Lecture delivered during the Teachers Enrichment Workshop held at IMSC between 26th November to 1st December 2018.

Good afternoon friends

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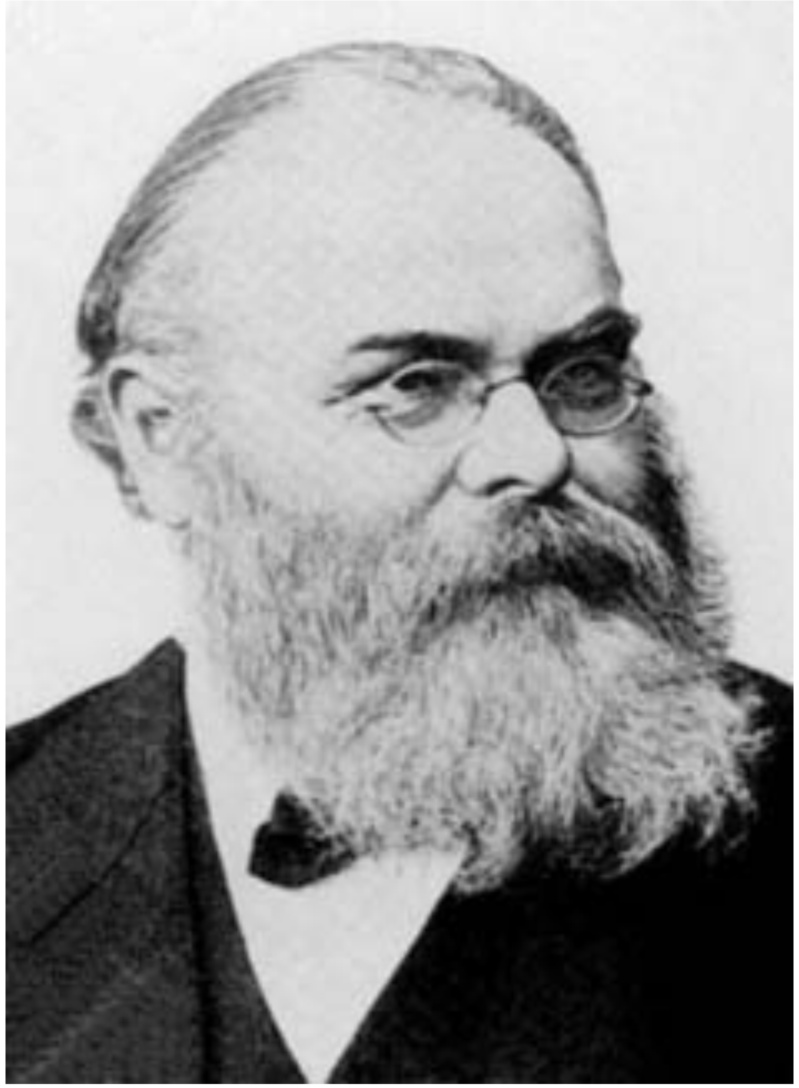

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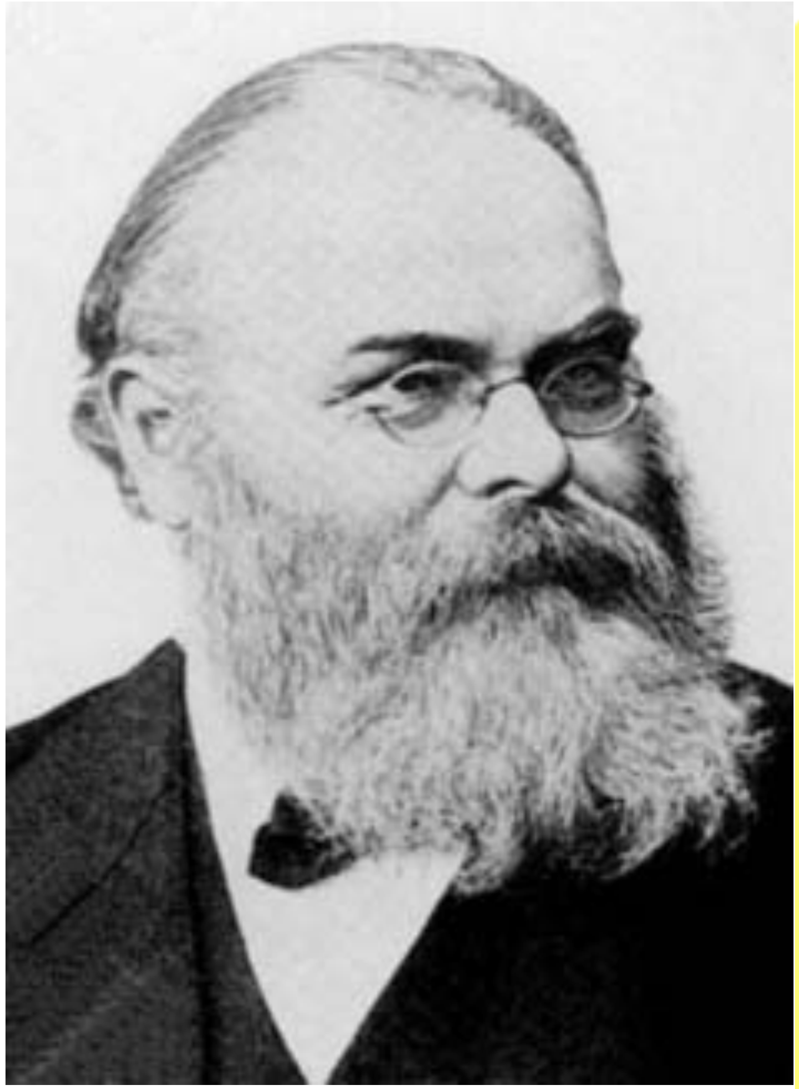
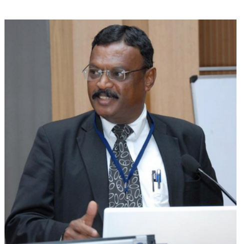


1. He worked on the conformal mapping of polyhedral surfaces onto the spherical surface and on a problem of the calculus of variation, namely surfaces of least area.



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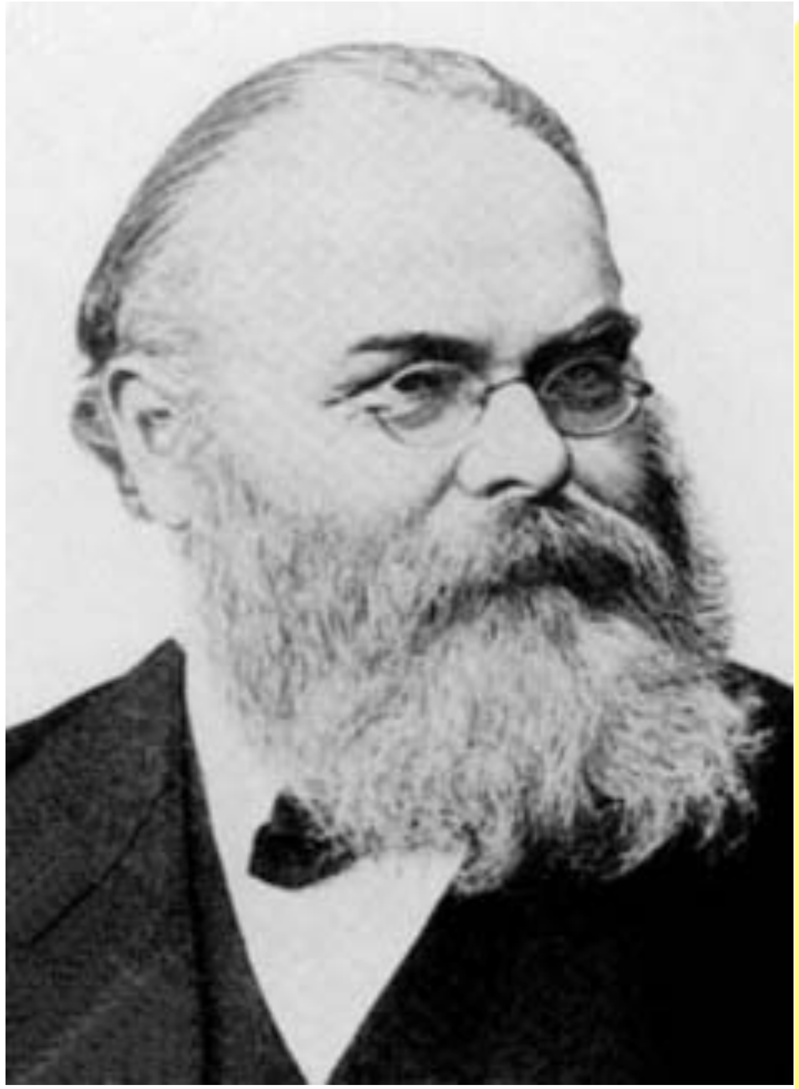

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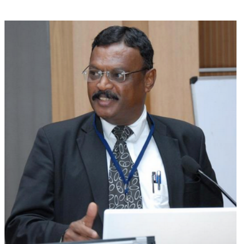

