Lecture delivered during the Teachers Enrichment Workshop held at IMSC between 26th November to 1st December 2018.

Good afternoon friends

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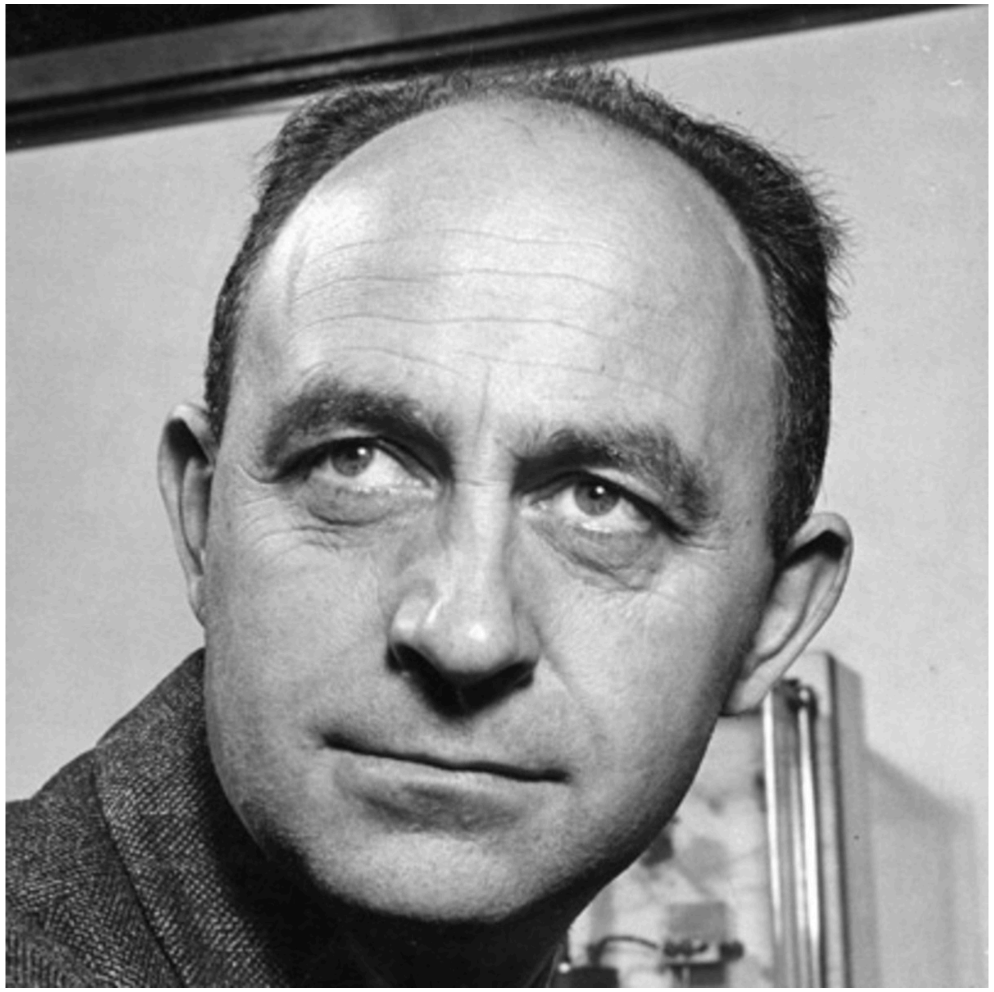

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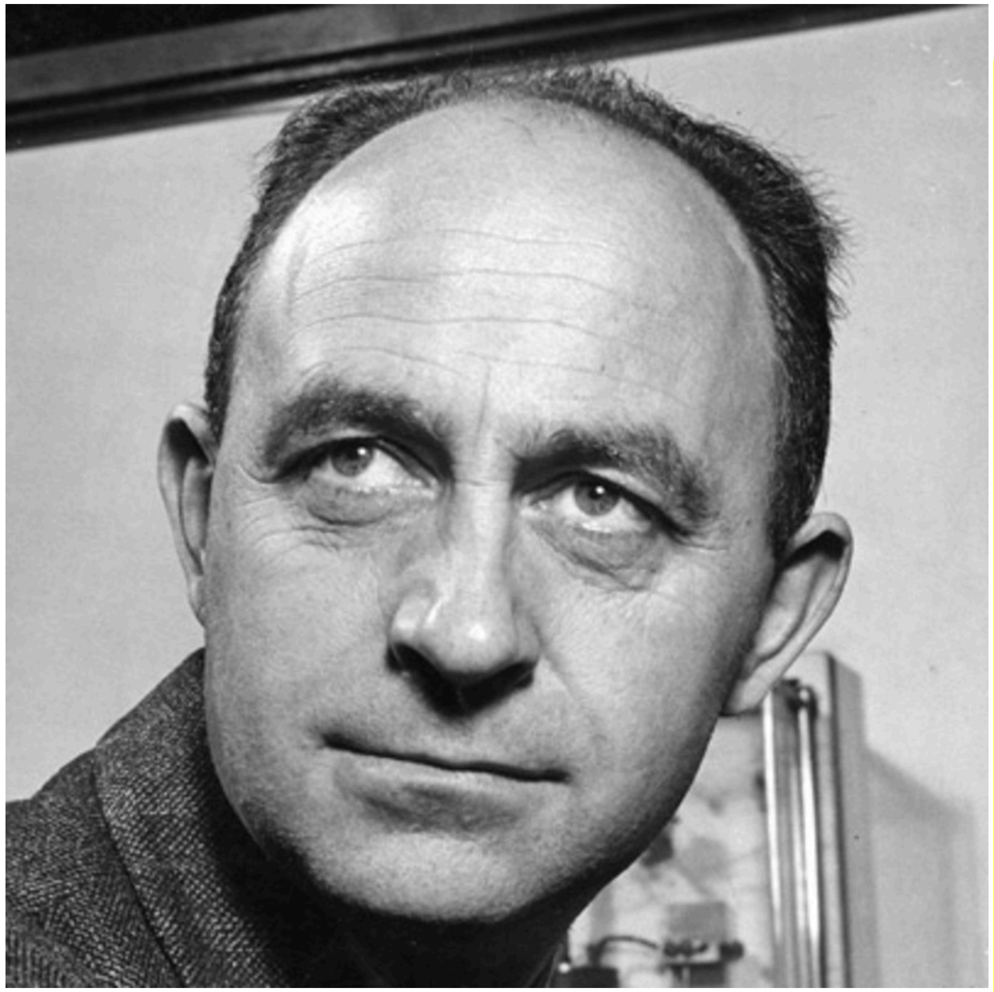
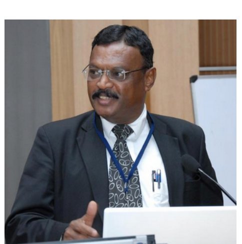


