From Phenomenon to Function

Teaching Math and Physics Together

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Agenda

- Why this chain matters
- Seven real-world problems, each with:
  - The problem itself and the math tools it needs (with plain-language definitions)
  - The governing equations, with every symbol explained
  - Step-by-step GeoGebra build instructions
  - A hands-on task to try
- Mid-talk discussion: sequencing across departments
- Cross-department takeaway
- Glossary and resources

Why This Chain Matters

- Physics problems **motivate** math tools; math tools **make physics solvable**
- Showing students this loop builds intuition, not memorization
- GeoGebra sits at the intersection: graphing calculator, dynamic geometry tool, and simulation environment in one
- Same file can serve two classrooms with two different emphases

Problem 1: Projectile Motion

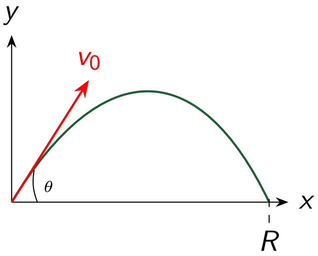
Real-world hook: basketball arc, javelin throw, water fountain

Math tools needed:

- **Parametric equations** — both coordinates (x and y) are written as separate functions of a third variable (here, time t), instead of y being written directly in terms of x
- **Quadratic functions** — functions where the variable appears to the power 2; their graph is a parabola
- **Vectors (velocity components)** — a quantity with both size and direction, split here into a horizontal part and a vertical part

Problem 1: Equations

$$x(t) = v_0 \cos\theta \cdot t$$
$$y(t) = v_0 \sin\theta \cdot t - \tfrac{1}{2} g t^2$$
$$R = \frac{v_0^2 \sin(2\theta)}{g}$$



What each symbol means:

- $x(t), y(t)$: horizontal / vertical position at time t
- v_0: launch speed
- θ: launch angle, from horizontal
- g: acceleration due to gravity (9.8 m/s^2)
- t: time elapsed since launch
- R: range — total horizontal distance travelled before landing

GeoGebra Instructions: Projectile Motion

1. Open GeoGebra Classic (`geogebra.org/classic`).
2. In the Input bar, type `v_0=Slider(0,50,1)` and press Enter. This creates a slider named v_0 from 0 to 50 in steps of 1; drag its dot to change the value.
3. Type `theta=Slider(0°,90°,1°)` and press Enter — GeoGebra automatically renders "theta" as θ and creates an angle slider.
4. Type `g=9.8` and press Enter. This is a fixed constant, not a slider, since gravity doesn't change during the demo.
5. Type the trajectory command: `Curve(v_0*cos(theta)*t, v_0*sin(theta)*t-0.5*g*t^2, t, 0, 5)` — draws the full parabolic path for t between 0 and 5 seconds.
6. To animate a moving point: `time=Slider(0,5,0.05)`, then `P=(v_0*cos(theta)*time, v_0*sin(theta)*time-0.5*g*time^2)`. (Don't name this point `Point` — reserved.) Right-click the `time` slider dot and choose "Animation On".
7. To display the range: `R=(v_0^2*sin(2*theta))/g`. Use the Text tool to display "Range = " with R inserted from the dropdown.

Hands-On: Projectile Motion

- Build the slider setup on your own device
- Fix $v_0 = 20$ m/s. Test angles 30°, 45°, 60°
- **Question:** which angle gives the greatest range? Confirm with the formula
- **Discuss with a neighbor:** where would you introduce this — math class first, or physics class first?

