MATSCIENCE REPORT No. 33

LECTURES ON

ORIGIN OF SYMMETRIES

E. C. G. SUDARSHAN

Visiting Professor, Matscience

Notes prepared by

T. S. SANTHANAM

33

THE INSTITUTE OF MATHEMATICAL SCIENCES, MADRAS-20 (INDIA)

Matscience Report No.32

THE INSTITUTE OF MATHEMATICAL SCIENCES

MADRAS-20 (India)

Lectures on

ORIGIN OF SYMMETRIES

E.C.G.Sudarshan *
Visiting Professor, Matscience

Notes by

T.S. Santhanam

* Department of Physics, University of Rochester, Rochester, U.S.A.

These notes have been perused by Prof. Sudarshan

CONTENTS

oooo0oooo

Lecture 1

General Introduction to the Existence of Symmetries and their relation with other fundamental postulates

As early as in 19th century, Hertz noticed that there existed a certain amount of arbitrariness in the description of the forces between particles. The laws which govern the forces between particles were arbitrary since there was no postulated mechanism which is responsible for mediating the forces between particles. This arbitrariness was absent for the special case of elastic collisions between particles. However not all interactions were simply elastic collisions; one had to be able to understand action-at-a-distance. The answer was the postulate about the existence of some free particles, which transmit the interaction between particles by going from one to the other as a 'messenger'. Nevertheless, in such a description, great deal of artifidality creeps in through the introduction of such unobserved particles.

In a sense, Hertz's point of view has been revived in this century by people who worked within a very different framework In this, one introduces ''fields'' and the mechanism of the interaction is explained as a result of the coupling between fields. The quantum field theory as it stands, does not provide a unique specification of the interaction between particles. Instead, a variety of interactions are possible. The form of the interaction includes in it dimensionless constants like (bare) coupling constant and the dimensional quantities like the (bare) mass of the field. The interaction as written in the Lagrangian form reads, for example (for fermion fields);

$$\mathcal{L} = \left(\overline{\psi} \left(\gamma_\mu \partial_\mu + m_0 \right) \psi \right) + \mathcal{L}_{interaction}$$

If one assumes this form for the interaction, one knows that the perturbation calculation leads to a rather unpleasant result that

$$| m - m_0 | = \infty$$

where m is the actual mass (i.e. mass when there is interaction present) and m_0 is the bare mass. This leads to a paradox as whether to abandon the calculation or to modify the form of L.

<u>How can we define particles</u> ?

One way is to postulate straightaway the existence of particles. The other way is to define them through interaction. But it is difficult to see how all particles could come out of an interaction since we should have something before the interaction so as to get particles out. There are two guide lines to get out of this difficulty.

(i) Consider for example the hydrogen atom which is a bound state of a proton and an electron. There is a difference between the free protons and electrons and those which form the hydrogen atom; while the latter have a structure for them, the former do not have them. But if one says that the particle that comes out is composite, one is really tempted to ask as which is really elementary and which is composite. If two particles go to make the third,

$$A + B \rightarrow C$$

in quantum field theory unlike in, say, atomic physics, one can also write a corresponding relation

$$A \rightarrow \bar{B} + C$$

where $\bar{B}$ and B are related by some standard operation. Then, A is a bound state of $\bar{B}$ and C ? To be more specific let us again take the case of deutron. The above two processes read in this case as

$$n + p \rightarrow d$$

$$n \rightarrow \bar{p} + d$$

The binding energy in the former case is only $\sim$ 2 Mev which is very small while in the latter case the binding energy between $\bar{p}$ and d is very high and is comparable to the mass of the constituent particles. This is only a formal way of writing down a bound state. In this sense, every particle is composite and is no more elementary. Various excited states of hydrogen atom or the various nuclear levels may be associated with new particles.

If in a system the bound state is not much more massive than any of its constituents, then the particle are self supporting self-consistently.

Even if we assume the conventional interaction, if one could neglect the effects of gravitational, weak and electromagnetic interactions, the natural candidate to describe any interaction is the strong interaction and in this if the mass of the particles is of the same order as the binding energy between them, then these particles are self-supporting and the interaction is self-consistent.

There are three ways of studying the self-consistency of the problem .

(a) One is the study of the qualitative and aesthetic description of the dynamics;

(b) The second is to actually compute and exhibit the self-consistency of the solution; and

(c) third is through the study of the symmetry of the system.

Since the third one explicitly involves only algebraic things, self-consistency, can be actually demonstrated and can be related to direct observations.

