Lecture delivered during the Teachers Enrichment Workshop held at IMSC between 26th November to 1st December 2018.

Good afternoon friends

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f = actual frequency
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v_o = velocity of the observer
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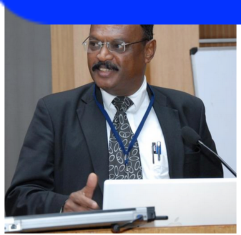
Christian Andreas Doppler

Born: 29 November 1803 in Salzburg, Austria

Died: 17 March 1853 in Venice (now Italy)



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Fourier Transform pairs

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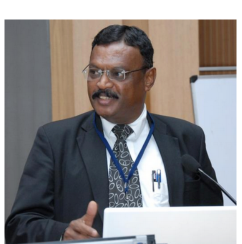

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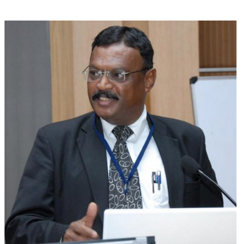

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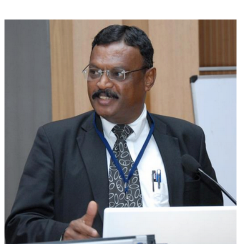

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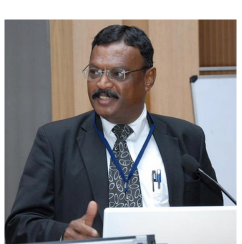

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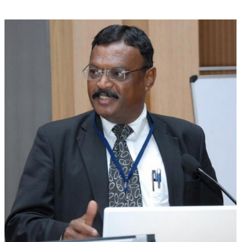

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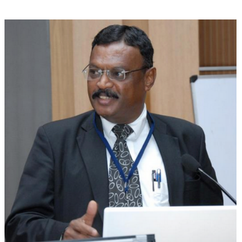

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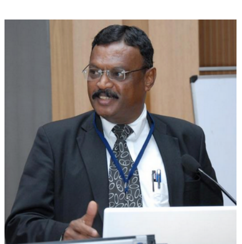

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0	∞	e^{-st}	**Laplace**
$-\infty$	∞	e^{-ist}	**Fourier**

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Transform	Symbol	K	$f(t)$	t_1	t_2	K^{-1}	u_1	u_2
Abel transform		$\frac{2t}{\sqrt{t^2-u^2}}$		u	∞	$\frac{-1}{\pi\sqrt{u^2-t^2}}\frac{d}{du}$	t	∞
Fourier transform	$\mathcal{F}$	$e^{-2\pi iut}$	L_1	$-\infty$	∞	$e^{2\pi iut}$	$-\infty$	∞
Fourier sine transform	$\mathcal{F}_s$	$\sqrt{\frac{2}{\pi}}\sin(ut)$	on $[0,\infty)$, real-valued	0	∞	$\sqrt{\frac{2}{\pi}}\sin(ut)$	0	∞
Fourier cosine transform	$\mathcal{F}_c$	$\sqrt{\frac{2}{\pi}}\cos(ut)$	on $[0,\infty)$, real-valued	0	∞	$\sqrt{\frac{2}{\pi}}\cos(ut)$	0	∞
Hankel transform		$t\,J_\nu(ut)$		0	∞	$u\,J_\nu(ut)$	0	∞
Hartley transform	$\mathcal{H}$	$\frac{\cos(ut)+\sin(ut)}{\sqrt{2\pi}}$		$-\infty$	∞	$\frac{\cos(ut)+\sin(ut)}{\sqrt{2\pi}}$	$-\infty$	∞
Hermite transform	H	$e^{-x^2}H_n(x)$		$-\infty$	∞		0	∞
Hilbert transform	$\mathcal{H}il$	$\frac{1}{\pi}\frac{1}{u-t}$		$-\infty$	∞	$\frac{1}{\pi}\frac{1}{u-t}$	$-\infty$	∞
Jacobi transform	J	$(1-x)^\alpha\ (1+x)^\beta\ P_n^{\alpha,\beta}(x)$		-1	1		0	∞
Laguerre transform	L	$e^{-x}\ x^\alpha\ L_n^\alpha(x)$		0	∞		0	∞
Laplace transform	$\mathcal{L}$	e^{-ut}		0	∞	$\frac{e^{ut}}{2\pi i}$	$c-i\infty$	$c+i\infty$
Legendre transform	$\mathcal{J}$	$P_n(x)$		-1	1		0	∞
Mellin transform	$\mathcal{M}$	t^{u-1}		0	∞	$\frac{t^{-u}}{2\pi i}$	$c-i\infty$	$c+i\infty$
Two-sided Laplace transform	$\mathcal{B}$	e^{-ut}		$-\infty$	∞	$\frac{e^{ut}}{2\pi i}$	$c-i\infty$	$c+i\infty$
Poisson kernel		$\frac{1-r^2}{1-2r\cos\theta+r^2}$		0	2π			
Weierstrass transform	$\mathcal{W}$	$\frac{e^{-\frac{(u-t)^2}{4}}}{\sqrt{4\pi}}$		$-\infty$	∞	$\frac{e^{\frac{(u-t)^2}{4}}}{i\sqrt{4\pi}}$	$c-i\infty$	$c+i\infty$

