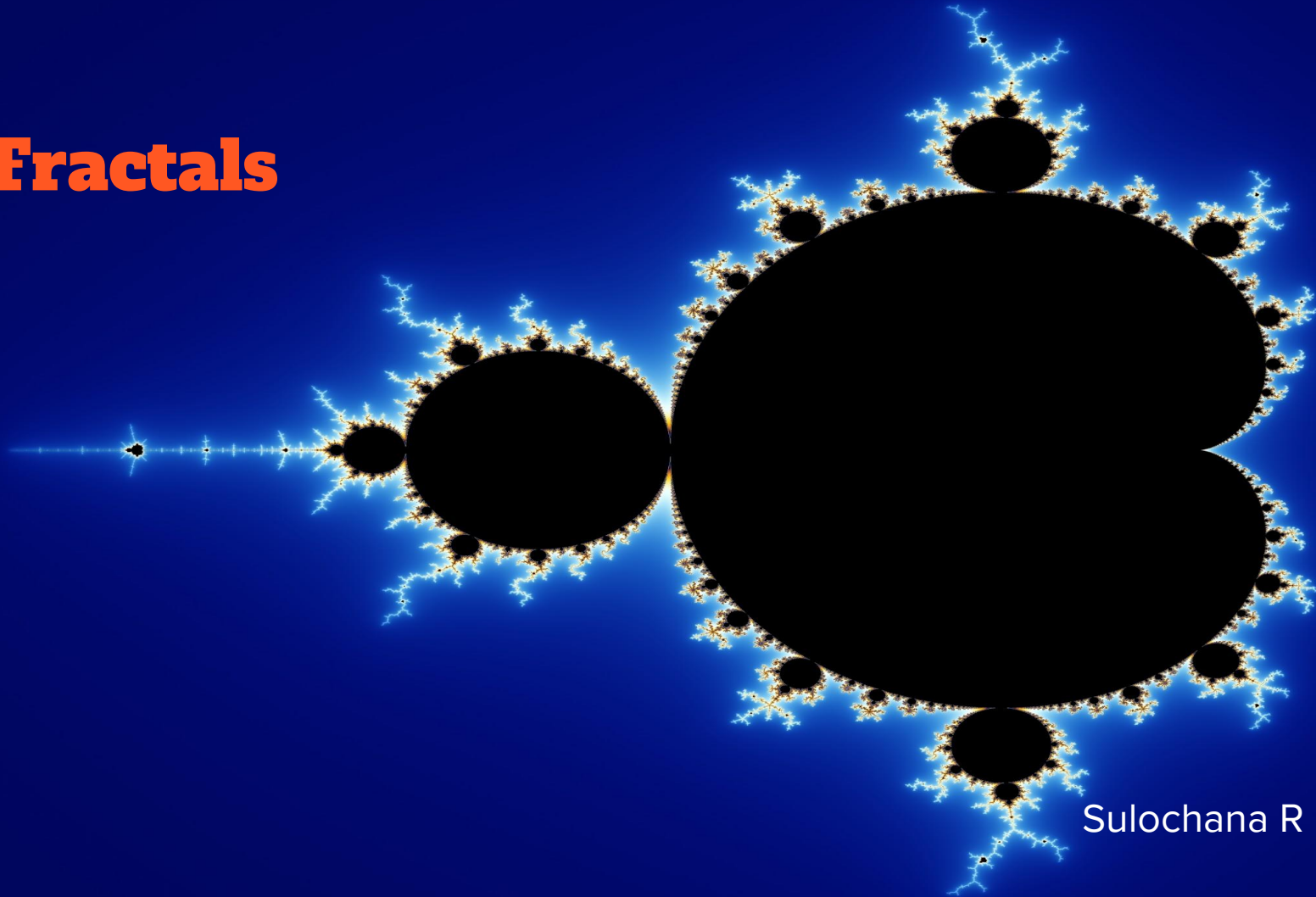


Fractals



Sulochana R , HBCSE

An Example - Cauliflower

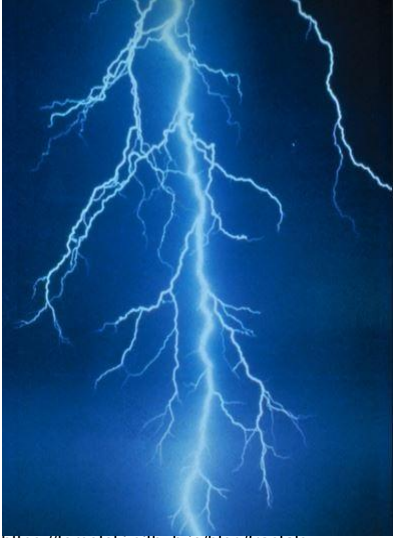


<http://zitogiuseppe.com/images/cavolo2.jpg>

What are fractals?

- Pattern repeats (Self-replicating)
- Smaller portions similar to bigger portion
- Self-similar objects - smaller parts of the object are exact scaled down copies or replicas of the whole object.
- The term "fractal" was coined by the mathematician Benoît Mandelbrot in 1975.

