# SYLLABI FOR MATHEMATICS COURSES

## Desired outcomes after completion of courses

The student should be able to

- define all concepts alluded to in the syllabi,
- give appropriate examples,
- state and prove all the theorems mentioned, and,
- apply these concepts and theorems in problem solving.

## 1. Algebra I

**Group theory**

- Group actions: Orbits, stabilisers, transitivity
- Lagrange, Cauchy, Sylow theorems in the language of group actions
- Direct and semidirect products
- symmetric and alternating groups

**Matrices, determinants and linear maps**

- Linear maps and matrices, dual = transpose
- determinants
- Equality of row, column and determinantal rank over a commutative ring

**Representations of a single endomorphism**

- Minimal and characteristic polynomials, eigenvalues and eigenvectors
- Rational and Jordan canonical forms
- S-N decomposition

**Bilinear forms and spectral theorems**

- Preliminaries and quadratic maps
- Symmetric forms, orthogonal basis, Sylvester's theorem
- Hermitian forms, polarization, Cauchy-Schwarz inequality
- Spectral theorems, polar decomposition

**Basic category theory**

- Categories and functors
- Universal properties
- Sums, products and limits

**Rings and modules over a PID**

- Finitely generated abelian groups
- PID $\Rightarrow$ UFD, $R$ UFD $\Rightarrow$ $R[X]$ UFD, Gauss' lemma
- Irreducibility criteria
- Modules over a PID

**Tensor products**

- Of vector spaces, modules over a ring, basic properties
- connection with Hom, of algebras
- tensor, symmetric and exterior algebras and connection with the determinant

## 2. Algebra II

**Group theory**

- simple, solvable and nilpotent groups
- Jordan-Holder theorem

**Galois theory**

- Finite extensions, algebraic extensions, algebraic closure
- Splitting fields and normal extensions
- separable extensions
- Finite fields
- Inseparable extensions
- Galois extensions
- Examples and applications
- Cyclotomic fields
- Independence of characters, norm and trace
- Cyclic extensions
- Solvable and radical extensions

Teacher's choice from among the following suggested topics or others.

**Semisimplicity**

- Schur's lemma and semisimple modules
- Jacobson density theorem, DCT
- Structure of semisimple rings
- Structure of simple rings

**Representations of finite groups**

- Maschke's theorem
- Characters
- Class functions
- Orthogonality relations

**Commutative algebra and Dedekind domains**

- Prime, maximal ideals, Zariski topology, CRT
- Localization and its properties
- Integral extensions
- Dedekind domains - characterizations
- Unique factorisation - failure and restoration

## 3. Analysis I

**Measure Theory**

- Measurable spaces, Caratheodory's theorem and construction of measures, Lebesgue measure, Riesz representation theorem for compact metric spaces

- Measurable mappings, various convergence concepts like almost sure, convergence in measure.
- Integration, MCT, DCT.
- Product measures, Fubini's theorem.
- Radon-Nikodym theorem, Lebesgue decomposition theorem.
- $L^p$ spaces: Basic theory, Holder's inequality, Minkowsky inequality, completeness, their duality.
- (*) Analysis on $\mathbb{R}^n$; convolutions; approximate identity; approximation theorems; Fourier transform; Fourier inversion formula; Plancherel theorem.

**Note:** Topics marked with asterisk are optional.

## 4. Analysis II

**Elementary functional analysis**

- Topological vector spaces; Banach spaces; Hilbert spaces.
- bounded linear transformation;linear functionals and dual spaces.
- Hahn Banach theorem and it's geometric meaning.
- Category theoem and it's applications like open mapping theorem, uniform boundedness principle, closed graph theorem.
- Weak and Weak-* topologies, Banach-Alaoglu's theorem.

Teacher's choice from among the following suggested topics or others.

**Distribution Theory**

- The spaces $D(\Omega), E(\Omega)$, for $\Omega$ open in $I\!R^n$.
- $S(\mathbb{R}^n)$ and their duals, convolution, Fourier transform.
- Paley-Wiener theorems; fundamental solutions of constant coefficient partial differential operators.

**Banach Algebras and Spectral Theory**

- Banach algebras, spectrum of a Banach algebra element, Holomorphic functional calculus, Gelfand theory of commutative Banach algebras.
- Hilbert space operators, $C^*$-algebras of operators, commutative $C^*$-algebras.
- Spectral theorem for bounded self-adjoint and normal operators. formulation).
- Spectral theorem for compact operators,(*) application to Peter-Weyl theorem.

**Note:** Topics marked with asterisk are optional.

## 5. Topology I

**Basic notions**

- Topological spaces, product topology, connectedness, path connectedness and related notions
- Compactness, Lebesgue number lemma
- Uniform continuity
- Tychonoff's theorem

**Quotient topology**

- Construction of surfaces as quotient spaces of polygons
- Adjunction spaces, cell attachments
- Group actions and orbit spaces

**One of the following two topics**

- Separation axioms, Urysohn lemma, metrizability, paracompactness, partitions of unity
- Topologies on function spaces, Baire category theorem

**Fundamental group**

- Covering spaces and fundamental groups - Examples
- Free groups, free products of groups
- Seifert-van Kampen theorem -examples and applications

## 6. Topology II

Instructors choice among the following topics. It is suggested that one topic from the first two and one from the last two be covered. Depending on the background of the students, time permitting more topics could be covered.

**Simplicial topology and homology**

- Simplicial complexes - basic notions, examples, subdivision
- Simplicial approximation theorem - applications
- Simplicial homology, review of homological algebra, (co)homology of a simplicial complex
- Eilenberg-MacLane axioms, examples and applications.

**Singular homology**

- Basic notions of singular homology - examples
- homotopy invariance of homology groups
- $H_1$ as abelianization of $\pi_1$
- Subdivision, Mayer-Vietoris - applications, excision
- Attaching cells and cellular homology, calculations for $CP^n$, $RP^n$
- Basic notions of Cohomology, Universal coefficients Theorem
- Eilenberg-Steenrod axioms, (Statement of) Kunneth formula.

**Differential topology**

- Differentiable manifolds - Basic notions
- Tangent bundle, vector fields
- Differential forms, de Rham cohomology
- Integration on manifolds. Stokes theorem
- statement of Poincaré duality - examples.

**Riemannian geometry**

- Basic notions - differentiable manifolds, tangent vectors, vector fields
- Lie derivative, Riemannian metric, geodesics, exponential map
- Levi-Civita connection, curvature tensor, sectional curvature
- Geometry of the plane, Riemann sphere, and the Poincaré upper half space
- Gauss-Bonnet theorem for surfaces (proof may be omitted)

## 7. Complex Analysis

- Analytic function, Cauchy-Riemann equations, power series, exponential and logarithmic function.
- Cauchy theorem on a disc, Integral formula, power series and Laurant series expansion. Product development, Weierstrass theorem, Homotopy version of Cauchy's theorem, Liouvilles theorem, residue theorem, Argument principle.
- Maximum modulus principle, Schwarz lemma, Phragmen-Lindelof method.
- Conformal mapping, Mobius transformation, Automorphism of disc and upper half plane. Riemann mapping theorem.
- Harmonic functions, Dirichlet problem, Mean value property
- Analytic continuation, Monodromy theorem
- (optional) Introduction to Hyperbolic geometry
- (optional) Elliptic functions, Gamma and Zeta functions