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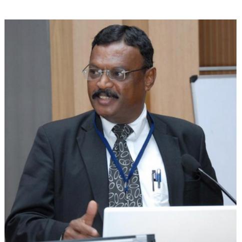
Good afternoon friends



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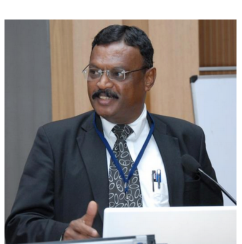
2. He was appointed to the Commission set up by the Royal Society to review the rival claims of Newton and Leibniz to be the discovers of the calculus.
3. He is famed for predicting the day of his own death. He found that he was sleeping 15 minutes longer each night and summing the arithmetic progression, calculated that he would die on the day that he slept for 24 hours. He was right!



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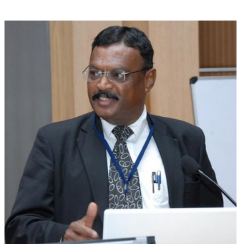

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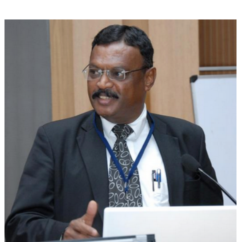
$$(\cos x + i\sin x)^n = \cos nx + i\sin nx$$



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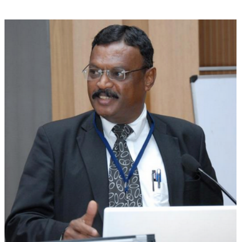
Abraham de Moivre

Born: **26 May 1667 in Vitry-le-François, Champagne, France**

Died: **27 November 1754 in London,**



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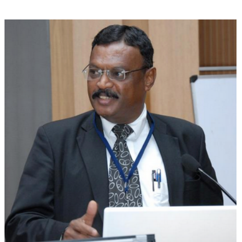
Dirichlet's existence criteria for the Fourier Series of a function



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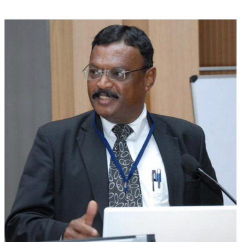

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$$where, \begin{cases} f(t^+) = \lim_{x \to t^+} f(x) & \text{(Right limit)} \\ f(t^-) = \lim_{x \to t^-} f(x) & \text{(Left limit)} \end{cases}$$

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There are functions for which Fourier series may not exists

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$$f(t)=\begin{cases} 0 \;\; if \;\; t\in(0,2\pi), \quad t \; is \; rational \\ 1 \;\; if \;\; t\in(0,2\pi), t \; is \; irrational \end{cases}$$

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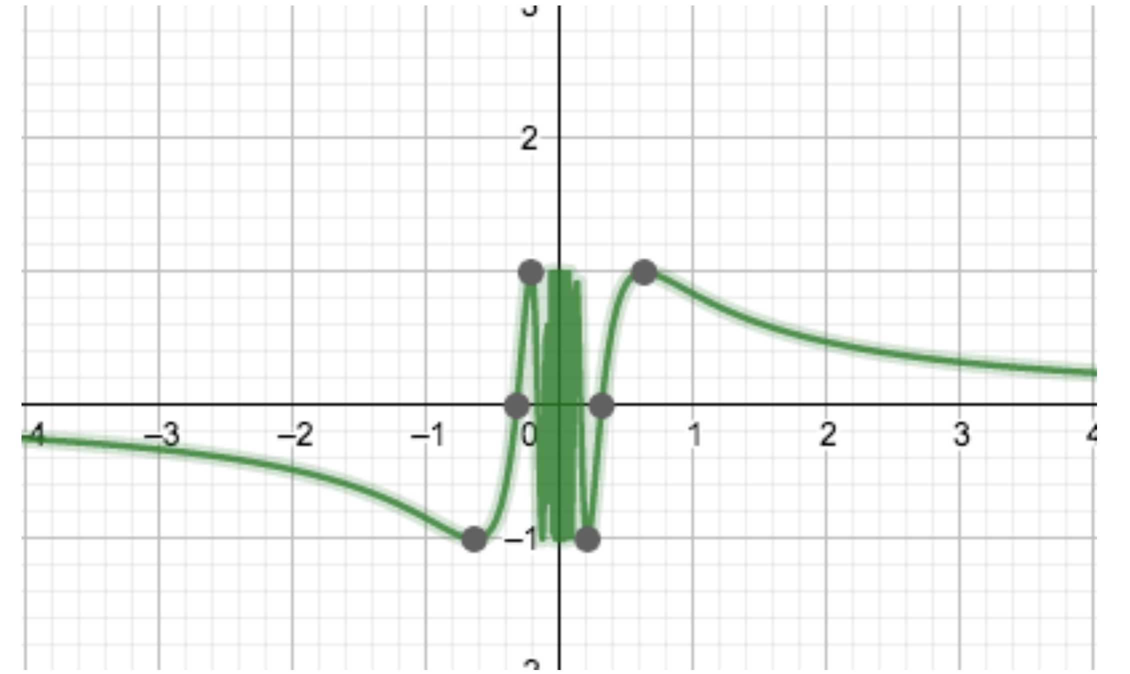

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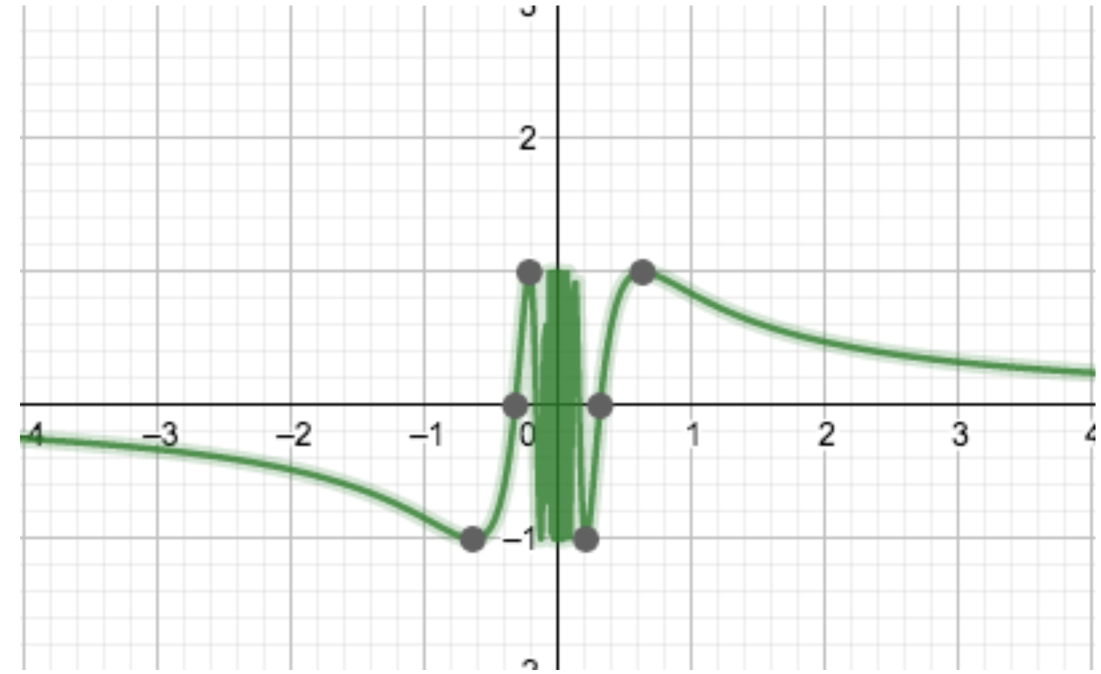

