Will It Sink or Float?

Buoyant Force: Governing Relations and Diagrams

The Central Question

A large iron ship floats; a small pebble sinks. Weight alone cannot explain this, since the ship is far heavier than the pebble.

The resolution lies in an upward force exerted by a fluid on any object immersed in it — the **buoyant force**.

This deck follows the same five-task structure:

- Apparent weight
- Origin of buoyant force (pressure and depth)
- Independence from weight
- Dependence on volume
- Density and the floating condition

Apparent Weight and Buoyant Force

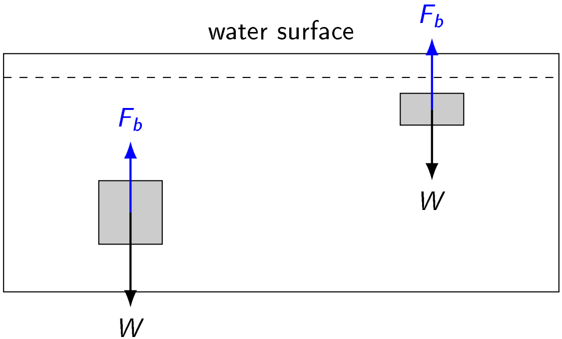
When an object is weighed in air and then in water, the reading decreases. The reduced reading is the **apparent weight**, and the difference between the two readings is the buoyant force F_b:

$$W_{\text{apparent}} = W_{\text{actual}} - F_b \qquad \Longrightarrow \qquad F_b = W_{\text{actual}} - W_{\text{apparent}}$$

For a floating object, the buoyant force alone supports the full weight:

$$\text{Sinking:} \quad F_b < W \qquad\qquad \text{Floating:} \quad F_b \geq W$$

Diagram: Forces on Sinking vs. Floating Objects



Sinking: $F_b < W$

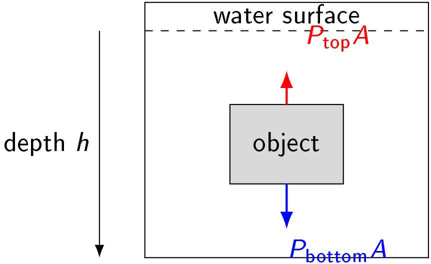
Floating: $F_b \geq W$

Pressure Increases with Depth

Pressure in a fluid of density ρ_{fluid} at depth h below the surface:

$$P(h) = P_0 + \rho_{\text{fluid}}\, g\, h$$

Because pressure increases with depth, a submerged object experiences greater pressure on its lower surface than on its upper surface. This asymmetry is the origin of the buoyant force.



Deriving the Buoyant Force

For a submerged shape of cross-sectional area A and height h, with top face at depth h_1 and bottom face at depth $h_2 = h_1 + h$:

$$F_b = P_{\text{bottom}}A - P_{\text{top}}A = \big[(P_0 + \rho_{\text{fluid}}gh_2) - (P_0 + \rho_{\text{fluid}}gh_1)\big]A$$

$$F_b = \rho_{\text{fluid}}\, g\,(h_2 - h_1)\,A = \rho_{\text{fluid}}\, g\, h\, A$$

Since hA is the volume of fluid displaced, V_{sub}:

$$\boxed{F_b = \rho_{\text{fluid}}\, g\, V_{\text{sub}}}$$

Independent of the object's own weight or material. Side forces cancel by symmetry.

Depth Independence

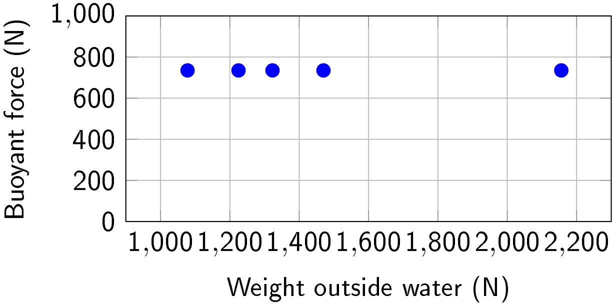
Because F_b depends only on V_{sub} and ρ_{fluid}, and not on the absolute depth h_1:

- The buoyant force on a fully submerged object of fixed volume does not change as it is lowered further into the fluid
- This holds provided the fluid's density does not vary with depth
- Measuring the weight of a stone at different depths underwater confirms this directly — the reading stays constant

Does Buoyant Force Depend on Weight?

Holding submerged volume fixed while varying mass (different numbers of marbles inside identical sealed containers of volume V):

$$F_b^{(1)} = F_b^{(2)} = \cdots = \rho_{\text{fluid}}\, g\, V \qquad \text{for all objects of equal volume } V$$

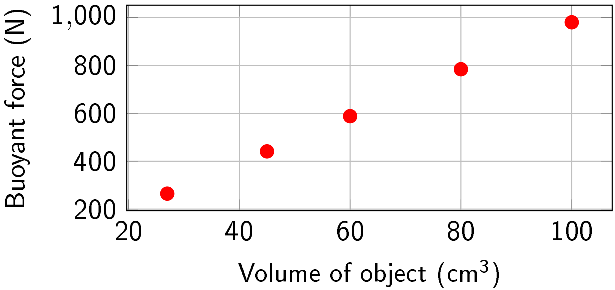


$F_b = 735$ N for every object, despite weights ranging from 1078 N to 2156 N — because all five share the same volume (100 cm^3).

