MATSCIENCE REPORT 5

COLLECTED TOPICS

ON

ELEMENTARY PARTICLE THEORY

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Collected topics on ELEMENTARY PARTICLE THEORY.

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(A series of lectures delivered in the spring 1962 summarising recent papers in high energy physics).

REPORT ON RECENT THEORETICAL WORK.

The observed resonances are:

	I - spin	Angular momentum and Parity	Decay mode and Mass.
1.	$T = 1$ (ρ)	$J = 1^-$	2π (750 Mev width 80 Mev
2.	$T = 0$ (η^0)	$J = 1^-$	3π (550 Mev $\pi + N \rightarrow \pi + \pi +$
3.	$T = 0$ (ω_0)	$J = 1^-$	2π (785 Mev $\pi^+ + d \rightarrow \pi^+ + \pi^-$ (width $<$ 30 Mev) $+ 2p$

In Sakurai's theory (is the vector theory of strong interactions) there was already a prediction for two distinct $T = 0$ vector mesons. He now identifies the two observed resonances ω^0 and η_0 (for the latter he assumed the spin party assignments) with the two vector mesons and argues out their respective roles.

A. It is shown that ω^0 with a larger mass should be coupled to the baryonic current while the η_0 with a lower mass couples the hyper-charge current on the following arguements:

In this model the pseudoscalar π and K should emerge as tightly bound states of $N\bar{N}$ and $N\bar{\Lambda}$ respectively bound by a heavy neutral vector mesons. Hence the coupling of the baryonic vector meson must be stronger than the coupling of the hyperchange vector meson since otherwise we should expect a very tightly bound $\bar{K} N$ system which does not exist. In principle a study of the KN scattering amplitude as a function of momentum transfer should reveal the mass of the comp-ling particle. If η were to be coupled to the hypercharge current bo the $T = 1$ and $T = 0$ KN amplitude must have poles at $t = m_\eta^2$ with equal residues but no poles should be present at

$t = m_\omega^2$. But this is not feasible at present. The better method could be to study the effect from nuclear forces. Since the baryonic V. m. is responsible for the strong binding between $N\bar{N}$ or $N\bar{\Lambda}$ the same would cause a repulsion between NN which accounts for the repulsive core. Breit has shown that with a vector meson mass of $4\,m_\pi$ the core radius would become too large in the sense that the central force would be dominated by the repulsion due to the vector meson. But the quantitative estimate is not at all rigorous. But Sakurai bases his conclusion of the ω being the strong coupling between the baryonic current only on this evidence i.e.

$m_\omega = 5.9\,m_\pi$ "is a better condidate" than

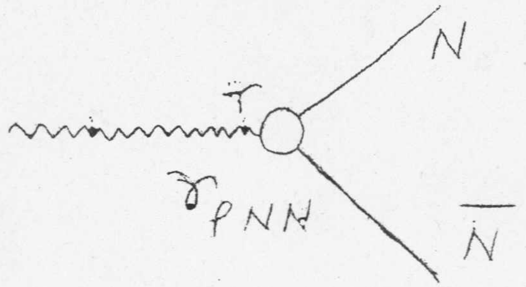
$m_\eta = 4\,m_\pi$ so that ω is the vector meson associated with the B_μ^B field while η_0 is the one associated with B_μ^Y field. But the more encouraging feature is that in $p\bar{p}$ and $\pi^+ d$ collisions ω mesons show up much more than η. He then draws attention to the fact that thus not all strong interactions are as strong as possible as assumed by Chew and Frautschi.

B. The η with a mass lower then ρ is helpful in the understanding of the isoscalar form factor of the nucleon.

The Stanford experiments have revealed that the neutron charge ~~xxx~~ cloud has a fairly large $+$vely charged 'fringe'. This means that the average mass state responsible for the isoscalar form factor must be lower than that responsible for the isovector form factor. The tentative figures were

$$F_1^S(s) = .5\,\frac{11\,m_\pi^2}{11\,m_\pi^2 - s} + .5$$

so that $s \sim 11\, m_\pi^2$ so that $\omega \{ m_\omega > m_\rho \}$ would be of no use but η_0 would do the job. The paradox that η which is less strongly coupled to $N\bar{N}$ then ω should be responsible for F_1^s (not ω) is only apparent. If the universal coupling constant of η^* to hypercharge current defined at $s = 0$ does not differ appreciably from $\eta N\bar{N}$ at $s = m_\eta^2$ then there must be a substantial contribution to F_1^s from the one η state since both η_0 and the 'isoscalar photons' are coupled to same conserved hypercharge current.



Moreover in a theory which is symmetric between N and Ξ isoscalar proton $\longrightarrow$ ω^0 is forbidden ($m_{(\Xi - N)} \sim 0$)

(Baryonic current is even under $N \rightleftarrows \Xi$ but h.c. current is odd)

We should expect

$$F_1^s(q^2) = \frac{\alpha_\eta m_\eta^2}{q^2 + m_\eta^2} + \frac{\alpha_\omega m_\omega^2}{q^2 + m_\omega^2} + \underbrace{[1 - \alpha_\eta - \alpha_\omega]}_{\text{other states}}$$

It is shown crudely

$$\alpha_\eta = 1\cdot 2 \; ; \; \alpha_\omega = -\cdot 7 = \frac{-\gamma_{\eta NN}}{\gamma_\eta} \sim 1$$

This seems to make the theory more plausible i.e. to introduce V. mesons in the beginning rather them give them a dynamical origin (Chew)

In his second paper he gives a theoretical basis for his arguments with an assumption of invariance under reflection in hypercharge space.

The transformations are

$$\begin{pmatrix} p \\ n \end{pmatrix} \rightleftarrows \begin{pmatrix} \Xi^- \\ \Xi^0 \end{pmatrix} ; \qquad \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix} \rightleftarrows \begin{pmatrix} \Sigma^- \\ \Sigma^0 \\ \Sigma^+ \end{pmatrix}$$

$$\Lambda \rightleftarrows \Lambda$$

$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \rightleftarrows \begin{pmatrix} \pi^- \\ \pi^0 \\ \pi^+ \end{pmatrix} \qquad \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \rightleftarrows \begin{pmatrix} -K^- \\ \bar{K}^0 \end{pmatrix}$$

$P_{N\Xi}$ even Ξ has spin $\frac{1}{2}$, R is 'good' to the extent $m_{(\Xi - N)} \sim 0$

1. Take $\pi \cdot \Sigma \times \Sigma$ i.e $\left(\bar{\Sigma}^+ \Sigma^+ - \bar{\Sigma}^- \Sigma^- \right) \pi^0 = 0$ since it changes sign under R. But $\pi \cdot \bar{\Lambda}\Sigma \ (\bar{\Sigma}^+\Sigma^+ + \bar{\Sigma}\pi^- + \bar{\Sigma}^0 \ldots$ can exist. It is known that $Y_1^* \longrightarrow \dfrac{\pi + \Sigma \ (T=1)}{\pi + \Lambda \ (T=1)} \sim 0 \cdot \Lambda$

i.e. $Y_1^* \longrightarrow \pi + N$ so that $\pi \cdot \bar{\Lambda} Y_1^*$ must be even consider R, i.e Y_1^* must transform like Σ

$$\pi \cdot \bar{\Sigma} \times Y_1^* = 0$$

2. R invariance forbids $\pi + \Lambda \rightleftarrows \pi + \Sigma$ and one pion exchange between N and Σ should be zero. It also predicts P_{33} ie $\Xi\pi$ resonance.

3. If ω is even under R then $\omega^0 \longrightarrow \pi^+ + \pi^- + \pi^0$ is "forbidden" (i.e. $\omega^0 \ \pi^{(1)} \cdot \pi^{(2)} \times \pi^{(3)}$ is odd under R) Thus the narrow width of ω^0 is explained.

4. $A_\mu \longrightarrow -A_\mu$ under R and the e.m. interactions of the strongly interacting particles is also invariant under R since both the I_Z and U changesign under R. This requires that Λ^0 and $\Sigma\Lambda$ anomalous mag moment should be 'zero'

$$\mu(\Xi^-) = -\mu(p)$$
$$\mu(\Xi^0) = -\mu(n)$$
$$\mu(\Xi^+) = -\mu(\Sigma^-)$$

5. If R is imposed on weak interactions as well (i.e. of Strongly interacting particles). We have

$$\alpha\,(\Xi^- \longrightarrow \pi^- + \Lambda) = -\alpha\,(\Lambda \rightarrow \pi^- + p)$$

(seems me to be true).

The question is whether R invariance is fundamental and if so why?

According to UTSI the fundamental vector couplings are invariant under R where ρ, coupled to the I - spin current transforms like

$$\begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix} \longrightarrow - \begin{pmatrix} \rho^- \\ \rho^0 \\ \rho^+ \end{pmatrix}$$

and the ω is even under R while η is odd. Since $\pi = N\bar{N}$ and $K = N\bar{\Lambda}$ they should have definite transformation properties under R tho' the 'plus' is ad hoc but $\pi \cdot \bar{\Lambda}\Sigma$ will vanish with a — for which is not borne out by hyperfragment

6. R does not explain narrowness of η but perhaps the smaller phase space and weaker $N\bar{N}\rho$ makes it narrow.

7. $\omega_0 \longrightarrow \pi^0 + \gamma$ is ' forbidden '

$\omega_0 \longrightarrow \pi^+ + \pi^- + \gamma$ is allowed only if two are in odd l state.

SAKURAI'S MODEL OF STRONG INTERACTIONS.

It is know that the various symmetry models proposed for explaining the strong interactions have not been successful in all points and Pais has pointed out that no internal symmetries stronger than those implied by charge independence works to all orders which are not contradicted by experiment. For example, the 'global' and cosmic symmetries have not led to any fruitful predictions. Sakurai feels that the Yukawa couplings may themselves be phenomenological manifestations of some other more fundamental coupling which should be derived by exploiting the existent symmetries.

The known conservations laws are:

1. Baryon number (B) conservation and which is exact and the baryneic charge can be taken as a dynamical attribute and

2. Conservation of I- spin (T) which implies charge independence which we assume is exact in the absence of e.m. and weak couplings.

In addition to B and T we need one more quantum no. to specify a particle which may either be Q or S or Y. Sakurai chooses Y since Q does not seem to characterise strong interaction since even leptons carry charge and also one of exact conservation laws. (2) is broken by the very coupling giving rise to conservation of . Between S and Y he chooses Y since systems of half-integral T can produce

To formulate the conservation laws of internal attributes for equation (1) we apply $|A\rangle = e^{iB\lambda}|A\rangle$ where λ is a real constant and $|A\rangle$ has a definite B which implies $\psi \longrightarrow e^{iB\lambda}\psi$ for the field operator ψ. And invariance of the Lagranigian under this leads to conservation of B. Similarly for spin we have $\psi \longrightarrow e^{iT\lambda}\psi$ where λ is a constant real vector in space. The important point is that the phase factor λ is not a function of space-time in contrast with the electro magnetic guage transformation, i.e,

$$\psi \longrightarrow e^{ie\lambda(x)}\psi$$

has to be counteracted by

$$A_\mu \longrightarrow A_\mu + \frac{\partial \Lambda}{\partial x_\mu}$$

if the guage transformation is to be local. Yang and Mills have shown that if the I- spin guage transformation is to be local then <u>we are forced to introduce a vector field with I spin unity</u> coupled universally to the I- spin current $\sim$ constructed out of all fields with I- spin.

Sakurai points out that a conservation law of an internal attribute implies a vector type interaction corresponding to it so that the law is consistent with the local concept. When this is generalized to B and Y conservation, we are led to 3 fundamental vector couplings which are the 'only' ones, i.e.,

$$\mathcal{L}_T = - f_T \vec{B}_\mu^T \cdot \vec{J}_\mu^T \qquad (1)$$

$$\mathcal{L}_Y = - f_Y \vec{B}_\mu^{\,Y} \cdot \vec{J}_\mu^{\,Y} \tag{2}$$

$$\mathcal{L}_B = - f_B \vec{B}_\mu^{\,B} \cdot \vec{J}_\mu^{\,B} \tag{3}$$

where $B^{T,Y,B}$ are the vector fields analogous to .

If the fields were bare we have

$$\begin{aligned}\vec{J}_\mu^{\,T} = {} & i \bar{\psi}_N \frac{\tau}{2} \gamma_\mu \psi_N - \bar{\psi}_\Sigma \times \gamma_\mu \psi_\Sigma \\ & + i \bar{\psi}_\Xi \frac{\tau}{2} \gamma_\mu \psi_\Xi + \phi_\pi \times \frac{\partial \phi_\pi}{\partial x_\mu} \\ & + i \left\{ \frac{\partial \phi_K^\dagger}{\partial x_\mu} \frac{\tau}{2} \phi_K - \phi_K^\dagger \frac{\tau}{2} \frac{\partial \phi_K}{\partial x_\mu} \right\} \\ & + f_{\mu\nu}^T \times B_\nu^T \end{aligned} \tag{4}$$

$$\begin{aligned} J_\mu^B = {} & i \bar{\psi}_N \gamma_\mu \psi_N + i \bar{\psi}_\Lambda \gamma_\mu \psi_\Lambda + i \bar{\psi}_\Sigma \gamma_\mu \psi_\Sigma \\ & + i \bar{\psi}_\Xi \gamma_\mu \psi_\Xi \end{aligned} \tag{5}$$

$$\begin{aligned} J_\mu^Y = {} & i \bar{\psi}_N \gamma_\mu \psi_N - i \bar{\psi}_\Xi \gamma_\mu \psi_\Xi + \\ & i \left\{ \frac{\partial \phi_K^\dagger}{\partial x_\mu} \phi_K - \phi_K^\dagger \frac{\partial \phi_K}{\partial x_\mu} \right\} \end{aligned} \tag{6}$$

$$f_{\mu\nu} \equiv \frac{\partial B_\mu^T}{\partial x_\nu} - \frac{\partial B_\nu^T}{\partial x_\mu} - f_T\, B_\mu \times B_\nu \tag{7}$$

where Ψ_Σ is the direct product of a 4 compt. Dirace spinor $\sim$ in Lorentz space and 3 compt. iso vector in iso space and so on.

