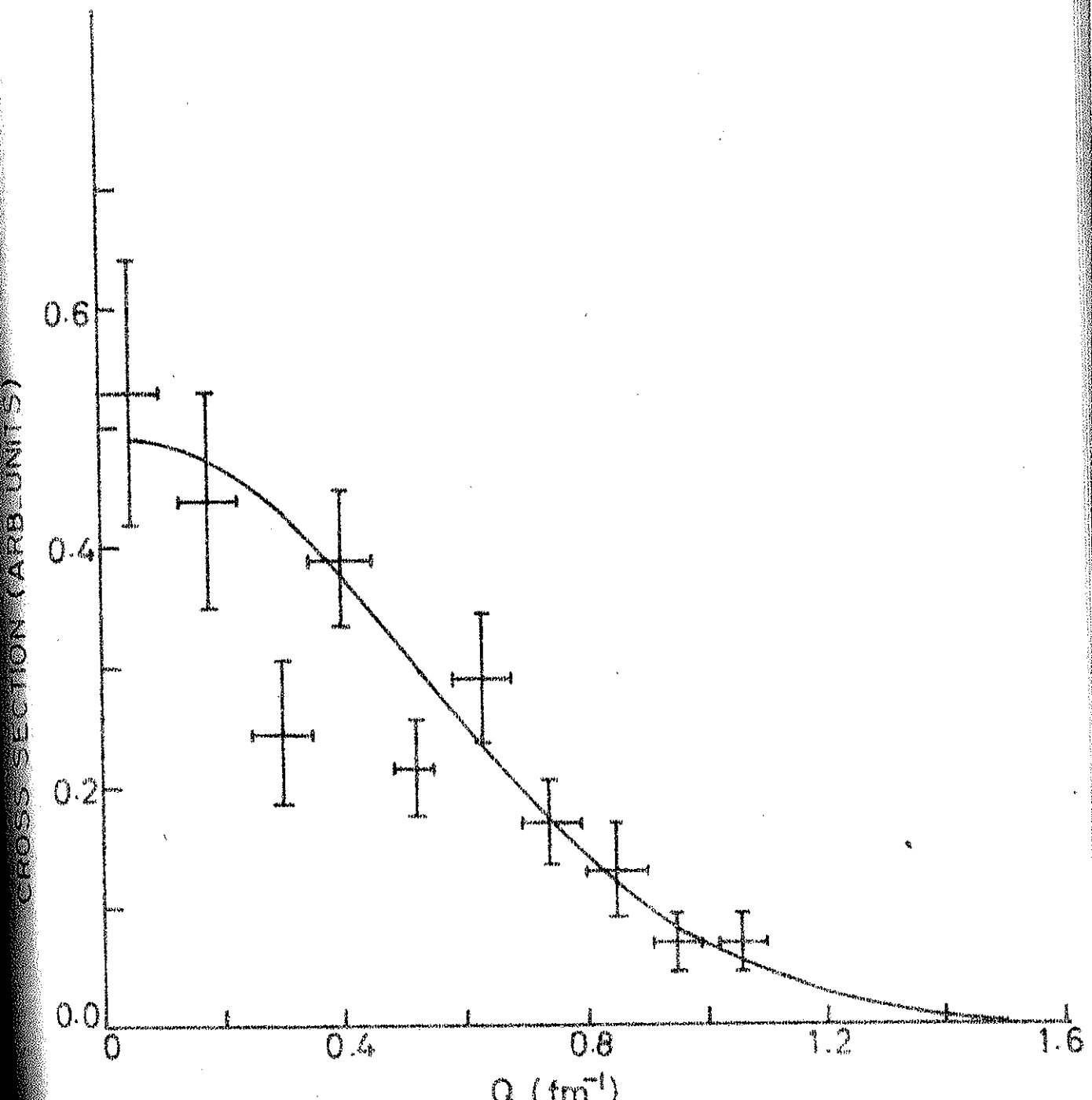


FIGURE . 5

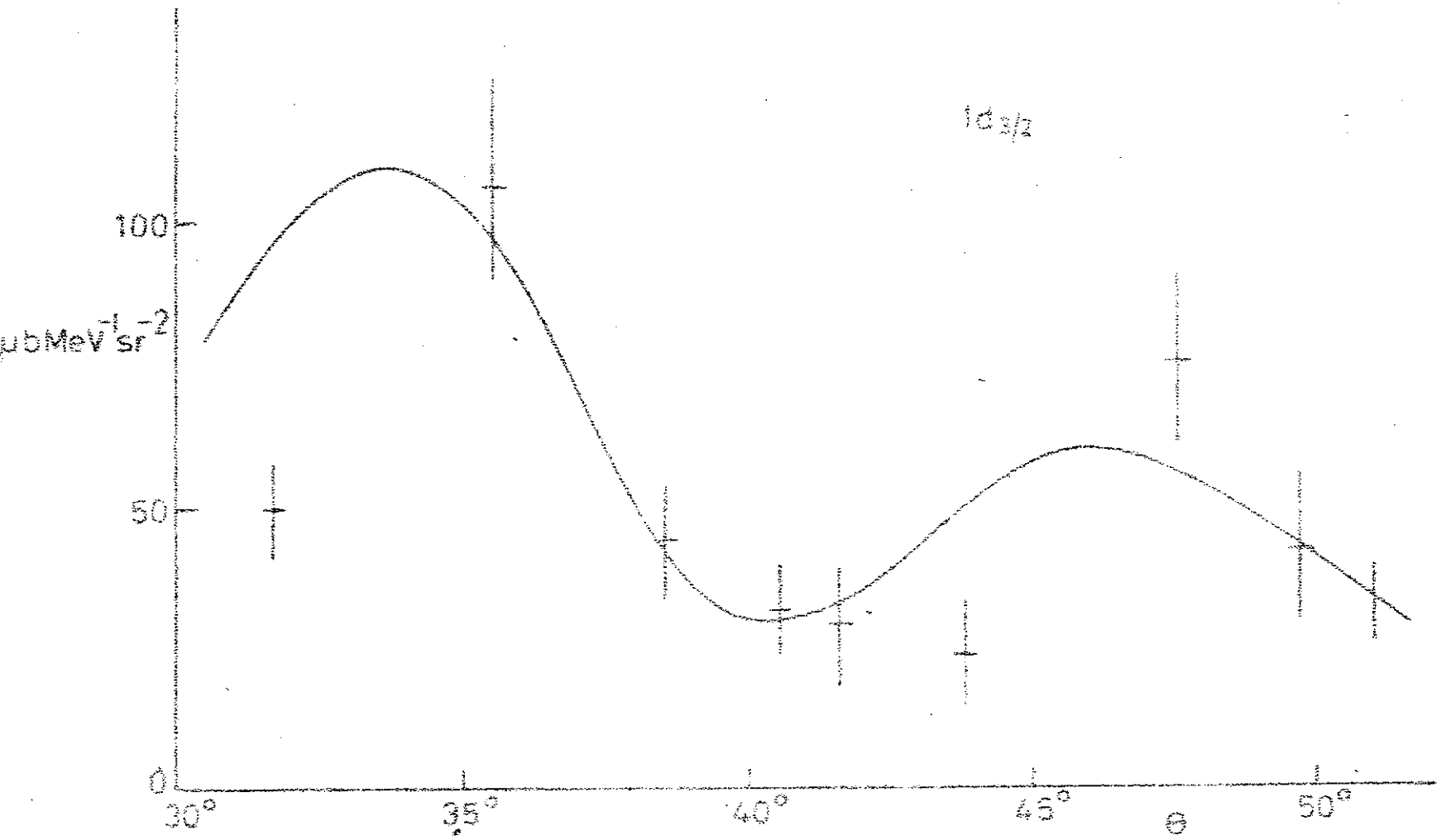


FIGURE . 6

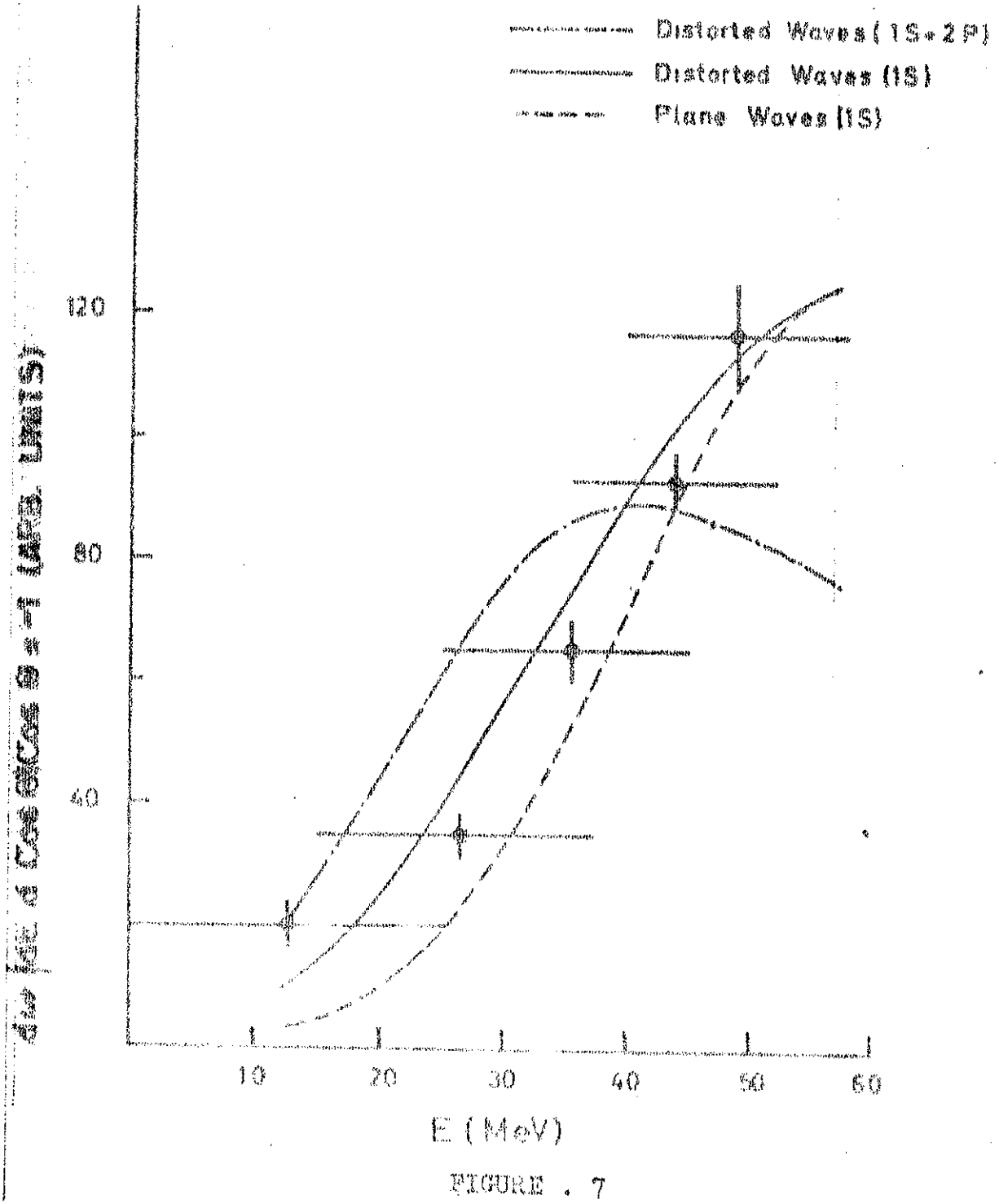


FIGURE . 7

Fig 7

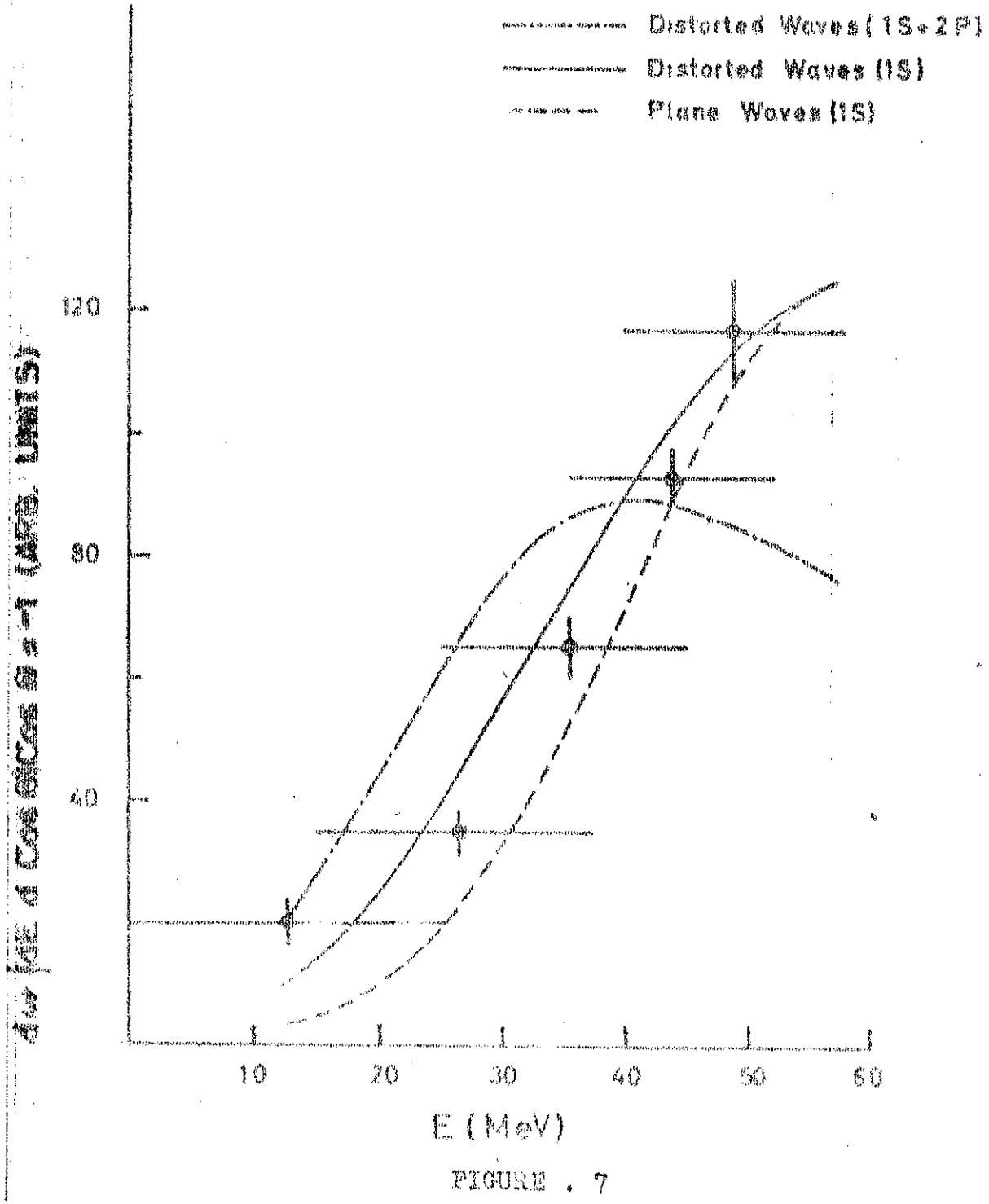


FIGURE . 7

Fig 7

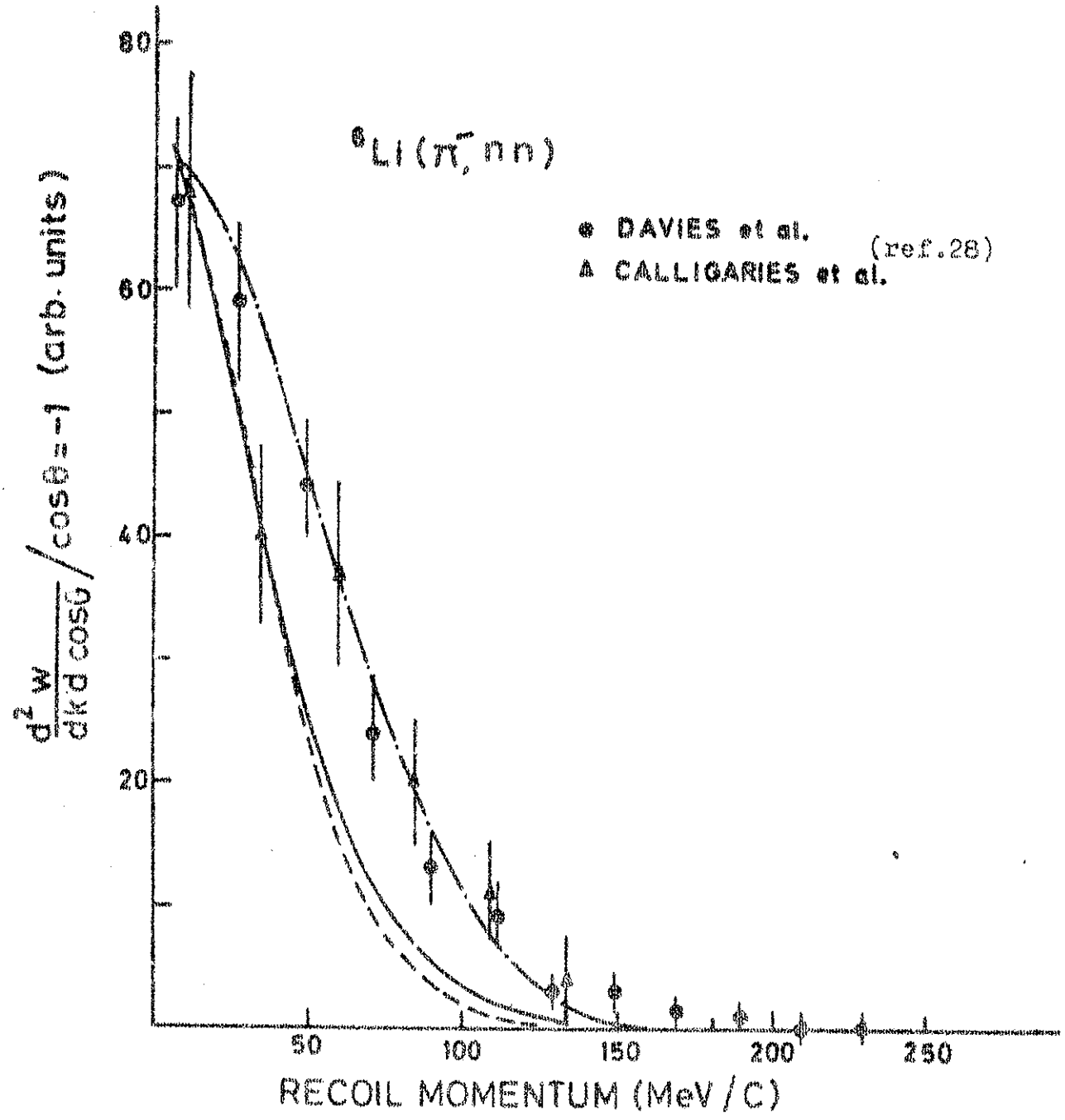
$^6Li(\pi^-, nn)$
● DAVIES et al.
▲ CALLIGARIES et al.
(ref.28)
$\frac{d^2W}{dk\,d\cos\theta}/\cos\theta=-1$ (arb. units)
80
60
40
20
50
100
150
200
250
RECOIL MOMENTUM (MeV/C)

FIGURE . 8

# EFFECTIVE INTERACTION IN NUCLEI AND REALISTIC INTERACTIONS

K.P.Joshi
School of Studies in Physics
Vikram University, Ujjain

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## 1. Introduction

Theoretical studies of the structure of nuclei necessarily need a knowledge of the nucleon-nucleon interaction. One chooses the interaction from either of the following two catagories: (a) realistic interactions and (b) phenomenological effective interactions. Two most commonly used realistic interactions were given in 1962. They are Yale and Hamada-Johnston potentials. Both are static potentials, they conform to the one-pion exchange potential (OPEP) at large inter-nucleon distances and are fitted to the observed nucleon-nucleon scattering and deuteron data. They have infinitely repulsive hard cores and this makes the perturbation calculations starting from single-particle wave functions impossible. One would expect the force among the nucleons in a nucleus to be different from the free two-body force because in the former case the presence of other nucleons shall alter it. The connection between the two was not clear until the calculation of nuclear properties of A=18 nuclei were done by Kuo and Brown[1] in 1965 using Hamada-Johnston potential. Because of the infinite-repulsion at short distances, the calculations are much involved and one has to introduce various approximations at various stages.