Does Buoyant Force Depend on Volume?

Holding weight fixed while varying volume (cylinders of equal mass, made from iron, ceramic, plastic, wood, foam):

$$F_b \propto V_{\text{sub}} \qquad \text{(weight held constant)}$$



Weight outside water is 980 N in every case; F_b rises from 264.6 N (iron, 27 cm^3) to 980 N (foam, 100 cm^3), where $F_b = W$ and the object floats.

Combining the Two Results

$$F_b = \rho_{\text{fluid}}\, g\, V_{\text{sub}}$$

- Buoyant force depends on the density of the fluid and the submerged volume of the object
- Buoyant force does **not** depend on the weight of the object

The Floating Condition

An object floats in equilibrium when:

$$F_b = W \quad \Longrightarrow \quad \rho_{\text{fluid}}\, g\, V_{\text{sub}} = \rho_{\text{obj}}\, g\, V_{\text{obj}}$$

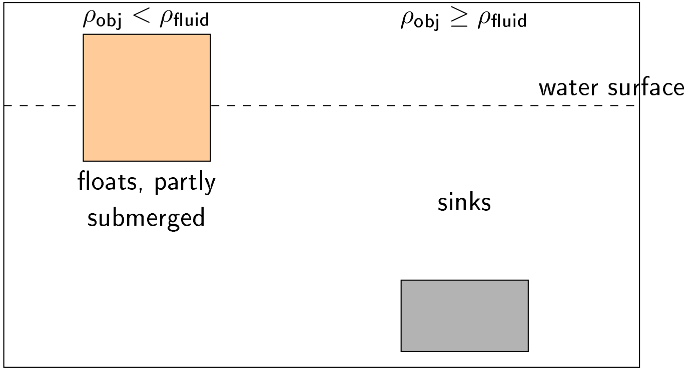
$$\Rightarrow \quad \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}}$$

This ratio is the fraction of the object's volume that must be submerged to float. Two limiting cases:

$$\rho_{\text{obj}} < \rho_{\text{fluid}} \;\Rightarrow\; \text{floats, partially submerged}$$

$$\rho_{\text{obj}} \geq \rho_{\text{fluid}} \;\Rightarrow\; \text{sinks}$$

Diagram: Density Comparison



Average Density

When mass is added to a rigid container of fixed outer volume V (marbles, sand, water), the container's material density does not change, but its **average density** does:

$$\rho_{\text{avg}} = \frac{m_{\text{container}} + m_{\text{added}}}{V_{\text{outer}}}$$

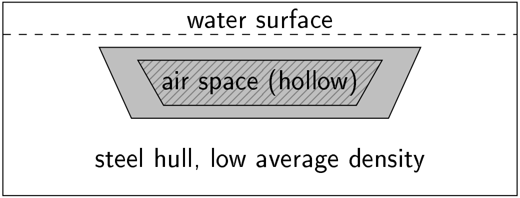
- The container floats while $\rho_{\text{avg}} < \rho_{\text{fluid}}$
- It sinks once $\rho_{\text{avg}} \geq \rho_{\text{fluid}}$
- V_{outer} stays fixed throughout — only the numerator changes

Why Large Ships Float

A steel hull has a material density several times that of water ($\rho_{\text{steel}} \approx 7850$ kg/m^3 versus $\rho_{\text{water}} \approx 1000$ kg/m^3), yet the ship floats because its hull encloses a large volume of air.

$$\rho_{\text{avg, ship}} = \frac{m_{\text{steel}}}{V_{\text{steel}} + V_{\text{air space}}}$$

Since $V_{\text{air space}}$ is large, $\rho_{\text{avg, ship}}$ drops well below ρ_{water}, even though the steel itself is far denser than water.



Summary of Relationships

Relation	Meaning
$F_b = W_{\text{actual}} - W_{\text{apparent}}$	Buoyant force from spring balance readings
$F_b = \rho_{\text{fluid}}\, g\, V_{\text{sub}}$	Buoyant force from pressure difference (depth-independent)
$F_b \geq W \Rightarrow$ float; $F_b < W \Rightarrow$ sink	Floating condition
$\rho_{\text{obj}} < \rho_{\text{fluid}} \Rightarrow$ float	Density comparison, uniform object
$\rho_{\text{avg}} = \dfrac{m_{\text{total}}}{V_{\text{outer}}}$	Average density governs composite/hollow objects

The buoyant force depends on the density of the fluid and the submerged volume of the object; it does not depend on the object's weight. Whether an object floats or sinks is governed by a comparison of average densities — of the object (or object-plus-contents) against the surrounding fluid.