MATSCIENCE REPORT 46

# SELECTED TOPICS IN WEAK INTERACTIONS


by

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THE INSTITUTE OF MATHEMATICAL SCIENCES, MADRAS-20 (INDIA)

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LECTURES ON

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(Lecture notes have not been revised by the author)

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## Lecture I

In these few lectures, let us discuss various aspects of the theory of weak interactions. The most general form of the weak interaction Hamiltonian may be written as

$$H_{Weak} = H_{lep} + H_{S.L} + H_{N.L}$$

where $H_{Lep}$ involves currents of only leptons (examples of such pure leptonic weak interaction being the proto type mu-decay and neutrino-lepton interactions). $H_{S.L}$ involves both leptons and hadrons (strongly interacting particles). The examples of semi-leptonic weak interactions are the familiar beta-decay or mu-capture (which conserve the strangeness quantum number) and $K_{\ell 3}$-decay (for which $\Delta S \neq 0$ ). The part $H_{N.L}$ does not involve leptons at all. It is very difficult to understand the $\Delta S = 0$ part of this non-leptonic weak interaction since it is usually masked by strong interaction. Few examples of non-leptonic $\Delta S \neq 0$ weak decays are the K-$\pi$ decays and Y-N$\pi$ decays ( Y here refers to the hyperons in general). The range of the weak coupling constant is $\sim 10^{-49}$ erg cm$^3$ and may be conveniently described by the dimensionless coupling constant $\sim 10^{-7} \hbar c \left(\frac{\hbar}{m_\pi c}\right)^2$. Most of the presently available information are about the semi-leptonic interactions.

The basic assumption made about the weak interaction is that it is the Fermi type local interaction. Though the Yukawa type non-local weak interaction through the exchange of heavy mass (indicating the smallness of the non-locality) intermediate vector bosons, it has so only been academic except in some neutrino reactions which we will discuss later. Let us first discuss about the beta-decay for which lot of experimental information is available.

Beta-decay $\quad n \rightarrow p + e^- + \bar{\nu}$

The most general Lorentz invariant interaction Hamiltonian for this decay may be written as

$$H = \sum_i \Big[ C_i (\bar{\psi}_p O_i \psi_n)(\bar{\psi}_e O_i \psi_\nu) + C_i' (\bar{\psi}_p O_i \psi_n)(\bar{\psi}_e O_i \gamma_5 \psi_\nu) \Big] + h.c. \qquad (1)$$

where $\psi$'s stand for the respective particle spinors (or fields) and the operators $O_i$'s stand for the five operators

$$O_S = 1$$

$$O_P = \gamma_5$$

$$O_A = i \gamma_\mu \gamma_5$$

$$O_T = -i [\gamma_\mu \gamma_\nu]$$

$$O_V = \gamma_\mu \qquad \mu, \nu = 1, 2, 3, 4$$

In writing down eq.(1) we have discarded terms of the kind

$$\sum_i D_i (\bar{\psi}_p O_i \bar{\psi}_n)(\bar{\psi}_c O_i \psi_\nu^c) + D_i' (\bar{\psi}_p O_i \bar{\psi}_n)(\bar{\psi}_c O_i \gamma_5 \psi_\nu^c)$$

where $\psi_\nu^c$ stands for the conjugate fields, by a redefinition of lepton number. Notice that these terms should be included on the basis of only Lorentz invariance. Let us come back to this point when we discuss the lepton number conservation. If time reversal invariance holds in this case, then the coefficients $C_i$'s are real Eq.(1) may be written in a slightly different form

$$H = \sum_i C_i \left[ (\bar{\psi}_p O_i \psi_n) \bar{\psi}_c O_i \left(1 + \frac{C_i'}{C_i} \gamma_5\right) + h.c \right] \qquad (2)$$

Let us go to the non-relativistic approximation of eq.(2). The scalar and vector interactions are of the Fermi type $u_p^+ u_p$ obeying the selection rule $\Delta J = 0$ and the tensor and axial vector interactions are of the Gamow-Teller type $\bar{u}_1 \vec{\sigma} u_n$, with selection rules $\Delta J = 0, 1 [(0 \to 0)$ transition being forbidden]. Here stands for the angular momentum and $\vec{\sigma}$ for the Pauli matrices. We omit the Pseudoscalar interaction which in the case of beta-decay contributes almost nothing. In this non-relativistic approximation

$$\psi = \begin{bmatrix} w_1 \\ -\dfrac{\vec{\sigma}\cdot\vec{p}}{E+M} w_1 \end{bmatrix} \tag{3}$$

In order to explicitly evaluate the coefficients occurring in (2), we resort to experiments of the following types, namely the determination of (a) the asymmetry of emitted from polarised nuclear or (b) the longtitudinal polarization of the electron from unpolarised nuclei.

Let us first discuss the famous Wu's experiment on the beta decay from the polarised $Co^{60}$ nucleus. The initial $Co^{60}$ has $J = 5$ . This decays as $Co^{60} \rightarrow Li^{60} + \bar{e} + \bar{\nu}$. The final $Li^{60}$ has J=4. Hence this is a pure Gamow Teller transition and thus the interaction should be a mixture of T and A. The angular distribution of electron is given by

$$\frac{d\sigma}{d\Omega} \sim (1 + \alpha \cos\theta)$$

where

$$\alpha = 2 \frac{\langle J_z \rangle}{J} \frac{v}{c} \frac{C_T C_T' - C_A C_A'}{|C_T|^2 + |C_{T'}|^2 + |C_A|^2 + |C_A'|^2}$$

The longitudinal polarization of the $\bar{e}$ is

$$P_{\bar{e}} = \frac{2v}{c} \frac{C_S C_S' - C_V C_V'}{|C_S|^2 + |C_S'|^2 + |C_V|^2 + |C_V'|^2}$$

if the transition is of the Fermi type

$$= 2\frac{v}{c}\,\frac{C_T C_T' - C_A C_A'}{|C_T|^2+|C_{T'}|^2+|C_A|^2+|C_{A'}|^2}$$

if the transition is of the Gamow Teller type. By CP invariance we know

$$P_{e^+} = -P_{e^-}$$

Experimentally one finds

$$P_{e^-} = -\frac{v}{c} = -P_{e^+}$$

This means that in eq.(2),

$$\frac{C_i'}{C_i} = +1 \qquad \text{for} \quad O = V, A$$

$$= -1 \qquad \text{for} \quad O = S, T$$

Thus, we are left with the possibilities

$$\psi_e^+ \gamma_4 O_{V,A} (1+\gamma_5) \psi_\nu$$

or

$$\psi_e^+ \gamma_4 O_{S,T} (1-\gamma_5) \psi_\nu \tag{4}$$

for $H_{int}$. From the commutation properties of the -matrices we know that

$$\gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4$$

$$\{\gamma_5, \gamma_\mu\}_+ = 0$$

$$\gamma_4 O_{V,A} \gamma_5 = \gamma_5 \cdot \gamma_4 O_{V,A}$$

$$\gamma_5 O_{S,T} \gamma_5 = -\gamma_5 \gamma_4 O_{S,T}$$

so that eq.(4) may be written as

$$H = \sum_i \left[ (\bar{\psi}_p O_i \psi_n)(\psi_e^+ (1+\gamma_5) \gamma_4 O_i \psi_\nu) + h.c \right]$$

for V and A

$$= \sum_i \left[ (\bar{\psi}_p O_i \psi_n)(\psi_e^+ (1-\gamma_5) \gamma_4 O_i \psi_\nu) + h.c \right]$$

for S and T (5)

<u>Theory of two component neutrino</u>

The Dirac equation for a spin 1/2 particle may be written as

$$\left[ \gamma_\mu \frac{\partial}{\partial x_\mu} + m \right] \psi = 0 \tag{6}$$

where $\gamma$'s are the usual (4 x 4) Dirac matrices and m is the mass of the particle. We can also write the above equation in a two component form

$$(\vec{\alpha} \cdot \vec{p} + \beta m) \psi = E \psi \tag{7}$$

where $\alpha$'s are $\beta$'s are the anticommuting idempotent matrices with

$$\gamma_K = -i\beta\alpha_K \qquad K = 1,2,3$$

$$\gamma_4 = \beta$$

$$\begin{aligned} \gamma_5 &= i\alpha_1\alpha_2\alpha_3 \\ &= -\alpha_1\sigma_1 \\ &= -\alpha_2\sigma_2 \\ &= -\alpha_3\sigma_3 \end{aligned} \tag{8}$$

Also

$$\alpha\gamma_5 = \gamma_5\alpha = \vec{\sigma}$$

where

$$\vec{\sigma}_m = -i\alpha_k\alpha_l \qquad k, l, m \text{ Cyclic}$$

$$[\gamma_5, \alpha] = 0$$

Thus from eq.(7), multiplying with $\gamma_5$ we find by using the properties (8)

$$\begin{aligned} \gamma_5 E\psi &= \gamma_5(\vec{\alpha}\cdot\vec{p} + \beta m)\psi \\ &= -(\vec{\sigma}\cdot\vec{p} - \beta m\gamma_5)\psi \end{aligned}$$

Therefore

$$\begin{aligned} (1 \pm \gamma_5)\psi &= \left[1 \mp \frac{\vec{\sigma}\cdot\vec{p}}{E}\right]\psi \\ &= \mp\frac{m}{E}\beta\gamma_5\psi \end{aligned} \tag{9}$$

In the case of neutrinos, we know

$$m_\nu = 0 \qquad |\vec{p}| = E$$

Thus, we get two equations

$$E > 0 \quad (1 \pm \gamma_5)\psi_\nu = (1 \mp \vec{\sigma}\cdot\vec{p})\psi_\nu$$
$$= (1 \pm \gamma_5)\psi_\nu \qquad (10)$$

$$(\vec{\sigma}\cdot\vec{p})\left[(1 \pm \gamma_5)\psi_\nu\right] = \mp(1 \mp \vec{\sigma}\cdot\vec{p})\psi_\nu$$

and for $E < 0$

$$(\vec{\sigma}\cdot\vec{p})\left[(1 \pm \gamma_5)\right]\psi_5 = \pm(1 \pm \gamma_5)\psi_\nu$$

where $\hat{p}$ is the unit vector $\dfrac{\vec{p}}{E}$

We then find that the projection $(1+\gamma_5)\psi$ gives a lefthanded neutrino (or) a righthanded antineutrino and gives a right $\nu$ or a left $\bar{\nu}$. In the language of second quantization the projection $(1+\gamma_5)\psi_\nu$ absorbs a left $\nu$ (or) emits a right $\bar{\nu}$ and the projection $\psi_\nu^+(1+\gamma_5)$ emits a left $\nu$ (or) absorbs a right $\bar{\nu}$ and the total four comp $\psi_\nu$ may then be written as

$$\psi_\nu = \tfrac{1}{2}(1+\gamma_5)\psi + \tfrac{1}{2}(1-\gamma_5)\psi$$

In the next lecture we will discuss the experiments which supported this two component theory of the neutrino.

## Lecture 2

In the last lecture we saw that the weak interaction Hamiltonian describing the beta-decay process may be written as

$$H_W = \sum C_i \left[ (\bar{\psi}_p O_i \psi_n)(\psi_e^+ (1-\gamma_5)\gamma_4 O_i \psi_\nu \right] + h.c$$

$$= \sum_{i=V,A} C_i \left[ (\bar{\psi}_p O_i \psi_n)(\bar{\psi}_e O_i (1+\gamma_5) \psi_\nu \right] + h.c$$

$$+ \sum_{i=S,T} C_i \left[ (\bar{\psi}_p O_i \psi_n)(\bar{\psi}_e O_i (1-\gamma_5) \psi_\nu ) \right] + h.c$$

We have neglected the negligible pseudoscalar term. The division of the $H_\omega$ into $(V, A)$ and $(S, T)$ combinations arise due to the fact that the operator $\gamma_4 O_i$ commutes with $\gamma_5$ if $i = V, A$ and anticommutes when $i = S, T$. There have been two kinds of experiments; first one has to test whether the combination $(V, A)$ or the combination $(S, T)$ is compatible with experiments on the determination of the helicity of the neutrino and the angular correlation of the electron and the neutrino.

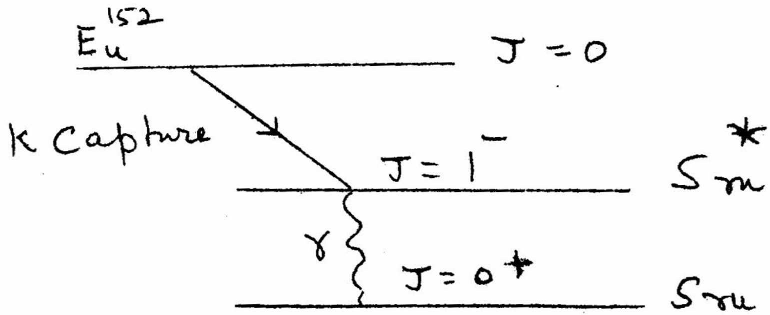
### Goldhaber's experiment on the direct measurement of the helicity of the neutrino

The reaction considered is the K-electron capture of the nucleus $Eu^{152}$

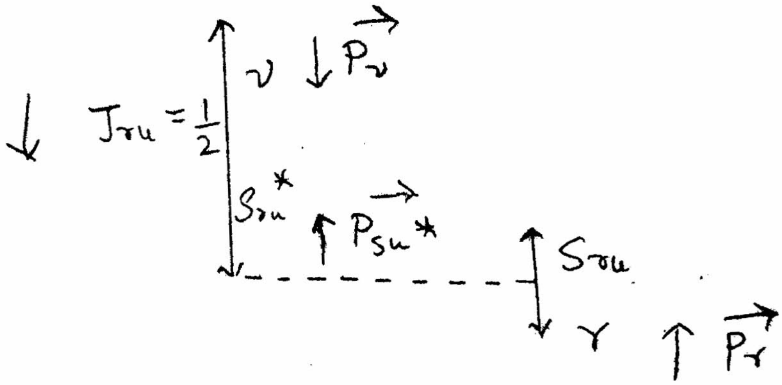
$$Eu^{152} + e^{-} \longrightarrow Sm^{152\,*} + \nu$$

$$Sm^{152\,*} \longrightarrow Sm^{152} + \gamma$$

The initial nucleus $Eu^{152}$ has $J = 0$. Hence the total initial spin is $J = \frac{1}{2}$




This is a pure Gamow-Teller transition and so we have only A or T in the interaction. We can give the spin-momentum balance of the reaction as follows



The experiment consists in selecting those $\gamma$ -rays which go opposite to the direction of $\nu$ (i.e. the $\gamma$ -rays that go in the direction of the recoil nucleus) by studying the resonant scattering of the $\gamma$ -rays.

The helicity of the $\gamma$ -ray will be the same as that of the neutrino. It was found experimentally that the belicity of the $\gamma$ -ray is negative. Thus the neutrino that is emitted must be lefthanded. We then have A and not T in the interaction Hamiltonian. The angular correlation between the electron and the neutron is given by

$$\sigma(\theta_{e\nu}) \sim 1 + a_F \frac{v}{c} \cos\theta_{e\nu}$$

$$a_F = \frac{|C_V|^2 - |C_S|^2}{|C_V|^2 + |C_S|^2}$$

The experiment on the angular $e\nu$ correlation in the beta-decay of $A^{35}$ (a pure Fermi transition) yields that $a_F > 0$, indicating that it is V and not S that enters the Hamiltonian. Thus we are left with a unique choice of the form of the Hamiltonian, namely, that it is a combination of V and A.