SYMMETRIES:

Three classes of symmetries are commonly observed. They are: 1) particle - antiparticle (pairing) symmetry

2) families of particles and

3) multiplets of particles.

About the families of particles, we know that the photon which has only electromagnetic interaction, forms a family by itself. The other families are the mesons (spin zero bosons) and the heavy baryons(spin half fermions). The members in a given family have almost the same mass, spin, etc. and perhaps the same kind of interactions (though significantly different). These are broad classifications. However, it is not completely clear, how an internal classification can be made within a given family without arbitrariness.

(e.g. the question whether to group the electrons leptons together and muon separately among the leptons, or to group charged leptons together and neutral separately.)

The baryons, though have the same kind of strong interaction, yet can be distinguished among themselves by their characteristic weak and electromagnetic interactions.

On the contrary, particle-antiparticle pairing symmetry seems to be basic in the relativistic quantum field theory.

Multiplets of particles, however, can be put in the quantum field theory through the existence of some group property among the members of the multiplets, i.e. through the existence of some local transformations between the components of the group. Using this, it is possible, to give the form of 'invariant interactions', invariant under the group transformations. But to define particles as manifestations of the interactions between fields really takes one outside the framework of quantum field theory. It should be noted that the form of interaction so constructed as to be invariant under a group of transformations is arbitrary if not completely. Building up of particles and the interpretation in field theory is the purpose of these lectures. The material to be used in this series of lectures are derived from the work of Cutkosky according to whom there exists a possible way in which the existence of symmetries and the mechanism of breaking the symmetry among particles of equal mass could be self-consistency derived; from the work of the present author in collaboration with Macfarlane and Mukunda regarding the application of Smushkevich method in explaining the symmetries; and a short paper of Sakurai in which it is explained that it is possible to derive relations between coupling constants purely from algebraic considerations (by equating the contributions from various self-energy diagrams of the pions) that will be consistent with the predictions of unitary symmetry model.

In discussing the free field, we are familiar with the following facts: The Dirac equation can be written in the form

$$\psi = \sum_{\alpha} \{ a_\alpha u_\alpha + b_\alpha^{+} v_\alpha^{*} \} ,$$

where a_α is the annihilation operator of a particle and b_α^{+} is the creation operator of a particle with negative energy and α refers to a complete set of functions. ψ can be thought of equally as a wave function of a single particle obeying the equation

$$(i \gamma_\mu \partial_\mu - m) \psi = 0.$$

where m is the mass of the particle or as an operator as before. Now what are the transformations of ψ that will leave this equation invariant? We are used to the scalar, vector and electromagnetic fields and we know how they transform $\psi \rightarrow \psi' = T\psi$ where T is some local transformation. These are transformations which are associated with the invariant structure of points. In a similar manner the transformations of quantized fields may be interpreted as the transformation of creation and annihilation operators. In such a description a_α and b_α^+ are no longer independent and are constrained by the requirement of local transformations of the field operators. Let us now see the relation between the transformation properties of a and b^+. For example, if we consider a fermion-antifermion pair, the intrinsic parity of this pair is negative and that of boson-antiboson pair is positive, i.e.

The intrinsic parity of	$(F\,\bar{F})$ is	$-$ ve	(1.1)
and that of	$(B\,\bar{B})$ is	$+$ ve	

These are direct consequences from the two important postulates taken in analogy from classical fields

(a) The expansion of the field operator contains both positive and negative frequency states (i.e. both particle and antiparticle and operators are put together to form a field).

(b) The quantized fields transform locally like the classical fields. However, one knows, that the results (1) about the intrinsic parity are borne out very well by experiments (positronium → photon experiments) which may be interpreted as a test of the correctness of these postulates from which these results have been deduced. In field theory one can write down an interaction between two or more fields to describe particles say a neutron (n) and proton (p). As far as interactions are concerned both p and n behave similarly (apart from electromagnetic interactions); but still we know that these two are different particles. Therefore, in describing these particles in terms of interactions (which we can write down), we want them to be equivalent, but not identical). (It should be noted that this again is a postulate and not a deduction). We then use the electromagnetic interaction as a tool to distinguish these two particles.

In the description of the particles, one often introduce three types of quantum numbers

a) Mechanical (Angular momentum, energy, etc.)

b) Additive (electric charge; baryon number, etc.)

and c) Non-additive 'Vector' quantum numbers (such as total angul momentum $\vec{J}$ total isospin $\vec{I}$, etc.):

The additive quantum numbers are those which are algebr ally conserved in any interaction under the simultaneous gauge transformation of the fields

$$\psi \longrightarrow \psi' = e^{i\alpha} \psi \qquad (1.2)$$

where ψ is real. Such a gauge transformation leaves the interaction invariant. A set of all these gauge transformations form 'gauge group'. It is however clear that the study of this group is not going to yield us any new result going beyond the postulat additive conservation law.