Though through these calculations a unified picture of the nuclear forces has emerged; there are many points yet to be clarified and investigated for a quantitative account to be possible. The uncertainties which need be examined more carefully are :

(a) One uses the two-particle ladder series or reaction matrix, G, in place of hard-core potential, V, following Brueckner's prescription[2]:

$$G(E) = V + V \frac{Q_{2p}}{E - H_0} G(E), \qquad (1)$$

where, $Q_{2p}$ is the Pauli's operator inhibiting the occupation of the already filled states. In practice e.g. (1) is solved by replacing G on the R.H.S. by V, i.e. taking terms upto second order only in the expansion. Calculations by Barrett et al[3] shows that the higher orders terms should also be included. Besides the calculation of G matrix for tensor force is possible only with some additional approximations[1].

(b) One is forced to work in a limited configuration space. The nuclear interaction, therefore, should be modified to include the effects of the neglected configurations. The model potential $\vartheta(E)$ is usually taken as

$$\vartheta(E) = G + G \frac{Q'}{E - H_0} \vartheta(E) . \qquad (2)$$

where $Q' = Q - Q_{2p}$. Here also there is a problem of double-counting of certain diagrams[1], the quantitative effect of which on the various nuclear properties are still uncertain.

The Sussex group derived the relative matrix elements of the interaction[4] directly from the observed phase shifts. This does not suffer from the uncertainties referred to in para (a) above. No explicit form for the interaction potential was assumed thus bypassing the usual intermediate step of deriving the potential first. This has the special advantage that the deficiencies in one's treatment of the many-body problem is not hidden by the adjustment of the force parameters to fit some particular properties.

On the other hand, there are numerous calculations wherein either the effective interaction potential, generally central potential of smooth shape, or the two-body matrix elements are adjusted to reproduce the observed nuclear properties. This approach had been quite useful to give us general nature of the effective nuclear force. However, the interaction depends on the configuration space chosen and there is always a danger that having chosen a wrong configuration the good agreement might have been forced by adjusting the various parameters of the interaction[5].

Let us remember at this point that many nuclear properties depend at best only on some averaged behaviour rather than on the detailed nature of the nuclear forces. Thus, for

example, the phase shifts are averages over the entire space. One should, therefore, not be surprised to find that different authors have used different phenomenological effective potentials (sometimes the same author uses different effective potentials) which have widely different natures and all the same they give equally good fits to the observed nuclear properties. In our study of the effective interactions in nuclei we examine the various interactions with a purpose to find out gross features of the nuclear interaction which a 'reasonable' effective interaction should possess. Towards this end, as a first step we derive equivalent potentials of simple form from the realistic interactions. The following realistic interactions which are representatives of different types of interactions, have been simulated:

(i) Yale Interaction,

(ii) Sussex Interaction, and

(iii) Tabakin Interaction.

The assumed simple forms of the equivalents also make it possible to compare these realistic interactions with one another. Next we compare the derived equivalents with the various phenomenological potentials. Of course, the effective interaction in nuclei is expected to be different from the free two-body force. However, the comparison is done with the hope that it shall help us give some gross features of the nuclear interactions.

In reaction 2, we give the assumed forms of the equivalent potentials. Here we also give an equivalent potential for the Sussex interaction. In reaction 3, the equivalents are compared with the various effective central potentials in widespread use. In the end of this section, we also summarize the gross features which a reasonable effective interaction should possess. Lastly, in section 4, we compare the effective interactions with relation to the single-particle energies of $16_O$.

2. Equivalent Potentials

In view of the fact that various authors have found reasonable agreements for the various nuclear properties using central forces alone, we have attempted to simulate the realistic interactions first with a central force alone: These are called set I of the equivalents. The central force is taken as:

$$V_c = ( A_W W + A_B B + A_M M + A_H H ) f(r), \quad (3)$$

where, W, B, M and H are Wigner, Bartlett, Majorana and Heisenberg operators respectively and the A's give their respective strengths. However, for the coupled channels, such as, $^3S - {}^3D$ state, a simple Wigner type force is used

$$V^{0D} = V_0 P^{0D} S_{12} g(r), \quad (4)$$

where, the operator $P^{OD}$ projects out only that part of the tensor operator, $S_{12}$, which operates in the off-diagonal states alone with $\ell' = \ell \pm 2$ . The radial dependences f(r) and g(r) are assumed to be Gaussian.

The parameters of the interaction, Eqs.(3) and (4) are determined by best fitting the relative matrix elements of the corresponding realistic interactions. Such a simple interaction is found to give level spectra reasonably similar to that obtained from the corresponding realistic interactions. However, there is, in general, a marked descrepancy for some levels, e.g., ($J = 0^+$, T=1), ($2^+$,1), and ($1^+$, 0). They are found to be more strongly bound for the equivalents. The reason is that the relative triplet states - $^3P$ and $^3D$ can be approximated by a central force only poorly. To rectify this, we next assume the equivalent potentials to be of the form central plus tensor forces. This called set II of equivalents central force is again of the form of eqs.(3) and the tensor force is

$$V_T = V_{0T} S_{12} g(r) . \qquad (5)$$

The parameters of the equivalent potentials are again determined by best fitting the relative matrix elements of the corresponding realistic interactions. Now the simulation is improved much. We have found that in the cases of all the three realistic interactions equivalent potentials of the form, central

plus tensor forces are reasonably good approximations.

We present below the results of the Sussex equivalent potential as a typical case. The other details are given elsewhere[6].

The parameters of the derived equivalent potential are given in table I. The low lying energy levels of $^{18}F$ obtained with the equivalent potential are compared with those from the Sussex interaction in fig.1. They have rms deviation of 314 KeV. The large deviations observed for the levels $J = 0^+_1$, T=1 and $(2^+_1, 0)$ result because of the assumed central forces in $^3D$ and $^3P$ states respectively. The subscript '1' denotes that it is the lowest level with these quantum numbers. Parallel effects were observed in a similar analysis for the simulation of Yale interaction made earlier[6]. If these levels are excluded the rms deviation becomes 221 KeV only.

The interaction ranges in different states (table I) have been kept the same, equal to 1.87 fm but for $^1P_1$ state where a shorter range of 1.35 fm has been found to be necessary. Also the potential is repulsive in this state consistant with the accepted view[7]. The potential in $^3P$ state is very weak but attractive. For Yale equivalent potential[6] also we have found it to be a weaker force compared to that in other states. This is clear from table II where the various potentials in common use are listed.

Note that the derived potential in $^1S_0$ state is different from that in $^1D_2$ state and so is the case for triplet-even states, $^3S$ and $^3D$. This shows that the potential is not purely central. This state dependence may result from the velocity dependence or non-locality of the interaction.

Finally, as is the common belief[7], the tensor force in triplet-odd state is weaker and repulsive as compared to the stronger attractive triplet-even state force. Also, it has longer range than the central potential.

3. Effective interaction and realistic interactions

Next we compare the derived Sussex and Yale equivalent potentials with different central potentials in widespread use. The parameters of the various interactions are given in table II. The choice of the interactions is such that the effective potentials derived by different considerations are represented. Thus, besides the most commonly used potentials, such as those of Elliott and Flowers[8], Inoue et al[9] and Ferrel and Visscher[10] were obtained for sd shell region and that of True[11] for A=14. Schmittroth's potentials[12] are derived by a least squares fit with the two-body matrix elements obtained by Talmi fit made by Arima et al[13] for sd shell region and by Cohen and Kurath[14] for p shell nuclei. The potentials of Gillet et al[15] were originally used for $^{12}C$ and $^{16}O$ but have been found satisfactory for $^{40}Ca$ and $^{208}Pb$ also. The interaction of Clark and Elliott[16] has been obtained by best fitting the observed spectra for wide range of nuclei.

Seemingly table II represents a haphazard collection of parameters. Still the potentials can give equally good fit to the observed properties because they enter into the calculations in particular combinations. Of course, one expects that the interaction in nuclei shall be different from the free two-body force. However, we hope that a comparison of the different effective potentials may still prove useful in giving information about the gross nature of the effective force, which a 'reasonable' potential should possess.

In all cases the triplet-even force is attractive and is generally stronger than that in the singlet-even state. In these states the equivalent Sussex and Yale potentials are similar. However, in Singlet-odd and in triplet-odd states the potentials are much diverse. Thus, the 'CAL' and 'COP' interactions of Gillet et al[15] have opposite signs in singlet-odd state and so is the case with Schmittroth's interactions[12] in p and sd shell nuclei. In analysing the structure of the TBME's, Schmittroth finds that $(2^+,0)$ level is very sensitive to this interaction e.g., if the $(2^+,0)$ matrix element is to be changed from -3.7 to -1.25, the potential strength, $V_{so}$ has got to be varied from -91 MeV to +10 MeV. Inoue et al[9] use for $V_{so}$ values ranging from 0 to 17 MeV. However, it is repulsive in both the equivalent interactions for Yale and Sussex.