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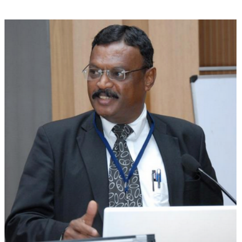

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$$\frac{1}{\sqrt{2\pi}}\left\{\left(\frac{2\cos s}{is}\right)+\left(\frac{2i\sin s}{s^2}\right)\right\}=\frac{i}{\sqrt{2\pi}}\left\{\left(\frac{2\sin s}{s^2}\right)-\left(\frac{2\cos s}{s}\right)\right\}$$

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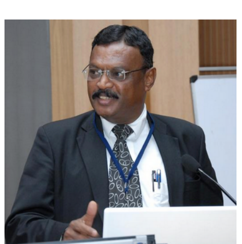

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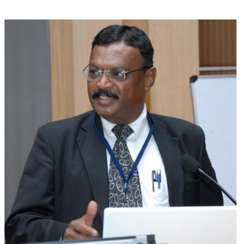
$$That\ is,\ \frac{i}{2\pi}\int_{s=-\infty}^{\infty}\left(\frac{\sin s - s\cos s}{s^2}\right)e^{-isx}\,ds = f(x)$$



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$$We\,have,\quad \widehat{f}_c(s) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t)\cos st\,dt \;=\sqrt{\frac{2}{\pi}}\int_0^{\infty} t^{29}\cos st\,dt$$

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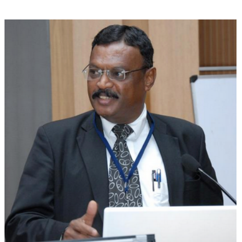

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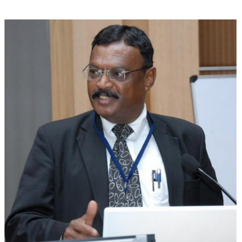

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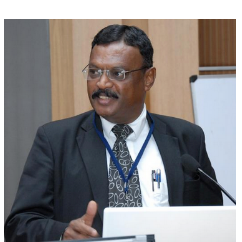
$$\text{Substituting, } ist=-\zeta \text{ we get, } \sqrt{\frac{2}{\pi}}\int_0^{\infty} t^{29}e^{ist}\,dt=\left(-\frac{1}{is}\right)^{30}\sqrt{\frac{2}{\pi}}\int_0^{\infty} e^{-\zeta}\zeta^{30-1}\,d\zeta$$



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$$=\left(-\frac{1}{is}\right)^{30}\sqrt{\frac{2}{\pi}}\int_0^\infty e^{-\zeta}\zeta^{30-1}\,d\zeta=\left(\frac{i}{s}\right)^{30}\sqrt{\frac{2}{\pi}}\Gamma(30)$$

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$$= \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2} \right)^{30} \sqrt{\frac{2}{\pi}}\, \frac{1}{s^{30}} \Gamma(30)$$

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$$\therefore\ \hat{f}_c(s)=\frac{\cos 15\pi}{s^{30}}\sqrt{\frac{2}{\pi}}\Gamma(30)=-\sqrt{\frac{2}{\pi}}\frac{29!}{s^{30}}$$

$$=\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)^{30}\sqrt{\frac{2}{\pi}}\,\frac{1}{s^{30}}\Gamma(30)$$

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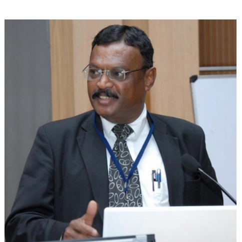
$\left(for\ \ n>0\right)$ Prove that

$$\widehat{f}_c(s)+i\widehat{f}_s(s)=\sqrt{\frac{2}{\pi}}\int_0^\infty t^{n-1}\,e^{ist}\,dt=\left(\cos n\pi+i\sin n\pi\right)\sqrt{\frac{2}{\pi}}\frac{1}{s^{n}}\Gamma(n)$$



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Find the Fourier cosine *transform of* $f(x)=e^{-x^2}$

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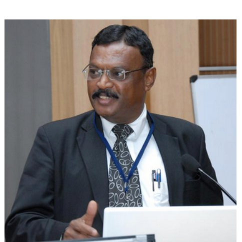
$$= -\int_0^\infty t e^{-t^2} \sin st\, dt = \frac{1}{2}\int_0^\infty \sin st\left(-2te^{-t^2}\right)dt = \frac{1}{2}\int_0^\infty \sin st\; d\left(e^{-t^2}\right)$$



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$$= \frac{1}{2}\left\{\sin st\,.e^{-t^2}\Big|_0^\infty - s\int_0^\infty \cos st\, e^{-t^2}\,dt\right\}$$

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$$=-\frac{s}{2}\int_0^\infty e^{-t^2}.\cos st\,dt=-\frac{s}{2}I$$