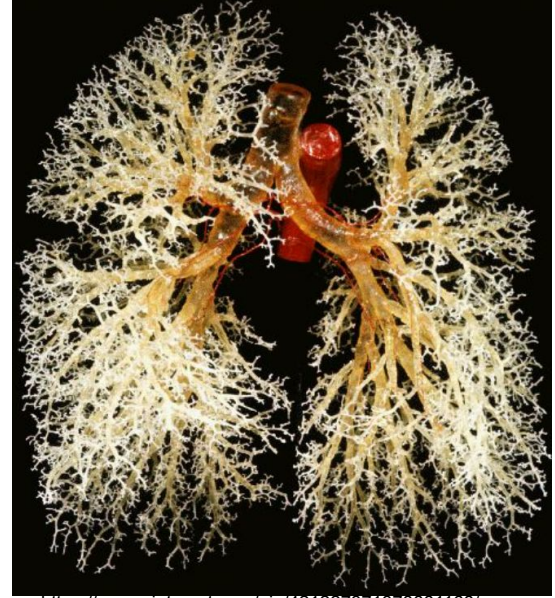
Some examples



<https://temetski.github.io/blog/fractals>

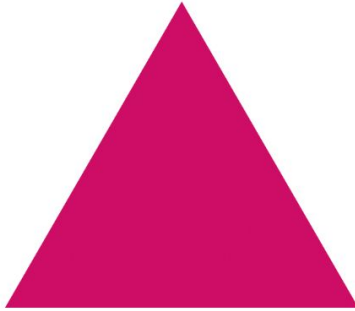


<http://www.5280math.com/nw-tree-fractals>



<https://www.pinterest.com/pin/421227371378881168/>

Sierpinski Triangle



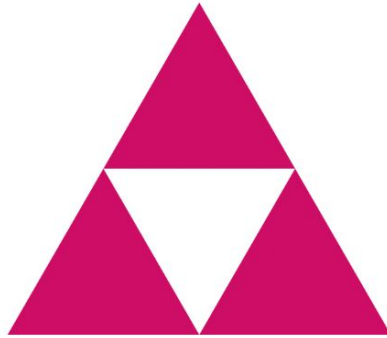
STAGE 0

Equilateral Triangle

Now, mark the mid-points of all sides and join them

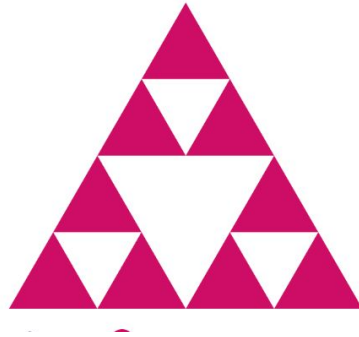
What do you get?