On the contrary, the non-additive quantum numbers (whic are like angular momentum in classical mechanics) add up vectoric and depend on the manner in which the wave functions are made. For example, in the rotations, one finds that only J_Z (third component of angular momentum) is additive (and this forms only an Abelian group). It is clear that to view the whole thing, one has to go beyond this one component description.

Let us now see the isotopic spin group. One has to specify the following things:

1) The group has to be specified. The representations of the group should be found and the assignments of the particle to various representations of the group have to be made. The assignment of the particles should be consistent with additive quantum numbers and dimensionality of representation taking into account the multiplicity of the state. For example, we assign the isospin of the particles

$$I = 0, \quad \Lambda, \omega \text{ etc.}$$
$$I = 1/2, \quad N, K \text{ etc.}$$
$$I = 1, \quad \Sigma, \pi, \text{ etc.}$$

The multiplicity of a given I state is (2I + 1) so that in assigning these particles one has to be consistent with the dimensionality of the representation.

2) We must require that the interaction be consistent with symmetry.

3) One should be able to explain the violation of the symmetry.

The cause for the violation has to be specified (for example we say that the charge independence is violated by electromagnetic interaction). The violation may be thought of as due to some interaction, or may be explained in a self-consistent way. This is because of the 'tendency' of the system to break the symmetry so that it can be more stable. One can calculate the solutions when there is a small violation of the symmetry.

4) In nature, not all, but only low lying representations are realized. The task is to find the allowed representations such that the interaction is self consistent.

One fact worth recalling is that the irreducible representations of a compact group are finite dimensional and are equivalent to unitarity representations.

Lecture 2

In this lecture, let us discuss the charge independence or isospin group. We will not worry about the relations between space-time symmetries and the isospin group. The discussion of isospin group will be mostly the discussion of the representations of the corresponding Lie algebra of the infinitesimal generators of the local group. By "local" we mean those transformations which are in the neighbourhood of the identity operation. All the irreducible representations of the isospin Lie algebra are known. There is one and only one irreducible representation corresponding to a given value of the isospin I with 2I = nonnegative integer. Any representation is a linear combination of all the irreducible representations. Let $\psi(I, \nu)$ be a complete set of orthonormal functions for any I (for which 2I is an integer) and $-I \leq \nu \leq I$. Then the multiplicity of this irreducible representation is (2I + 1) corresponding to all the possible values that ν can take. If $\psi(I_1, \nu_1)$ and $\psi(I_2, \nu_2)$ are two given irreducible representations, the direct product.

also furnishes a basis of the representation. Thus just as $\psi(I_1,\nu_1)$ and $\psi(I_2,\nu_2)$ form complete basis for the irreducible representation, so also $\psi(I,\nu)$ given by

$$\psi(I_1,\nu_1) \otimes \psi(I_2,\nu_2) = \sum_{I=|I_1-I_2|}^{I_1+I_2} C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu}\, \psi(I,\nu) \tag{2.1}$$

forms a complete basis for the irreducible representation. The coefficients $C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu}$ are called the Clebsh-Gordan coefficients. They have certain nice properties;

$C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu} = 0$ unless

(1) $\nu_1 + \nu_2 = \nu$

(2) $I_1 + I_2 + I$ is an integer

(3) $|I_1 - I_2| \leq I \leq I_1 + I_2$

(4) $-I \leq \nu \leq I$ (2.2)

(5) $-I_1 \leq \nu_1 \leq I_1$

(6) $-I_2 \leq \nu_2 \leq I_2$

The Clebsh-Gordan coefficients may be always chosen <u>real</u>: eqn.(2.1) can be written as

$$|I_1\nu_1\rangle|I_2,\nu_2\rangle = \sum_{I\,\nu} C^{I_1\,I_2\,I}_{\nu_1\,\nu_2\,\nu}|I_1 I_2 I\nu\rangle \quad (2.1a)$$

Formula (2.1a) may be inverted thus.

$$|I_1 I_2 I\nu\rangle = \sum_{\nu_1\nu_2} C^{I_1\,I_2\,I}_{\nu_1\,\nu_2\,\nu}|I_1\nu_1\rangle|I_2\nu_2\rangle \quad (2.3)$$