Half life of neutron and $O^{14}$

The experiments which fix the relative sign between V and A consist in comparing the relative $(ft)$ values of $O^{14}$ and a neutron. The beta-decay in $O^{14}$ is a pure fermi transition and thus involves only the vector interaction. $O^{14} \rightarrow N^{14*}$

Both the initial and final nuclei have $J = 0$, the transition thus being a pure fermi one. The half life time is given by

$$\frac{1}{\tau_0^{14}} \sim |C_V|^2 \left| \int 1 \right|^2_{O_{14}}$$

$$\left| \int 1 \right|^2_{O_{14}} \propto \left| \langle T_z' T' | \sum_i \frac{\tau^-}{2} | T_z T \rangle \right|^2$$

$O^{14} (J = 0 \; T = 1 \; T_z = 1)$ and $N^{*14} (J = 0, T' = 1, T_z' = 0)$

Then

$$\left| \int 1 \right|^2_{O_{14}} = 2 \qquad \text{using Wigner Echart theorem}$$

Thus $|C_V|^2_{O_{14}}$ is known. For the beta-decay of the neutron we have

$$\frac{1}{\tau_n} \sim |C_V|^2_n \left| \int 1 \right|^2 + |C_A|^2 \left| \int \vec{\sigma} \right|^2$$

$$\sim |C_V|^2 |M_F|^2 + |C_A|^2 |M_{G.T}|^2$$

$$\sim |C_V|^2 1 + |C_A|^2 \cdot 3$$

From these two measurements, it is found that

$$|C_A| = 1.2 |C_V|$$

However, it should be remembered that the determination of $C_V$ involves nuclear matrix elements, and thus carries with it the usual uncertainties. Hence the following corrections are needed.

(1) Finite distribution of the charge of the nucleus for the $e\gamma$ phase-space Fermi function F

(2) Screening effect of the atomic electrons. For the $0 \rightarrow 0$ transition this has been computed to be

$$|C_V|^2 |M_F|^2 (ft) = \frac{\pi^3 \hbar^3 \log 2}{m_0^5 c^4}$$

from which $C_V$ could be determined.

(3) Radiative corrections: For a free neutron that correction to the radiative effects has been estimated to be

$$\frac{\Delta(ft)}{(ft)} \simeq 1.7\ \%$$

while for the neutron in a nucleus, it is still not yet completely clear how one can determine this correction unambigously.

(4) Isotopic impurities due to coulomb potential. This modifies the determination of $|M_F|^2$ to the extent of

$$\frac{\Delta |M_F|^2}{|M_F|^2} < 0.25\ \%.$$

(5) Isotopic impurity due to electromagnetic non-coulomb effect. This correction seems to be rather small.

The confirmation on the relative magnitude deduced from the experiments on the $(ft)$ values of $O^{14}$ and $n$ comes from the measurement of the electron and neutrino asymmetries from the polarized neutrons. The angular distribution of the electron from polarized neutron is

$$\sigma(\theta) \sim 1 + \alpha \cos\theta$$

$$\alpha = - \frac{\langle J_z \rangle}{J(J+\frac{1}{2})} \frac{v}{c} \frac{|C_A|^2 \left|\int \vec{\sigma}\right|^2 + 2 C_A C_V \sqrt{J(J+1)} \cdot \left|\int \vec{\sigma}\right| \left|\int 1\right|}{|C_A|^2 \left|\int \vec{\sigma}\right|^2 + |C_V|^2 \left|\int 1\right|^2}$$

where $\theta$ is the direction of the electron with respect to the incident polarised neutron.

With $J_z = \frac{1}{2}$, $\left|\int \vec{\sigma}\right|^2 = 3$, $\left|\int 1\right|^2 = 1$, we have

$$\alpha \sim \left(|C_A|^2 + C_A C_V\right)$$

Experiment shows almost no asymmetry for the electron while the antineutron prefers to come out in the direction of $\langle J_z \rangle_N$. This fixes that $C_A = -1.2\, C_V$

and thus the interaction to be $(V-A)$. We may therefore write the beta-decay interaction as

$$H \sim C_V \left\{ \left[ \left( \bar{\Psi}_p \gamma_\mu (1 + 1.25\gamma_5) \Psi_n \right] \left[ \bar{\Psi}_e \gamma_\mu (1+\gamma_5) \Psi_\nu \right) \right] \right\} + h.c$$

$$C_V = \frac{G}{\sqrt{2}}$$

From the experiments on $O^{14}$, $Al^{26}$, and many other nuclei, it has been found that

$$G = 1.4029 \pm 0.0022 \times 10^{-49} \text{ Erg Cm}^3$$

Lecture 3

## Conserved Vector Current Hypothesis

Comparison between the $\mu$-decay and beta decay rates form the basis of the conserved vector current (C.V.C) hypothesis.

The existence of two neutrinos has been established beyond doubt. Then the conservation of lepton number breaks up into separate conservations of electron number (electron number for $e^-, \nu_e = 1$ and for $\mu^-, \nu_\mu = 0$ ) and muon number (muon number for $e^-, \nu_e = 0$ and for $\mu^-, \nu_\mu = 1$ ). We will assume the locality of the four Fermi weak interaction and the two component neutrino theory. We will also assume CP and T invariances. With all these assumptions, the $\mu$-decay interaction may be written as

$$H = g_\mu \left[ \left( \bar{\psi}_e \gamma_\lambda (1+\gamma_5) \psi_e \right) \left( \bar{\psi}_{\nu_\mu} \gamma_\lambda (1+\gamma_5) \psi_\mu \right) \right] + h.c$$

The $\mu$-decay probability could be calculated to be

$$\frac{dN^\pm}{d\Omega\, dx} \sim x^2 \Big\{ 3(1-x) + 2\rho\left(\tfrac{4}{3}x - 1\right) \mp P \xi \cos\theta \left[ 1 - x + 2\delta\left(\tfrac{4}{3}x - 1\right) \right] \Big\}$$

where $\rho$ is the Michel parameter characterizing the shape of the spectrum. P is the polarization of the $\mu$. $\theta$ is the angle between the polarized direction of the initial $\mu$ and the momentum of the emitted electron.

$$x = \frac{\text{electron energy}}{E_{max}}$$

and $\delta$ is the variation of the energy with asymmetry $\xi$ = asymmetry in the direction of the electron. If one assumes $m_e = 0 = m_\nu$, (V-A) theory gives the following sets of values $\delta$ =3/4, $\rho$ =3/4; $\xi$ = -1

$$\frac{dN}{d\Omega} \sim 1 + \frac{1}{3}(\hat{p}_\mu \cdot \hat{p}_e)\xi$$

corrections to the $\mu$-decay calculations.

Radiative correction: In the case of $\mu$-decay, the radiation correction is finite. The experimental values are $\rho$ =07780 ± 0.25, $|\xi| \geq$ 0.975 ± 0.054 (Bardon),

$\geq$ 0.96 ± 0.05 (Plano)

The second value of $|\xi|$ is calculated assuming $\delta$ =3/4.

The electron polarization given by the (V-A) theory is

$$Pe_\pm = \mp \xi \frac{v}{c}$$ (purely longitudinal)

Polarized electron gives rise to circular polarization of Bremstrahlung photons. This is measured by looking at the compton scattering on magnetised iron.

Remarks

(a) $H \sim \sum_i g_i \left[ (\bar{\psi}_e O_i \psi_\mu)(\bar{\psi}_{\nu_\mu} O_i (1 + \frac{g_i'}{g_i}\gamma_5)\psi_{\nu_e}) \right] + h.c$

may be put in the previous form by means of the Fierz transformation on H.

Fierz transformation is defined by

$$L_i(a,b,c,d) = (\bar{\psi}_a O_i \psi_b)(\bar{\psi}_c O_i \psi_c)$$
$$= \sum_j a_{ij} L_j(a,d,c,b)$$

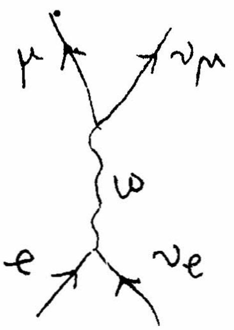
where

$$a_{ij} = \begin{array}{c|ccccc} & S & V & T & A & P \\ \hline S & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ V & 1 & -1/2 & 0 & \frac{1}{2} & -1 \\ T & 3/2 & 0 & -1/2 & 0 & 3/2 \\ A & 1 & 1/2 & 0 & -1/2 & -1 \\ P & \frac{1}{4} & -1/4 & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \end{array}$$

(b) Though (V-A) theory with $m_e = 0 = m_{\nu_e}$ gives the values $\rho = 3/4 = \delta$, $\xi' = -1$, these values do not imply (V-A) theory.

(c) <u>Effect of non-locality</u>

If an intermediate vector boson mediates weak interaction, then ( V-A) theory gives



$$\rho = \frac{3}{4} + \frac{1}{3}\left(\frac{m_\mu}{m_\omega}\right)^2$$

$$\xi = -1 - \frac{3}{5}\left(\frac{m_\mu}{m_\omega}\right)^2$$

However, at present experiments indicate that if a $\omega$ meson exists, then

$$m_\omega \geqslant 1.6 \text{ GeV}.$$

(d) If $m_{\nu_\mu} \neq 0$ (still with $m_{\nu_e} = 0$) then the spectrum gets changed a little. But the experimental estimates on $m_{\nu_\mu}$ are not yet conclusive. If one could measure the photon energy rather accurately in a radiative $\mu$-decay of the charged pion, then one may possibly determine $m_\nu$. At present $m_{\nu_\mu}$ seems to be $\lesssim 1$ MeV.

<u>Life time of the Muon</u>

$$\frac{1}{T_\mu^\pm} = g_\mu^2 \frac{m_\mu^5 c^4}{192 \pi^3 \hbar^7}$$

The radiative correction gives $\frac{\Delta \tau_\mu}{\tau_\mu} \sim -0.42\,\%$

implying $\Delta g_\mu / g_\mu \simeq +0.21\,\%$

The best value for $g_\mu$ we have at present is

$$g_\mu = 1.4350 \pm 0.0011 \times 10^{-49} \text{ erg cm}^3$$

Remarks: (a) If we have $(V-A)$ law + intermediate boson

$$\tau_\mu^{w} = \tau_\mu \left[1 - \frac{3}{5}\left(\frac{m_\mu}{m_w}\right)^2 + \frac{3\alpha}{\pi}\log\frac{M_w}{m_N}\right]$$

$$m_w \gtrsim 1.6 \text{ GeV}$$

This implies a correction

$$\frac{\Delta g}{g} \sim 0.1\,\%.$$

Lecture 4

In this lecture, we shall discuss the conserved vector current (C.V.C.) hypothesis. This was suggested on the ground that weak vector coupling constant remains unrenormalized due to strong interaction effects.

Let us now take the $\beta$ -decay process and $\mu$ -decay process. The weak interaction hamiltonian can be written in the form $H = J_\mu \ell_\mu$

where

$$J_\mu = J_\mu^V + J_\mu^A$$

$$\ell_\mu = \bar{\psi}_e \gamma_\mu (1+\gamma_5)\psi_\nu$$

and

$$J_\mu^V = g_V \bar{\Psi}_N \gamma_\mu \tau_+ \psi_N$$

$$J_\mu^A = \frac{g_V}{\sqrt{2}} \bar{\Psi}_N \gamma_\mu \gamma_5 \tau + \psi_N$$

$$\tau_+ = \frac{\tau_1 + i\tau_2}{\sqrt{2}}$$

If we compare the weak vector part of the strongly interacting particles $J_\mu^V$ with the electromagnetic current of the nucleons which we shall write as

$$H^{e.m} \sim c\, J_\mu^{e.m}\, A_\mu$$

where

$$J_\mu^{e.m} = i\left\{\bar{\psi}_N \gamma_\mu \frac{1+\tau_3}{\sqrt{2}} \psi_N + \cdots\cdots \right\}$$

$$= J_\mu^{e.m} \quad \text{(Isoscalar)}$$

$$+ J_\mu^{e.m} \quad \text{(Isovector)}$$

$$J_\mu^{e.m} \text{ (Isovector)}$$

$$= i\left\{\bar{\Psi}_N \gamma_\mu \frac{\tau_3}{\sqrt{2}} \psi_N + \left[\vec{\pi} + \frac{\partial \pi}{\partial x_\mu}\right]_0 + \cdots\cdots \right\}$$

Here $A_\mu$ is the vector field of the photon and $\pi$ is the field of the pion. Now if we compare $J_\mu^{(Weak)}$ (vector) with $J_\mu^{e.m}$ (Isovector) we should observe that these two currents, viz.

$$J_\mu^{(\text{Weak})}(\text{vector}) = \frac{g_V}{\sqrt{2}}\left(\bar{\psi}_N \gamma_\mu \tau_\pm \psi_N\right)$$

$$\text{and } J_\mu^{c.m}(\text{Isovector}) = \frac{e}{\sqrt{2}}\bar{\psi}_N \gamma_\mu \tau_3 \psi_N$$

may be thought of as

components $J^\pm$, $J_0$ of the isospin current

$$J_\mu^{V(+)} = i\Big\{\bar{\psi}_p \gamma_\mu \psi_N + \sqrt{2}\Big(\pi^{+*}\frac{\partial \pi^0}{\partial x_\mu} - \pi^0 \frac{\partial \pi^+}{\partial x_\mu}\Big)$$

$$\sqrt{2}\,\bar{\psi}_{\Sigma^+}\gamma_\mu \psi_{\Sigma^0} + \cdots\cdots$$

$$+\Big(\bar{\phi}_{K^+}\frac{\partial \phi_{K^0}}{\partial x_\mu} - \phi_{K^0}\frac{\partial \phi_K^*}{\partial x_\mu}\Big) + \cdots\Big\}$$

However, we know from gauge invariance of electromagnetic interaction that this current $J_\mu^{c.m}$ is conserved. Now, in analogy with electromagnetic interaction, we <u>assume</u> that just as $J_\mu^{c.m}$ <u>each component of the isospin current, viz. $J_\mu^{+/-}$ (weak)</u> is conserved as was postulated first by Feynman and Gell-Mann. This principle viz. that each component of the isospin current is called the conserved vector current hypothesis (C.V.C.). Of course we neglect the electromagnetic corrections to the weak vector currents when we talk to about the conservation of $J_\mu^V$ (weak). This principle seems to be highly favoured by experiments.