Sierpinski Triangle



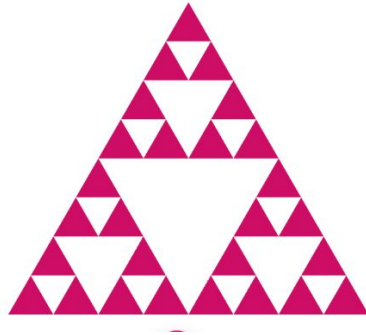
STAGE 1

Sierpinski Triangle



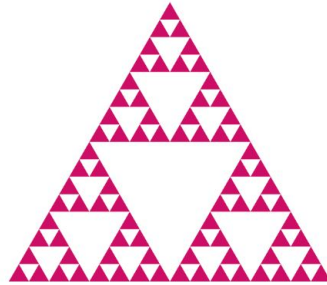
STAGE 2

Sierpinski Triangle



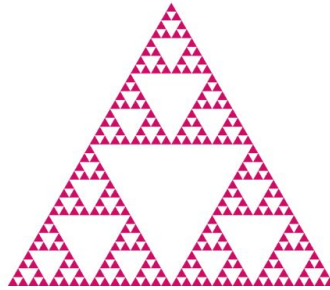
STAGE 3

Sierpinski Triangle



STAGE 4

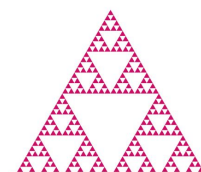
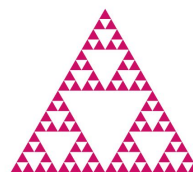
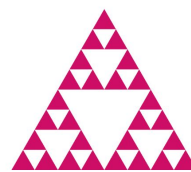
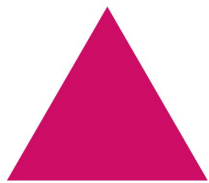
Sierpinski Triangle



STAGE 5

Investigating our construction

STAGES



How many
triangles?

1

...

...

...

...

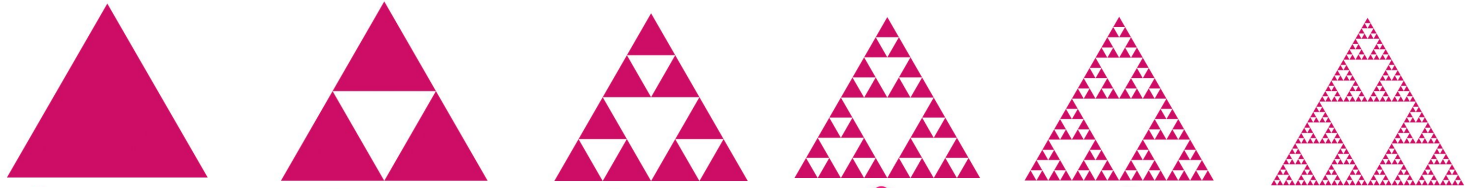
...

... n

Is there a pattern to this?

Copies of Stages

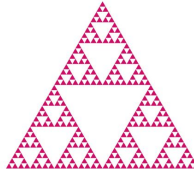
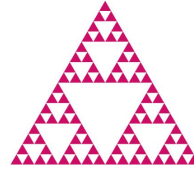
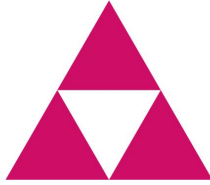
STAGES



How many copies of Stage 0 is in Stages 1, 2, 3, 4, 5?

Copies of Stages

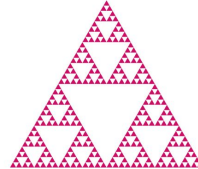
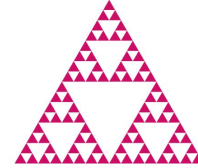
STAGE 1



How many copies of Stage 1 is in Stages 2, 3, 4, 5?

