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LECTURES ON BROKEN SYMMETRY AND GOLDSTONE BOSON

E. YAMADA
Visiting Member, Matscience

THE INSTITUTE OF MATHEMATICAL SCIENCES, MADRAS-20, INDIA.

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Professor E. Yamada* **
Visiting Member,
MATSCIENCE, Madras.

* Permanent Address: Research Institute for Fundamental Physics, Kyoto University, Kyoto, Japan.

** Visiting Member under the Council of Scientific and Industrial Research Scheme.

1. Introduction

Recently, much work has been done using "self-consistent" method. We can get nonperturbative solutions with this kind of approximation. This feature makes it quite suitable to discuss, for instance, the break down of symmetry.

The essense of the method is quite simple: first, we assume an indefinite value for some physical quantity. Then we calculate this quantity using the given value, and equate the result with the given value. This procedure gives an equation for a value of the quantity.

We shall give some examples which make use of this method.

a) Nambu Model[1)]

Let us consider a fermion field with self interaction. The calculation of the observable mass to the lowest order in new Tamm-Dancoff approximation gives an equation for mass m in the following form,

$$m = m_0 + m f(m) \tag{1}$$

Here, $f(m)$ is a divergent integral which depends on m and m_0 is a bare mass which should vanish if the Lagrangian is invariant under γ_5-transformation. For $m_0 \neq 0$, the above equation expresses an ordinary mass renormalization. In the case of γ_5 -transformation invariance, we have a "trivial" solution $m = 0$. Then there is no violation of symmetry. Equation (1) can have, however, other kind of solutions when we introduce cut-off

to make $f(m)$ finite. The solution is given by, $f(m) = 1$
If this equation has non-vanishing solution, it gives a "non-trivial" solution which vilates γ_5 -invariance.

b) Goldstone Model[2) 3)]

The model is composed of a spinless boson field ϕ with $\lambda\phi^4$ -type self interaction. Since the theory is invariant under the transformation $\phi \to -\phi$, the vacuum expectation value of the field $\langle\phi\rangle$ should vanish. If we calculate $\langle\phi\rangle$ in some approximation (e.g. Fig. 1), treating $\langle\phi\rangle$ as if it has non-vanishing value, we get an equation

Fig. 1.

$$\langle\phi\rangle_0 = \langle\phi\rangle_0 f(\langle\phi\rangle_0). \tag{2}$$

Here f is a divergent integral depending on $\langle\phi\rangle_0$ (f in (2) is different from f in (1)). In this case too, it is possible to have a non-trivial solution besides the trivial one.

This model can be generalized to include several bosons, and Goldstone suggested the existence of Goldstone boson in the generalized version.

c) Nishijima's theory of weak interaction[10)]

First, we assume that the strongly interacting particles (baryons and mesons) satisfy charge independence and strangeness conservation. Though the matrix elements which violate the conservation.laws should vanish, we calculate those matrix elements with similar method as in the above examples (e.g. Fig. 2). When we use the lowest order approximation for weak interaction, we get linear homogeneous equations for the matrix elements. With suitable choice of coupling constants for strong interactions, it is possible to have a non-vanishing matrix elements as the solutions of the equations, although we cannot determine the strength of weak interaction in this theory.

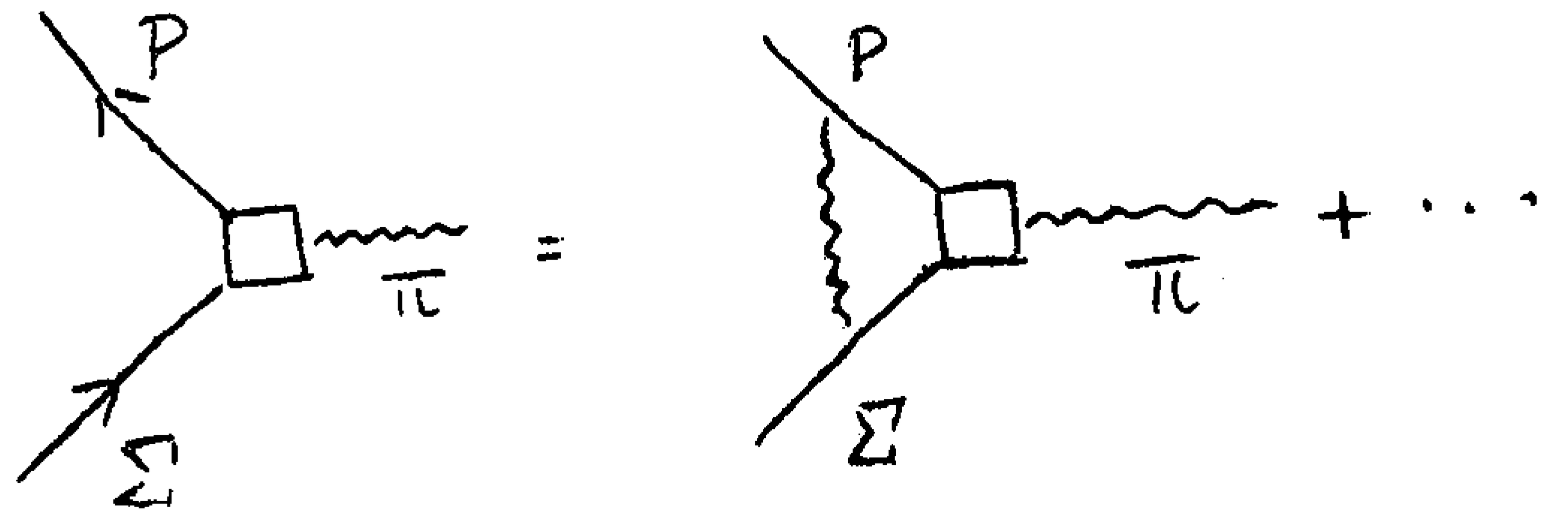

Fig.2.

Above three examples are concerned with broken symmetry. Bootstrap process is another kind of self-consistent method. We shall discuss here the self-consistent procedure which causes a spontaneous break down of symmetry. We note that there is always a trivial solution which preserves the symmetry.

We may ask the following questions in these problems:

1) Which is the correct solution, trivial or nontrivial one? And what is the criterion to pick up the correct solution?

In case of BCS-Bogoliubov theory of superconductors, the criterion is provided by the energy gap. But this cannot be directly translated into the case of elementary particles, for there can be no transition between the states which belong to mutually inequivalent representations.