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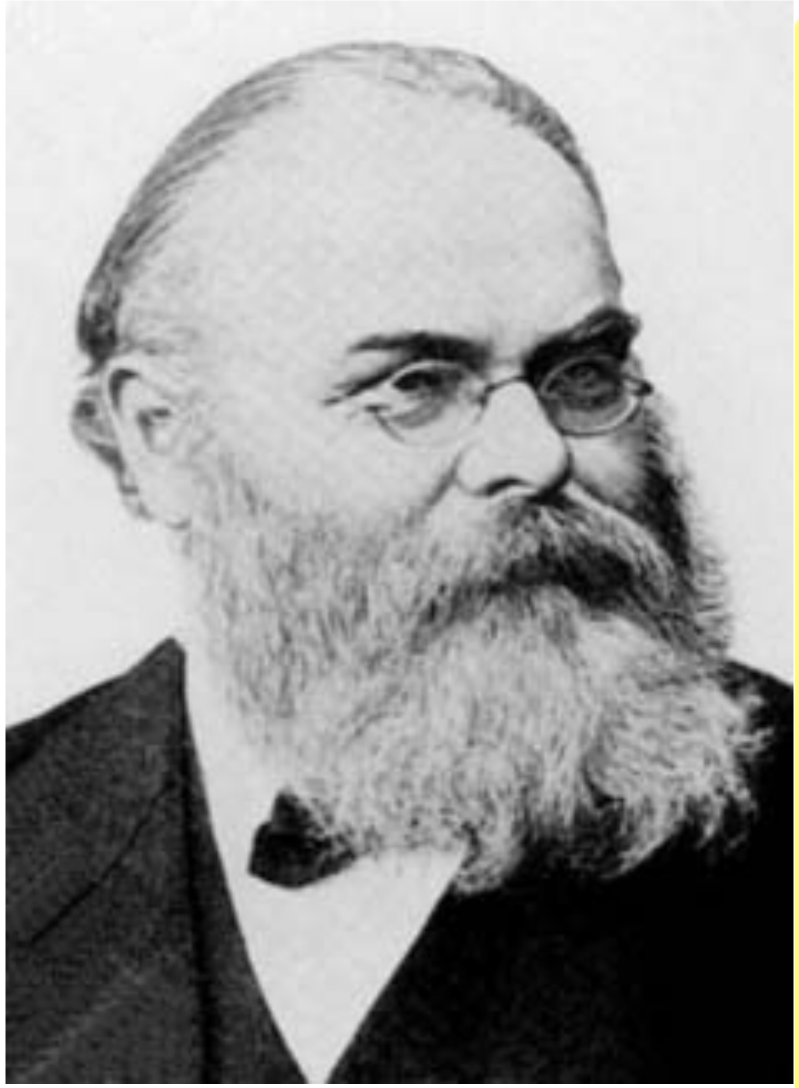

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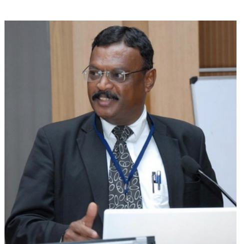

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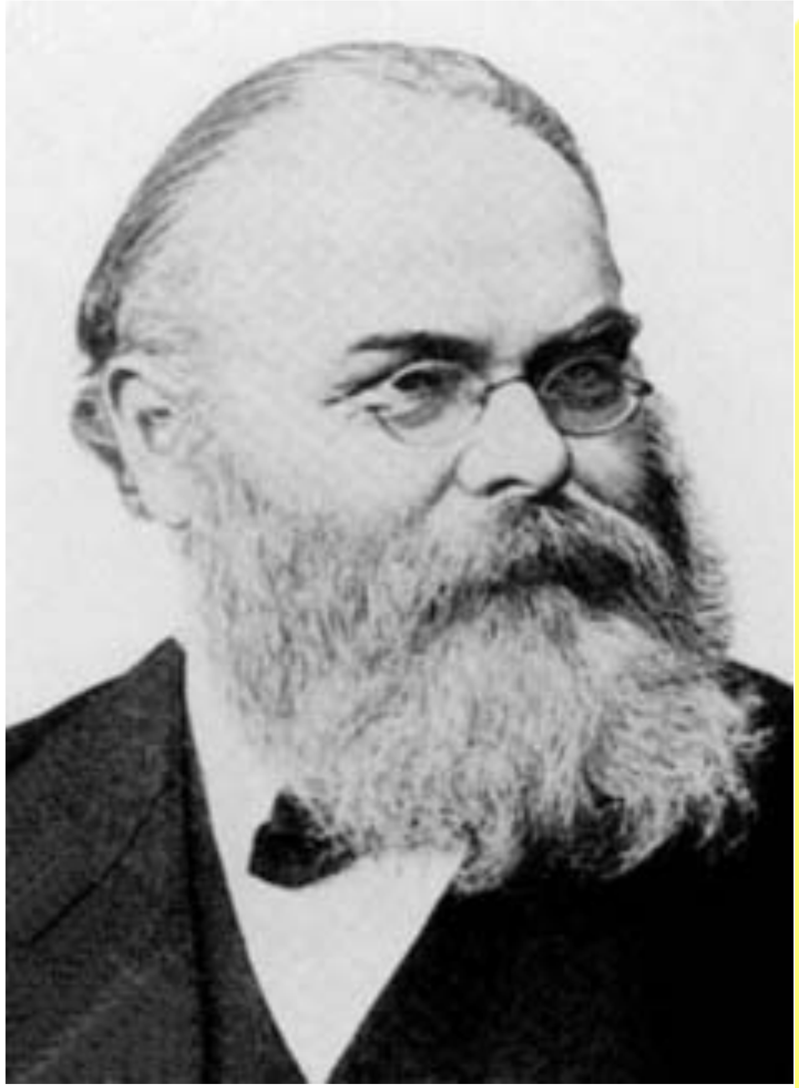

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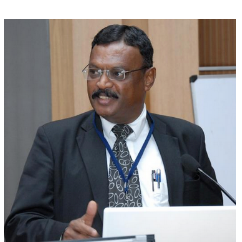

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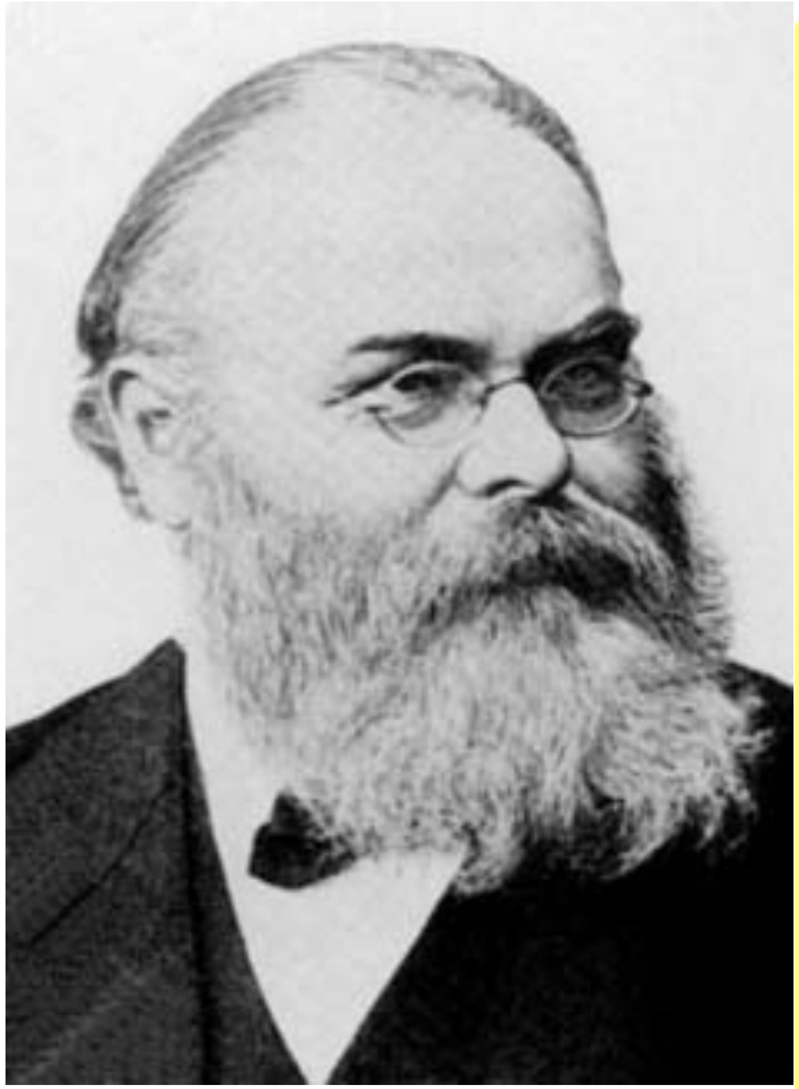

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Hermann Amandus Schwarz

Born: 25 January 1843 in Hermsdorf, Silesia (now Poland)

Died: 30 November 1921 in Berlin, Germany

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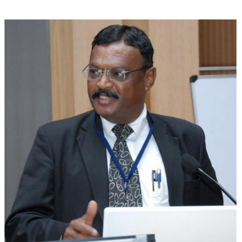

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Convolution/cross correlation of two functions

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Convolution of two functions	$f*g(t)=\int\limits_{-\infty}^{\infty} f(u)g(t-u)\,du$

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Convolute

$$f(t)=\begin{cases}1 & ,\ |t|<1\\ 0 & otherwise\end{cases} \quad and \quad g(t)=\begin{cases}1 & ,\ |t|<1\\ 0 & otherwise\end{cases}$$

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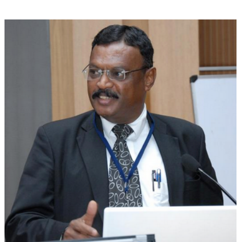

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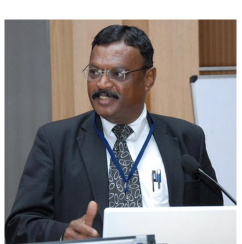

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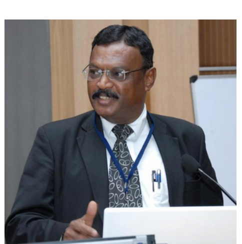
$$h(t) = \begin{cases} 0 & t < t_1 + t_2 \\ \left(\int_{-\infty}^{\infty} f(u) g(t-u)\, du \right) & t_1 + t_2 < t \ < t < T_1 + T_2 \\ 0 & t > T_1 + T_2 \end{cases}$$



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Geometrical representation of Convolution of functions

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The graphical presentation of the convolution integral involves the following steps:

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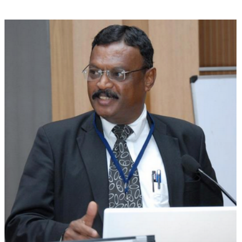

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Geometrical representation of Convolution of functions

The graphical presentation of the convolution integral involves the following steps:

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2: Flip about the vertical axis one of the signals (the one that has a simpler form (shape) since the commutativity holds), that is, represent one of the signals in the time scale .