Copies of Stages

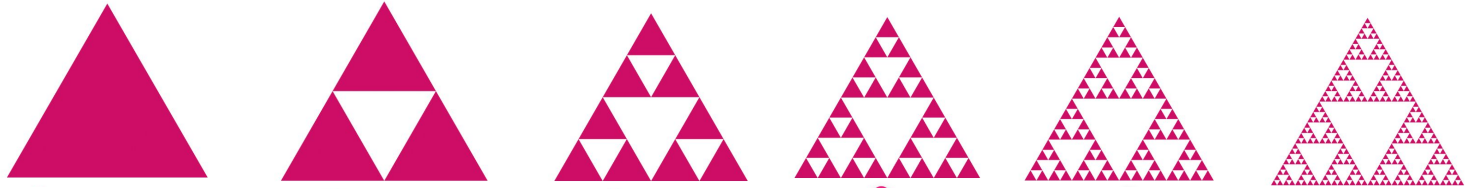
STAGE 2



How many copies of Stage 2 is in Stages 3, 4, 5?

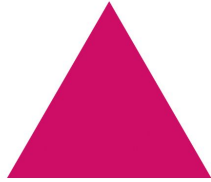
Copies of Stages

STAGES



In general how many copies of the previous stages will you find in stage n ?

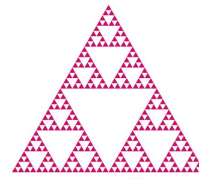
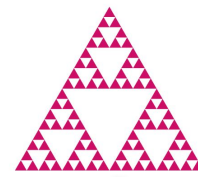
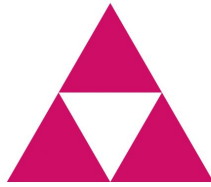
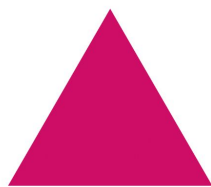
Area



Let the area of the shaded triangle at **stage 0** be **A** sq. unit. What will be the *area of shaded triangles* in other stages?

Area

STAGES



Area, sq. units

A

...

...

...

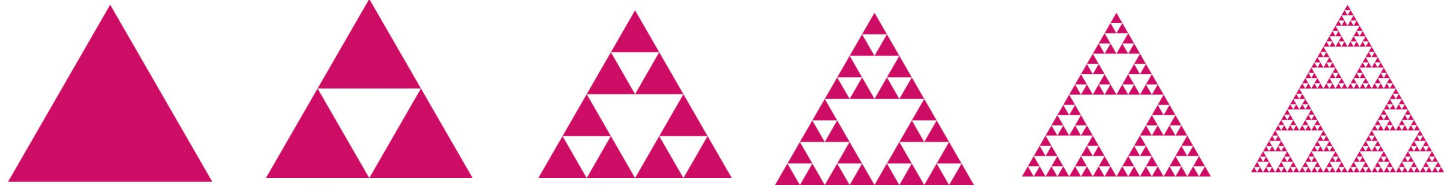
...

...

... n

Area

STAGES



Area, sq. units

A

...

...

...

...

...

... n

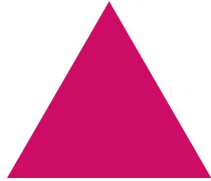
What constant multiplier can be used to go from one stage to the next? Express the area of shaded triangles as are powers of a given fraction.

Area

What happens to the area of the shaded triangles as n increases indefinitely?

Can you express the shaded area at any given stage in terms of the shaded area of the previous stage?

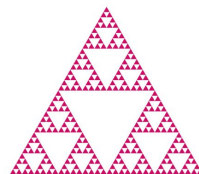
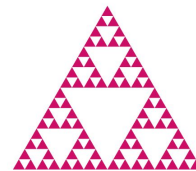
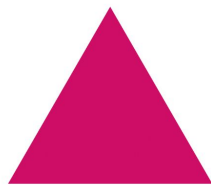
Perimeter



Let us assume this triangle to have a side of length **1** unit. What will be the *perimeter of shaded triangles* of other stages?

Perimeter

STAGES



Perimeter, units

3

...

...

...