Some people argue as follows. Though there is no transition between inequivalent states, we may imagine the existence of a small perturbation which connects these states in an enlarged Hilbert space containing both the mutually orthogonal subspaces. Then, using the perturbation, the stability of each state can be investigated. This is in analogy with the theory of superconductivity.

Maki[4)] calculated the S-matrix element for fermion-antifermion scattering in Nambu model, and found that there was a pole in non-physical region on physical sheet for ^1S state in case of trivial solution. (The pole goes into non-physical region when the coupling becomes strong enough to cause the non-trivial solution). Then he concluded that the non-trivial solution is the right one in this case. The conclusion is probably dependent on the approximation. It is desirable to have at least a physical explanation for the whole process, though there may be no possibility to derive this conclusion without any approximation.

2) What is the mechanism of the violation of the symmetry?

Usual explanation is the inequivalence between symmetric and asymmetric states. This should certainly be true in mathematical sense. If there is an asymmetric solution for symmetric theory, there should exist other states which are obtained by the symmetry transformation from the original one. These new states should have same energy and momentum etc. When all of them belong to same Hilbert space and can be transformed into each other by suitable unitary transformation, no difficulty of symmetry violation arises. It seems however, the inequivalence is merely a mathematical statement on the symmetry violation and it does not explain anything about the physical process which causes the violation.

Inequivalent representation itself is rather a familiar thing in quantum field theory. Renormalization and true vaçuum are typical examples. In the case of symmetry violation, there arises inequivalence due to the limiting processes to infinity of normalization volume. Actual volume which contains total system is finite but it is very big compared with the wavelength in question. The inner product of vectors belonging to "inequivalent" subspace is not zero but infinitesimal for large but finite normalization volume. Thus the situation should be explained in the terms which has no singularity for infinite normalization volume.

The last question is:

3) After all, is this explanation of broken symmetry is the right one?

In the following the question 2) will be discussed. A brief discussion will be made also on the question 3) in conclusion. First, a few examples will be used to show that a boson with vanishing mass (Goldstone boson) appears accompanying the break down of the symmetry. Then the role played by the boson in the break down process will be discussed. Unfortunately, the proof of the argument cannot be given, because it is not able to avoid to use some approximation in the course of discussion.

The difficulty to give a proof lies at least partially on the nature of the problem. Since the self-consistent method does not provide a diagonalization of Hamiltonian, there remains ambiguity, for instance, in the definition, of vacuum. Usually self-consistent equation is obtained with some approximation. To be consistent in the argument, it is necessary to use <u>same</u> approximation for various processes. But the concept of same approximation for different processes is not at all clear one.

There is another problem concerning this point, With some approximations, we get selfconsistent equations which allow non-trivial solutions. It is, however, not certain whether the correct equations have such solutions. There are identities which turn into equations when some approximations are made.[5)]

2, Example: Spinless Bosons

Let us consider two spinless bosons with the following Lagrangian density,

$$L = -\frac{1}{2}\sum_{i=1,2}\left[(\partial_\mu \phi_i)^2 + \mu_0^2 \phi_i^2\right] - \frac{1}{4}\lambda\left(\sum_{i=1,2}\phi_i^2\right)^2 \tag{3}$$

The Lagrangian is invariant under O_2,[+)]

$$\phi_1' = \cos\theta\cdot\phi_1 + \sin\theta\cdot\phi_2$$

$$\phi_2' = -\sin\theta\cdot\phi_1 + \cos\theta\cdot\phi_2 \tag{4}$$

+) Generalization to the case of n bosons which is invariant under O_n is straightforward.

So that the physical mass for two fields ϕ_1 and ϕ_2 should be equal when the symmetry is preserved.

Field equation is given by,

$$(\Box - \mu_0^2)_i(x) = \lambda \Big(\sum_{j=1,2} \phi_j(x)^2\Big)\, \phi_i(x) \tag{5}$$

We define causal propagator by,

$$\Delta_i^c(x-y) = \langle 0 | T\phi_i(x)\, \phi_i(y) | 0 \rangle \tag{6}$$

The propagator satisfies the equation,

$$(\Box - \mu_0^2)\, \Delta_i^c(x-y) = -i\,\delta(x-y) +$$

$$+ \lambda \sum_{j=1,2} \langle 0 | T\phi_j(x)\phi_j(x)\,\phi_i(x)\,\phi_i(y) | 0 \rangle \tag{7}$$

Then the following approximations are made,

$$\langle 0 | T\phi_k(x)\,\phi_\ell(x)\,\phi_i(x)\,\phi_j(y) | 0 \rangle \sim \langle 0 | \phi_k(x)\,\phi_\ell(x) | 0 \rangle\, \Delta_i^c(x-y)$$

$$+ \langle 0 | \phi_\ell(x)\,\phi_i(x) | 0 \rangle\, \Delta_k^c(x-y) + \langle 0 | \phi_i(x)\,\phi_k(x) | 0 \rangle\, \Delta_\ell^c(x-y),$$

$$\langle 0 | \phi_k(x)\,\phi_\ell(x) | 0 \rangle \sim \sum_{i=1,2} \int d^3p\, \langle 0 | \phi_k(x) | p, i \rangle \langle p, i | \phi_\ell(x) | 0 \rangle$$

$$= \delta_{k\ell} \int \frac{d^3p}{2\omega_p^k} = \delta_{k\ell} \int d^4p\, \theta(p_0)\, \delta(p^2 + m^2)$$

$$= \frac{1}{2}\,\delta_{k\ell}\, \Delta^{(1)}_{\mu_k}(0) = \delta_{k\ell}\, \Delta^c_{\mu_k}(0)$$

Here, $\omega_p^{4} = \sqrt{\underset{\sim}{p}^2 + \mu_K^2}$ and

$$\Delta_\mu^c(0) = \frac{-i}{(2\pi)^4} \int_{CF} \frac{d^4p}{p^2+\mu^2} \tag{8}$$

With these approximations, Eq.(7) becomes,

$$(\Box - \mu_i^2)\, \Delta_i^c(x-y) = -i\,\delta(x-y) \tag{7'}$$

Where μ_i^2 is defined as,

$$\mu_i^2 = \mu_0^2 + \lambda \left[\Delta_{\mu_i}^c(0) + \sum_{j=1,2} \Delta_j^c(0) \right] \tag{9}$$

The equation (7') means that physical mass for the field ϕ_i is μ_i .