1. was an Italian physicist who created the world's first nuclear reactor.



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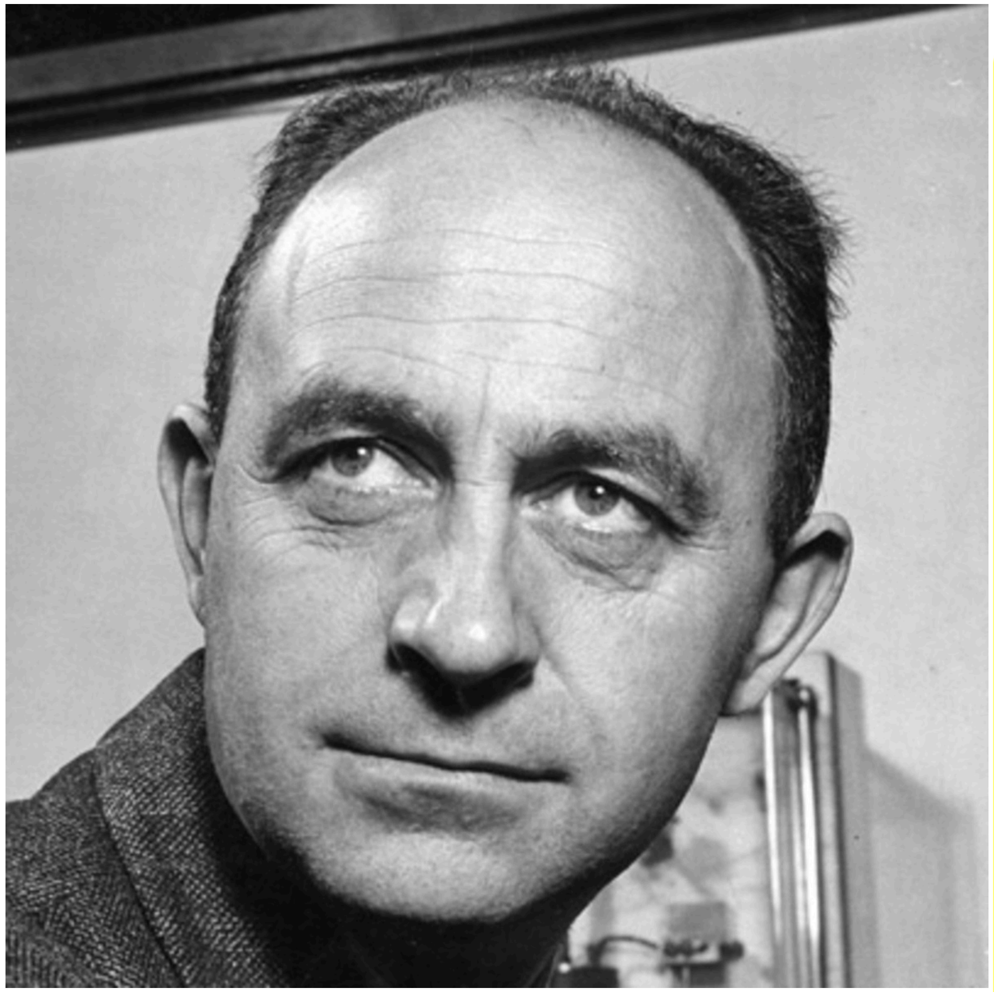