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$$\therefore \int_0^\infty e^{-t^2} \cos st \, dt = k e^{-\frac{s^2}{4}}$$

$$k = \int_0^\infty e^{-t^2} \, dt = \frac{\sqrt{\pi}}{2}$$

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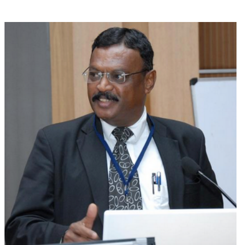

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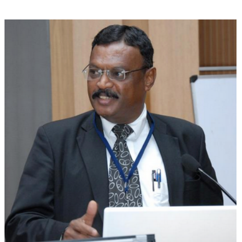

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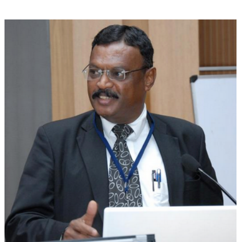
$$Therefore \quad \widehat{f_c}(s) = \sqrt{\frac{2}{\pi}} \times I = \sqrt{\frac{2}{\pi}} \times \frac{\sqrt{\pi}}{2} e^{-\frac{s^2}{4}} = \frac{1}{2} e^{-\frac{s^2}{4}}$$



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$$\textit{Find the Fourier } \sin e\, \textit{transform of } f(x)=\frac{e^{-\alpha x}}{x}$$

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$$Find\ the\ Fourier\ \text{sine}\ transform\ of\ \ f(x)=\frac{e^{-\alpha x}}{x}$$

$$We\,have,\quad \hat{f}_s(s)=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t)\sin st\,dt\ =\sqrt{\frac{2}{\pi}}\int_0^{\infty}\frac{e^{-\alpha t}}{t}\sin st\,dt$$

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Let $F(s)=\int_0^\infty \dfrac{e^{-\alpha t}}{t}\sin st\,dt$ then,

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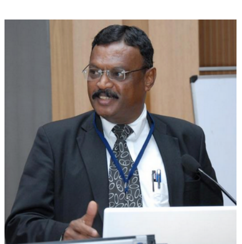

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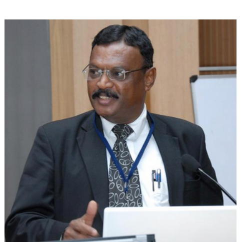

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Properties of Fourier transformations

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Properties of Fourier transformations

1. *Linearity*

$$F\left(\alpha f(t)+\beta g(t)\right)=\alpha F\left(f(t)\right)+\beta F\left(g(t)\right)$$

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2. *Change of scale*

$$F\left(f(\alpha t)\right)=\frac{1}{\alpha}F\left(\frac{s}{\alpha}\right),\ \alpha\neq 0$$

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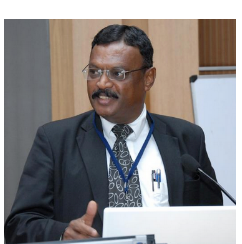
$$F\left(f(\alpha t)\right)=\frac{1}{\alpha}F\left(\frac{s}{\alpha}\right),\ \alpha\neq 0$$

3. *Shifting*

$$F\left(f\left(t-a\right)\right)=e^{ias}F\left(f(t)\right)$$



Lecture delivered during the Teachers Enrichment Workshop held at IMSC between 26th November to 1st December 2018.

Properties of Fourier transformations

$1.\ Linearity$

$$F\left(\alpha f(t)+\beta g(t)\right)=\alpha F\left(f(t)\right)+\beta F\left(g(t)\right)$$

$2.\ Change of\ scale$

$$F\left(f(\alpha t)\right)=\frac{1}{\alpha}F\left(\frac{s}{\alpha}\right),\ \alpha\neq 0$$

$3.\ Shifting$

$$F\left(f\left(t-a\right)\right)=e^{ias}F\left(f(t)\right)$$

$4.\ Modulation$

$$F\left(f(t)\cos\alpha t\right)=\frac{1}{2}\left\{F\left(s+a\right)+F\left(s-a\right)\right\}$$

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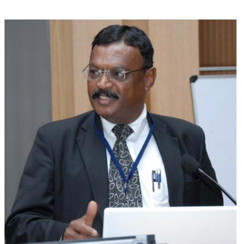
Properties of Fourier transformations



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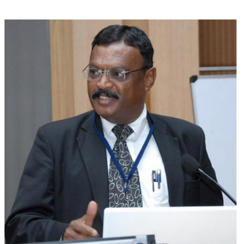
Properties of Fourier transformations

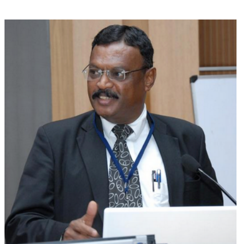
Establish that $F_s\left(t.f(t)\right) = -\frac{d}{ds}F_c\left(f(t)\right)$



Properties of Fourier transformations

$$\textit{Establish that } F_s\left(t.f(t)\right) = -\frac{d}{ds}F_c\left(f(t)\right)$$

$$\textit{Establish that } F_c\left(t.f(t)\right) = \frac{d}{ds}F_s\left(f(t)\right)$$



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