Next, we shall calculate the scattering $\phi_{i_1} + \phi_{i_2} \rightarrow \phi_{j_1} + \phi_{j_2}$

The S-matrix element is given by,

$$\langle q_1 j_1 ; q_2 j_2 | S | p_1 i_1 ; p_2 i_2 \rangle = -\frac{1}{(2\pi)^3} \frac{1}{\sqrt{4\omega_{q_1}^{j_1} \omega_{q_2}^{j_2}}} \iint_{-\infty}^{\infty} dx\, dy\, e^{-i(q_1 x + q_2 y)}$$

$$\times (\Box_x - \mu_{j_1}^2)(\Box_y - \mu_{j_2}^2) \langle 0 | T\, \phi_{j_1}(x)\, \phi_{j_2}(y) | p_1 i_1 ; p_2 i_2 \rangle \tag{10}$$

Using the same approximation as in the calculation of masses, we get from Eq.(5),

$$(\Box - \mu_{j_1}^2)(\Box_y - \mu_{j_2}^2)\langle 0|T\phi_{j_1}(x)\phi_{j_2}(y)|P_1 i;\ P_2 i_2\rangle$$
$$\simeq -2i\delta(x-y)\lambda\langle 0|T\phi_{j_1}(x)\phi_{j_2}(y)+\delta_{j_1 j_2}\sum_{k=1,2}T\phi_k(x)\phi_k(y)|$$
$$P_1 i;\ P_2 i_2\rangle$$

When we define the following quantities,

$$\langle 0|T\phi_{j_1}(x)\phi_{j_2}(y)|P_1 i_1;\ P_2 i_2\rangle \equiv \frac{e^{i(P_1+P_2)(x+y)/2}}{2(2\pi)^2\sqrt{\omega_{P_1}^{i_1}\omega_{P_2}^{i_2}}}\int d^4k\, e^{ik(x-y)}\mathcal{F}_{i_1 i_2}^{j_1 j_2}(k)$$

$$\mathcal{G}_{ij}^{kl} \equiv \int d^4p\,\mathcal{F}_{ij}^{kl}(p)$$

$$G_{ij}^{kl} \equiv \frac{1}{2\pi}\left[\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\right].$$

S-matrix element is expresses as,

$$\langle q_1 j_1;\ q_2 j_2|S|P_1 i_1;\ P_2 i_2\rangle =$$
$$= i\frac{\lambda}{4\pi}\delta(P_1+P_2-q_1-q_2)\left[2\mathcal{G}_{i_1 i_2}^{j_1 j_2}+\delta_{j_1 j_2}\sum_{k=1,2}{}^{'}\mathcal{G}_{i_1 i_2}^{kk}\right] \tag{11}$$

The quantity $\mathcal{G}$ satisfies the following relation

$$\mathcal{G}_{i_1 i_2}^{j_1 j_2} = G_{i_1 i_2}^{j_1 j_2} + \lambda R_{j_1 j_2}\left[2\mathcal{G}_{i_1 i_2}^{j_1 j_2}+\delta_{j_1 j_2}\sum_{k=1,2}\mathcal{G}_{i_1 i_2}^{kk}\right] \tag{12}$$

where $R_{j_1 j_2}$ is defined by $P = P_1 + P_2$ being the total momentum of the state,

$$R_{j_1 j_2}(P) = \frac{i}{(2\pi)^4} \int_{C_F} d^4k \left[(P/2 + k)^2 + \mu_{j_1}^2\right]^{-1} \left[(P/2 - k)^2 + \mu_{j_2}^2\right]^{-1} \tag{13}$$

The pole of the scattering amplitude for $\phi_1 + \phi_2 \to \phi_1 + \phi_2$ is determined by the condition,

$$2\lambda R_{12}(P) = 1 \tag{14}$$

To compare the conditions (9) and (14), we define the functions $K_n(a)$

$$K_n(a) = \frac{-i}{(2\pi)^4} \int_{C_F} \frac{d^4p}{(p^2+a)^n} \tag{15}$$

Since the integral is divergent for $n \leqq 2$, the cut-off is introduced. We use the same cut-off factor for all n. With a reasonable choice of cut-off factor, $K_n(a)$ is positive definite and satisfied the relation.

$$\frac{d}{da} K_n(a) \equiv K_n'(a) = -n K_{n+1}(a). \tag{16}$$

The quantities $\Delta_\mu^c(0)$ and R_{ij} can be written in terms of the functions $K_n(a)$

$$\Delta_\mu^c(0) = K_1(\mu^2) \tag{17}$$

$$R_{ij}(P) = -\frac{1}{2} \int_{-1}^{1} d\alpha\, K_2\left(\frac{1}{2}(\mu_i^2 + \mu_j^2) + \frac{1}{2}(\mu_i^2 - \mu_j^2) + \frac{1}{4}(1-\alpha^2)P^2\right) \tag{18}$$

We look for the solution of Eq.(9) which gives the different values for two masses, $\mu_1^2 \neq \mu_2^2$ From Eq. (9),

$$\mu_1^2 - \mu_2^2 = 2\lambda\left[K_1(\mu_1^2) - K_1(\mu_2^2)\right]$$

If $\mu_1^2 = \mu_2^2$, this gives the condition for λ

$$1/\lambda = 2\left[K_1(\mu_1^2) - K(\mu_2^2)\right]/\left[\mu_1^2 - \mu_2^2\right] \tag{19}$$

Since K_n is a monotonically decreasing function, Eq.(19) indicates that λ should be negative.

Next, we expand the intgrand on the right hand side of Eq.(18) in the power series of P^2. Then, up to the first order,

$$R_{ij}(P) \sim -\frac{1}{2}\int_{-1}^{1} d\alpha\Big[K_2\big(\tfrac{1}{2}(\mu_i^2+\mu_j^2) + \tfrac{1}{2}\alpha(\mu_i^2-\mu_j^2)\big) - \\ -\frac{1}{2}(1-\alpha^2)P^2 K_3\big(\tfrac{1}{2}(\mu_i^2+\mu_j^2) + \tfrac{1}{2}\alpha(\mu_i^2-\mu_j^2)\big)\Big]$$

$$= \left[K_1(\mu_i^2) - K_1(\mu_j^2)\right]/\left[\mu_i^2-\mu_j^2\right] - \frac{1}{4}P^2\langle 1-\alpha^2\rangle_3 \frac{K_2(\mu_i^2)-K_2(\mu_j^2)}{\mu_i^2-\mu_j^2} \tag{20}$$

where $\langle 1-\alpha^2\rangle_n$ is defined by

$$\langle 1-\alpha^2\rangle_n = \\ \int_{-1}^{1} d\alpha\,(1-\alpha^2)K_n\big(\tfrac{1}{2}(\mu_i^2+\mu_j^2)+\tfrac{1}{2}\alpha(\mu_i^2-\mu_j^2)\big)\Big/ \\ \int_{-1}^{1} d\alpha\,K_n\big(\tfrac{1}{2}(\mu_i^2+\mu_j^2)+\tfrac{1}{2}\alpha(\mu_i^2-\mu_j^2)\big)$$