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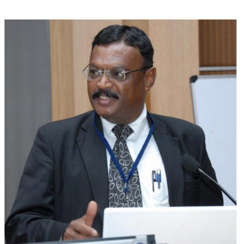
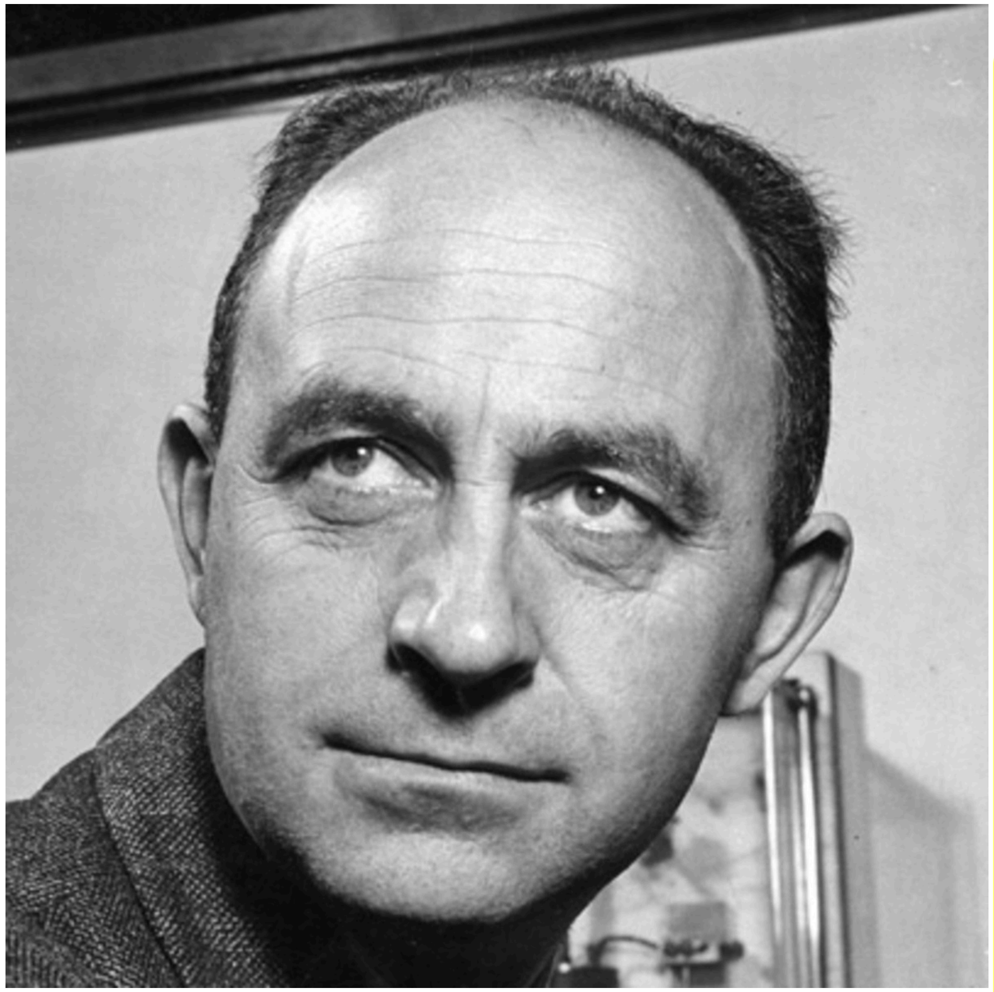
2. He made important contributions to the development of quantum theory, nuclear and particle physics and to statistical mechanics.





1. was an Italian physicist who created the world's first nuclear reactor.
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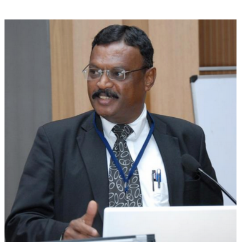

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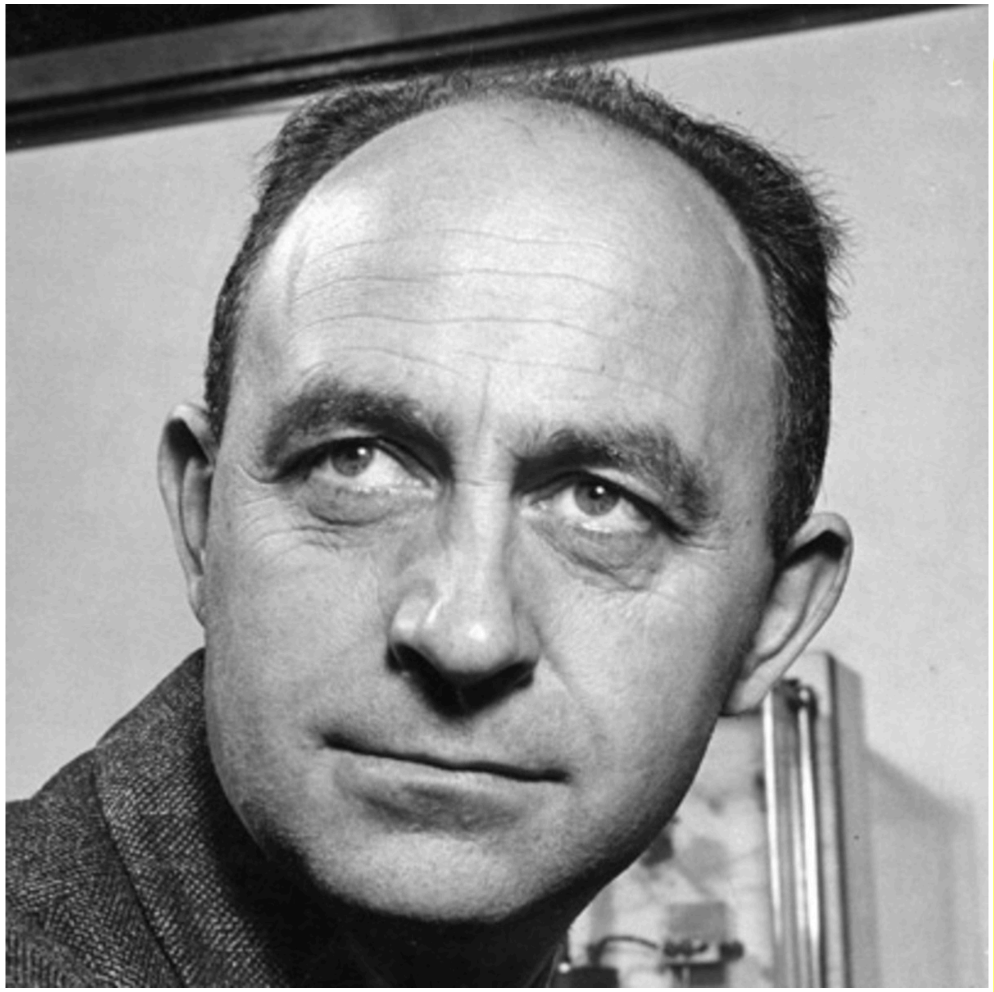

