SPONTANEOUS CHIRAL SYMMETRY BREAKING (SCSB) IN QUARK CONFINEMENT

By

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BONAFIDE CERTIFICATE

Certified that this dissertation titled **SPONTANEOUS CHIRAL SYMMETRY BREAKING (SCSB) IN QUARK CONFINEMENT** is the bonafide work of **Mr. Alok Kumar** who carried out the project under my supervision. Certified further, that to the best of my knowledge the work reported herein does not form part of any other dissertation of the basis of which a degree or award was conferred on an earlier occasion for this or any other candidate.

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Abstract

The non-perturbative aspects of QCD, the gauge theory of strong interactions are studied. The phenomenon of confinement of quarks and the dynamical chiral symmetry breaking are examined. The phenomenological linear confining potential is consistent with quantum gauge Yang-Mills theory. Such a potential emanates from $\frac{1}{q^4}$ propagator for gluons in the infrared region. The dynamical chiral symmetry breaking in the above region studied using Schwinger-Dyson equation in the ladder approximation. We exhibit dynamical chiral symmetry breaking. The parameters in the analysis are determined by evaluating the quark condensate and comparing with its recent values. The numerical value of M(0) for quark is found to be 0.63 GeV in agreement with lattice QCD estimates. With this, we predict the dual gluon mass to be 980 MeV which is in reasonable agreement with earlier estimate of 848 MeV.

திட்டப்பணிச் சுருக்கம்

அடிப்படை இடைவினைகள் நான்கில் ஒன்றான வலுவுள்ள இடைவினை, உடையாத SU(3) குழுவினை உடைய அளவுக் கொள்கையின் வாயிலாக விளக்கப்பட்டுள்ள நிலையில், அக்கொள்கையின் உலைவு அற்ற விளைவுகளை இந்த பட்டமேற்படிப்பு ஆராய்ச்சி விளக்க முற்பட்டுள்ளது. இவ்விளைவுகள், எப்படி குவார்க்குகள் புரோட்டான் அல்லது நியூட்ரானுள் வெளியே வராமல் என்றென்றும் அடங்கிக் கிடக்கிறது என்பதும், எப்படி கைரல் சமச்சீர் குவாண்டம் முறையில் உடைக்கப் படுகிறது என்பதும் ஆகும். இதில் முன் விளைவு, குவார்க்குகளுக்கு இடையே உள்ள நிலையாற்றல் அவைகளின் இடைவெளிக்கு நேர்போக்கில் இருப்பின், நிகழ வாய்ப்புள்ளது. அத்தகு நிலையாற்றல் $\frac{1}{q^4}$ முறையில், புற அகச்சிவப்பு குளுவான்கள் கடக்கும் என்றும், இதைப் பயன்படுத்தி, சுவின்கர் - டைசன் சமன்பாடு வாயிலாக, கைரல் சமச்சீர் குவாண்டம் முறையில் உடைக்கப்படும் என்றும் அறியப்பட்டுள்ளது. இதில் ஒரு அளபுறு M(O). இது குவார்க்குகளின் சுருக்கம் மதிப்பீட்டை பயன்படுத்தி, 0.63 GeV என்று மதிப்பிடப்பட்டுள்ளது. இந்த மதிப்பீடு, கணினி வழியாக உருவாக்கப்பட்ட மதிப்பீட்டை ஒத்துள்ளது. மேலும் டூவல் குளுவானின் நிறை 980 MeV என்று மதிப்பிடப்பட்டுள்ளது. இந்த மதிப்பீடு ஏனைய மதிப்பீட்டுடன் ஒத்துள்ளது.

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Chapter 1

INTRODUCTION

All hadrons (mesons and baryons) are made of quarks and gluons and they interact among themselves via strong interactions. Quarks have three types of color charges and come in six flavours (u,d,s,c,t,b). Unlike photon which is chargeless and the force mediator between electrons, gluons are the force mediator between quarks and have color charges. Hence gluons can interact with gluons which is the source of the non-Abelian nature of the force [1]. Due to this non-Abelian nature, the strong coupling constant which is scale-dependent is low at high energy scale (i.e. less than one). At high energy-scale (short distance-scale) the coupling constant tends to zero and the quarks and gluons start moving freely inside the hadrons. This phenomena is called "Asymptotic Freedom". Owing to small coupling constant in high-energy zone, the perturbative method calculation is satisfactory [2]. The property of QCD that led directly to its discovery as a candidate of the strong interaction is asymptotic freedom. Thus color (chromo) force provides a mechanism for binding quarks together into hadrons, and the dynamics of this interactions is governed by Quantum Chromodynamics (QCD). The position of QCD as the candidate theory of strong interactions was strengthened still further in 1974 when Gross and Wilczek went on to prove mathematically that only non-Abelian gauge field theories can give rise to asymptotically free behaviour [2] .

"QCD is an non-Abelian gauge theory (an example of Yang-Mills Theory) of strong interactions among quarks and gluons inside the hadrons."

Quarks and gluons are non-interacting at short distances, but have never been observed as free (asymptotic) particles. This fact had lead to the conjecture that colored particles never appear in asymptotic states. This is called confinement of quarks and gluons confinement should be derivable from the QCD Lagrangian. In fact, if we pull the quarks apart, the energy that would be put in, in order to seperate them is used to create new hadrons between the seperated particles. This process is called hadronisation.

In general, if the potential between two quarks is proportional to the distance between them, then two quarks can never be seperated. So phenomenologically, confinement implies $V(r) \sim \sigma r$ where σ is a constant (GeV/fm). If we try to seperate the quarks by force, then the restoring force of the linear potential between them grows sufficiently rapidly to prevent them from being seperated. Thus they can never be seperated if they are bound by a linear potential. Similarly, if the quark potential asymptotically becomes a constant or decreases with distance, then the potential is not sufficient to confine the quarks.

One of the aspects studied here is a possible explanation of linear potential from quantum field theory point of view. A related feature of the non-perturbative aspect of QCD is 'Chiral Symmetry Breaking', which is signalled by quarks acquiring mass, is studied using Schwinger-Dyson equation.

1.1 Chiral Symmetry and its Spontaneous Breaking

Let us consider the chiral transformation of quark field $\psi(x)$ as

$$\psi(x) \to \psi'(x) = e^{i\theta\gamma_5}\psi(x) \tag{1.1}$$

where θ is independent of x i.e. $\theta \in R$ and is a constant.

Under this transformation $\overline{\psi}(x) = \psi^\dagger(x)\gamma_0$ will transform as

$$
\begin{aligned}
\overline{\psi}(x) &= \psi^\dagger(x)\gamma_0 \\
\overline{\psi}(x) \to \overline{\psi}'(x) &= (e^{i\theta\gamma_5}\psi(x))^\dagger\gamma_0 \\
&= \psi^\dagger(x)e^{-i\theta\gamma_5}\gamma_0 \\
&= \psi^\dagger(x)\gamma_0 e^{+i\theta\gamma_5} \\
& \quad \{\gamma_5\gamma_0 + \gamma_0\gamma_5 = 0\} \\
&= \overline{\psi}(x)e^{+i\theta\gamma_5}
\end{aligned}
\tag{1.2}
$$

so, $\overline{\psi}(x)\psi(x)$ would transform as

$$
\begin{aligned}
\overline{\psi}'(x)\psi'(x) &= \overline{\psi}(x)e^{i\theta\gamma_5}e^{i\theta\gamma_5}\psi(x) \\
&= \overline{\psi}(x)e^{i2\theta\gamma_5}\psi(x)
\end{aligned}
\tag{1.3}
$$

This means a mass term $\overline{\psi}(x)\psi(x)$ in QCD Lagrangian for quarks is not invariant under global chiral transformation [3] [4]. So, in the chiral limit (quark mass goes to zero)

$$\mathcal{L}_{cl} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \overline{\psi}\gamma^\mu(i\partial_\mu + gA^a_\mu t^a)\psi \tag{1.4}$$

is invariant under global chiral transformation. If mass of the quark is introduced at the Lagrangian itself, the chiral symmetry is broken explicitly. If we start with quark mass as zero initially in the Lagrangian, the quantum correction generates mass for quark and the global chiral symmetry get broken, this is called spontaneous chiral Symmetry Breaking. This is studied here using Schwinger-Dyson equation.

1.2 Outline of Dissertation

In chapter 2, the theoretical aspects of QCD as quantum gauge field theory are given to motivate that a linear potential between quarks is compatible with relativistic quantum field theory. Consequently, the gluon propagator in the infrared (confining) region is $(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2})\frac{m^2}{k^4}$. In chapter 3, we analyse Schwinger-Dyson equation for quarks without bare mass in the above gluon background in the ladder approximation. We explicitly demonstrate chiral symmetry breaking using $\frac{1}{q^4}$ propagator. The quark condensate $< \overline{\psi}\psi >$ is evaluated and numerical results are presented, by fitting the parameters with quark condensate.

Chapter 2

NON-ABELIAN GAUGE QUANTUM FIELD THEORY:A THEORETICAL PERSPECTIVE

Gauge quantum field theories are the only quantum field theories relevant to elementary particle physics. So it will be of physical importance to analyze the structure of these theories, without relying on perturbation theory. This is particularly useful to address the issue of confinement of quarks and gluons in QCD as the confining regime is in the infrared region where the QCD coupling is large so that perturbative methods cannot be reliably employed. The first assumption we make here is, in the confining region, the QCD coupling (large) is a constant, g(q)=g(0). This assumption has been suggested by Gribov [5].

Gauge quantum field theories have different properties from standard quantum field theories. An example is the Abelian field theory in which the indefinite metric in the definition of scalar product plays crucial role. Further, in non-Abelian gauge quantum field theories, the cluster property does not necessarily hold, although such a property holds good for Abelian gauge quantum field theory. So non-Abelian gauge quantum field theory has two important features, indefinite metric structure and failure of cluster property when the gauge group is not broken.

The definition of physical space $V_{phys} \subset V_{total}$ such that the norm V_{phys} is positive semi-definite i.e. $(\phi, \phi) \geq 0; \phi \in V_{phys}$ is another distinguishing property of gauge quantum field theory. As the matrix elements between two physical states $\phi_1, \phi_2 \in V_{phys}$ do not change by adding to ϕ_1 and/or to ϕ_2, states $\chi \in V_{phys}$ with vanishing norm $< \chi, \chi >= 0$, as these are also orthogonal to ϕ_1, ϕ_2 $(|(\phi, \chi)| \leq |(\phi, \phi)|^{1/2}(\chi, \chi)|^{1/2}$ by Schwarz inequality and as $(\chi, \chi) = 0$, it follows $(\phi, \chi) = 0)$, it is convenient to characterize the physical state corresponding to ϕ by the equivalence class $[\phi]$. The quotient $V_{phys} = V_{phys}/V_0$, $V_0 = \{\chi \in V_{phys}; (\chi, \chi) = 0\}$ will be called the space of physical states and the scalar product is positive-definite, in V_{phys}/V_0.