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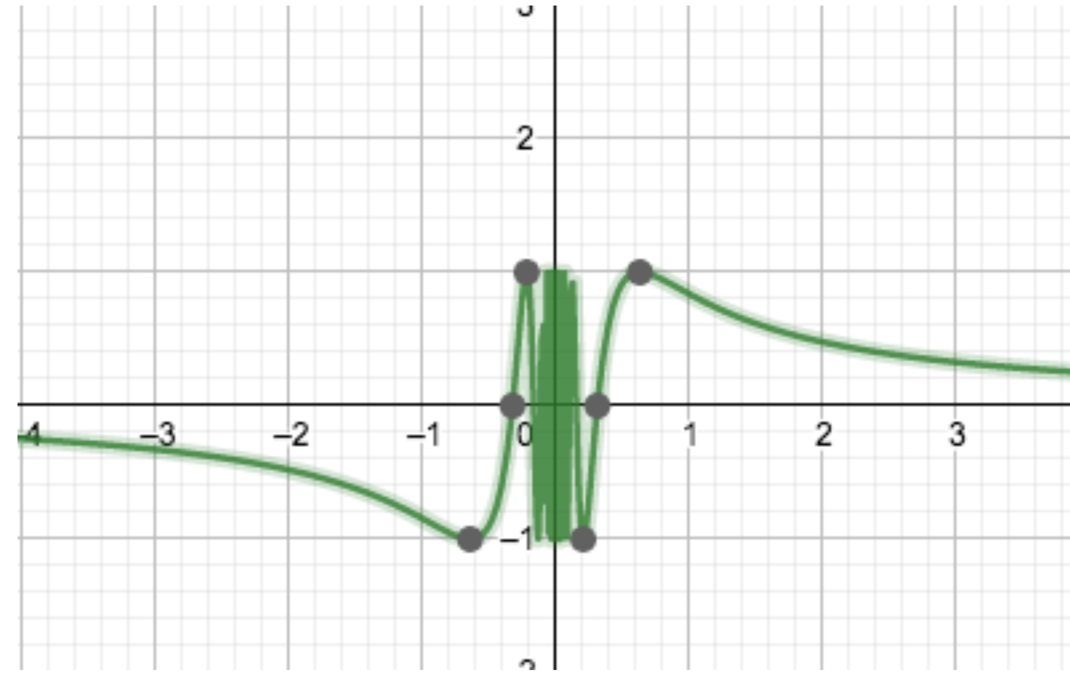

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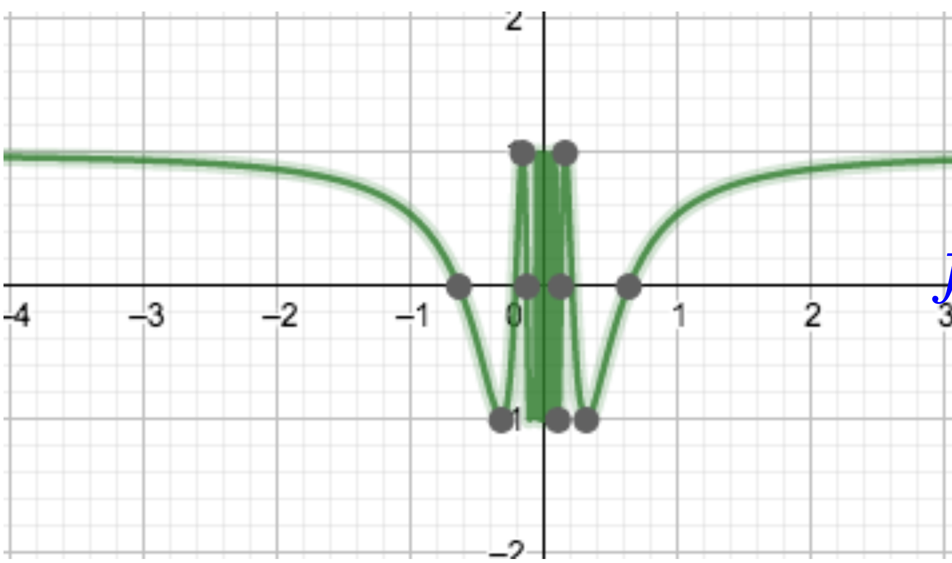

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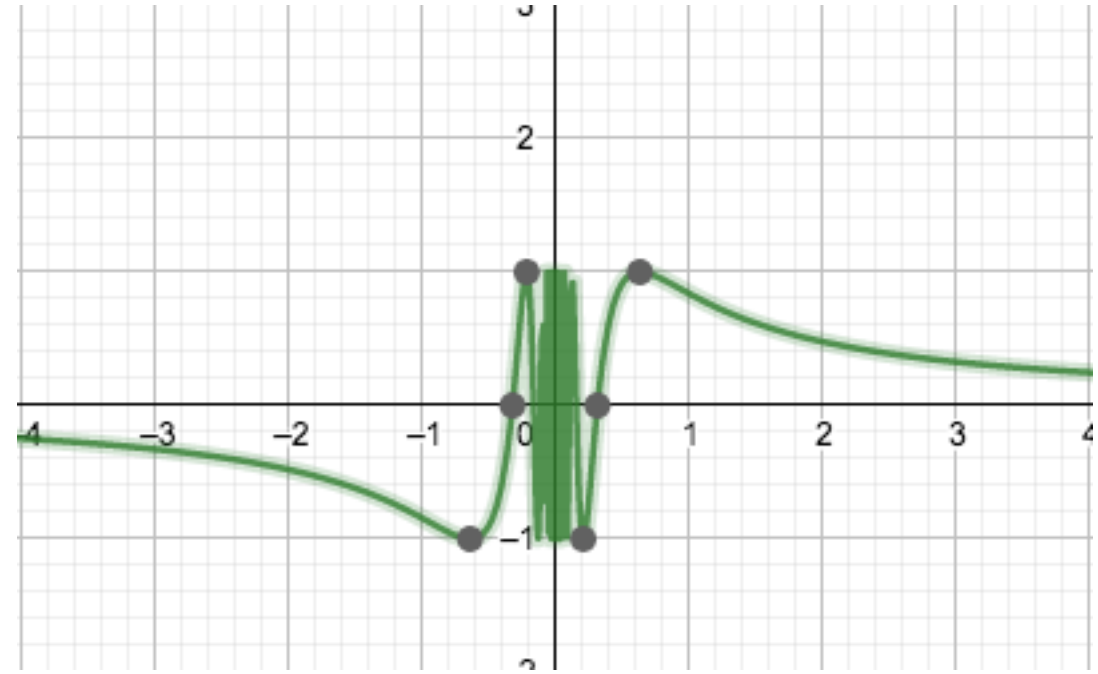

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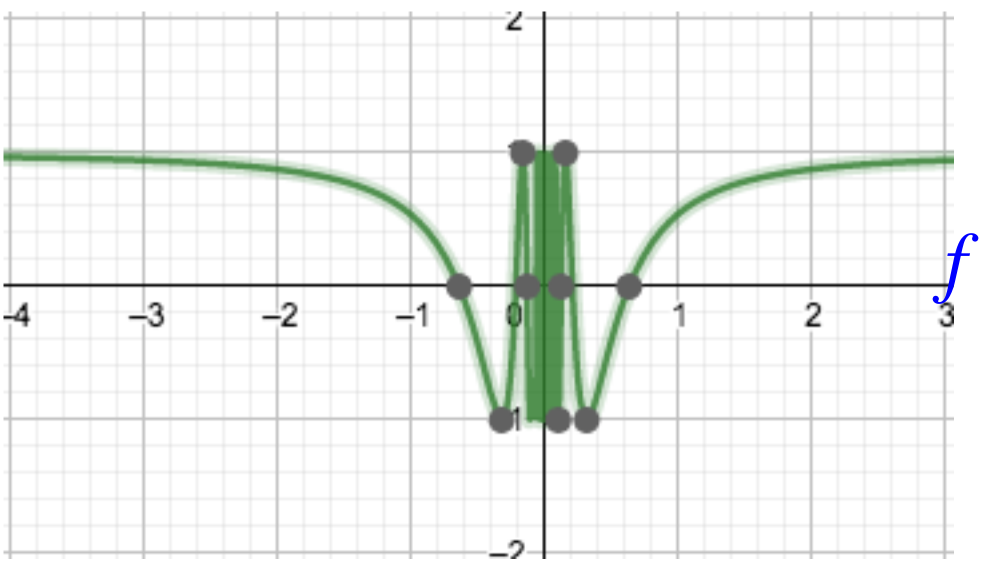