Enrico Fermi

Born: 29 September 1901 in Rome, Italy

Died: 28 November 1954 in Chicago, Illinois, USA

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Practical Harmonic Analysis

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$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt = 2\times\frac{1}{2\pi}\int_{0}^{2\pi} f(t)\,dt$$

$$= 2\left[mean\,value\,of\ f(t)\,in\,(0,2\pi\right]$$

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$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\ dt = 2\times\frac{1}{2\pi}\int_0^{2\pi} f(t)\cos nt\ dt$$

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$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\ dt = 2\times\frac{1}{2\pi}\int_0^{2\pi} f(t)\sin nt\ dt$$

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Various harmonics

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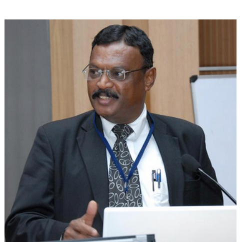

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A harmonic is a signal or wave whose frequency is an integral (whole-number) multiple of the frequency of some reference signal or wave.

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$$a_1 \cos x + b_1 \sin x - \textit{the fundamental harmonic}$$

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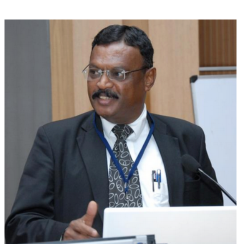
$$a_1 \cos x + b_1 \sin x - \textit{the fundamental harmonic}$$

$$a_2 \cos 2x + b_2 \sin 2x - \textit{second harmonic}$$



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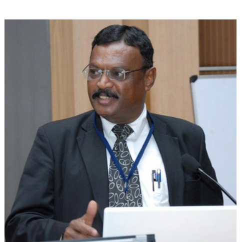
$$a_n \cos nx + b_n \sin nx - n^{th} \textit{ harmonic}$$



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$$y = \frac{a_0}{2} + a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta$$

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Obtain the first 3 coefficients in the Fourier cosine series for y where y is given by the table

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Obtain the first 3 coefficients in the Fourier cosine series for y where y is given by the table

x
y

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Obtain the first 3 coefficients in the Fourier cosine series for y where y is given by the table

x	0
y	4

$$y = \frac{a_0}{2} + a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta$$

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Obtain the first 3 coefficients in the Fourier cosine series for y where y is given by the table

x	0	1
y	4	8

$$y = \frac{a_0}{2} + a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta$$

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Obtain the first 3 coefficients in the Fourier cosine series for y where y is given by the table

x	0	1	2
y	4	8	15

$$y = \frac{a_0}{2} + a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta$$

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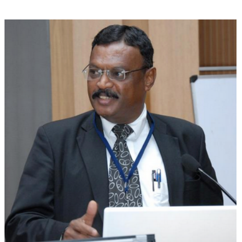
Obtain the first 3 coefficients in the Fourier cosine series for y where y is given by the table

x	0	1	2	3
y	4	8	15	7

$$y = \frac{a_0}{2} + a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta$$



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Obtain the first 3 coefficients in the Fourier cosine series for y where y is given by the table

x	0	1	2	3	4
y	4	8	15	7	6

$$y = \frac{a_0}{2} + a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta$$

Obtain the first 3 coefficients in the Fourier cosine series for y where y is given by the table

x	0	1	2	3	4	5
y	4	8	15	7	6	2

$$y = \frac{a_0}{2} + a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta$$

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Obtain the first 3 coefficients in the Fourier cosine series for y where y is given by the table

x	0	1	2	3	4	5
y	4	8	15	7	6	2

Taking the intervals as 60 degree we get

$$y = \frac{a_0}{2} + a_1 \cos\theta + a_2 \cos 2\theta + a_3 \cos 3\theta$$

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θ	0	60	120	180	240	300

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Taking the intervals as 60 degree we get

θ	0	60	120	180	240	300
x	0	1	2	3	4	5
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The Fourier cosine series for y is

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θ	$cos\theta$	$cos2\theta$	$cos3\theta$	y	$ycos\theta$	$ycos2\theta$	$ycos3\theta$

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θ	$\cos\theta$	$\cos 2\theta$	$\cos 3\theta$	y	$y\cos\theta$	$y\cos 2\theta$	$y\cos 3\theta$
0	1	1	1	4	4	4	4

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θ	$\cos\theta$	$\cos 2\theta$	$\cos 3\theta$	y	$y\cos\theta$	$y\cos 2\theta$	$y\cos 3\theta$
0	1	1	1	4	4	4	4
60	0.5	-0.5	-1	8	4	-4	-8