The last term in (4) is because the B_μ^T field possesses I-spin so that it can interact with itself. If the field operators are 'dressed', the coupling is still unaffected in the low-energy limit and the f's are not renormalised. by this process. The coupling between a bare proton and B_μ^T is same as that between a 'dressed' proton and B_μ since B_μ^T can interact with the I- spin of η as well as π^+ (?). This universality is compared to the conserved vector current of Feynman and Gell-Mann. Secondly, the universality is undisturbed by the other two cuplings unlike in the global symmetry model.

Thirdly, the coupling guarantees PCT invariance.

Properties of the B fields:-

1. Under G conjugation we have

$$G B_\mu^T G^{-1} = B_\mu^T \qquad \text{(a)} \qquad (2\pi)\ \rho$$

$$G B_\mu^Y G^{-1} = -B_\mu^Y \qquad \text{(b)} \qquad (3\pi)\ \eta$$

$$G B_\mu^B G^{-1} = -B_\mu^B \qquad \text{(c)} \qquad (3\pi)\ \omega$$

In (a) there is a minus sign from G say on $G\pi G^{-1} = -\pi$ and so J_μ^T has a minus sign since it is a vector while J_μ^B and J_μ^Y do not, so that G on B_μ^T does not give a minus sign. This implies that if $M_{B^T} > 2\pi$ it decays into 2π strongly while $B_{B,Y,\mu} > 3\pi$ they will decay to 3π if $< 3\pi$ the decay will be $\pi^0 + \gamma$ and $2\pi + \gamma$.

Now since terms like $\mu^2 B_\mu^2$ are not there the B field cannot be massive and this term if introduced does not satisfy the guage principle. Barring the B_μ^T field which at least interacts with itself and so can produce a self-mass the B ~~and~~ fields cannot generat a mass at all. He suggests that an effective interaction between B^B and B^Y may give rise to a mass.

If the coupling constants $f^2/4\pi$ were small an exchange of a single B quantum between two currents would lead to an effective

$$H = \frac{f_T^2}{4\pi\mu^2} \vec{J}_\mu^T \cdot \vec{J}_\mu^T$$

i.e.,

$$\frac{f^2}{4\pi} J_\mu^T \left\{ g_{\mu\nu} - \frac{K_\mu K_\nu}{M^2} \right\} \frac{J_\nu^T}{K^2 + M^2} \approx H$$

for small momentum transfers. This seems similar to the current $V - A$ weak coupling.

Predictions of the theory.

1. It is known that low-energy $\pi - N$ S-wave $T = \frac{1}{2}$ interaction is not attractive while $T = 3/2$ is not repulsive. Here the S-State potential is $\propto$ to $T_\pi \cdot \tau_{N/2}$ is a +ve quantity.

$$T_\pi \cdot \frac{T_N}{2} = \frac{1}{2}\left\{ (T_\pi + T_N)^2 - T_N^2 - T_\pi^2 \right\}$$

$$= \frac{1}{2}\left\{ \frac{1}{2}(1/2 + 1) - 2 - 3/4 \right\}$$

This is because here

$$\left\langle \frac{1}{2} \right| \left(\phi \times \frac{\partial \phi}{\partial x_0}\right) u_n^\dagger \frac{T}{2} u_n \left| \frac{1}{2} \right\rangle = \left\langle \frac{1}{2} \right| \tau \cdot T \left| \frac{1}{2} \right\rangle$$

while in general the form is $a + b\, T \cdot T$ so that the spin dependence is not sufficient to explain this. <u>But here the form</u> <u>$J_\mu^T \cdot J_\mu^T$ gives this directly.</u> But this makes the $T = 1/2$ too attractive while $T = 3/2$ is too repulsive that a term like $\lambda \phi_\pi^2 u_N^\dagger u_N$ with $\lambda > 0$ is necessary so that again as in the Yukawa case there is no quantitative agreement. The p-wave is same as in static theory.

2. The $\pi \Sigma \times \Lambda$ state gives $\frac{1}{2} T_\pi \cdot T_\Sigma = -2$ for $T = 0$ and -1 for $T = 1$ and 1 for $T = 2$ so that $T = 0$ state must be very attractive which predicts an S state $T = 0$ resonance.

3. As for the ρ resonance which can be predicted from the Yang Mills B_T quantum $(J = 1, T = 1)$ decaying into we here can have two three π resn. in $T = 0, J = 1$ state corresponding to B_B and B_Y.

4. The 2 higher resonances in πN in $T = 1/2$ and one in $T = 3/2$ state may be related to the fact that there are two kinds of B quanta with $T = 0$ and one with $T = 1$.

5. The $\pi^+ p$ and $\pi^- p$ total cross sections are flat above the 3 higher resonances. If these resonances could be produced by the same mechanism of the resonance we should expect higher resonance.

6. The width of the three higher resonances are too narrow to be accounted for by the usual mechanisms.

K interactions.

1) While π had no Y, K has both Y and T

$K\ N$ have $+ve\ Y$ while $\bar{K}$ have $-ve\ Y$

So $K N$ interaction is repulsive while $\bar{K} N$ is attractive provided Yukawa interaction $\sim$ play unimportant roles.

The s wave $T = 0$, $T = 1$, KN interactions are repulsive — seen also from $K^+ p$ data.

2) The spin part gives $T_N / 2 \ \frac{T_K}{2} = 1/4$ for $T = 1$ and $-3/4$ for $T = 0$ so that (1) should be undisturbed

$$f_Y^2/4\pi\mu_Y^2 > f_T^2/4\pi\mu_T^2$$

With this assumption the $T = 1$ state for KN has to be more repulsive than $T = 0$. Which agrees with experiment. From experiments he arrives at

$$f_B^2/4\pi > f_Y^2/4\pi > f_T^2/4\pi$$

He than tackles the question of the distinction of a fermion from an anti-fermion. For a massless field

$$\psi \longrightarrow a\psi + b\gamma_5 C \bar{\psi}^T$$

$$C\gamma_\mu C^{-1} = -\gamma_\mu\ ; \quad |a|^2 + |b|^2 = 1$$

leaves H invariant. In this theory since we assume that all masses are due to strong and e.m. interactions, when these are switched off we cannot distinguish between fermon and anti-fermion even for baryons and leptons since the coupling constants go to zero and hence the internal attributes also go to zero. To write the conserved current for fermionic charge when $m = 0,\ f_B = f_y = f_T = e = 0$ we must project the true fermion state. The fermionic charge operator Q_F gives

$$\langle Q_F | \Psi \rangle_F = - Q_F \, C | \Psi \rangle$$

We try to find Γ^F such that

$$C \left(\overline{\Gamma^F \Psi} \right)^T = - \Gamma_F \, C \, \overline{\Psi}^T$$

where Γ_F is a linear combination of the 16 independent Deric matrices, we can show that $\Gamma_F = a i + b \gamma_5$ where are real. Sinc eigen values of matrix must be we are led to the only possibility $b = \pm 1$ (and $a = 0$ we can define the true fermion state with $b = 1$ and so leptons and baryons are fermions. The fact that γ_5 disgonalises the Q_F means that as e.m. and strong interaction disappear matter and anti-matter can be made only via the sign of γ_5. So that the conserved curr. is

$$\overline{\Psi} \gamma_\mu (1 + \gamma_5) \Psi$$

which is $V - A$ of weak interaction

Yang and Mills argument.

Invariance under guage transformation of the 1st kind is equivalent to the statement that the phases are irrelevant. But locality demands that relative phases should be unrelated

$$\Psi \longrightarrow e^{i e \varepsilon} \Psi$$

ε is a function of . But demand of guage transformation necessitates the existence of A_μ, a vector field which transforms as

$$A_\mu \rightarrow A_\mu + \partial_\mu \varepsilon$$

In the Lag. We have

$$\partial_\mu \psi^* \partial_\mu \psi + ie\, \partial_\mu \varepsilon\, \partial_\mu \psi^*$$
$$\rightarrow \partial_\mu \psi^* \partial_\mu \psi - ie\, \partial_\mu \varepsilon\, \psi^* \partial_\mu \psi$$
$$+ e^2 \partial_\mu \varepsilon\, \partial_\mu \varepsilon\, \psi^* \psi .$$

But if we allow A_μ to exist and replace $\partial_\mu \rightarrow \partial_\mu - ie A_\mu$ and then transformation is automatically obtained.

G. Parity.

$$G = C I_2 = C e^{-i\pi(T_2 + U/2)}$$

T is an iso pseudo vector

$$I_2 T_1 I_2^{-1} = -T_1$$
$$I_2 T_2 I_2^{-1} = T_2$$
$$I_2 T_3 I_2^{-1} = -T_3$$

Therefore $G T_1 G^{-1} = T_1$; $G \vec{T} G^{-1} = \vec{T}$

B_μ^T which is an isovector

LIE GROUP DYNAMICAL FORMALISM AND THE RELATION BETWEEN QUANTUM AND CLASSICAL MECHANICS.

(T.F. Jordan & E.C.G. Sudarshan.)

In this paper, the authors claim to have developed a formal theory of generalized dynamics which includes the classical and quantum mechanics as special cases. The dynamical structure of the theory is that of a Lie algebra of functions of the basic dynamical variables which provides the infinitesmal generators of the group of dynamical transformations. The particular representation, i.e., real variables or operators chosen for the algebra is relevant only for a physical interpretation and not for the dynamical atructure analysis.

It is well-known that the formal relationship between quantum and classical mechanics is in the analogy between commutators and Poisson brackets and the Heisenberg and Hamilton's equations of motion. Moyal has formulated quantum mechanics in terms of functions in classical phase space such that corresponding to the commutator of two operators there is a complicated function which is not the Poisson bracket. The present authors show that this bracket has the properties of a Lie bracket. Thus functions on the phase space form a Lie algebra with this bracket. The elements of this algebra act as generators for the dynamical transformations which are elements of the corresponding Lie group. And the operator representation provides the usual formulation of quantum mechanics.

Classical Mechanics.

The classical statistical mechanical state of a system is describe by a probability distribution function $\rho(M)$ on the phase space $\{M\}$ of the system where M is $2n$ dimensional space of n pairs of q_i, p_i (canonical coordinates and momenta)

And

$$\int \rho(M)\, dM = 1 \qquad . \tag{1}$$

The expectation value of a function $A(M)$ of the dynamical variable

$$\langle A \rangle = \int A(M)\, \rho(M)\, dM \tag{2}$$

And a state is pure if

$$\rho_{M'}(M) = \delta(M - M') \tag{3}$$

and the time evolution of the functions $A(M)$ is

$$\frac{\partial}{\partial t} A_t(M) = \left[A_t(M), H(M) \right]_{P.B.} \tag{4}$$

where $H(M)$ is the Hamiltonian and $[\quad]_{P.B.}$ is the Poisson bracket. The expectation value at time of the physical quantity A is

$$\langle A \rangle_t = \int \rho(M)\, A_t(M)\, dM \tag{5a}$$

where $\rho(M)$ is constant in time. Here we allow each point in phase space to move along the trajectory given by (4) and average over the initial distribution since we know by Liouville's theorem that the amount of density at an infinitesimal element of phase space remains constant as that element moves along its trajectory. But we may as well consider the physical function A to be constant and average with respect to a distribution which has undergone the inverse time transformation

$$\langle A \rangle_t = \int \rho_t(M)\, A(M)\, dM \tag{5 (b)}$$

$$\frac{\partial}{\partial t} \rho(M) = - \left[\rho_t(M), H(M) \right]_{P.B} \tag{4 (b)}$$

By (4) H generates an element of the group of canonical transformation. We limit ourselves to cases where H is a power series expans and take for simplicity a system with one pair of q & p i.e.,

$$H(p, q) = \alpha_{mn}\, q^m p^n$$

so that (4) is

$$\frac{\partial}{\partial t} A(p, q) = \left[A, q^m p^n \right]_{P.B} \tag{6 (a)}$$

Similarly for ρ or we can absorb the time dependence into

$$A(q, p) = \left[A(q, p), q^m p^n \right]_{P.B} \alpha_{m,n} \tag{7a}$$

<u>The properties of the P. B.</u>

For real numbers a & b and functions A, B, C,

$$\left[A, aB + bC \right]_{P.B} = a \left[A, B \right]_{P.B} + b \left[A, C \right]_{P.B} \tag{8a}$$

(Linearity)

$$[A, A]_{P.B} \equiv 0 \quad \text{anti-symmetry} \tag{8b}$$

i.e.,

$$[A, B]_{P.B} = -[B, A]_{P.B} \tag{8c}$$

$$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] \equiv 0 \quad \text{(Jacobi identity)}$$

Thus the real linear space power series in dynamical variables forms an infinite dimensional Lie algebra with P.B. as Lie bracket.

Operator representation.

