COURSEWORK AND SYLLABI FOR THE PH.D. AND INTEGRATED PH.D. PROGRAMS IN MATHEMATICS AT IMSC

All courses in mathematics carry 8 credits except for the seminar course which carries 4 credits and the research methodology course which is a pass/fail course with no credits earned.

The Ph.D. program requires a total of 9 courses including the seminar course and the research methodology course (for a total of 60 credits).

The I.Ph.D. program requires 13 courses including the seminar course and the research methodology course along with a 28 credit master's thesis (for a total of 120 credits).

The courses are chosen from among the courses listed below along with other courses offered from time to time either at IMSc or at other institutes with which HBNI has an MoU such as CMI. Topics courses in mathematics may be repeated for credit and will be shown on the transcript with suffixes such as I, II, III etc.

1. Algebra I - 8 credits

Group theory

- Group actions: Orbits, stabilisers, transitivity
- Lagrange, Cauchy, Sylow theorems in the language of group actions
- Direct and semidirect products
- symmetric and alternating groups

Matrices, determinants and linear maps

- Linear maps and matrices, dual = transpose
- determinants
- Equality of row, column and determinantal rank over a commutative ring

Representations of a single endomorphism

- Minimal and characteristic polynomials, eigenvalues and eigenvectors
- Rational and Jordan canonical forms
- S-N decomposition

Bilinear forms and spectral theorems

- Preliminaries and quadratic maps
- Symmetric forms, orthogonal basis, Sylvester's theorem
- Hermitian forms, polarization, Cauchy-Schwarz inequality
- Spectral theorems, polar decomposition

Basic category theory

- Categories and functors
- Universal properties
- Sums, products and limits

Rings and modules over a PID

- Finitely generated abelian groups
- PID $\Rightarrow$ UFD, R UFD $\Rightarrow$ $R[X]$ UFD, Gauss' lemma

- Irreducibility criteria
- Modules over a PID

Tensor products

- Of vector spaces, modules over a ring, basic properties
- connection with Hom, of algebras
- tensor, symmetric and exterior algebras and connection with the determinant

2. Algebra II - 8 credits

Group theory

- simple, solvable and nilpotent groups
- Jordan-Holder theorem

Galois theory

- Finite extensions, algebraic extensions, algebraic closure
- Splitting fields and normal extensions
- separable extensions
- Finite fields
- Inseparable extensions
- Galois extensions
- Examples and applications
- Cyclotomic fields
- Independence of characters, norm and trace
- Cyclic extensions
- Solvable and radical extensions

Instructor's choice from among the following suggested topics or others.

Semisimplicity

- Schur's lemma and semisimple modules
- Jacobson density theorem, DCT
- Structure of semisimple rings
- Structure of simple rings

Representations of finite groups

- Maschke's theorem
- Characters
- Class functions
- Orthogonality relations

Commutative algebra and Dedekind domains

- Prime, maximal ideals, Zariski topology, CRT
- Localization and its properties
- Integral extensions
- Dedekind domains - characterizations
- Unique factorisation - failure and restoration

3. Analysis I - 8 credits

Measure Theory

- Measurable spaces, Caratheodory's theorem and construction of measures, Lebesgue measure, Riesz representation theorem for compact metric spaces
- Measurable mappings, various convergence concepts like almost sure, convergence in measure.
- Integration, MCT, DCT.
- Product measures, Fubini's theorem.
- Radon-Nikodym theorem, Lebesgue decomposition theorem.
- L^p spaces: Basic theory, Holder's inequality, Minkowsky inequality, completeness, their duality.
- (*) Analysis on $\mathbb{R}^n$; convolutions; approximate identity; approximation theorems; Fourier transform; Fourier inversion formula; Plancherel theorem.

Note: Topics marked with asterisk are optional.

4. Analysis II - 8 credits

Elementary functional analysis

- Topological vector spaces; Banach spaces; Hilbert spaces.
- bounded linear transformation;linear functionals and dual spaces.
- Hahn Banach theorem and it's geometric meaning.
- Category theoem and it's applications like open mapping theorem, uniform boundedness principle, closed graph theorem.
- Weak and Weak-* topologies, Banach-Alaoglu's theorem.

Instructor's choice from among the following suggested topics or others.

Distribution Theory

- The spaces $D(\Omega), E(\Omega)$, for Ω open in $I\!R^n$.
- $S(\mathbb{R}^n)$ and their duals, convolution, Fourier transform.
- Paley-Wiener theorems; fundamental solutions of constant coefficient partial differential operators.

Banach Algebras and Spectral Theory

- Banach algebras, spectrum of a Banach algebra element, Holomorphic functional calculus, Gelfand theory of commutative Banach algebras.
- Hilbert space operators, C^*-algebras of operators, commutative C^*-algebras.
- Spectral theorem for bounded self-adjoint and normal operators. formulation).
- Spectral theorem for compact operators,(*) application to Peter-Weyl theorem.

Note: Topics marked with asterisk are optional.