Another distinguishing feature of non-Abelian gauge quantum field theory is related to Wightman functions. In the standard QFT, with positive metric, the positivity property ensures that quantized fields can always be constructed once vacuum expectation values are the given set of Wightman functions. In the case of the indefinite- metric case, this is not possible in general. However, using V_{phys} this can be circumvented. The translation invariance of the Wightman functions requires the space-time translation operators U(a) are unitary, now with respect to the indefinite product, i.e. $U(a)^\dagger \eta U(a) = \eta$. This means, the Fourier transform of the two point function need not be a measure. We now consider observability condition in general. In a local gauge quantum field theory, with local symmetry group G unbroken, its generators Q^i commute with all the observables. A necessary condition for an operator A to describe an observable is $< \phi|[Q^i, A]|\phi >= 0$. Consequently, in the Abelian gauge theory, Q corresponds to electric charge and so $(\psi_i, Q^i\psi_i) = q^i(\psi_i, \psi_i)$ an observable. For QCD, $[Q^a, Q^b] = if^{abc}Q^c$ and so color charges cannot be observed. A deeper issue is whether a non-Abelian gauge quantum field theory has asymptotic particle-like states with non-vanishing colour. Such non-perturbative characteristic questions can be addressed now. The non-observability of quarks means that quarks are associated with a basic set of fields $\psi_i(x)$ but no particle like aymptotic states exist with quark quantum numbers. The validity of the cluster property becomes important in the existence of the asymptotic limit of a field operator. The failure of the cluster property for the quark fields ψ_i is strictly related to the fact that the states $\psi_i|0>$ do not have an asymtotic limit belonging to V_{phys}. (In 2-d QED, the cluster property fails and one views the dipole states as bound states of electrons intracting through a potential increasing at infinity). So the question of a mechanism of confinement is the cluster property of gauge quantum field theory [6].

2.1 Quantum Yang-Mills Theory

QCD is an $SU(3)_c$ unbroken gauge theory whose classical Lagrangian density is

$$\mathcal{L}_{cl} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \overline{\psi}\gamma^\mu(i\partial_\mu + gA^a_\mu t^a)\psi \tag{2.1}$$

with $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$

and t^a's are hermitian generators of SU(3). The classical equation of motion is $D^{ab}_\mu F^{\mu\nu b} = -gj^{\nu a}$; $D^{ab}_\mu = \partial_\mu\delta^{ab} + gf^{acb}A^c_\mu$. It is to be noted that the current $j^{\nu a}$ is covariently conserved i.e.

$$D^{ab}_\nu j^{\nu b} = 0. \tag{2.2}$$

By rewriting the classical equation of motion as

$$\partial_\mu F^{\mu\nu a} = -g\{j^{\nu a} + f^{acb}A^c_\mu F^{\mu\nu b}\} \equiv -gJ^{\nu a} \tag{2.3}$$

$j^{\nu a}$ is ordinarily conserved , i.e. $\partial_\nu J^{\nu a} = 0$. $J^{\nu a}$ contains a piece $f^{acb}A^c_\mu F^{\mu\nu b}$, a contribution from the gauge fields. In quantising the theory, we need to fix the gauge, as the momentum canonically conjugate to A^a_0 vanishes. In fixing the gauge, we encounter the Gribov ambiguity [7], that is, we cannot fix the gauge uniquely. This is an inherent problem which is not yet solved. Nevertheless we fix a gauge as Lorentz gauge and use Faddeev-Popov [3] method of writing the quantum Lagrangian. This is a standard procedure and so we do not give the details.

The equation of motion from the quantum YM theory is

$$D^{\mu ab}F^b_{\mu\nu} = \partial_\nu B^a - gj^a_\nu - ig(\partial_\nu\overline{C} \times C)^a = \partial_\nu B^a - gj^a_\nu - igf^{abc}\partial_\nu\overline{C}^b C^c \tag{2.4}$$

where c's are the anti-commuting FP ghost fields and B^a's are the Lagrange multipliar fields in $\mathcal{L}_{GF}$ and satisfy

$$D^{\mu ab}(\partial_\mu B^b) = igf^{abc}\partial_\mu\overline{C}^b.(D^{\mu cd}C^d) \tag{2.5}$$

Having fixed the gauge, the Lagrangian has no local gauge symmetry. However it has global symmetries, called BRS symmetry [8]. The BRS transformations leaving the quantum Lagrangian invariant, are

$$\left.\begin{array}{rcl} \delta A^a_\mu &=& [iQ_B, A^{\mu a}] \\ &=& (D^{ab}_\mu C^b) \\ \delta\psi^\alpha &=& [iQ_B, \psi^\alpha] \\ &=& igC^a(\tau^a)^\alpha_\beta\psi^\beta \\ \delta B^a &=& [iQ_B, B^a] \\ &=& 0 \\ \delta C^a &=& [iQ_B, C^a] \\ &=& -\frac{g}{2}C^bC^c \\ \delta\overline{C}^a &=& [iQ_B, \overline{C}^a] \\ &=& iB^a \end{array}\right\} \tag{2.6}$$

where the BRS-charge is given by

$$Q_B = \int d^3x\{B^a(D^{ab}_0C^b) - B^aC^a + i\frac{g}{2}f^{abc}\overline{C}^aC^bC^c\} = Q^\dagger_B \tag{2.7}$$

It is easy to show

$$\delta F^a_{\mu\nu} = gf^{acd}F^c_{\mu\nu}C^d \tag{2.8}$$

Now writing, $\delta F^a_{\mu\nu} = [iQ_B, F^a_{\mu\nu}]$,

$$\begin{array}{rcl} [Q_B, F^a_{\mu\nu}] &=& -igf^{acd}F^c_{\mu\nu}C^d \\ &=& igf^{adc}C^dF^c_{\mu\nu} \\ &=& ig(C\times F_{\mu\nu})^a \neq 0 \end{array} \tag{2.9}$$

The quantum equation of motion can be written as

$$\partial^\mu F^a_{\mu\nu} = -gJ^a_\nu + \{Q_B, D^{ab}_\nu\overline{C}^b\} \tag{2.10}$$

where [9]

$$\begin{aligned} J^a_\mu &= f^{abc}A^{\nu b}F^c_{\nu\mu} + j^a_\mu + f^{abc}A^b_\mu B^c - if^{abc}\overline{C}^b(D^{cd}_\mu C^d) + if^{abc}(\partial_\mu \overline{C}^b)C^c \\ &= j^a_\mu + f^{abc}A^{\nu b}F^c_{\nu\mu} - \{Q_B, f^{abc}A^b_\mu \overline{C}^c\} + if^{abc}(\partial_\mu \overline{C}^b)C^c. \end{aligned}$$

and

$$[Q_B, J^a_\mu] = -i\partial^\nu f^{abc}C^b F^c_{\mu\nu} \tag{2.11}$$

The total state vector space V in a covariant formulation of gauge theory necessarily contians negative norm states i.e. V has an indefinite metric. The impossivility of covariantly quantising electromagnetic field A_μ in the positive-metric Hilbert space has been demonstrated by Mathews, Seetharaman and Simon [10]. Since the positivity of the metric is vital to the probabilistic interpretation of the quantum theory, we need to define a suitable subspace of V in such a way that the physical S-matrix, defined by restricting the total S-matrix to the physical subspace, is unitary.

In non-Abelian gauge theory, the physical subspace $V_{phys} \subset V$ is specified by the subsidiary condition

$$Q_B\,|phys\rangle = 0; V_{phys} = \{|\phi>; Q_B|\phi>= 0\} \tag{2.12}$$

This is due to Kugo and Ojima [11], also by Curci and Ferrari [12]. As Q_B generates BRS transformations which are infinitesimal local gauge transformation, the above condition essentially expresses the gauge invariance of the physical states belonging to V_{phys}. The vacuum is taken to be annihilated by Q_B, $Q_B|0>= 0$ and so $|0\rangle \subset V_{Phys}$. Unphysical particles violating norm positivity are 'confined' in the sense they appear in V_{phys} only in the zero norm combinations. In defining a physical S-matrix between physical states with positive norm, we have to have

1. hermitian Hamiltonian ;

2. time invariance of physical subspace, i.e.

$$HV_{phys} \subseteq V_{phys} \tag{2.13}$$

H being the generator of time translation. This is assured using $[H, Q_B] = 0$, as let $H|phys> = |\psi>$. Then

$$\begin{aligned} Q_B|\psi> &= Q_B H|phys> \\ &= H Q_B|phys> \\ &= 0. \end{aligned}$$

Since Q_B annihilates $|\psi>$, (2.13) is assured.

and (3) positive-semi definiteness,

$$|\psi> \subset V_{phys} \Rightarrow <\psi|\psi> \geq 0 \tag{2.14}$$

2.2 Observables in Quantum Yang-Mills Theory

The physical space V_{phys} contains states with zero norm. Let us introduce a Hilbert space of physical states as V_{phys}/V_0 where V_0 is a space of zero-norm states. Any zero norm state $|\chi\rangle$ in V_0 is orthogonal to states in V_{phys}. The transition probability between physical states

$T(\phi_1|\phi_2) = |<\phi_1|\phi_2>|^2$ has the property

$T(\phi_1|\phi_2) = T(\phi_1 + \chi_1|\phi_2 + \chi_2) = |<\phi_1|\phi_2>|^2$.