To every real function $A(M)$ we shall a Hermitian operator A on a Hilbert space so that we have a linear correspodence and to P.B. there corresponds a commutator and

$$\int A(M)\, B(M)\, dM = Tr\,(A B) \tag{9}$$

We shall also need operators $\rho_{M'}$ corresponding to pure to state distribution and all other states are superpositions so that all that we need is

$$A(M) = Tr\,[A\, L(M)] \tag{10a}$$

$$A = \int A(M)\, L(M)\, dM \tag{10b}$$

where

$$L(M) = \int \rho_{M'} \in \rho_{M'}(M)\, dM' = \rho_{M'}\, \delta(M-M')\, dM' \tag{11}$$

Then since

$$\int \rho_{M'}(M)\,\rho_{M''}(M)\,dM = \int \delta(M-M')\,\delta(M-M'') = \delta(M'-M'')$$

we need

$$Tr\,(\rho_{M'}\,\rho_{M''}) = \delta\,(M'-M'') \tag{12}$$

and (9) is also satisfied and

$$A = \int A(M')\,\rho_{M'}\,dM'$$

$$A(M') = Tr\,(A\,\rho_{M'})$$

If $\rho_{M'}$ is Hermitian then operators corresponding to real functions are Hermitian and if

$$\int \rho(M')\,dM' = 1 \tag{13}$$

then

$$\int A(M)\,dM = Tr\,A \tag{14}$$

so that (1) to $Tr\,(\rho) = 1$ (15)

So that ρ the state of the system

$$\langle A \rangle = Tr\,(A\rho) \tag{16}$$

The commutator is

$$\frac{1}{i}\,(AB-BA) = [A,B]_- = \int A(M)\,B(M')\,[\rho_M,\rho_{M'}]\,dM\,dM'$$

so that we need the commutator rules for operator in order to get the commutator corresponding to P.B. For a pair q, p we let

$$\left[f_{q,p}, f_{q',p'} \right] = f_{q',p} \frac{\partial}{\partial q} \delta(q-q') \frac{\partial}{\partial p'} \delta(p-p') - f_{q,p'} \frac{\partial}{\partial q'} \delta(q-q') \frac{\partial}{\partial p} \delta(p-p') \tag{17}$$

so that

$$[A, B]_- = \int A(q,p)\, B(q',p') \left\{ f(q',p \ldots \right\} dq\, dp\, dq'$$

5

and reducing this and assuming f_{qp} is such that

$$q^m p^n f_{q,p} \longrightarrow 0 \quad \text{as} \quad q, p \longrightarrow \infty$$

$$[A, B]_- = \int \left\{ \frac{\partial}{\partial q} \frac{\partial}{\partial p'} - \frac{\partial}{\partial q'} \frac{\partial}{\partial p} \right\} A(q,p)\, B(q',p')\, f_{q,p}\, \delta(q-q')\, \delta(p-p')\, dq\, \cancel{dp}\, dq'\, dp' \tag{18}$$

(19)

which corresponds to the P.B. Then the corresponding operators $(q^m p^n)_{op}$ will satisfy

$$\left[(q_{op})^{m_1} (p_{op})^{n_1}, (q_{op})^{m_2} (p_{op})^{n_2} \right]_- = (m_1 n_2 - m_2 n_1) \left\{ q^{m_1 + m_2} p^{n_1 + n_2 - 1} \right\} \tag{19}$$

In general (10) does not preserve multiplication. In fact (19) are inconsistent with assigning the operators $(q_{op})^m$ to q^m and $(p_{op})^n$ to p^n

which can be simply verified. So the relation between functions and operators will not preserve multiplication. It is also seen that $\rho_{M'}$ cannot be $+ve$ definite and have a discrete spectrum. If so we would have $Tr\,(\rho_{M'})^2 \leq Tr\,\rho_{M'} = 1$ which contradicts (12) so that $\rho_{M'}$ cannot be $+ve$ definite and since $\rho_{M'}^2 \neq \rho_{M'}$ this operator cannot be a projection. In other words, each vector cannot be associated with a physical state of the system, i.e., a pure state cannot be associated with a vector in Hilbert space.

Quantum Mechanics.

To each quantum mechanical state of a system one can associate ((Moyal) a quasi-probability distribution function $\rho(M)$ on the phas space $\{M\}$. similarly to any physical quantity represented by operator A there corresponds a function A ([illegible]) which classically represents this quantity. To the $[A, B]$ there corresponds a function called the Moyal bracket of fns $A(M)$ and $B(M)$ denoted by $\sin\left[A(M), B(M)\right]$ where

$$\frac{2}{\hbar}\sin\frac{\hbar}{2}\left[\frac{\partial}{\partial q_A}\frac{\partial}{\partial p_B} - \frac{\partial}{\partial q_B}\frac{\partial}{\partial p_A}\right]A(q,p)\,B(q,$$

$$= 2\sum^{\infty}\sum^{2n+1}\frac{(-1)^{n+k}}{\quad k!}\left(\frac{1}{2}\right)^{2n+1}\left[\frac{\partial^{2n+1}}{\partial q_A^{2n+1-k}\,\partial} A(\right.$$

where $\hbar = 1$ and as $\hbar \to 0$ only the 1st term (which is P.B.) in $[\quad]$ remains. Thus $[\quad]_M \longrightarrow [\quad]$ P.B. if one of the functions A and B is of quadratic or lower order in q and p. The pure states are represented by fns $f(M)$ where

$$f(M) = \frac{1}{2} \cos [f(M), f(M)]$$

where $\cos[\quad] = \sin[\quad]_+$. It is shown that the Moyal bracket satisfies linearity, anti-symmetry and Jacobi identity. Hence the linear space of power series in forms a Lie algebra with the Moyal bracket as the Lie bracket.

The operator representation leads to the usual formalism. In equation (10) $L(M)$ is now given by

$$L(M) = \sum_i A_i^\dagger A_i(M)$$

where operators A_i form a basis in the linear space of operators with

$$Tr\,(A_i A_j^\dagger) = \delta_{ij}$$

$$\text{and} \quad \int A_i(M)\, A_j^*(M)\, dM = \delta_{ij} \tag{28}$$

For <u>pure state</u> density operators $f^2 = f$ so that we can identify the pure states with vectors of Hilbert space. But the functions $f(M)$ in general take $-ve$ values (hence quasi-probability) so that not all distributions represent physical states. This is related to the uncertainly principle. (?)

The equations of motion etc., are the same. The quantum formalism differs from the classical only in the properties of the operators. It is interesting that (20) which is usually called the quentum condition is characteristic of both classical and quantum mechanics. The dynamics can be completed with

$$\left[\left(q^{m_1}p^{n_1}\right)_{op},\left(q^{m_2}p^{n_2}\right)_{op}\right] = C^{mn}_{m_1 n_1, m_2 n_2}\left(q^{m}p^{n}\right)_{op} \qquad (29)$$

where
$$C^{mn}_{m_1 n_1/m_2 n_2}\left(q^{m}p^{n}\right) \qquad (30)$$

$$= \mathrm{Sin}\left[q^{m_1}p^{n_1}, q^{m_2}p^{n_2}\right] \qquad (\quad)$$

where (29) is the quantum analogue of (19) . SInce this determines the structure of the Lie Algebra corresponding to power series of q, p , it determines the dynamics of the formalism. For example if

$H = p^2 + q^2$ then $\left(H^2\right)_{sp} \neq H_{op}$

so that while these studies facilitate a comparison of quantum to classical mechanics, the phase-space formulation is not suitable for a physical interpretation of quantum mechanics.

<u>Classical approximation to quantum mechanics.</u>

Since in this scheme the difference between real variable and operator formalism is only due to a choice of the representation of the group, the equations of motion (and not the P.B.) are taken as the starting point, i.e,

$$i\hbar\frac{\partial}{\partial t}\rho = H\rho - \rho H$$

And if ρ is a pure state corresponding to eigen vector state

The R.H.S. if of the same order in $\hbar$ as commutator $\times \hbar$ and so $\hbar \rightarrow 0$ limit will not retain the operator props. Instead we consider the Moyal bracket which reduces to P.B. as $\hbar \rightarrow 0$ in the classical approximation. We retain terms of order $\hbar$ and of zero order ~~with~~ which is what the W.K.B. approximation does.

Summarizing we have seen that both classical and quantum mechanics get into a formal scheme. Suitable dynamical variables and a quantity to describe the probable distribution are chosen. The bracket relation between functions of the dynamic variables is assumed - leading to a Lie algebra. The representation for these allows the formulation of a physically meaningful kinematical scheme.

ON THE DISPERSION THEORY OF DIRECT NUCLEAR REACTIONS.

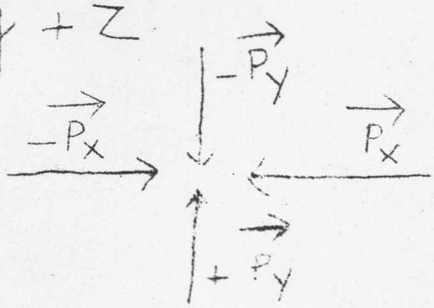
(I.S. Shapiro)

The aim is to develop a dispersion theoretic approach to problems like

$$A + x \longrightarrow B + y$$

and

$$A + x \longrightarrow B + y + z$$



Kinematics.

Two independent variables can be selected from

a) The K.E. of the colliding particle, E

b) The square of the momentum transfer

$$q^2 = \left(\vec{P}_y - \vec{P}_x\right)^2 \qquad (1)$$

c) The square of the sum of momenta

$$P^2 = \left(\vec{P}_x + \vec{P}_y\right)^2 \qquad (2)$$

In the c.m. frame

$$q^2 + p^2 = 2\left(\vec{P}_x^{\,2} + \vec{P}_y^{\,2}\right) = 4\left[m_{xA} + m_{yB}\right]E + 4\,m_{yB}\,Q$$

where $Q = m_A + m_x - m_B - m_y$ is the threshold (3)

(Note: $E = \dfrac{P_x^2}{2m_{xA}} = \dfrac{P_y^2}{2m_{yB}} - Q$

$$\left(P_x^2 + P_y^2\right) = 4\,m_{xA}E + 4\,m_{yB}E + 4\,m_{yB}\,Q\,)$$

Either q^2, E or E, p^2 are selected as independent variables

II. Unitarity and Analyticity.

From $SS^\dagger = 1$

where $S = 1 + i(2\pi)^4 T$

and
$$T = iA + B, \quad A = A^\dagger; \quad B = B^\dagger$$

and
$$A_{if} = \frac{1}{2}(2\pi)^4 \sum_n T_{in} T^\dagger_{nf} \tag{4}$$

The matrices T and A are of the form

$$T_{kl}(q^2, E) = M_{kl}(q^2, E)\,\delta_{\lambda k}\,\delta_{\lambda l}\,\delta^4(l-k)$$
$$A_{kl}(q^2, E) = A_{kl}(q^2, E)\,\delta_{\lambda k}\,\delta_{\lambda l}\,\delta^4(l-k) \tag{5}$$

where the arguments of the function denote the momenta of the states k and l and λ denotes an aggregate of discrete quantum numbers. A_{kl} is the absorptive part of M_{kl}. The main postulate is that M_{kl} is an analytic function of q^2, E except for poles and branch points. Though $M_{kl}(q^2, E)$ is a many valued function we deal with only one of its sheets i.e. the physical sheet and

$$M_{kl}(z^*) = M^\dagger_{kl}(z) \tag{6}$$

Pole Graphs.

If we take the case when in $n \rightarrow f$ one particle b is absorbed when b is emitted in $i \rightarrow n$ then (4) is

$$A_{if} = 2\pi m_b \, \delta\left(p_b^2 - 2 m_b E_b\right),$$

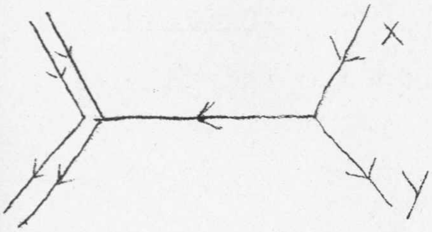
$$\sum_{spins} M_{ib} M_{bf}^{\dagger}$$ ~~spins~~ (7) when $p_b^2 = 2 m_b E_b$

the amplitude M_{if} has a pole and

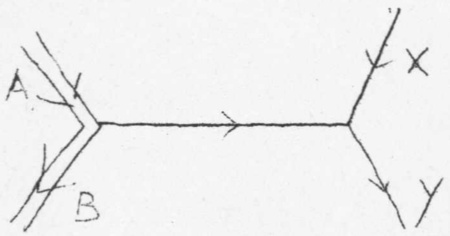
$$M_{if} = 2 m_b \sum_{spin\ b} \frac{M_{ib} M_{bf}^{\dagger}}{p^2 - 2 m_b E_b - i\eta} \qquad \eta \to 0$$

(8)

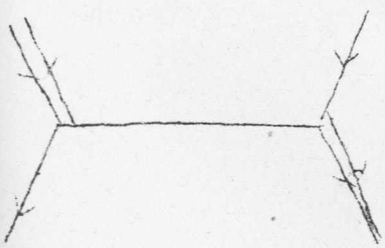
These can be represented by pole graphs - i.e, Feynmann graphs with one internal line



(a) 'Stripping' reaction.

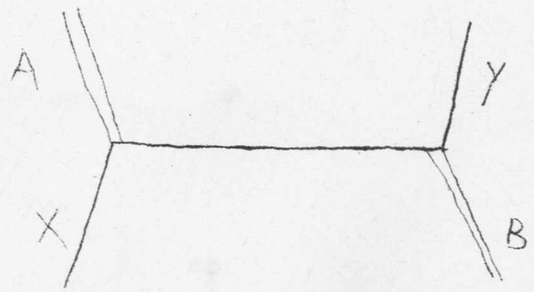


(b) 'Pick-Up' reaction.



(C)

'Exchange stripping and heavy pick up'



(D)

Quasi compound process.

Examples:-

(a) (d,p) $b \equiv n$ (He^3,p), $b \equiv d$

' Stripping '

(b) (p,d) $b \equiv n$ (n,d) $p = b$. (pick up)

(c) $B^{11}(d,n)C^{12}$ here $b = B^{10}$ (heavy pick up)

(d) Corresponds to formation of compound nucleus. This corresponds to complex poles in unphysical sheet while in the physical sheet it corresponds to involved Feynmann graphs with branch point singularities. But for (a)..(d) we assume have poles on the real axis and correspond to such states of b from which emission of nuclear particle is impossible. In this sense (d) is a direct process. These exhaust the pole graphs, for direct reactions.