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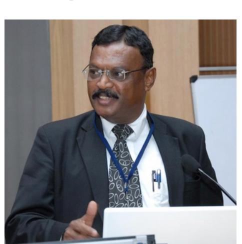
3: Vary the parameter from -infinity to infinity , that is, slide the flipped signal from the left to the right, look for the intervals where it overlaps with the other signal and evaluate the integral of the product of two signals in the corresponding intervals.



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Convolute the two rectangular signals

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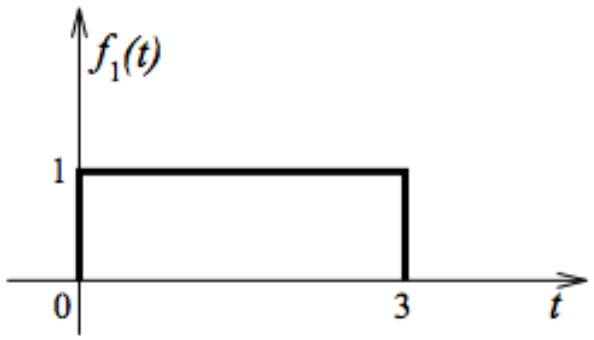

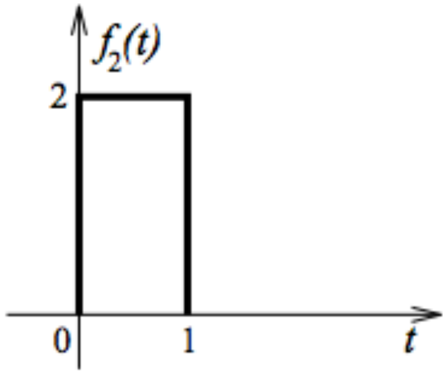

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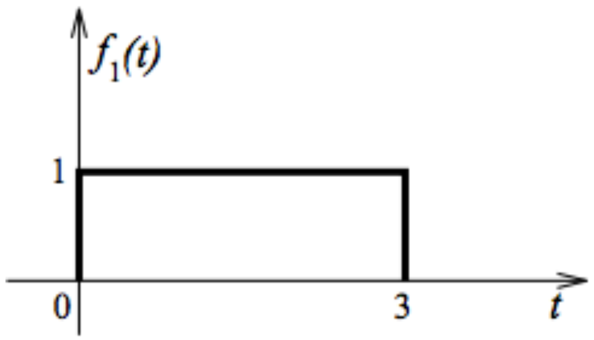

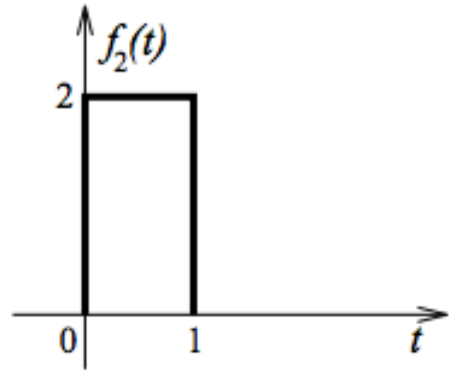

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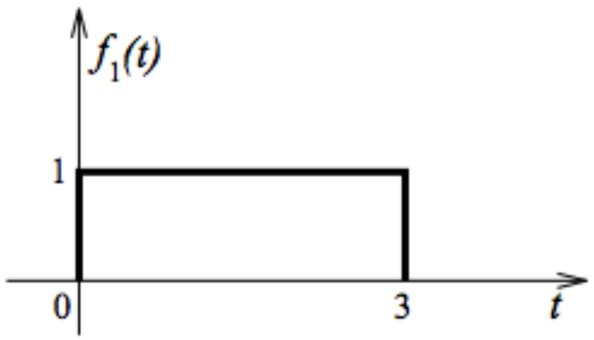

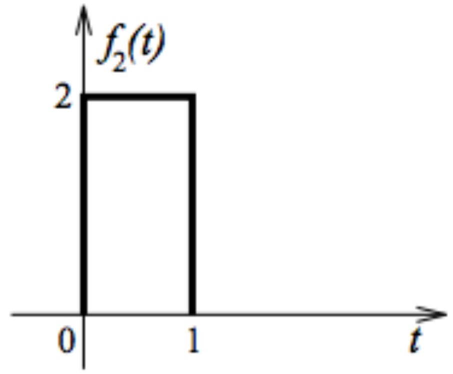

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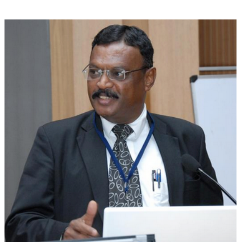

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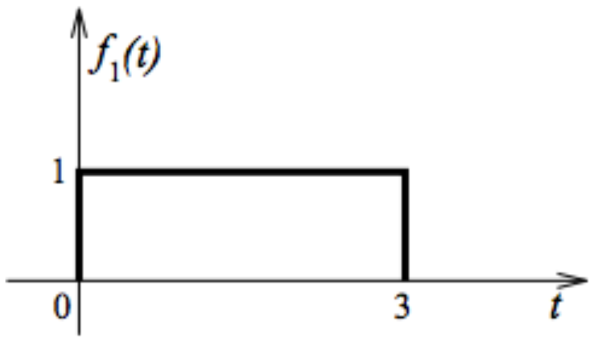

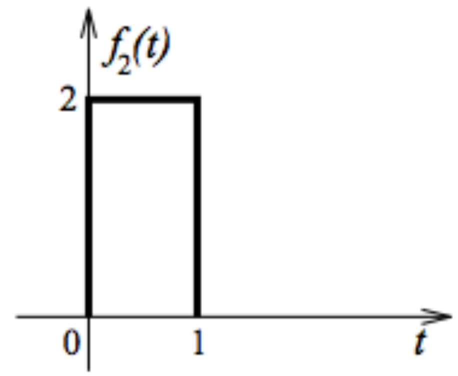

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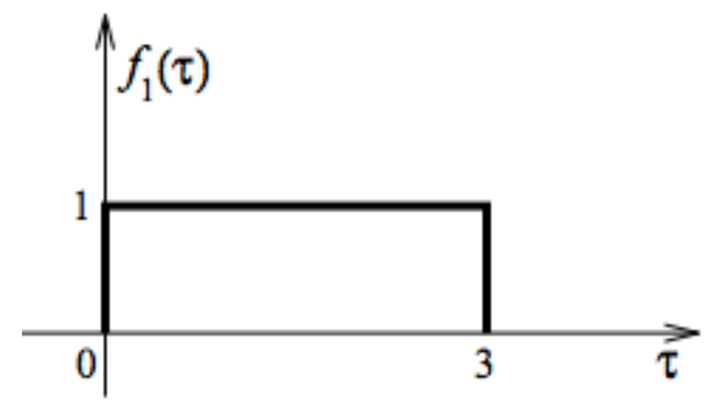

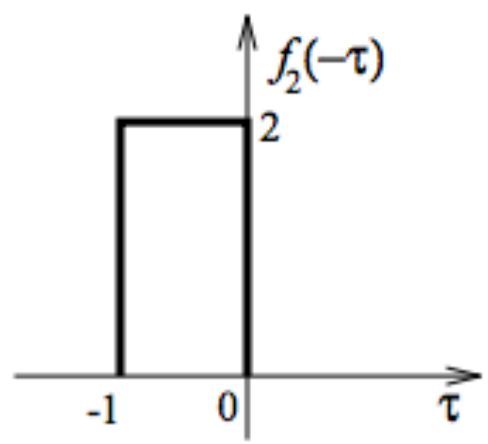

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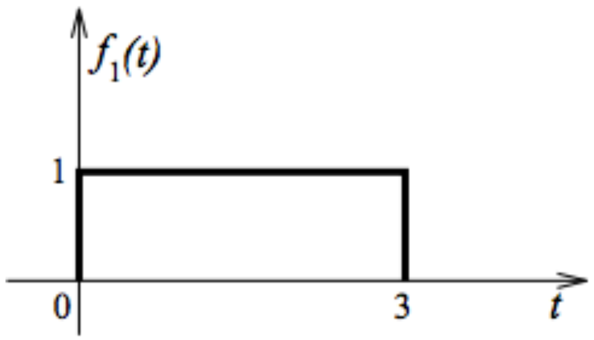