Besides transition probability, physical quantities to be measured such as 4-momentum P_μ etc, must be such that any state $\in V_0$ should make no physical effect in the measurement. To ensure this, if a zero norm state $|\chi> \in V_0$ were transformed by a physical quantity R into $|\chi'> = R|\chi>$, such that $<\psi|\chi'> = <\psi|R|\chi> \neq 0$ for some $|\phi> \subset V_{phys}$,

then the measurement of R could not be described consistantly. So, we require the physical quantity to satisfy

$$<\phi|R|\chi> = <\chi|R|\phi> = 0 \ \forall \ |\phi> \in V_{phys}, \ \forall \ |\chi> \in V_0 \tag{2.15}$$

This can be understood by considering the situation in the usual quantum theory with positive definite inner product. In here

$$T(\psi|\phi) = | < \psi|\phi > |^2 =< \phi|\psi >< \psi|\phi >= E(P_\psi|\phi) \tag{2.16}$$

where the expectation value of an observable R in the state ϕ is defined to be

$$E(R|\phi) =< \phi|R|\phi > \tag{2.17}$$

so that

$$E(P_\psi|\phi) =< \phi|P_\psi|\phi >=< \phi|\psi >< \psi|\phi > . \tag{2.18}$$

Now returning to YM theory, every observable R which is self-adjoint, admits spectral decomposition i.e. $R = \sum_n a_n P_{\phi_n} = \sum_n a_n |\phi_n >< \phi_n|$ with $R|\phi_m >= a_m|\phi_m >$. Then the expectation value of R in $|\phi >$ is

$$\begin{aligned} E(R|\phi) &= < \phi|R|\phi > \\ &= \sum_n a_n < \phi|\phi_n >< \phi_n|\phi > \\ &= \sum_n a_n T(\phi_n|\phi) \end{aligned} \tag{2.19}$$

As adding a zero-norm state χ is not to change this,

$$E(Q|\phi + \chi) = E(R|\phi) \ for \ |\phi >\subset V_{phys}; \chi \subset V_0 \tag{2.20}$$

and this implies

$$< \chi|R|\phi >=< \phi|R|\chi >= 0. \tag{2.21}$$

(Also $< \chi|R|\chi >= 0$, since if $R|\chi >= |\chi' >\subset V_0$, then $< \chi|\chi' >= 0$. If $R|\chi >= |\phi >\notin V_0$ but $\in V_{phys}$, then $< \chi|\phi >= 0$).

These provide a definition for observable. An operator R is observable, if

$< \chi|R|\phi >= 0 =< \phi|R|\chi >$

Equivalently,

$< \phi + \chi|R|\phi + \chi >=< \phi|R|\phi >= E(R|\phi)$.

(As an example, let us find whether P_μ is observable or not. Since Q_B is a translationally invariant scalar

$[Q_B, P_\mu] = 0$

So, $P_\mu|\phi>=|\psi>$(say). $|\phi> \subset V_{phys}$;$Q_B|\phi>=0$. $Q_B|\psi>=Q_BP_\mu|\phi>=P_\mu(Q_B|\phi>)=0$. So $|\psi>\subset V_{phys}$. Since $|\chi>\subset V_0$ is orthogonal to $|\phi>\subset V_{phys}$ and since $|\psi>\subset V_{phys}$, it follows that

$<\chi|\psi>=<\chi|P_\mu|\phi>=0$

So, P_μ is an observable.)

In summary, we can say if an operator $\hat{O}$ commutes with Q_B then $\hat{O}$ is an observable.

In the case Abelian gauge theory, Stroocchi and Wightman [13] introduced four notions of the gauge invariance for operators

1. gauge independance : $<\phi_1+\chi_1|R|\phi_2+\chi_2>=<\phi_1|R|\phi_2>$ for all $|\phi>\in V_{phys}$ and $|\chi>\in V_0$;
2. weak gauge invariance : $RV_0 \subseteq V_0$;
3. gauge invariance : $RV_{phys} \subseteq V_{phys}$;
4. strict gauge invariance : (3) with $[R, \partial^\nu F_{\nu\mu}+ej_\mu]=0$.

Following Nakanishi and Ojima [14], for non-Abelian gauge quantum theory it is seen that, (1) implies (3) and (2). In non-Abelian gauge quantum theory, the physical subspace is characterized by $Q_B|\phi>=0$: $|\phi>\in V_{phys}$; $Q_B^2=0$.

If an operator R satisfies (1), then $<\phi|R|\chi>=0$ for $|\phi>\in V_{phys}$; $|\chi>\in V_0$. Let $|f>\in V$. Then for any $|\phi>\in V_{phys}$, consider $<f|Q_BR|\phi>$. This is zero, as $Q_B|f>\in V_0$ (since, $Q_B|f>=|n>$:$<n|n>=<f|Q_B^2|f>=0$). So, $<f|Q_BR|\phi>=0$.This implies $Q_BR|\phi>=0$ and therefore $R|\phi>\in V_{phys}$, which is (3). Next, from (1), $<\phi|R|\chi>=0$, let $R|\chi>=|\pi>$. If $|\pi>\in V_{phys}$, then $<\phi|\pi>=0$, which means there can be no transition between two physical states. This is not possible as $|\pi>$

is arbitrarily but $\in V_{phys}$. So, the only possibility is $|\pi>\in V_0$. So $RV_0 \subset V_0$ which is (2).

Now, the conditon (3) which implied by (1), gives,

$RV_{phys} \subset V_{phys}$

$Q_B R V_{phys} = 0$

$R Q_B V_{phys} = 0$

$R|\phi>=|\phi'>\in V_{phy}$

$Q_B R|\phi>=Q_B|\phi'>=0$

$RQ_B|\phi>=0$

So

$$[Q_B, R]_{\pm}|V_{phys}>=0 \tag{2.22}$$

or if R is an observable, then $[iQ_B, R]=0$.

(The converse is true. If $[Q_B, R]=0$, $R|\phi>=|\phi'>;|\phi>\in V_{phys}$. $Q_B R|\phi>= Q_B|\phi'>=RQ_B|\phi>$. So $|\phi'>\in V_{phys}$.So $RV_{phys}\subset V_{phys}$. If $[Q_B, R]_+=0$, let $R|\phi>= |\phi'>;|\phi>\in V_{phys}$. Then $Q_B R|\phi>=Q_B|\phi'>=-RQ_B|\phi>=0 \Rightarrow |\phi'>\in V_{phys}$ or $RV_{phys}\subset V_{phys}$.)

2.3 Confining Potential and Cluster Property

In describing the potential for quarks, we need a potential not decreasing at infinity to confine quarks.This implies a failure of the cluster property for the vacuum expectation value of two point function.This cluster property is

$<0|\phi_1(x_1)\phi_2(x_2)|0>\longrightarrow<0|\phi_1(x_1)|0><0|\phi_2(x_2)|0>$ as $|x_1-x_2|\rightarrow\infty$

Araki, Hepp and Ruelle [15] proved that the cluster property should hold in a Lorentz covariant local field theory with unique vacuum. In this case, the potential cannot be linearly rising. So, there appears a contrdiction between linear (confining) potential and gauge quantum theory. However, in a non-Abelian gauge theory, the possible failure of cluster property has been pointed out by Strocchi [16] . To understand this, we use the inequality derived by Araki, Hepp and Ruelle, on the assumption of covariance under translation, local commutativities, uniqueness of vacuum and spectral condition that

$$| < 0|\phi_1(x_1)\phi_2(x_2)|0 > - < 0|\phi_1(x_1)|0 >< 0|\phi_2(x_2|0 > | \leq C[\xi]^{-\frac{3}{2}}e^{-M[\xi]}[\xi]^{2N}(1 + \frac{|\xi^0|}{[\xi]})$$

$$or$$

$$| < 0|\phi_1(x_1)\phi_2(x_2)|0 > - < 0|\phi_1(x_1)|0 >< 0|\phi_2(x_2|0 > | \leq C'[\xi]^{-2}[\xi]^{2N}(1 + \frac{|\xi^0|}{[\xi]})(2.23)$$

when there is mass gap or no mass gap. $\xi = x_1 - x_2$ and N is a non-negative integer depending upon $\phi's$.

We first consider Abelian gauge theory (QED) where there is no mass gap, then

$| < 0|\phi_1(x_1)\phi_2(x_2)|0 > - < 0|\phi_1(x_1)|0 >< 0|\phi_2(x_2|0 > | \leq C'[\xi]^{-2}[\xi]^{2N}(1 + \frac{|\xi^0|}{[\xi]})$

The role of Q_B in QED is played by B(x)= -$\partial^\mu A_\mu(x)$.

Using $[A_{\mu(x)}, A_\nu(y)] = -i\eta_{\mu\nu}D(x - y)$

We have

$[\partial_\mu A_\nu(x), \partial^\lambda A_\lambda(y)] = -i\partial_\mu\partial^\lambda\eta_{\nu\lambda}D(x - y)$

$[\partial_\nu A_\mu(x), \partial^\lambda A_\lambda(y)] = -i\partial_\nu\partial^\lambda\eta_{\mu\lambda}D(x - y)$ and so

$$[F_{\mu\nu}(x), B(y)] = 0 \tag{2.24}$$

so that $F_{\mu\nu}(x)$ in QED is an observable. Next, using

$< 0|A_\mu(x)A_\nu(y)|0 >= \eta_{\mu\nu}F(x - y) + \partial_\mu\partial\nu G(x - y)$

where G is a gauge artifact,

we have

$< 0|F_{\mu\nu}(x)F_{\rho\sigma}(y)|0 >= -\{\eta_{\mu\rho}\partial_\nu\partial_\sigma - \eta_{\nu\rho}\partial_\mu\partial_\sigma - \eta_{\mu\rho}\partial_\nu\partial_\rho + \eta_{\nu\sigma}\partial_\mu\partial\rho\}F(x - y)$.

Let F(f)=$\int d^4xF_{\mu\nu}(x)f^{\mu\nu}(x)$

where $f^{\mu\nu}(x)$ is some function in R^4.

Then the state $F(f)|0>$ will be in V_{phys} since $F^{\mu\nu}(x)$ is an observable. Therefore

$$< 0|F^{\dagger}(f)F(f)|0 > \geq 0 \tag{2.25}$$

This implies the Fourier transform of $< 0|F_{\mu\nu}(x)F_{\rho\sigma}(y)|0 >$ is a measure. So N =0. Thus $F(x-y) \rightarrow [x-y]^{-2}$, which $\rightarrow 0$ as $|x-y| \rightarrow \infty$. Therefore the cluster property holds good and the potential also $\rightarrow 0$ as $|x-y| \rightarrow \infty$.

For QCD, which is a non-Abelian gauge quantum field theory, there is no mass gap. In here, we have

$$[iQ_B, F^a_{\mu\nu}] = if^{abc}C^bF^c_{\mu\nu} \tag{2.26}$$

and therefore $F^a_{\mu\nu}$ is not an observable (in contrast to $F_{\mu\nu}$ of Abelian theory). Consequently, the state

$F(f)|0 >= \int d^4xF^a_{\mu\nu}f^{a\mu\nu}(x)|0 >$

will not be in V_{phys}. This implies that the Fourier transform of $< 0|F^a_{\mu\nu}(x)F^b_{\rho\sigma}(y)|0 >$ will not be a measure and so N $\neq$0. The cluster property fails. Thus, the possibility of linear confining potential (not vanishing at spatial infinity) and quantum field theory are compatible. Also reasoning from a quark-antiquark confining linear potential, it has been realised that most suggestions of confinement mechanism imply a gluon propagator diverging faster than $\frac{1}{q^2}$ as $q^2 \rightarrow 0$, and typically as $'\frac{1}{q^4}'$. In describing confinement in QCD as dual Meissner effect [17], the dual gluon propagator is $\frac{1}{q^2-m^2}$ where m^2 is related to monopole effect. Then the gluon propagator will go as $\frac{m^2}{q^4}$. Let us consider quark-quark scattering.