Discussion.

(a) Energy momentum conservation leads to

$$p_b^2 - 2m_b E_b = q^2 + 2m_b\Big[\mu_{YB}\,\varepsilon^{B}_{Ab} + (1-\mu_{YB})\,\varepsilon^{X}_{YB} + \mu_{YB} - \mu_{xA}\Big]E \tag{a}$$

where $\varepsilon^{\alpha}_{\beta\gamma} = m_\beta + m_\gamma - m_\alpha$ = B.E of β and γ in α

and $\mu_{\alpha\beta} = m_{\alpha\beta}/m_\alpha$

(Note. At vertex (2) $P_Y - P_X = P_b$ and

$$P_X^2/2m_X = E_b + \frac{P_X^2}{2m_Y} - (m_b + m_Y - m_X)$$

PRODUCTION CROSS-SECTIONS OF INTERMEDIATE BOSONS BY NEUTRINOS IN THE COULOMBS FIELD OF AND .

(T.D. Lee, P. Markstein and C.N.Yang).

This paper is a report on the numerical computation of the cross sections for the above processes. Therefore a brief survey of weak interaction theory with particular emphasis on the intermediate vector boson hypothesis is necessary.

It is well known that a generalisation of the four-fermion interaction leads to the current-current type of an interaction Lagrangian given by

$$\mathcal{L}_{int} = J^{\dagger} J + h.c$$

where

$$J = (\bar{\nu} e) + (\bar{\nu} \mu) + (\bar{p} n) + (\bar{p} \Lambda)$$

The pairs have been constructed on the basis of the known experimental fact that all observed decays satisfy the

$\Delta S / \Delta Q = 0, + 1$ rule; i.e. $\Sigma^{+} \rightarrow n + e^{+} + \nu$

with $\Delta S / \Delta Q = -1$; i.e $(\bar{n} \Sigma)(\bar{\nu} e)$

is not observed. The inclusion of the self-interaction terms like $(\bar{\nu} e)\ (\bar{e} \nu)$ leads to the scattering

$$\nu + e^{-} \longrightarrow \nu + e^{-}$$

with a cross-section proportional to $G^2 \sim 10^{-45}\ cm^2$

while in the absence of a $J^{\dagger} J$ picture it would have been of the order G^4. Making use of this fact the following experiment

The self-interaction term $(\bar{n}p)(\bar{p}n)$ will give a weak parity violating contribution to the nuclear force in the first order (in the usual theory it is of second order only) and can be detected from the multipole nature of the γ ray in polarized $n +$ nucleus $\longrightarrow$ nuclues$' + \gamma$. Again, on the basis of experimental evidence, neutral currents like $\bar{e}e$, $\bar{\nu}\nu$, $\bar{\mu}e$ etc. have not been included in J. For example $\bar{\mu}e$ coupled to baryonic current will lead to

$$\mu^- + p \longrightarrow e^- + p \ , \ \mu^\pm \longrightarrow e^\pm + \gamma$$

which are very rare (10^{-5} normal modes) Similarly the $|\Delta S| = 1$ neutral currents would lead to

$$K^+ \longrightarrow \pi^+ + \mu^+ + e^-$$

which are rarer than

$$K^+ \longrightarrow \pi^0 + e^+ + \nu$$

Now, there is no reason why the J^+J interaction should not result from the exchange of charged bosons $W^\pm$ with a large mass M_B. This would mean that the fermi interaction is really a second order effect due to the intermediate bosons. $W^\pm$.

The following properties of W become immediately obvious.

1. Both $+ve$ and $-ve$ charge states should exist. (consider $\mu^\pm \rightarrow e^\pm + \nu + \bar{\nu}$ for example).

2. W should have spin 1 (i.e. vector meson) in order to transmit the observed vector or axial vector form of weak interaction.

3. The mass should be quite large i.e. $M_W \gtrsim m_K$.

   Otherwise the favourite decay mode of K would be

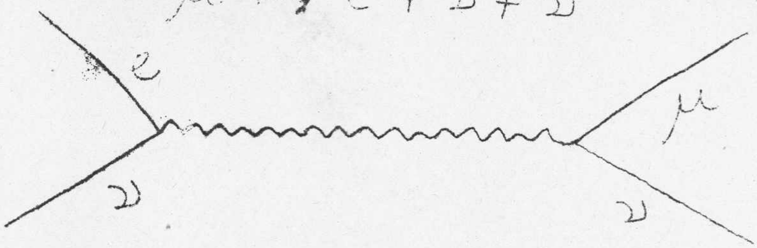
$$K \longrightarrow W + \gamma$$

and there would have been no difficulty

4. The fundamental weak coupling is

$$g_W \; \Phi_\alpha \left(J_\alpha + J_\alpha^{s.c} + \cdots \right) + h.c$$

where Φ_α is the field operator for W. The process $\mu \rightarrow e + \nu + \bar{\nu}$ is now



The finite mass of W implies a non-locality in the interaction extended over a dimension $\frac{1}{m_W}$. The contribution to the matrix element has the form

$$\frac{g_W^2}{m_W^2 + (\Delta k)^2} \, , \qquad G \approx \frac{g_W^2}{m_W^2}$$

for

$$m_x^2 \sim m_\rho^2 \, ; \qquad \frac{g^2}{4\pi} \sim 10^{-6}$$

and decay life ~~fune~~ time is 10^{-16} secs. The decay modes would be

$$W \longrightarrow 2\pi$$
$$\longrightarrow \mu\,(e) + \nu$$

etc.

5. In $\mu \longrightarrow e + \nu + \bar{\nu}$ decay assuming a four-fermion point interaction the electron spectrum was given by

$$P(p_e) = p_e^2 \left[W - p_e + \frac{2}{9} \rho (4 p_e - 3W) \right]$$

W = max momentum of e^-

where the Michel parameter ρ is a function of the various coupling strengths and $\simeq 3/4$. But the introduction of W modifies the spectrum so that the effective ρ is

$$\rho - \frac{3}{4} = \frac{1}{3} \left(\frac{m_\mu}{m_W} \right)^2 \leq .015$$

which is consistent with existing data.

6. In case we want to include neutral currents also, we need to postulate the existence of neutral vector mesons W^0. It is easy to see that $W^0 \neq \overline{W_0}$. If there was only one neutral W^0 then the field Φ_α is hermitian and therefore couples with a hermitian current. Hence the $\Delta S = 1$ part of the neutral current will contain both $(\bar{n}\Lambda), (\bar{\Lambda} n)$ and $(\bar{n}\Lambda)(\bar{\Lambda} n)$ would lead to $\Delta S = 2$ transitions which is forbidden.

7. Another important consequence of the non-locality is the question of $\mu \rightarrow e + \gamma$ and the identity of the neutrinos accompanying e and μ.

Let the μ neutrinos be ν' and the e neutrino ν. Then $\pi^- \rightarrow \mu^- + \bar{\nu}'$ and the absorption process

$$\bar{\nu}' + p \rightarrow \mu^+ + n$$

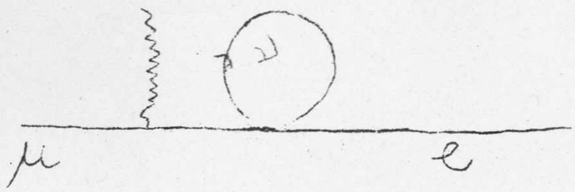
is allowed while

$$\bar{\nu} + p \rightarrow e^+ + n$$

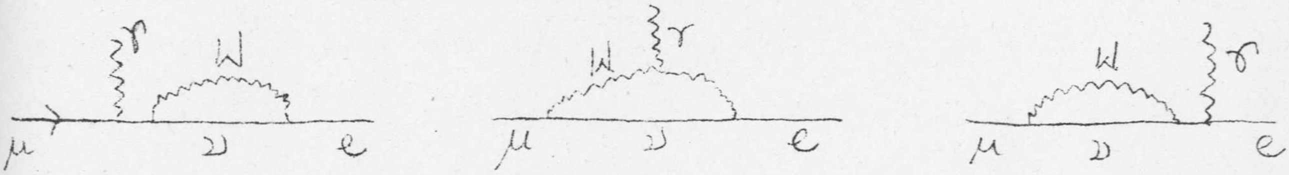
is forbidden. An indirect way which may probably suggest the existence of W is the readiative decay of μ. If W does not exist then

$$\mu^{\pm} \longrightarrow e^{\pm} + \gamma$$

can go through



which is of 1st order in a point interaction. Now the ν loop transforms as a vector under proper Lorentz transformations which would imply a vector character for the vacuum, unless it vanishes. But this process can occur if there exists a W as follows:-



This is of 2nd order in the W type interaction which is effectively of 1st order in the point interaction. Actually the decay matrix element is logarithmically divergent and using a cut-off Λ, the decay matrix element is

$$\frac{\Gamma\,(\mu \rightarrow e + \gamma)}{\Gamma\,(\mu \rightarrow e + \quad)} = F\left(\frac{m_W^2}{\Lambda^2}\,;\ \frac{\mu_W}{\mu_{BW}}\right)$$

where μ_W is the magnetic moment of W. If

$$m_W^2 \gg \Lambda^2\,, \quad F \rightarrow 0$$

which agrees with the results of a point interaction. If $\mu_W = 2$ Bohr magnetons $\times$ μ_{BW} ; $F \simeq \frac{1}{1200}$ and F is logarithimically divergent if $\mu_W \neq 2\mu_{BW}$ and

$$F \sim 10^{-3} \text{ to } 10^{-4} \text{ for } \mu_W = 1\ \mu_{BX},$$

$$\log \frac{\Lambda^2}{m_W^2} \sim 1.$$

Experimentally $F < 10^{-5}$ so that either W does not exist or m_W is too large. If we allow the possibility of the existence of separate neutrinos ν and ν' then there is still room for W. For, we cannot have a ν loop at all so that the 1st diagram forbids the process even if W exists. Thus according to Lee, the absence of $\mu \longrightarrow e + \gamma$ seems to indicate the dual neutrino theory independent of the existence of W.

Production of W.

W-s can be produced by strongly interacting particles - i.e.,

$$\pi^- + p \longrightarrow W^- + p$$

with a cross-section $\sim 10^{-32}\ cm^2$. But the identification is very difficult because of the background of strong interactions. Similarly the photo-production of W-pairs is also difficult.

But the best experiment would be with high energy neutrino beams incident on a nucleus of charge. Ze ; $\nu' + Z \rightarrow W^+ + \mu^- + Z$ (in diffierent states) (1)

$$\overline{\nu}' + Z \longrightarrow W^- + \mu^+ + Z \qquad (2)$$

Since the neutrinos can interact only weakly, when W can be produced energetically. This will be the main mode. Thus we can look for the dissociation of ν into W^+ and μ^- in the presence of a nucleus, to conserve energy and momentum. Thus we can have

and cross-section $\sim (\alpha Z)^2$. The present paper gives a numerical computation of the cross-sections for Fe. The two processes have the same differential cross-sections by the following theorem.

a) For process (2) consider a mirror reflection of (1) with all momenta and helicities reversed. The differential cross-section to lowest order in e and g are the same. To prove this we perform the CP operation on the leptons and the P operation on the nucleon which leaves the system invariant.

b) In (1) if helicities are held fixed a mirror reflection of all momenta leaves the cross-sections unchanged. To prove this we perform the time reversal operation which changes. the matrix element into ~~t~~its complex conjugate.

The interaction Lagrangian of W with Leptons is

$$\mathcal{L}_{int} = ig\,\bar{\Psi}\,\gamma_\lambda(1+\gamma_5)\,\Psi_{\nu'}\,\Phi^*_\lambda + h.c. \qquad (3)$$

But $G_V \approx \sqrt{2}\,\dfrac{g^2}{m_W^2}$ ie $g = m_W\, 2^{-1/4}\, G_V$.

$$G_V \approx 10^{-5}\, m_p^{-2}$$

The electro-magnetic ~~interacting~~ interaction of W with A_μ is taken to be

$$\mathcal{L}_{WA} = -\frac{1}{2}\left(\frac{\partial A_\mu}{\partial x_\mu}\right)^2 - \frac{1}{2} G^*_{\mu\nu} G_{\mu\nu} - m_W^2 \phi^*_\mu \phi_\mu$$
$$- i e K F_{\mu\nu} \phi^*_\mu \phi_\nu \tag{5}$$

where $$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu}$$

$$G_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$$

and $$\partial_\mu = \frac{\partial}{\partial x_\mu} - i e A_\mu$$

$$e = \left(\frac{4\pi}{137}\right)^{1/2}$$

and the magnetic moment of W along its spin is $\frac{e\hbar}{2mc}(1+K)$

The e.m. interaction with the nucleus contributes a factor $\frac{1}{\beta}$ which for coherent process when the nucleus recoils as a whole is

$$V_4 = \frac{i 2 e F^2(q^2)}{q^2} \qquad V_1 = V_2 = V_3 = 0 \tag{9}$$

where $q = p' - p$. The form-factor

$$F^2 = \left(1 + \frac{1}{12}\alpha^2 q^2\right)^{-2}$$

with

$$\alpha^2 = \frac{3}{5}\left[1.3 \times 10^{-13} A^{1/3}\right]^2 \text{cm}^2$$

(11)

For the production from a free proton or incoherent production

$$V_\beta = \frac{ie}{q^2} u_{p'}^{+} \gamma_4 \left\{ F_1 \gamma_\beta + i F_2 K \frac{1}{2} \left(\gamma_\lambda \gamma_\beta - \gamma_\beta \gamma_\lambda\right) (p-p')_\lambda u_{p'} \right\}$$

with the normalization

$$u_{p'}^{+} u_{p'} = u_p^{+} u_p = 1$$

(12)

for the Diract spinors and

$$K = \frac{1}{2}(\mu_p) = .8948$$

F_1 and F_2 are the form factors of the proton taken from Stanford experiments. The differential cross-section

$$d\sigma = (32\pi^5)^{-1} \left|\alpha_a + \alpha_b\right|^2 d^3\mu \, d^3W \, \delta\left(E_\mu + E_W + E_{p'} - E_\nu - E_p\right)$$

where α_a and α_b are the contributions from the two diagrams.