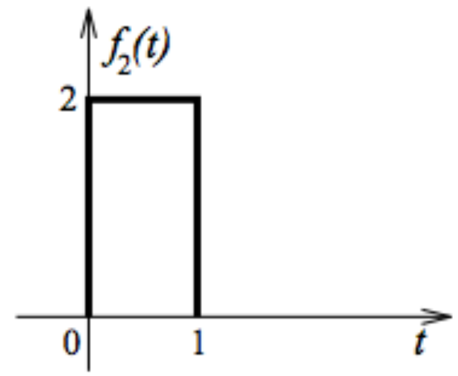

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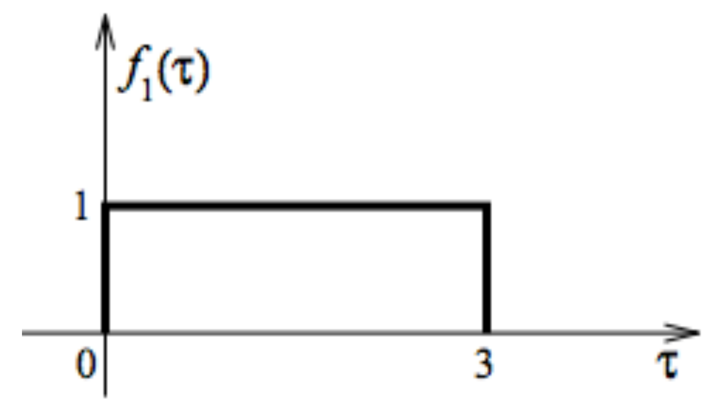

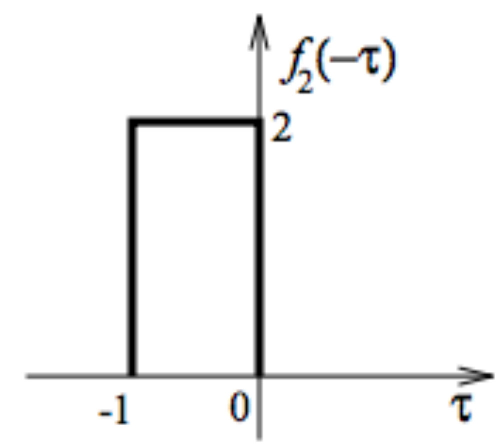

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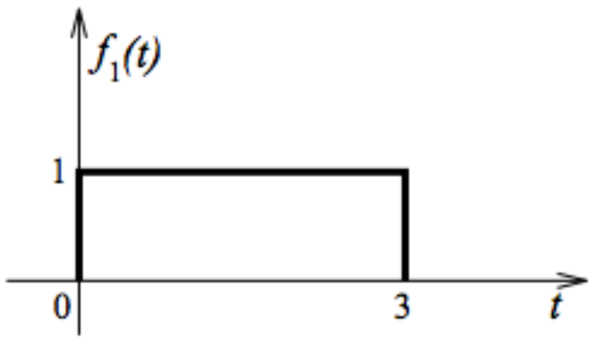

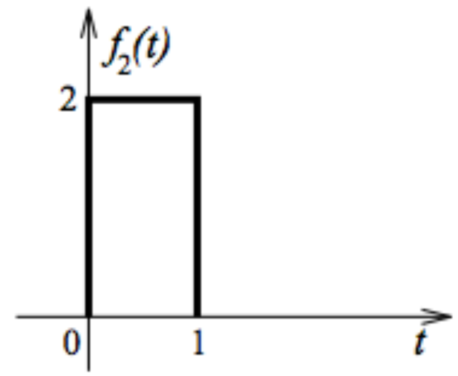

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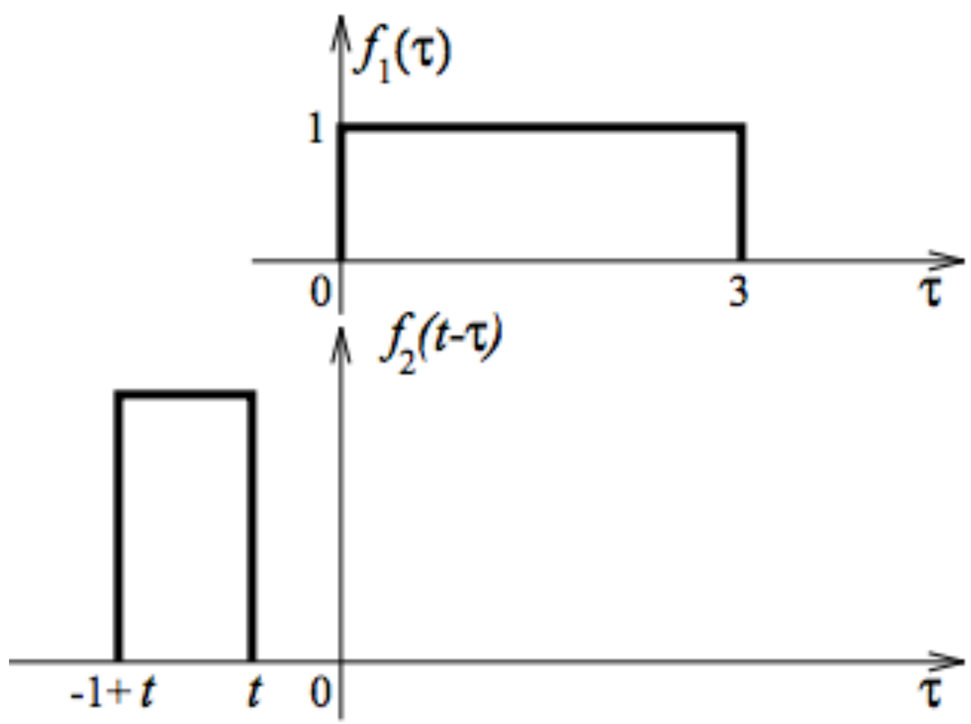

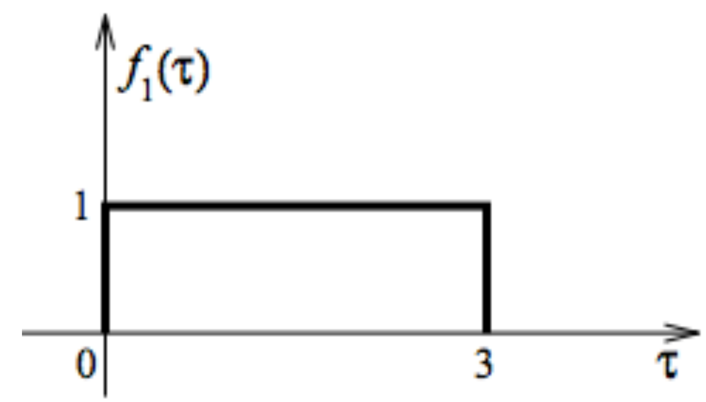

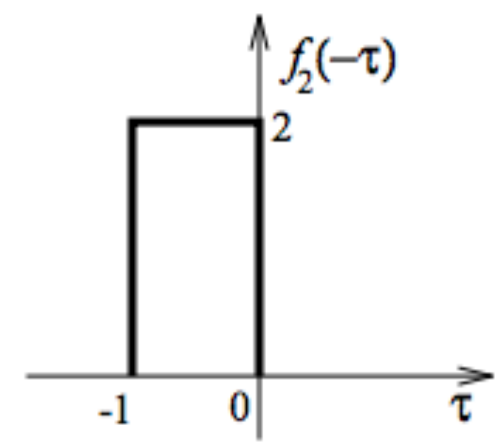

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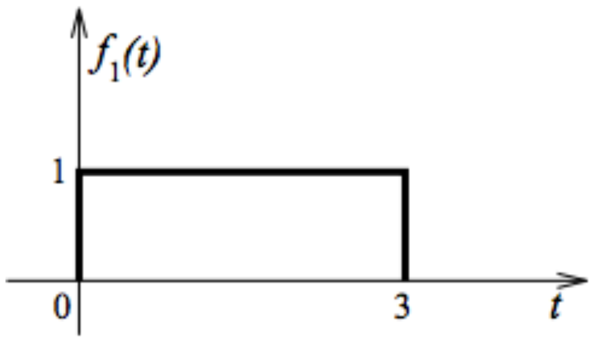

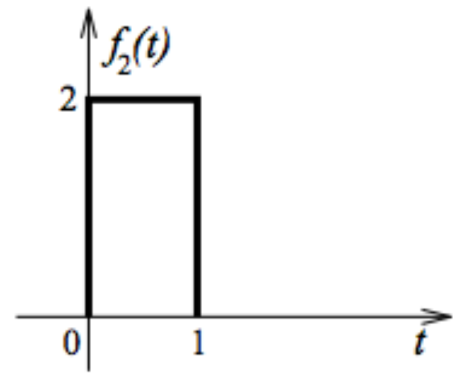

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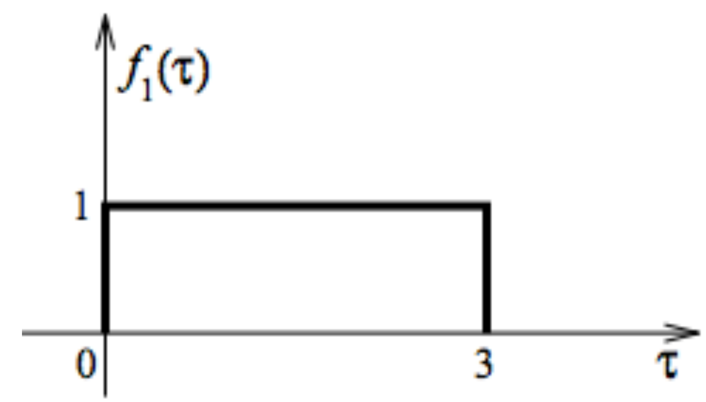