Problem 2: Optimization

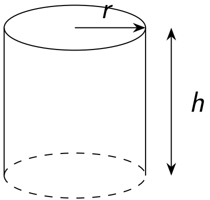
Real-world hook: cheapest packaging (fixed volume), or fastest path of light (Snell's law)

Math tools needed:

- **Derivatives** — the rate at which a function's output changes as its input changes; geometrically, the slope of the curve at a point
- **Critical points** — points where the derivative equals zero (the slope is flat), which are candidates for a maximum or minimum
- **Constraint equations** — an equation that fixes a relationship between variables (here, that the volume must stay constant) so only one variable is truly free to change

Problem 2: Equations

$$S(r) = 2\pi r^2 + \frac{2V}{r}$$
$$S'(r) = 4\pi r - \frac{2V}{r^2} = 0$$
$$\Rightarrow \; r = \left(\frac{V}{2\pi}\right)^{1/3}$$



What each symbol means:

- $S(r)$: total surface area, as a function of radius r
- r: radius of the cylinder (solving for this)
- V: volume of the cylinder — fixed (the constraint)
- $S'(r)$: derivative of S; setting it to zero finds the minimum

GeoGebra Instructions: Optimization

1. Start a new file. This problem reuses short names (x) used differently elsewhere — building in the same file can cause silent errors.
2. Type `V=500` and press Enter. This fixes the volume as a constant.
3. Type `r=Slider(0.1,10,0.1)` and press Enter — the variable you'll drag to search for the minimum.
4. Type `S(x)=2*pi*x^2+2*V/x` and press Enter to define the surface-area function; it plots automatically. (Use `x`, not `r` — `r` already exists as a slider.)
5. To trace the point as you drag: `P=(r,S(r))`. As you move the r slider, P moves along the curve.
6. Type `Derivative(S)` to create $S'(x)$ as a new function and let GeoGebra plot it automatically.
7. Use the Point tool to click where the derivative curve crosses the x-axis — this r-value is the minimum-area radius. Compare it to $r = (V/2\pi)^{1/3}$ computed by hand.

Hands-On: Optimization

- Set $V = 500$ cm^3. Drag r toward the minimum visually
- Verify your visual answer against $r = (V/2\pi)^{1/3}$
- **Question:** what happens to the optimal shape if the container has no lid (only one circular face)?

Problem 3: Exponential Growth/Decay

Real-world hook: radioactive decay, Newton's Law of Cooling, capacitor discharge

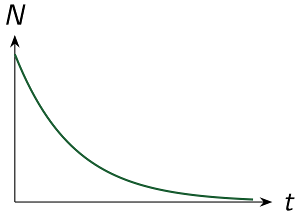
Math tools needed:

- **Exponential functions** — functions of the form $a \cdot b^t$ (or e^{kt}), where the variable appears in the exponent, producing rapid growth or decay
- **Logarithms** — the inverse operation of exponentiation; used to solve for t when it's trapped in an exponent
- **Differential equations** (rate $\propto$ amount) — an equation relating a quantity's rate of change to the quantity itself, rather than giving the quantity directly

Problem 3: Equations

$$\frac{dN}{dt} = -kN \;\Rightarrow\; N(t) = N_0 e^{-kt}$$

$$t_{1/2} = \frac{\ln 2}{k}$$



What each symbol means:

- $N(t)$: amount remaining at time t
- N_0: initial amount, at $t = 0$
- k: decay constant — larger k means faster decay
- $t_{1/2}$: half-life
- **Caveat:** decays to zero — exact for radioactive decay, capacitor discharge. Newton's cooling decays to *ambient* temperature instead.

GeoGebra Instructions: Exponential Growth/Decay

1. Start a new file, to avoid collisions with names reused elsewhere (e.g. `k`, `f`).
2. Type `N_0=Slider(1,200,1)` and press Enter to create a slider for the initial amount.
3. Type `k=Slider(0.01,2,0.01)` and press Enter to create a slider for the decay constant.
4. Type `f(t)=N_0*exp(-k*t)` and press Enter — the decay curve plots automatically.
5. To bring in real data: open the Spreadsheet view. Type time values in column A, measured values in column B. Cooling data decays toward *ambient* temperature, not zero — subtract the ambient reading first.
6. Select both columns, right-click, choose "Create List of Points" to plot them on the graph.
7. Drag the k slider until the curve visually passes through the data points (fit-by-eye).
8. Optionally, select the points and type `FitExp(list1)` to have GeoGebra compute the best-fit curve automatically.