$$\alpha_a = -2egi\left\{(\nu-\mu)^2 + m_W^2\right\}^{-1} \times \left\{2[e\phi][WV] + [LV][q\phi] - [Lq][\phi V]\right\}$$

and

$$\alpha_b = 2egi\left\{(\nu-W)^2_+ m_\mu^2\right\}^{-1} \times$$
$$\left\{2[\ell\phi][\mu\nu] + [\ell\nu][q\phi] - [\ell q][\phi\nu] + \det\right\} \quad (15)$$

where

$$L = -(1+x)\ell - [\ell\mu]\left\{(L-\Psi)W - x_q\right\} m_W^{-2} \quad (16)$$

ϕ = amplitude of W

$(2E_W)^{\frac{1}{2}}\ \vec{\phi}$ Unit vector $\perp \vec{W}$ for transverse W

W $= E_W (m_W)^{-1}$ (Unit vector $\parallel \ell$ to $\vec{W}$) for Longitudinal W

$(2E_W)^{\frac{1}{2}}\ \phi_4 = 0$ for transverse W

$= i|\vec{W}|(m_W)^{-1}$ for longitudinal W (17)

The four vector

$$\ell_\lambda = u_\mu^\dagger \gamma_4 \gamma_\lambda u_\nu \quad (18)$$

The det is the determinant formed by components of ϕ, ν, ℓ and q

$$\det = \varepsilon_{\xi\eta\zeta\lambda}\, \phi_\xi \nu_\eta \ell_\zeta q_\lambda \quad (19)$$

The results are obtained by numerical computation. The total coherent cross-section is σ_Z (coherent) and on protons it is σ_p. The total σ_Z (total) is computed from

$$\sigma_Z \text{ (total)} = Z\,\sigma_p + \left(1 - \frac{1}{Z}\right)\sigma_Z \text{ (coherent)} \tag{20}$$

The correction term $\left(-\frac{1}{Z}\right)\sigma_Z$ (coherent) is to substract out the contributions from those incoherent processes. Included in $Z\,\sigma_p$ which gives rise to small momentum transfers and hence prohibited as incoherent processes. It is seen that for high energy ν' the contribution from coherent process dominates since the minimum of momentum transfer becomes smaller and the coherent process dominates. The energy distribution is carried by W so that the μ from W decay will be more energetic than the accompanying μ.

(<u>Note</u>. To arrive at the interaction between a complex vector field with non-vanishing rest-mass and the electromagnetic field we see that the Lag. of the non-interacting fields is

$$L_1 + L_2 = -\frac{1}{2}\,\partial_\mu A_\nu\,\partial_\nu A_\mu - \partial_\mu \phi_\nu^*\,\partial_\mu \phi_\nu$$

We now replace $\partial_\mu \phi$ by $(\partial_\mu - ieA_\mu)\phi \quad -m^2\phi_\mu^*\phi_\mu$

$$= -\frac{1}{4} F_{\mu\nu}F_{\mu\nu} - (\partial_\alpha + ieA_\alpha)\phi_\nu^*(\partial_\alpha - ieA_\alpha)\phi_\nu$$
$$-m^2\phi_\mu^*\phi_\mu$$

$$= -\frac{1}{2}(\partial_\mu A_\nu)^2 - (\partial_\mu + ieA_\mu)\phi_\nu^*(\partial_\mu - ieA_\mu)\phi_\nu$$
$$-(\partial_\nu + ieA_\nu)\phi_\mu^*(\partial_\nu - ieA_\nu)\phi_\mu$$

$$-\left(\partial_\mu + ie A_\mu\right)\phi_\nu^* \left(\partial_\nu - ie A_\nu\right)\phi_\mu$$

$$-\left(\partial_\nu + ie A_\nu\right)\phi_\mu^* \cdot \left(\partial_\mu - ie A_\mu\right)\phi_\nu$$

$$-m^2 \phi_\mu^* \phi_\mu$$

$$= -\frac{1}{2}\left(\partial_\mu A_\nu\right)^2 - \frac{1}{2} G_{\mu\nu}^* G_{\mu\nu}$$

$$-m^2 \phi_\mu^* \phi_\mu$$

$$+ \text{magnetic moment term}$$

$$= \lambda F_{\mu\nu} \phi_\mu^* \phi_{:\nu}$$

RELATIVISTIC MODEL FIELD THEORY WITH FINITE SELF-MASSES.

The main objective of such model field theories appears to be the solution of problems under approximations which violate a minimum number of assumptions of the complete field theory. In this model, the axiom that is violated is that of crossing symmetry and therefore the Mandelstam representation. The dispersion relations in energy are assumed to hold for all amplitudes and unitarity gives the absorptive parts in the physical regions. The absorptive parts in the unphysical region are assumed to be zero which violates crossing symmetry. Then the dispersion relations form an infinite set of coupled integral equations for all amplitudes and an exact solutions to this set in some simple cases can now be found in which the self-masses are finite and it is shown that this is equivalent to summing a certain class of Feynman diagrams. We shall deal with 2 types of spinless bosons A and B for simplicity and B is distinct from $\bar{B}$ and A is scalar so that $\bar{B}+B \to A$ in S-state. The masses of A and B are and respectively. To construct a conventional field theory one needs a Lagrangian density but instead it is here assumed that all amplitudes satisfy dispersion relations, thereby avoiding the need for any unobservable quantities such as bare masses or coupling constant. We thus define the S-matrix as

$$S_{ij} = \delta_{ij} - i(2\pi)^4 \delta^4 (p_i - p_j) \frac{T_{ij}}{N_i N_j} \tag{1}$$

where p_i, p_j, are the total four momenta of states i and j and N_i & N_j are the normalization factors.

$$N_i = \Pi (2E)$$

T_{ij} is a function of the independent variables that can be constructed from the momenta in and we assume

$$T_{ij}(s) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{Im\, T_{ij}(s')}{s' - s - i\epsilon} \tag{2}$$

The unitarity condition gives

$$i\left(T_{ij} - T_{ji}^{*}\right) = \sum_{n} \frac{1}{N_n} T_{in} T_{jn}^{*} (2\pi)^4 \delta^4 (P_i - P_n) \tag{3}$$

which condition must hold for physical s values i.e. $s \geq \max\left(M_i^2 \cdot M_j^2\right)$

where $M_{i,j}$ are the total mass of i & j .

Actually in the derivation of the above expression for any particular case field theoretically we find by using the commutator of current operators there exists a second term which due to the δ - function does not contribute to the R.H.S. in the physical region. For instance in the case of πN scattering it is given by

$$i\left[T^{\times}\left[(pq;p'q')\right]-T(p'q';pq)\right]$$

$$= -(2\pi)^4 \sum_n \left[\frac{\delta(p_n - p - q)}{N_n^2 V^{r}} T^{\times}(n, p'q') T(n, p, q) - \frac{\delta(p_2 - p + q')}{N_n^2 V^{n}} T^{\times}(n, p - q') T(n, p - q') \right]$$

where the second term contributes only for

$$p_n = p - q'$$

since

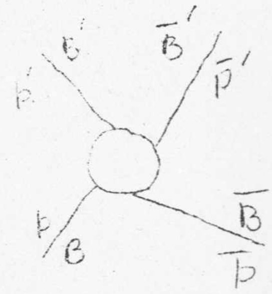
$$-p_n^2 = +M_n^2 = +\left[(E+\omega)^2 - 2k^2(1+\cos\theta)\right]$$

is always less than $(M+\mu)^2$ this can contribute only unphysical region but evidently this will be the in the physical region of the crossed process. An inclusion of this would lead us to a Mandelstam representation employing the principle of crossing symmetry.

But in the present case we assume $\operatorname{Im} T_{ij}(s) = 0$ for $s < \max(M_i^2, M_j^2)$ which together with (2) & (3)

defines our theory. But even so, a solution of this set of equations cannot be proved to be unique. But we may guess a suitable one such that all $T_{ij} = 0$ except those for which (2) and (3) differ only in that any number of A particles in have been replaced by particles in j and vice versa and for which and contain at least one A or one $B\bar{B}$ pair. This is consistent it is possible to obtain an exact solution for this. (would this amount to a one meson approximate say for πN ? I think it is, with the added restriction that NN or $\bar{N}\bar{N}$ scattering amplitudes are 0) The case of $B\bar{B}$ scattering is now discussed in detail.

<u>$B\bar{B}$ scattering:</u>



The amplitude is
$$T_{p\bar{p}';\,p\bar{p}} = T(s,t)$$
$$s = (p+\bar{p})^2$$

and $t = (p'-p)^2$ and by (3)

$$\mathrm{Im}\,T(s,t) = -\frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\frac{1}{2\omega_q} T_{p\bar{p}q}\, T^*_{p\bar{p}q}(2\pi)^4\,\delta(q-p-\bar{p})$$

$$-\frac{1}{2}\int\frac{d^3p''}{(2\pi)^3}\int\frac{d^3\bar{p}''}{(2\pi)^3}\frac{1}{2E_{p''}}\frac{1}{2E_{\bar{p}''}}T_{p'\bar{p}',p''\bar{p}''}\,T^*_{p\bar{p},p''\bar{p}''}$$

The first term on the R.H.S. is $(2\pi)^4\,\delta(p''+\bar{p}''-p-\bar{p})$

$$-\frac{2\pi}{2}\int\frac{d^3q}{2\omega_q}\,\delta^3(\vec{q}-\vec{p}-\vec{\bar{p}})\,\delta(\omega_q-E_p-E_{\bar{p}})\,(TT^*)$$

$$= -2\pi\int\frac{d^3q}{2\omega_q}\,\delta^3(\vec{q}-\vec{p}-\vec{\bar{p}})\,\delta\left(\left((\vec{p}+\vec{\bar{p}})^2+\mu^2\right)^{1/2}-E_{\bar{p}}-E_p\right)$$

$$= -2\pi \int \frac{d^3q}{2\omega_q}\, \delta^3\left(\vec{q} - \vec{p} - \vec{\bar{p}}\right) \delta\left[\left[(\vec{p}+\vec{\bar{p}})^2 + \mu^2\right]^{1/2} - E_{\bar{p}} - E_p\right] T T^*$$

$$= \frac{-2\pi}{2} \frac{1}{2\omega_{p+\bar{p}}}\, \delta\left(\left[(\vec{p}+\vec{\bar{p}}) + \mu^2\right]^{1/2} - E_{\bar{p}} - E_p\right) T T^*$$

$$= -\frac{2\pi}{2}\, \delta\left(s - \mu^2\right) T_{p\bar{p}}\,(p+p')\, T^*_{p\bar{p},\, p+\bar{p}}$$

and $T\left(p\bar{p}, p+\bar{p}\right) = g$ a number since there are no independent variables that can be formed of $p\bar{p}$ and q.

$$1^{st} \text{ term} = -{}^{2\pi}/_2\, \delta\left(s - \mu^2\right) g^2$$

The second term in the R.H.S. of (5) is

$$\frac{-(2\pi)^4}{2\,(2\pi)^6} \int \frac{d^3p''\, d^3\bar{p}''}{2E_{p''}\, 2E_{\bar{p}''}} \left(T T^*\right) \delta^{\circ}\left(\vec{p}'' + \vec{\bar{p}}'' - \vec{p} - \vec{\bar{p}}\right)$$
$$\delta\left(E_{p''} + E_{\bar{p}''} - E_p - E_{\bar{p}}\right)$$

$$= -\frac{1}{2(2\pi)^2} \int \frac{d^3p''}{2E_{p''}\, 2E_{\bar{p}''}} \left(T T^*\right) \delta\left(\sqrt{(p+\bar{p}-p'')^2 + M^2} + \sqrt{p''^2 + m^2} - E_p - E_{\bar{p}}\right)$$

$$-\frac{1}{2(2\pi)^2}\, 4\pi \int \frac{p''^2\, dp''}{2E_{p''}\, 2E_{\bar{p}''}} \left(T T^*\right) \delta\left(E_{p''} + E_{\bar{p}''} - E_p - E_{\bar{p}}\right)$$

$$= -\frac{1}{2\pi}\,\frac{p''^2}{2E_{p''}\,2E_{\bar{p}''}}\,\frac{TT^*}{(\partial E/\partial p'')} = [\quad] \times \frac{TT^*}{p''\left(\frac{E_{p''}+E_{\bar{p}''}}{E_{p''}\,E_{\bar{p}''}}\right)}$$

$$= -\frac{1}{8\pi}\,\frac{p''}{(E_{p''}+E_{\bar{p}''})} = -\frac{1}{16\pi}\left(\frac{s-4M^2}{s}\right)^{1/2} TT^* \quad \text{In C.M. } p'' = \frac{\sqrt{s-4M^2}}{2}$$

Hence (5) reduces to

$$I_m T(s) = -g^2\,\delta(s-\mu^2) - \frac{1}{16\pi}\left(\frac{s-4M^2}{s}\right)^{1/2} |T(s)|^2$$

and inserting (6) into (8) we get

$$T(s) = \frac{g^2}{s-\mu^2} - \frac{1}{16\pi^2}\int_{4M^2}^{\infty}\left(\frac{s'-4M^2}{s'}\right)^{1/2}\frac{|T(s')|^2}{s'-s-i\epsilon}\,ds'$$

Similar considerations may be applied for AB scattering as well with $s = (p+q)^2$ and $t = (p-p')^2$ and the states $n = AB$ and $n = BB\bar{B}$ will contribute. This however leads to an integral equation coupling AB scattering to itself and the process $A+B \to B+B+\bar{B}$ and so it does not lead to a singe uncoupled equation but is one of three amplitudes describing the process

$$A+B \to A+B \longleftrightarrow B+B+\bar{B}$$

$$B+B+\bar{B} \longleftrightarrow B+B+\bar{B}$$

<u>Solution for $B\bar{B}$ Scattering.</u>

Equation (8) is

$$T(s) = \frac{g^2}{s-\mu^2} - \frac{1}{16\pi^2}\int_{4M^2}^{\infty}\left(\frac{s'-4M^2}{s}\right)^{1/2}\frac{|T(s')|^2}{s'-s-it}\,ds' \tag{1}$$

To solve this, define

$$D(s) = g^2/s-\mu^2/T(s) \quad [N/D \text{ method}].$$

(2)

The analytic properties of $T(s)$ follow from (1) together with the assumption that $T(s)$ has no zeros so that the analytic properties of $D(s)$ can be deduced. Thus we infer

$$D(s) = 1 + \frac{s-\mu^2}{16\pi^2} g^2 \int_{4M^2}^{\infty} \left(\frac{s'-4M^2}{s'}\right)^{1/2} \frac{ds'}{(s'-\mu^2)^2(s'-s-i\epsilon)}$$

(3)

$D(s)$ has no **poles** so that the assumption (3) that $T(s)$ has no zeros is consistent. As $s \to \pm\infty$

$$D(s) \to 1 - g^2/16\pi^2 \int_{4M^2}^{\infty} \left(\frac{s'-4M^2}{s'}\right) \frac{ds'}{s'-\mu^2)^2}$$

Thus if is too large, $D(s)$ will have a zero for some $s < \mu^2$ which implies a pole of $T(s)$ which was not present in the original equation. Thus (2) and (3) are solutions of (1) only for small value of g^2.