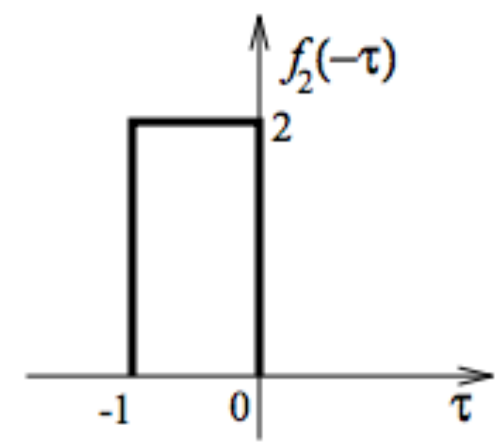

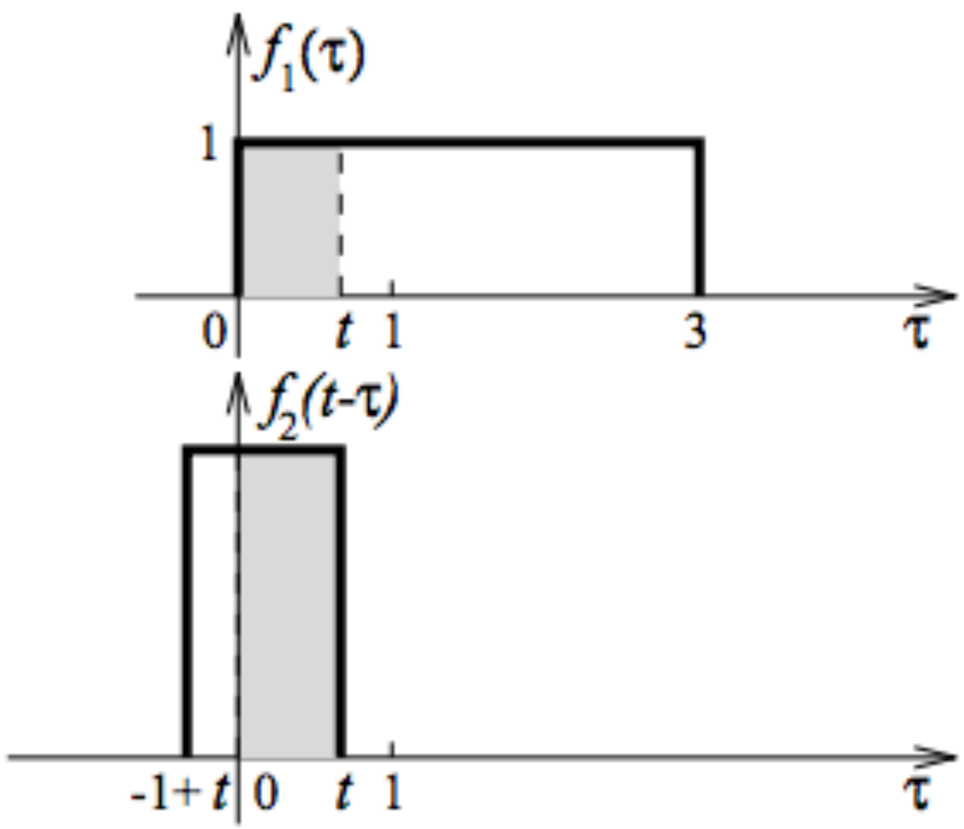

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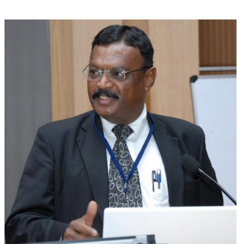
http://eceweb1.rutgers.edu/~gajic/solmanual/slides/chapter6C.pdf



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Convolution Theorem

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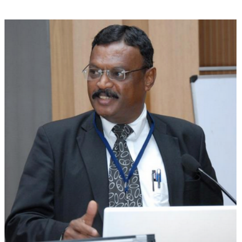
$$F\{f(t)*g(t)\}=F(s).G(s)$$



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Proof of this theorem was illustrated on the Board

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Parseval’s theorem

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Parseval's theorem

$$\int_{-\infty}^{\infty} f(t)\overline{g(t)}\,dt = \int_{-\infty}^{\infty} \hat{f}(s)\overline{\hat{g}(s)}\,ds$$

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Parseval's theorem

$$\int_{-\infty}^{\infty} f(t)\overline{g(t)}\, dt = \int_{-\infty}^{\infty} \hat{f}(s)\overline{\hat{g}(s)}\, ds$$

$$Taking\ g(t) = f(t)\ \ gives \int_{-\infty}^{\infty} \left|f(t)\right|^2 dt = \int_{-\infty}^{\infty} \left|\hat{f}(s)\right|^2 ds$$

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Relation between Laplace and Fourier transformation

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Relation between Laplace and Fourier transformation

$$If\ f(t)=\begin{cases} e^{-\alpha t}g(t) & t>0 \\ 0 & t<0 \end{cases}$$

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Relation between Laplace and Fourier transformation

$$If\ f(t) = \begin{cases} e^{-\alpha t} g(t) & t > 0 \\ 0 & t < 0 \end{cases}$$

$$Then\ L\big(g(t)\big) = F\big(f(t)\big)$$

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Relation between Laplace and Fourier transformation

$$If\ f(t) = \begin{cases} e^{-\alpha t} g(t) & t > 0 \\ 0 & t < 0 \end{cases}$$

$$Then\ L(g(t)) = F(f(t))$$

$$F(f(t)) = \int_{t=-\infty}^{\infty} f(t)\, e^{ist} dt = \int_{t=0}^{\infty} e^{-\alpha t}\, e^{ist} g(t)\, dt$$

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Relation between Laplace and Fourier transformation

$$If\ f(t)=\begin{cases} e^{-\alpha t}g(t) & t>0 \\ 0 & t<0 \end{cases} \qquad Then\ L\left(g(t)\right)=F\left(f(t)\right)$$

$$F\left(f(t)\right) = \int_{t=-\infty}^{\infty} f(t)\,e^{ist}dt = \int_{t=0}^{\infty} e^{-\alpha t}\,e^{ist}g\left(t\right)dt$$

$$= \int_{t=0}^{\infty} e^{-(\alpha-is)t}g\left(t\right)dt$$

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Relation between Laplace and Fourier transformation

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$$= \int_{t=0}^{\infty} e^{-(\alpha - is)t} g(t)\,dt = \int_{t=0}^{\infty} e^{-yt} g(t)\,dt$$

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$$= L\big(g(t)\big)$$

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Transforms of Derivatives

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$$We\ have\ F\left(u(x,t)\right)=\int_{-\infty}^{\infty}u(x,t)e^{isx}\,dx$$

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$$We\ have\ F\big(u(x,t)\big) = \int_{-\infty}^{\infty} u(x,t)e^{isx}\,dx$$

$$\therefore\ F\left(\frac{\partial^2 u}{\partial x^2}\right) = \int_{-\infty}^{\infty} \frac{\partial^2 u}{\partial x^2} e^{isx}\,dx$$

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$$=-s^2F(u)$$

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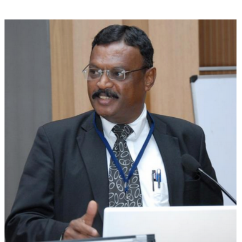
$$F_s\left(\frac{\partial^2 u}{\partial x^2}\right) = \int_0^{\infty} \frac{\partial^2 u}{\partial x^2} \sin sx\,dx = s\,u(0,t) - s^2 F_s(u)$$



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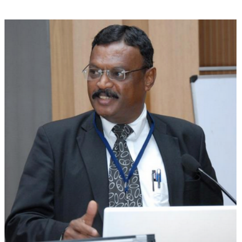
$$F_c\left(\frac{\partial^2 u}{\partial x^2}\right) = \int_0^{\infty} \frac{\partial^2 u}{\partial x^2} \cos sx\,dx = -\frac{\partial u(0,t)}{\partial x} - s^2 F_c(u)$$



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$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}, \quad a = \sqrt{\frac{T}{\rho}}$$