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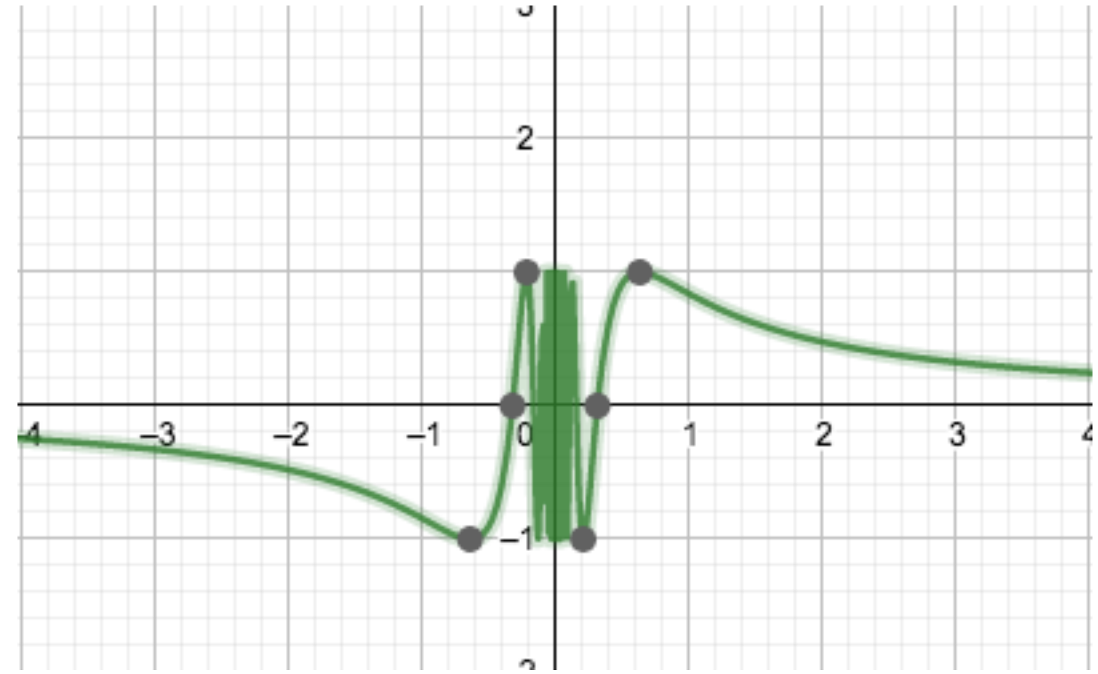

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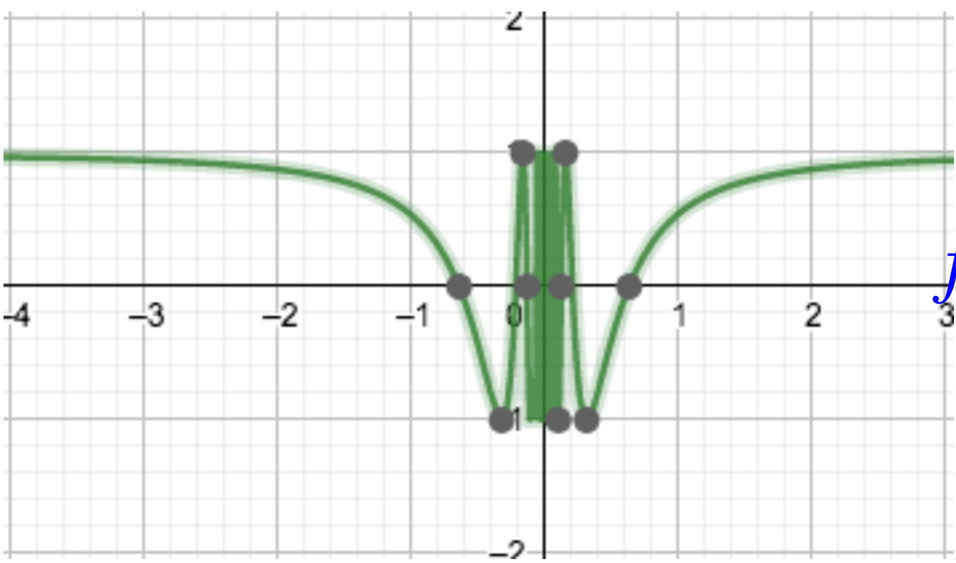

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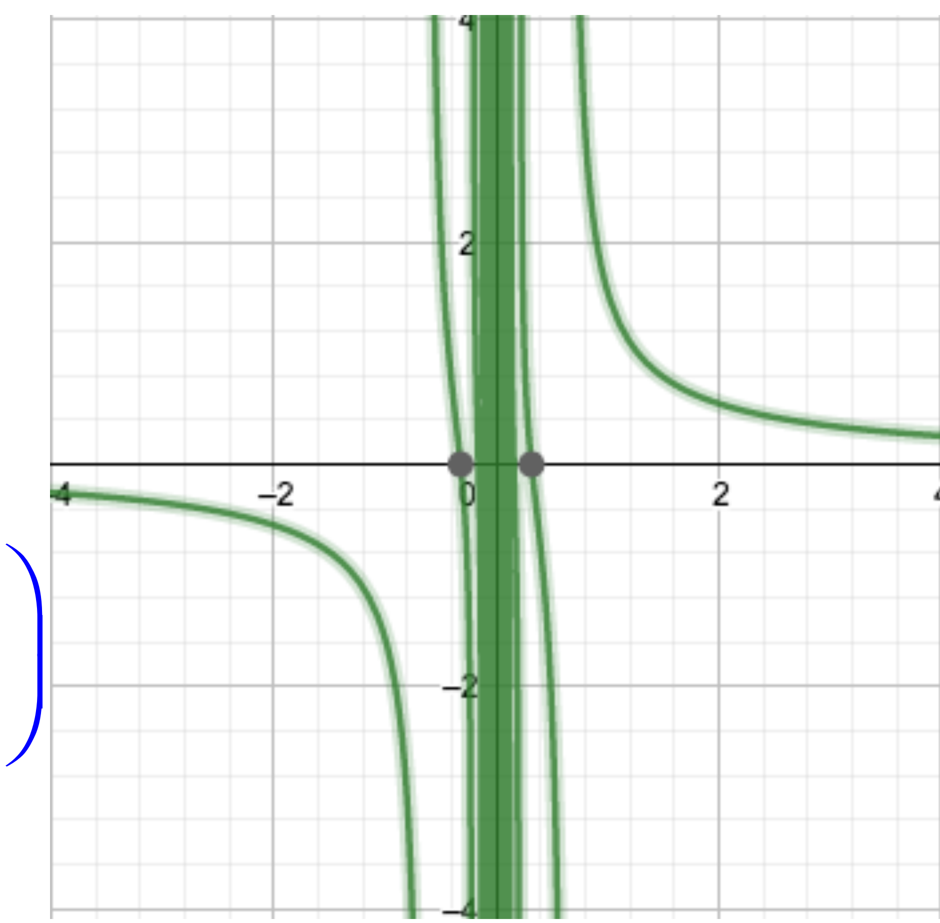

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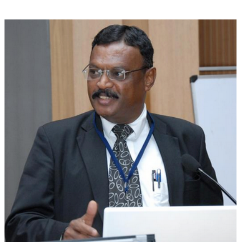

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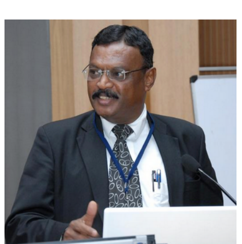

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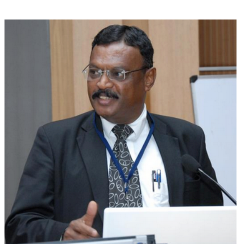

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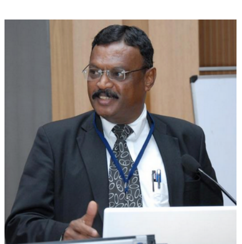

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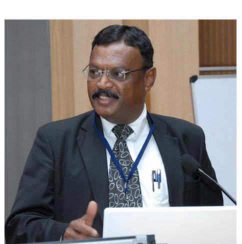

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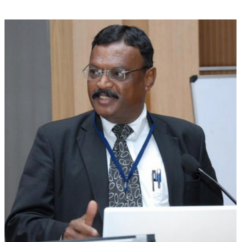

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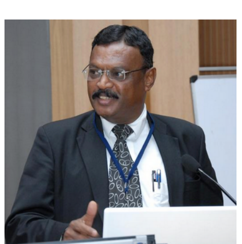
Where, "suffix " denote the integration and "super fix dashes" denote the integration



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Expansion 1 *Find the Fourier series of the function*