Since the dimension of the representation $D(I,\nu)$ is equal to (2I + 1) corresponding to the number of components $-I \leq \nu \leq I$ Eq.(1a) gives a relation

$$(2I_1+1)\ (2I_2+1) = \sum_{I}(2I+1) \quad (2.4)$$

Let us represent the C in eq. (2.3) by a unitary matrix (since the transformation is unitary) U as

$$C^{I_1\,I_2\,I}_{\nu_1\,\nu_2\,\nu} = U^{I_1\,I_2}_{\nu_1\,\nu_2,\,I\nu}, \quad (2.5)$$

Since U is unitary we know that

$$UU^{+} = I = U^{+}U$$

which takes the form

$$\sum_{I,\nu} U_{\nu_1 \nu_2 ; I \nu} U^*_{\nu_1' \nu_2' ; I \nu} = \delta_{\nu_1 \nu_1', \nu_2 \nu_2'}$$

$$\sum_{\nu_1 \nu_2} U^*_{\nu_1 \nu_2 ; I \nu} U_{\nu_1 \nu_2 ; I', \nu'} = \delta_{I I', \nu \nu'}$$

That is

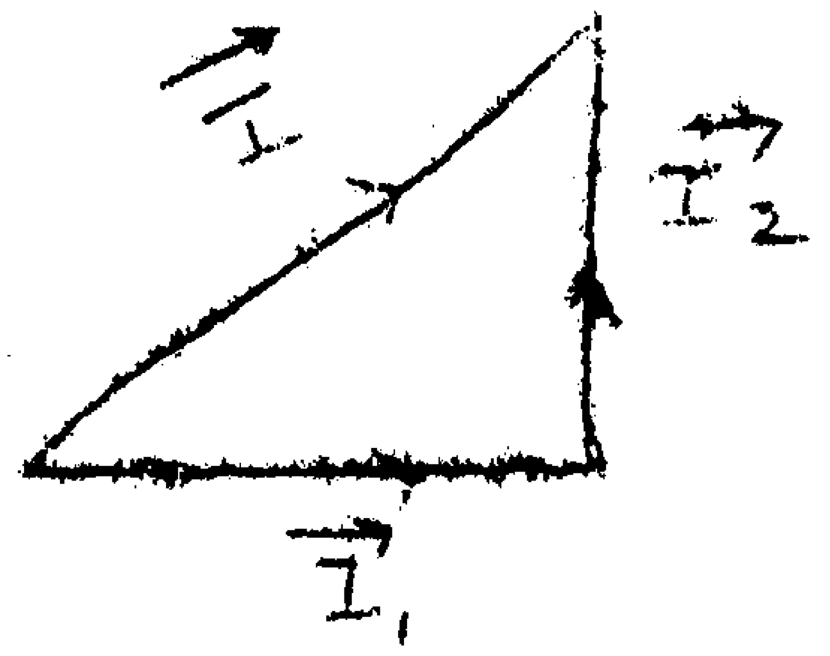
$$\sum_{I,\nu} C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu} \left(C^{I_1 I_2 I}_{\nu_1' \nu_2' \nu} \right)^* = \delta_{\nu_1 \nu_1', \nu_2 \nu_2'}$$

$$\sum_{\nu_1, \nu_2} C^{I_1 I_2 I'}_{\nu_1 \nu_2 \nu'} \left(C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu} \right)^* = \delta_{I \nu I' \nu'}$$

(2.6)

Eqns. (2.6) are usually referred to as orthogonality properties of Clebsh-Gordan coefficients. The coupling scheme, eq.(2.3), corresponds to

$$\vec{I_1} + \vec{I_2} = \vec{I}$$



However, this is not the most symmetric one we can have. We should look for a coupling related to $\vec{I}_1 + \vec{I}_2 + \vec{I} = 0$