We shall see later how this principle is supported by experiments. The essence of C.V.C. hypothesis may be summarized as

$$\partial J_\mu^{\text{Vector}}(+,-,0)/\partial x_\mu = 0$$

<u>Consequences of the C.V.C. hypothesis and its experimental verification</u>

On the basis of just Lorentz invariance one could write down for the matrix element of the electromagnetic current

$$\langle p' | J_\mu^{e.m} | p \rangle$$
$$= i\,\bar{u}_{p'} \left\{ F_1(q^2)\gamma_\mu - \frac{\mu}{2M}\sigma_{\mu\nu} q_\nu \right\} u_p + F_3(q^2) q_\mu$$

where $p$ and $p'$ are the four momenta of the incident and emerging nucleous, $q = p' - p$ and $\mu = \mu_p - \mu_n$. In principle, there are five terms for this matrix element and it is an easy matter to show using Dirac equation that only three of these five are independent and we could choose these three as $\gamma_\mu$, $\sigma_{\mu\nu} q_\nu$ and $q_\mu$. These are terms we have written in the above expression. Gauge invariance of $J_\mu^{e.m}$ would imply that $F_3(q^2)$ is zero.while this term will be present in the case of weak interaction currents. The conservation of

$J_\mu^{c.m}$ will mean that

$$F_1(0) = 1, \quad F_2(0) = 1$$

and $\mu = (\mu_p - \mu_n) \approx 3.70$

That is, at $q \simeq 0$, the charge is not renormalized by strong interaction. In the case of weak interaction, the vector current of the hadrans may be written in its most general form as

$$\langle p' | J_\mu^V | p \rangle = i \bar{u}_{p'} \{ F_1(q^2) \gamma_\mu - \frac{\mu}{2M} F_2(q^2) \sigma_{\mu\nu} q_\nu + i h_v(q^2) q_\mu \} u(p)$$

The coefficients are so chosen to make $F$'s and $h$ real:
The consequences of the C.V.C. hypothesis are

(a) $F_1$ and $F_2$ are the same as in the electromagnetic case

(b) At $q = 0$, there is no renormalization of the vector coupling constant

and (c) the appearance of the 'weak magnetism' term.

<u>Experimental Verification of C.V.C. hypothesis</u>

(1) The experimental result that the coupling constant of the $\mu$-decay $(g_\mu)$ and the vector coupling constant in the $\beta$-decay $(g^V_\beta)$ are equal i.e.

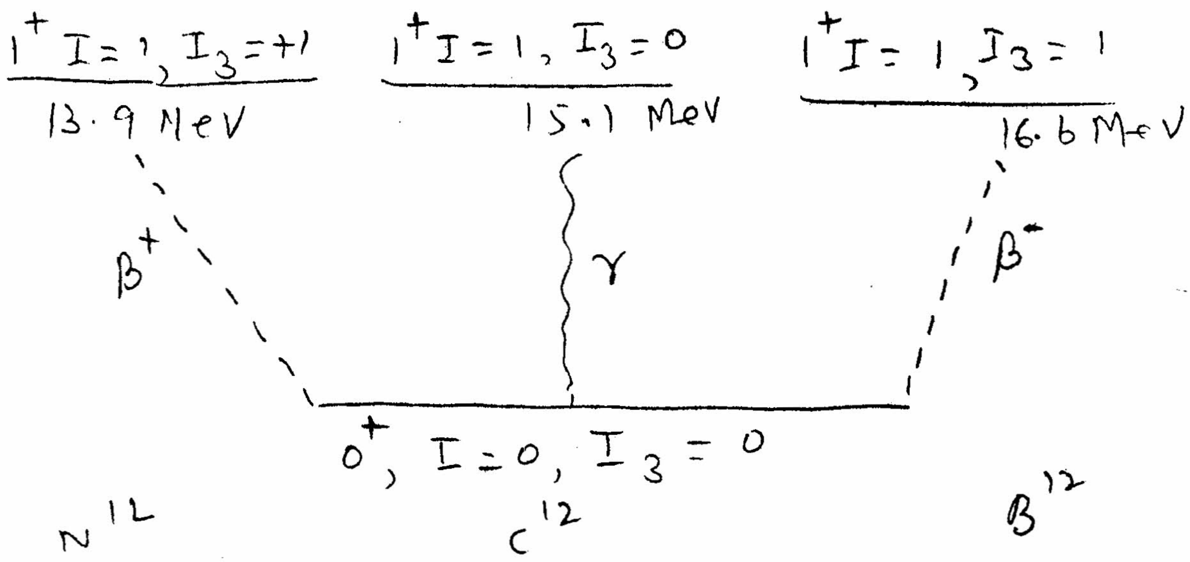
$$(g^V_\beta) = g_\mu \qquad \text{(2 discrepancy)}$$

is a very good experimental verification of the C.V.C. hypothesis. This mean that inspite of the fact that $\beta$ -decay involves strong interaction effects while $\mu$ -decay is purely weak, the vector coupling constant is not all renormalized by strong interaction effects at zero momentum transfer (The weak interaction till now is considered to be local and only the first order is taken into account though recently), there have been attempts by Feinberg and Pais to show that higher order effects are finite and should be taken into account). As a matter of fact, it is this equality viz. $g_\mu = g^V_\beta$ that actually gave rise to the C.V.C. hypothesis.

<u>Evidence of the weak magnetism term</u>

In order to test the C.V.C. hypothesis by experiments, it is essential for us to select a situation where one could isolate the vector part alone from the rest. A very important experiment was suggested by Gell-Mann in which one has to study the nuclear beta-decays of $B^{12}$ and $C^{12}$. The system we are going to consider is the isotopic triplet $(N^{12}, C^{12}, B^{12})$

Consider now the following transitions



The test consists in comparing the electromagnetic transition $C_p^{12*} \rightarrow C^{12}$ with the Gamow-Teller $(G.T)$ $\beta$-decays $B^{12} \rightarrow C^{12}$ and $N^{12} \rightarrow C^{12}$. According to the C.V.C. hypothesis, the matrix elements for all these three transitions should be the same except for minor coulomb corrections. Since the energy release in the $\beta$ -transitions are sufficiently large, in addition to the allowed G.T. axial vector transitions, the first forbiddent A-V interference term also will contribute to these transitions and the separation of a single term becomes difficult. However, fortunately these interference terms are of opposite signs when we consider both $\beta^+$ and $\beta^-$ transition so that by taking both $\beta^+$ and $\beta^-$ transitions one could eliminate these interference terms and compare the weak magnetism term in $\beta$ -decay with the corresponding ones in electromagnetic transition. For the $\beta^-$ decay spectrum we have

$$\frac{dN^-}{dE} = \frac{(dN^-)}{(dE)_{allowed}}\left[1+\frac{8}{3}a\left(E-\frac{E_0}{2}-\frac{m^2}{2E}\right)+\frac{2}{3}b\left(E_0-\frac{m^2}{E}\right)\right]$$

The first term on the right hand side corresponds to the allowed transitions, the second term to the first forbidden term and the third term to the second forbiddent term. If we neglect the third term we will have

$$\frac{dN^-}{dE} = \left(\frac{dN^s}{dE}\right)_{allowed}\left(1+\frac{8}{3}aE\right)$$

~~allowed~~ for $\beta^-$ decay

Similarly

$$\frac{dN^+}{dE} = \left(\frac{dN}{dE}\right)_{\text{allowed}} \left(1 - \frac{8}{3} a E\right) \text{ for } \beta^+ \text{ decay}$$

It should be remembered that 'a' comes from weak magnetism term. Its value for electromagnetic transitions is

$$a = 1 + \mu_p - \mu_N \approx 4.7$$

The value of a for $\beta$ transitions should be the same as that for electromagnetic transitions if C.V.C. hypothesis holds. Careful experimental measurements of 'a' in the $\beta$-transition have shown that the C.V.C. hypothesis is correct.

## $\beta$-decay of the $\pi$-meson

Consider the decay

$$\pi^+ \longrightarrow \pi^0 + e^+ + \nu$$

In the absence of the C.V.C. hypothesis, the above process can go only through a virtual $N\bar{N}$ pair loop and thus one has to know much about strong interaction effects. However, C.V.C. predicts a definite rate for the above decay and one can compare this with experiments. It is a very easy matter to show that only vector part contributes to the above decay and the matrix element may be written as

$$m \sim \frac{g_V}{\sqrt{2}} \langle \pi^0 | J_\mu^V | \pi^+ \rangle \left[\bar{\psi}_e \gamma_\mu (1+\gamma_5) \psi_\nu\right]$$

The procedure consists in relating the matrix element $\langle \pi^0 | J_\mu^V | \pi^+ \rangle$ with the matrix element of the isospin operator $I^-$. We shall only quote the result. The branching ratio according to the C.V.C. hypothesis is

$$R_{Theory} = \frac{\Gamma(\pi^+ \to \pi^0 + e + \nu)}{\Gamma(\pi^+ \to \mu + \nu)} = 1.07 \pm 0.02 \times 10^{-8}$$

and

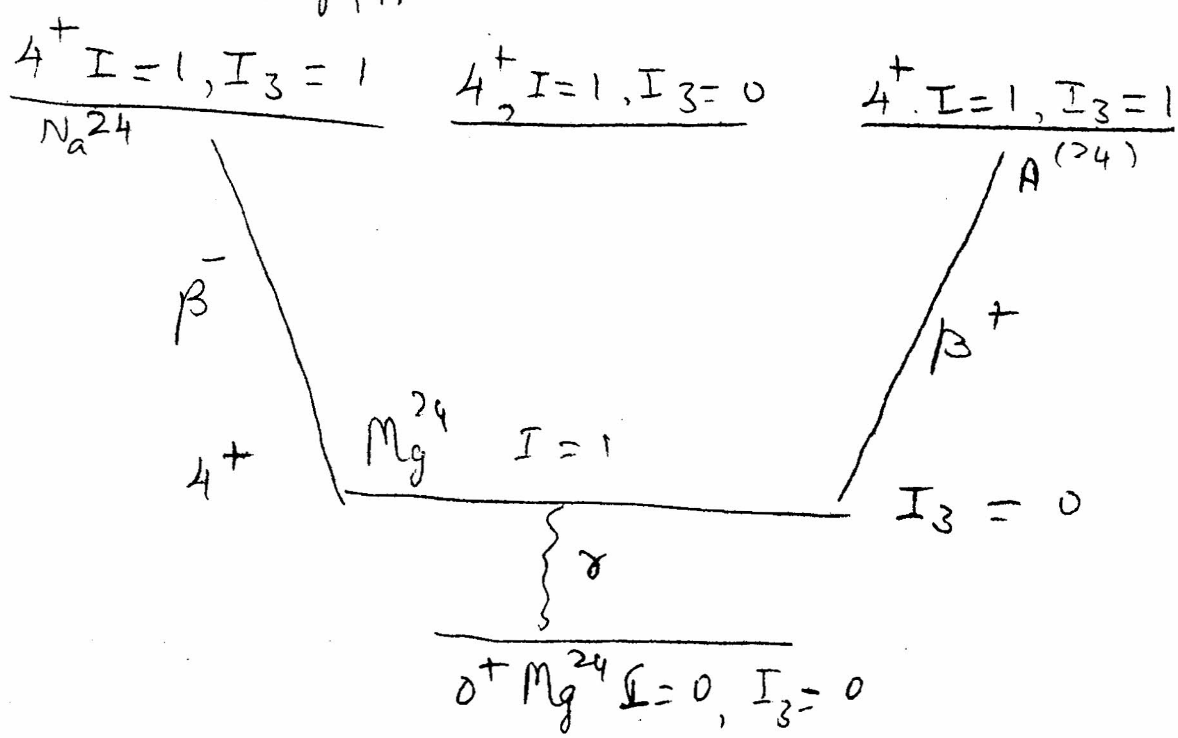
$$R_{Exp} = 1.15 \pm 0.22 \quad 10^{-8}$$

$$= (1.08 \pm 0.22) \quad \text{theory}$$

Later experiments have shown even closer fit thus establishing the correctness of the C.V.C. hypothesis.

## $\beta$-$\gamma$ angular correlation

Bouchiat has suggested the following test. This test consists in the analysis of transitions of the isotopic triplet $Na^{24}, Mg^{24}, A^{(24)}$. The transitions are given by



In this case, both Fermi and G.T. transitions occur. By studying the $\beta-\gamma$ angular correlation, we can get the Fermi part of the matrix element (For Fermi transition we have transitions). Of course, some $\Delta I =$ transition will be present due to isotopic impurities of the nucleus. If these impurities are due to only coulomb interaction, then C.V.C. will hold good. The angular correlation is given by

$$W^{\pm}(\theta) = 1 + A^{\pm} \cos\theta$$

where

$$\theta = \angle \gamma \text{ ray and } \vec{P_\beta}$$

and

$$A \sim M_{Fermi} / M_{G.T}$$

Under charge conjugation

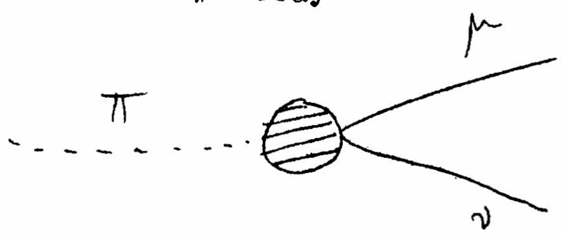
$$M_{G.T} \rightarrow M_{G.T}$$
$$M_F \rightarrow -(M_F)$$

so that C.V.C. hypothesis predicts $A^+ + A^- = 0$. Experimentally this is satisfied.

## Lecture 5

### Partially Conserved Axial Vector Current and Goldberger-Treimen Relation

In the last lecture, we were considering only the vector part of the weak interaction current. In this lecture we shall discuss about the axial vector current. For this let us take the $\pi$-decay




The black box contains all strong interaction effects since the pion is pseudoscalar, only axial vector current contributes. The matrix element for this process can be written as

$$A(\pi \to \mu\nu) \sim \langle 0 | J_\lambda^A | \pi \rangle \, \ell_\lambda$$

$$\ell_\lambda = \bar{\psi}_\mu O_\lambda \psi_\nu$$

$$O_\lambda = \gamma_\lambda (1 + \gamma_5)$$

and

$$\langle 0 | J_\lambda^A | \pi \rangle = - i \, g_\pi \, P_\lambda^\pi$$

since $p_\lambda^{\pi}$ is the only vector available. $g_\pi$ has the dimension $(1/m)$. By conservation of momentum, we have

$$P_\pi = P_\mu + P_\nu$$

Hence

$$A(\pi \longrightarrow \mu \nu) \sim g_\pi m_\mu \bar{\psi}_\mu (1+\gamma_5) \psi_\nu$$

and therefore

$$R(\pi \rightarrow \mu \nu) = \frac{g_\pi^2 m_\mu^2}{4\pi m_\pi^3} (m_\pi^2 - m_\mu^2)^2$$

Experimentally $\mu^+$ was found to have left helicity. Experimentally $\nu_\mu$ helicity was found to be correct and

$$g_\pi^2 = \frac{2.2 \times 10^{-4}}{m_\pi^2}$$

Also

$$\frac{R(\pi \rightarrow \mu \nu)}{R(\pi \rightarrow \mu \nu)} = \frac{g^2_{\pi e \nu}}{g^2_{\pi \mu \nu}} \left(\frac{m_e}{m_\mu}\right)^2 \left[\frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2}\right]$$

$$= 1.2 \times 10^{-14} \left(\frac{g_{\pi e \nu}}{g_{\pi \mu \nu}}\right)^2$$

The experimental value is $1.2 \times 10^{-4}$ thus implying

$$g_{\pi e\nu} = g_{\pi\mu\nu}$$

This is the so-called $\mu - e$ universality. It should be remarked that even though the pseudoscalar coupling is allowed by parity, yet such a coupling would lead to the unphysical result $\left[ m_e / m_\mu \right]^2 \to 1$ so that pion decay is a strong evidence for the non-existence of pseudoscalar interaction.