Obviously,

$$1 \geqq \langle 1-\alpha^2 \rangle_n \geqq 0$$

When Eq.(19) is satisfied,

$$1 - 2\lambda R_{12}(P) \sim \frac{1}{2}\lambda P^2 \langle 1-\alpha^2 \rangle_3 \frac{K_2(\mu_1^2) - K_2(\mu_2^2)}{\mu_1^2 - \mu_2^2} \tag{21}$$

Thus the scattering amplitude for $\phi_1 + \phi_2 \rightarrow \phi_1 + \phi_2$ has a pole at $P^2 = 0$ with a residue $(1/2\pi^2)\langle 1-\alpha^2\rangle_3^{-1}[(\mu_1^2-\mu_2^2)/(K_2(\mu_1^2)-K_2(\mu_2^2))]$ This means the existence of Goldston boson as a bound state of ϕ_1 and ϕ_2 +).

There are two trivial solutions of Eq.9) when the value of λ permits non-trivial solution. One of them gives the scattering amplitude of wrong analyticity, a pole at $P^2 > 0$. The other solution does not show such a difficulty.

3. Example; Nambu Model

The model is that of spin one half fermion with self interaction which is invariant under γ_5 -transformation. The Lagrangian density is given by,

$$L = \bar{\psi}\gamma\partial\psi - \frac{1}{2}g(J_S^2 - J_P^2) \tag{22}$$

+) Since $\mu_1 \neq \mu_2$ one of the ϕ_i can decay into the lighter one and Goldstone boson. But the decay process occurs in the higher order.

where,

$$J_S = \frac{1}{2} \sum_\alpha [\bar{\Psi}_\alpha \Psi_\alpha - \Psi_\alpha \bar{\Psi}_\alpha]$$

$$J_P = \frac{1}{2} \sum_\alpha [\bar{\Psi}_\alpha (\gamma_5 \Psi)_\alpha - (\gamma_5 \Psi_\alpha)_\alpha \bar{\Psi}_\alpha] \tag{23}$$

A similar approximation as in the preceding example gives the self-consistent equation for mass,

$$m = 4g \Delta^c_m (0) m \tag{24}$$

The scattering amplitude for the fermion and the antifermion in $^1S_0$ state in the ladder approximation is,

$$T = \frac{g}{(2\pi)^2} [a_p a_A + 8g^2 m^2 P^2 S_2]^{-1} [a_A + \frac{2m^2}{P^2} a_p + 8g m^2 S_2] \tag{25}$$

where

$$a_p = 1 - 4g (S_0 - 2 S_1 + 2m^2 S_2)$$

$$a_A = 1 + g (S_0 + S_1 - 4m^2 S_2) \tag{26}$$

The quantities S_k'S are defined as,

$$S_0 \equiv \frac{1}{2}\int_{-1}^{1} d\alpha\, K_1(m^2 + \frac{1}{4}(1-\alpha^2)P^2)$$
$$= K_1(m^2 + \frac{1}{4}\theta_0 P^2)$$
$$S_1 \equiv \frac{1}{2}\int_{-1}^{1} d\alpha\, [m^2 + \frac{1}{4}(1-\alpha^2)P^2]\, K_2(m^2 + \frac{1}{4}(1-\alpha^2)P^2)$$
$$= [m^2 + \frac{1}{4}\theta_1 P^2]\, K_2(m^2 + \frac{1}{4}\theta_1 P^2)$$
$$S_3 \equiv \frac{1}{2}\int_{-1}^{1} d\alpha\, K_2(m^2 + \frac{1}{4}(1-\alpha^2)P^2)$$
$$= K_2(m^2 + \frac{1}{4}\theta_2 P^2)$$

These expressions containing θ_k's are obtained by mean[27] value theorem of integration. The θ_k's are functions of m^2 and take values between zero and one,

$$0 < \theta_k < 1$$

In the limit of $P^2 \to 0$

$$a_P = 1 - 4g K_1(m^2) + O(P^2),$$
$$a_A = 1 + g(K_1(m^2) - 3K_2(m^2)) + O(P^2) \qquad (28)$$

Since the mass equation (24) can be written as,

$$m[1 - 4g K_1(m^2)] = 0, \qquad (24')$$

the scattering amplitude has a pole at $p^2 = 0$ when the mass m of the fermion is the non-trivial solution of Eq.(24'). This shows the appearance of Goldstone boson as a bound state of fermion and antifermion.

Next the trivial solution $m = 0$ is examined when there exists a non-trivial solution. The solution of $4gK_1(m_0^2) = 1$ is denoted as m_0 . The scattering amplitude for vanishing mass is given by;

$$T = \frac{1}{(2\pi)^2}\, g\, a_p^{-1} \tag{25'}$$

where,
$$\begin{aligned} a_p &= 1 - 4g\,(S_0 - 2S_1) \\ &= 4g\,[K_1(m_0^2) - K_1(\tfrac{1}{4}\theta_0 p^2) + \tfrac{1}{2}\theta_1 p^2 K_2(\tfrac{1}{4}\theta_1 p^2)] \\ &\equiv 4g\,F(p^2) \end{aligned} \tag{29}$$

The pole of the scattering amplitude is determined by the zero point of the function $F(p^2)$, Since $F(p^2)$ satisfies the inequalities,

$$F(0) = K_1(m_0^2) - K_1(0) < 0, \tag{30}$$

and
$$F\left(\frac{4m_0^2}{\theta_0}\right) = 2\frac{\theta_1}{\theta_0}\, m_0^2\, K_2\left(\frac{\theta_1}{\theta_0} m_0^2\right), \tag{30'}$$

the zero point of $F(p^2)$ exists in the region,

$$0 < p^2 < 4m_0^2/\theta_0$$

This shows that the scattering amplitude has the wrong analyticity, that is, the existence of a pole on the negative real axis corresponding to imaginary mass.