In other words the question is given the relation,

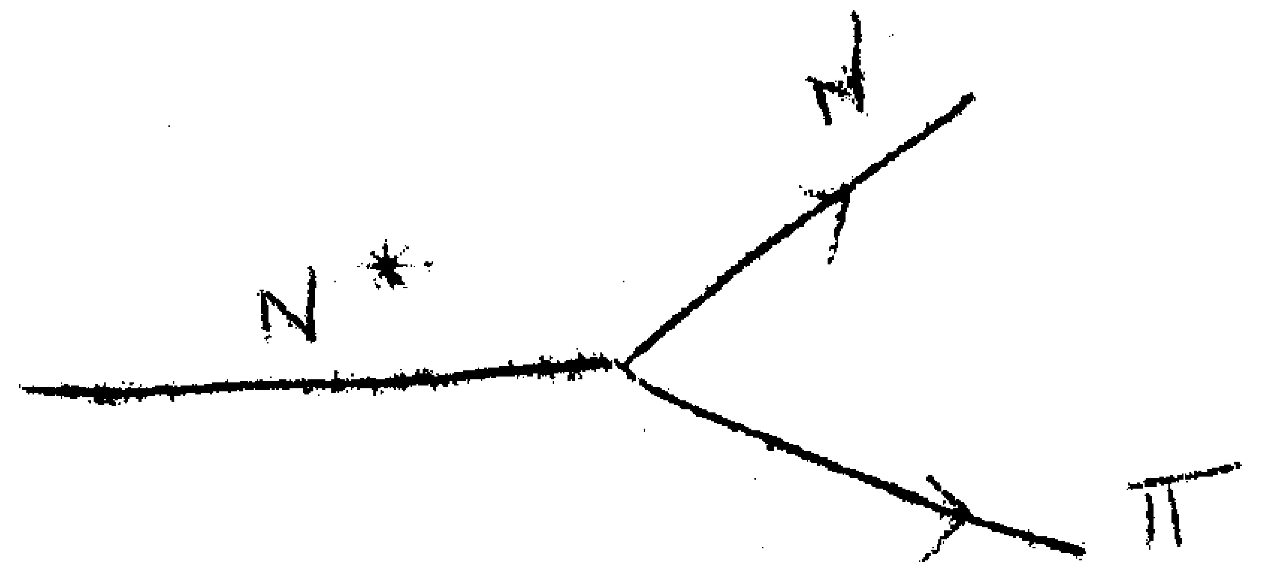
$$\psi_1(I_1, \nu_1) \otimes \psi_2(I_2, \nu_2) = \sum_{I, \nu} C^{I_1 \, I_2 \, I}_{\nu_1 \, \nu_2 \, \nu} \psi(I \nu)$$

what can we do to treat ψ_1, ψ_2 and ψ on an equal footing? Can we get ψ_2 as the product of ψ_1^* and ψ ? The special feature of the group R_3 is that if we consider a representation C, $\bar{C}$ also furnishes a representation with the same dimension i.e. I_2 and $\bar{I}_2$ representations are identical apart from isomorphism. Hence the same set of coefficients $C^{I_1 \, I_2 \, I}_{\nu_1 \, \nu_2 \, \nu}$ (apart from phases and normalization) serve also for these couplings. As an example for this symmetry, consider the multitude of reactions

$$N_* \longrightarrow N + \pi$$

where N_* is the (3,3) resonance of the pion nucleon system with I = 3/2 and therefore having 4 components. Actually the number of independent amplitudes we can have is 4 x 2 x 3 = 24 while most of them should be zero because of the restrictions put forth by the C.G. coefficients. (Thus only a sub-set of the 24 combinations ($N^* N \pi$)) can be non-zero. The product of the wave-functions of the nucleon and pion should yield the wave function of the N^*, ie

$$g \left(\Psi^{I=3/2}_{N^*}(\nu) \right) = \sum_{\nu_1, \nu_2} C^{\frac{1}{2}\ 1\ \frac{3}{2}}_{\nu_1\ \nu_2\ \nu} N(\nu_1) \times \pi(\nu_2) \quad (2.7)$$



g can be interpreted as the partial width for this given process. The matrix element for this transition $(I, \nu) \rightarrow$ $\rightarrow (I_1, \nu_1), (I_2 \nu_2)$ is therefore proportional to

$$M(\nu \rightarrow \nu_1, \nu_2) = g\, C^{\frac{1}{2}\ 1\ \frac{3}{2}}_{\nu_1\ \nu_2\ \nu} \quad (2.8)$$

Since $C'S$ are just constants, the cross-section for this process

$$\sigma \propto |g|^2 \tag{2.9}$$

with constant of proportionality being purely kinematic. From (2.8) we see that the orthogonality properties of $C'S$ are also satisfied by the matrix elements. We can get relations between $M' (\nu \to \nu_1, \nu_2)'s$ using the orthogonality properties of the C.G. coefficients with I = I' as we consider fixed value of I. In the first order we see that from Eq.(2.8)

$$\sum_{\nu_1,\nu_2} M(\nu \to \nu_1, \nu_2)\, M^*(\nu^* \to \nu_1, \nu_2) = |g|^2 \delta_{\nu\nu'} \tag{2.10}$$

for fixed value of I. Thus

$$\sum_{\nu_1\nu_2} |M(\nu \to \nu_1, \nu_2)|^2 = g^2 \quad \text{independent of } \nu \tag{2.11}$$