Hands-On: Exponential Growth/Decay

- Plot $N_0 = 100$, adjust k until half-life ≈ 5 time units
- Read the half-life off the graph, then check with $\ln 2/k$
- **Question:** how would you explain "half-life" to a student who has not yet learned logarithms?

Problem 4: Related Rates

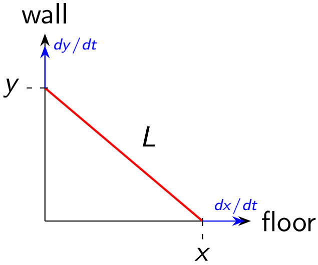
Real-world hook: a ladder sliding down a wall, a balloon being inflated, an oil spill spreading

Math tools needed:

- **Implicit differentiation** — differentiating an equation that relates two variables (x and y) directly, without first solving for one in terms of the other
- **Chain rule** — the rule for differentiating a function of a function; here it's what lets us differentiate y^2 with respect to time even though y is itself a function of time
- **Geometric constraints** — a fixed geometric relationship (here, the Pythagorean theorem) that ties the changing quantities together

Problem 4: Equations

$$x^2 + y^2 = L^2$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$\Rightarrow \quad \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$



What each symbol means:

- x: distance from the wall to the base of the ladder
- y: height of the top of the ladder on the wall
- L: length of the ladder (fixed)
- dx/dt: speed at which the base slides away
- dy/dt: speed at which the top slides down (what we solve for)

GeoGebra Instructions: Related Rates

1. Start a new file. This problem defines `x` as a persistent slider and A, B as points — avoid leftover collisions with a later file.
2. Type `L=5` and press Enter to fix the ladder length as a constant.
3. Type `x=Slider(0,L,0.05)` and press Enter — the distance of the base from the wall.
4. Type `y=sqrt(L^2-x^2)` and press Enter. This computes the height automatically from the Pythagorean constraint.
5. Use the Point tool (or type `A=(x,0)` and `B=(0,y)`) to mark the base and top of the ladder, then use the Segment tool to connect them.
6. Right-click the `x` slider dot and choose "Animation On" to make the base slide outward automatically.
7. Select point B, right-click, choose "Trace On". Run the animation and watch the trace — notice it is not a straight line.
8. To see the numeric rates: `dxdt=1`, then `dydt=-(x/y)*dxdt` to compute the top's speed at the current position.

Hands-On: Related Rates

- Set $L = 5$ m. Animate the bottom sliding out at constant speed
- **Question:** does the top of the ladder fall at a constant speed? Watch the trace
- **Discuss:** why does the top accelerate downward even though the bottom moves at constant speed?

Problem 5: Measurement Error and Line of Best Fit

Real-world hook: timing a pendulum's period, spring displacement vs. mass — any physics lab with repeated trials

Math tools needed:

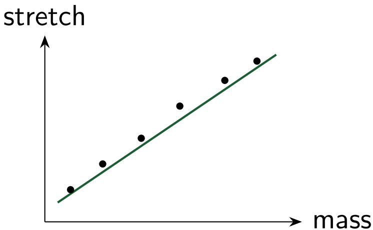
- **Linear regression** — a method for finding the straight line that best fits a set of scattered data points
- **Correlation coefficient** — a number between -1 and 1 that measures how closely the data follows a straight-line pattern
- **Residuals and uncertainty** — a residual is the vertical gap between a data point and the fitted line; residuals capture the measurement error not explained by the line

Problem 5: Equations

$$\hat{y} = mx + b$$
$$r = \text{correlation} \in [-1, 1]$$

What each symbol means:

- $\hat{y}$: predicted value from the fitted line
- m: slope of the best-fit line
- b: y-intercept of the best-fit line
- x: the independent variable (e.g., hanging mass)
- r: correlation coefficient; r close to ± 1 means the points lie nearly on a straight line