4. Example: Goldstone Models

Goldstone's first model is a neutral spinless field with the following Lagrangian density,

$$L = -\frac{1}{2}[(\partial_\mu \phi)^2 + \mu_0^2 \phi^2] - \frac{\lambda}{4}\phi^4 \tag{31}$$

The system is invariant under the transformation $\phi \to -\phi$ When the symmetry is preserved, the vacuum expectation value of the field $\langle \phi \rangle_0$ should vanish. We shall look for the symmetry violating solution. We define ϕ' by,

$$\phi(\underset{\sim}{x}) = \langle \phi \rangle_0 + \phi'(\underset{\sim}{x}) \tag{32}$$

The vacuum expectation value $\langle \phi \rangle_0$ should be constant due to translational invariance. The hamiltonian of the system is,

$$H = \frac{1}{2}\int d^3x\, [\pi^2 + (\underset{\sim}{\nabla}\phi)^2 + \mu_0^2 \phi^2 + \frac{\lambda}{2}\phi^4] \tag{33}$$

where $\pi(x)$ is a canonical conjugate of $\phi(x)$. By the definition (33), it is clear that $\phi'(x)$ too satisfies the canonical commutation relation with $\pi(x)$. Inserting the definition (32) into the Hamiltonian, we get,

$$H = \frac{1}{2} V(\mu_0^2 \langle\phi\rangle_0^2 + \frac{\lambda}{2} \langle\phi\rangle_0^4) + \int d^3x [\mu_0^2 \langle\phi\rangle_0 + \lambda \langle\phi_0\rangle^3]\phi' + \tag{34}$$

$$+ \frac{1}{2} \int d^3x [\pi^2 + (\nabla \phi')^2 + (\mu_0^2 + 3\lambda \langle\phi\rangle_0^2) \phi'^2] +$$

$$+ \lambda \int d^3x [\langle\phi\rangle_0 \phi'^3 + \frac{1}{4} \phi'^4]$$

Here, V is a normalization volume. Since the second term on the right hand side of Eq. (34) is a "dangerous" term, the coefficient of ϕ' should vanish. the condition gives the equation for $\langle\phi\rangle_0$

$$\langle\phi\rangle_0 (\mu_0^2 + \lambda \langle\phi\rangle_0^2) = 0 \tag{35}$$

With the value of $\langle\phi\rangle_0$ which is determined by Eq.(35), the effective mass of ϕ' -field is given as,

$$\mu^2 = \mu_0^2 + 3\lambda \langle\phi\rangle_0^2 \tag{36}$$

The trivial solution of Eq.(35) is $\langle\phi\rangle_0 = 0$, which respects the parity invariance. The non-trivial solution $\langle\phi\rangle_0^2 = -\frac{1}{\lambda}\mu_0^2$ gives a negative value for μ^2

$$\mu^2 = -2\mu_0^2 \tag{37}$$

His second model is composed of two spinless bosons with the symmetrical Lagrangian density,

$$L = -\frac{1}{2}\sum_{i=1,2}\left[(\partial_\mu \phi_i)^2 + \mu_0^2 \phi_i^2\right] - \frac{\lambda}{4}\left(\sum_{i=1,2}\phi_i^2\right)^2 \qquad (38)$$

The system is invariant under two dimensional rotation

$$\phi_1 \rightarrow \cos\theta\cdot\phi_1 + \sin\theta\cdot\phi_2$$
$$\phi_2 \rightarrow -\sin\theta\cdot\phi_1 + \cos\theta\cdot\phi_2\,, \qquad (39)$$

besides the parity transformation $\phi_i \rightarrow -\phi_i$. Thus the vacuum expectation value of the fields $\langle\phi_i\rangle_0$ should vanish. It is possible, however, to have a non-trivial solution in this case too.

We assume that the solution has a form,

$$\phi_i = \phi_i' + \langle\phi_i\rangle_0 \qquad (40)$$

The expectation value $\langle\phi_i\rangle_0$ must be constant. With the transformation (39) $\langle\phi_2\rangle_0$ can be made to be zero. Inserting (40) into the Hamiltonian of the system which is derived from the Lagrangian (38), we get,

$$H = \frac{1}{2}V\left[\mu_0^2\langle\phi_1\rangle_0^2 + \frac{\lambda}{2}\langle\phi_1\rangle_0^4\right] + \int d^3x\,\langle\phi_1\rangle_0\left[\mu_0^2 + \lambda\langle\phi_1\rangle_0^2\right]\phi_1' +$$
$$+ \frac{1}{2}\int d^3x\left[\sum_{i=1,2}\left\{\pi_i^2 + (\nabla\phi_i')^2 + (\mu_0^2 + \lambda\langle\phi_1\rangle_0^2)\,\phi_i'^2\right\} + 2\lambda\langle\phi_1\rangle_0^2\,\phi_1'^2\right] +$$
$$+ \int d^3x\left[\lambda\langle\phi_1\rangle_0\phi_1'\left(\sum_i\phi_i'^2\right) + \frac{1}{4}\lambda\left(\sum_i\phi_i'^2\right)^2\right] \qquad (41)$$

The condition to eliminate the "dangerous" term is,

$$\langle\phi_1\rangle_0\,[\mu_0^2+\lambda\langle\phi_1\rangle_0^2]=0 \tag{42}$$

The trivial solution of Eq.(42) is $\langle\phi_1\rangle_0$ while the non-trivial one is $\langle\phi_1\rangle_0^2=-\frac{1}{\lambda}\mu_0^2$ The masses are given by,

$$\begin{aligned}\mu_1^2&=\mu_0^2+3\lambda\langle\phi_1\rangle_0^2,\\ \mu_2^2&=\mu_0^2+\lambda\langle\phi_1\rangle_0^2\end{aligned} \tag{43}$$

Eqs. (43) give $\mu_1^2=-2\mu_0^2$ and $\mu_2^2=0$ for non-trivial solution. Goldstone concluded from the results that there appears a massless boson ______Goldstone boson_______ when the symmetry given by conginuous transformation is violated. We note, however, that the non-trivial solutions in these models give negative values for the square of mass, that is imaginary mass. If these solutions ar rejected as physically unacceptable, there is no violation of symmetry and no Goldstone boson.

Umezawa and Kamefuchi[6) criticized the Goldstone's argument pointing out that the effect of field reaction invalidate his conclusion. The field reaction can be calculated using Feynman diagram method. But we shall use a different method here.