Eq. (2.10) follows from the fact that

$$C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu} = 0 \quad \text{unless} \quad \nu_1 + \nu_2 = \nu.$$

Eq. (2.11) has a nice interpretation, viz. the total transition probability for any component is independent of ν. In other words life time of transition is independent of ν. If we take the crossed orthogonality property

$$\sum_{\nu_1 \nu'} M(\nu \to \nu_1, \nu_2)\, M^*(\nu \to \nu_1' \nu_2') = |g|^2$$

$$\times \sum_{\nu, \nu'} C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu}\, C^{*\, I_1 I_2' I}_{\nu_1 \nu_2' \nu}$$

$$= |g|^2 \frac{(2I+1)}{(2I_2+1)}\, \delta_{\nu_2, \nu_2'} \qquad (2.12)$$

$$= (\text{constant})\ \delta_{\nu_2 \nu_2'}$$

when, $I_2 = I_2'$

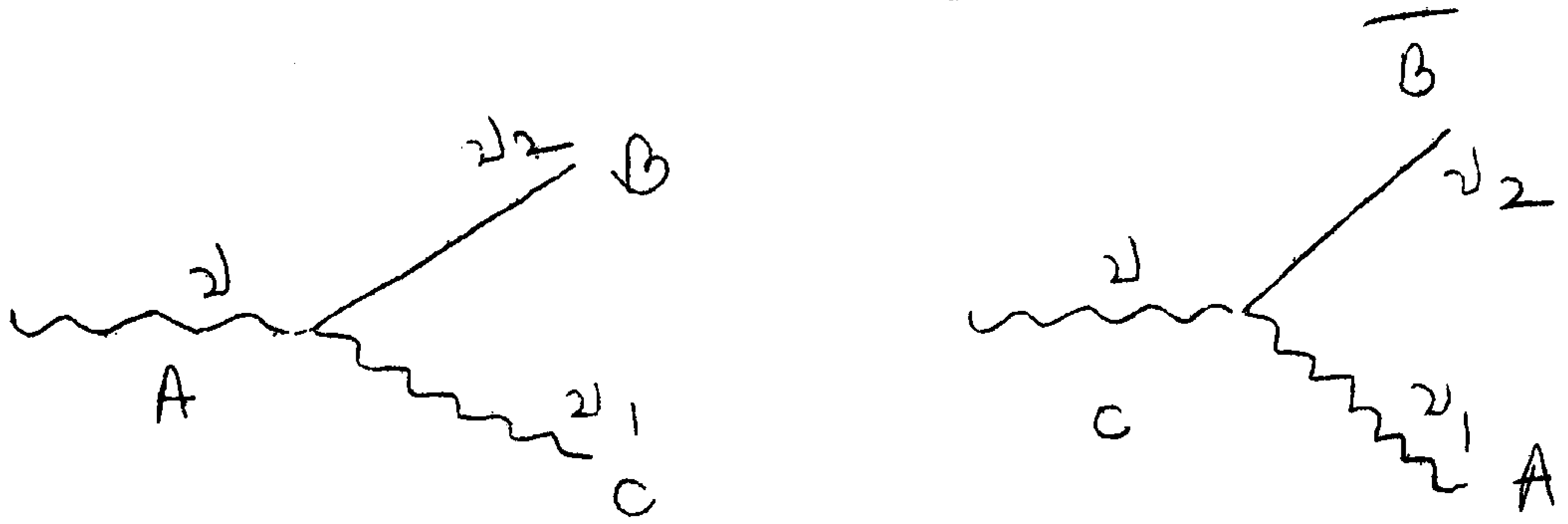
Thus

$$\sum_{\nu_1 \nu_1'} |M(\nu \to \nu_1, \nu_2)|^2 = g^2 \left(\frac{2I+1}{2I_2+1}\right) \qquad (2.13)$$

$$= \text{independent of } \nu_2$$

It is easy to see that Eq.(2.13) is the immediate consequence of charge independence, Eq. (2.13) has an interpretation

It is easy to see that Eq. (2.13) is the immediate consequence of charge independence Eq.(2.13) has an interpreation as follows: Consider the following diagrams



Eq.(2.13) says that the life time of transition $C \longrightarrow A + \overline{B}$ of the various members of the C multiplet are the same if the reactions are energetically possible. In other words, in all physically allowed processes total transitions from each component are the same.