In the triplet-odd state the various forces have much more variation. However, one observes that in general the force

in this state is weak. For Sussex and Yale equivalents it is seen to be attractive, though much weaker in the former case.

In terms of exchange mixtures, Wigner force component is in all cases positive but for Rosenfeld mixture and equivalent Sussex potential. However, they are weak. Rosenfeld mixture was derived to explain the singlet-triplet splittings for deuteron without the use of the tensor force and also gives saturation to nuclear binding energy. It is near to the free two-body force. However, Rosenfeld force has been found unsuitable by Abulaffio[17] to explain the level order in $^{22}Ne$, $^{24}Mg$ and $^{26}Mg$. They assume the force of the form

$$V = (W + M)\, f(r) \qquad (3)$$

and find that $M \geq .8$ (with normalization, $W + M = 1$). It has been pointed out by Parikh and Bhatt[18] that Kuo and Brown's renormalised force[1] including core-excitation, has essentially a Majorana character. Schmittroth's force for the sd shell is, on the contrary, has very small Majorana force content. This is surprising as it has been derived from the Talmi fit matrix elements and as such should include the core renormalization effects.

From the inelastic scattering experiments in the sd shell nuclei, Anderson et al[19] have found that M and H components are both attractive and that M is about four times stronger than H. From the table II it is observed that for the

various forces generally M > H though not as much as observed by Anderson et al[19]. The other components, B and H have much diverse values.

As pointed out earlier, particular combinations e.g. $\frac{B - M}{B + M}$, rather than the individual components are important for most of the nuclear properties. We also tabulate this parameter for various interactions in table II.

To summarize, we have derived an equivalent potential of simple form to the Sussex interaction. This interaction is compared with the equivalent potential derived from Yale interaction by a similar method and with other central effective interactions in common use. The interaction in even states is attractive and strong. In odd states it is weaker. In singlet-odd state the interactions have varying signs but it is repulsive for both Sussex and Yale equivalents. In triplet-odd state both Yale and Sussex have still weaker attractive force. In terms of the exchange components, the Majorana force is predominant consistant with the findings from the scattering experiments[12,19] and with Kuo and Brown's force[1].

## 4. Effective interactions and $^{16}O$

In this section, we compare the various effective interactions with relation to the single-particle energies and binding energy of $^{16}O$. The phenomenological interactions used are the ones which have been employed successfully to calculate the various nuclear properties, particularly energy levels in

sd shell. One can get some idea about the 'goodness' of these interactions by calculating properties other than energy levels using them. As our purpose is to compare the various interactions we calculate the single-particle energies and binding energy by taking the single-particle wave functions to be those of a harmonic oscillator.

Results and Discussion

As our aim is to compare the various interactions, we calculate the single-particle energies and the binding energies of $^{16}O$ in harmonic oscillator approximation. In table III we present the results for the various interactions. For comparison, the experimental values and for the single-particle energies the Hartree-Fock values have been included for some cases.

A look at the table shows that in the case of some interactions even the 0S state is unbound. Of the phenomenological interactions used no interaction gives satisfactory values for single-particle energies and binding energies. Rosenfeld and the Inoue et al[9] interactions give the best values for the separation of the berycentres of 0p and 0s and between 0d and 0p levels. However, they fail to bind $^{16}O$! This is also the case with surface - delta and with Elliott and Flowers[3] interactions.

The other effective interactions examined are the equivalents of Yale and Sussex. These interactions are derived by best fitting the relative matrix-elements of the respective realistic interactions. The equivalents I comprise of a central force plus a Wigner type force operating only in states off-diagonal

in relative angular momentum, for example, $^3S-^3D$ state and equivalents II have the form of central plus tensor forces. The results for these interactions clearly show the advantage over other phenomenological potentials for use in nuclear structure calculations. This is particularly so for the equivalent potentials II.

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TABLE I

Parameters of the equivalent Sussex interaction

| Relative state | T S | Potential Strength (MeV) | Potential Range (Gaussian) (fm) |
|---|---|---|---|
| $^1S_0$ | 10 | -26.45 | 1.87 |
| $^1D_2$ | 10 | -21.96 | 1.87 |
| $^3S_1$ | 01 | -33.47 | 1.87 |
| $^3D$(central) | 01 | -27.52 | 1.87 |
| $^1P_1$ | 00 | 74.43 | 1.35 |
| $^3P$(central) | 11 | - 1.49 | 1.87 |
| $^3S_1 - {}^3D_1$ | 01 | -101.92[a] | 2.18 |
| $^3P_2 - {}^3F_2$ | 11 | 20.41[a] | 2.18 |

a. This force operates only in the states off-diagonal in orbital angular momentum.

TABLE II

Comparison of the parameters of the different normalized nucleon-nucleon central potentials in widespread current use. The normalization used is $^{31}V = -40$MeV and W+M+B+H=1

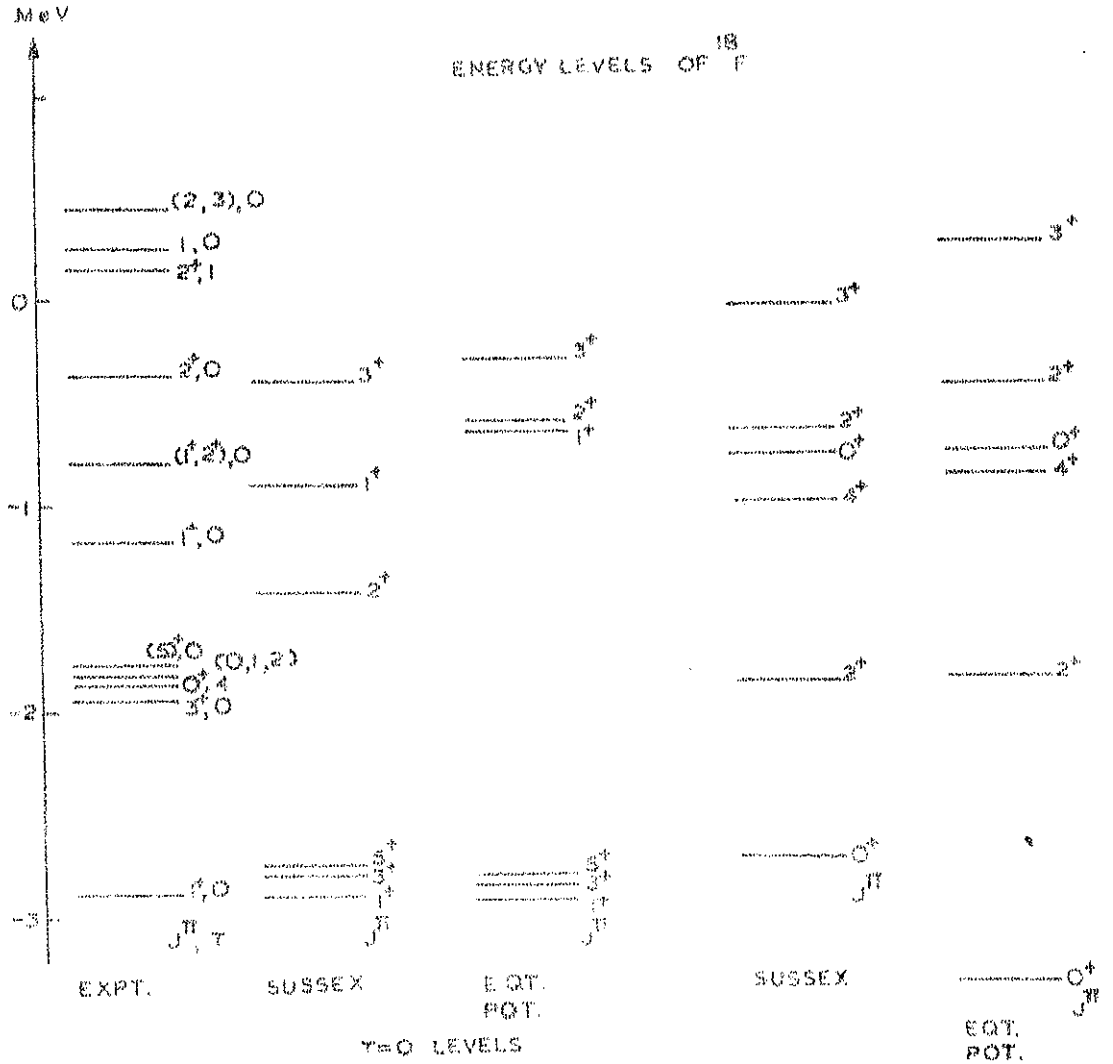
| Interactions | $^{11}V$ (MeV) | $^{13}V$ (MeV) | $^{33}V$ (MeV) | W | M | B | H | $\frac{B-M}{B+M}$ |
|---|---|---|---|---|---|---|---|---|
| Serber | 0 | -40 | 0 | 0.5 | 0.5 | 0 | 0 | -1 |
| Rosenfeld | 72 | -24 | 14 | -0.13 | 0.93 | 0.46 | -0.26 | -0.58 |
| Soper | 16 | -20 | - 4 | 0.30 | 0.43 | 0.27 | 0 | -1 |
| Elliott and Flowers[a] | -28.57 | -57.14 | 14.86 | 0.485 | 0.365 | -0.125 | 0.265 | -1.92 |
| Inoue et al[b] | 0 | -60.67 | 46.67 | 0.32 | 0.57 | -0.07 | 0.18 | -1.43 |
| Ferrel and Visscher[c] | -3 | -25 | 13 | 0.55 | -0.75 | 0.31 | 0.01 | -2.41 |
| True[d] | 0 | 52.00 | 0 | 0.44 | 0.44 | 0.06 | 0.06 | -0.77 |
| Schmittroth[e] | | | | | | | | |
| (i) For p shell[f] | 18.77 | -46.67 | 17.11 | 0.27 | 0.66 | 0.04 | 0.03 | -0.89 |
| (ii) For sd shell[g] | -151.97 | -68.82 | 21.89 | 0.87 | -0.08 | -0.53 | 0.74 | 0.74 |
| Gillet et al[h] | | | | | | | | |
| (i) 'CAL' | 26 | -20 | 6 | 0.35 | 1.15 | 0.00 | -0.50 | -1 |
| (ii) 'COP' | -24 | -24 | 25 | 0.66 | 0.68 | -0.68 | 0.34 | 00 |
| Clark and Elliott[j] | 52 | -40 | 8 | 0.125 | 0.875 | 0.275 | -0.275 | -0.52 |
| Hale Equivalent[k] | 50.46 | -52.44 | -19.73 | 0.29 | 0.59 | 0.30 | -0.27 | -0.2 |
| Present | 112.56[l] | -50.62 | - 2.25 | 0.1 | 0.99 | 0.62 | -0.51 | -0.23 |