The field quantities ϕ' and π are expanded as,+)

$$\phi'(\underset{\sim}{x}) = \sum_{\underset{\sim}{k}} \frac{1}{\sqrt{2\omega_{\underset{\sim}{k}} V}} e^{i\underset{\sim}{k}\underset{\sim}{x}} (a_{\underset{\sim}{k}} + a^*_{-\underset{\sim}{k}}),$$

$$\pi(\underset{\sim}{x}) = \frac{1}{i} \sum_{\underset{\sim}{k}} \sqrt{\frac{\omega_k}{2V}} e^{i\underset{\sim}{k}\underset{\sim}{x}} (a_{\underset{\sim}{k}} - a^*_{-\underset{\sim}{k}}) \tag{44}$$

where $\omega_k = \sqrt{\underset{\sim}{k}^2 + \mu^2}$ $a_{\underset{\sim}{k}}$ and $a^*_{\underset{\sim}{k}}$ satisfy the commutation relation $[a_{\underset{\sim}{k}}, a^*_{\underset{\sim}{k}'}] = \delta_{\underset{\sim}{k}\underset{\sim}{k}'}$. With these expressions the Hamiltonian (34) takes the form after ordering,

$$H = \frac{1}{2} V(\mu_0^2 \langle\phi\rangle_0^2 + \frac{\lambda}{2} \langle\phi\rangle_0^4) + \sqrt{\frac{V}{2}} [\langle\phi\rangle_0 (\mu_0^2 + \lambda\langle\phi\rangle_0^2 + 3\lambda\Delta_\mu^C(0)) a_{\underset{\sim}{0}} + h.c] +$$

$$+ \frac{1}{2} \sum_{\underset{\sim}{k}}' [\omega_k + \frac{1}{\omega_k}(\underset{\sim}{k}^2 + \mu_0^2 + 3\lambda\langle\phi\rangle_0^2 + 3\lambda\Delta_\mu^C(0))] a^*_{\underset{\sim}{k}} a_{\underset{\sim}{k}} +$$

$$+ \frac{1}{4} \sum_{\underset{\sim}{k}}' \{[-\omega_k + \frac{1}{\omega_k}(\underset{\sim}{k}^2 + \mu_0^2 + 3\lambda\langle\phi\rangle_0^2 + 3\lambda\Delta_\mu^C(0))] a_{\underset{\sim}{k}} a_{-\underset{\sim}{k}} + h.c\} \tag{45}$$

+(higher power than terms in $a_{\underset{\sim}{k}}$ and a^*_k)

The second and the fourth terms are "dangerous" terms. The condition to eliminate these terms are given by,

$$\langle\phi\rangle_0 (\mu_0^2 + \lambda\langle\phi\rangle_0^2 + 3\lambda\Delta_\mu^C(0)) = 0 \tag{46}$$

$$\mu^2 = \mu_0^2 + 3\lambda\langle\phi\rangle_0^2 + 3\lambda\Delta_\mu^C(0) \tag{47}$$

+) We can use more general form for the expansion, but it gives same results as expansion (44)

Since $\Delta_\mu^c(0)$ is a positive quantity, the non-trivial solution of Eq.(46) is possible only for negative λ. Then μ^2 becomes negative.

The same method gives the conditions for the second Goldstone model.

$$\langle\phi_1\rangle_0[\mu_0^2 + \lambda\langle\phi_1\rangle_0^2 + 3\lambda\Delta_{\mu_1}^c(0) + \lambda\Delta_{\mu_2}^c(0)] = 0,$$

$$\mu_1^2 = \mu_0^2 + 3\lambda\langle\phi_1\rangle_0^2 + 3\lambda\Delta_{\mu_1}^c(0) + \lambda\Delta_{\mu_2}^c(0), \qquad (48')$$

$$\mu_2^2 = \mu_0^2 + \lambda\langle\phi_1\rangle_0^2 + \lambda\Delta_{\mu_1}^c(0) + 3\lambda\Delta_{\mu_2}^c(0). \qquad (49)$$

For the non-trivial solution,

$$\langle\phi_1\rangle_0^2 = -\frac{\mu_0^2}{(-\lambda)} - (3\Delta_{\mu_1}^c(0) + \Delta_{\mu_2}^c(0)),$$

Eqs. (49) are written as,

$$\mu_1^2 = -2\mu_0^2 - 2\lambda(3\Delta_{\mu_1}^c(0) + \Delta_{\mu_2}^c(0)) = 2\lambda\langle\phi_1\rangle_0^2,$$

$$\mu_2^2 = -2\lambda(\Delta_{\mu_1}^c(0) - \Delta_{\mu_2}^c(0)). \qquad (49')$$

If the symmetry is preserved for internal lines, μ_2^2 vanishes even when field reaction is taken into account, because $\Delta_{\mu_1}^c(0) = \Delta_{\mu_c}^c(0)$). It seems, however, to be more consistent to include the symmetry breaking effect for the internal lines too. Then μ does not vanish, and there appears no Goldstone boson as pointed out by Umezawa and Kamefuchi.

Although the model does not satisfy Goldstone theorem, we can reject this counter example on the ground that it gives imaginary mass for ϕ_1' field, that is physically unacceptable solution.+) It is obvious from Eq.(48), only negative value for λ enables to have non-trivial solution. In other words, Goldstone model can say nothing about the validity of Goldstone theorem.

+) The imaginary mass occurs in vanishing μ_2^2 case too.

5. Mechanism of Symmetry Violation.

In above discussions we have shown a few examples where Goldstone boson appears accompanying the symmetry violation. Unfortunately, the discussion is far from a proof of the theorem. Even the examples are shown to satisfy the theorem after using some approximations. In spite of the incompleteness of the argument, we believe that the theorem should be true, because it seems that only the existence of Goldstone boson provides a natural explanation of the symmetry violation.

At first, we state the meaning of the symmetry, though it may be rather self-evident. The symmetry of the system does not mean the symmetry of the state, but that of Hamiltonian (or Lagrangian). The eigenstate of the energy-momentum changes to a different state by a transformation belonging to the symmetry. The symmetry requires that the transformed state is also an eigenstate of the energy-momentum with same eigehvalue. It also requires that the matrix elements of the transformation operator is independent of time.

To see the mechanism of symmetry violation, let us consider a simple example of an one dimensional crystal. The distance between the nearest neighbour is denoted by a. Then the ground state of the crystal is invariant under the translation $x \rightarrow x + \delta$, only when δ is a multiple of a. If the system is invariant under translation the state obtained by the translation $x \rightarrow x + \delta$ from the original state has the same energy and thus describes another ground state. A small displacement of atoms from the equilibrium gives a sound wave in the crystal. Now a sound wave with finite intensity and infinite wave length, that is, infinitesimal wave number is equivalent to a translation of the whole crystal. Since the translated crystal has the same energy as the original one, the quantum of the sound wave should have an energy spectrum $E(k)$ such that $E(k) \to 0$ for $k \to 0$. In relativistic case, this kind of energy spectrum corresponds to vanishing mass. When the total length of the crystal is infinite, the finiteness of the intensity of sound wave for infinitesimal wave number requires infinite number of phonons and thus it gives an 'inequivalent' state. Physically, this 'inequivalence' corresponds to the 'macroscopic operation' which is necessary to make the translation of the infinite crystal.