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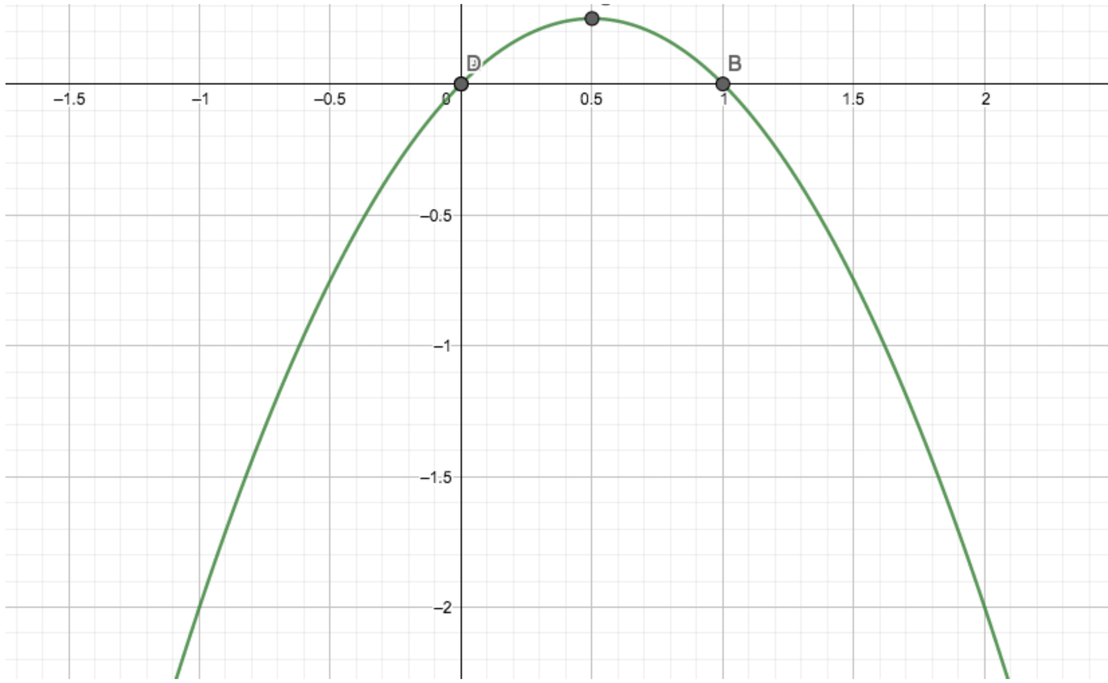

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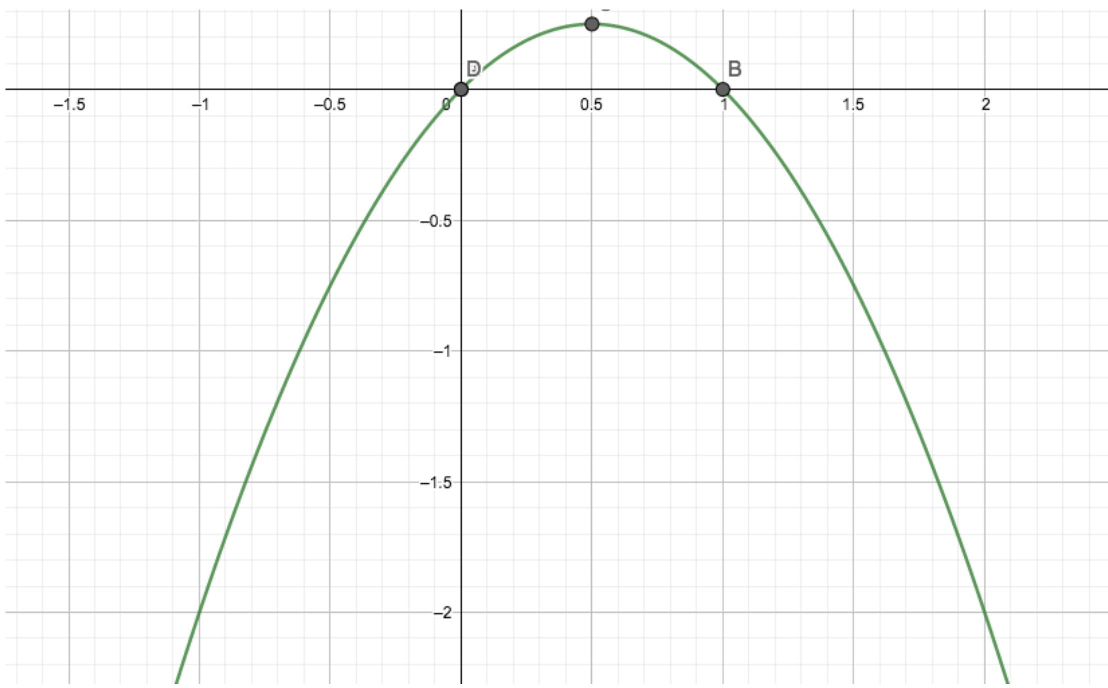

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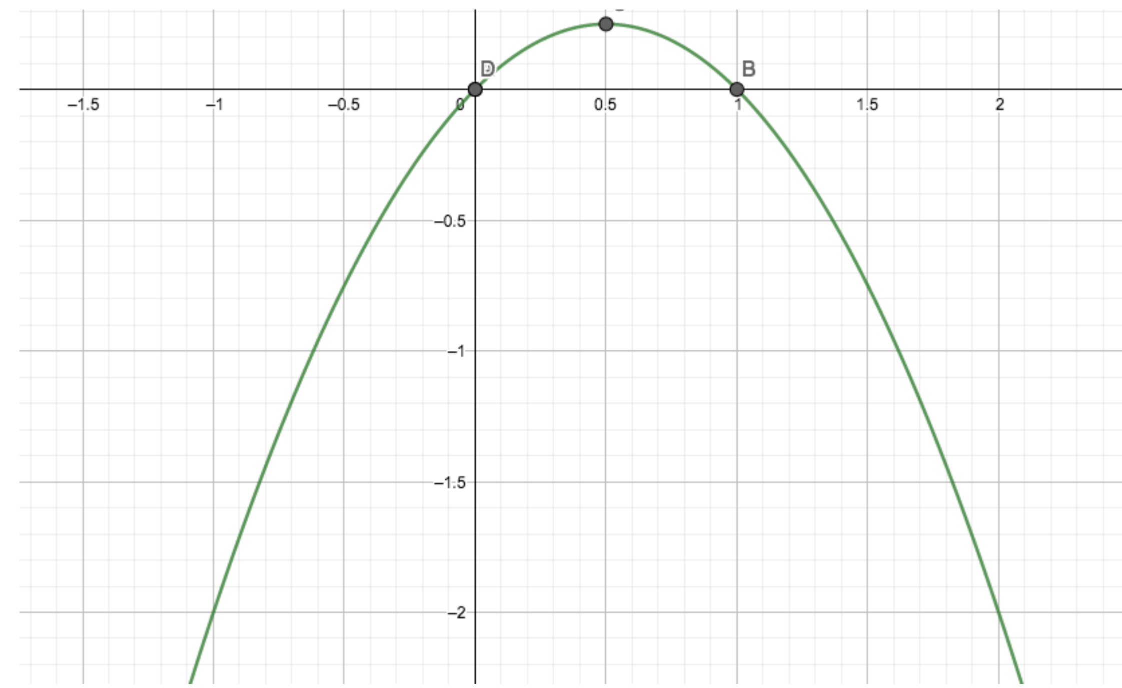

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a_0	$\frac{1}{\pi}\int_{-\pi}^{\pi}\left(x - x^2\right)dx$	$-\frac{2}{3}\pi^2$

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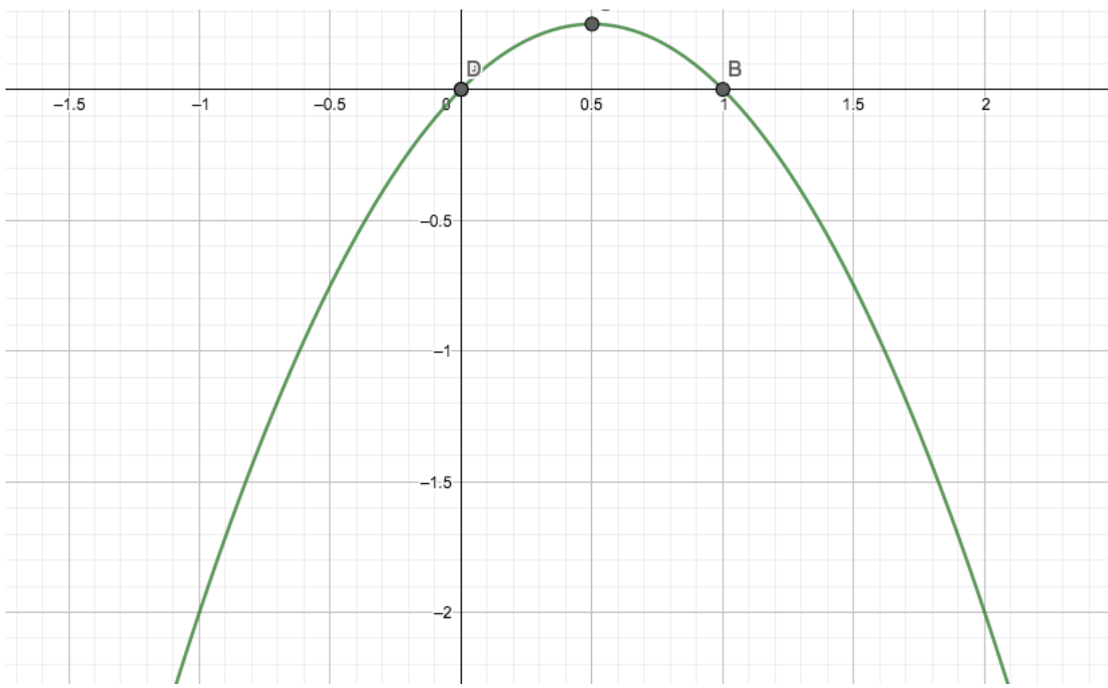