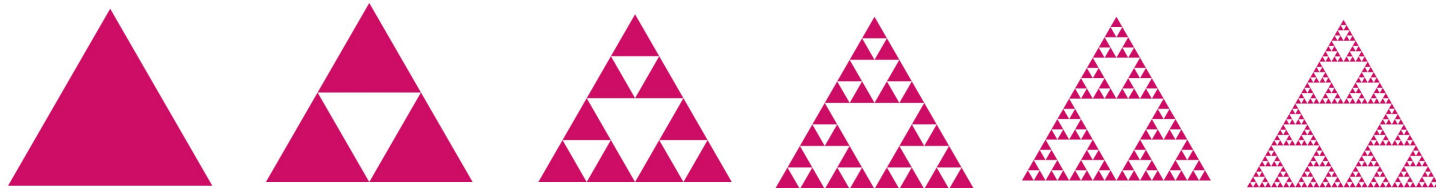
...

...

... n

Perimeter

STAGES



Perimeter, units

3

...

...

...

...

...

... n

What constant multiplier can be used to go from one stage to the next?

Perimeter

What happens to the length of the boundary of the shaded triangles as n increases?

Can you express the length of the boundary of the shaded triangles at any given stage in terms of the length of the boundary of the shaded triangles of the previous stage?

Some Questions

What happens to the area and Perimeter as the number of stages increases?

What other observations can we make from the areas and perimeters we have found?