Generalizing the argument of the above example, we may say that the energy-momentum degeneracy of the ground state, which contains an infinite number of degrees of freedom in space and time, corresponds to the existence of exicitations with energy spectrum of the form $E(k) \to 0$ for $k \to 0$.

This does not mean, however, any violation of symmetry. When there is an impurity in the crystal, the force exerted on the impurity depends on the relative place of it within a cell. Since the translation makes the whole system including the impurity move by the same distance, the transformed state still has the same energy. On the other hand, let us suppose that we have missed to recognize the difference in the state of crystal. In this case, the translation causes the change in the location of the impurity relative to the crystal lattice. Then those states are clearly different, and, for instance, they will have different energies. Thus we may consider that the translation invariance is violated. 'Forgetting' the existence of the crystal is not so simple as supposed at first. The replacement of the effect of lattice by phonons is an usual procedure. The above consideration indicates that we should not forget the possibility of having infinite number of zero energy phonons in ground states. (In usual circumstances, of course, it does not give any important effect for many electron problem.) The inclusion of an infinite number of zero energy bosons does not change the energy of ground state, but it can cause long range correlation because of the infiniteness of their wavelength.

More realistic case is that of Bogoliubov's theory of superfluidity of liquid helium[7]. Since helium atom (that of He^4) satisfies Bose statistics, Bose-Einstein condensation occurs at low temperature. Then there exists a macroscopic number of helium atoms in zero energy state. Bogoliubov argued that it would be a better approximation to treat the operators relating to this state, a_0 and a_0^*, as classical quantities than the approximations where all the states were considered on the same footing. For the 'physical system' which is treated quantum mechanically, is not closed, there is no conservation of number of helium atoms. The transition of helium atom between zero and finite momenta corresponds to the creation and annihilation of the atom. In this formalism it is clear that the 'vacuum' expectation value of the field describing the helium atom does not vanish.

The ground state is not a pure state but a mixed state with various number of helium atoms in this formalism. Actually, $\langle \phi(\underset{\sim}{x}) \rangle_0$ means a collection of $\langle v | \phi(\underset{\sim}{x}) | v' \rangle$, where v and v' denote components of ground state containing different number of helium atoms. Thus the violation of symmetry (number conservation) is a false one. But, if we forget the true nature of the ground state, it appears as though the number is not conserved. In this sense, helium atom itself plays a role of Goldstone boson which satisfies the condition $E(k) \to 0$ for $k \to 0$*.

If we want to discuss the problem in terms of quantum mechanics, we can express the situation in a little different way. Let us consider the quantity,

$$\langle 0 | \phi(\underset{\sim}{x}) \phi(\underset{\sim}{y}) | 0 \rangle = \sum_{\underset{\sim}{k}} \sum_{\alpha} \langle 0 | \phi(\underset{\sim}{x}) | \underset{\sim}{k}, \alpha \rangle \langle \underset{\sim}{k}, \alpha | \phi(\underset{\sim}{y}) | 0 \rangle$$

$$= \sum_{\underset{\sim}{k}} \sum_{\alpha} | c_\alpha(\underset{\sim}{k}) |^2 e^{i \underset{\sim}{k} (\underset{\sim}{x} - \underset{\sim}{y})} \tag{50}$$

where

$$c_\alpha(\underset{\sim}{k}) = \langle 0 | \phi(0) | \underset{\sim}{k}, \alpha \rangle$$

If $c_\alpha(\underset{\sim}{k})$ is a smooth function of $\underset{\sim}{k}$,

$$\sum_{\underset{\sim}{k}} \sum_{\alpha} | c_\alpha(\underset{\sim}{k}) |^2 e^{i \underset{\sim}{k} (\underset{\sim}{x} - \underset{\sim}{y})} \to 0 \quad \text{for } | \underset{\sim}{x} - \underset{\sim}{y} | \to \infty .$$

* For constant density, $N_0 = a_0^* a_0$ becomes unbounded when normalization volume goes to infinity, thus it causes 'inequivalence'.

We rewrite Eq.(50),

$$\langle 0|\phi(\underset{\sim}{x})\phi(\underset{\sim}{y})|0\rangle = \sum_{\alpha}|C_\alpha(0)|^2 + \sum_{\alpha}{}'|C_\alpha(0)|^2 + \sum_{\alpha, k\neq 0}|C_\alpha(\underset{\sim}{k})|^2 e^{i\underset{\sim}{k}(\underset{\sim}{x}-\underset{\sim}{y})} \qquad (50')$$

The second term on the right hand side of Eq.(50') goes to zero when $|\underset{\sim}{x}-\underset{\sim}{y}|$ goes to infinity, while the first term remains constant. Since $C_\alpha(\underset{\sim}{k})$ is proportional to $1/\sqrt{V}$, this constant tends to zero when the normalization volume becomes infinite, which is necessary to have the limiting process $|\underset{\sim}{x}-\underset{\sim}{y}| \to \infty$. As is seen by replacing summation by integration, the first term gives negligible contribution unless $C_\alpha(\underset{\sim}{k})$ has a singularity higher than $k^{-3/2}$ at the origin. When there occurs Bose-Einstein condensation, it is possible to have such a singularity. Strictly speaking, if we make the limit $V \to \infty$ at first, $C_\alpha(\underset{\sim}{k})_{\underset{\sim}{k}=0}$ gives vanishing contribution but if we exchange the order of sum and the limit, we may expect finite answer. Or, in other words, the contribution from the region $|\underset{\sim}{k}| < \varepsilon$ does not vanish even when we use infinite normalization volume, and the contribution remains finite in the limit $\varepsilon \to 0$ after the integration. The situation is analogous to the case of infrared divergence. It is due to infinite degeneracy for $\underset{\sim}{k} = 0$ caused by zero energy atoms. Here it should be noted that $\langle n-1|\phi(\underset{\sim}{x})|n\rangle$ is proportional to $\sqrt{n}$. Physically the finiteness of two body correlation function at infinite distance indicates the existence of long range correlation accompanied by Bose-Einstein condensation. We write the function in the form,

$$\langle 0|\phi(\underset{\sim}{x})\,\phi(\underset{\sim}{y})|0\rangle = |\langle 0|\phi(0)|0\rangle|^2 + \overline{\langle 0|\phi(\underset{\sim}{x})\,\phi(\underset{\sim}{y})|0\rangle}$$

The second term behaves in ordinary manner. Since the ground state is not a pure state but a mixed state, the first term must be replaced by $Tr\,(\rho_0\,\phi^{\dagger}(0)\,\rho_0\,\phi(0))$, where ρ_0 is a density matrix for the ground state. When we forget about such a complexity of the ground state, the trace term is treated as though it is the square of 'vacuum' expectation value $|\langle 0|\phi(0)|0\rangle|^2$. Then it looks as if the expectation value does not vanish violating the originally assumed symmetry.