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θ	$\cos\theta$	$\cos 2\theta$	$\cos 3\theta$	y	$y\cos\theta$	$y\cos 2\theta$	$y\cos 3\theta$
0	1	1	1	4	4	4	4
60	0.5	-0.5	-1	8	4	-4	-8
120	-0.5	-0.5	1	15	-7.5	-7.5	15

θ	$\cos\theta$	$\cos 2\theta$	$\cos 3\theta$	y	$y\cos\theta$	$y\cos 2\theta$	$y\cos 3\theta$
0	1	1	1	4	4	4	4
60	0.5	-0.5	-1	8	4	-4	-8
120	-0.5	-0.5	1	15	-7.5	-7.5	15
180	-1	1	-1	7	-7	7	-7

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θ	$cos\theta$	$cos2\theta$	$cos3\theta$	y	$ycos\theta$	$ycos2\theta$	$ycos3\theta$
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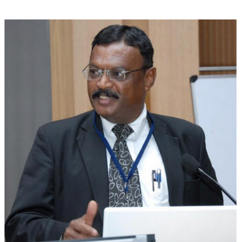
60	0.5	-0.5	-1	8	4	-4	-8
120	-0.5	-0.5	1	15	-7.5	-7.5	15
180	-1	1	-1	7	-7	7	-7
240	-0.5	-0.5	1	6	-3	-3	6



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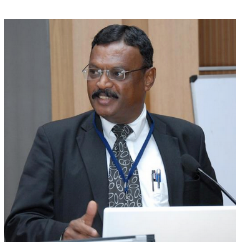
180	-1	1	-1	7	-7	7	-7
240	-0.5	-0.5	1	6	-3	-3	6
300	0.5	-0.5	-1	2	1	-1	-2



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180	-1	1	-1	7	-7	7	-7
240	-0.5	-0.5	1	6	-3	-3	6
300	0.5	-0.5	-1	2	1	-1	-2
Total				42	-8.5	-4.5	8

θ	$cos\theta$	$cos2\theta$	$cos3\theta$	y	$ycos\theta$	$ycos2\theta$	$ycos3\theta$
0	1	1	1	4	4	4	4
60	0.5	-0.5	-1	8	4	-4	-8
120	-0.5	-0.5	1	15	-7.5	-7.5	15
180	-1	1	-1	7	-7	7	-7
240	-0.5	-0.5	1	6	-3	-3	6
300	0.5	-0.5	-1	2	1	-1	-2
Total				42	-8.5	-4.5	8
Two	times	mean	value	14	-2.83333333333333	-1.5	2.66666666666667

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240	-0.5	-0.5	1	6	-3	-3	6
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Total				42	-8.5	-4.5	8
Two	times	mean	value	14	-2.83333333333333	-1.5	2.66666666666667
				a_0	a_1	a_2	a_3

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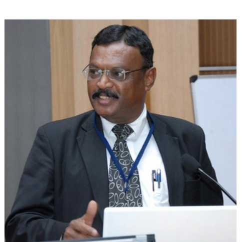
300	0.5	-0.5	-1	2	1	-1	-2
Total				42	-8.5	-4.5	8
Two	times	mean	value	14	-2.83333333333333	-1.5	2.66666666666667
				a_0	a_1	a_2	a_3

$$y = 7 - 2.8\cos\theta - 1.5\cos 2\theta + 2.7\cos 3\theta$$



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We have,

$$f(x)=\frac{a_0}{2}+\sum_{1}^{\infty}a_n\ cos\left(\frac{n\pi x}{c}\right)+\sum_{1}^{\infty}b_n\ sin\left(\frac{n\pi x}{c}\right)$$

$$=\left\{\begin{array}{l}\frac{1}{2c}\int_{-c}^{c}f(t)dt+\frac{1}{c}\sum_{1}^{\infty}\left(\int_{-c}^{c}f(t)cos\left(\frac{n\pi t}{c}\right)dt\right)\cos\left(\frac{n\pi x}{c}\right)+\\ \qquad +\frac{1}{c}\sum_{1}^{\infty}\left(\int_{-c}^{c}f(t)\sin\left(\frac{n\pi t}{c}\right)dt\right)\sin\left(\frac{n\pi x}{c}\right)\end{array}\right.$$

$$=\frac{1}{2c}\int_{-c}^{c}f(t)dt+\frac{1}{c}\sum_{1}^{\infty}\int_{-c}^{c}f(t)cos\left(\frac{n\pi\overline{t-x}}{c}\right)dt$$

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Fourier integrals

We have,

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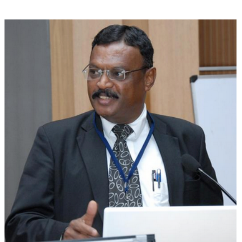
$$=\left\{\begin{array}{l}\displaystyle\frac{1}{2c}\int_{-c}^{c}f(t)dt+\frac{1}{c}\sum_{1}^{\infty}\left(\int_{-c}^{c}f(t)cos\left(\frac{n\pi t}{c}\right)dt\right)\cos\left(\frac{n\pi x}{c}\right)+\\ \displaystyle\qquad\qquad+\frac{1}{c}\sum_{1}^{\infty}\left(\int_{-c}^{c}f(t)\sin\left(\frac{n\pi t}{c}\right)dt\right)\sin\left(\frac{n\pi x}{c}\right)\end{array}\right.$$

$$=\frac{1}{2c}\int_{-c}^{c}f(t)dt+\frac{1}{c}\sum_{1}^{\infty}\int_{-c}^{c}f(t)cos\left(\frac{n\pi\overline{t-x}}{c}\right)dt$$