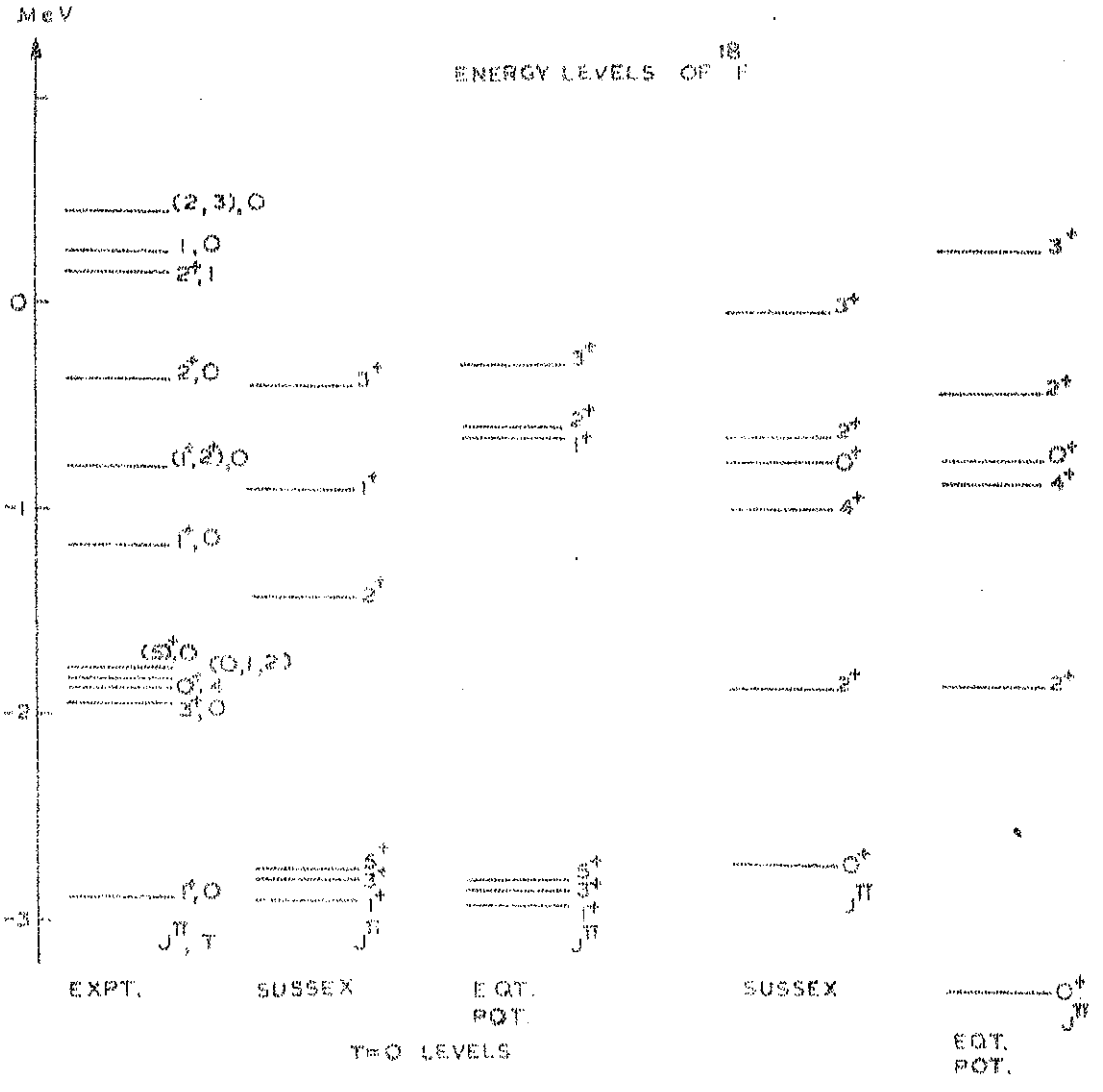
a. Ref. 8
b. Ref. 9
c. Ref. 10
d. Ref. 11
e. Ref. 12
f. Ref. 13
g. Ref. 14
h. Ref. 15
j. Ref. 16
k. Ref. 6
l. Range is 1.35 fm whereas in other states it is 1.87 fm.

Table III

Comparison of the single-particle energies and ground state energy, $E_o$ of $^{16}O$ for various interactions. The energies (in MeV) are calculated in the harmonic oscillator approximation.

| nteraction | $0s_{1/2}$ | $0p_{3/2}$ | $0p_{1/2}$ | $0d_{5/2}$ | $1s_{1/2}$ | $0d_{3/2}$ | $E_o$ |
|---|---|---|---|---|---|---|---|
| xperiment | -51.0 | -21.81 | -15.65 | -5.02 | -4.15 | 0.06 | -142 |
| ale(HF) | -44.85 | -18.95 | -13.78 | 0.67 | -1.59 | 4.70 | |
| ale | -39.26 | -16.82 | -12.54 | 3.04 | 5.38 | 8.73 | -46.75 |
| ale equivalent | | | | | | | |
| I | -51.74 | -26.22 | -26.22 | -5.85 | -10.05 | -5.85 | -146.84 |
| II | -40.76 | -17.67 | -15.10 | 2.06 | 3.37 | 5.80 | -61.974 |
| ussex(HF) | -51.08 | -23.41 | -17.13 | -1.87 | -0.68 | 3.40 | |
| ussex | -39.36 | -18.45 | -14.87 | -0.66 | 0.01 | 4.13 | -56.973 |
| ussex equivalent | | | | | | | |
| I | -36.52 | -15.37 | -15.37 | 3.02 | 0.99 | 3.02 | -46.013 |
| II | -40.38 | -19.38 | -16.41 | -0.38 | -0.21 | 2.87 | -65.168 |
| urface delta | 0.64 | 7.41 | 7.41 | 15.39 | 14.06 | 15.39 | 161.24 |
| lliott and lowers | 1.09 | 5.08 | 5.08 | 15.90 | 19.70 | 15.90 | 152.594 |
| osenfeld | -13.32 | -6.38 | -6.38 | 0.94 | 19.69 | 0.94 | 30.136 |
| noue-Arima | -23.05 | -0.87 | -0.87 | 15.62 | 18.51 | 15.62 | 74.806 |


MeV
ENERGY LEVELS OF $^{18}$F
0
-1
-2
-3
(2,3),0
1,0
2⁺,1
2⁺,0
(1⁺,2⁾,0
1⁺,0
(5)⁺,0
(0),1,2)
0⁺,4
3⁺,0
4⁺,1
J^π, T
EXPT.
3⁺
1⁺
2⁺
SUSSEX
EQT. POT.
T=0 LEVELS
SUSSEX
EQT. POT.
T=1 LEVELS


MeV
ENERGY LEVELS OF $^{19}$F
0
-1
-2
-3
(2,3),0
1,0
2⁺,1
2⁺,0
(1⁺,2⁺),0
1⁺,0
(5/2⁺),0
(0,1,2)
0⁺,4
3⁺,0
1⁺,0
J^π, T
EXPT.
3⁺
1⁺
2⁻
5/2⁺
3/2⁺
1/2⁺
J^π
SUSSEX
3⁺
2⁺
1⁺
3/2⁺
5/2⁺
1/2⁺
J^π
EQT. POT.
T=0 LEVELS
3⁺
3⁺
2⁺
0⁺
4⁺
2⁺
0⁺
J^π
SUSSEX
3⁺
2⁺
0⁺
4⁺
2⁺
0⁺
J^π
EQT. POT.
T=1 LEVELS

# ON GENERALIZED CLIFFORD GROUPS*

N.R.Ranganathan and R.Jagannathan
MATSCIENCE, The Institute of Mathematical Sciences
Madras-600020

Since the development of L-Matrix Theory by Alladi Ramakrishnan there has been an extensive investigation of the various mathematical aspects and physical applications of Clifford Algebra and its generalizations[1] by the group at Matscience. Recently, we have applied the techniques of generalized clifford algebra (G.C.A.) to the study of Magnetic Translation group (M.T.G) which is the symmetry group of the Hamiltonian of a Bloch electron in a magnetic field.[2,3,4] In [2] we have shown that using the basis elements of G.C.A. denoted by $C_n^N$, we can construct a group, denoted by $G_n^N$, called the generalized clifford group (G.C.G) and we have also studied their irreducible representations. In [3], we have established the relation between the G.C.G and M.T.G and have studied the irreducible representations in [2]. The Hamiltonian of the problem being non-invariant under the ordinary translation group, Brown and Zak[4] introduced the magnetic translation group whose elements obey a commutation relation of the type $AB = \omega BA$, $\omega$ being a general root of unity depending on the magnetic field. Since this commutation relation is the same as that between the elements of G.C.G, it makes it possible to directly employ the techniques of G.C.G to the representations of MTG. In view of the recent

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* Presented by N.R.Ranganathan

application of Clifford algebra to the problem of nuclear energy levels by Boon[5], we are at present investigating the possible physical applications of G.C.A. and G.C.G.