Goldberger-Treiman Relation

It is a very easy matter to show that the axial vector current cannot be conserved. This is so because

$$\langle 0 | \partial J_\lambda^A | \pi \rangle = -i\, g_\pi (p_\lambda^\pi)^2$$
$$= -i\, g_\pi\, m_\pi^2$$

so that if $\partial J_\lambda^A = 0$ then $g_\pi = 0$

That is pion is not coupled to leptons at all. This is contradictory to experiments. So

$$\partial \bar{J}_\lambda^A = 0$$

The most general matrix element for -decay for the axial vector interaction part can be written as

$$\langle p | J_\lambda^A | n \rangle = i\, \bar{u}_p \left[ F_A(q^2) \gamma_\mu \gamma_5 - i F_p(q^2) q_\mu \gamma_5 - i h_A(q^2)(P_\mu + n_\mu) \gamma_5 \right] u_n$$

If one assumes G-invariance of this nonstrange decay matrix element (the invariance holds since $[G, S] = 0$, $\hat{S}$ being the strangeness operator) then it is easy to show that

$$G(\bar{\Psi}_N \gamma_\mu \gamma_5 \tau^+ \Psi_N) G^{-1} = -(\bar{\Psi}_N \gamma_\mu \gamma \tau^+ \Psi_N)$$

$$G(\bar{\Psi}_N q_\mu \gamma_5 \tau^+ \Psi_N) G^{-1} = -(\bar{\Psi}_N q_\mu \gamma_5 \tau^+ \Psi_N) G^{-1}$$

while

$$G(\bar{\Psi}_N (P_\mu + n_\mu) \gamma_5 \Psi_N) G^{-1}$$

$$= + \bar{\Psi}_N (P_\mu + n_\mu) \gamma_5 \Psi_N$$

Here

$$G = C\, e^{i \pi I_2}$$

C being the charge conjugation operator.

When $q = 0$ (as in the case of -decay) the dominant term is odd under $G$ so that it is highly reasonable to assume

$$G J_\lambda^{\text{axial vector}} G^{-1} = - J_\lambda^A$$

and so

$$h_A = 0$$

by $G$ invariance.

The axial vector term even under $G$ is called the irregular term and in fact the presende of these terms may be responsible for a slight violation of time-reversal invariance. Thus

$$\langle p \left| \frac{\partial J_\mu^A}{\partial x_\mu} \right| n \rangle = i (\bar{u}_p \gamma_5 u_n) \left[ 2 m_N F_A(q^2) - q^2 F_p(q^2) \right]$$

If $\dfrac{\partial J_\mu^A}{\partial x_\mu} = 0$ then

$$F_p(q^2) = \frac{2M}{q^2} f_p(q^2)$$

which when substituted back in the $\beta$-decay matrix element leads to the wrong result viz. that the pseudoscalar interaction is more dominant that the axial vector interaction. Thus, even here we find

$$\langle p \left| \frac{\partial J_\lambda^A}{\partial x_\mu} \right| n \rangle \neq 0$$

Let us write $\chi(q^2)$ as

$$\chi(q^2) = 2 M f_A(q^2) - q^2 f_p(q^2)$$

and since $\dfrac{\partial J_\mu^A}{\partial x_\mu}$ is a pseudoscalar, let us write down a dispersion relation for $\chi(q^2)$ with one pion pole dominant approximation. Then

$$2MF_A(q^2) - q^2 F_p(q^2)$$

$$= \frac{\sqrt{2}\, G_\pi g_\pi m_\pi^2}{q^2 + m_\pi^2} + \int_{(3m_\pi)^2}^{\infty} \frac{\sigma(q'^2)\, q'^2\, dq'^2}{(q'^2 + q^2 - i\epsilon)}$$

Assume now that

$$\lim_{q^2 \to 0} q^2 F_A(q^2) = 0$$

Then, at $q^2 = 0$, we have

$$2MF_A(0) = \sqrt{2}\, G_\pi g_\pi + \int \sigma(q'^2)\, d\cdot q'^2$$

As a third assumption let us neglect the higher terms and retain only the pole term. Then

$$2MF_A(0) = -\sqrt{2}\, G_\pi g_\pi \qquad \text{ie } F_A(0) = \frac{-G_\pi g_\pi}{\sqrt{2}\, M}$$

$G_\pi$ is the pion-nucleon coupling constant and $g_\pi$ is determined from pion-decay. Now if $F_A(q^2)$ is well behaved in the region $0$ to $m_\pi^2$ and when single pion pole term is due to $F_p$ essentially then

$$F_p(q^2) = \frac{-\sqrt{2}\, G_\pi g_\pi}{q^2 + m_\pi^2} = \frac{2MF_A}{q^2 + m_\pi^2}$$

This is the Goldberger-Trieman relation and this is very good agreement with experiment. The axial vector current is conserved in the limit $m^2_\pi \to 0$.

## Lecture 6

### High Energy Neutrino Reactions

The study of weak interactions at high energies may throw more light on the structure of weak interactions. For this, the best reaction is the high energy neutrino reactions. Neutrinos are produced through the familiar decay modes of the $\pi$ and K mesons which are copiously produced from accelerators. Since, the cross-section for such a reaction (for example $\nu + N \to N + \ell$) is very small ($\simeq 10^{-38}\,cm^2$, when $E_\nu \sim 1$ GeV), so that we need intense beams of very high energy neutrinos and consequently massive detectors. However, the advancement made in this field is really magic thanks to the expensive schemes of CERN and Brookhaven Laboratories. In the following few lectures, we shall discuss some experimental details and theoretical conclusions, one could draw from these experiments.

Fast $\pi$ and k meson beams are produced by energetic protons accelerated by the large synchrotrons. (energy of the protons $\sim$ 25 GeV). To produce neutrinos, these mesons have to be allowed to decay. $\mu$-mesons are essentially stopped by ionization loss. With the energy attained by the $\mu$'s originating from $\pi$'s and $K$'s produced by a 25 GeV proton beam, a thickness of iron $\sim$ 20m is required. Consequently, the distance between target and detector has to be large ($\sim$ several tens of metres). The spread of neutrinos is avoided through focussing arrangements. The convention devices are of least use in such high momentum intervals. A new device has been designed by S.Van der Meer called the 'magnetic horn'. Two types of detectors have been used, a large heavy liquid bubble chamber and a massive spark chamber. The bubble chamber that has been used, contained $\sim$ 750 K.G. of $CF_3Br$ (Freon) and was equipped with a magnetic field of $\sim$26 K.G. The CERN spark chamber consisted of an 18 ton brass or Aluminium spark chamber with plates 0.5 thick. Behind this, there was a 50 ton spark chamber consisting of lead plates 5 to 10 c.m. thick. Between two sections, a magnetic field of 3 K.G. strength was placed, extending 1 metre along the beam direction. There have been some recent suggestions by G.Bernardini to use magnetised iron plates between two adjascent gaps. The events obtained are (CERN)

| | | | |
|---|---|---|---|
| 1963 run | - | 18 events/hr | in spark chamber |
| 1964 run | - | 40 events/hr | in spark chamber |

This has to be compared with 10 events/day using bubble chamber. However, the precision in the analysis of the events obtainable with a spark chamber is far below that of a bubble chamber, where coordinates can be measured with the precision of a fraction of a millimetre. The experimental results obtained give us good deal of information in the following physical problems.

(a) Existence of two neutrinos

(b) Intermediate Vector bosons

(c) Weak neutral currents

(d) Elastic neutrino reactions and weak form factors

(e) Inelastic neutrino reactions

(f) Neutrino flip hypothesis

Existence of two neutrinos

The CERN experiments have confirmed the earlier Brookhaven experiments. The beams were obtained from $\pi$-decay. The following reactions were analysed

$$\nu + n \rightarrow p + \mu^- \qquad (1)$$

$$\nu + n \rightarrow p + e^- \qquad (2)$$

The ratio of the two reaction was found to be

$$\frac{R(2)}{R(1)} = 1.7 \pm 0.5\ \%$$

This then confirms the existence of two neutrinos one associated with electron and the other one with the muon. As a remark, we mention, that if we accept this two neutrino hypothesis, the absence of the reaction $\mu \rightarrow e + \gamma$ could be reasonably understood.

Neutrino Flip hypothesis

It was earlier suggested that the neutrino associated with the muon in $\pi$ decay corresponds to the neutrino associated with the electron in k-decay i.e.,

$$\pi \rightarrow \pi + \nu_1 \qquad K \rightarrow \mu + \nu_2$$
$$\pi \rightarrow e + \nu_2 \qquad K \rightarrow e + \nu_1$$

It then follows that just as $\pi$ decays dominantly to $\nu_\mu$, K mesons will dominantly decay into $\nu_e$. If this hypothesis were true, out of 2100 spark chamber events, 200 should be electron events while experimentally, the number of electron events was only 44. Even among this, half of the events can be attributed to electrons coming from $K_{e3}$ decay.

Thus the neutrino-flip hypothesis cannot be accepted.

Conservation of leptons

The lepton number conservation may be divided as the independent conservations of electron number and muon number respectively viz.

$$l = l_e + l_\mu$$
$$l_e = N_e - N_{\bar{e}}$$
$$l_\mu = N_\mu - N_{\bar{\mu}}$$

From the angular of the charged lepton production by the ,
there has been a good evidence for the conservation of lepton number to a very good accuracy.

Neutral Currents leptonic

If neutral lepton currents were coupled with the same strength as the charged ones, the ratio of the neutral current elastic events

$$\nu + p \rightarrow \nu + p$$

to the 'charged' elastic events

$$\nu + n \rightarrow p + \mu^-$$

should be one. Experimentally

$$\frac{R(\nu + p \rightarrow \nu + p)}{R(\nu + n \rightarrow p + \mu^-)} \sim 7\%$$

Thus we can give an upper limit to the cross-section in neutral (current) reactions to be $10^{-39}\ cm^2$. Even if $\nu + p \rightarrow \gamma + p$ is seen, we cannot say whether it is due to neutral lepton currents or due to electromagnetic properties of the neutrino. Then we should study the vertex $(\nu - \gamma - \nu)$. The most general matrix element may be written as

$$\langle \gamma | J_\mu^{c.m} | \nu \rangle \sim \bar{u}_\nu \gamma_\mu (1+\gamma_5) u_\nu \frac{e}{6} q^2 f(q^2)$$

$$\langle \gamma | J_\mu^{\gamma} | \nu \rangle = F_1 \gamma_\mu + F_2 \sigma_{\mu\nu} q_\nu + F_1' \gamma_\mu \gamma_5 + F_2' \sigma_{\mu\nu} q_\nu \gamma_5$$

According to the two component neutrino theory $F_2 = f_2' = 0$. This is because the projections for neutrino and anti-neutrino are $(1+\gamma_5)$ and $(1-\gamma_5)$ respectively, and $(1+\gamma_5)(1-\gamma_5)=0$. That is, the magnetic moment of a two-component neutrino is zero. The electric form factor of the neutrino can be calculated. It is found that

$$\langle f^2 \rangle^{1/2}_{theory} \sim 10^{-34} \text{ to } 10^{-35} \text{ cm}^2$$

theoretically, while experimentally,

$$\langle f^2 \rangle^{1/2}_{Exp} \leqq 3 \times 10^{-32} \text{ cm}^2$$

Thus, the electromagnetic form factors of the neutrinos, if they exist, should be too small to produce observable effects.

The absence of lepton neutral currents can also be understood from pair production of k-mesons. Experimentally

$$\frac{R(K_2^0 \to \mu^+ + \mu^-)}{R(K^+ \to \mu^+ + \nu)} = \frac{4\,(G_{\mu\mu})^2 m_K^3 (m_K^2 - 4m_\mu^2)}{|G_{\mu\nu}|^2 (m_K^2 - m_\mu^2)}$$

$$< 10^{-3}$$

This leads to the conclusion that

$$\frac{(G_{\mu\mu})^2}{(G_{\mu\nu})^2} < 2.5 \times 10^{-4}$$

Similarly from the experiments on the reactions it is found that

$$\frac{|G_{e-pair}|^2}{|G_{e\nu}|} \lesssim \frac{1}{2} \times 10^{-2}$$

and from the experiments on $K^+ \to \pi^+ + 2\nu_e$ or $2\nu_\mu$ it is found that

$$\frac{|G_{\nu_e \nu_e}|^2 + |G_{\nu_\mu \nu_\mu}|^2}{|G_{e\nu}|^2} \leq \frac{1}{6}$$

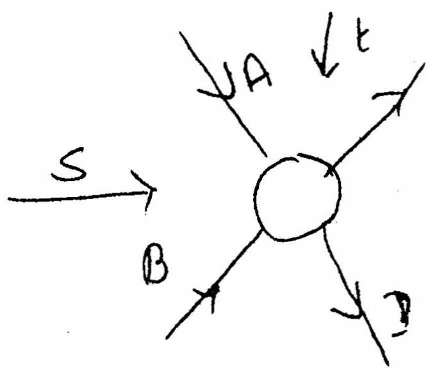
Lecture 7

## Elastic Neutrino Reactions

The reactions

$$\nu + n \to p + e^-$$
$$\bar{\nu} + p \to n + e^+$$

are called elastic neutrino (antineutrino) reactions



Define the invariants

$$s = -(p_A + p_B)^2$$
$$t = -(p_A + p_c)^2$$
$$u = -(p_B + p_c)^2 :$$

We are eventually interested in the $s$-channel. The differential cross-section can be written as

$$\frac{d\sigma}{dt} = \frac{\sum_{pol} |T|^2}{\left[s-(M_A+M_B)^2\right]\left[s-(M_A-M_B)^2\right](2S_A+1)(2S_c+1)}$$

$$\sum_{Spin} |T|^2 = \sum_{\ell=0}^{2J_{max}} A_\ell(t)\, P_\ell(\cos\phi)$$

Here J is the maximum angular momentum in the t-channel. $S_A, S_t$ are the spin of the particles A and C respectively. The angle $\phi$ (the scattering angle in the t-channel) is defined by

$$\cos\phi = \frac{t(s-u)-(M_A^2-M_c^2)(M_B^2-M_D^2)}{\left\{\left[t-(M_A+M_c)^2\right]\left[t-(M_A-M_c)\right]^2\left[t-(M_B+M_c)\right]^2\left[t-(M_B-M_c)\right]\right\}^{1/2}}$$

Let us suppose that there exists some analytic continuation from the t-channel to the s-channel.

$$\frac{d\sigma}{dt} \doteq \frac{\sum_{J=0}^{2J_{max}} B_\ell(t)\,(s-u)^\ell}{\left[s-(m_A+m_B)^2\right]\left[s-(m_A-m_B)^2\right](2S_A+1)(2S_C+1)}$$

In the s-channel (in which we are interested in) being the momentum transfer ~~in~~ the reaction, then for reaction

$$\nu + B \rightarrow \ell^- + D$$

we find

$$\frac{d\sigma_\nu}{dq^2} = \frac{1}{k_\nu^2}\left[A(q^2) + B(q^2)(s-u) + C(q^2)(s-u)^2\right]$$

and for the reaction

$$\bar{\nu} + D \longrightarrow \ell^+ + B$$

we find

$$\frac{d\sigma_{\bar{\nu}}}{dq^2} = \frac{1}{k_\nu^2}\left[A(q^2) - B(q^2)(s-u) + C(q^2)(s-u)^2\right]$$

where we have assumed PC Conservation. This formula looks like exactly the same as Rosenbluth formula for electron scattering but for the term $B(q^2)(s-u)$. This absence of this term actually signifies the party conservation in the electron scattering process. This term arises due to the interference between the vector and axial vector terms. At high energies, ( $\sim$10 GeV), the term $C(q^2)$ dominates. Thus,

$$\lim_{k^2 \to \infty} d\sigma_\nu = d\sigma_{\bar{\nu}} \sim C(q^2)(s-u)^2$$

and the parity violating term, $B(q^2)$ loses its importance. In other words, at high energies, the neutrino reaction proceeds as though parity is conserved.