GeoGebra Instructions: Regression on Lab Data

1. Start a new file, to avoid collisions with `x` or g left over from an earlier problem.
2. Open the Spreadsheet view (View menu $\rightarrow$ Spreadsheet).
3. Type your measured x-values (e.g., hanging mass) into cells A1, A2, A3, ..., and the corresponding y-values into B1, B2, B3, ...
4. Select all filled cells, right-click, choose "Create List of Points". This creates `list1`.
5. Type `FitLine(list1)` and press Enter — the best-fit line is drawn and its equation appears in the Algebra view.
6. Type `PearsonCorrelation(list1)` and press Enter to compute r.
7. To show residuals: use the Segment tool to draw a vertical segment from each point to the fitted line.
8. To compare with a manual guess: `m=Slider(-5,5,0.1)` and `b=Slider(-5,5,0.1)`, then `g(x)=m*x+b`.

Hands-On: Regression on Lab Data

- Enter 6–8 rough data points from a real or invented spring experiment
- Try to guess the best-fit line by eye using a slider, then compare to `FitLine`
- **Question:** what does a low r value suggest about the experiment (measurement error vs. a genuinely non-linear relationship)?

Problem 6: Standing Waves and Resonance

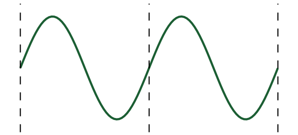
Real-world hook: guitar strings, wind instruments, a wine glass shattering from sound

Math tools needed:

- **Trigonometric functions** — functions like sin and cos that describe repeating, wave-like patterns
- **Superposition** — the principle that when two waves overlap, the resulting displacement is simply the sum of the two individual displacements
- **Wavelength/frequency** — wavelength is the physical distance between repeating points on a wave; frequency is how many repetitions occur per second; the two are linked by the wave's speed

Problem 6: Equations

$$
\begin{aligned}
y &= A\sin(kx - \omega t) + A\sin(kx + \omega t) \\
&= 2A\sin(kx)\cos(\omega t)
\end{aligned}
$$



What each symbol means:

- A: amplitude — max displacement
- k: wave number, $k = 2\pi/\lambda$
- ω: angular frequency, $\omega = 2\pi f$
- x: position along the string; t: time
- Nodes: points where $\sin(kx) = 0$

GeoGebra Instructions: Standing Waves

1. Start a new file, to avoid collisions with `k`, `x`, `f`, or `g` left over from earlier problems.
2. Type `A=1`, `k=2`, and `omega=2`, to fix starting values for amplitude, wave number, and angular frequency.
3. Type `tt=Slider(0,10,0.05)` to create a time slider. (Name it `tt`, not `t` — a global `t` slider would freeze the wave.)
4. Type `f(x,t)=A*sin(k*x-omega*t)` to define the first traveling wave.
5. Type `g(x,t)=A*sin(k*x+omega*t)` to define the second, oppositely-travelling wave.
6. Type `h(x)=f(x,tt)+g(x,tt)` — the standing wave sum, evaluated at the slider's current time.
7. Right-click the `tt` slider dot and choose "Animation On". Watch $h(x)$: some points stay fixed at zero (nodes) while others oscillate.
8. To try different harmonics, type `k=Slider(1,6,1)` instead of a fixed value.

Hands-On: Standing Waves

- Animate t and locate the nodes (points that never move)
- Change the wavelength slider to produce 2 nodes, then 3 nodes
- **Question:** how does node count relate to the harmonic number in a vibrating string?

Problem 7: Vectors and Forces in Equilibrium

Real-world hook: bridge trusses, cable tension, tug-of-war

Math tools needed:

- **Vector addition** — combining two or more vectors by placing them tip-to-tail; the sum is called the resultant
- **Components** — splitting a vector into its horizontal (x) and vertical (y) parts, which can be added separately
- **Trigonometry** — using sin and cos of a vector's angle to compute its components

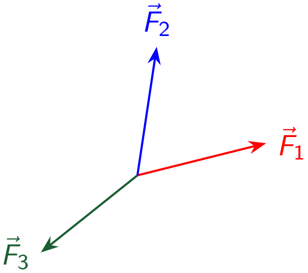
Revisits Problem 1's trig/vectors — deliberately: same tool, new context.