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a_0	$\frac{1}{\pi}\int_{-\pi}^{\pi}\left(x-x^2\right)dx$	$-\frac{2}{3}\pi^2$
a_n	$\frac{1}{\pi}\int_{-\pi}^{\pi}\left(x-x^2\right)\cos nx\,dx$	$(-1)^{n+1}\frac{4}{n^2}$

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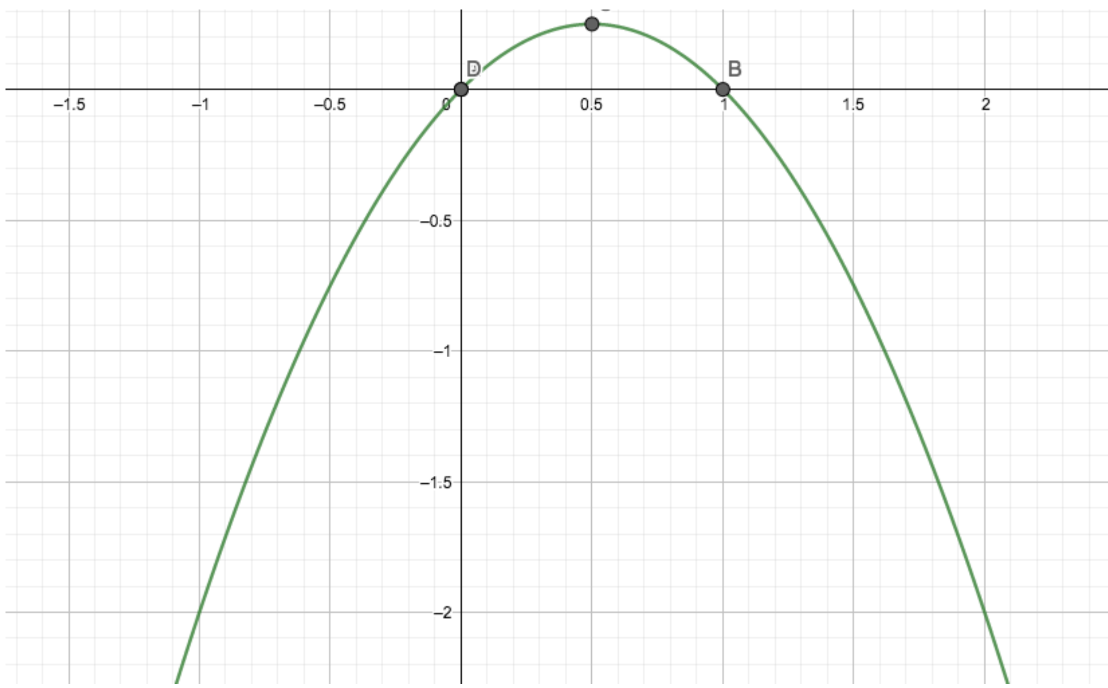
Expansion 1 Find the Fourier series of the function

$$f(x) = x - x^2 \quad in \ (-\pi, \pi)$$



$$Let, \ f(x) = \frac{a_0}{2} + \sum_{1}^{\infty} a_n \cos nx + \sum_{1}^{\infty} b_n \sin nx$$

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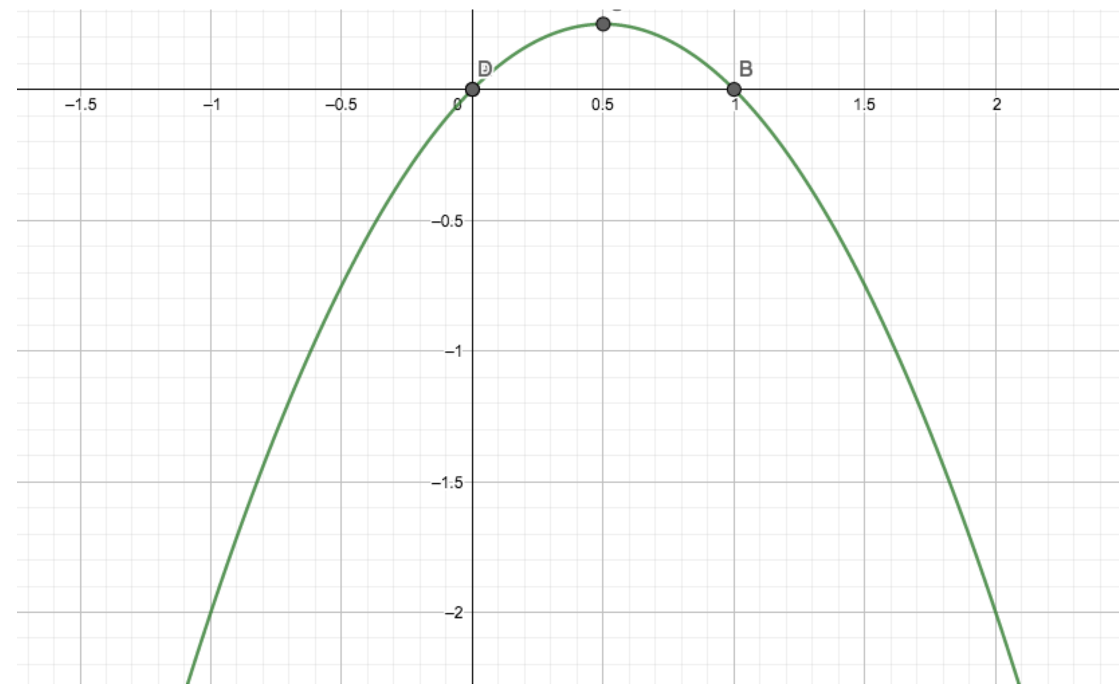

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$$\therefore \ x - x^2 = -\left(\frac{\pi^2}{3}\right) + \sum_{1}^{\infty}\left((-1)^{n+1}\frac{4}{n^2}\right)\cos nx + \sum_{1}^{\infty}\left((-1)^{n+1}\frac{2}{n}\right)\sin nx$$

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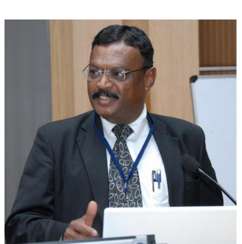
$$\Rightarrow 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{12}$$



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Problem for Tutorial

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Problem for Tutorial

Expansion 1a *Find the Fourier series of the function*

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Expansion 2 *Find the Fourier series of the function*