Weak Form Factors

It was earlier shown that the Hamiltonian for the weak leptonic process may be written as

$$H_\omega \sim g J_\mu l_\mu^+ + h.c$$

where

$$J_\mu = i\bar{\nu}\gamma_\mu(1+\gamma_5)l$$

The weak form factors arise actually due to the strong current. The matrix element of $J_\mu$ (the weak current of strong interacting particles) can then be written as

$$\langle k'|J_\mu|k\rangle = i\bar{u}_p(k')$$
$$\Big[F_1\gamma_\mu - \frac{\mu}{2M}F_2\sigma_{\mu\nu}q_\nu + ih_\nu q_\mu$$
$$+ \lambda F_A\gamma_\mu\gamma_5 - \frac{ib}{m}F_p\gamma_5$$
$$- ih_A(k_\mu + k'_\mu)\gamma_5\Big]u_n(k)$$

$K$ and $K'$ are the four momenta of the initial and final nucleons $M$ is the mass of the nucleon $m$ is the mass of the lepton $F's$ and $h's$ are functions of $q^2$, $q$ being the momentum transfer i.e. $q = (K' - K)$. Invariance under time reversal implies that $F's$ and $h's$ are real. From C.V.C. theory we have

$$h_V = 0$$

$$F_1 = F_1^{C.m} \qquad F_1(0) = 1$$

$$F_2 = F_2^{C.m} \qquad F_2(0) = 1$$

$$\mu = (\mu_p - \mu_n) \approx 3.01$$

= anamalous magnetic moment

$h_V = 0 = h_A$ by G-invariance. From $\beta$ -decay we know that

$$\lambda F_A(0) = 1.25$$

From the analysis of $\mu$ -capture, we find that the induced pseudoscalar term

$$h \sim 8\lambda$$

We have already seen that

$$\frac{d\sigma_{\nu\bar{\nu}}}{dq^2} = \frac{G^2 m^2}{16\pi} \frac{1}{K^2_{lab}} \left\{ A(q^2) \pm B(q^2)(s-u) + C(q^2)(s-u)^2 \right\}$$

where

$$A(q^2) = q^2 \Big\{ \Big(1 + \frac{q^2}{4M^2}\Big)\Big( -F_1^2 + \lambda^2 F_A^2 - \frac{q^2}{4M^2}\mu^2 F_2^2 \Big)$$

$$+ \frac{q^2}{2M^2}(F_1 + \mu F_2)^2 - \frac{m}{M} \cdot \lambda b F_A F_p$$

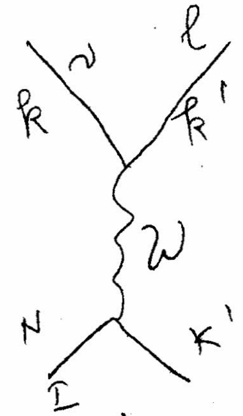
$$+ \frac{q^2}{4M^2} b^2 F_2^2 \Big\}$$

and $$B(q^2) = \frac{q^2}{M^2}(F_1 + \mu F_2)\lambda F_A$$

$$C(q^2) = \frac{1}{4M^2}\Big( F_1^2 + \lambda^2 F_A^2 + \frac{q^2}{4M^2}\mu^2 F_2^2 \Big)$$

$$G \approx \frac{10^{-5}}{M^2 p} \qquad g_V = \sqrt{2}\, G$$

If we suppose that the above reaction goes via an intermediate boson




then

$$H \sim g\,(l_\mu W_\mu^+ + J_\mu W_\mu^+) + h \cdot c$$

The matrix element

$$\langle F|T|i\rangle = g^2 \langle F|J_\mu|I\rangle \frac{\delta_{\mu\nu} - q_\mu q_\nu}{(q^2 + M_\omega^2)} \langle f|l_\nu^+|i\rangle$$

where

$$q = (K - K') = (k' - k)$$

Thus

$$\langle F|T|i\rangle \frac{g^2}{m_\omega^2} \langle F|J_\nu'|I\rangle \langle f|l_\nu^+|i\rangle$$

where

$$J_\nu' = \left(J_\nu - \frac{J_\mu q_\mu q_\nu}{M_\omega^2}\right)\left(1 + \frac{q^2}{M_\omega^2}\right)^{-1}$$

so that the scattering cross-section is exactly the same as in the previous case of point interaction, but now

$$F_{12}^A \rightarrow F_{1,2}'^A = F_{12}^A\left[1 + \frac{q^2}{M_\omega^2}\right]^{-1}$$

$$F_p \rightarrow F_p' = F_p + \frac{2\lambda m}{b} \frac{m}{M_\omega} F_A\left(1 + \frac{q^2}{m_\omega^2}\right)^{-1}$$

$$h_A^B \rightarrow h_A' = h_A\left(1 + \frac{q^2}{m_\omega^2}\right)^{-1}$$

It is very easy to see that as

$$m_\omega \longrightarrow \infty \qquad F' = F$$

Comparison with Experiments

Usually the reaction that has to be considered is not the isolated reaction $\nu + N \rightarrow N + \ell$ , but rather $\nu +$ Nucleus $\rightarrow$ (Nucleus)$'$ + $\ell$ so that assumption have to be made regarding the nuclear models to be used. Secondly, experimentally one does not know the clear-cut neutrino spectrum and thirdly the information is not quite resourceful about the form factors.

As for as the nuclear model is concerned, mostly calculations are made using Fermi Statistical Model so that for the reaction on a bound nucleon we have to make the following correction

$$\left(\frac{d\sigma}{d\Omega}\right)_N = \frac{3}{4\pi p_F^3} \int d\vec{k} \left(\frac{d\sigma}{d\Omega}\right)_{free}$$

$p_F$ is the Fermi momentum actually in experiments, the average recoil of the proton $\sim$ 100 to 500 MeV.

Form Factors and Comparison with Experiment

In order to determine the form factors pertinent to the elastic reactions, the four momentum transfer $q$ was determined for each event. The resulting distribution (experimental) was compared with the theoretical cross-section $\frac{d\sigma}{dq^2}$ in which the vector form factors had been taken to be equal to the electromagnetic form factors, the pseudoscalar term neglected. Then $d\sigma/dq^2$ is determined except for the axial form factor which was taken to be of the form

$$F_A \sim \left(1 + \frac{q^2}{M_A^2}\right)^{-2}$$

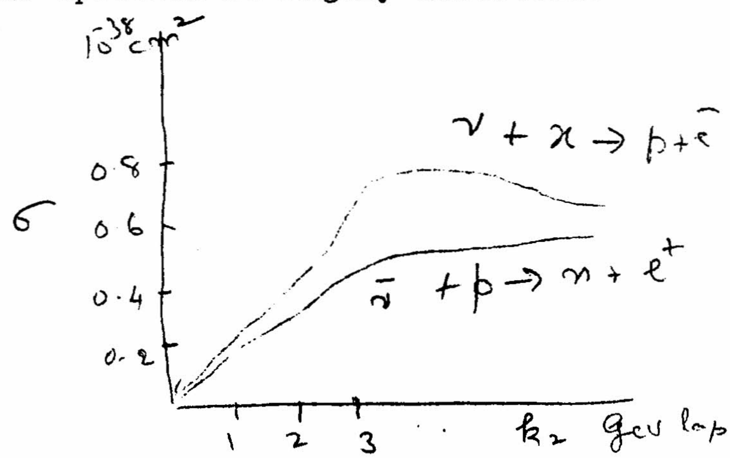
Fitting the theoretically predicted curve on the experimental data allowed one to determine the value of $M_A$. It was found that

$$M_A = 0.85 + 0.5 - 0.45 \ \mathrm{Gev}/c^2$$

The results are completely in consistency with theory except for a small region where neutrino spectrum is highly uncertain. The future plane is to find out

$$(d\sigma_\nu - d\sigma_{\bar{\nu}})$$

which will be a measure of $B(q^2)$ and so $F_A$ can be calculated.




The proton polarization is being measured at CERN.

Theoretically, it has been shown that for the reaction

$$\nu + \ell \longrightarrow \ell + \text{anything}$$

the transverse polarization must be zero if Time reversal invariance holds

$$\lim_{k_r \to \infty} P_L = F \frac{x}{\sqrt{1+x^2}} \frac{2\lambda F_A (F_1 - \mu F_2)}{\lambda^2 F_A^2 + F_1^2 + \mu^2 F_2^2 x^2}$$

where

$$x^2 = \frac{\xi^2}{(2m)^2}$$

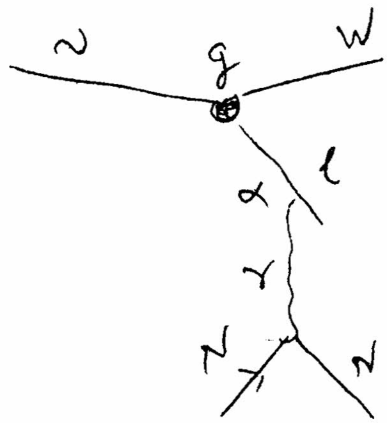
Lecture 8

## Intermediate Vector Boson

One problem which had mostly been of academic interest is the existence of Intermediate Vector Bosons (IVB) mediating weak interactions just as $\pi$ mesons mediate strong interaction between nucleons. The coupling of IVB with baryons and leptons may be written in a general form as

$$H_w \sim g(\ell_\mu W_\mu^+ \; J_\mu W_\mu^+) + h.c$$

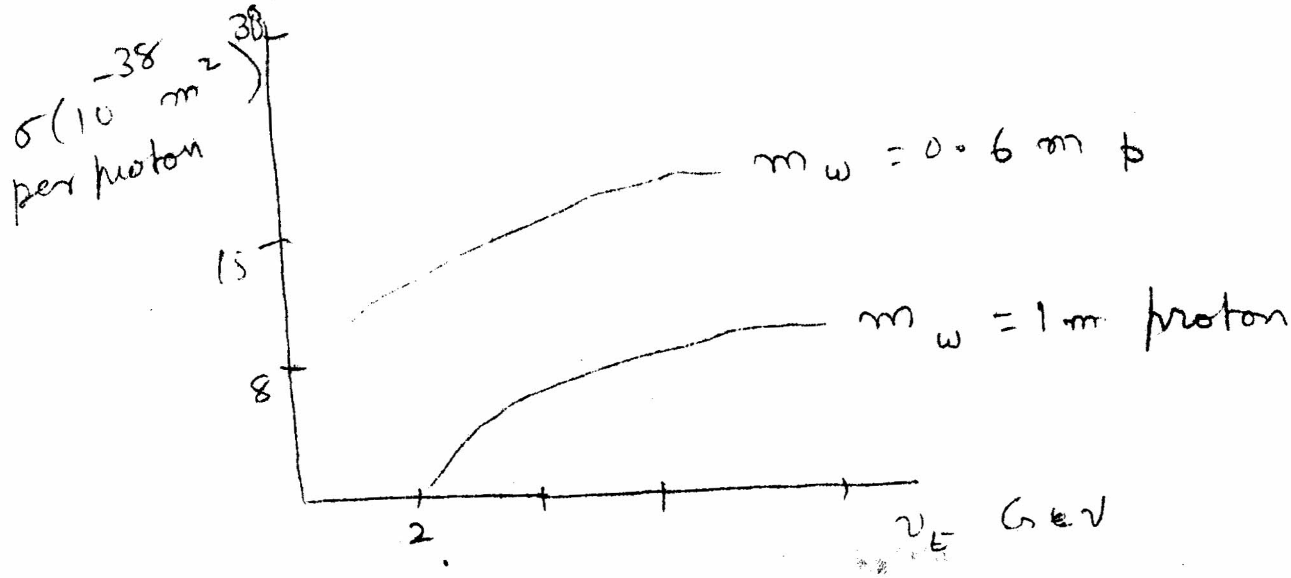
where $\ell_\mu$ is the current of the leptons and $J_\mu$ is the hadronic current. Let us look at first order processes in which $\omega$ may be created. One such possible reaction is the semi-weak neutrino reaction




Here $g$ is the coupling constant for the IVB with the leptons (semi-weak) this is related to the universal Fermi coupling constant, through the relation

$$\frac{g^2}{m_w^2} = \frac{G^F}{\sqrt{2}}$$

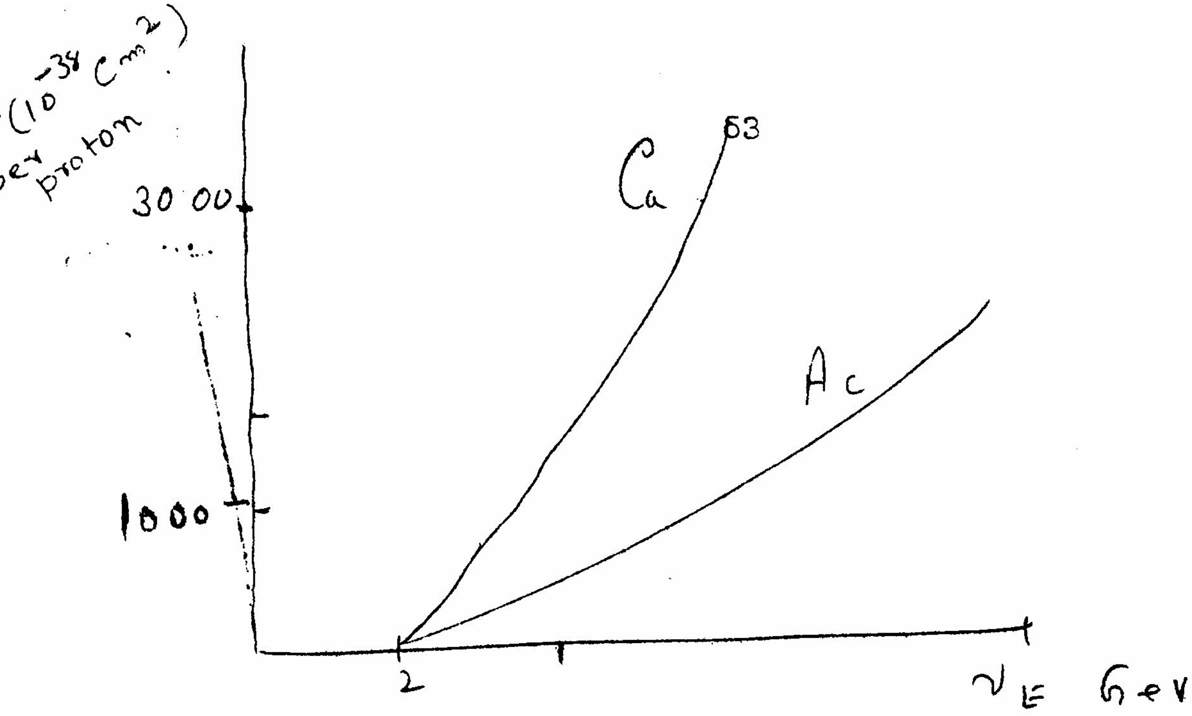
Calculating of cross-section for these reactions have been made by Lee and others. We will have to distinguish two types of reactions. Coherent reactions are those in which the nucleus recoils as a whole and incoherent reactions are those in which a particular nucleon inside the nucleus recoils. For incoherent reaction, usually calculations are made using Fermi Model for the nucleon inside the nucleus. The resulting cross-section obtained for the above process is given by




For the coherent reaction, the photon is attached to the nucleus itself and the nucleus recoils as a whole. For the nucleus, Fermi charge distribution

$$\rho(r) \propto \left\{ 1 + e^{-\left(\frac{r-R}{b}\right)} \right\}^{-1}$$

is assumed where R = mean sq. radius $\sim 1.07A^{\frac{1}{3}} \times 10^{-13}$ cm and b = impact perameter $\sim 0.568 \times 10^{-13}$ cm. The cross-section in this case is given by the plot




It is clear that the cross-section for coherent reaction is very much larger than for incoherent reaction. This is because of the electromagnetic interaction.