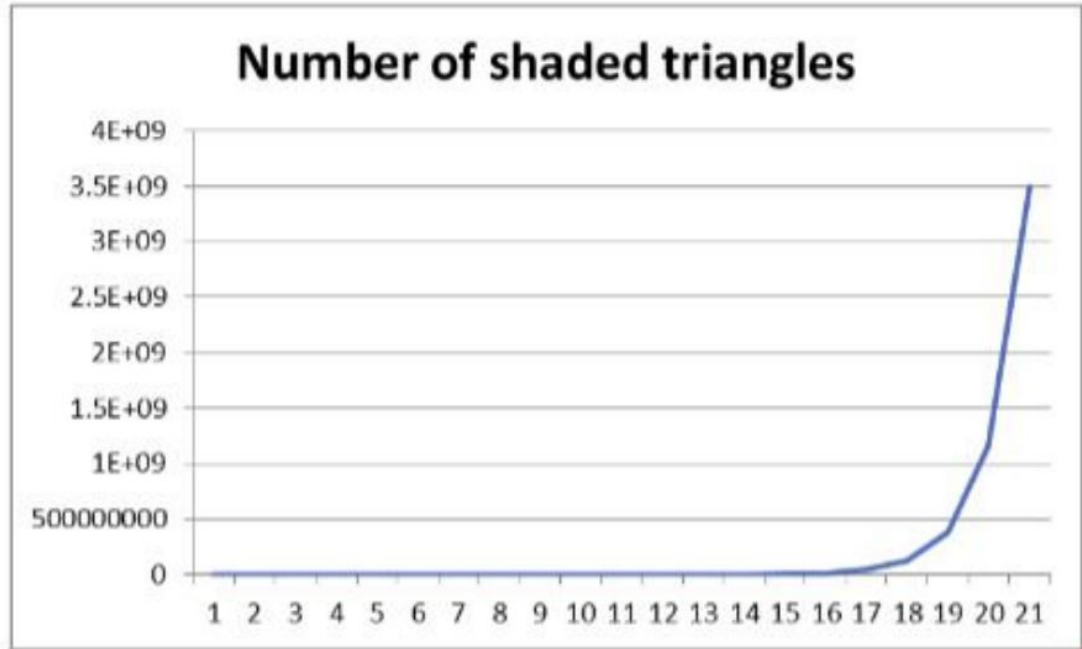
Plotting

No. of Stages	Area	Perimeter
0	A	3
1		
2		
3		
4		
5		

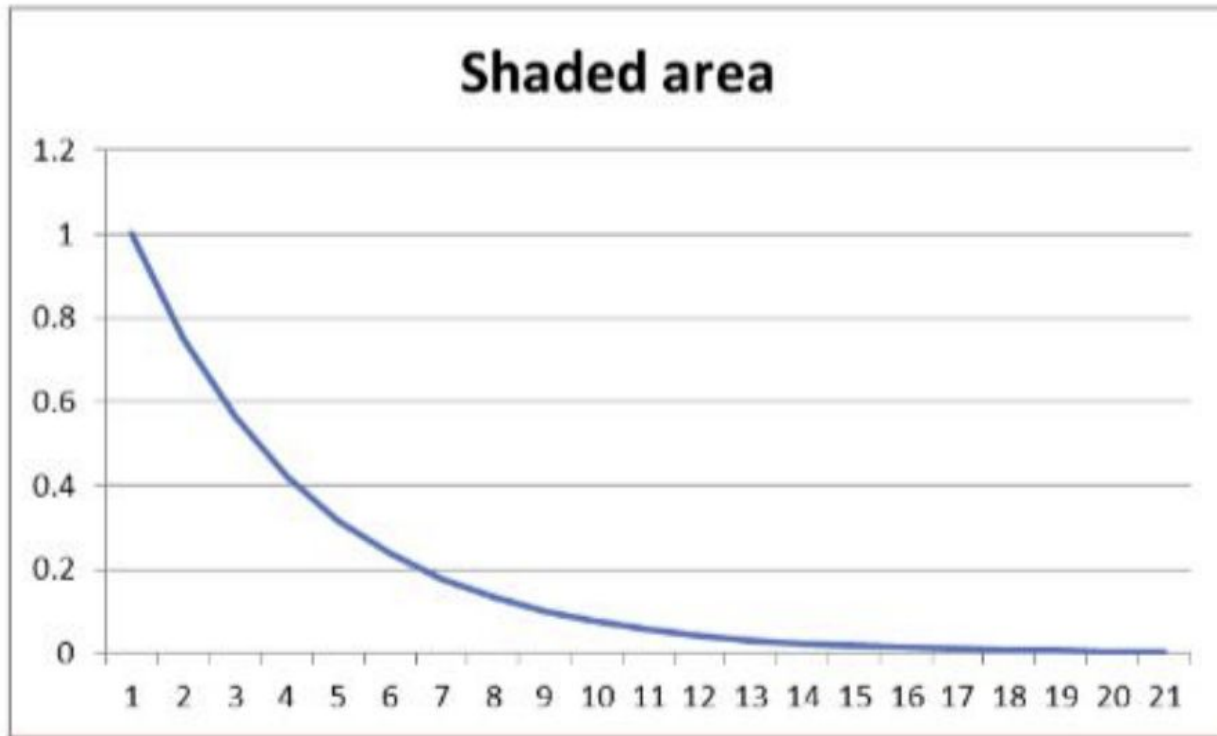
Plotting

	A	B	C	D
1	Stage number	Number of shaded triangles	Shaded area	Perimeter
2	0	1	1	3
3	1	3	0.75	4.5
4	2	9	0.5625	6.75
5	3	27	0.421875	10.125
6	4	81	0.31640625	15.1875
7	5	243	0.237304688	22.78125
8	6	729	0.177978516	34.171875
9	7	2187	0.133483887	51.2578125
10	8	6561	0.100112915	76.88671875
11	9	19683	0.075084686	115.3300781
12	10	59049	0.056313515	172.9951172
13	11	177147	0.042235136	259.4926758
14	12	531441	0.031676352	389.2390137
15	13	1594323	0.023757264	583.8585205
16	14	4782969	0.017817948	875.7877808
17	15	14348907	0.013363461	1313.681671
18	16	43046721	0.010022596	1970.522507
19	17	129140163	0.007516947	2955.78376
20	18	387420489	0.00563771	4433.67564
21	19	1162261467	0.004228283	6650.51346
22	20	3486784401	0.003171212	9975.77019