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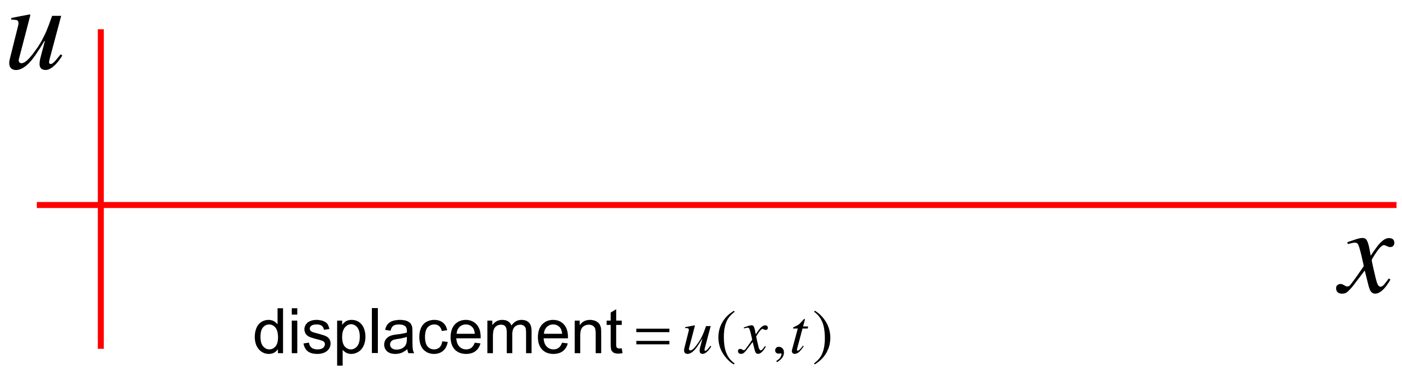

displacement $= u(x,t)$

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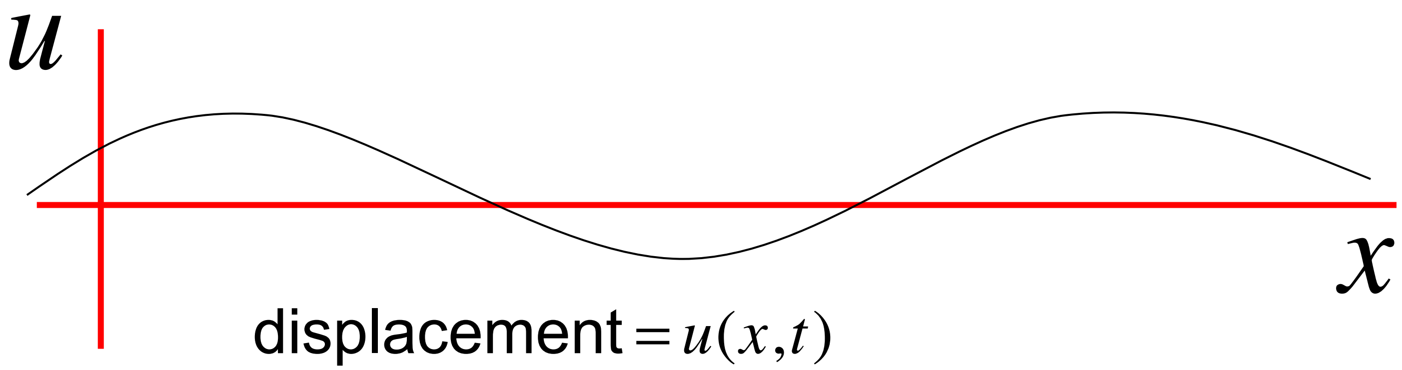

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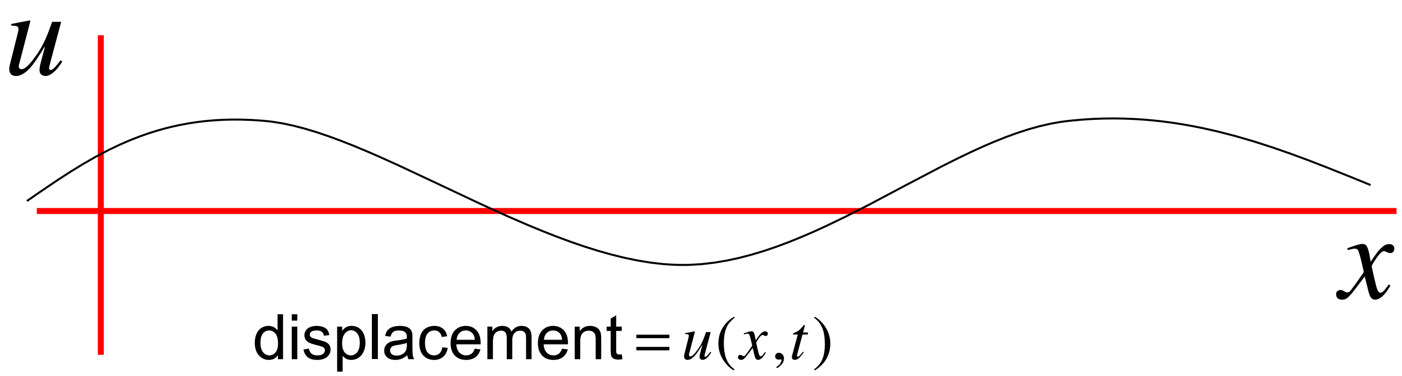
Wave equation



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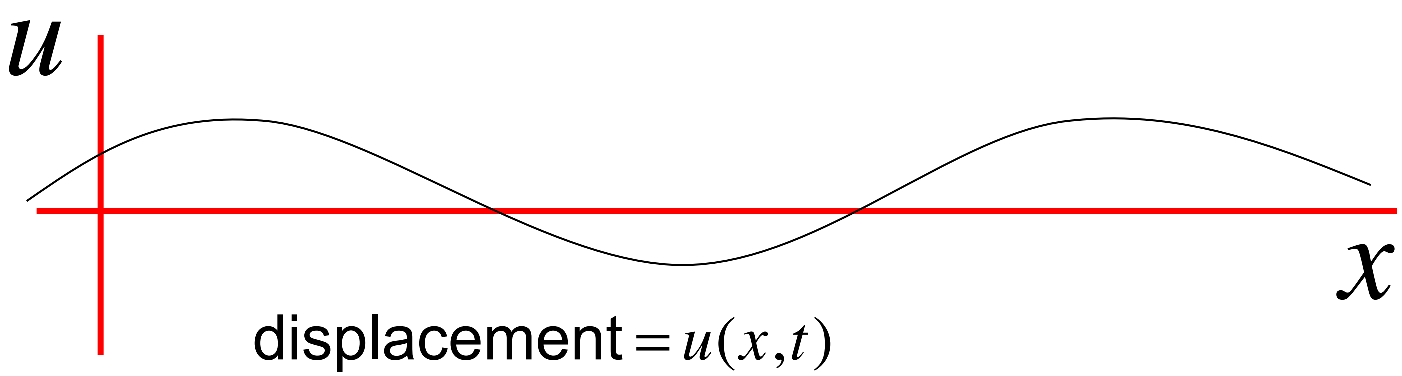

Assuming that a stretched string is vibrating. The wave equation says that, at any position on the string, acceleration in the direction perpendicular to the string is proportional to the curvature of the string.

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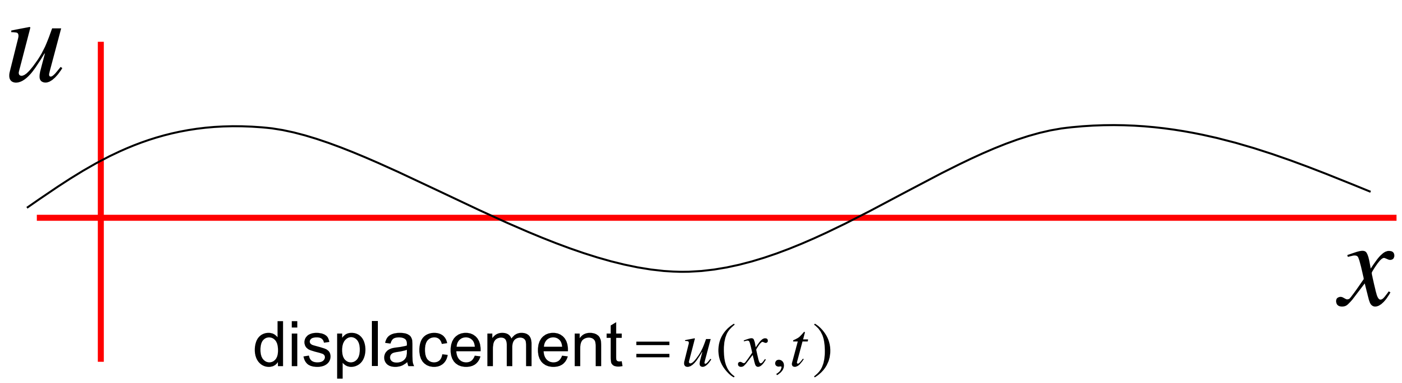

Let

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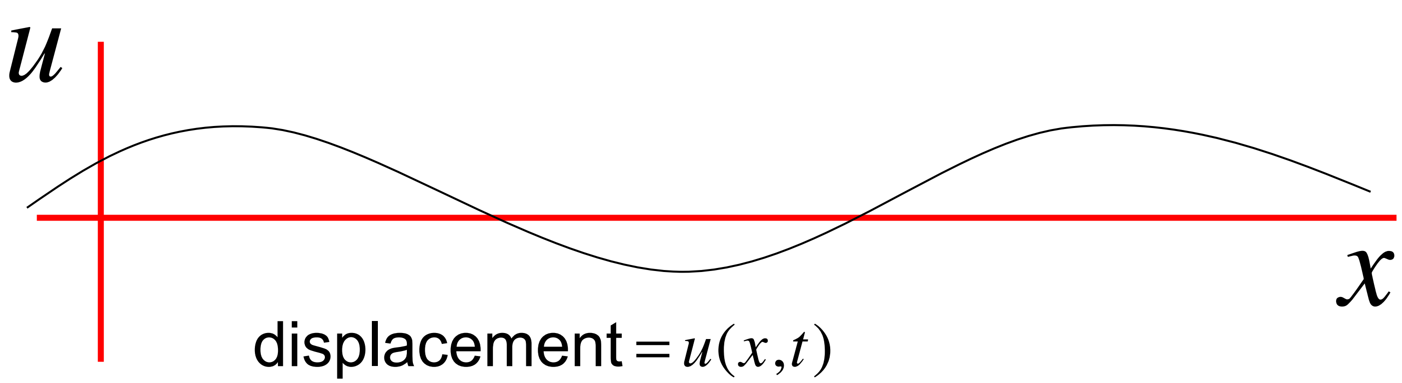