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# A GENERAL METHOD TO DETERMINE PARTICLE DENSITY IN A SELF-CONSISTANT FASHION FOR A MANY-BODY SYSTEM

V. Radhakrishnan
MATSCIENCE, The Institute of Mathematical Sciences
Madras-20, INDIA.

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We propose here a general technique in a formal way to determine, the particle density or local potential in a Many-Body System. I have shown before[1] how the many-body problem of crystallisation in statistical mechanics as well as other phase transition can be ultimately types of/reduced to the solution of a non-linear integral equation of Hammerstein type. Here a very close problem is examined. Before studying the problem of crystallisation one should have at hand, a method of obtaining an equation to determine in a self-consistent way the particle density of the given many-body system. This is indicated in a general way in this lecture. We assume that the particles are confined to a volume under the action of a self-consistent potential $V(\vec{r})$. In a very general way we can take that the particle density $\rho(\vec{r})$ and $V(\vec{r})$ are connected in a linear fashion. That is

$$V(\vec{r}) = \int K(\vec{r}, \vec{r}')\, \rho(\vec{r}')\, d\vec{r}' \qquad (1)$$

where the kernel K is to be explicity calculated according to the model we adopt for the system.

At a given point $\vec{r}$ in the system, the force acting on the particle due to the potential $V(\vec{r})$ can be taken to be

$$\lambda \rho(\vec{r}) \vec{\nabla} V(\vec{r})$$

where $\lambda$ is some constant. This force is balanced by the pressure gradient acting on the particle density at $\vec{r}$, namely $\mu \vec{\nabla} \rho(\vec{r})$, where $\mu$ is a constant. Balancing we get

$$\frac{\vec{\nabla}\rho(\vec{r})}{\rho(\vec{r})} = \lambda' \vec{\nabla} V(\vec{r}) \quad , \quad \lambda' = \frac{\lambda}{\mu} . \qquad (2)$$

Integrating (2) we get

$$\log \rho(\vec{r}) = \lambda' V(\vec{r}) + \text{const.} \qquad (3)$$

or

$$\rho(\vec{r}) = e^{\Lambda V(\vec{r})} \quad , \text{ where } \Lambda \text{ is constant} \qquad (4)$$

We substitute this in eq. (1), eliminate $V(\vec{r})$ and obtain

$$\log \rho(\vec{r}) = \Lambda \int K(\vec{r}, \vec{r}') \rho(\vec{r}') \, d\vec{r}'. \qquad (5)$$

If we had elimiated $\rho(\vec{r})$, then we would obtain

$$V(\vec{r}) = \int K(\vec{r}, \vec{r}') e^{\Lambda V(\vec{r}')} d\vec{r}'. \qquad (6)$$

Equation (5) determines the density $\rho(\vec{r})$ and eq. (6) determines the potential $V(\vec{r})$ both self-consistently. This is the solution of the problem posed in the begining. Equations (5) as well as (6) are non-linear integral equations of Hammerstein type.

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# PROSPECTS OF FUSION POWER

N.Ananthakrishnan,
Seethalakshmi Ramaswami College,
TIRUCHY-18

The true nature of energy and power still lies at the very core of the profound mystery of our existence.

In this didactic poem on "the Marriage of Heaven and Hell", William Blake states:

> " Man has no Body distinct from his Soul; for that called Body is a portion of the Soul discerned by the five senses, the chief inlets of the Soul in this age. Energy is the only life and is from the Body, and Reason is the bound or outward circumférence of energy. Energy is eternal delight".

I am sure, you will all agree, that Blake's definition of energy is certainly more satisfying than the definitions given in Text-books on Physics. Even within the frame-work of physical science, energy has a transcendental quality. No matter what language the physicists may use, energy will remain the lord and the giver of life, a reality truly transcending our mathematical descriptions. Its nature lies at the very heart of the fundamental mystery of our existence as animate beings in an inanimate universe.

Recent advances in the performance of several experimental plasma containers have brought the proppect of fusion power

almost close to the final "break even" level of scientific possibility. The achievement of a practical fusion reactor is sure to produce a profound impact on almost every aspect of human society. The most revealing indication of such a possibility is given by the fact that the main emphasis in recent meetings and discussions involving workers in the field has been on a consideration of the practical, technological, economic, and social aspects of power generation by controlled fusion reactors.

Let us briefly examine the powerful energy options of the future. To begin with, there are the known energy sources:- the fossil fuels, and nuclear fuels $U^{235}$ and $U^{238}$. It is generally agreed that these sources are being used up at a very rapid rate - a rate that may be even accelerated by increases in population and living standards. In addition, environmental considerations such as atmospheric pollusion by burnt gases and nuclear radiations could further restrict the use of these energy sources. Certain other sources of energy such as water-power, tidal power, geo-thermal power, and wind power are calculated to be insufficient to meet the needs of the future. Direct solar radiation resulting from fusion reactions securing in the core of the sun is an abundant source of energy; but the practical difficulty is to find out an economic way of concentrating the available low energy density of solar radiation.

Controlled fusion is potentially an infinite source of energy. Its energy out-put results from the reduction in the total mass of a nuclear system that accompanies the fusion of two light nuclei. Nuclear fusion may be considered as the reverse of nuclear fission. It can lead to the release of energy in such cases where the total mass of the product nuclei is less than the total mass of the reactants. This is always the case for light nuclei such as the proton, deuteron, triton. In 1939, Hans Bethe (Nobel Laureate of 1970) developed a theory of evolution of stellar energy in which protons by suitable nuclear reactions are fused into a helium nucleus, thereby releasing energy which is transformed into radiation. One series of reactions is the well-known proton-proton chain reaction in which the energy released during the production of each He-nucleus is 26.7 MeV. However it is important to realise that, in order that two protons may interact, they must have enough kinetic energy to overcome the electrostatic repulsion which tends to keep them apart. Simple calculation shows that for the lightest nuclei (the isotopes of hydrogen) the energy needed for the nuclear reaction to take place is 0.1 MeV. Experiments with particle accelerators have shown that energies of this order are essential to cause interaction among nuclei of very low atomic number, while larger amounts of energy are needed for nuclei of higher atomic number. Now the energy acquired by

a charged particle in an accelerator is kinetic in nature and the same energy could be acquired by sufficient increase of temperature. But the temperature will have to be as high as 1000 million degrees kelvin, if the mean energy of the particles is to be 0.1 MeV. In other words 0.1 MeV is equivalent to temperature of 1000 million degrees kelvin !

The uncontrolled release of a large amount of fusion energy was achieved in 1952 with the first thermo-nuclear test explosion. This test showed that fusion energy could be relaased on a large scale by raising the temperature of a high density plasma to about 50 million degrees. This is the story of the hydrogen bomb - uncontrolled release of energy by thermo-nuclear fusion.

The search for a controlled release of fusion energy was begun almost coincident with the successful development of the hydrogen bomb or Hell-bomb in the United States of America, in Britain, in Europe, and in the USSR. This search essentially is to devise a practical way of maintaining a relatively low density plasma at a temperature high enough so that the out-put of fusion energy derived from the plasma is greater than the input of some other kind of energy applied to the plasma. Since no solid material can exist at the very high temperature range required for a useful energy out-put, the principal emphasis from the beginning has been on the use of magnetic fields to confine or "contain" the plasma. When it is realised that average

magnetic fields of 75 kilogauss and peak fields of upto 150 kilogauss may be required for which super-conducting windings at liquid helium temperature may be needed„ the extreme complexity of the variety of "magnetic bottles" designed for the successful plasma containment can very well be imagined. I can only mention some of the linear and toroidal types of plasma containing machines developed at various plasma physics research centres in the world - the Stellarator, the Tokomak, the Theta-Pinch types of machines. Experimental plasmas which range in temperature from 50 million degrees to 1000 million degrees C have been realised particle or ion-densities from $10^9$ to $10^{22}$ ions per C.C. have been realised also. It may be mentioned that a minimum ion-density of $\sim 10^{13}$ ions per C.C. is needed for fusion to occur at a useful rate.

Typical fusion reactions are the following:-

$$d + d \rightarrow {}^3He + n + 3.2\ MeV$$

$$d + d \rightarrow t + p + 4.0\ MeV$$

$$d + t \rightarrow {}^4He + n + 17.6\ MeV$$

$${}^6Li + p \rightarrow {}^3He + {}^4He + 4.0\ MeV$$

$${}^6Li + {}^3He \rightarrow {}^4He + {}^4He + p + 16.9\ MeV$$

$${}^6Li + d \rightarrow {}^4He + {}^4He + 22.4\ MeV$$

$${}^7Li + p \rightarrow {}^4He + {}^4He + 17.5\ MeV$$

The most likely fuel for a fusion power energy source appears to be deuterium, an abundant heavy isotope of hydrogen, easily separated from sea-water. For a d-t fuel mixture, Lawson found that, at temperatures higher than the ignition temperature, the product of ion-density and confinement time must be equal to $10^{14}$ see per C.C. in order to achieve "the break-even" condition. This criterion defines a surface in 3d-space, the coordinates of which are the logarithmic values of density, temperature, and confinement time. A successful release of energy will be achieved when the set of conditions for a given machine reaches this surface. The exact location and shape of the surface is dependent both on the fuel cycle used and the recovery efficienty of the reactor system. The d-t fuel needs the minimum temperature for break-even power balance, while fuels other than d-t mixture would need higher temperatures. The d-t fuel reactor releases 80% of its fusion energy in the form of highly energetic neutrons. The reactor can generate electric power by absorbing the neutron energy in a liquid-lithium shield, circulating the liquid lithium to a heat-exchanger and thereby heating water to produce steam and thus drive a conventional steam generator electric power plant. The reactor core may be toroidal or linear.