We may also construct more solutions i.e. define

$$D(s) = \lambda + g^2/s-\mu^2 \;/T(s)$$

(4)

Where λ is at present an arbitrary number and on the assumption that $T(s)$ has no zeros except possibly $s_s = \mu^2 - g^2/\lambda$ (where Numerator of $(4) = 0$) but this zeros does not imply a pole in D and we have

$$D(s) = 1 + \frac{s-\mu^2}{16\pi^2} \int_{4M^2}^{\infty} \left(\frac{s'-4M^2}{s'}\right)^{1/2} \left(\lambda + \frac{g^2}{s'-\mu^2}\right) \frac{ds'}{\frac{s'-\mu^2}{(s'-s-i\epsilon)}}$$

(5)

which is an explicit form for $D(s)$, Reversing the roles of D and T in (4) constitutes a more general solution of (1) from which the earlier case is got by seating $\lambda = 0$ i.e. (5) is a solution of (1) for such values of that $D(s)$ has no zeros and hence $T(s)$ has no poles not allowed by (1). This restricts the range λ.

In (5) $D(s)$ has no poles and so $T(o)$ no zeros except possibly atso. From (1) it is seen that the only place where a zerod could occur for $T(s)$ is on the real axis above μ . Therefore if is to be a zero we have

$$s_0 = \mu^2 - \frac{g^2}{\lambda} > \mu^2$$

(6)

so that

$$\lambda < 0$$

(7)

If $T(s_0) \neq 0$ on the other hand we must have $D(s_0) = 0$ and from (5) we see that if $D(s_0) = 0$ then

$$g^{-2} = \frac{1}{16\pi^2} \int_{4M^2}^{\infty} \left(\frac{s'-4M^2}{s'}\right)^{1/2} \frac{ds'}{(s'-\mu^2)^2}$$

(8)

$$e) \quad -1 = \frac{s_0-\mu^2}{16\pi^2} \int_{4M^2}^{\infty} \left(\frac{s'-4M^2}{s'}\right)^{1/2} \left[\frac{-g^2}{s_0-\mu^2} + \frac{g^2}{s'-\mu^2}\right) \frac{ds'}{(s'-\mu^2)(s'-s_0-i\epsilon)}$$

$$= \frac{-s_0-\mu^2}{16\pi^2} \int_{4M^2}^{\infty} \left(\frac{s'-4M^2}{s'}\right)^{1/2} \left[\frac{g^2(s_0-s')}{(s_0-\mu^2)(s'-\mu^2)}\right] \frac{ds'}{(s'-\mu^2)(s'-s_0-i\epsilon)}$$

$$= -\frac{1}{16\pi^2} \int_{4M^2}^{\infty} \left(\frac{s'-4M^2}{s'}\right)^{1/2} \left[\frac{g^2}{s'-\mu^2)^2}\right]$$

And λ is related to $T(s)$ around $s = \mu^2$ as follows. From (4) and (5), near $s = \mu^2$,

$$T_s = \frac{g^2}{s-\mu^2} + \lambda - \frac{g^2}{16\pi^2} \int_{4M^2}^{\infty} \left(\frac{s'-4M^2}{s'}\right)^{1/2} \left[\lambda + \frac{g^2}{s'-\mu^2} \frac{ds'}{(s'-\mu^2)^2}\right.$$

$$+ o(s-\mu^2)$$

(9)

Now expanding (1) about $s = \mu^2$ we get

$$\lambda - \frac{g^2}{16\pi^2} \int_{4M^2}^{\infty} \left(\frac{s'-4M^2}{s'}\right)^{1/2} \left(\lambda + \frac{g^2}{s'-\mu^2}\right) \frac{ds'}{(s'-\mu^2)}$$

$$= \bar{\lambda} = -\frac{1}{16\pi^2}\int_{4m^2}^{\infty}\left(\frac{s'-4m^2}{s'}\right)^{1/2}\left(\frac{|T(s')|^2}{s'-\mu^2}\right)^2 ds' \tag{10}$$

$\therefore \bar{\lambda} < 0$ and $\lambda \geq \bar{\lambda}$. Thus the range of λ cannot be outside the interval $\bar{\lambda}$ to 0.

Now if $\lambda \neq 0$ (5) shows that as $s \longrightarrow \infty$
$D(s) \longrightarrow -\lambda/16\pi^2 \log|s|$

and from (4) we see that $T(s) \longrightarrow -16\pi^2/\log|s|$ independent of the sign of λ. This is sufficient to guarantee the existence of the unsubstracted dispersion relation, (5). Since T has no poles by definition except at $s = \mu^2$, so D should have no zeros but if we allowed for $\bar{K}N$ bound systems. There would be additional poles so that D would have the corresponding zeros. But D should never have zeros for negative s i.e. ghost states. From (5) we see that the λ term in D is +ve and ($\lambda < 0$ for $s > \mu^2$)) while g^2 term is −ve. For sufficiently large −ve s the λ term dominates. But if g^2 is too large, there may be region where D becomes −ve. A restriction on the size of g^2 relative to λ is hence necessary to ensure that D will have no zeros. Then the solutions for $\bar{B}\bar{B}$ scattering depends on two coupling constants λ and g^2 and is valid within certain ranges of these.

We now note that since crossing symmetry does not exist we have only one non-zero phase shift is only S-wave scattering exists. Define

$$T(s) = -16\pi \left(\frac{s}{s-4M^2}\right) \sin\delta(s)\, e^{i\delta(s)} \tag{11}$$

for $s > 4M^2$

$$\mathrm{Im}\, D(s) = -\frac{1}{16\pi}\left(\frac{s-4M^2}{s}\right)^{1/2}\left(\lambda + \frac{g^2}{(s-\mu^2)}\right) \tag{12}$$

and $\mathrm{Im}\, D(s) = 0$ for $s < 4M^2$ from (5) we get

$$\sin\delta(s)\, e^{i\delta(s)} = \frac{-\mathrm{Im}\, D(s)}{D(s)} \tag{13}$$

and so $D(s)$ is the conventional determinantal function for S-wave $\overline{B}B$ scattering. (13) is also equivalent to

$$\mathrm{Im}\, D(s) = -\tan\delta(s)\ \mathrm{Re}\, D(s) \tag{14}$$

which along with the analytic properties of D gives the integral equation

$$D(s) = 1 - \frac{s-\mu^2}{16\pi^2}\int_{4M^2}^{\infty} \frac{\mathrm{Re}\, D(s')\tan\delta(s')}{(s'-\mu^2)\,(s'-s-i\epsilon)}\, ds'$$

(15)

to which the solutions reads,

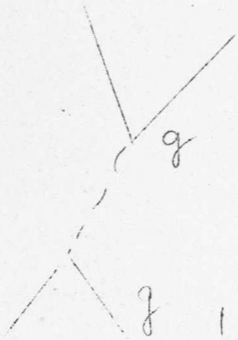
$$\mathcal{D}(s) = \exp \left\{ - \frac{s-\mu^2}{\pi} \int_{4M^2}^{\infty} \frac{ds'}{(s'-\mu^2)(s'-s-i\epsilon)} \right.$$

(16)

Comparison with perturbation expansion.

We have defined $\mathcal{D}(s)$ in (5) and from (12) we see that corresponds to the following two Feynman diagrams.

|(a)|



|(b)|

It is hence to be expected that the entire scattering $T(s)$ is equivalent to a set of Feynman graphs formed by all chains built out of (a) and (b). This can be verified by computing the scattering produced by the sum of such diagrams by the usual Feynman technique and comparing this with the $d.$ relations.

In lowest order the Feynman amplitude by graphs in (1) is

$$F_1 = -i \left(\lambda_0 + \frac{g_0^2}{(s-\mu^2)} \right.$$

(1)

where λ_0 and g_0 are the unrenormalised coupling constants and μ the physical mass. The phase shift in lowest order

$$\left(\sin\delta\, e^{i\delta}\right)_1 = -\frac{1}{16\pi}\,\frac{q}{\omega}\left(\lambda_0 + \frac{g_0^2}{s-\mu^2}\right) \tag{2}$$

where δ is the c.m. phase shift and q and ω are the c.m. momentum and energy of the particle; $\omega = \left(\frac{s^2}{4}\right)^{1/2}$; $q = \left(\frac{s^2}{4} - M^2\right)^{1/2}$

Thus in lowest order (By the determinantial method)

$$\mathrm{Im}\, D_1(s) = \frac{1}{16\pi}\left(\frac{s-4M^2}{s}\right)^{1/2}\left(\lambda_0 + \frac{g_0^2}{s-\mu^2}\right) \tag{3}$$

and

$$D_1(s) = 1 + \frac{s-\mu^2}{\pi}\int \frac{\mathrm{Im}\, D_1(s')}{(s'-s-i\epsilon)(s'-\mu^2)}\, ds'$$

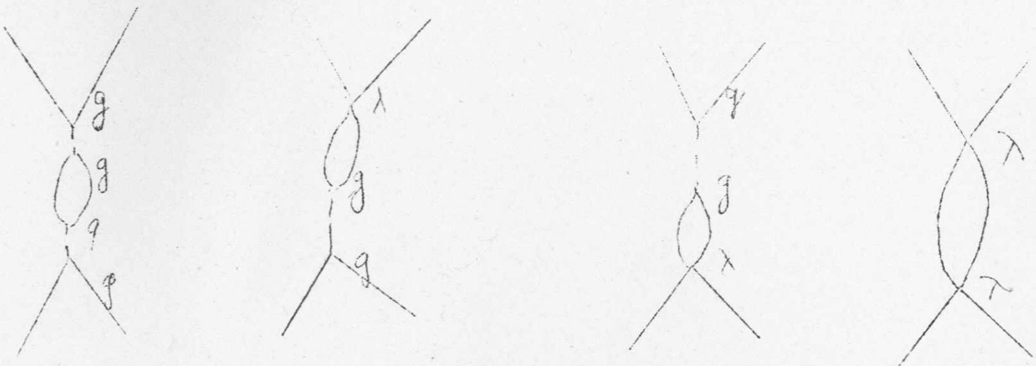
$$= 1 + \frac{(s-\mu^2)}{\pi}\int_{4M^2}^{\infty}\left(\frac{s'-4M^2}{s'}\right)^{1/2}\left(\lambda_0 + \frac{g_0^2}{(s'-\mu^2)}\right)\frac{ds'}{(s'-\mu^2)(s'-s-i\epsilon)} \tag{4}$$

and the scattering amplitude in lowest order is

$$T(s) = 16\pi\left(\frac{s}{s-4M^2}\right)^{1/2}\frac{\mathrm{Im}\, D(s)}{D(s)} \tag{5}$$

which is identical with the previous solution except that we now have λ_0 and g_0. It can be shown that the succeeding orders serve to alter (5) by replacing λ_0 and g_0 by the renormalised ones λ and g. To see this we consider this second order determinantal approximation.