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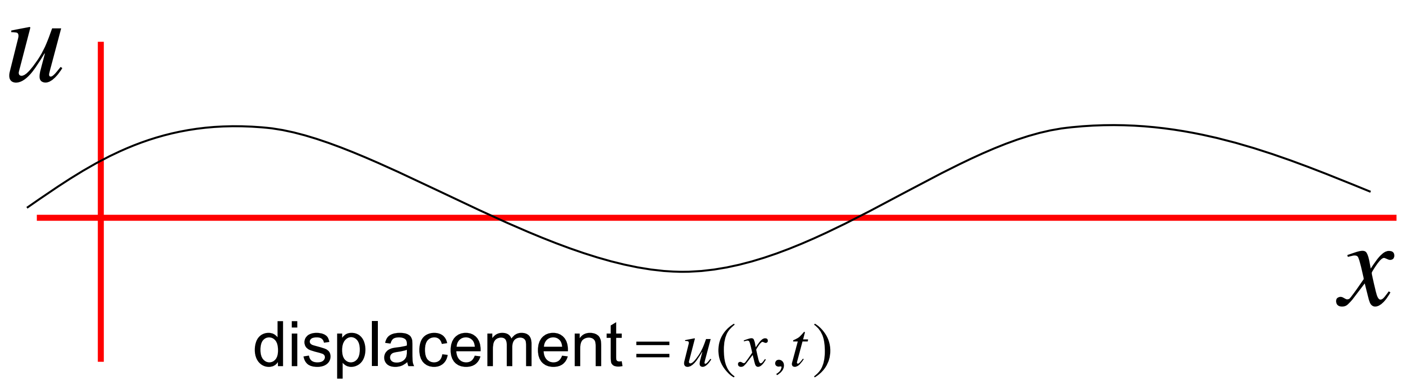

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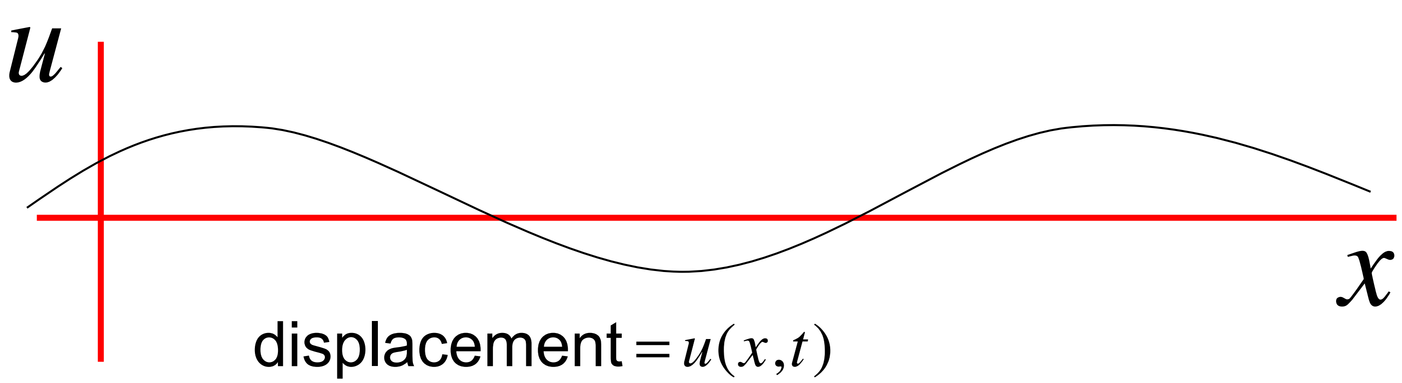

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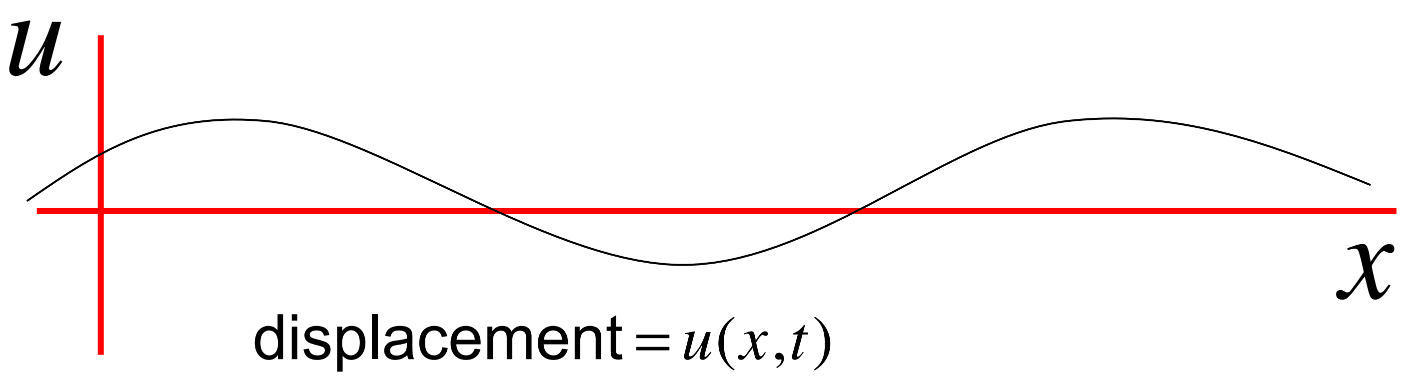

Let

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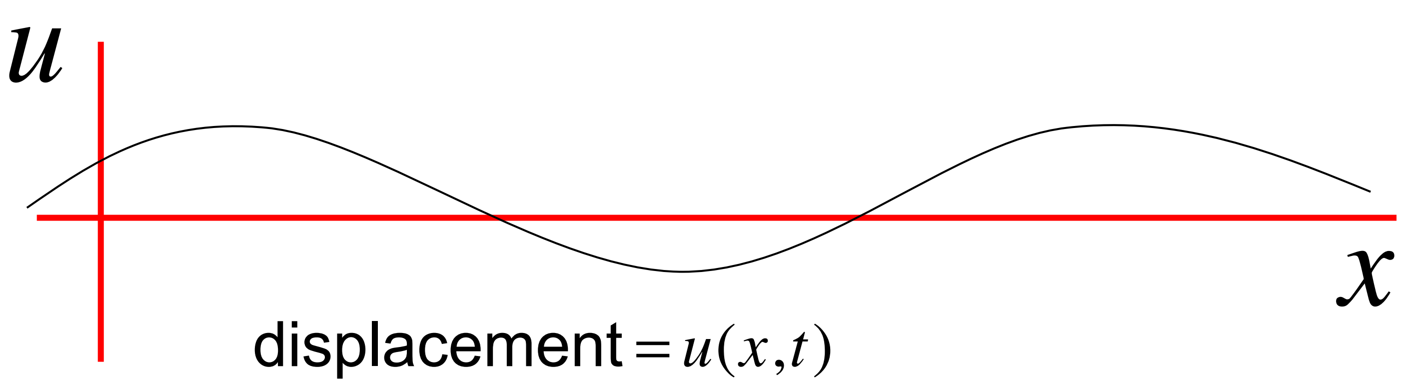

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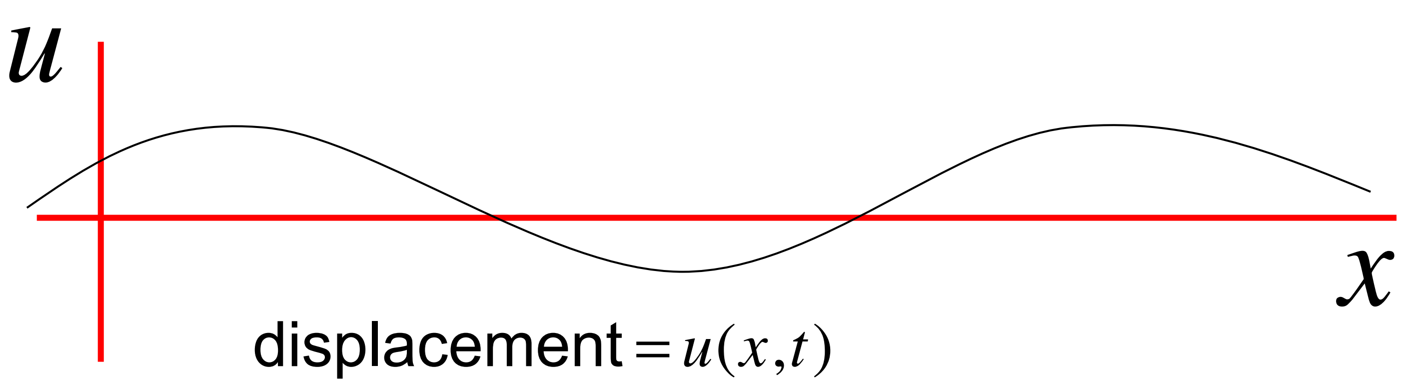

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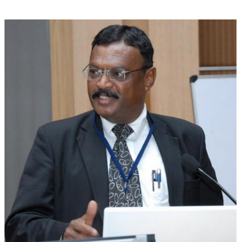