This general approach can also lead to a promising new technique for converting the world's reserves of $U^{238}$ and Thorium 232 to suitable fuels for fission reactors. By using the abundance of the inexpensive energetic neutrons produced from the d-t cycle to synthesise heavy nuclei, cheap fuel might be made for existing fission reactors with a high degree of efficiency.

The fundamental requirements of a controlled and meaningful release of fusion energy in a reactor can be summarised briefly as follows:-

(i) The plasma must be hot enough for the production of fusion energy at a rate exceeding that of the energy loss due to bremstrahlung (Radiation loss due to collisions within the plasma). The temperature at which this occurs is the iginition temperature. For the fuel cycle based on the fusion reaction between deuterium and tritium nuclei, the ignition temperature is about 40 million degrees C.

(ii) The plasma must be contained long enough for the release of a substantial net out-put of energy.

(iii) The energy must be recovered in a useful form.

The problem of confining a plasma long enough to release a reasonable net amount of energy has proved even more difficult than the problem of achieving the ignition temperature plasma instabilities are the main cause of this rapid plasma leakage.

However practically ideal plasma containment has been achieved in several machines.

The characteristics of a large-scale fusion reactor would depend not only on the fuel cycle but also on the plasma confinement configuration and the plasma density range chosen. Thus we may have a number of different forms of fusion reactors. For example, medium density magnetic-mirror reactors and high density laser ignited reactors can operate at power levels as low as 5-50 MW. For central station power supply system, the medium density reactors can easily operate on d-t fuel cycle beacuse of the mixtures low ignition temperature. Further the high neutron output ( ~80% of the fusion energy) associated with this fuel can be advantageously utilised for a heat cycle conversion system. Such a reactor can efficiently operate with a power output in the billion watt range. Before a large scale reactor of this type is built, it is essential to be sure that the plasma will remain stable when the present experimental devices are sealed up to huge reactor sizes and temperatures. Ticklish problems to be encountered then would be: the long term equilibrium of the plasma, interaction of unruly plasma particles with walls of the container, the efficient working of the heat-exchanger system, and the need for carefully pumping large quantities of liquid lithium across the magnetic field.

It is true that difficult technological problems do exist and the time-table for fusion power reactor is difficult to predict. But the advantages of fusion power far out-weigh the difficulties that have to be tackled. We have abundant and cheap fuel supplies and the possibilities of safer reactors. If eventually fusion reactors operate only with deuterium fuel, we have enough material, readily available in the oceans of the world to supply all conceivable power requirements. There is no hazard of a runaway chain-reaction as in a fission reactor. The most advantageous aspect of the whole reactor system is that the fuels which have to be transported into a fusion reactor after it has been set up are deuterium and enriched lithium, neither of which is radioactive. There are no radiation hazards at all with the fusion reactor.

How soon fusion power? One may safely predict that the time it would take to have a typical large scale fusion reactor in operation may range from 10 to 50 years. If fusion power could be taken up as a national objective, expanded and intensive programmes can be carried out over the entire density range accompanied by necessary programmes of research on engineering and material problems to determine as quickly as possible the best and most economical design for practical fusion power sytems.

Nature is notoriously a naughty, naughty dame, demanding from man the highest sacrifices and sufferings for revealing any one of her inner most secrets. But the mind of man knows no bounds and hope springs eternal in the human breast. Let me best conclude with the valuable remarks of the U.K.Atomic Energy Authority:

> "Fusion technology is now at the stage of conceptual designs, although much more information is essential for detailed assessment of the prospects or cost of fusion power. Unexpected difficulties may yet delay or prevent eventual success; or unexpected discoveries hasten the harnessing of a new and exciting source of power".

# "EQUATIONS OF MOTION METHOD"

R.Sridhar
MATSCIENCE, The Institute of Mathematical Sciences
Madras-600020

Abstract

The method of equation of motion, extensively used in many-body theory may be stated as follows: If an operator $X_k^\dagger$ satisfies the equation

$$[H, X_k^\dagger]\,|\Phi\rangle = \omega(k)\, X_k^\dagger\,|\Phi\rangle , \qquad (1)$$

where $|\Phi\rangle$ is the ground state which is assumed to be known, then $X_k^\dagger|\Phi\rangle$ is an excited eigenstate of the Hamiltonian with excitation energy $\omega(k)$. If $|\Phi\rangle$ is assumed to be the physical vacuum then $X_k|\Phi\rangle = 0$, which in turn may determine $|\Phi\rangle$. Thus $X_k^\dagger$ creates independent quasi-particle excitations of the system. This method has been used with great advantage in determining the elementary excitations in Superconductors[1] and in liquid HeII[2]. In nuclear physics also this method has been used; for example in determing the particle-hole excitation energies[3]. If $a_m^\dagger a_i$ creates a particle-hole pair and $a_i^\dagger a_m$ annihilates one, then the most general operator linear in particle - hole excitations may be constructed as follows

$$X_k^\dagger = e^{-i\omega(k)t} \sum_{m,i} \left(x_{m,i}\, a_m^\dagger a_i - y_{m,i}\, a_i^\dagger a_m\right)$$

where the x and y are the amplitudes.

In general, equation (1) can not be solved exactly. One has to linearise this equation - most commonly used procedure of linearisation being the Random Phase Approximation (R.P.A) though other methods of linearisation may as well be employed based on physical arguments. Once $|\Phi\rangle$ and $\omega(k)$ are determined, in the framework of some approximation procedure, all other necessary physical properties of the system can be determined by straightforward procedures.

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# CORIOLIS AND ASYMMETRY EFFECTS ON MOMENTS OF INERTIA OF ODD-ODD DEFORMED NUCLEI*

Jagdev Gargi and S.D.Sharma
Department of Physics
Punjabi University, Patiala, Punjab.

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## ABSTRACT

The effect of k-coupling in odd-odd nuclei is being studied in order to study the systematics of moments of inertia from nucleus to nucleus and also under various excitations causing the rotational patterns. The preliminary work on $\Delta k=\pm 1$ mixing was presented in A.E.E.T. symposium, 1976. The present study deals also with the asymmetry effects on moments of inertia , which causes the mixing of $\Delta K=\pm 2$ states. The results are based on the assumption that the asymmetry-effects can be treated as perturbation on the symmetric core. The results are illustrated for few cases and are found to be consistent with the classical picture of the model. The moments of inertia show the predicted trends reasonably well.

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* Presented by Jagdev Gargi.

## Introduction

Davidov and Filippov's non-axial rotor model was designed to work for even-even nuclei[1] and it was extended to odd A nuclei by J.P.Davidson[2]. This model was modified by S.D.Sharma to apply it to odd-odd nuclei with symmetric and asymmetric core assumptions[3]. The modified model has been used to study the moment of inertia of odd-odd nuclei. The effect of K-coupling in odd-odd nuclei is being investigated in order to study the systematics of moment of inertia from nucleus to nucleus and also under various excitations causing the rotational patterns. The present study deals with the asymmetry effects on moment of inertia which causes the mixing of $\Delta K = \pm 2$ states and also the effect of coriolis coupling which is responsible for mixing of $\Delta K = \pm 1$ states. The moment of inertia shows the predicted trends reasonably well. Here the asymmetry effects will be treated as perturbation on symmetric core and the effect of these first and second types of coupling will be studied simultaneously.

## Theory:

The model consists of rotating core inert to vibrations with two odd nucleons attached to it. The Hamiltonian has the $H_R$ rotational part. $H_p$ and $H_n$ the Hamiltonian for proton and neutron moving in the harmonic oscillator potential wells, with l.s and $l^2$ terms. $v_{pn}$ the residual neutron-proton interaction.

The part of Hamiltonian for the coriolis coupling is

$$H_{r.p.c} = \frac{\hbar^2}{2\mathcal{J}_0}\left(I^+\mathcal{J}^- + I^-\mathcal{J}^+\right)$$

and the secular equation for $\Delta K = \pm 1$ mixing is

$$\begin{vmatrix} E_{IK} - E' & \langle K|H_{r.p.c}|K+1\rangle \\ \langle K+1|H_{r.p.c}|K\rangle & E_{IK+1} - E' \end{vmatrix} = 0$$

In case of odd-odd nuclei this yields A' the effective moment of inertia parameter for $\Lambda_n = 1/2$

$$A' = A\left[1 \mp \frac{2K+1}{E^0_{IK}} \pm \left\{\left(\frac{2K+1}{E^0_{IK}}\right)^2 + \frac{(I-K)(I+K+1)\left(\mathcal{J}_n+\frac{1}{2}\right)}{(E^0_{IK})^2}\right\}^{1/2}\right]$$