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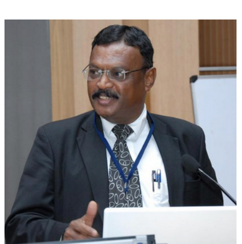

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$$= \frac{1}{2c}\int_{-c}^{c} f(t)\,dt + \frac{1}{c}\sum_{1}^{\infty}\int_{-c}^{c} f(t)\cos\left(\frac{n\pi\overline{t-x}}{c}\right)dt$$

$$Now\text{, assuming } \int_{-c}^{c}|f(t)|\,dt < \infty,\ c \xrightarrow{\lim} \infty \left|\frac{1}{2c}\int_{-c}^{c} f(t)\,dt\right|$$

$$\leq c \xrightarrow{\lim} \infty \left(\frac{1}{2c}\int_{-c}^{c}|f(t)|\,dt\right) = 0$$

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$$Now, \text{assuming} \int_{-c}^{c}\left|f(t)\right|dt < \infty,\ \lim_{c\to\infty}\left|\frac{1}{2c}\int_{-c}^{c} f(t)\,dt\right|$$

$$\le \lim_{c\to\infty}\left(\frac{1}{2c}\int_{-c}^{c}\left|f(t)\right|dt\right) = 0$$

$$Taking\ \delta\lambda = \frac{\pi}{c}\ we\ get,$$

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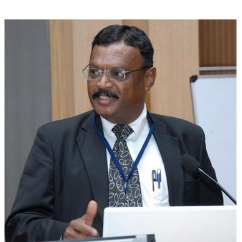
$$Taking\ \ \delta\lambda = \frac{\pi}{c}\ we\ get,$$

$$c \xrightarrow{\lim} \infty\left\{\frac{1}{c}\sum_{1}^{\infty}\int_{-\infty}^{\infty} f(t)\cos\left(\frac{n\pi\overline{t-x}}{c}\right)dt\right\}$$

$$= \delta\lambda \xrightarrow{\lim} \infty\left\{\frac{1}{\pi}\sum_{1}^{\infty}\delta\lambda\int_{-\infty}^{\infty} f(t)\cos n\,\delta\lambda(t-x)\,dt\right\}$$



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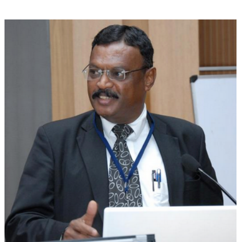
Hence,

$$\lim_{\delta\lambda \to \infty}\left\{\frac{1}{\pi}\sum_{1}^{\infty}\delta\lambda\int_{-\infty}^{\infty} f(t)\cos n\,\delta\lambda\left(t-x\right)dt\right\}=\frac{1}{\pi}\int_{\lambda=0}^{\infty}\int_{-\infty}^{\infty} f(t)\cos\lambda\left(t-x\right)dt\,d\lambda$$



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$$W.k.that,\ \delta\lambda \xrightarrow{\lim} \infty \sum_{n=1}^{\infty} F(n\delta\lambda) = \int_{0}^{\infty} F(\lambda)\, d\lambda$$

Hence,

$$\delta\lambda \xrightarrow{\lim} \infty \left\{ \frac{1}{\pi} \sum_{1}^{\infty} \delta\lambda \int_{-\infty}^{\infty} f(t) \cos n\delta\lambda (t-x)\, dt \right\} = \frac{1}{\pi} \int_{\lambda=0}^{\infty} \int_{-\infty}^{\infty} f(t) \cos \lambda (t-x)\, dt\, d\lambda$$

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Which is the Fourier Integral of the function f(x)

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$$f(x) = \frac{1}{\pi} \int_{\lambda=0}^{\infty} \int_{-\infty}^{\infty} f(t) \cos \lambda (t - x) \, dt \, d\lambda$$

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$$f(x)=\frac{1}{\pi}\int_{\lambda=0}^{\infty}\int_{-\infty}^{\infty} f(t)\cos\lambda(t-x)\,dt\,d\lambda$$

$$=\begin{cases}\dfrac{1}{\pi}\displaystyle\int_{\lambda=0}^{\infty}\int_{-\infty}^{\infty} f(t)\cos\lambda t\cos\lambda x\,dt\,d\lambda + \\ \\ \dfrac{1}{\pi}\displaystyle\int_{\lambda=0}^{\infty}\int_{-\infty}^{\infty} f(t)\sin\lambda t\sin\lambda x\,dt\,d\lambda\end{cases}$$

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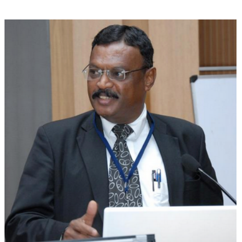
If f(x) is an odd function



If f(x) is an odd function

$$f(x)=\frac{2}{\pi}\int_{\lambda=0}^{\infty}\int_{0}^{\infty}f(t)\sin\lambda t\sin\lambda x\,dt\,d\lambda$$

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If f(x) is an odd function

$$f(x) = \frac{2}{\pi} \int_{\lambda=0}^{\infty} \int_{0}^{\infty} f(t) \sin \lambda t \sin \lambda x \, dt \, d\lambda$$

is the Fourier Sine Integral of f(x)