5. Topology I - 8 credits

Point-set topology

- Quotient topology including the construction of standard topological spaces such as surfaces and real and complex projective spaces as quotient spaces
- The notion of attachment of a cell to a topological space
- Group actions and orbit spaces
- Topologies on function spaces
- Baire category theorem
- Arzelà-Ascoli theorem

Fundamental groups and covering spaces

- Fundamental groups, covering spaces and their relationship
- Free groups, free products of groups
- Seifert-van Kampen theorem - examples and applications

Introduction to homology

- Definition of homology groups
- Homotopy invariance of homology groups
- The first homology group as the abelianization of the fundamental group
- Review of homological algebra necessary to introduce the Mayer-Vietoris sequence
- Mayer-Vietoris sequence and its applications in computing homology groups of surfaces, complex projective spaces, real projective spaces etc.

Applications of fundamental groups and homology groups

Instructors choice among the following topics or other topics at this level.

- Jordan curve theorem
- Winding number of a closed curve
- Brouwer's fixed point theorem
- Fundamental theorem of algebra
- Nielsen-Schreier theorem

6. Topology II - 8 credits

Instructors choice among the following topics. It is suggested that one topic from the first two and basic notions of differential topology be covered in addition to some of the advanced topics.

Homology theory

- Quick review of homology theory
- Relative homology and the associated long exact sequence
- Excision theorem and its applications
- Characterisation of homology theory by the Eilenberg-Steenrod axioms
- Homology with coefficients

Cohomology theory and introduction to homotopy groups

- Basic notions of cohomology
- Universal coefficient theorem
- Künneth formula
- Cup product and the cohomology ring, Borsuk-Ulam theorem

Basic notions of differential topology

- Differentiable manifolds, tangent bundle, vector fields, flows
- Differential forms and de Rham cohomology
- Integration on manifolds
- Stokes theorem
- Poincaré duality using differential forms.

Advanced topics

- Higher homotopy groups and the Hurewicz theorem
- H-spaces, suspensions, fibre bundles
- Cap product and various forms of duality with integral coefficients
- Bott periodicity theorem
- Topics in differential geometry such as:
  - Smooth vector bundles
  - Notions of connection, curvature and parallel transport
  - Definition of Riemannian manifold
  - Gauss-Bonnet formula
  - Notion of geodesic and Hopf-Rinow theorem
- Obstruction theory and introduction to characteristic classes:
- Topics in Morse theory such as:
  - Definition and genericity of Morse functions
  - Lemma of Morse
  - Cell structure associated to a Morse function and Morse homology
  - Morse-Smale-Witten complex

7. Complex Analysis - 8 credits

- Analytic function, Cauchy-Riemann equations, power series, exponential and logarithmic function
- Cauchy theorem on a disc, Integral formula, power series and Laurent series expansion Product development, Weierstrass theorem, Homotopy version of Cauchy's theorem, Liouville's theorem, residue theorem, Argument principle
- Maximum modulus principle, Schwarz lemma, Phragmen-Lindelof method
- Conformal mapping, Mobius transformation, Automorphisms of the disc and upper half plane, Riemann mapping theorem
- Harmonic functions, Dirichlet problem, Mean value property
- Analytic continuation, Monodromy theorem
- (optional) Introduction to Hyperbolic geometry
- (optional) Elliptic functions, Gamma and Zeta functions

8. Credit seminar - 4 credits

The topic of the seminar will be chosen by the student in consultation with an assigned faculty member. The student will read recent research papers as assigned by the faculty member, and present the results in a formal seminar.

9. Research Methodology - Pass/Fail

An introduction to the methods and techniques of academic research through a project and presentations - both oral and written.

The following courses are advanced level research courses whose content will be decided by the instructor based on current research and the requirements of the individual students.

10. Topics in analytic number theory - 8 credits

Course content varies according to instructor's choice.

11. Topics in algebraic number theory - 8 credits

Course content varies according to instructor's choice.

12. Topics in commutative algebra - 8 credits

Course content varies according to instructor's choice.

13. Topics in modular forms - 8 credits

Course content varies according to instructor's choice.

14. Topics in elliptic curves - 8 credits

Course content varies according to instructor's choice.

15. Topics in algebraic curves - 8 credits

Course content varies according to instructor's choice.

16. Topics in diophantine geometry - 8 credits

Course content varies according to instructor's choice.

17. Topics in transcendental number theory - 8 credits

Course content varies according to instructor's choice.

18. Topics in algebraic groups - 8 credits

Course content varies according to instructor's choice.

19. Topics in infinite dimensional Lie algebras - 8 credits

Course content varies according to instructor's choice.

20. Topics in functional analysis - 8 credits

Course content varies according to instructor's choice.

21. Topics in non-commutative geometry - 8 credits

Course content varies according to instructor's choice.

22. Topics in Lie groups - 8 credits

Course content varies according to instructor's choice.

23. Topics in algebraic geometry - 8 credits

Course content varies according to instructor's choice.

24. Topics in differential geometry - 8 credits

Course content varies according to instructor's choice.

25. Topics in partial differential equations - 8 credits

Course content varies according to instructor's choice.

26. Topics in mathematical physics - 8 credits

Course content varies according to instructor's choice.

27. Topics in algebra - 8 credits

Course content varies according to instructor's choice.

28. Topics in operator algebras - 8 credits

Course content varies according to instructor's choice.

29. Topics in representation theory - 8 credits

Course content varies according to instructor's choice.

30. Topics in algebraic combinatorics - 8 credits

Course content varies according to instructor's choice.

31. Topics in topology - 8 credits

Course content varies according to instructor's choice.

32. Topics in symplectic geometry - 8 credits

Course content varies according to instructor's choice.

33. Programming for mathematicians - 8 credits

Course content varies according to instructor's choice.