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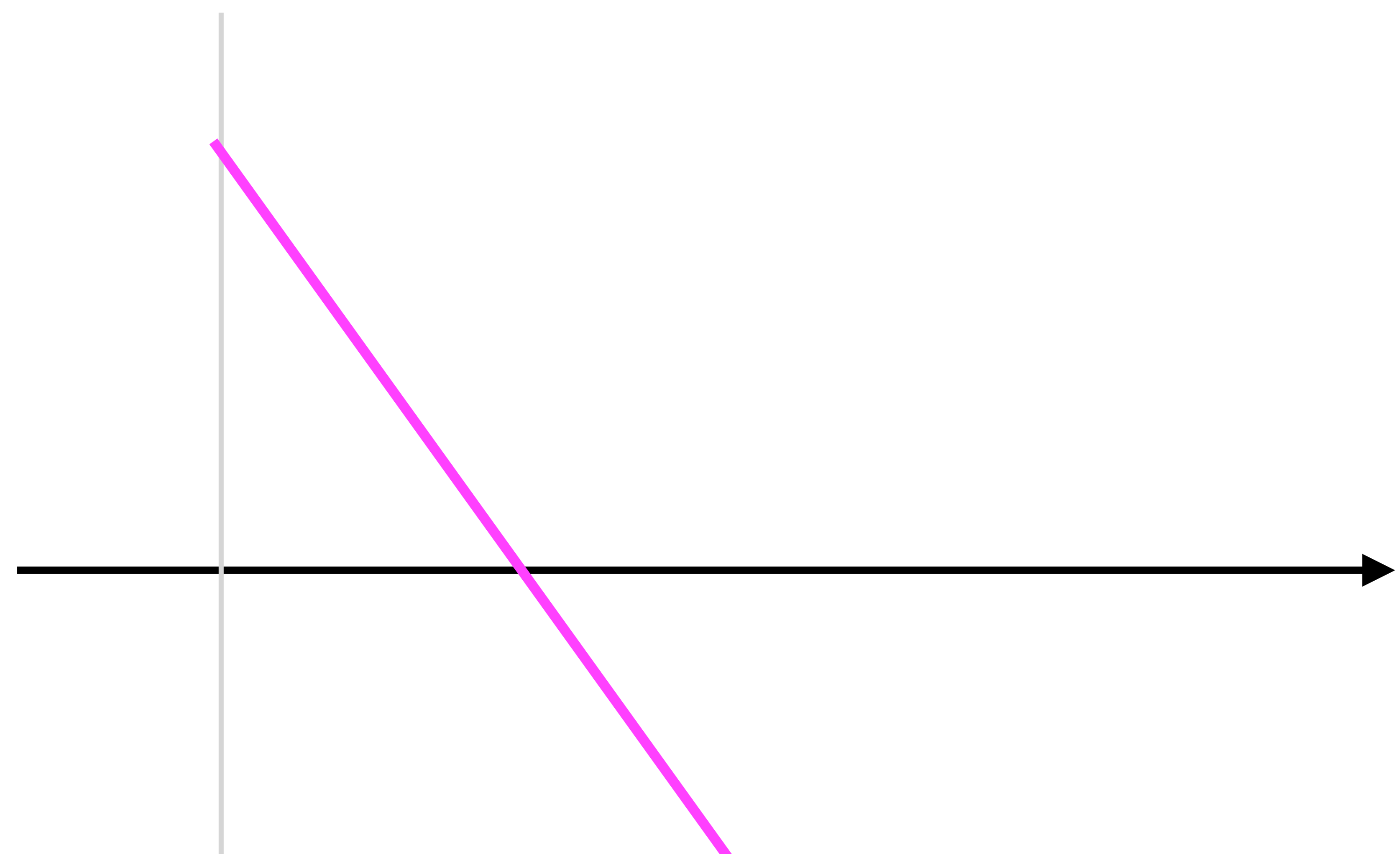

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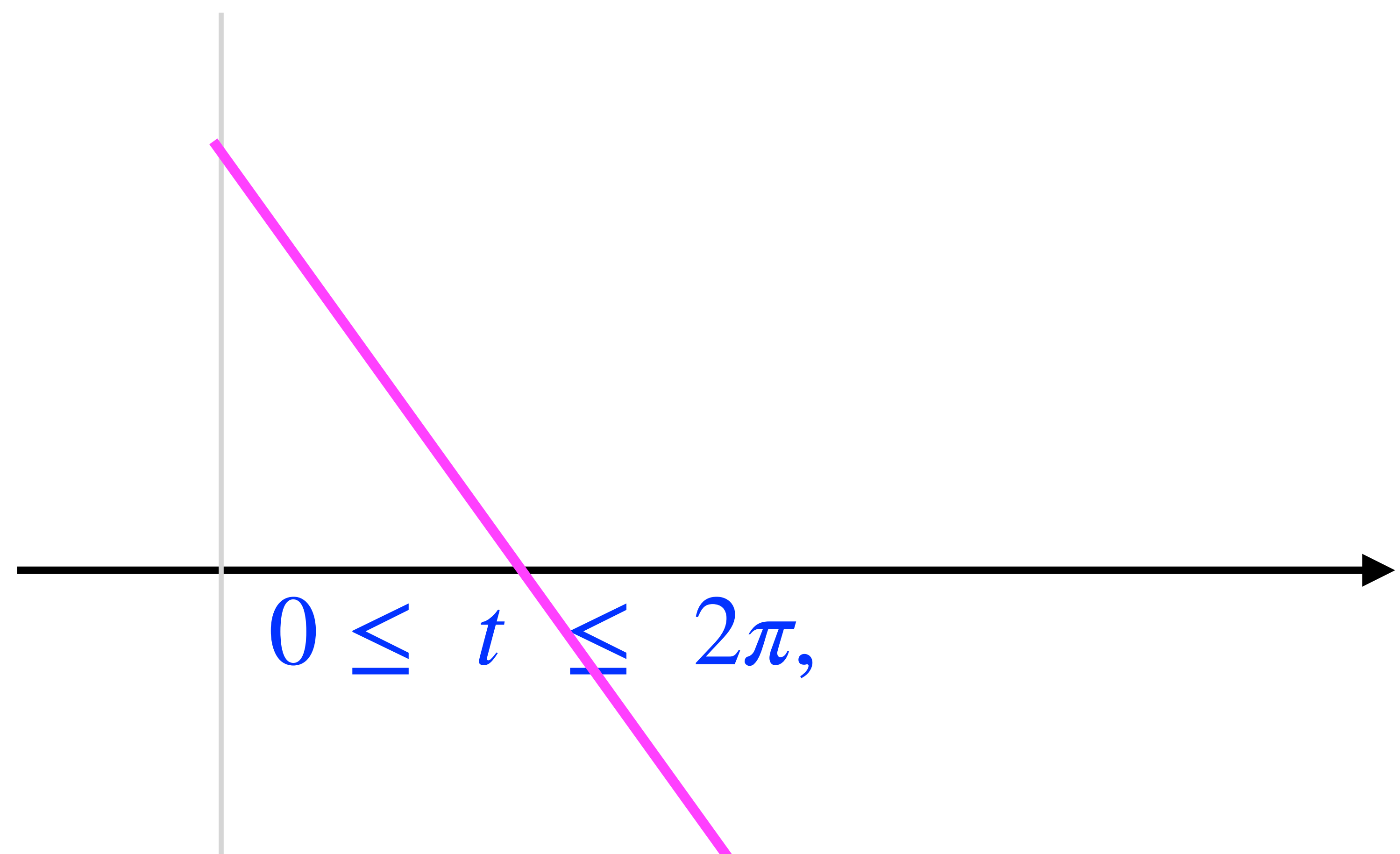

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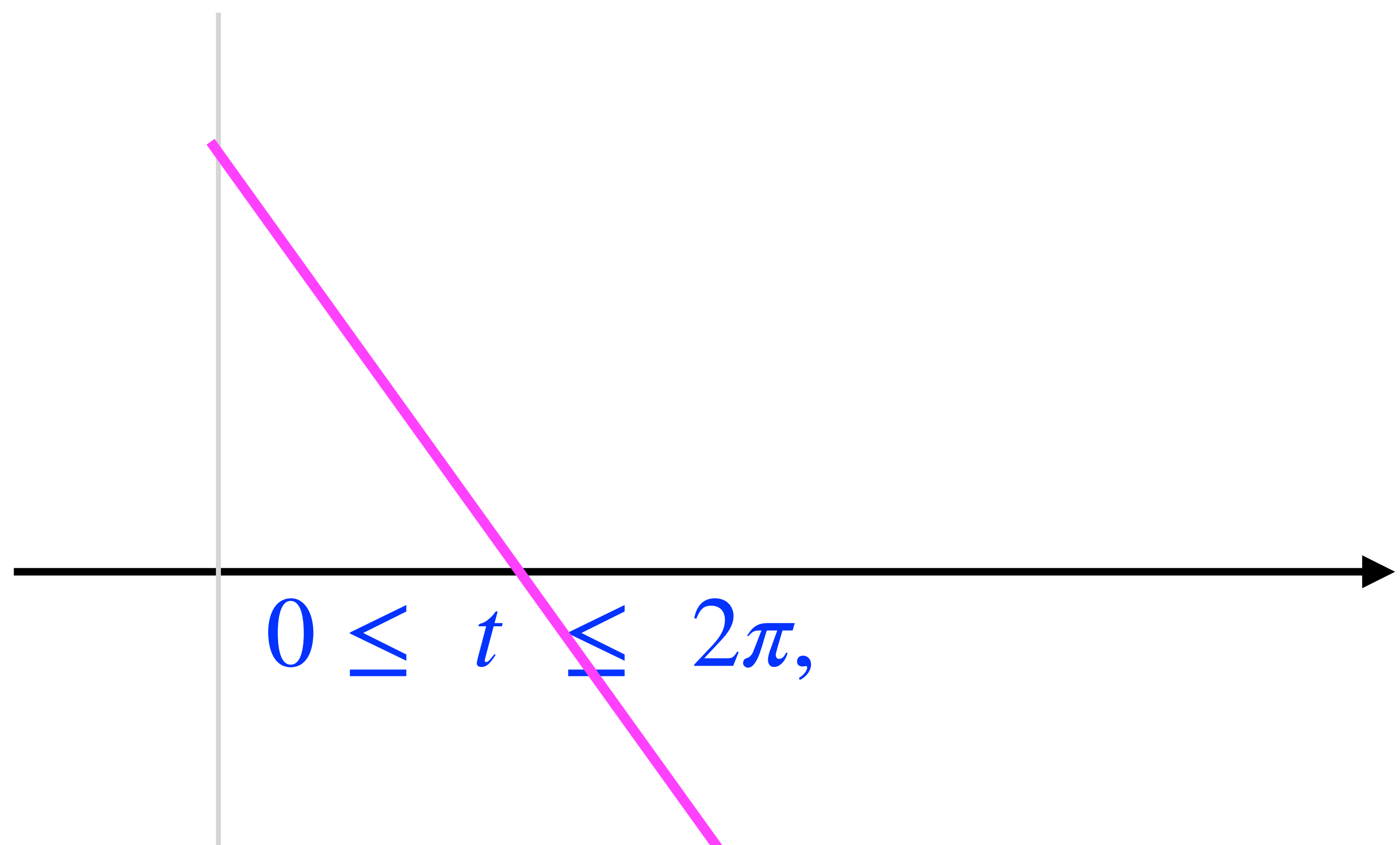