The Feynman amplitute for the Feynman diagrram (2.1)

$$\begin{aligned} iM &= (ig)^2\overline{u}(p')\gamma^{\mu}t^a u(p)\frac{-ig_{\mu\nu}\delta^{ab}}{(p'-p)^4}\overline{u}(k')\gamma^{\nu}t^b u(k) \\ &= ig^2\overline{u}(p')\gamma^{\mu}t^a u(p)\frac{g_{\mu\nu}\delta^{ab}}{(p'-p)^4}\overline{u}(k')\gamma^{\nu}t^b u(k) \end{aligned}$$

To evaluate the amplitute in the nonrelativistic limit, we keep terms only to lowest

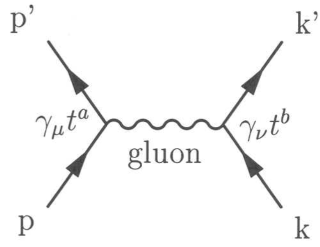

Figure 2.1: **Feynman diagram for Quark-quark Scattering**

order in 3-momenta,

$$p = (m, \vec{p}) \qquad k = (m, \vec{k})$$
$$p' = (m, \vec{p'}) \qquad k' = (m, \vec{k'})$$

Using these expressions, we have

$$(p'-p)^2 = -|\vec{p'} - \vec{p}|^2 + O((\vec{p})^4) \; ; \; U^s(p) = \sqrt{m}\begin{bmatrix} \xi^s \\ \xi^s \end{bmatrix}, \; etc.$$

So, in the nonrelativistic limit,

$$\overline{u}(p')\gamma^0 u(p) = u^\dagger(p')u(p) \approx 2m\xi'^\dagger\xi$$
$$[since\ u^\dagger u = 2E_{\vec{p}}\xi^\dagger\xi]$$
$$For\ \vec{p'} = \vec{p} = 0$$
$$\overline{u}(p')\gamma^i u(p) = 0$$

Therefore, they can be neglected compared to $\overline{u}(p')\gamma^0 u(p)$ in the nonrelativistic limit. Thus we have

$$iM \approx \frac{ig^2}{-(\vec{p'}-\vec{p})^4}(2m\xi'^\dagger\xi)_p(2m\xi'^\dagger\xi)_k g_{00}$$
$$= \frac{ig^2}{-(\vec{p'}-\vec{p})^4}(2m\xi'^\dagger\xi)_p(2m\xi'^\dagger\xi)_k$$

This amplitude should be compared with the Born approximation to the scattering amplitute in nonrelitivistic Quantum Mechanics, written in terms potential function V(r):

$$< p'|iT|p > = -i\tilde{V}(q)(2\pi)\delta(E_{\vec{p'}} - E_{\vec{p}}) \;\; (\vec{q} = \vec{p'} - \vec{p})$$

So, apparently for linear potential

$$\tilde{V}(q) = \frac{g^2m^2}{q^4}$$

$$\begin{aligned}
V(x) &= g^2m^2 \int \frac{d^3q}{(2\pi)^3}\frac{e^{i\vec{q}.\vec{x}}}{q^4} \rightarrow g^2m^2 \int \frac{d^3q}{(2\pi)^3}\frac{e^{i\vec{q}.\vec{x}}}{(q^2-i\epsilon)^2} \quad \epsilon > 0 \\
&= g^2m^2 \int \frac{d^3q}{(2\pi)^3} \int q^2 dq \int_{-1}^{1} d(cos\theta) \int_0^{2\pi} d\phi \frac{e^{iqxcos\theta}}{(q^2-i\epsilon)^2} \\
&= \frac{g^2m^2}{4\pi^2} \int_0^\infty q^2 dq \left[\frac{e^{iqxcos\theta}}{iqx}\right]_{-1}^{1} \frac{1}{(q^2-i\epsilon)^2} \\
&= \frac{g^2m^2}{4\pi^2} \int_0^\infty dq \frac{e^{iqx}-e^{-iqx}}{iqx} \frac{q^2}{(q^2-i\epsilon)^2} \\
&= \frac{g^2m^2}{4\pi^2 ix} \int_{-\infty}^\infty dq \frac{qe^{iqx}}{(q^2-i\epsilon)^2} \\
& \quad let\ qx = t \Rightarrow dq = \frac{dt}{x} \\
&= \frac{g^2m^2}{4\pi^2 ix} \int_{-\infty}^\infty \frac{t}{x}\frac{dt}{x} \frac{x^4 e^{it}}{(t^2+i\epsilon x^2)^2} \quad x^2 > 0\ ;\ \epsilon > 0\ ;\ \epsilon' > 0 \\
&= \frac{g^2m^2x}{4\pi^2 i} \int_{-\infty}^\infty dt \frac{te^{it}}{(t^2+i\epsilon')^2}
\end{aligned}$$

If $I = \int_{-\infty}^{\infty} e^{iax}dx$ satisfy the following:

(1) f(z) is analytic in upper half plane except for a finite no. of poles;

(2) Limit f(z)$\rightarrow$ 0 as $|z| \rightarrow \infty$

then $I = \int_{-\infty}^{\infty} e^{iax}dx = 2\pi i(sum\ of\ residues\ in\ upper\ half\ plane)$ [18].Therefore,

$$\begin{aligned}
V(x) &= \frac{g^2m^2x}{4\pi^2 i} \int_c dt \frac{e^{it}}{t^3} \\
& \quad [\,pole\ of\ order\ three] \\
&= \frac{g^2m^2x}{4\pi^2 i}.2\pi i \frac{1}{2!}\frac{d^2}{dt^2}\left(t^3\frac{e^{it}}{t^3}\right)_{t\rightarrow 0} \\
&= -\frac{g^2m^2x}{4\pi^2}\frac{2\pi}{2} = -kx
\end{aligned}$$

which is a linear potential.

The validity of the cluster property plays a crucial role in the existence of asymptotic limit of the field operator. The failure of the cluster property is thus related to the fact quark and gluon states donot have asymptotic limit, $\notin V_{phys}$.

Now, we illustrate how quark fields are not asymptotic. A quark field with color index α, satisfies

$[Q_B, \psi^\alpha] = gC^a(\tau^a)^\alpha_\beta \psi^\beta$

Since $[Q_B, \psi^\alpha] \neq 0$, ψ^α is not an observable. Let

$|\alpha >= \psi^\alpha|0>$

from which it follows $Q_B|\alpha >\neq 0$ and so $|\alpha >\notin V^{phys}$. Since the S-matrix is defined for physical states, $|\alpha>$ cannot be an asymptotic. On the other hand, the color singlet combination will be seen to be asymptotic. We find

$$\begin{aligned}\delta_{BRST}\epsilon^{\alpha\beta\gamma}\psi^\alpha\psi^\beta\psi^\gamma &= \epsilon^{\alpha\beta\gamma}\{(\delta\psi^\alpha)\psi^\beta\psi^\gamma + \psi^\alpha(\delta\psi^\beta)\psi^\gamma + \psi^\alpha\psi^\beta(\delta\psi^\gamma)\} \\ &= gC^a\epsilon^{\alpha\beta\gamma}\{(\tau^a)^\alpha_\delta\psi^\delta\psi^\beta\psi^\gamma + \psi^\alpha(\tau^a)^\beta_\delta\psi^\delta\psi^\gamma + \psi^\alpha\psi^\beta(\tau^a)^\gamma_\delta\psi^\delta\}\end{aligned}$$

By explicitly computing this for a=1 to 8 with Gell-Mann SU(3) matrices we notice $\delta_{BRST}\epsilon^{\alpha\beta\gamma}\psi^\alpha\psi^\beta\psi^\gamma = 0$ which implies

$[Q_B, \epsilon^{\alpha\beta\gamma}\psi^\alpha\psi^\beta\psi^\gamma] = 0.$

Consequently, the operator $\epsilon^{\alpha\beta\gamma}\psi^\alpha\psi^\beta\psi^\gamma$ is an observable which is hadron. Also, the state $\epsilon^{\alpha\beta\gamma}\psi^\alpha\psi^\beta\psi^\gamma|0>$ has the property of being annihilated by Q_B and so $\in V_{phys}$. So it is an asymptotic state.

Chapter 3

SCHWINGER-DYSON EQUATION

Schwinger-Dyson equations provide a nonperturbative approach to solve quantum field theory [19]. The best known DSE is the simplest 'gap equation' which describes how the propagation of a fermion is modified by its interactions with the medium being traversed.

$$S^{-1}(p) = Z_2(i\gamma.p + m_b) + Z_1 \int \frac{d^4q}{(2\pi)^4} g^2 \gamma^\mu \frac{\lambda^a}{2} S(q) \Gamma^{b\nu} D^{ab}_{\mu\nu}(p-q) \tag{3.1}$$

where

$$\begin{aligned}
S(p) &: \ Full\ quark\ propagator; \\
D^{ab}_{\mu\nu} &: \ Dressed-gluon\ propagator; \\
\Gamma^{b\nu}(q;p) &: \ Dressed-quark-gluon\ vertex; \\
m_b &: \ \Lambda-dependent\ current\ quark\ bare\ mass; \\
Z_1 &: \ Quark-gluon-vertex\ renormalisation\ constant; \\
Z_2 &: \ Quark\ Wave\ function\ renormalisation\ constant;
\end{aligned}$$

In applying the above to QCD in the infrared region, we invoke the following assumptions.

1. In the infrared region, the strong coupling constant g is taken as constant following the Girbov [5] i.e. g=g(0). So it can be taken out of the integral.

2. Under rainbow approximation,
   $\Gamma_\nu^b = \gamma_\nu \lambda^b/2$.

3. $Z_2 = Z_1 = 1$.