Then the total cross-section for the above process is given by the sum of coherent and incoherent cross-sections.

Total cross-section in $10^{-38}$ / per nucleons

| $E_\nu$ | $\sigma$ (Cn) $m_w = 1 m_p$ | $\sigma_{total}$ (AC) $M_w = 0.6 M_p$ |
|---|---|---|
| 2 BeV | 4.34 | 118.4 |
| 4 BeV | 144.00 | 1083.0 |

<u>Decay of W -meson</u>

The principal decay modes of W-meson are

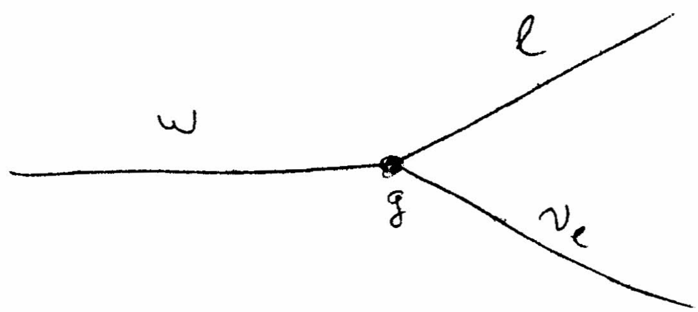
$W \to 2$ leptons

$W \to 2\pi$

etc. The general interaction is given by

$$H_W \sim g(l_\mu + J_\mu) W_\mu^+ \quad h.c$$

where $l_\mu$ and $J_\mu$ are the current of the leptons and hadrons respectively. Let us first consider the decay of $W$ into two leptons




Here no strong interaction is involved. The coupling is only semi-weak. The rate for this decay is given by (just the same as in $\pi \rightarrow \mu\nu$ decay)

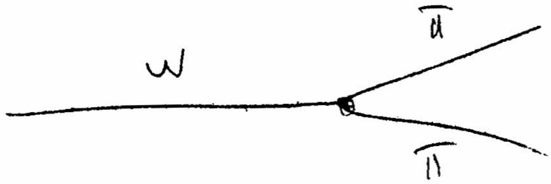
$$R(W^\pm \rightarrow l^\pm + \nu) = \frac{g_{l\nu}^2}{6\pi} M_W$$

$$= \frac{G_F}{6\pi\sqrt{2}} M_W^3$$

One thing which could be said about $m_W$ is that $m_W < m_K$. Otherwise, $W$ must have decayed into a $K$ meson and must have been observed. Using the approximate value $m_W \simeq m_K$, we find that

$$R\,(\omega^{\pm} \longrightarrow \ell^{\pm} + \nu_e) \geq 10^{17}\ sec^{-1}$$

Another decay mode of $\omega$ is $\omega \rightarrow 2\pi$



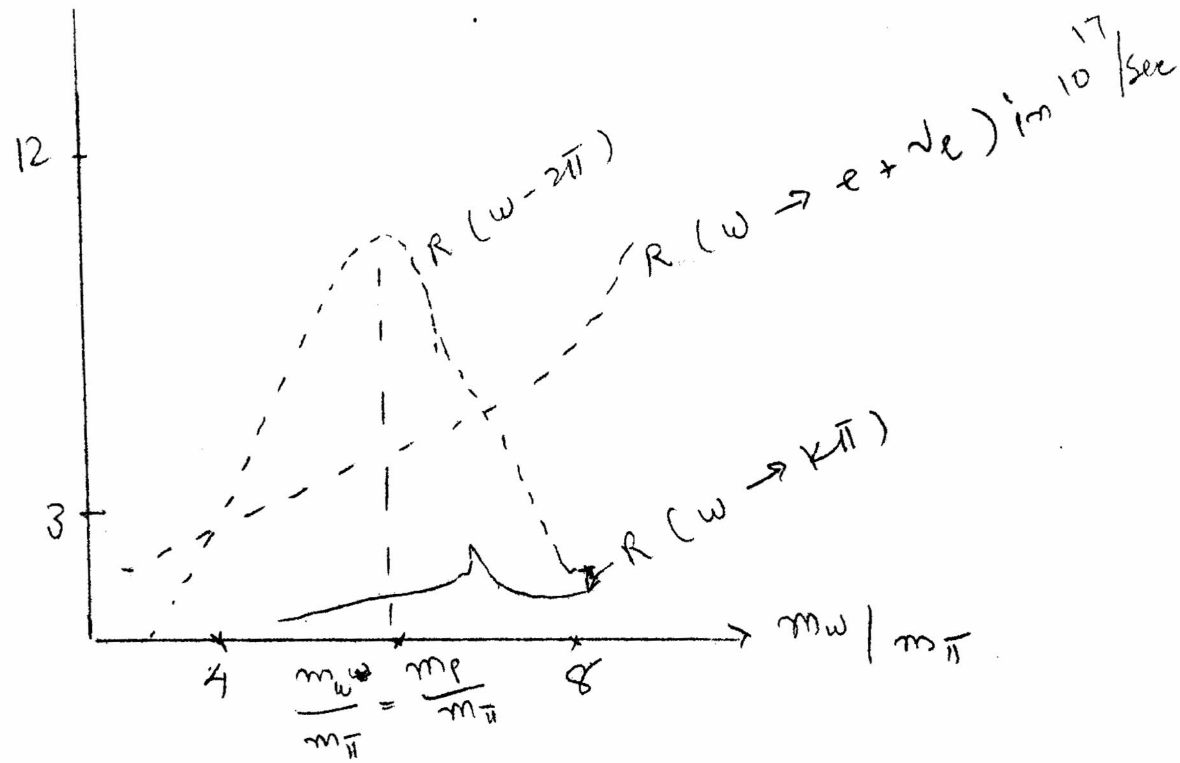

The $\omega_{\pi\pi}$ vertex is known from the C.V.C. hypothesis. The coupling is then through the isospin-current. The amplitude for $\omega \rightarrow 2\pi$ is given by

$$A(\omega \rightarrow 2\pi) = \frac{\sqrt{2}\, g_{\ell\nu}(p^{+} - p^{0})_{\lambda}\, \epsilon_{\lambda}\, F_{\pi}(m_{\omega}^{2})}{(8 m_{\omega} E_{\pi}^{2})^{1/2}}$$

is the polarization of $\omega$ . $F_{\pi}(m_{\omega}^{2})$ is the same as electromagnetic form factor of $\pi^{+}$ if we assume C.V.C. hypothesis. Then, from the known $K\ell_3$ form factors one finds that

$$R(\omega \rightarrow 2\pi) = \frac{g_{i\nu}^{2}}{24\pi} \left| F_{\pi}(m_{\omega}^{2}) \right|^{2} \left(1 - \frac{4 m_{\pi}^{2}}{m_{\omega}}\right)^{3/2}$$

The various decay rates may be plotted as




The bump in $\omega \to K\pi$ is due to the $K^*$ resonance. The peak in $\omega \to 2\pi$ is due to the $\rho$ meson, dominance. We also find that

$$\frac{R(\omega^+ \to \mu^+ + \nu_\mu)}{R(\omega^+ \to e^+ + \nu_e)} \lesssim 1$$

$$0 \lesssim \frac{R(\omega^+ \to \pi^+ + \pi^0)}{R(\omega^+ \to e^+ + \nu_e)} \lesssim 12$$

and

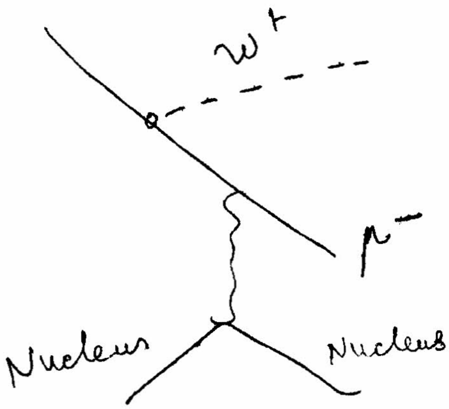
$$0 \lesssim \frac{R(\omega \to K\pi)}{R(\omega \to \ell\nu)} \lesssim 0.5$$

Lecture 9

In this lecture, we shall discuss the experimental indication of the existence or otherwise of the W-mesons from CERN experiment. The I.V.B. could be created in reactions discussed in the previous lecture

$$\nu + (Z, A) \rightarrow (Z, A) + \ell^- + W^+$$

then decays into a lepton and a neutrino




Then the search for the $W$ is linked to finding charged lepton pairs (since by $e$-$\mu$ universality electrons may be equally produced). For this, the spark chamber was a far better instrument than the bubble chamber, as the bubble chamber dimensions are comparable to the interaction length for pions which will make the detection of muons very difficult. In the spark chamber, out of 5200 events they have selected the events in which there were two long tracks (which will correspond to $\mu$-pairs).

**Selection:** The spark chamber contained several interaction lengths and could thus be used to make an analysis of the mean free path of the particles produced in interactions which were candidates for lepton pair interpretation. Select out events in which there are only two tracks of visible range $> .5\ \Lambda_0$ (geometrical range) such that one track $> 1.5\ \Lambda_0$ and the other $> 0.8\ \Lambda_0$. In other words the angle of two tracks should be small. This selection leads to 350 events (out of 5200 possible $\mu$-pairs). One has to make sure that the long track chosen is not due to any strongly interacting particle (such as $\pi$-mesons). They assume to start with that one of the pairs is due to strongly interacting particles and measure the expected track length on this assumption. This is given by

$$L_{\text{expected}} = \frac{\sum_i L_i}{\lambda_i} \qquad i = \text{different strongly interacting particles.}$$

where $L_i$'s are the actual track lengths measured and $\Lambda_i$ is the interaction length of the strongly interacting particles assumed.

$L_i$'s are got from experiments. They have to put the value of $\lambda_i$. The important thing is the calibration of the track lengths of the strongly interacting particles. Then they minimise the number of expected interaction ( I expected), i.e. they put themselves in the least favourable situation. The result is the following

| Experiment | $I_{expected}$ if there is one strongly interacting particle | $I_{observed}$ |
|---|---|---|
| 1963 | 63 | 56 |
| 1964 | 33 | 36 |
| Total | 96 | 92 |

Earlier, due to some wrong calculation, they found that $I_{observed} > I_{expected}$ assuming that there is one strongly interacting particle. This made them to think that $W$ exists. Later, more refined, analysis have been made through better measurements of ranges. From kinematics, $\mu^+$ will have the highest energy compared to $\mu^-$ since $\omega$ is massive. Now, the criterion for selection becomes more restrictive, as

$$\mu^+ \text{ candidate has range } > 7\Lambda_0$$

$$\mu^- \text{ candidate has range } > 2.4\Lambda_0$$

Theoretically the number of such pairs can be predicted, by assuming the neutrino spectrum at high energies and the branching ratio $\dfrac{R(\omega \rightarrow \mu\nu)}{R(\omega \rightarrow a\nu)}$ . The expected and observed

$\mu$ pairs are as follows:

| $M_W$ GeV | Expected $\mu$ pairs. Van der Meer spectrum | Low $q^2$ spectrum | Number of observed $\mu$ pairs |
|---|---|---|---|
| 1.3 | 21 | 51 | |
| 1.5 | 11 | 26 | 0 |
| 1.8 | 4 | 9 | |

The conclusions from the results of the last 1964 Cern neutrino experiments is that if the IVB exists then its mass

$$m_W > 1.8 \text{ GeV}$$

As for the $\mu$. $e$ events, the results were

| $M_W$ GeV | ($\mu$ - $e$) pair Van der Meer spectrum | Low $q^2$ spectrum | Observed |
|---|---|---|---|
| 1.8 | 6 | 16 | $\leq 3$ |

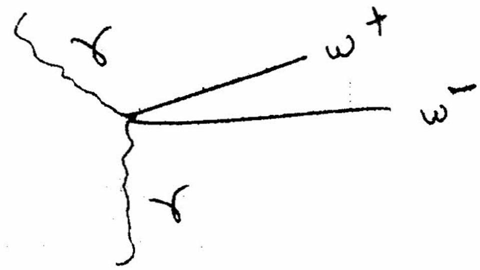
which strengthened the above conclusion..

As a remark, we may say, that it might be that one must have looked into a lepton pair and a pion and not just a lepton-pair alone. This is because a pion may be produced at the strong vertex. Or virtually from the nuclear interactions. The polarization of $\omega$ is the same as the neutrino.

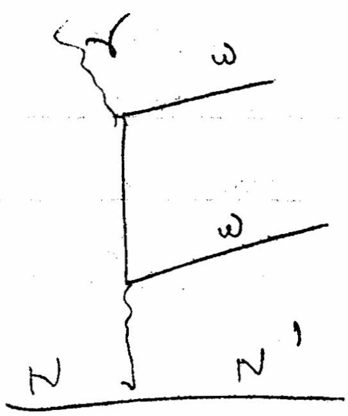
-production by Strongly Interacting Particles or Photons

(a) Photon pair production

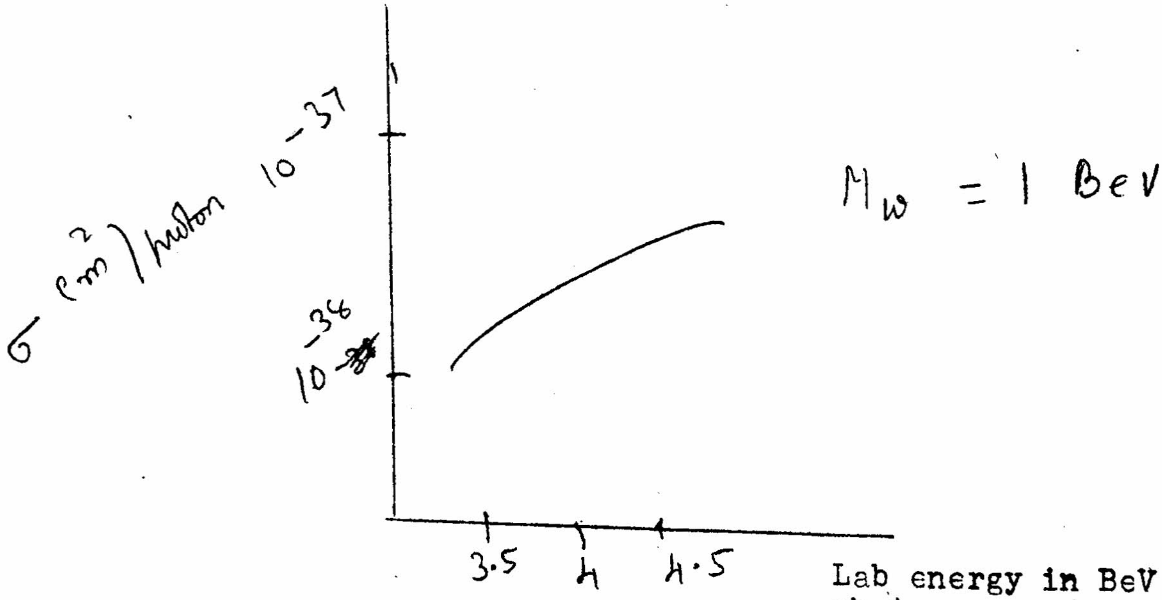
Since $\omega$ has both electromagnetic and weak interactions and since the coupling strength in electromagnetic interaction is larger, one might naturally ask whether $\omega$ could be produced in electromagnetic interaction?