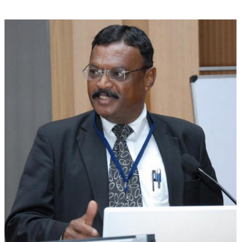

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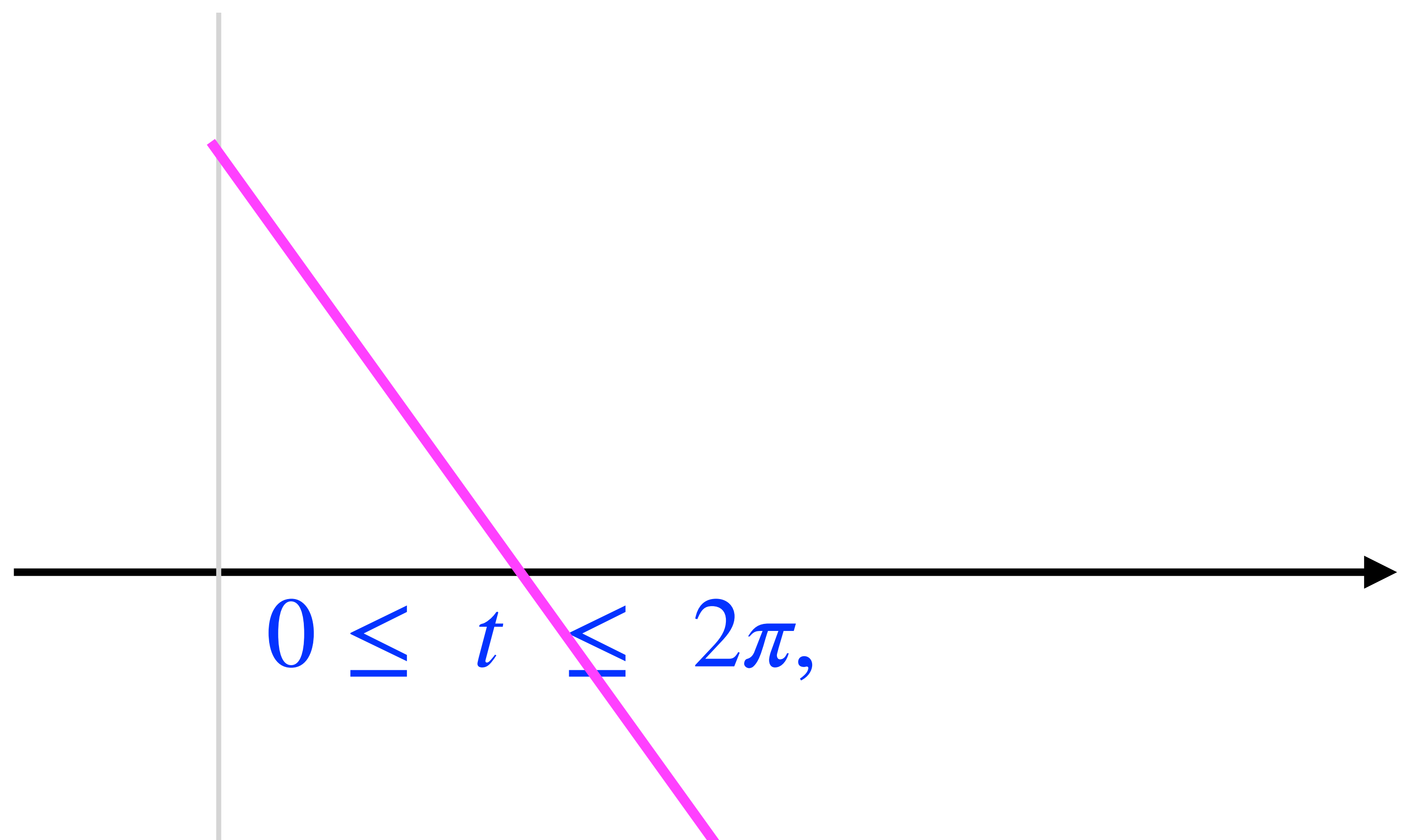

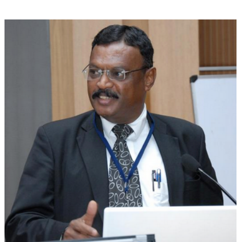

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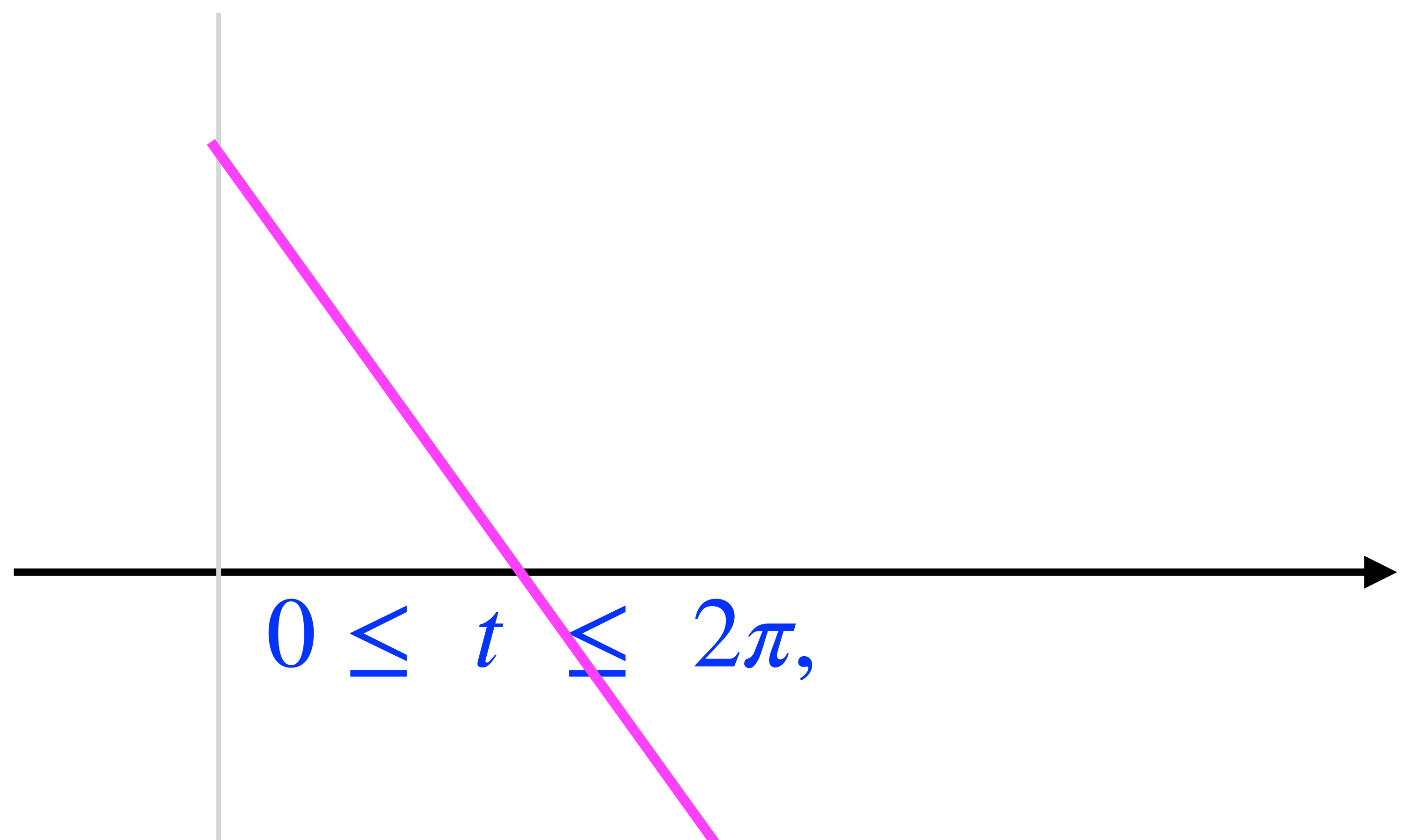