Using all the assumption in equation (3.1) we have

$$S^{-1}(p) = S_0^{-1}(p) + g^2 \int \frac{d^4q}{(2\pi)^4} \gamma_\mu \frac{\lambda^a}{2} S(q) \frac{\lambda^b}{2} \gamma_\nu D_{ab}^{\mu\nu}(p-q)$$

$$D_{ab}^{\mu\nu} = \delta^{ab} D^{\mu\nu}(p-q)$$

$$S^{-1}(p) = S_0^{-1}(p) + g^2 \int \frac{d^4q}{(2\pi)^4} \gamma_\mu S(q) \gamma_\nu D^{\mu\nu}(p-q)$$

$$S^{-1}(p) = S_0^{-1}(p) + g^2 \int \frac{d^4q}{(2\pi)^4} \gamma_\mu S(p-q) \gamma_\nu D^{\mu\nu}(q) \tag{3.2}$$

One of the form of the solution of SDE can be in terms of unknown functions $A(p^2)$ and $M(p^2)$ to be determined, as

$$\begin{aligned} S(p) &= \frac{1}{i\not{p}A(p^2) + M(p^2)} \\ &= i\not{p}C(p^2) + D(p^2) \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} C(p^2) &= \frac{-A(p^2)}{p^2A^2(p^2) + M^2(p^2)} \\ D(p^2) &= \frac{M(p^2)}{p^2A^2(p^2) + M^2(p^2)} \end{aligned} \tag{3.4}$$

Now from above equations, we have

$$p^2C^2(p^2) + D^2(p^2) = \frac{1}{p^2A^2(p^2) + M^2(p^2)} \tag{3.5}$$

We are interested in spontaneous breaking of chiral symmetry. So we set $m_b = 0$ here after. Substituting S(p) from equation (3.3), and taking $A(p^2) = 1$.

$$\begin{aligned}\frac{1}{i\not{p}C(p^2) + D(p^2)} &= i\not{p} + g^2 \int \frac{d^4q}{(2\pi)^4}\gamma^\mu\{i(\not{p} - \not{q})C((p-q)^2) + D((p-q)^2)\} \\ &\quad \gamma^\nu D_{\mu\nu}(q) \end{aligned} \tag{3.6}$$

Now taking trace over γ matrices and using the fact the trace odd no. of γ matrices vanish, we obtain

$$\begin{aligned}\frac{-D(p^2)tr(I)}{-p^2C^2(P^2) - D^2(p^2)} &= g^2 \int \frac{d^4q}{(2\pi)^4} tr(\gamma^\mu\gamma^\nu)D_{\mu\nu}(q)D((p-q)^2) \\ &\quad [tr(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}] \\ \frac{D(p^2)}{p^2C^2(P^2) + D^2(p^2)} &= g^2 \int \frac{d^4q}{(2\pi)^4} D^\mu_\mu(q)D((p-q)^2) \end{aligned} \tag{3.7}$$

Again, multiplying both sides of equation (3.6) with γ^λ and taking the trace over gamma matrices and using the fact that terms with odd no. of gamma matrices would not contribute, we have

$$\begin{aligned}\frac{\gamma^\lambda\{ip_\mu\gamma^\mu C(p^2) - D(p^2)\}}{-p^2C^2(p^2) - D^2(p^2)} &= i\gamma^\lambda\gamma^\mu p_\mu + g^2 \int \frac{d^4q}{(2\pi)^4}\gamma^\lambda\gamma^\mu\{i(\not{p} - \not{q})C((p-q)^2) \\ &\quad + D((p-q)^2)\}\gamma^\nu D_{\mu\nu}(q)\end{aligned}$$

upon taking trace

$$\begin{aligned}\frac{4ig^{\lambda\mu}C(p^2)p_\mu}{-p^2C^2(p^2) - D^2(p^2)} &= 4ip_\mu g^{\lambda\mu} + g^2 \int \frac{d^4q}{(2\pi)^4} i(p-q)_\rho tr(\gamma^\lambda\gamma^\mu\gamma^\rho\gamma^\nu) \\ &\quad C((p-q)^2)D_{\mu\nu}(q) \\ &\quad using\ tr(\gamma^\lambda\gamma^\mu\gamma^\rho\gamma^\nu) = 4\{g^{\lambda\mu}g^{\rho\nu} - g^{\lambda\rho}g^{\mu\nu} + g^{\lambda\nu}g^{\mu\rho} \\ &= 4ip_\mu g^{\lambda\mu} + g^2 \int \frac{d^4q}{(2\pi)^4} i(p-q)_\rho C((p-q)^2)4\{g^{\lambda\mu}g^{\rho\nu} \\ &\quad -g^{\lambda\rho}g^{\mu\nu} + g^{\lambda\nu}g^{\mu\rho}\}D_{\mu\nu}(q)\end{aligned}$$

Contracting both sides with p_λ

$$\begin{aligned}\frac{4ip^2C(p^2)}{-p^2C^2(p^2) - D^2(p^2)} &= 4ip^2 + 4ig^2 \int \frac{d^4q}{(2\pi)^4} C((p-q)^2)\{p^\mu(p-q)^\nu \\ &\quad -p^\rho(p-q)_\rho g^{\mu\nu} + p^\nu(p-q)^\mu\}D_{\mu\nu}(q)\end{aligned}$$

Using $D_{\mu\nu} = D_{\nu\mu}$ and exchanging dummy indices

$$\frac{p^2C(p^2)}{-p^2C^2(p^2) - D^2(p^2)} = p^2 + g^2 \int \frac{d^4q}{(2\pi)^4} C((p-q)^2)\{2p^\mu(p-q)^\nu - p.(p-q)g^{\mu\nu}\}D_{\mu\nu}(q) \tag{3.8}$$

The consideration in Ch.2 have resulted in the $\frac{1}{q^4}$ behaviour for the gluon propagator in the infrared region of QCD and including the tensorial structure we take the gluon propagator in Feynman gauge as in [4].

$$D^{ab}_{\mu\nu}(q) = -\frac{m^2}{q^4}(g_{\mu\nu})\delta^{ab} \tag{3.9}$$

Now from equation (3.9)

$$D^\mu_\mu(q) = -4\frac{m^2}{q^4}$$

and so (3.7) becomes

$$\begin{aligned} \frac{D(p^2)}{p^2C^2(p^2) + D^2(p^2)} &= -g^2 \int \frac{d^4q}{(2\pi)^4} D((p-q)^2) 4\frac{m^2}{q^4} \\ &= -4m^2g^2 \int \frac{d^4q}{(2\pi)^4} \frac{D((p-q)^2)}{q^4} \end{aligned} \tag{3.10}$$

Now from equation (3.8) and equation(3.9)

$$\begin{aligned} \frac{p^2C(p^2)}{-p^2C^2(p^2) - D^2(p^2)} &= p^2 - g^2 \int \frac{d^4q}{(2\pi)^4} C((p-q)^2)\{2p^\mu(p-q)^\nu - p.(p-q)g^{\mu\nu}\}\frac{m^2}{q^4}g_{\mu\nu} \\ &= p^2 - m^2g^2 \int \frac{d^4q}{(2\pi)^4} \frac{C((p-q)^2)}{q^4}\{2p.(p-q) - 4p.(p-q)\}\frac{1}{q^4} \\ &= p^2 + 2m^2g^2 \int \frac{d^4q}{(2\pi)^4} \frac{C((p-q)^2)}{q^4}\{2p.(p-q)\} \end{aligned} \tag{3.11}$$

Let us consider the equation (3.10) and euclideanize.

Substitute $(p-q)_\mu = k_\mu$.

Then,

$$\begin{aligned}
\frac{D(p^2)}{p^2C^2(p^2)+D^2(p^2)} &= -4m^2g^2\int\frac{d^4q}{(2\pi)^4}\frac{D(k^2)}{((p-k)^2)^2} \\
&= -4m^2g^2\int\frac{dk}{(2\pi)^4}k^3d\Omega_k\frac{D(k^2)}{(p^2-2p.k+k^2)^2} \\
&= -4m^2g^2(2\pi)\int\frac{dk}{(2\pi)^4}k^3Sin^2(\omega)Cos(\theta)d\theta d\omega\frac{D(k^2)}{(p^2-2p.k+k^2)^2} \\
&= -4m^2g^2(\pi)^2\int k^3\frac{dk}{(2\pi)^4}\left[\frac{1}{2|p||k|(p^2+k^2-2|p||k|Cos(\theta))}\right]_{-1}^{1} \\
&= -8m^2g^2(\pi)^2\int k^3\frac{dk}{(2\pi)^4}\frac{D(k^2)}{(p^2-k^2)^2} \qquad (3.12)
\end{aligned}$$

We shall take $D(k^2)$ as analytic function and use its Cauchy-Riemann representation [20]

$$D(k^2)=\int_{-\infty}^{\infty}d\alpha\frac{\overline{D}(\alpha)}{k^2-\alpha} \qquad (3.13)$$

Then,

$$\frac{D(p^2)}{p^2C^2(p^2)+D^2(p^2)} = -8m^2g^2\pi^2\int k^3\frac{dk}{(2\pi)^4}\int_{-\infty}^{\infty}d\alpha\frac{\overline{D}(\alpha)}{(k^2-\alpha)(p^2-k^2)^2} \qquad (3.14)$$

Using Feynmen parameterization

$$\begin{aligned}
\frac{1}{ab} &= \int_0^1 dx\frac{1}{(xa+(1-x)b)^2} \qquad (3.15) \\
Differentiating\ w.r.t.\ a & \\
\frac{1}{a^2b} &= \int_0^1 dx\frac{2x}{(xa+(1-x)b)^3} \qquad (3.16) \\
So, & \\
\frac{1}{(k^2-p^2)^2(k^2-\alpha)} &= \int_0^1 dx\frac{2x}{[k^2-xp^2-(1-x)\alpha]^3}
\end{aligned}$$

Thus equation (3.14) can be expressed as

$$\frac{D(p^2)}{p^2C^2(p^2)+D^2(p^2)}=-8m^2g^2\pi^2\int_{-\infty}^{\infty}d\alpha\int_0^1 dx\int\frac{dk}{(2\pi)^4}\frac{\overline{D}(\alpha)2xk^3}{[k^2-xp^2-(1-x)\alpha]^3} \qquad (3.17)$$

Let us consider the integral

$$
\begin{aligned}
I &= \int_0^\infty dk \frac{k^3}{[k^2 - xp^2 - (1-x)\alpha]^3} \\
& \textit{let us put } k^2 - xp^2 - (1-x)\alpha = z \\
& \Rightarrow k^2 = z + xp^2 + (1-x)\alpha \\
& \Rightarrow kdk = \frac{dz}{2} \\
I &= \frac{1}{2}\int_{-xp^2-(1-x)\alpha}^\infty dz \left[\frac{z + xp^2 + (1-x)\alpha}{z^3}\right] \\
&= \frac{1}{2}\int_{-xp^2-(1-x)\alpha}^\infty dz \left[\frac{1}{z^2} + \frac{xp^2 + (1-x)\alpha}{z^3}\right] \\
&= \frac{1}{2}\left[-\frac{1}{z} - \frac{xp^2 + (1-x)\alpha}{2z^2}\right]_{-xp^2-(1-x)\alpha}^\infty \\
&= \frac{1}{2}\left[\frac{1}{-xp^2 - (1-x)\alpha} + \frac{xp^2 + (1-x)\alpha}{2(-xp^2 - (1-x)\alpha)^2}\right] \\
&= \frac{1}{2}\left[\frac{-xp^2 - (1-x)\alpha}{2(-xp^2 - (1-x)\alpha)^2}\right] \\
I &= \frac{1}{4}\left[\frac{1}{-xp^2 - (1-x)\alpha}\right]
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\frac{D(p^2)}{p^2C^2(p^2) + D^2(p^2)} &= -8m^2g^2\pi^2 \int_{-\infty}^\infty d\alpha \int_0^1 dx \frac{2x\overline{D}(\alpha)}{4(2\pi)^4(-xp^2 - (1-x)\alpha)} \\
&= \frac{16m^2g^2\pi^2}{4(2\pi)^4} \int_{-\infty}^\infty d\alpha \overline{D}(\alpha) \int_0^1 dx \frac{x}{xp^2 + (1-x)\alpha} \qquad (3.18)
\end{aligned}
$$