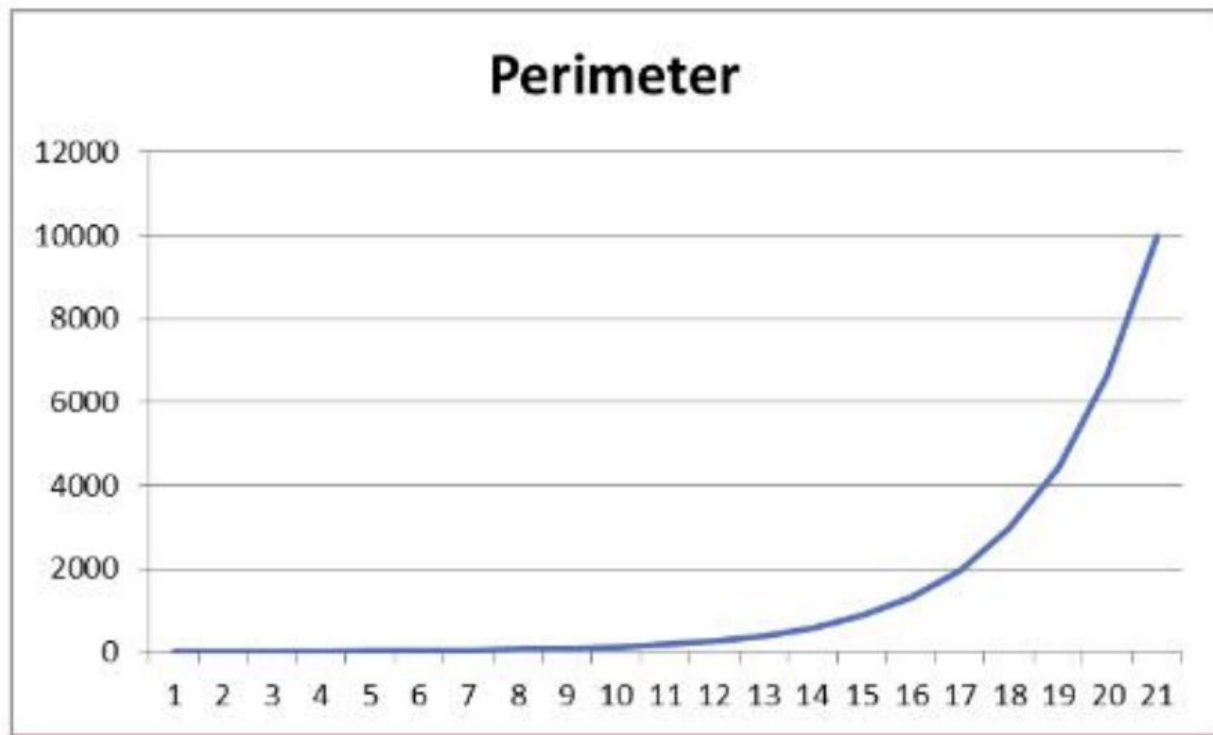
Plotting



Plotting



Plotting



Other Fractals we can create

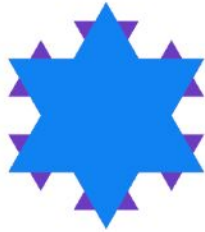
Koch snowflake



0



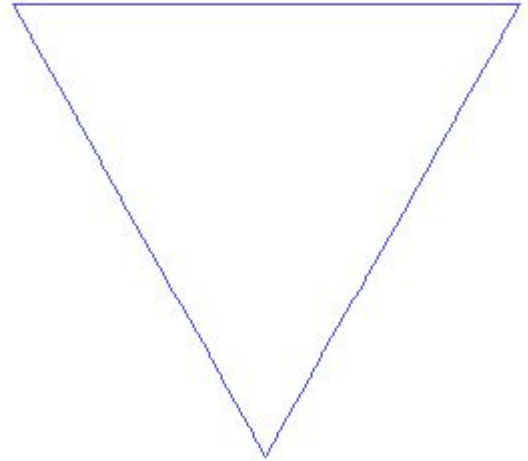
1



2



3



Resources

<https://mathigon.org/course/fractals/introduction>

<http://zitogiuseppe.com/project/nfractals.html>