For the violation of symmetry, the important thing is not existence of Goldstone boson but occurrence of long range correlation. The Goldstone boson is suitable as a medium for such correlation. It should be pointed out that a finite effect can be expected only if the density of zero energy particles, which can mediate long range correlation, is finite. Thus the number of the zero energy boson should be infinite for the infinite normalization volume. This means that there must be Bose-Einstein condensation. The occurrence of the phenomenon requires that the particle in question should satisfy Bone statistics.*)

The similar situation arises in relativistic quantum field theory too. A famous example is infrared divergence in

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*) It may be true that the ground state should be treated macroscopically to explain the actual violation of symmetry, that is, not as a pure state but as a mixed state. The introduction of random phase approximation, that is, the neglection of the interference terms, may be justified on this ground.

quantum electrodynamics. The infrared catastrophy is caused by the neglect of states with infinite number of soft photons. Here, we shall not discuss the infrared effect in more detail, but refer to the books by Umezawa[8] or Jauch and Rohlich[9]. We only note that, for getting a finite contribution, it is necessary to make the energy width of the ground state not zero but infinitesimal.

We may expect similar infrared effect which causes the violation of the conservation law of additive quantum numbers in the above examples. If such an effect is realized, there occurs symmetry violation. In those examples, there are currents which satisfy the continuity equation,

$$\partial_\mu j_\mu = 0 , \tag{51}$$

where,

$$j_\mu = i(\phi_1 \partial_\mu \phi_2 - \phi_2 \partial_\mu \phi_1) \quad \text{(Example in § 2)}$$
$$j_\mu = i\bar{\psi}\gamma_5\gamma_\mu\psi \quad \text{(Example in § 3)} \tag{52}$$

Then the total charge

$$Q = \int d^3x \, j(\underset{\sim}{x})$$

should be conserved. But a part of the charge can be brought away, if there is a boson coupled with the current (52). The mean distance of the removed charge from the origin will be of an order of the inverse of the boson mass. Thus the total charge enclosed within a arbitrary large volume will change when there exists a massless boson coupled with the current. In general, the magnitude of the removed charge will fluctuate in time causing the violation of the conservation law. If this is the case, we can understand the mechanism of the symmetry violation. With the approximation used in §§ 2 and 3, we can show that the continuity equation (51) does not hold for non-trivial solution. The results show that the divergence of the current contains the term which is not localized even when the source of the current is localized. The non-localizable term appears as the contribution from Goldstone boson, suggesting the correctness of the above conjecture.

6. Conclusion.

There is a possibility to have a physically reasonable explanation of the villation of symmetry, if there appears a massless boson coupled with the conserved current. The essential point of the effect is the long range correlation mediated by the massless boson. If such a correlation brings a part of conserved quantity out of the region containing the whole system in question, and if the removed quantity fluctuates in time, we shall have an apparent violation of symmetry. Since the role of Goldstone boson is to mediate the long range correlation, there is some restrictions in the features of the boson which can cause the violation. In solid state physics, the existence of the boson is not indispensable if the long range correlation appears by some other reason. Usually such a boson appears through the phenomenological quantization of some kind of excitation is solid state physics. The correlation can be caused by the excitation. It is necessary to look into the actual nature of the excitation to see whether the approximation in the form of boson is good one for the problem or not. On the other hand only the existence of massless boson can cause the long range correlation in relativistic quantum field theory unless the degeneracy of vacuum is assumed.

In the above discussions, we did not give any proof. It is difficult to give any rigorous proof in this problem. For example, we have used the field equation to derive the self consistent mass equation which has the symmetry breaking solution. The consistent use of the field equation gives the continuity equation for a current, which assures the preservation of the symmetry.*) Thus the symmetry breaking solution contradicts this result derived from the field equation. A logically rigorous proof cannot tolerate such a contradiction.

For a rigorous discussion of the problem, the existence of massless boson, especially the possibility of having an infinite number of soft bosons, should be taken into account from the first stage of the calculation.

Then the field corresponding the boson does not vanish at the surface of an arbitrary large volume. If this is the case, the surface integral terms of the variation of action integral cannot be neglected. Thus the field equation becomes different

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*) The proof of the conservation law does not contain any state, so that the inequivalence cannot play any important role in the contradiction.

from the case where the massless boson does not exist. The integral of three dimensional divergence of current over whole space does not vanish too. These facts invalidate the proof of conservation law. Until we succeed to construct a relativistic quantum field theory which takes these effects into consideration, we cannot give any proof for this problem. Our argument on the mechanism of the symmetry violation is based merely on the physical intuition.

Even if we have assumed that the above explanation of the symmetry violation is the correct one, we are not sure that such an explanation can be used for the realistic problems of SU_3 - symmetry or γ_5 -invariance. There are at least two reasons to doubt the validity of such an explanation.

One of them is rather an empirical one. If there is a massless boson, it can exist in states with finite energy-momentum. Though such states do not contribute to the symmetry violation, the boson in these states can be observed easily. Though at present no effort is made to observe such a boson, it should be discovered because the neglect of the existence of the boson will result in the violation of energy-momentum conservation. So far no evidence of such boson has been found.

The other reason is an academic one. The motion of a particle interacting with Goldstone boson is analogous to the motion of the particle in the external classical field. For instance, the mass of the γ_5 -invariant particle appears as the effective mass of the particle in some kind of 'medium'. The fact suggests that the 'elementary' particle should not be treated as an independent entity but as a inseparable part of the whole system including the vacuum. It may be possible to construct such type of quantum field theory, but the drastic change in fundamental concepts will be needed for that.

The conclusion is as follows. Although it is possible to understand the violation of symmetry in terms of massless boson, it is quite improbable that such kind of violation is causing the actual broken symmetry which can be found in the law of elementary particles.

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