We carry out the x-integration

$$\begin{aligned}
\frac{D(p^2)}{p^2C^2(p^2)+D^2(p^2)} &= \frac{16m^2g^2}{64\pi^2}\int_{-\infty}^{\infty} d\alpha\overline{D}(\alpha)\frac{1}{p^2-\alpha}\int_0^1 dx\frac{(p^2-\alpha)x+\alpha-\alpha}{(p^2-\alpha)x+\alpha} \\
&= \frac{16m^2g^2}{64\pi^2}\int_{-\infty}^{\infty} d\alpha\overline{D}(\alpha)\frac{1}{p^2-\alpha}\int_0^1 dx\left[1-\frac{\alpha}{(p^2-\alpha)x+\alpha}\right] \\
&= \frac{m^2g^2}{4\pi^2}\int_{-\infty}^{\infty} d\alpha\frac{\overline{D}(\alpha)}{(p^2-\alpha)} - \frac{m^2g^2}{4\pi^2}\int_{-\infty}^{\infty} d\alpha\frac{\overline{D}(\alpha)}{p^2-\alpha}\int_0^1 \frac{\alpha}{(p^2-\alpha)x+\alpha} \\
&= \frac{m^2g^2}{4\pi^2}D(p^2) - \frac{m^2g^2}{4\pi^2}\int_{-\infty}^{\infty} d\alpha\frac{\overline{D}(\alpha)\alpha}{(p^2-\alpha)^2}\left[log(x+\frac{\alpha}{p^2-\alpha})\right]_0^1 \\
&= \frac{m^2g^2}{4\pi^2}D(p^2) - \frac{m^2g^2}{4\pi^2}\int_{-\infty}^{\infty} d\alpha\frac{\overline{D}(\alpha)\alpha}{(p^2-\alpha)^2}log\left[\frac{1+\frac{\alpha}{p^2-\alpha}}{\frac{\alpha}{p^2-\alpha}}\right] \\
&= \frac{m^2g^2}{4\pi^2}D(p^2) - \frac{m^2g^2}{4\pi^2}\int_{-\infty}^{\infty} d\alpha\frac{\overline{D}(\alpha)\alpha}{(p^2-\alpha)^2}log\left(\frac{p^2}{\alpha}\right)
\end{aligned}$$

Let us consider

$$I = \int_{-\infty}^{\infty} d\alpha\frac{\overline{D}(\alpha)\alpha}{(p^2-\alpha)^2}(logp^2 - log\alpha)$$

We assume $\overline{D}(\alpha)$ is a smooth function of α. Then $\alpha = p^2$ is a double pole.($\alpha = 0$ is a singularity which will not contribute).

$$\begin{aligned}
I &= \frac{1}{1!}\frac{d}{d\alpha}\left[\alpha\overline{D}(\alpha)log\left(\frac{p^2}{\alpha}\right)\right]_{\alpha=p^2} \\
&= \left[\overline{D}(\alpha))log\left(\frac{p^2}{\alpha}\right) + \alpha\overline{D}'(\alpha)log\left(\frac{p^2}{\alpha}\right) - \overline{D}(\alpha)\right]_{\alpha=p^2} \\
&= -D(p^2)
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{D(p^2)}{p^2C^2(p^2)+D^2(p^2)} &= \frac{m^2g^2}{2\pi^2}D(p^2) \\
\frac{1}{p^2C^2(p^2)+D^2(p^2)} &= \frac{m^2g^2}{2\pi^2} \qquad (3.19)
\end{aligned}$$

So, from equation (3.5)

$$p^2 + M^2(p^2) = \frac{m^2g^2}{2\pi^2} \qquad (3.20)$$

We examine the effect of introducing an ultra-violet cut-off to the q-integration in (3.2). This results in,

$$\begin{aligned}\frac{D(p^2)}{p^2C^2(p^2)+D^2(p^2)} &= -4m^2g^2\int_0^\Lambda \frac{dkk^3}{(2\pi)^4}\int d\Omega_k \frac{D(k^2)}{(p^2+k^2-2|p||k|cos\theta)^2} \\ &= -4m^2g^2\pi^2\int_0^\Lambda \frac{dkk^3}{(2\pi)^4}\int_{-1}^{1} d(cos\theta)\frac{D(k^2)}{(p^2+k^2-2|p||k|cos\theta)^2} \\ &= -8m^2g^2\pi^2\int_0^\Lambda \frac{dkk^3}{(2\pi)^4}\frac{D(k^2)}{(p^2-k^2)^2} \qquad (3.21)\end{aligned}$$

As before, using $D(k^2) = \int_{-\infty}^{\infty} d\alpha \frac{\overline{D}(\alpha)}{k^2-\alpha}$

$$\frac{D(p^2)}{p^2C^2(p^2)+D^2(p^2)} = \frac{-8m^2g^2}{(16\pi^2)}\int_{-\infty}^{\infty} d\alpha \int_0^1 dx \int_0^\Lambda \frac{dk}{(2\pi)^4}\frac{\overline{D}(\alpha)2xk^3}{[k^2-xp^2-(1-x)\alpha]^3}$$

using Feynman parametrization

$$\frac{1}{(k^2-p^2)^2(k^2-\alpha)} = \int_0^1 dx \frac{2x}{(k^2-xp^2+(1-x)\alpha)^3}$$

So,

$$\frac{D(p^2)}{p^2C^2(p^2)+D^2(p^2)} = -\frac{m^2g^2}{\pi^2}\int_0^\Lambda dk \int_{-\infty}^{\infty} d\alpha \overline{D}(\alpha)k^3 \int_0^1 dx \frac{x}{(k^2-p^2x-(1-x)\alpha)^3}$$

Now let us consider the integration

$$
\begin{aligned}
I &= \int_0^\Lambda dk \frac{k^3}{[K^2 - p^2x - (1-x)\alpha]^3} \\
&\quad \textit{putting } k^2 - p^2x - (1-x)\alpha = z \Rightarrow 2kdk = dz \\
&\quad \textit{when } k = 0, z = -(p^2x + (1-x)\alpha); \\
&\quad \textit{when } k = \Lambda, z = \Lambda^2 - (p^2x + (1-x)\alpha); \\
I &= \frac{1}{2}\int_{-(p^2x+(1-x)\alpha)}^{\Lambda^2-(p^2x+(1-x)\alpha)} dz \frac{z + p^2x + (1-x)\alpha}{z^3} \\
&= \frac{1}{2}\int_{-(p^2x+(1-x)\alpha)}^{\Lambda^2-(p^2x+(1-x)\alpha)} dz \left[\frac{1}{z^2} + \frac{p^2x + (1-x)\alpha}{z^3}\right] \\
&= \frac{1}{2}\left[-\frac{1}{z} - \frac{1}{2}\frac{p^2x + (1-x)\alpha}{z^2}\right]_{-(p^2x+(1-x)\alpha)}^{\Lambda^2-(p^2x+(1-x)\alpha)} \\
&= \frac{1}{2}\left[-\frac{1}{\Lambda^2 - p^2x - (1-x)\alpha} - \frac{1}{p^2x + (1-x)\alpha}\right. \\
&\quad \left. - \frac{p^2x + (1-x)\alpha}{2(\Lambda^2 - p^2x - (1-x)\alpha)^2} + \frac{p^2x + (1-x)\alpha}{2(p^2x + (1-x)\alpha)^2}\right] \\
&= \frac{1}{2}\left[-\frac{1}{2(p^2x + (1-x)\alpha)} - \frac{1}{\Lambda^2 - p^2x - (1-x)\alpha}\left(\frac{2\Lambda^2 - p^2x - (1-x)\alpha}{2(\Lambda^2 - p^2x - (1-x)\alpha)}\right)\right] \\
&= -\frac{1}{4}\left[\frac{1}{(p^2x + (1-x)\alpha)} + \frac{2\Lambda^2 - p^2x - (1-x)\alpha}{(\Lambda^2 - p^2x - (1-x)\alpha)^2}\right]
\end{aligned}
$$

So,

$$
\begin{aligned}
\frac{D(p^2)}{p^2C^2(p^2) + D^2(p^2)} &= \frac{m^2g^2}{4\pi^2}\left[\int_{-\infty}^{\infty} d\alpha\overline{D}(\alpha)\int_0^1 dx \frac{x}{p^2x + (1-x)\alpha}\right. \\
&\quad \left. + \int_{-\infty}^{\infty} d\alpha\overline{D}(\alpha)\int_0^1 dx \frac{x(2\Lambda^2 - p^2x - (1-x)\alpha)}{(\Lambda^2 - p^2x - (1-x)\alpha)^2}\right]
\end{aligned}
$$

First Integral:

$$
\begin{aligned}
\int_0^1 dx \frac{x}{p^2x + (1-x)\alpha} &= \frac{1}{p^2 - \alpha}\int_0^1 dx \frac{(p^2 - \alpha)x + \alpha - \alpha}{(p^2 - \alpha)x + \alpha} \\
&= \frac{1}{p^2 - \alpha}\left[1 - \frac{\alpha}{p^2 - \alpha} log(x + \frac{\alpha}{p^2 - \alpha})\right]_0^1 \\
&= \frac{1}{p^2 - \alpha}\left[1 - \frac{\alpha}{p^2 - \alpha} log(\frac{p^2}{\alpha})\right]
\end{aligned}
$$