The graphs



The Feynman amplitude is

$$F_2 = \int \frac{d^4 p''}{(2\pi)^4} \frac{i}{p''^2 - M^2} \frac{i}{(p''-p-\bar{p})^2 - M^2}$$

$$\left[(-i\lambda_0)^2 + 2(-i)^2 \lambda_0 g_0^2 \frac{i}{s-\mu^2} + (-ig_0)^4 \left(\frac{i}{s-\mu^2}\right)^2 \right] \qquad (6)$$

And by the usual Feynman parametrization we find

$$\left(\sin\delta e^{i\delta}\right)_2 = \frac{1}{16\pi} \frac{q}{\omega} \left\{ \left[I - I(\mu^2) \right] \right.$$

$$\times \left[\left(\frac{\lambda_0}{4\pi}\right)^2 + \frac{2}{s-\mu^2} \frac{\lambda_0}{4\pi} \frac{g_0^2}{4\pi} + \left(\frac{g_0^2}{4\pi}\right)^2 \left(\frac{1}{s-\mu^2}\right)^2 \right]$$

$$- i\alpha \left[\left(\frac{\lambda_0}{4\pi}\right)^2 + 2\left(\frac{\lambda_0}{4\pi}\right)\left(\frac{g_0^{\,2}}{4\pi}\right) \frac{1}{s - \mu^2} \right] \Bigg\} \tag{7}$$

where

$$I(s) - I(\mu^2) = \int_0^1 dx \log \frac{x(1-x)s/4 - M^2}{x(1-x)\mu^2/4 - M^2}.$$

so that

$$I(s) = \left(\frac{(s - 4M^2)}{s}\right)^{1/2} \left[2 \cosh^{-1}\left(\frac{s}{4M^2}\right)^{1/2} - i\pi \right] \tag{8}$$

and α is a constant given by

$$\alpha = \int_0^1 dx \int d^4q \left(\frac{1}{q^2 + x(1-x)\mu^2/4 - M^2} \right)^2 \tag{9}$$

And now

$$\mathrm{Im}\, D_2 = -(\sin\delta\, e^{i\delta})_2 - (\sin\delta\, e^{i\delta})_1\, D_1 \tag{10}$$

and from (4) and by evaluating the integral

$$D_1(s) = 1 + \left[\frac{\lambda_0}{16\pi^2} + \frac{g_0^2}{16\pi^2}\cdot\frac{1}{s-\mu^2}\right]\left[I(s) - I(\mu^2)\right] - \frac{g_0^2}{16\pi^2}\beta$$

(11)

where β is constant

$$\beta = \int_{4M^2}^{\infty} \left(\frac{s'-4M^2}{s'}\right)^{1/2} \frac{ds'}{(s'-\mu^2)^2}$$

(12)

The function $I(s)$ in (11) $\equiv$ with the function defined by (8) and now substituting (11) and (7) into (10) we get

$$\mathrm{Im}\, D_2(s) = \frac{1}{16\pi}\left(\frac{s-4M^2}{s}\right)^{1/2}\left(\lambda + \frac{g^2}{s-\mu^2}\right)$$

(13)

where

$$\lambda = \lambda_0 + i\alpha\left(\frac{\lambda_0}{4\pi}\right)^2, \quad g^2 = g_0^2 + 2i\alpha\frac{\lambda_0}{4\pi}\frac{g_0^2}{4\pi} - \beta\frac{g_0^2}{4\pi}$$

$$-\beta\left(\frac{\lambda_0}{4\pi}\frac{g_0^2}{4\pi}\right)$$

(14)

Thus $\mathrm{Im}\, D_2$ has the same form of D_1 in terms of λ and g^2. Thus summing all Feynman graphs of (2)

$$T(s) = \frac{\lambda + g^2/s-\mu^2}{1 + \frac{s-\mu^2}{16\pi^2}\int_{4M^2}^{\infty}\left(\lambda + g^2/s'-\mu^2\right)^{1/2}\frac{ds'}{(s'-\mu^2)(s'-s-i\epsilon)}}$$

(15)

which is identical with the dispersion theoretic result. But whereas in the latter approach λ and g^2 were resricted in this method we have not yet got any restrictions. This is because, the form of the d relations used did not allow for $B\bar{B}$ bound states. But if for example $\lambda > 0$ in (15) $T(s)$ <u>must</u> have a pole below μ^2 and may have a pole above μ^2. The upper pole exists if D_{11} vanishes for $\mu^2 < s < 4M^2$. Above $4M^2$ only the real part of the D_{11} can vanish in which case $T(s)$ has a resonance. The lower pole must represent a bound state if it occurs for $s > 0$ and an unphysical state if at $s = 0$ or below.

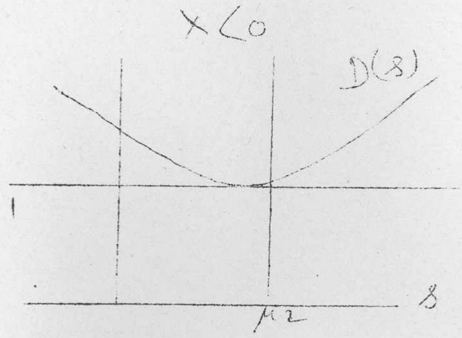
We know

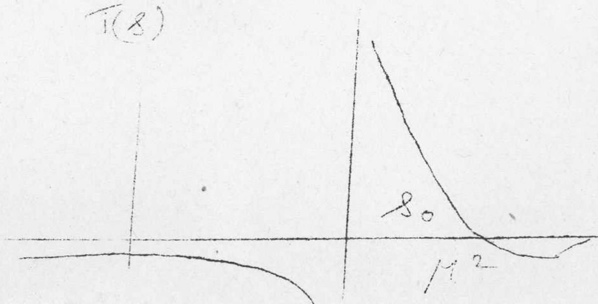
$$D(s) \longrightarrow -\frac{\lambda}{16\pi^2}\log|s| \quad \text{as } s \to \infty$$

$$T(s) \longrightarrow -\left[\frac{\lambda}{16\pi^2}\log|s|\right]^{-1}$$

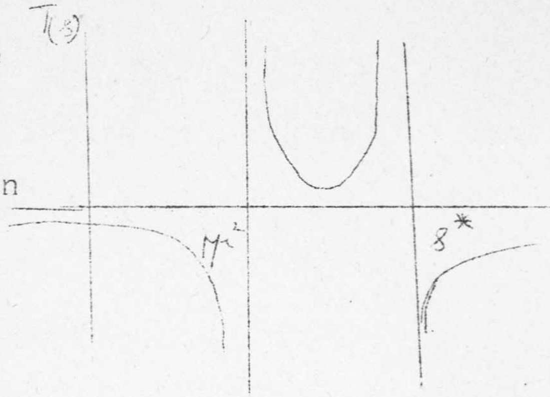
and so $T(s) \to 0$ from below at both $+$ and $-\infty$. T has a pole at $s = \mu^2$ with a +ve residue so $T \to -\infty$ below the pole and $+\infty$ above. If $\lambda < 0$ and g is small then D has no zeros so μ^2 is the only pole in T (see (a)). If $\lambda > 0$ $\mathrm{Re}\, D$ must have zeros above and below μ^2. If one of these — i.e. lower since $s_0 < \mu^2$ coincides with s_0 ~~xx~~ we have see (b).

(a)





These may become unphysical say for large g^2 in (a) D can be made to have 2 one of them in $-S$. We may also obtain the regions of λ and g^2 for which these difficulties arise as follows.



$$D(0) = 1 + \frac{1}{16\pi^2}\left(\lambda + \frac{g^2}{(s-\mu^2)}\right)\left[I(s) - I(\mu^2)\right] - \frac{g^2}{16\pi^2}\beta$$

$$\left[I(s) - I(\mu^2)\right] \to -\infty \text{ as } s \to I\infty \qquad (16)$$

$\longrightarrow$ increases below μ^2 and becomes +ve at μ^2.
$\longrightarrow$ It remains the upto $s^\times > 4M^2$ and then decreases - haming a single max. bet. μ^2 and $4M^2$.

The following conclusions may now be drawn:

1. If $g^2 = \frac{16\pi^2}{\beta}$, the only possible zeros are s_0 and $s^\times$. The former does not give a pole to T and $s^\times > 4M^2$ so only the $Re\, D$ vanishes and so represents a resonance in T.
2. If $g^2 < \frac{16\pi^2}{\beta}$, a zero of D implies that

$$\left[\lambda + g^2/s-\mu^2\right]\left[I(s) - I(\mu^2\right] \text{ is +ve at the zero.}$$

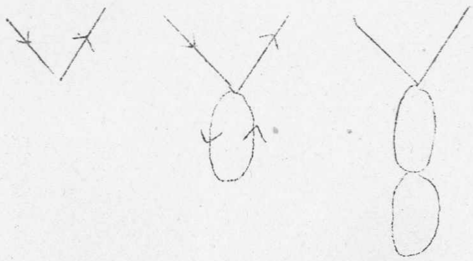
Now $\lambda + g^2/s-\mu^2 < 0$ for $s < s_0$ (i.e $s_0 > \mu^2$)

and > 0 if $s > s_0$ $\left[I(s) - I(\mu^2)\right]$ is +ve only bet μ^2 & s^*. Thus if $\lambda < 0$ zeros of D must be confined to the region $\mu^2 < s < s_0$ and if ɱn . Or else they can exist only for

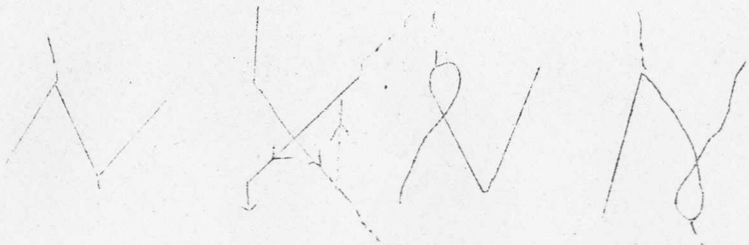
$s > s^* > 4M^2$ when they correspond to resonances. Thus the only time when it can become unphysical is if $\lambda > 0$ and $s_0 < 0$ i.e. $0 < \lambda < g^2/M^2$. And if $g^2 > \frac{16\pi^2}{\beta}$ zeros must be confined to complementary regions.

If should be stressed that in this approach one avoids ghost difficulties, since the assumed dispersion relation has no poles in the $-ve$ axis. But if we externed the theory for all values of λ and g^2 then this difficulty would arise. For cértain ranges of and it agrees with the original theory but for others the extended theory agrees with a modified dispersion of $B\bar{B}$ bound states. But if D has zeros for $s < 0$ then we have ghosts.

Sets of Feynman graphs for other processes in the model can be identified as built out of basic Feynman graphs.



The Feynman graphs for AB Scattering are



which when stretched out are

RARE DECAY MODES OF $\omega(\eta)$ MESON

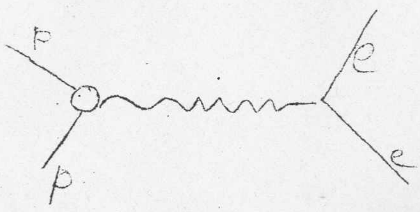
The work decay modes of resonances are discussed and it is shown that the effect of final state interaction may enhance these modes so that they are comparable to the strong modes.

The decays considered are

$$\omega \longrightarrow e^+ + e^- \qquad (1)$$
$$\longrightarrow \mu^+ + \mu^- \qquad (2)$$
$$\longrightarrow \pi^+ + \pi^- \qquad (3)$$

The (3) may be relatively frequent due to the possibility of the ρ resonance i.e. the $T=1$, $J=1$, $\pi\pi$ resonance may enhance the mode. The strong decay mode of ω (780 Mev. $I=0$) is 3π mode and similarly for η (550 Mev). These are said to be responsible for the isoscalar form factor of the nucleon.

It is now suggested that if ω and η participate in form factor experiments, it would mean that ω and η is coupled both to the nucleon and the electron. The coupling is order e^2 since it is mediated by a virtual photon. And so we should expect a similar coupling mechanism between $\omega(\eta)$ and any other changed particle. This means the above three weak decays may occur. The first two will differ only by the mass difference of μ and e while (3) violates I spin. The coupling may be different due to the intrinsic strong interaction. of $\omega(\eta)$ with baryon pairs but it should exist.



Let the vector (change) coupling of ω to nucleon and to e^- be G & g. Let the contribution of in the dispersion theoretic expression for the isoscalar form factor $F_1(q^2)$ of the form Cm^2/q^2+m^2

Then

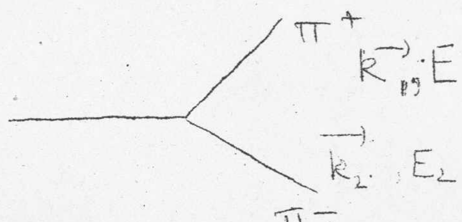
$$Gg = \frac{1}{2} e^2 \frac{q^2+m^2}{q^2} F_1(q^2) \Big|_{q^2=-m^2}$$

$$= \tfrac{1}{2} e^2 C$$

$$\text{or} \quad \frac{g^2}{4\pi} = \frac{\alpha}{4} \cdot \frac{C^2}{G^2/4\pi} \qquad (1)$$

And assuming the same g the partial widths for the 3 processes are found as follows.