In case of $\Delta K = \pm 2$ mixing the part of the Hamiltonian is

$$H_A = P\hbar\omega\left(\frac{S_1 - S_2}{4}\right)\left(I^+I^+ + I^-I^-\right)$$

where

$$P = \hbar^2/(8BK\hbar\omega_0)$$

and

$$S_K = \left[4B\,\mathrm{Sin}^2(\gamma - 2\pi K/3)\right]^{-1}$$

and the secular equation is

$$\begin{vmatrix} E_{IK} - E' & \langle K|H_A|K+2\rangle \\ \langle K+2|H_A|K\rangle & E_{IK+2} - E' \end{vmatrix} = 0$$

which yields the moment of inertia parameter

$$A' = A\left[1 \mp \frac{4(K+1)}{E^{\circ}_{IK}} \pm \left\{\frac{16(K+1)^2}{(E^{\circ}_{IK})^2} + \frac{(I-K)(I-K-1)(I+K+1)(I+K+2)}{(E^{\circ}_{IK})^2}\left(\frac{S_1-S_2}{4}\right)^2\right\}^{\frac{1}{2}}\right]$$

where

$$E^{\circ}_{IK} = I(I+1) - 2K^2.$$

It is clear that in one band the effective moment of inertia will be less and in other one greater than the value without coupling. It is also clear from the above expressions that for higher I values, the moment of inertia will have a tendency to approach the unaffected magnitudes and this is quite expected from classical notions. In the previous report the values (theoretical and experimental) for $Tb^{158}$ have been quoted. We can report $Re^{186}$ which is a good example for $\Delta K = \pm 1$ mixing. In case of $Re^{186}$ there are three different sets of configurations[4]

1. p [402 ↑] ± n [512 ↓]
2. p [402 ↑] ± n [510 ↑]
3. p [402 ↑] ± n [503 ↑]

giving rise to various bands with proton configuration kept in tact. In case of 1st and 3rd configuration moment of inertia parameter comes out to be about 15 keV same for all I values. But in case of second configuration the effective moment of inertia in k = 2 and K = 3 bands is perturbed due to coriolis coupling. The moment of inertia parameter for K = 2 and I = 3 is A' = 18.6 keV; and I = 4, is A' = 18.2 keV.

But on the other band K = 3 the effective moment of inertia parameter comes out to be very large (21.7 keV) which is not predicted by the theory. This may be due to the effects of excitation of neutron simultaneously with perturbation caused by coriolis coupling. In case of $Ho^{166}$, $K = 1^+$ band is obtained from $[523\uparrow]_p - [523\downarrow]_n$ in which moment of inertia parameter is about 9.5 keV for all I values[5]. But $[523\uparrow]_p \pm [521\downarrow]_n$ configuration gives $K = 3^+$ and $K = 4^+$ bands are perturbed due to the coriolis coupling. In one band ($K=3^+$) the moment of inertia is smaller and in another band ($K=4^+$) greater than the unperturbed value. For $K = 3^+$ band

and I = 4, A' = 8.71 keV and I = 5, A' = 8.76 keV.

and I = 6, A' = 8.79 keV and I = 7, A' = 8.82 keV.

For $K = 4^+$ band

and I = 5, A' = 9.9 keV and I = 6, A' = 9.8 keV.

It is clear that in both bands the effective moment of inertia is approaching the uneffected value as I increases.

Having a closer look at the Nilsson's levels in various ranges of the periodic table one can easily conclude that the $\Delta K = \pm 2$ mixing will not be more likely possible for high Z nuclei beyond 2p - 1f shell, as the levels with differing j-values intervene the lower levels and thus giving rise to the bands which do not have same j-values in the nucleonic configuration and thus avoiding the possibility of giving rise to $\Delta K=\pm 2$ mixing. On the contrary in the lower region (say 2s-1d shell) one can easily get bands which arise from different nucleonic configurations but still with the same angular momenta for the odd nucleons. It is to be pointed out that this mixing cannot exist between the two bands $K_1 = \Lambda_2 + \Lambda_1$ , $K_2 = \Lambda_2 - \Lambda_1$ with the same basic for the odd nucleons. Thus we are to work mainly in 2s-1d and 2p-1f shell for this mode of study. This very point was pincoded by Sharma while scanning the whole of the periodic table for the study of spectra and other properties of odd-odd nuclei[3].

Here we point out the example of $Na^{22}$ and $P^{30}$ for this type of mixing. There are two bands K=3 and K=1 in $Na^{22}$ which has the ground state configuration $3/2+[211\uparrow]_p$ , $3/2+[211\uparrow]_n$ and the first available Nilsson state at the best fit deformation $\beta = 0.10$ is $5/2+[202\uparrow]$ . So we have $3^+$ ground state and $1^+$ this first excited state and there is found a perturbed rotational sequence $3^+$, $4^+$, $5^+$ and other one $1^+$, $2^+$, $3^+$. The results improve under the assumption of asymmetric core but we need more experimental data for the verification of our results on thus nucleus.

In case of $p^{30}$ the ground state configuration is $1/2 + [211\downarrow]_p$, $1/2 + [211\downarrow]_n$ and a band $K = 1^+$ at about 0.7 MeV is found to be mainly from the configuration $1/2 + [200\uparrow]_p$, $3/2 + [202\downarrow]_n$. It has three members and another one with $K = 3^+$ from $3/2^+ [202\downarrow]_p$, $3/2^+ [202\downarrow]_n$ It has only two members. These two bands with $\Delta K = 2$ are mixed. But one has to treat the problem simultaneously studying the effect of coriolis coupling. On the whole the diagonalization of total Hamiltonian has led to good results, but in this region due to large K-mixing the moment of inertia seems to loose its significance. But simultaneously considering the two effects one can certainly arrive at the conclusion that inspite of all these perturbation one can still talk of moment of inertia of the nuclei in this region. This study also explains the complex nature of spectra in the 2s-1d and 2p-1f shells in comparison with those of the nuclei beyond 2p-1f shell.

It is found that l-mixing has no effect on moment of inertia but the effect of residual interaction of the two odd nucleons is also responsible for the complex nature of the spectra. In case of $p^{30}$ the coriolis effects, Asymmetry effects and the interaction-effects for $K = 0^+$ bands are all called into play simultaneously, this is the reason that the spectrum is very complicated. The over all study of moments of inertia of other nuclei is being continued. The investigations are expected to reveal all secrets of this quantity.

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# ON SUPERLUMINAL INERTIAL FRAMES*

G.Alagar Ramanujam** & N.Namasivayam
Department of Physics, N.G.M.College, Pollachi

Abstract

Lorentz type transformations, in four dimensions, between two inertial frames with a relative velocity greater than the velocity of light, are obtained here. Though the results look very similar to those given by Olkhovsky, Recami and Mignani in recent papers, there are places where the results given here do not agree with those of the above authors. For example, the "transversal" velocities which become imaginary in their case, remain real in ours. More-over, the validity of Einstein's law of addition of velocities for superluminal particles has not been assumed, but on the contrary has been proved here. Finally, a Hamiltonian for a Dirac tachyon of spin $\frac{1}{2}$ is obtained, by constructing a relevant Foldy-wouthuysen type operator.

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## List of Participants

| | |
|---|---|
| Alladi Ramakrishnan | MATSCIENCE, Madras |
| Ananthakrishnan, N | S.R.College, Trichi |
| Anantharaman, J | National Institute of Engineering, Mysore |
| Arthur, B.M. | Madras Christian College, Madras |
| Aruna, P | MATSCIENCE, Madras |
| Bhargava, V.T.A. | Madras Christian College, Madras |
| Devanathan, V | Madras University, Madras |
| Jagannathan, R | MATSCIENCE, Madras |
| Jagathesan, B | Pachaiyappa's College for Men, Kanchipuram |
| Jagdev Gargi | Punjabi University, Patiala |
| Jain, B.K. | Bhabha Atomic Research Centre, Trombay |
| Joshi, K.P. | Vikram University, Ujjain |
| Krishnamurthy, T.V. | Engineering College, Madras |
| Kuppuswamy, R. | Central Polytechnic, Madras |
| Mahalanabis, J | Saha Institute of Nuclear Physics Calcutta |
| Mani, S | MATSCIENCE, Madras |
| Mariwalla, K.H. | MATSCIENCE, Madras |
| Nagarajan, T | Madras University, Madras |

| | |
|---|---|
| Parikh, J | Atomic Minerals Division, DAE, New Delhi |
| Parthasarathy, R | Madras University, Madras |
| Puttaswamy, N.G. | Bangalore University, Bangalore |
| Radhakrishnan, V | MATSCIENCE, Madras |
| Raghavan, S.S. | Engineering College, Madras |
| Ramanujam, A. | N.G.M.College, Pollachi |
| Ramaswamy, A | Vaishnav College, Madras |
| Ramaswamy, P | Madras University, Madras |
| Ranganathan, N.R. | MATSCIENCE, Madras |
| Rangarajan, C | Vaishnav College, Madras |
| Rangarajan, S | Vaishnav College, Madras |
| Rao, A.B.P. | Sholapur College, Sholapur |
| Rao, V.B. | Jain Hind College, Bombay |
| Ray, S.S. | Saha Institute of Nuclear Physics, Calcutta |
| Sabesan, R | National College, Trichi |
| Santhanam, T.S. | MATSCIENCE, Madras |
| Satpathy, L | Berhampur University, Berhampur |
| Satyanarayana, T | MATSCIENCE, Madras |
| Shanta, R | Indian Institute of Technology, Bombay |
| Sharma, S.D. | Punjabi University, Patiala |
| Sheela, M.L. | Mysore University, Mysore |
| Shubha | Mysore University, Mysore |

| | |
|---|---|
| Sridhar, R | MATSCIENCE, Madras |
| Srinivasa Rao, K | MATSCIENCE, Madras |
| Subramaniam, P.R. | Madras University, Madras |
| Swaminathan, K.M. | Engineering College, Madras |
| Tegli, S.J. | Sholapur College, Sholapur |
| Tekumalla, A.R. | MATSCIENCE, Madras |
| Teli, M.T. | Shivaji University, Kolhapur |
| Thuse, P.V. | Sholapur College, Sholapur |
| Tiwari, N | Saha Institute of Nuclear Physics, Calcutta |
| Venugopalan, E | Jamal Mohamed College, Trichi |
| Vijayakumar, A | MATSCIENCE, Madras |
| Viswanathan, K.K. | Government Arts College, Madras |

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