

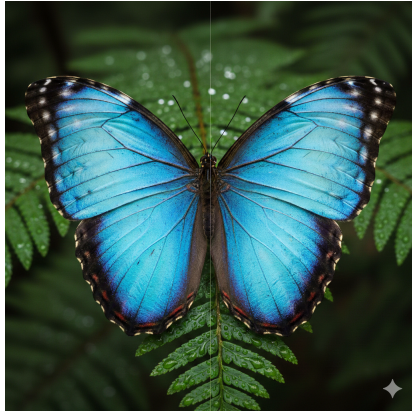
# Symmetry

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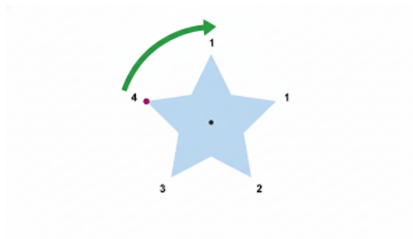
What is symmetry?



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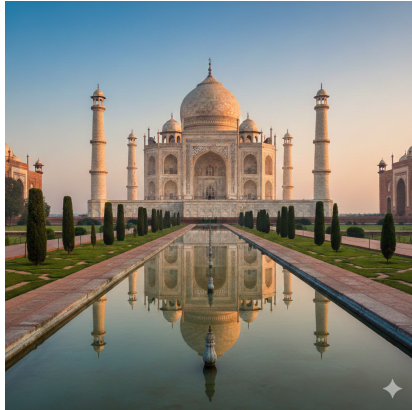
# What is symmetry?



# What is Symmetry?



# Symmetry and architecture



# Rigid motions

Transformation of a plane or space that preserves the distance between every pair of points

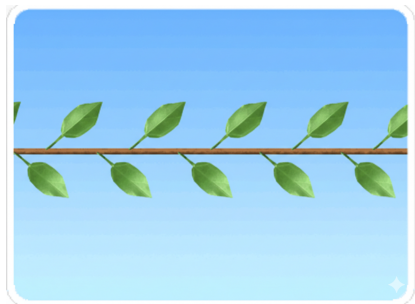
# Rigid motions

## **Theorem:**

Any rigid motion of the plane is one among the following:

1. Rotation
2. Reflection
3. Translation
4. Glide reflection

# Glide Reflection



# Definition

A **symmetry** of an object is an operation that leaves it invariant.

# Abstract symmetry

## Palindromic Polynomials

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where,  $a_{n-i} = a_i$  for all  $0 \leq i \leq n$ .

**Theorem:** If  $\alpha$  is a root, then  $1/\alpha$  is also a root.

## Abstract symmetry

$$\binom{n}{0}x^ny^0 + \binom{n}{1}x^{n-1}y^1 + \dots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}x^0y^n$$

# Abstract symmetry

## Binomial Formula

$$(x+y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y^1 + \dots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}x^0y^n$$

is symmetric in the variables  $x$  and  $y$ .

**Theorem:**

$$\binom{n}{k} = \binom{n}{n-k}$$

# Fractals

- Self-similar structures

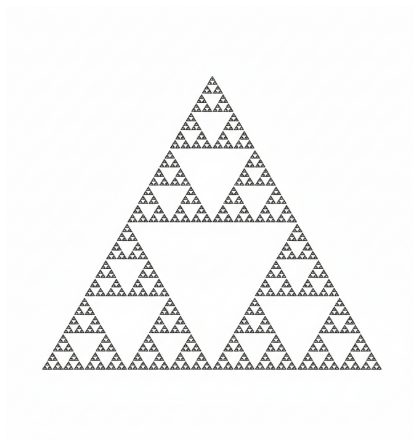


Figure 1: Sierpinski Triangle

# Laws of Physics

**Noether's theorem** For every symmetry in nature, there is a conservation law.

i.e., if the laws of physics don't change when you perform a certain "operation" (a symmetry), there is a physical quantity that will never change (a conservation law).

# Law of conservation of momentum

If you conduct an experiment in one spot and then slide the entire setup to a different location, the results remain the same. This is spatial translation symmetry.

Because the system doesn't care where it is, the momentum of the system must remain constant unless acted upon by an outside force.

# Law of conservation of energy

If the laws of physics today are the same as they were yesterday and will be tomorrow, the system has time translation symmetry.

If physics changed based on the time of day, you could theoretically "create" energy by running a process when it's "cheaper" and harvesting it when it's "stronger." Since physics is constant, energy is conserved.