Problem 7: Equations

$$\sum \vec{F} = 0 \iff \sum F_x = 0,\ \sum F_y = 0$$

What each symbol means:

- $\vec{F}$: a force vector — has both magnitude and direction
- $\sum \vec{F}$: the vector sum (resultant) of all forces
- F_x, F_y: horizontal and vertical components
- Equilibrium: the resultant force is exactly zero



GeoGebra Instructions: Vectors and Equilibrium

1. Start a new file. Problem 4 defines A and B as dependent points; new free points with the same names will fail if those definitions are still present.
2. Use the Point tool to place a starting point O at the origin.
3. Place two or three more points (A, B, C) anywhere — these define the tips of your force vectors.
4. Type `v_1=Vector(O,A)` to create the first force vector from O to A. Repeat for `v_2=Vector(O,B)` and `v_3=Vector(O,C)`.
5. Type `Sum({v_1,v_2,v_3})` to compute the resultant vector; it draws starting at the origin by default.
6. Right-click the resultant vector and change its color so it's easy to distinguish from the individual forces.
7. Drag points A, B, C around and watch the resultant vector change length. When the resultant shrinks to a single point, the forces are in equilibrium.

Hands-On: Vectors and Equilibrium

- Drag vectors until the resultant vanishes
- Read off the angles and magnitudes at equilibrium
- **Question:** could three unequal forces ever be in equilibrium? Test it

Summary: Seven Problems, Seven Math Tools

Problem	Core Math Tool
Projectile motion	Parametric equations, quadratics, vectors
Optimization	Derivatives, critical points
Exponential decay	Exponentials, logarithms
Related rates	Implicit differentiation, chain rule
Regression on lab data	Linear regression, correlation
Standing waves	Trigonometry, superposition
Vectors & equilibrium	Vector addition, trigonometry

Vectors/trig appears twice (Problems 1 & 7) by design — same tool, two different physical contexts.

Glossary: Math Tools at a Glance

- **Parametric equations** — x and y each written as separate functions of a third variable
- **Quadratic functions** — variable raised to the power 2; graph is a parabola
- **Vectors** — quantities with both magnitude and direction
- **Derivatives** — the rate of change of a function; the slope at a point
- **Critical points** — where the derivative is zero; candidate maxima/minima
- **Constraint equations** — a fixed relationship that removes one degree of freedom
- **Exponential functions** — variable appears in the exponent; rapid growth or decay
- **Logarithms** — the inverse of exponentiation; solves for a trapped exponent
- **Differential equations** — relate a rate of change to the quantity itself
- **Implicit differentiation** — differentiating without first isolating a variable
- **Chain rule** — differentiating a function of a function
- **Linear regression** — finding the best-fit straight line through scattered data
- **Correlation coefficient** — a -1 to 1 measure of how linear a dataset is
- **Residuals** — the gap between a data point and the fitted line
- **Trigonometric functions** — sin, cos, etc.; describe repeating wave patterns
- **Superposition** — overlapping waves add together directly
- **Vector addition / components** — combining vectors tip-to-tail, or splitting into x/y parts

Discussion: Sequencing Across Departments

Talk with a neighbor

- For your own school's curriculum: which math topics currently arrive *after* the physics topic that needs them?
- Would a shared timeline between departments fix this?
- Which of today's seven problems would you use first with your students?

Cross-Department Takeaway

- The same GeoGebra file can serve two lessons
- Physics teacher emphasizes the **phenomenon**
- Math teacher emphasizes the **function or technique**
- Suggestion: build a shared file library between departments so both build on the same visual models

Resources

- GeoGebra: `geogebra.org` — free, browser-based, no install needed
- PhET Interactive Simulations: `phet.colorado.edu`
- Today's example files: shared via department email / shared drive after the session
- Suggested next step: pick one problem, build it together as a joint department planning task

Questions?