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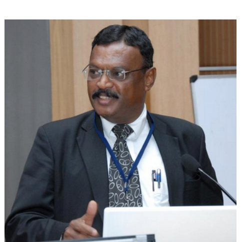

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If f(x) is an odd function

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If f(x) is an even function

$$f(x) = \frac{2}{\pi} \int_{\lambda=0}^{\infty} \int_{0}^{\infty} f(t) \cos \lambda t \cos \lambda x \, dt \, d\lambda$$

is the Fourier Cosine Integral of f(x)

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General Fourier integral representation

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General Fourier integral representation

$$F(x) = \frac{1}{\pi} \int_{\lambda=0}^{\infty} \left\{ A(\lambda)\cos\lambda x + B(\lambda)\sin\lambda x \right\} d\lambda$$

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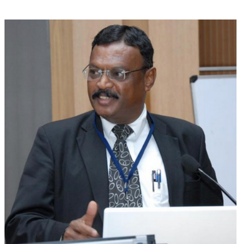

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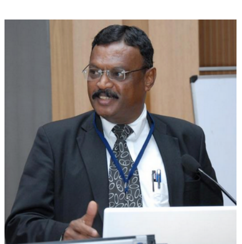

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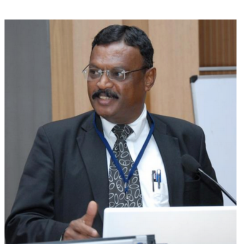

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Problems of finding the Fourier integrals

1) *Find the Fourier Integral of* $f(x)=\begin{cases}1 & |x|<1\\ 0 & otherwise\end{cases}$

$$Now, \int_{-\infty}^{\infty} f(t)\cos\lambda(t-x)\,dt=\int_{-\infty}^{-1}0\,dt+\int_{-1}^{1}\cos\lambda(t-x)\,dt+\int_{1}^{\infty}0\,dt$$

$$=\left.\frac{\sin\lambda(t-x)}{\lambda}\right|_{t=-1}^{t=1}=\frac{1}{\lambda}\big(\sin\lambda(1-x)-\sin\lambda(-1-x)\big)$$

$$=\frac{1}{\lambda}\big(\sin\lambda(1-x)+\sin\lambda(1+x)\big)=\frac{2\sin\lambda\cos\lambda x}{\lambda}$$

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$$f(x) = \frac{2}{\pi} \int_{\lambda=0}^{\infty} \left[\frac{\sin\lambda \cos\lambda x}{\lambda} \right] d\lambda$$

$$f(x) = \frac{2}{\pi} \int_{\lambda=0}^{\infty} \left[\frac{\sin\lambda \cos\lambda x}{\lambda} \right] d\lambda$$

$$Therefore \int_{\lambda=0}^{\infty} \left[\frac{\sin\lambda \cos\lambda x}{\lambda} \right] d\lambda = \begin{cases} \frac{\pi}{2} & |x| < 1 \\ 0 & otherwise \end{cases}$$

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$$f(x)=\frac{2}{\pi}\int_{\lambda=0}^{\infty}\left[\frac{\sin\lambda\cos\lambda x}{\lambda}\right]d\lambda$$

$$\textit{Therefore}\quad\int_{\lambda=0}^{\infty}\left[\frac{\sin\lambda\cos\lambda x}{\lambda}\right]d\lambda=\begin{cases}\dfrac{\pi}{2} & |x|<1\\ 0 & \textit{other wise}\end{cases}$$

$$\textit{Hence}\quad\int_{\lambda=0}^{\infty}\left[\frac{\sin\lambda}{\lambda}\right]d\lambda=\frac{\pi}{2}$$

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1a) *Find the Fourier sine integral of* $f(x)=\begin{cases}1 & 0\le x\le\pi\\ 0 & x>\pi\end{cases}$

and hence show that $\displaystyle\int_0^\infty \frac{\sin x}{x}\,dx=\frac{\pi}{2}$

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2) *Find the Fourier Integral of* $f(x)=\begin{cases}1-x^2 & |x|\le 1\\ 0 & otherwise\end{cases}$

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2) *Find the Fourier Integral of* $f(x)=\begin{cases}1-x^2 & |x|\le 1\\ 0 & otherwise\end{cases}$

$$f(x)=\frac{1}{\pi}\int_{\lambda=0}^{\infty}\int_{-\infty}^{\infty} f(t)\cos\lambda(t-x)\,dt\,d\lambda$$

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$$Now, \int_{-\infty}^{\infty} f(t)\cos\lambda(t-x)\,dt=\int_{-1}^{1}\left(1-x^2\right)\cos\lambda(t-x)\,dt$$

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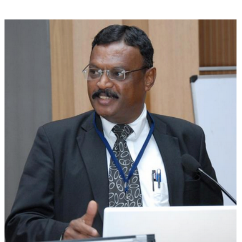
$$=\int_{-1}^{1}\left(1-t^2\right)d\left(\frac{\sin\lambda(t-x)}{\lambda}\right)dt$$

$$=\left(1-t^2\right)\left(\frac{\sin\lambda(t-x)}{\lambda}\right)-(-2t)\left(-\frac{\cos\lambda(t-x)}{\lambda^2}\right)+(-2)\left(\frac{\sin\lambda(t-x)}{\lambda^3}\right)\Bigg|_{t=-1}^{t=1}$$



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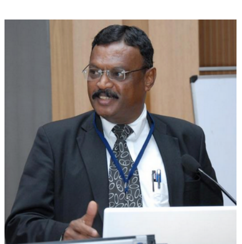