Second Integral:

$$\begin{aligned}\int_0^1 dx\frac{x(2\Lambda^2-p^2x-(1-x)\alpha)}{(\Lambda^2-p^2x-(1-x)\alpha)^2} &= \left[\int_0^1 dx\frac{x}{\Lambda^2-p^2x-(1-x)\alpha}\right.\\ &\quad \left.+\int_0^1 dx\frac{x\Lambda^2}{(\Lambda^2-p^2x-(1-x)\alpha)^2}\right]\\ &= I_1+I_2\end{aligned}$$

$$\begin{aligned}I_1 &= \int_0^1 dx\frac{x}{\Lambda^2-p^2x-(1-x)\alpha}\\ &= -\frac{1}{p^2-\alpha}\int_0^1 dx\frac{x(p^2-\alpha)+\alpha-\Lambda^2-(\alpha-\Lambda^2)}{x(p^2-\alpha)+\alpha-\Lambda^2}\\ &= -\frac{1}{p^2-\alpha}\left[1-\frac{\alpha-\Lambda^2}{p^2-\alpha}log\left(\frac{p^2-\Lambda^2}{\alpha-\Lambda^2}\right)\right]\\ & and\\ I_2 &= \int_0^1 dx\frac{x\Lambda^2}{(\Lambda^2-p^2x-(1-x)\alpha)^2}\\ &= \frac{\Lambda^2}{p^2-\alpha}\int_0^1 dx\frac{(p^2-\alpha)x+\alpha-\Lambda^2-(\alpha-\Lambda^2)}{(\Lambda^2-p^2x-(1-x)\alpha)^2}\\ &= \frac{\Lambda^2}{p^2-\alpha}\int_0^1 dx\left[\frac{1}{\Lambda^2-p^2x-(1-x)\alpha}-\frac{(\alpha-\Lambda^2)\Lambda^2}{p^2-\alpha}\frac{1}{(\Lambda^2-p^2x-(1-x)\alpha)^2}\right]\\ &= \frac{\Lambda^2}{p^2-\alpha}\left[\frac{1}{p^2-\alpha}log\left(x+\frac{\alpha-\Lambda^2}{p^2-\alpha}\right)-\frac{(\alpha-\Lambda^2)\Lambda^2}{(p^2-\alpha)^2}\left(-\frac{1}{x+\frac{\alpha-\Lambda^2}{p^2-\alpha}}\right)\right]_0^1\\ &= \frac{\Lambda^2}{p^2-\alpha}\left[\frac{1}{p^2-\alpha}log\frac{p^2-\Lambda^2}{\alpha-\Lambda^2}+\frac{(\alpha-\Lambda^2)\Lambda^2}{(p^2-\alpha)^2}\left(\frac{p^2-\alpha}{p^2-\Lambda^2}-\frac{p^2-\alpha}{\alpha-\Lambda^2}\right)\right]\\ &= \frac{\Lambda^2}{(p^2-\alpha)^2}log\left(\frac{p^2-\Lambda^2}{\alpha-\Lambda^2}\right)-\frac{\Lambda^2}{(p^2-\alpha)(p^2-\Lambda^2)}\end{aligned}$$

Finally, substituting the values of all integrals in equation

$$
\begin{aligned}
\frac{D(p^2)}{p^2C^2(p^2)+D^2(p^2)} &= \frac{m^2g^2}{4\pi^2}\int_{-\infty}^{\infty} d\alpha \frac{\overline{D}(\alpha)}{p^2-\alpha}\left[1-\frac{\alpha}{p^2-\alpha}log\left(\frac{p^2}{\alpha}\right)-1\right. \\
&\left.+\frac{\alpha-\Lambda^2}{p^2-\alpha}log\left(\frac{p^2-\Lambda^2}{\alpha-\Lambda^2}\right)+\frac{\Lambda^2}{p^2-\alpha}log\left(\frac{p^2-\Lambda^2}{\alpha-\Lambda^2}\right)-\frac{\Lambda^2}{p^2-\Lambda^2}\right] \\
&= \frac{m^2g^2}{4\pi^2}\int_{-\infty}^{\infty} d\alpha \frac{\overline{D}(\alpha)}{p^2-\alpha}\left[-\frac{\Lambda^2}{p^2-\Lambda^2}+\frac{\alpha}{p^2-\alpha}log\left(\frac{p^2-\Lambda^2}{\alpha-\Lambda^2}\right)\right. \\
&\left.-\frac{\alpha}{p^2-\alpha}log\left(\frac{p^2}{\alpha}\right)\right] \\
&= \frac{m^2g^2}{4\pi^2}\int_{-\infty}^{\infty} d\alpha \frac{\overline{D}(\alpha)}{p^2-\alpha}\left[-\frac{\Lambda^2}{p^2-\Lambda^2}+\frac{\alpha}{p^2-\alpha}log\left(\frac{\alpha(p^2-\Lambda^2)}{p^2(\alpha-\Lambda^2)}\right)\right] \\
&= -\frac{m^2g^2}{4\pi^2}\left(\frac{\Lambda^2}{p^2-\Lambda^2}\right)D(p^2) \\
&+\frac{m^2g^2}{4\pi^2}\int_{-\infty}^{\infty} d\alpha \frac{\overline{D}(\alpha)}{p^2-\alpha}\left[\frac{\alpha}{p^2-\alpha}log\left(\frac{\alpha(p^2-\Lambda^2)}{p^2(\alpha-\Lambda^2)}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{D(p^2)}{p^2C^2(p^2)+D^2(p^2)} &= \frac{m^2g^2}{2\pi^2}\left(\frac{\Lambda^2}{\Lambda^2-p^2}\right)D(p^2) \\
From\ equation\ (3.5) & \\
p^2+M^2(p^2) &= \frac{m^2g^2}{2\pi^2}\left(\frac{\Lambda^2}{\Lambda^2-p^2}\right) \qquad (3.22)
\end{aligned}
$$

It is clear that when Λ tends to ∞ equation (3.22) would become equation (3.20)

We examine (3.20) first.

$$
\begin{aligned}
p^2+M^2(p^2) &= \frac{m^2g^2}{2\pi^2}=C \\
M(p^2) &= (C-p^2)^{\frac{1}{2}} \qquad (3.23) \\
& and\ so \\
M(0) &= (C)^{\frac{1}{2}}=\frac{m^2g^2}{2\pi^2}\neq 0 \qquad (3.24)
\end{aligned}
$$

As we have restricated to infrared region, the relation is not valid for all momentum region. In the zero momentum limit, mass term $M(p^2)$ is nonzero, which means the global chiral symmetry is broken spontaneously in the infrared region. Since we have used the propagator for gluon in the S-D equation which is corresponds to confining potential that is taken linear potential, it is a manifestation of the believed fact that both infrared phenomena : chiral symmetry breaking and confinement are related. The above relation involves an unknown parameter $C(=\frac{m^2g^2}{2\pi^2})$. In order to estimate a value for C we consider quark condensate. Let us hence use the relation between chiral condensate parameter and mass term $M(p^2)$ [21].

$$
\begin{aligned}
<\bar{q}q> &= -\frac{12}{16\pi^2}\int dp^2 \frac{p^2M(p^2)}{(p^2+M^2(p^2))} && (3.25)\\
& \quad\ let\ p^2 = x \\
&= -\frac{12}{16\pi^2}\int dp^2 \frac{x(C-x)^{1/2}}{C} \\
<\bar{q}q> &= -\frac{12}{16\pi^2 C}\int dp^2 x(C-x)^{1/2} && (3.26)
\end{aligned}
$$

In the case where a cut-off is issued, (3.22) gives

$$
\begin{aligned}
p^2 + M^2(p^2) &= \frac{m^2g^2}{2\pi^2}\left(\frac{\Lambda^2}{\Lambda^2-p^2}\right) \\
&= C\left(\frac{\Lambda^2}{\Lambda^2-p^2}\right) \\
M(p^2) &= \left(C\left(\frac{\Lambda^2}{\Lambda^2-p^2}\right)-p^2\right)^{\frac{1}{2}} && (3.27)\\
\textit{and we have} & \\
<\bar{q}q> &= -\frac{N_c}{2\pi^2 C\Lambda^2}\int x dx\{(\Lambda^2-x)(C\Lambda^2-\Lambda^2x+x^2)\}^{1/2} && (3.28)\\
& \quad\ where\ p^2 = x
\end{aligned}
$$

Integrating equation (3.26)

$$\begin{aligned} <\bar{q}q> &= -\frac{12}{16\pi^2 C}\int_0^C dx x\sqrt{C-x} \\ & putting C-x=z \Rightarrow dx=-dz \\ &= -\frac{12}{16\pi^2 C}\int_C^0 dz(C-z)\sqrt{z} \\ &= -\frac{12}{16\pi^2 C}\left[\frac{2}{3}Cz^{\frac{3}{2}}-\frac{2}{5}z^{\frac{5}{2}}\right]_0^C \\ &= -\frac{12}{16\pi^2 C}\left[\frac{2c}{3}C^{\frac{3}{2}}-\frac{2}{5}C^{\frac{5}{2}}\right] \\ &= -\frac{12}{16\pi^2}\sqrt{C}\left[\frac{2C}{3}C^{-}\frac{2}{5}C\right] \\ &= -\frac{C^{\frac{3}{2}}}{5\pi^2} \end{aligned}$$

Using the lattice estimate [22].

$<\bar{u}u>=<\bar{d}d>=-(250MeV)^3$

we have

$$\begin{aligned} <\bar{q}q> &= -\frac{C^{\frac{3}{2}}}{5\pi^2} \\ M(0) &= (C)^{1/2}GeV = 0.917GeV \\ and\ so & \\ M(p^2) &= \{0.84-p^2\}^{\frac{1}{2}} \end{aligned} \tag{3.29}$$

This is plotted in Figure (3.1). The value $M(0) \approx 0.917GeV$ is large when compared with lattice estimation. So we use (3.28).

let us choose

$$<\bar{q}q> = -(220MeV)^3 = -0.0107GeV^3\ and\ N_c = 3$$

and equation is

$$\frac{1}{C\Lambda^2}\int xdx\left\{(\Lambda^2-x)(C\Lambda^2-\Lambda^2 x+x^2)\right\}^{\frac{1}{2}} = 0.07040 \tag{3.30}$$