1. In the rest frame $E_1 = E_2 = \frac{m\omega}{2}$



$$\Gamma = \frac{1}{(2\pi)^2} \int \frac{g^2}{8\omega E_1 E_2} |\vec{k}_1)_\mu \epsilon_\mu|^2 \, d^3k_1 \, d^3k_2 \, \delta^4(k_1+k_2-m\omega)$$

$$= \frac{1}{(2\pi)^2} \int \frac{g^2}{8\omega E_1 E_2} (4)(k^2 \cos^2\theta) \, d^3k_1 \, d^3k_2 \, \delta^4(k_1+k_2-m_\omega)$$

$$= \frac{4}{(2\pi)^2} \cdot 2\pi \; \frac{2}{3} \; \frac{g^2}{8} \int \frac{k^4 dk}{M_\omega E_1 E_2} \; \delta\left(E_1 + E_2 - M_\omega\right)$$

$$= \frac{1}{4\pi} \cdot 2 \cdot \frac{g^2}{3} \left[\frac{k^4}{M_\omega} \; \frac{k^3}{E_1 E_2} \qquad \right]$$

$$= \frac{2}{4\pi} \; \frac{g^2}{3} \; \frac{k^3}{M_\omega^2}.$$

$$\Gamma = \frac{g^2}{4\pi} \; \frac{m_\omega}{12} \left[1 - \frac{4 m_\pi^2}{m^2_\omega} \right]^{3/2} \left| F_\pi(q^2) \right|$$

2. For leptons

$$\Gamma = \frac{1}{(2\pi)^2} \int \delta^4 \left(E_1 + E_2 - M_\omega\right) \frac{g^2 m_p^2}{2\omega E_f^{\,2}} \sum_{\substack{\text{spin} \\ + \\ \text{pole}}} \left| \bar{u} \gamma_\mu (1+\gamma_5) u \, E_\mu \right|^2 d^3k_1 \, d^3k_2$$

And

$$\Sigma |\ |^2 = T_r \left[(1+\gamma_5)\gamma_\mu \frac{(-i k_1 + m_\rho)}{2 m_\rho} \gamma_\mu \times (1+\gamma_5) \frac{(-i k_2 + m_\rho)}{m_\rho} \right.$$

$$= \frac{1}{4m_\rho^2} T_r \left[k_1 k_2 + m_\rho^2 \right.$$

$$= \frac{1}{4m^2\rho} \left[4m^2_\rho + 4|k|^2 + k_0^2 \right] = \frac{m^2\omega}{m^2\rho} \left(1 + \frac{2m_\rho^2}{m_\omega^2}\right)$$

$$\Gamma = \frac{g^2}{4\pi} \frac{m_\omega}{3} \left(1 + \frac{2m^2\rho}{m_\omega^2}\right)\left(1 - \frac{4m^2\rho}{m^2\omega}\right)^{1/2}$$

Where $F_\pi(q^2)$ is the pion e.m. form factor and in terms o of ρ_2

$$F_\pi(q^2) = \frac{m^2\rho}{q^2 + m^2\rho - i m\rho \Gamma\rho}$$

(3)

m_ρ = 750 Mev and $\Gamma\rho \sim$ 100 Mev

and at $q^2 = -m^2$ where for ω; ... m = 780 Mev

$m_\rho^2 - m^2$ is small and

$$\Gamma(\omega \to e^+ + e^-) = 3 \times 10^{-3} \times \frac{c^2}{G^2/4\pi} \text{ Mev.}$$

$$\Gamma(\omega \to \pi^+ + \pi^-) = 7 \times 10^{-4}\ G^2/G^2/4\pi \left| F_\pi(-m^2) \right|^2$$

(5)

and it has been estimated that $C \sim -7$ and $G^2/4\pi$ $\sim 2-5$ and $|F_\pi(-m^2)|^2 \sim 40$ and

$$\Gamma\,(\text{leptonic}) = 5 \times 10^{-4} \text{ Mev}$$

$$\Gamma(\omega \rightarrow \pi^+ + \pi^-) = 5 \times 10^{-3} \text{ Mev} \tag{6}$$

To obtain the branching ratios for the rare modes we must buow the partial width for the strong decay mode ($\omega \rightarrow 3\pi$). Experimentally only its upper limit is given $\Gamma < 20$ Mev. If can be calculated if $\eta \rightarrow \pi^0 + \gamma$ is known and this again can be found by the Gell-Mann Zachariasen method. If we set $\pi^0\gamma / \pi^+\pi^-\pi^0 \sim 3$ (for η) and since the Gell-Mann estimate for $\pi^0 + \gamma$ is $.03$ Mev so that the partial width for the 3π mode is $.01$ Mev.

To compare this with the width for ω if other conditions are equal, the $\pi^+\pi^-\pi^0$ state of a neutral vector meson varies as Q^4 ($Q = Q$ values of the decay) which is 50 for ω. Therefore

Where we have taken into account the reduction in the ω width due to invariance under hypercharge reflection i.e.. If both ω and π are even under R, this may reduce the ~~width~~ ~~the~~ by a factor of 10.

There we see that the leptonic decay modes occur say for 1% then the $\pi^+ \pi^-$ may occur about 10% of the time.

For η however no enhancement due to final state interaction is possible.

$$\Gamma(\eta \to e^+ + e^-) = 2 \times 10^{-3} \; c^2/G^2/4\pi$$

$$\Gamma(\eta \to \pi^+ + \pi^-) = 3 \times 10^{-4} \; c^2/G^2/4\pi \text{ Mev.}$$

and similarly we obtain about 3% for mode and 5% for the $\pi^+ \pi^-$ mode.

<u>Concluding we have the following results:-</u>

1. There exist Goldberger-Treiman type relations for ω (η) meson which predict the rare decay modes $\omega \to \pi^+ + \pi^-, e^+ + e^-$ with branching ratios 10% and 1% respectively provided ω width is as narrow as .05 . The ρ resonance enhancement is responsible for the comparative increase in the width of the 2π mode. This is not possible for the η meson however.

2. In the previous paper by Sakurai we saw that invariance 'forbids' $\omega \to \pi^+ + \pi^- + \pi^0$ mode ($\pi^1 (\pi^2 \times \pi^3)$ is odd under R) and since $A_\mu \to -A_\mu$ $\omega^0 \to \pi^0 + \gamma$ is forbidden by R invariance if $\pi^+ \pi^-$ is in even relative orbital state and by change conjugation in invariance for odd relatives ρ state. Under such circumstances the most favoured e.m. decay is perhaps the $\pi^+ \pi^=$ mode. And if this is enhanced and if ω width

is very narrow we should expect a spwions ρ^0 peak in the $\pi^+ \pi^-_x Q$ value distribution but not for the $\pi^{\pm} \pi^0 Q$ value. This can be investigated in

$$p + \bar{p} \to 2\pi^+ + 2\pi^- + \pi^0$$

$$\pi^- + p \to \pi^+ + \pi^- + p$$

Of course, the real ρ^0 meson may also have intrinsic weak decay modes

$$\rho^0 \to e^+ + e^- \qquad \text{but} \qquad \frac{\Gamma(\rho_0 \to e^+ + e^-)}{\Gamma(\pi^+ + \pi^-)}$$

would be much less favourable than for ω because of the presumably large width $\Gamma \rho \to 2\pi \sim 100$ Mev.

3. In spite of the small branching ratios the leptornic decay modes may be observable in a spark chamber.

TESTS OF THE SINGLE PION EXCHANGE MODEL

This paper suggests a simple and yet an experimentally feasible test of the single-pion exchange model.

Consider the collision between particles p and k, resulting in two groups of outgoing particles ($p_1' \dots p_m'$ and $k_1' \dots k_m'$) loe resbict ourselves to configurations where the outgoing particles as viewed in the barycentric system form well defined cones consisting of $\{p_i'\}$ and $\{k_i'\}$ and assume that selection rules permit the exchange of a single pion.

$$p + k \to \{p_i'\} + \pi + k \to \{R'\} + \{k_i\}$$

And the invariant momentum

$$\Delta = p - \sum p_i' = \sum k_i' - k$$

Regarded as a function of $f_n\ \Delta^2$, the transition amplitude has a pole at $\Delta^2 = -\mu^2$ and the residues involves a product of the amplitudes $M(P + \pi \rightarrow \{p_i'\})$ and $M(k + \pi \rightarrow \{k_i'\})$

which deuribe the respective physical processes. The point $\Delta^2 = -\mu^2$ occurs outside the physical domain for . $p + k \rightarrow \{p_i'\} + \{k_i'\}$ but in the model it is assumed that the main contribution arised from the pole.

It is now suggested that even if the Δ^2 dependence is unspecified and even if the vertex functions are regarded as unknown the diagram gives rise to testable prediction on the reaction spectrum. This is because the structure of (1) implies that there is no correlation between the two groups of the particles; $\{p_i'\}$ and $\{k_i'\}$

beyond what follows from kinsematics. The result depends on the fact that the exchanged π has no spin.

The differential cross section $d\sigma$ is given by

$$\int d\sigma = f \prod_i dp_i' \, \delta(p_i'^2 + m_i^2) \prod_i dk_j' \, \delta(k_j'^2 + \mu_j^2)$$

$$\times \delta(p + k - \sum_i p_i' - \sum_j k_j')$$

other J is the relative current of the incident particles, of the square of the invariant transition amplitude. Now for the peripheral collision picture.

$$f = G(p, p_i) H(k, k_i') \tag{2}$$

This implies

1) In the system in which p is at rest, $d\sigma$ showuld be invariant under the simultaneous rotation of all three vectors $\vec{p}_i'$ about the vector q of the virtual meson and $q' = \vec{k} - \sum_i \vec{k}_i' = \sum_i p_i'$

2) Similarly in the k rest frame, it should be invariant under rotation of $\vec{k}_i'$ about $\vec{q} = -\sum_i \vec{k}_i'$

If can be proved that this is exhaustive for fixed incoming energy.

$$\pi(k) + N(p) \rightarrow \pi(k_1') + \pi(k_2') + N(p')$$

In the rest frame ~~x~~of pion, for given $\vec{p}$ and $\vec{p}'$ $d\sigma$, should be independent of the orientation of the plane defined by k_1' and k_2' about the line $\vec{q} \Rightarrow -k_1' - k_2' = +\vec{p}' - \vec{p}$

If $\vec{p}$ and $\vec{p}_x'$ are collinear this is trivial if $\{p_i'\}$ contains only one member. If $\vec{p}$ and $\vec{p}'$ are not collinear one could out of this model, envisage a correlation between the directions defined by $\vec{k}_1' \times \vec{k}_2$ and $\vec{p} \times \vec{p}_1$. If this were to be detected, it would weigh heavily against the single pion exchange model.

HIGH ENERGY NEUTRINO EXPERIMENTS

In this paper, general forms for the cross-sections for neutrino and antineutrino reactions are obtained assuming a 'point' interaction for leptons, the form of the strong interaction current being immaterial. At the energy ranges considered, the Fermi form for the strong interaction current is not expected to hold while the lepton current form is expected to have a wider range of applicability. The recent possibility of doing high energy neutrino experiment makes it feasible to extablish the validity of this particular form of lepton currents to the Bev region. Thus the results presented provide a method of verifying experimently the validity of the assumption of a 'point' interaction for leptons. That one is forced to employ a target involving strongly interacting particles and not say, electrons can be seen as follows:

We know that the effective Lagrangian for β - decay is

$$-\mathcal{L}_{eff} = \frac{G}{\sqrt{2}} \left[j_\lambda(x) \right]_\ell \left[j^*_\lambda(x) \right]_\mu + c \cdot c \qquad (1)$$

where

$$\left[j_\lambda(x) \right]_\ell = i \psi^+_e \gamma_4 \gamma_\lambda (1+\gamma_5) \psi_{\nu} \qquad (2)$$

and

$$\left[j^*_\lambda(x) \right]_\ell = i \psi^+_\nu \gamma_4 \gamma_\lambda (1+\gamma_5)_\ell \qquad (3)$$

and the experimental value for

$$G = 10^{-5}/m_p^2$$

This Lagrangian is of course, phenomenological and gives correct results only when used upto 1st order in perturbation theory. In fact the higher orders diverge. And it can be shown that this holds only for low momentum transfer. Consider,

$$\bar{e} + \nu \rightarrow \bar{\mu} + \nu \qquad (4)$$

The crosssections is

$$d\sigma = \frac{G^2}{\pi^2}\left(p_\nu^2 \; d\Omega\right)_{c.m.} \qquad (5)$$

where p_ν and $d\Omega$ are the momentum and solid angle subtended by the outgoing leptons in the c.m. system. Now G has dimensions of (length (Note: In units of $\hbar = c = 1$ (energy) $= L^{-1}$ and (mass) $= L^{-1}$ and so $[\phi] = L^{-1}$ .and $[\phi] = L^{-3/2}$ i.e. $G L^{-B-3F} = L^{-4}$

Here $B = 0$, $F = 2$ $\therefore [G] = L^{-4+B+3F} = L^2$

From this the structure of (5) is self-evident. Since $(G) = L^2$ so is σ ; so we need L^{-2} which can be p_ν^2 since this is the only independent momentum in the problem. Now, from the unitarity condition, for $S-$ waves,

$$\sigma_e = 0 < \frac{\pi\lambda^2}{2} \qquad (6)$$

where the $1/2$ factor arises due to averaging over the spin states of the electron. Thus Fermi's theory would be wrong for momenta (C.M.s)

$$(p_\nu)_{C.m.} > \left[\frac{\pi^2}{8G^2}\right]^{1/4} \sim 300 \text{ Gev} \tag{7}$$

(from (5) and (6)

$$\frac{4G^2}{\pi} p_\nu^2 = \sigma \qquad < \pi / p_\nu^2$$

since otherwise (5) would exceed the limit set by unitarity; But quite possibly deviations from Fermis' theory set in at lower energies.

Suppose we assume that the theory is correct for momenta,

$$p_\nu < 1/L$$

where L is some characteristic length so that one has two parameter G and L so we can choose them as

1) A dimensionless coupling constant

$$g^2 = GL^{-2} \tag{8}$$

and

2) the characteristic length. Thus the effect of weak interactions in any process must their depend on a function of two dimensionless quantities, g^2 and pL where p is a momentum connected with a process and for a given process cannot exceed a certain p max. Thus the statement, that weak interactions are 'weak' may imply either that g^2 or p max L or both are small. That p max L is small can however only be true with cirtainity when p max is either a real momentum of the process or a cut-off determined by the strong interactions

The weak interactions may mainfest themselves through virtual processes involving leptons and for those the only natural cut-off is given by L istself. For such virtual processes one can have p max L so that one is led to conclude that weak interactions are weak because g^2 is small that is we require

$$g^2 \ll e^2 \tag{9}$$

and so

$$\frac{1}{L} = \sqrt{\frac{g^2}{G}} < \sqrt{\frac{e^2}{10^{-5}}}\, m_p^2 \sim 30 \text{ Mev} \tag{10}$$

Therefore:

at 300 Gev there will definitely be from the Fermi theory at

$\leq$ 300 GeV there will probably be deviations.

But 1 Gev c.m. energy of ν in (4) corresponds to a 1 lb energy of 4000 Gev due to the smallness of the electron mass. Thus we are forced to consider reactions with heavies target particles. Which naturally leads to complications due to strong interactions. One can expect the Fermi's theory to be applicable at low momentum transfers

(that is at high energies)

that is