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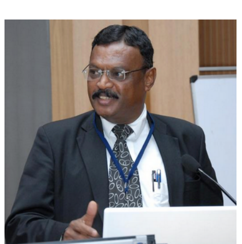
$$=\left(1-t^2\right)\left(\frac{\sin\lambda\left(t-x\right)}{\lambda}\right)-\left(-2t\right)\left(-\frac{\cos\lambda\left(t-x\right)}{\lambda^2}\right)+\left(-2\right)\left(\frac{\sin\lambda\left(t-x\right)}{\lambda^3}\right)\Bigg|_{t=-1}^{t=1}$$

$$=\frac{2}{\lambda^2}\left(-\cos\lambda\left(1-x\right)+\cos\lambda\left(-1-x\right)\right)-\frac{2}{\lambda^3}\left(\sin\lambda\left(1-x\right)-\sin\lambda\left(-1-x\right)\right)$$



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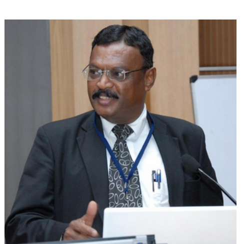
$$=\frac{2}{\lambda^2}\left(-\cos\lambda(1-x)+\cos\lambda(-1-x)\right)-\frac{2}{\lambda^3}\left(\sin\lambda(1-x)-\sin\lambda(-1-x)\right)$$

$$=\frac{2}{\lambda^2}\left(-\cos\lambda(1-x)+\cos\lambda(1+x)\right)-\frac{2}{\lambda^3}\left(\sin\lambda(1-x)+\sin\lambda(1+x)\right)$$



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$$=\frac{2}{\lambda^{2}}\left(-\cos\lambda(1-x)+\cos\lambda(1+x)\right)-\frac{2}{\lambda^{3}}\left(\sin\lambda(1-x)+\sin\lambda(1+x)\right)$$

$$=\frac{2}{\lambda^{2}}(2\cos\lambda\cos\lambda x)-\frac{2}{\lambda^{3}}(2\sin\lambda\cos\lambda x)$$

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$$=\left(1-t^2\right)\left(\frac{\sin\lambda(t-x)}{\lambda}\right)-(-2t)\left(-\frac{\cos\lambda(t-x)}{\lambda^2}\right)+(-2)\left(\frac{\sin\lambda(t-x)}{\lambda^3}\right)\Bigg|_{t=-1}^{t=1}$$

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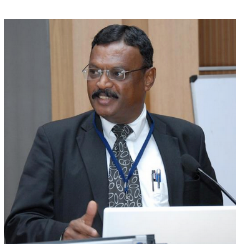
$$=\frac{2}{\lambda^2}\left(-\cos\lambda(1-x)+\cos\lambda(1+x)\right)-\frac{2}{\lambda^3}\left(\sin\lambda(1-x)+\sin\lambda(1+x)\right)$$

$$=\frac{2}{\lambda^2}(2\cos\lambda\cos\lambda x)-\frac{2}{\lambda^3}(2\sin\lambda\cos\lambda x)$$

$$=\frac{4}{\lambda^3}(\lambda\cos\lambda-\sin\lambda)\cos\lambda x$$



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$$=\frac{4}{\lambda^3}(\lambda\cos\lambda-\sin\lambda)\cos\lambda x$$

$$\therefore f(x)=\frac{4}{\pi}\int_{\lambda=0}^{\infty}\left(\frac{(\lambda\cos\lambda-\sin\lambda)\cos\lambda x}{\lambda^3}\right)d\lambda$$

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3) *Find the Fourier sine & cosine integral of* $f(x)=e^{-\alpha x},\ x>0$

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3) $Find\ \ the\ \ Fourier\ \sin e\ \ \&\cos ine\, integral\ of\quad f(x)=e^{-\alpha x},\ x>0$

Cosine integral

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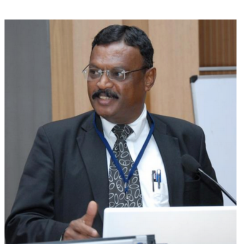
3) *Find the Fourier* sine & cosine *integral of* $f(x) = e^{-\alpha x},\ x > 0$

Cosine integral

$$f(x) = \frac{2}{\pi} \int_{\lambda=0}^{\infty} \int_{0}^{\infty} f(t) \cos \lambda t \cos \lambda x \, dt \, d\lambda$$



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3) *Find the Fourier sine & cosine integral of* $f(x) = e^{-\alpha x},\ x > 0$

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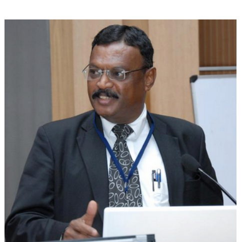

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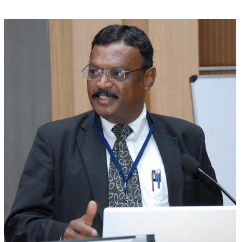

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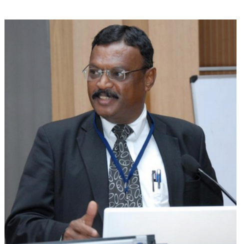

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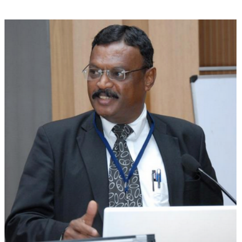

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Complex form of a Fourier Integral

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