Nucleus Nucleus coherent photoproduction



The cross-section is found to be

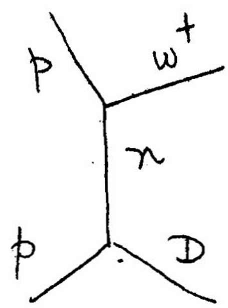



Lab energy in BeV of the photon

Coherent photo-pair production $[I_r(56),\ \gamma,\ Fe^{56}]$

(b) $\omega$-production from strongly interacting particles

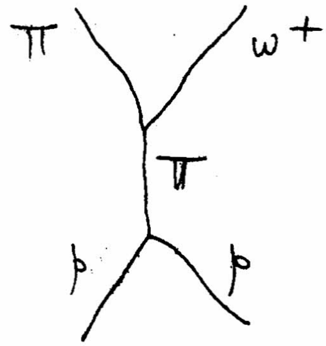
$$p + p \longrightarrow D + \omega^+$$




It is found that

$$\sigma \sim 10^{-35}\ cm^2 \text{ for } m_\omega \sim 1 \text{ Gev}$$

(c) $\pi + p \rightarrow \omega + p$




($\omega \pi \pi$)vertex is defined through the C.V.C. hypothesis. Then it is found that

$$\sigma \sim 10^{-33}\,\mathrm{cm}^2$$

for $m_\omega \sim 1$ GeV

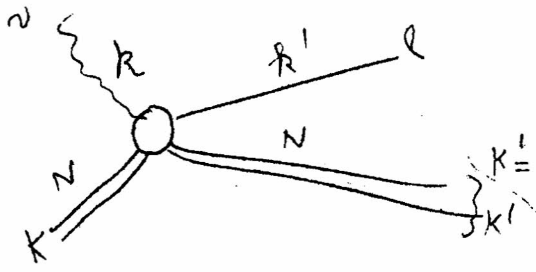
More complex interactions resulting in charged lepton pairs can be postulated but with little theoretical justification.

Lecture 10

## Inelastic neutrino reactions

In this lecture we shall discuss few important theorems connected with the inelastic reactions

$$\nu(\bar{\nu}) + N \rightarrow \ell + N + n\pi$$




$K' =$ total four momentum of all the outgoing strongly interacting particles.

Now $K'^2 = M_F^2$ is a variable now. This is the (mass)$^2$ of the final, state which involves $n$ pions. The differential cross-section is given by

$$\frac{d\sigma\pm}{dq^2} = \frac{1}{k_\nu^2}\left[A(q^2) \pm B(q^2)(s-u) + C(q^2)(s-u)^2\right]$$

where
$$q = (k' - K)$$
$$s+u+t = \sum M^2$$
$$s+u = m_\ell^2 + M^2 + 2M(e - e')$$

$e$ and $e'$ are energies of the incoming $\nu$ and outgoing lepton in the labsystem so that $(e - e')$ is the loss of energy of the lepton in the lab system.

It should be noted that $(s+u)$ is not a function of t, but is a new variable. A, B, and C are functions of two variables so that for such inelastic scattering the general formula has to be written as

$$\frac{d^2\sigma\pm}{dq^2 dK'^2} = \frac{1}{k_\nu^2}\left[A' \pm B'(s-u) + C'(s-u)^2\right]$$

where A', B' and C' are functions of t and the new variable $(s+u)$. In the lab system, we find that

$$\frac{d^2\sigma}{dq^2 dK'^2} \sim \left(\frac{A''}{e^2} + \frac{B''}{e} + C''\right)$$

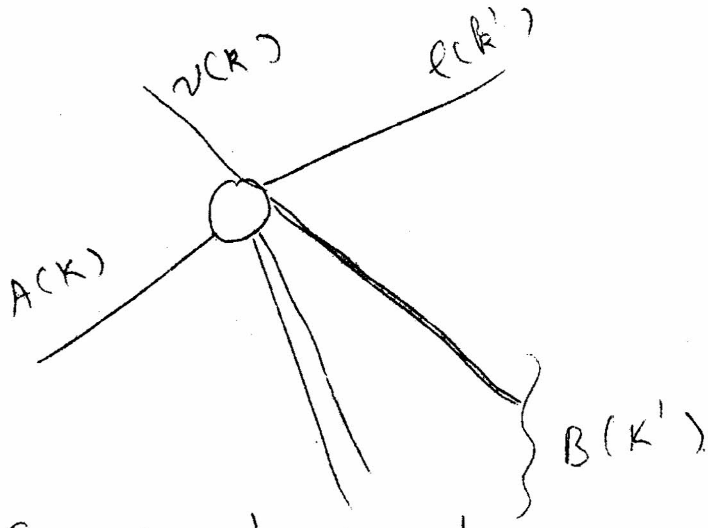
A'', B'', and C'' are functions of two variables.

ADLER THEOREM: (General theorem on inelastic lepton-baryon scattering). The theorem allows the possibility of testing the conserved vector current hypothesis, by looking into the forward inelastic scattering of neutrinos.

Consider the reaction

$$\nu + A \rightarrow \ell + B$$

where $B$ consists of a number of strongly interacting particles (say pions)




$$q = k - k' = K - K'$$

The scattering matrix for the above process can be written as

$$T \sim \underbrace{\bar{u}_\ell \gamma_\lambda (1+\gamma_5) u_\nu}_{\ell_\lambda} \langle B | J_\lambda^V + J_\lambda^A | A \rangle$$

When we find $|T|^2$ to calculate the cross-section

$$\langle |T|^2 \rangle \sim \langle B | J_\lambda^V + J_\lambda^A | A \rangle \langle B | J_\sigma^V + J_\sigma^A | B \rangle \, l_\lambda \, l_\sigma$$

$$= T_{\lambda\sigma} L_{\lambda\sigma}$$

where

$$T_{\lambda\sigma} = k'_\lambda k_\sigma + k'_\sigma k_\lambda - (k' k)\, \delta_{\lambda\delta}$$
$$+ \epsilon_{\lambda\sigma\nu\sigma} k_\gamma k'_\delta$$

The last term arises due to non-conservation of parity.

Suppose now we consider 'PARALLEL CONFIGURATION'. This configuration may be proved to be a Lorentz invariant concept

$$\text{i.e.} \quad \vec{k}' \parallel \vec{k}$$

$$\text{or} \quad k' = \alpha \vec{k}$$

and assume that

$$K^2 = M_A^2 \neq K'^2 = M_B^2$$

(i.e. it means that we must have an inelastic scattering) and assume that $k^2 = k'^2 = 0 = m_c^2$ since $q^2 = 0$ and $q_0 \neq 0$ we can put

$$k = k_0 \frac{q}{q_0}, \quad k' = k_0' \frac{q}{q_0}$$

(This easily checks with the conditions

$$k^2 = 0 = k_0^2 \quad ; \quad \text{at} \quad q^2 = 0$$

Using these expressions for $k$ and $k'$ we see that

$$T_{\lambda\sigma} = \frac{2 k_0 k_0'}{q_0^2} q_\lambda q_\sigma$$

(i.e. the parity violating term drops out).

Consider the matrix element of the divergence of the weak inter current

$$\left\langle B \left| \frac{\partial J_\lambda}{\partial x_\lambda} \right| A \right\rangle = -i q_\lambda \langle B | J_\lambda | A \rangle$$

Then

$$\langle B | T^2 | A \rangle \sim \frac{2 k_0 k_0'}{q^2} \left| \left\langle B \left| \frac{\partial J_\lambda^V}{\partial x_\lambda} + \frac{\partial J_\lambda^A}{\partial x_\lambda} \right| A \right\rangle \right|$$

This is Adler's theorem which states that in the parallel configuration the cross-section for inelastic neutrino scattering is directly proportional to the square of the divergence of the current if one neglects the lepton mass.

The parity violating term drops out in the parallel configuration since parity violating term is proportional to the interference term between $J_\mu^V$ and $J_\mu^A$ and since the current is conserved when this interference term is zero, this may be method of testing the conserved vector current hypothesis by observing the interference term in the parallel configuration. As an example, consider the reaction

$$\nu(K) + N \rightarrow \ell(k') + N'(q_1) + \pi(q_2) + \pi(q_3)$$

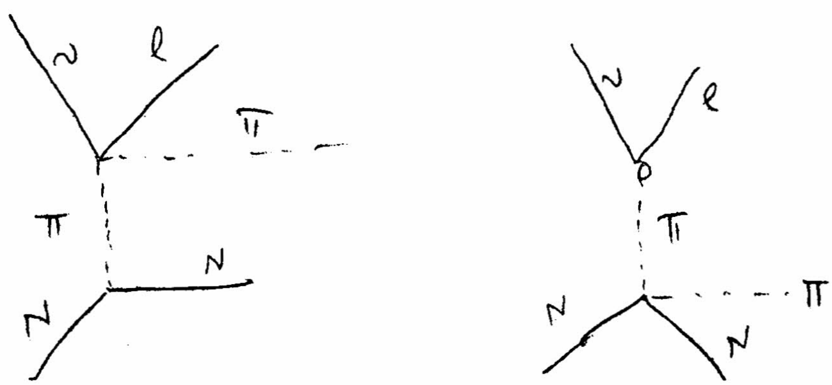
The term that is responsible for parity violation is $\vec{k}'(\vec{q}_2 \times \vec{q}_3)$. If this exists, interference term between $J^V$ & $J^A$ exists and the current will not be conserved.

This theorem is theoretically true but practically there is the difficulty in looking at the forward direction of the neutrino at zero angle (i.e. parallel configuration).

A main inelastic scattering is the reaction

$$\nu + N \rightarrow \ell + N + \pi.$$

(viz) the peripheral model

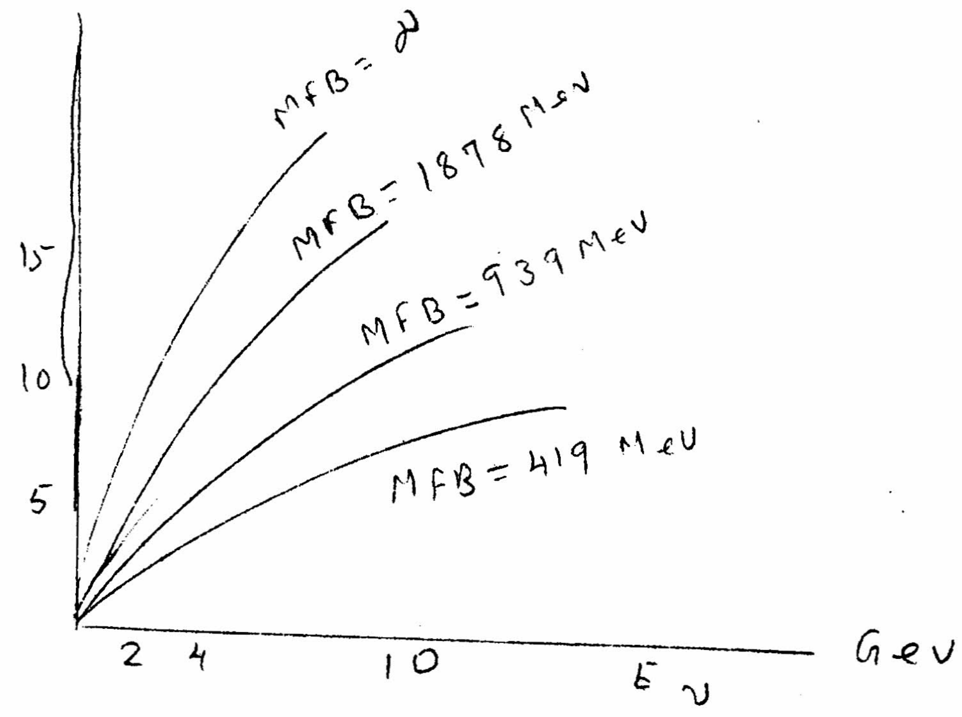


For the $(\pi\ \pi\ \ell\ \nu)$ vertex, we can use the c.v.c. hypothesis. For $(\pi\ \ell\ \nu)$ vertex we may use the Goldberger-Treiman relation. We may assume a general behaviour for the form factors at the lepton and baryon vertices as

$$F_\ell(q^2) \sim \frac{1}{1 + q^2/M_{F\ell}^2}$$

$$F_B(q^2) \sim \frac{1}{1 + q^2/M^2_{F\ell}}$$

Using these we obtain for the cross-section a behaviour like the following



Thus, theoretically we get on an average

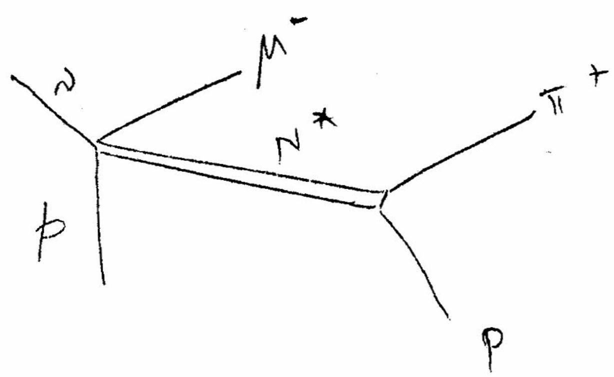
$$\sigma_{th} \sim 1.6 \times 10^{-40} \; Cm^2$$

Experimentally

$$\sigma_{Exp} \sim 20 \times 10^{-40} \; Cm^2$$

Isobar production in neutrino reactions

Recently Berman and Veltman (CERN preprint) have studied the isobar production in neutrino reactions




They find for the ratio $\pi^+ / \pi^0 \sim 5$ while experimentally this ratio is about 3. For $N^*$, the wavefunction used is the Rarita-Schwinger wavefunction since $N^*$ has a spin 3/2.