Lecture 3

In the last lecture we saw that for any process $(I\ \nu) \longrightarrow (I_1, \nu_1), (I_2, \nu_2)$, the transition matrix element is

$$M(\nu \rightarrow \nu_1, \nu_2) = g\ C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu} \tag{3.1}$$

If we use the orthogonality property of the Clebsh-Gordan coefficients, viz.

$$\sum_{\nu_1, \nu_2} C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu} \left(C^{I_1 I_2 I'}_{\nu_1 \nu_2 \nu'} \right)^* = \delta_{II'}\, \delta_{\nu\nu'}$$

we get

$$\sum_{\nu_1, \nu_2} M\ (\nu \rightarrow \nu_1, \nu_2)\ M^*(\nu' \rightarrow \nu_1\ \nu_2) = |g|^2 \delta_{\nu\nu'} \tag{3.2}$$

and for the same I we have

$$\sum_{\nu_1 \nu_2} |M\ (\nu \rightarrow \nu_1, \nu_2)|^2 = g^2 = \quad \text{independent of } \nu \tag{3.3}$$

which means that the total width for any component to decay is the same for any other component of the same multiplet. Also from the crossed orthogonality property of the C.G. coefficients viz.

$$\sum_{\nu_1,\nu} C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu} \left(C^{I_1 I_2' I}_{\nu_1 \nu_2' \nu} \right)^* = \frac{(2I+1)}{(2I_2+1)} \delta_{(I_2,I_2'),(\nu_2,\nu_2')}$$

we saw that

$$\sum_{\nu,\nu_1} |M\ (\nu \rightarrow \nu_1 \nu_2)|^2 = \text{constant independent of } \nu_2 \qquad (3.4)$$

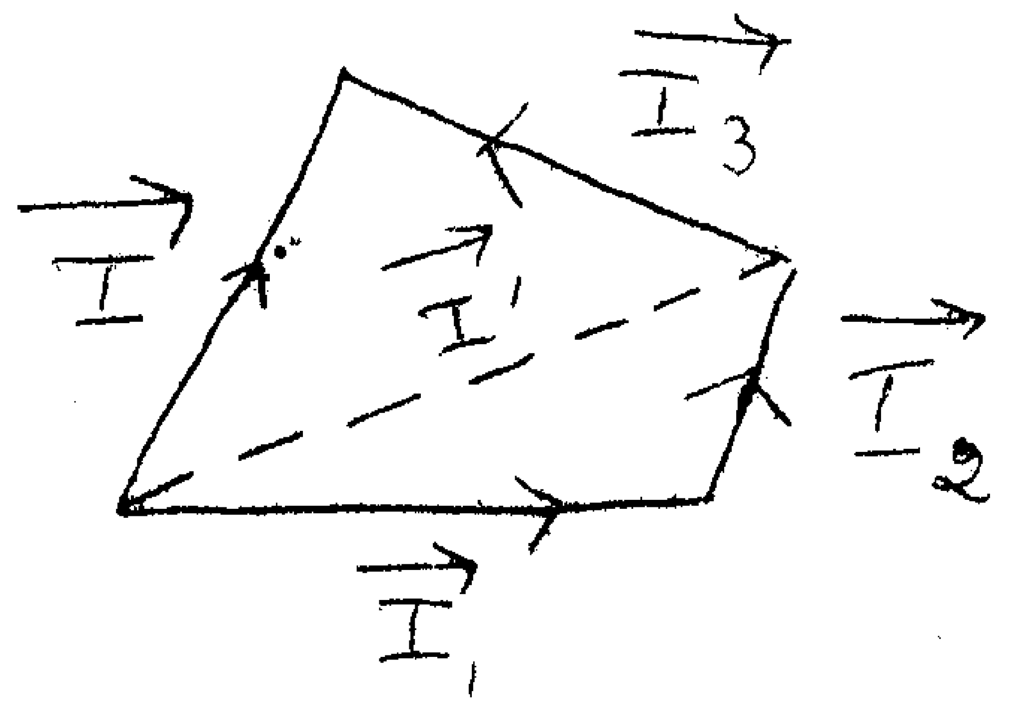
If we consider a single particle decaying into three particles $\nu \longrightarrow (\nu_1, \nu_2, \nu_3)$ this principle says that

$$\sum_{\nu_1 \nu_2 \nu_3} |M(\nu \rightarrow \nu_1, \nu_2, \nu_3)|^2 \quad \text{constant indopondont of } \nu \qquad (3.5)$$

In this case we have the relation

$$\vec{I_1} + \vec{I_2} + \vec{I_3} = I \qquad (3.6)$$

Corresponding to the diagram



the matrix element for the transition $(\nu_1, \nu_2) \longrightarrow (\nu_3, \nu_4)$ is

$$\sum_{I,\nu} g(I)\; C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu} \left(C^{I_3 I_4 I}_{\nu_3 \nu_4 \nu} \right)^* = M(\nu_1 \nu_2 \to \nu_3 \nu_4) \tag{3.7}$$