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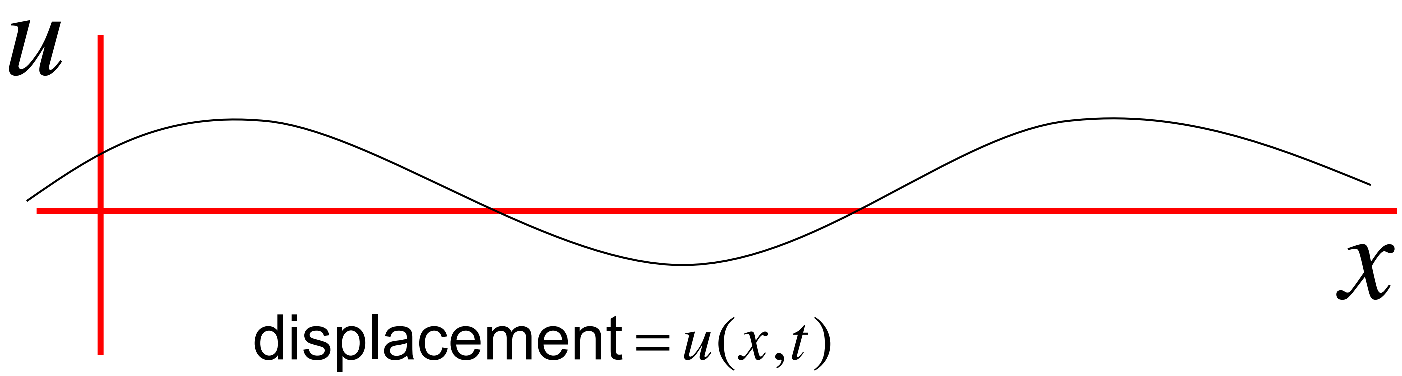
Wave equation



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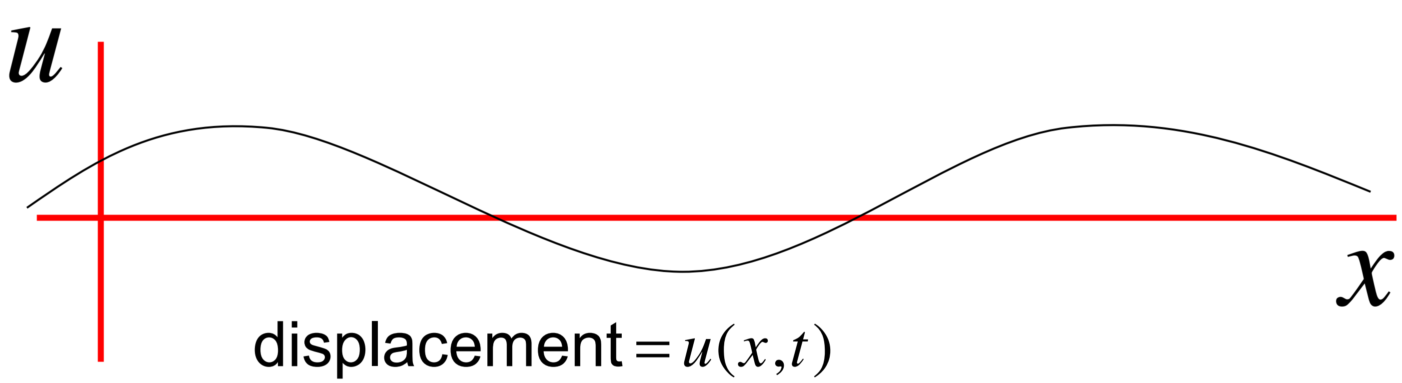
Wave equation



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Then

$$\rho \frac{\partial^2}{\partial t^2} u(x,t) = T \frac{\partial^2}{\partial x^2} u(x,t)$$

Assuming that a stretched string is vibrating. The wave equation says that, at any position on the string, acceleration in the direction perpendicular to the string is proportional to the curvature of the string.

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}, \quad a = \sqrt{\frac{T}{\rho}}$$

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Heat equation

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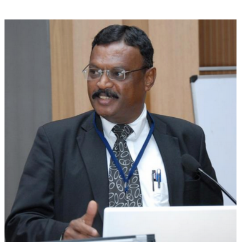
Heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$



Heat equation

- **The heat equation is a partial differential equation (PDE):**

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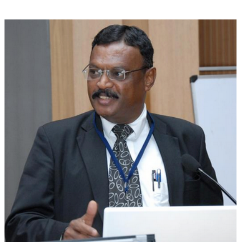
Heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

- The heat equation is a partial differential equation (PDE):
- c is the diffusion coefficient.
- Assume the initial distribution is a spike at x=0 and is zero elsewhere.



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Solve, using *Fourier transformation* $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ $(x, t > 0)$

with $u(x,0) = f(x)$ *and* $u(0,t) = 0$

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Solve, using *Fourier transformation* $\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2} \; (x, t > 0)$

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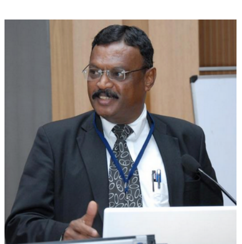
Taking FST we have,



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$$with\ u(x,0) = f(x)\quad and\ u(0,t) = 0$$

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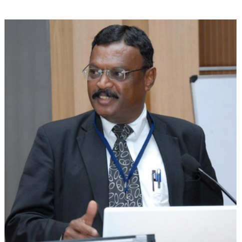

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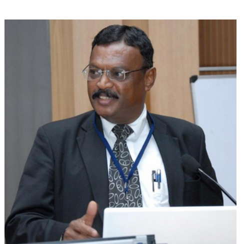
$$\therefore \frac{d}{dt}\left(F_s(u)\right) = c^2\left[s\,u(0,t) - s^2 F_s(u)\right]$$



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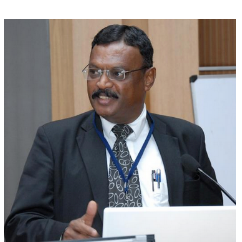
Taking FST we have, $\quad F_s\left(\dfrac{\partial u}{\partial t}\right) = F_s\left(\dfrac{\partial^2 u}{\partial x^2}\right) = s\,u(0,t) - s^2 F_s(u)$

$$\therefore \frac{d}{dt}\left(F_s(u)\right) = c^2\left[s\,u(0,t) - s^2 F_s(u)\right] \Rightarrow \frac{d}{dt}\left(F_s(u)\right) + c^2 s^2 F_s(u) = 0$$



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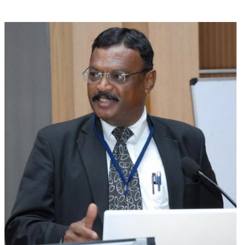
Also, $F_s(u) = \hat{f}(s)$ *at* $t = 0$



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Solve, using *Fourier transformation* $\dfrac{\partial u}{\partial t}=c^2\dfrac{\partial^2 u}{\partial x^2}\ (x,t>0)$

with $u(x,0)=f(x)$ *and* $u(0,t)=0$

Taking FST we have, $F_s\left(\dfrac{\partial u}{\partial t}\right)=F_s\left(\dfrac{\partial^2 u}{\partial x^2}\right)=s\,u(0,t)-s^2F_s(u)$

$$\therefore \frac{d}{dt}\left(F_s(u)\right)=c^2\left[s\,u(0,t)-s^2F_s(u)\right]\ \Rightarrow \frac{d}{dt}\left(F_s(u)\right)+c^2s^2F_s(u)=0$$

Also, $F_s(u)=\hat{f}(s)$ *at* $t=0$ *Hence* $F_s(u)=e^{-c^2s^2t}\hat{f}(s)$

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Solve, using *Fourier transformation* $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ $(x, t > 0)$

with $u(x,0) = f(x)$ *and* $u(0,t) = 0$

Taking F ST we have, $F_s\left(\frac{\partial u}{\partial t}\right) = F_s\left(\frac{\partial^2 u}{\partial x^2}\right) = s\,u(0,t) - s^2 F_s(u)$

$$\therefore \frac{d}{dt}(F_s(u)) = c^2\left[s\,u(0,t) - s^2 F_s(u)\right] \Rightarrow \frac{d}{dt}(F_s(u)) + c^2 s^2 F_s(u) = 0$$

Also, $F_s(u) = \hat{f}(s)$ *at* $t = 0$ *Hence* $F_s(u) = e^{-c^2 s^2 t}\hat{f}(s)$

Taking IF ST we have,

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Solve, using *Fourier transformation* $\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2} \; (x,t>0)$

with $u(x,0) = f(x)$ *and* $u(0,t) = 0$

Taking FST we have, $F_s\left(\dfrac{\partial u}{\partial t}\right) = F_s\left(\dfrac{\partial^2 u}{\partial x^2}\right) = s\,u(0,t) - s^2 F_s(u)$

$$\therefore \frac{d}{dt}\left(F_s(u)\right) = c^2\left[s\,u(0,t) - s^2 F_s(u)\right] \Rightarrow \frac{d}{dt}\left(F_s(u)\right) + c^2 s^2 F_s(u) = 0$$

Also, $F_s(u) = \hat{f}(s)$ *at* $t = 0$ *Hence* $F_s(u) = e^{-c^2 s^2 t}\hat{f}(s)$

Taking IFST we have,

$$u(x,t) = \frac{2}{\pi}\int_0^\infty e^{-c^2 s^2 t}\hat{f}(s).\sin st\, ds$$

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Left as an exercise

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Solve, using *Fourier transformation* $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \ (x, t > 0)$

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