Lecture 11

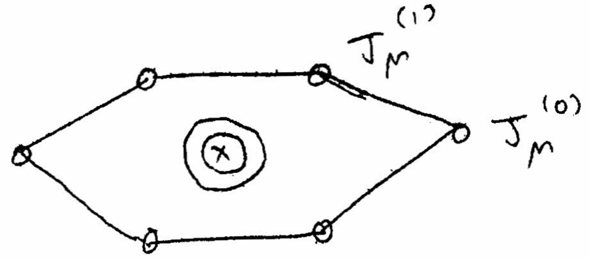
## Weak Interactions and $SU_3$

One starts by postulating that the weak interaction vector currents are not only in the adjoint representation of $SU_3$ but are in fact the components of the unitary spin current generated by the gauge transformations of the group. Thus they are directly related to the infinitesimal generators of the group. The eight currents have among them the three isotopic spin currents. Thus the weak interaction vector currents are in the same represantation as the electromagnetic current. One automatically gets the weak interaction selection rules

$$|\Delta \vec{I}| = \frac{1}{2} \quad \text{for} \quad |\Delta S| = 1$$

$$|\Delta \vec{I}| = 1 \quad \text{for} \quad \Delta S = 0$$

It is also assumed that the axial vector currents are also in the regular representation. Diagramatically ome represents this currents as



where $J_\mu(0)$, the $\Delta S = 0$ part behaves like a $\pi$ meson and the $J_\mu^{(1)}$ the $\Delta S = 1$ part behaves like $k^+$ or $K^-$. Of course for each $J_\mu$ we have a vector and axial vector part. In Gell-Mann's notation we have

$$J_\mu^{V(0)} = \mathcal{J}_\mu^{(1)} + i\, \mathcal{J}^{(2)} \qquad \text{with } F_i = -i \int \mathcal{J}_4^{(i)}\, d\vec{x}$$

$$J_\mu^{V(1)} = \mathcal{J}_\mu^{(4)} + i\, \mathcal{J}_\mu^{(5)}$$

One can explicitly write the currents as

$$\begin{aligned}
J_\mu^{V(1)} = (\approx K^+) = \frac{1}{\sqrt{2}} \Big\{ & -\sqrt{3}\, \bar{\Lambda} \gamma_\mu p + \sqrt{3}\, \bar{\Xi}^- \gamma_\mu \Lambda \\
& - \bar{\Sigma}^0 \gamma_\mu p - \sqrt{2}\, \bar{\Sigma}^- \gamma_\mu n \\
& + \bar{\Xi}^- \gamma_\mu \Sigma^0 + \sqrt{2}\, \bar{\Xi}^0 \gamma_\mu \Sigma^+ \\
& + \sqrt{3}\, K^+ \partial_\mu \eta_0 - \sqrt{3}\, \eta_0 \partial_\mu K^+ \\
& + K^+ \partial_\mu \pi^0 - \pi^0 \partial_\mu K^+ + \sqrt{2}\, K_0 \partial_\mu \pi^- \\
& - \sqrt{2}\, \pi^+ \partial_\mu K^0 \Big\}
\end{aligned}$$

$$\begin{aligned}
J_\mu^{V(0)} = \sim(\pi^+) = \Big\{ & \bar{n} \gamma_\mu p - \bar{\Xi}^0 \gamma_\mu \Xi^- - \sqrt{2}\, \bar{\Sigma}^0 \gamma_\mu \Sigma^+ \\
& + \sqrt{2}\, \bar{\Sigma}^- \gamma_\mu \Sigma^0 - K^+ \partial_\mu \bar{K}^0 + \bar{K}^0 \partial_\mu K^+ \\
& + \sqrt{2}\, \pi^+ \partial_\mu \pi^0 - \sqrt{2}\, \pi^0 \partial_\mu \pi^+ \Big\}
\end{aligned}$$

(i) Semileptonic meson decays

We shall apply this first to the semileptonic decays. We write

$$H_{S.L} = G J_\mu l_\mu^+$$

It is known that $G_{\Delta S=0}$ and $G_{\Delta S=1}$ are not the same but the latter is down by a factor of $\frac{1}{20}$. So Cabbibo modified the definition of universality to read as

$$J_\mu^V = J_\mu^{V(0)} \cos\theta + J_\mu^{V(1)} \sin\theta$$

Thus if G is the coupling constant which occurs in $\mu$ decay then the coupling constants for neutron $\beta$ decay and $(\Lambda\text{-}\beta)$ decay are given by $G\cos\theta$ and $G\sin\theta$ respectively.

Similarly one can write

$$J_\mu^A = J_\mu^{A(0)} \cos\theta' + J_\mu^{A(1)} \sin\theta'$$

One can determine $\theta$ and $\theta'$ knowing the ratio of decay rates for the $Kl_3$ and $\pi l_3$ decay and that of $K_{\mu_2}$ and $\pi_{\mu_2}$ decays. They are related because of unitary symmetry. The ratio of amplitudes for $Kl_3$ and $\pi l_3$ assuming that they are at zero momentum transfer (since the currents are conserved the bare coupling constant is same as the renormalized coupling constant at zero momentum transfer) is given by

Ratio of

$$\text{amp}\ \frac{\langle \pi^0 | J_\mu^{V(1)} | K^+ \rangle\ (\bar{\ell}\nu)_\mu}{\langle \pi^+ | J_\mu^{V(0)} | \pi^0 \rangle (\bar{\ell}\nu)_\mu}$$

$$\sim \frac{1/\sqrt{2}\ G \sin\theta}{\sqrt{2}\ G \cos\theta} \qquad \text{(phase space factors)}$$

$$= \frac{1}{2} \tan\theta \qquad \text{(phase space factors)}$$

From the experimental decay rates one finds that value of $\theta=0.26$. To determine $\theta'$ one has to find the ratio of decay rates of $K \to \ell_\nu$ and $\pi \to \ell\nu$ (i.e.)

$$\frac{\langle 0 | J_\mu^A | K \rangle (\bar{\ell}\nu)_\mu}{\langle 0 | J_\mu^A | \pi \rangle (\bar{\ell}\nu)_\mu} \sim \tan\theta'$$

The experimental values of decay rates gives

$$\theta' = 0.27$$

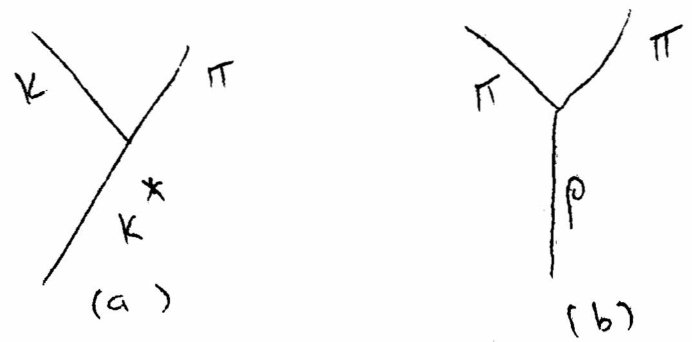
It is surprising that $\theta$ and $\theta'$ should be the same.

Thus one gets the coupling constant for $\beta$ decay to be

$$G_\beta = G_\mu \cos\theta$$

$$= 0.966\ G_\mu$$

All the above arguments are true only at zero momentum transfer (i.e.) when there is no renormalization effects. However even at zero momentum transfer there is the $SU_3$ symmetry breaking and the $\Delta S = 1$ vector current is not divergence-less. Sakurai tried to improve the above approximate formula by assuming that vector mesons are coupled to other particles through the conserved currents




For the above diagrams (a) and (b) the renormalization factors which enter at the vertices are given respectively by

$$\sqrt{\frac{Z_2(K)\,Z_2(\pi)\,Z_3(K^*)}{Z(K^*K\pi)}} \quad \text{and} \quad \sqrt{\frac{Z_2(\pi)\,Z_2(\pi)\,Z_3(\rho)}{Z_1(\pi\pi\rho)}}$$

If one makes the assumptions that

$$Z_3(K^*) = Z_3(\rho)$$

and

$$\frac{Z_1(\pi\pi\rho)}{Z_1(\pi K K^*)} = \frac{Z_1(\pi\ \pi e\nu)}{Z_1(K\pi e\nu)}$$

then Sakurai obtains

$$\theta_{\text{Sakurai}} = 0.81 \quad \theta_{\text{Cabbibbo}}$$

and hence $\cos\theta_{SuK} = 0.98$

which is closer to the experimental value.

(ii) <u>Hyperon decays</u>.

Using these $\theta$'s let us study the consequences of eqn.( ) for hyperon decays. We have to study

$$\langle A | j_\mu'^{V(i)} + j_\mu'^{A(i)} | B \rangle (\bar{e}\nu)_\mu$$

where A and B belong to the same representation. Then

$$\langle A | j_\mu'^{V(i)} + j_\mu'^{A(i)} | B \rangle = \bar{u}_A \left[ f^i_{AB} O_\mu + d^i_{AB} E_\mu \right] u_B$$

where $O_\mu$ and $E_\mu$ correspond to the unsymmetrical F coupling and the symmetrical D coupling respectively and $f$ and $d$ are the generalized SU(3) Clebsch-Gordon coefficients.

We have $O_\mu = F^V \gamma_\mu + F^A \gamma_\mu \gamma_5$

$$E_\mu = D^V \gamma_\mu + D^A \gamma_\mu \gamma_5$$

It is known that $J_\mu^{(0)}$ and $J_\mu^{(1)}$ respectively behave like the $T_2^{\,1}$ and $T_3^{\,1}$ components of a mixed tensor of rank two. The F and D coupling of the baryons to these currents is easily calculated by considering the combinations

$$\bar{B} B T + \bar{B} T B \longrightarrow \text{D coupling}$$

$$\bar{B} B T - \bar{B} T B \longrightarrow \text{F coupling}$$

We also have

$$M = \begin{pmatrix} \left(\frac{\pi^0}{\sqrt{2}} - \frac{\eta^0}{\sqrt{6}}\right) & \pi^+ & K^+ \\ \pi^- & \left(-\frac{\pi}{\sqrt{2}} - \frac{\eta}{\sqrt{6}}\right) & K_0 \\ K^- & \bar{K}_0 & \frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

$$B = \begin{bmatrix} \left(\frac{\Sigma^0}{\sqrt{2}} - \frac{\Lambda^0}{\sqrt{6}}\right) & \Sigma^+ & p \\ \Sigma^- & \left(-\frac{\Sigma^0}{\sqrt{2}} - \frac{\Lambda^0}{\sqrt{6}}\right) & n \\ \Xi^- & -\Xi^0 & \frac{2\Lambda^0}{\sqrt{6}} \end{bmatrix}$$

and $\bar{B} = B^+ \gamma_4$

Then for $J_\mu^{(o)}$, $(B\bar{B})^2$ is the coefficient corresponding to the d-f combination.and $(\bar{B}B)^2$ is the coefficient corresponding to the d+f combination. Similarly for $J_\mu^{(1)}$, $(\bar{B}B)^3$ is the coefficient for d-f combination and $(B\bar{B})^3$, is the coefficient for d+f combination.

Thus the coefficients d and f can be calculated easily without looking into tables. For the vector current we have then

$$\langle A | J_\mu^{V(i)} | B \rangle \longrightarrow \langle A | F^i | B \rangle$$
$$= i \, \bar{u}_A \, f^i_{AB} \, u_B$$

We have $F^V = 1$ and $D^V = 0$.

For the axial vector part we have from $\beta$ decay

$$F^A + D^A = 1.15 \pm 0.04 \text{ (from experimental value)}$$

Also from $\Sigma_1^- \to \Lambda^0 + \bar{e} + \bar{\nu}$ we have

$$\cos\theta \langle \Lambda | J_\mu^{V(0)} + J_\mu^{A(0)} | \Sigma^- \rangle$$
$$= \bar{u}_\Lambda \cos\theta \sqrt{2/3}\, E_\mu\, u_{\Sigma^-}$$
$$= \bar{u}_\Lambda \cos\theta \sqrt{2/3}\, D^A \gamma_\mu \gamma_5\, u_{\Sigma^-}$$

From the rate $(\sim (D^A)^2)$ we have $D^A = \pm\, 0.9 \pm 0.2$. Then we have for

$$\Sigma_1^- \to n + \bar{l} + \bar{\nu}$$
$$\langle n | J_\mu^{V(1)} + J_\mu^{A(1)} | \Sigma^- \rangle$$
$$= \bar{u}_n \left[ -O_\mu + E_\mu \right] u_{\Sigma^-}$$
$$= \bar{u}_n \left[ -\gamma_\mu + (-F^A + D^A) \gamma_\mu \gamma_5 \right] u_{\Sigma^-}$$

Substituting the two values of $D^A$ obtained from $\Sigma \to \Lambda + \bar{e} + \bar{\nu}$ decay we find

$$-F^A + D^A = -0.65 \pm 0.4 \quad (D^A = 0.9 \pm 0.2)$$

or

$$-F^A + D^A = 2.95 \pm 0.4 \quad (D^A = -0.9 \pm 0.2)$$

The second value gives a rate much too large (i.e.) it gives

$$\frac{\Gamma(\Sigma^- \to n + \bar{e} + \bar{\nu})}{\Gamma(\Sigma^- \to all)} = 1.6\%$$ while the experimental value is only 0.16% . Thus we finally have

$$D^A = 0.9 \pm 0.2 \quad \text{and} \quad F^A = 0.35 \pm 0.2$$

$$D^V = 0 \quad \text{and} \quad F^V = 1$$

Then we can calculate the matrix elements and branching ratios for other decay as shown in table.

| | M. E. | | Exp. |
|---|---|---|---|
| $\Lambda \to p + \bar{e} + \bar{\nu}$ | $G \sin\theta (-\frac{\sqrt{3}}{2}) [\gamma_\mu + F^A + \frac{1}{3} D^A] \gamma_\mu \gamma_5$ | $0.5 \pm 0.1 \times 10$ | $0.8 \pm 0.1 \times 10^{-3}$ |
| $\Sigma^- \to n + \bar{e} + \bar{\nu}$ | $G \sin\theta [\gamma_\mu + (F^A - D^A) \gamma_\mu \gamma_5]$ | $1.9^{+1.8}_{-1.0} \times 10^{-3}$ | $1.3 \pm 0.2 \times 10^{-3}$ |
| $\Sigma^- \to \Lambda + \bar{e} + \bar{\nu}$ | $G \sin\theta \frac{\sqrt{3}}{2} [\gamma_\mu + F^A - \frac{1}{3} D^A) \gamma_\mu \gamma_5]$ | $0.35 \pm 0.55 \times 10^{-3}$ | $2.4 \pm 1.4 \times 10^{-3}$ |

At the Dubna Conference Pais gave the results

$$\frac{D^A}{F^A} = \frac{0.68 \pm 0.03}{0.32 \pm 0.03}$$

(iii) Non-leptonic decays

I am not going into the details but just mentioning the concept of Octet enhancement. Since the currents are in the Octet representation $H_{N.L}$ should behave like a representation in the product 8 x 8. If one weants the $|\Delta I| = \frac{1}{2}$ rule one has to make the postulate that $H_{N.L}$ behaves like an Octet. In the product 8 x 8 we have '27' which contains $|\Delta I| = 3/2$ also. Thus we have to say that the octet in 8 x 8 dominates over the '27'. Then the main results are

1) $$A(\Lambda \to p\pi^-) + 2A(\Xi^- \to \Lambda\pi^-) = \sqrt{3}\, A(\Sigma^+ \to p\pi^0)$$

For this we assume CP invariance and that the currents have normal C properties

2) $K_1^0 \to 2\pi$ forbidden.

(iv) Weak vector Bosons

If we write the weak interaction Lagrangian as

$$\mathcal{L} = \Sigma J_\mu W_\mu^+$$

then L+L' transforms like the fundamental representation '3' ($3^* \times 3 \to 1+8$). Then since $J_\mu$ is in an octet we can easily see that $W_\mu$ should belong to 3,6 or 15. The usual assignment is that the $W$ mesons belong to the '3' representation

$$W^0_{\frac{1}{2}} \times \qquad \times\, W^+_{1/2}$$
$$\times\, W^0_0$$

With the antiparticles there are six bosons but only five are coupled.

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ALLADI RAMAKRISHNAN

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