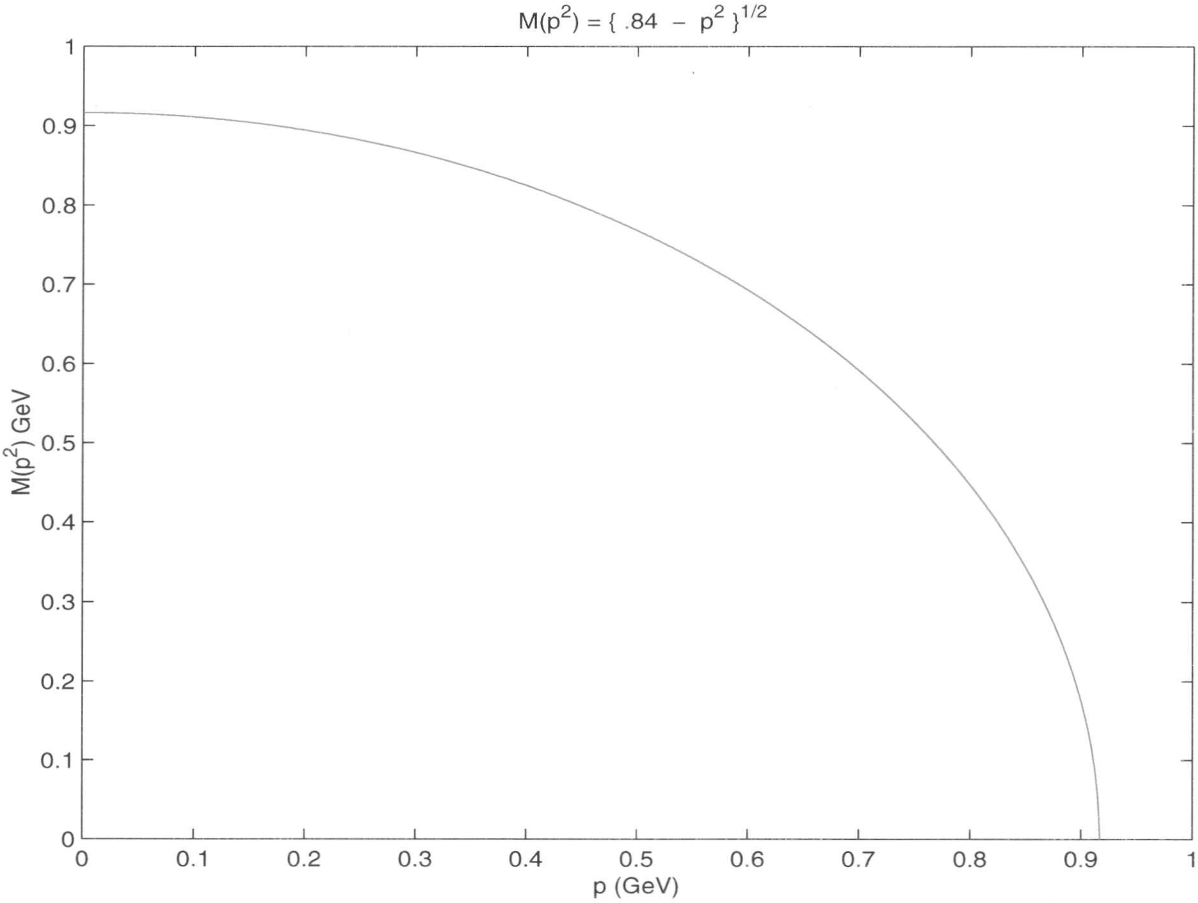

Figure 3.1: **Variation of Mass function with Momentum scale**

Table 3.1: **Data from Numerical Calculation**

$\Lambda(GeV)$	$C(GeV^2)$	Value of Integration
3	0.6	0.131571
3	0.65	0.148908
3	0.63	0.14188
3	0.62	0.138412
3	0.628	0.141184
3	0.625	0.140836
3	0.627	0.140836
3	0.4	0.0704602
3	0.399	0.07019
3	0.3995	0.070325
3	0.3998	0.0704061
3	0.39979	0.0704034***

After performing numerical integration for different values of C and Λ for equation (3.30) we the following data:

The choice of $C = 0.39979 GeV^2, \Lambda = 3GeV.$ (as indicated by three stars in the above data table) gives the correct lattice value of $< \overline{q}q >$. Then

$$\begin{aligned} M(0) &= \sqrt{C} = (0.39979)^{\frac{1}{2}} = 0.63228949 GeV \\ M(p^2) &= \left\{ C\left(\frac{\Lambda^2}{\Lambda^2 - p^2}\right) - P^2 \right\}^{\frac{1}{2}} \end{aligned}$$

This is plotted in Figure (3.2). The value M(0)=0.63 GeV is in reasonable agreement with lattice calculation [23]. m is the dual gluon mass. In order to estimate its value, we need to know g^2 at low momentum (non-perturbative). These are not known. However using $g^2 = 8.05$ [20], we find the dual gluon mass $\sim$ 980 MeV which is in due agreement with 848 MeV prediction by [20].

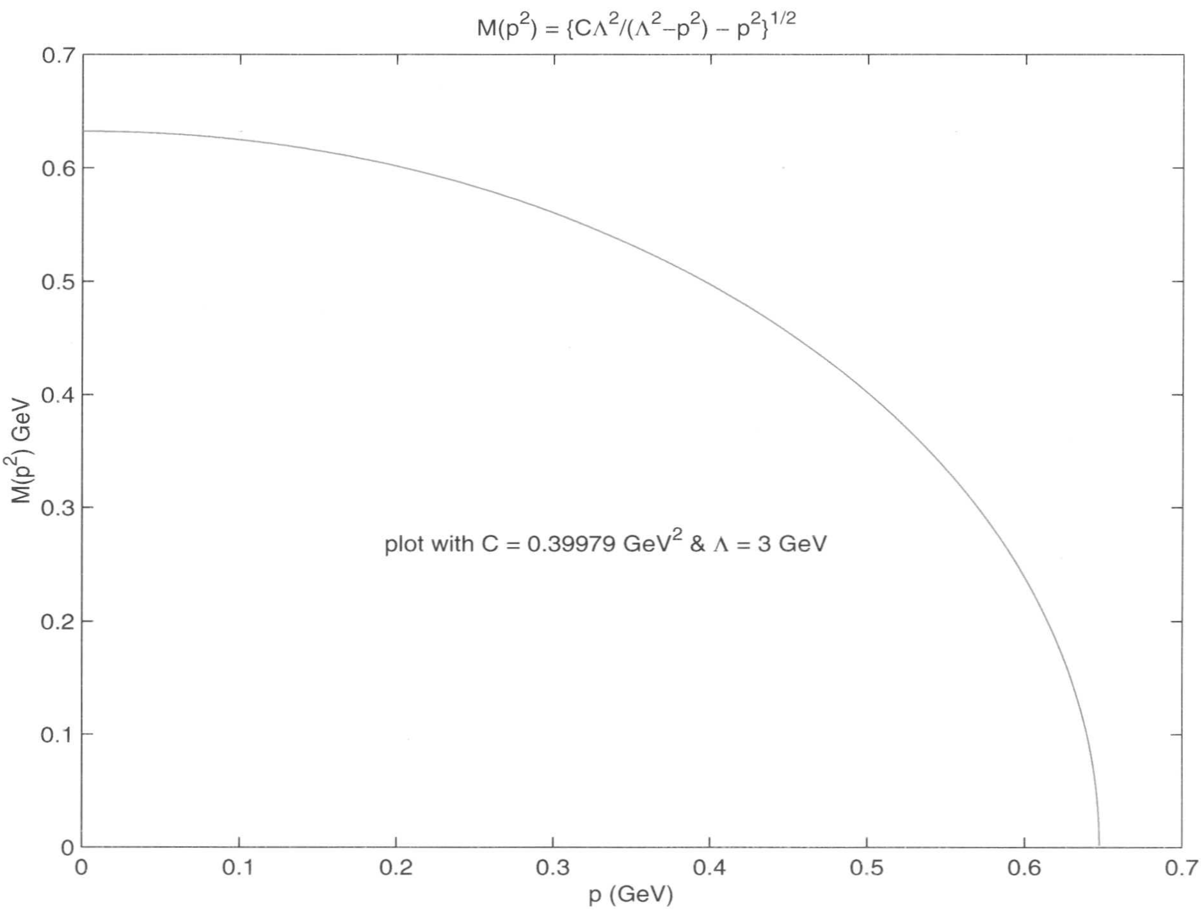

Figure 3.2: **Variation of Mass function with Momentum scale**

Chapter 4

CONCLUSION

In this thesis, we have considered the Yang-Mills theory with unbroken colour SU(3) symmetry, namely QCD, in the non-perturbative infrared regime. After a brief introduction, the formal aspects of a quantum Yang-Mills theory is analysed with the specific motivation of reconciling the linear confining potential with relativistic quantum field theory. This is possible mainly due to the unobservability of quarks and gluons. Using a criterian for unobservability, namely, the nonvanishing of the commutator of the BRST charge and the field in physical space, the difficulty of having a linearly rising potential is avoided. This has been utilised to exhibit how coloured quarks do not exit as asymptotic states while colour singlet hadrons exit as asymptotic states. Such a linear potential arises from $\frac{1}{q^4}$ propagator for gluons in the infrared region. This propagator follows from a dual Meissner effect decription of QCD with infrared region. Using static configuration, we demonstrate how the linear potential comes about. This involves one parameter, the dual gluon mass, which has been determined subsequently.

We have then considered the phenomenon of spontaneous chiral symmetry using Schwinger-Dyson equation. The input here is the $\frac{1}{q^4}$ propagator for the gluons and the Gribov's conjecture of QCD coupling being a constant at the near infrared region. Usually Schwinger-Dyson equation is very difficult to solve exactly. So we use ladder approximation. A solution of the Schwinger-Dyson equation is taken as parametrised by $D(q^2)$ and the Chauchy-Riemann representation for $D(q^2)$ is used. Then Feynman's parametrization of the integral is employed. A very useful relation

$p^2 + M^2(p^2) = \frac{m^2 g^2}{2\pi^2}$

is derived with in the approximation, where $M(p^2)$ is the momentum dependent quark current mass, m is the dual gluon mass and g is the QCD coupling in the infrared region. An improvement of this relation is obtained by introducing on cut-off Λ as

$$p^2 + M^2(p^2) = \frac{m^2 g^2}{2\pi^2} \left(\frac{\Lambda^2}{\Lambda^2 - p^2} \right)$$

The parameter above are determined by evaluating $< \bar{q}q >$ condensate as $\sim -(250MeV)^3$. Using the relation Λ (or $\Lambda \to \infty$), the quantum correction to the quark mass is found to be M(0)=0.9 GeV. Which is large compared to the lattice QCD estimate of ~0.6 GeV. So, we have used the second relation and find for Λ=3 Gev, M(0) is found to be 0.63 GeV. The variation of $M(p^2)$ with p is plotted. The result that $M(0) \neq 0$, shows spontaneous chiral symmetry breaking. Since $\frac{1}{q^4}$ behaviour of the propagator for gluon results in confinement via linear potential, we show confinement implies chiral symmetry breaking.

As a prediction, we have found the dual gluon mass M=980 MeV which is in good agreement with the earliar prediction of 848 MeV by Baker and his collaborators.

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