Hence the total transition rate sum (cf Rg.(5) is

$$\sum_{\nu_2, \nu_3, \nu_4} g(I)\, g^*(I') \left\{ C^{I_1 I_2 I}_{\nu_1 \nu_2 \nu} \; C^{*\, I_3 I_4 I}_{\nu_3 \nu_4 \nu} \right\} \cdot \left\{ C^{I_1 I_2 I'}_{\nu_1, \nu_2 \nu'} \; C^{*\, I_3 I_4 I'}_{\nu_3 \nu_4 \nu'} \right\} .$$

$$= \sum_{\nu_2, \nu_3, \nu_4} \left| M(\nu_1, \nu_2 \longrightarrow \nu_3, \nu_4) \right|^2$$

$$= \text{independent of } \nu_1 \tag{3.8}$$

This is, as we know, a consequence of the charge independence of strong interactions. Suppose for example, we consider the elastic process

$$\Sigma \pi \longrightarrow \Sigma \pi$$

With two isovectors, we can have independent amplitudes corresponding to I = 2, 1, 0 and the cross-sections can be expressed. in terms of the five algebraically independent quantities

$$|g_1|^2, \; |g_2|^2, \; |g_0|^2, \quad \arg(g_1^* g_2), \; \arg(g_2^* g_0),$$

Now let us see the electromagnetic properties of an isomultiplet. We know the relation

$$Q = I_3 + \frac{Y}{2}$$

in which the hypercharge Y is unchanged by isospin rotations and only I_3 is changed by such an isospin rotation. The currents that can be formed is

$$\begin{aligned} j_\mu^\alpha(x) &= e\, \overline{\Psi}_\alpha(x)\, \gamma_\mu\, \Psi_\alpha(x) \\ &= Q\, \overline{\Psi}(x)\, \gamma_\mu\, \Psi(x) \\ &= e\, \overline{\Psi}(x)\, \gamma_\mu \left(\frac{Y}{2} + I_3\right) \Psi(x) \end{aligned} \tag{3.9}$$

It should be remembered that Y remains invariant under isospin rotations.

Also

$$\int j_\mu(x)\, e^{ikx}\, d^4x = F_\mu(k)$$

(3.10)

has the same transformation properties as $j_\mu^{(x)}(x)$.

For example, let us compute the magnetic moment

$$\langle I\nu |\, j_\mu(x) \,| I\nu\rangle = \langle I\nu \,|\, a + bT_3 \,|\, I\nu\rangle$$

(3.11)

where the first term is a isospin scalar corresponding to the invariant hypercharge occurring in the charge operator while the second transforms like the third component of a vector (in isospin space). It is clear that the second term is proportional to ν since

$$C^{\,I\ 1\ I}_{\nu,\,0\ \nu} \propto \nu$$

Thus the current operator transforms like the sum of a scalar and the third component of a vector viz as $a + b\nu$. For example, consider the magnetic moment of the nucleons $(I = 1/2)$ we have from (3.11)

$$\begin{aligned} \mu_p &= a + 1/2b \\ \mu_n &= a - 1/2b \end{aligned} \tag{3.12}$$

In the case of Σ particles we have

$$\begin{aligned} \mu_{\Sigma^+} &= a + b \\ \mu_{\Sigma^0} &= a, \\ \mu_{\Sigma^-} &= a - b. \end{aligned} \tag{3.13}$$

so that we get a relation

$$\mu_{\Sigma^+} + \mu_{\Sigma^-} = 2\mu_{\Sigma^0} \tag{3.14a}$$

In general for any charge triplet

$$\mu_+ + \mu_- = 2\mu_0 \tag{3.14}$$

Consider the magnetic moments of N_* (I = 3/2).

$$\begin{aligned} \mu(N^*_{++}) &= a + 3/2\,b \\ \mu(N^*_{+}) &= a + 1/2\,b, \\ \mu(N^*_{0}) &= a - 1/2\,b, \\ \mu(N^*_{-}) &= a - 3/2\,b, \end{aligned} \tag{3.15}$$

so that we get two relations between the magnetic moments of these four particles. We can also extend this $\Delta\vec{I} = 1$ rule for electromagnetic interaction to the $\Delta\vec{I} = \frac{1}{2}$ rule of the weak interactions. We can also extend this to second order electromagnetic interactions which we will see in the next lecture.