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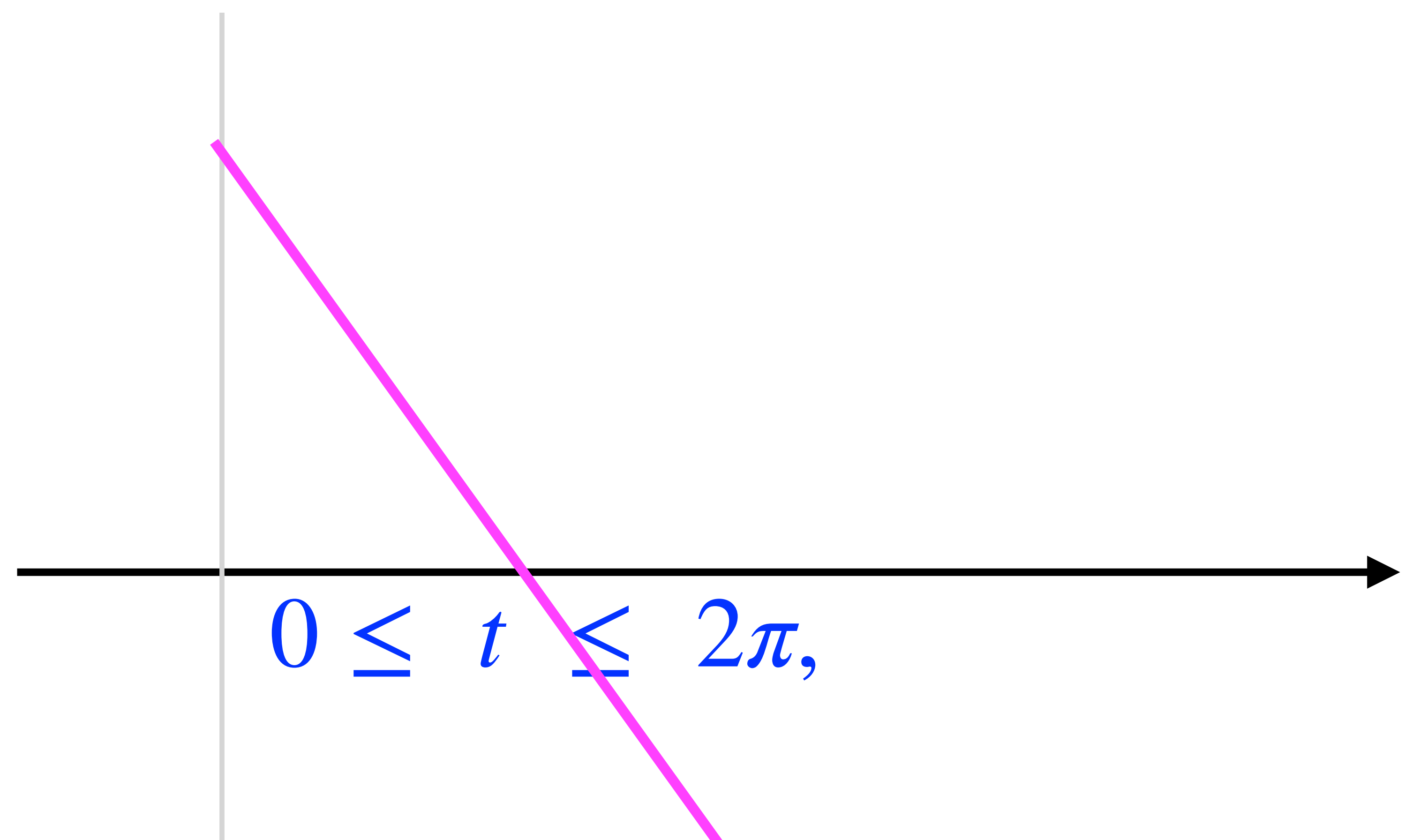

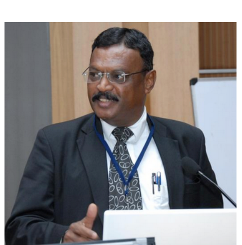

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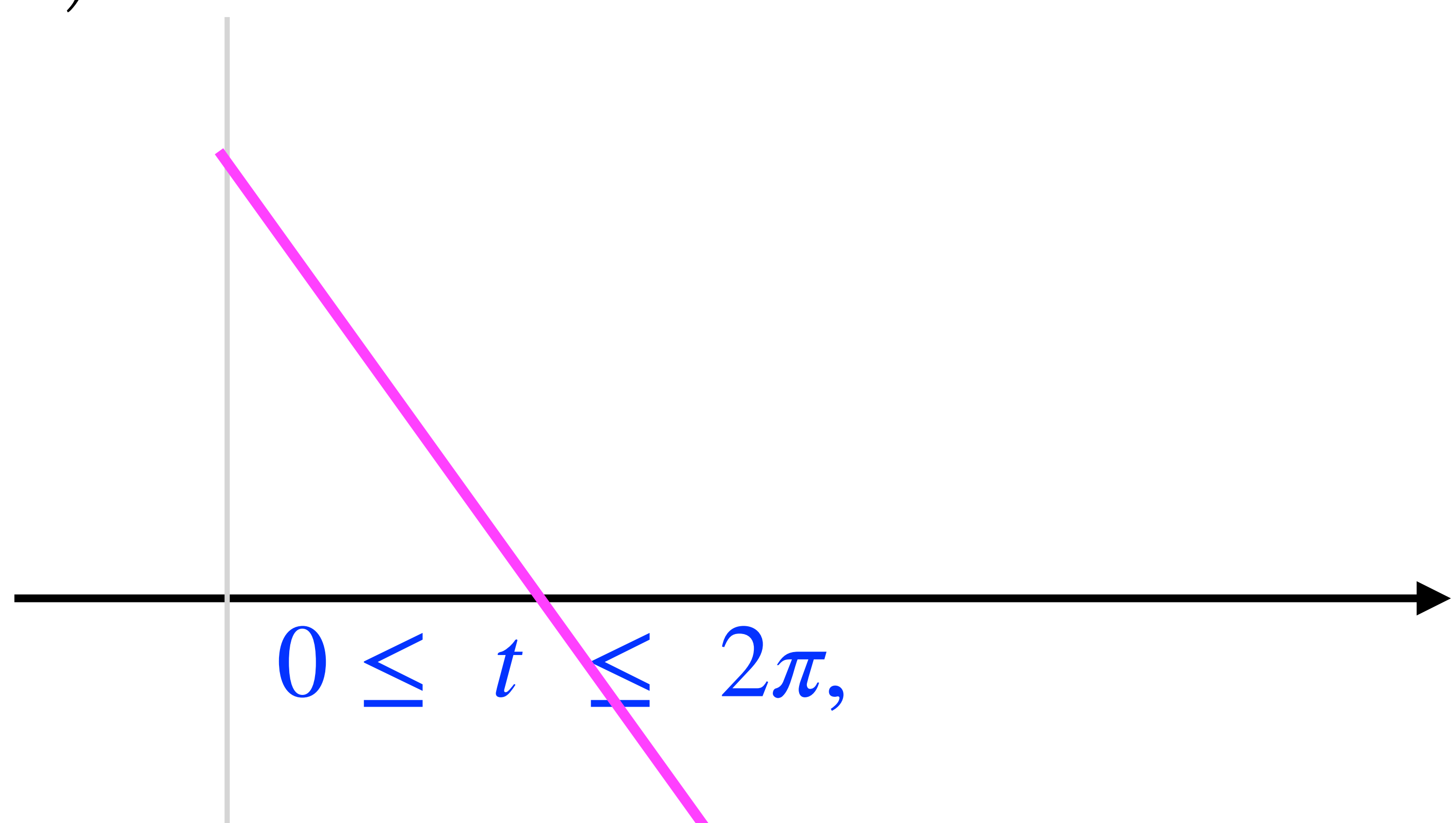

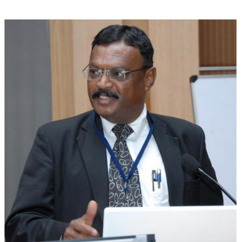

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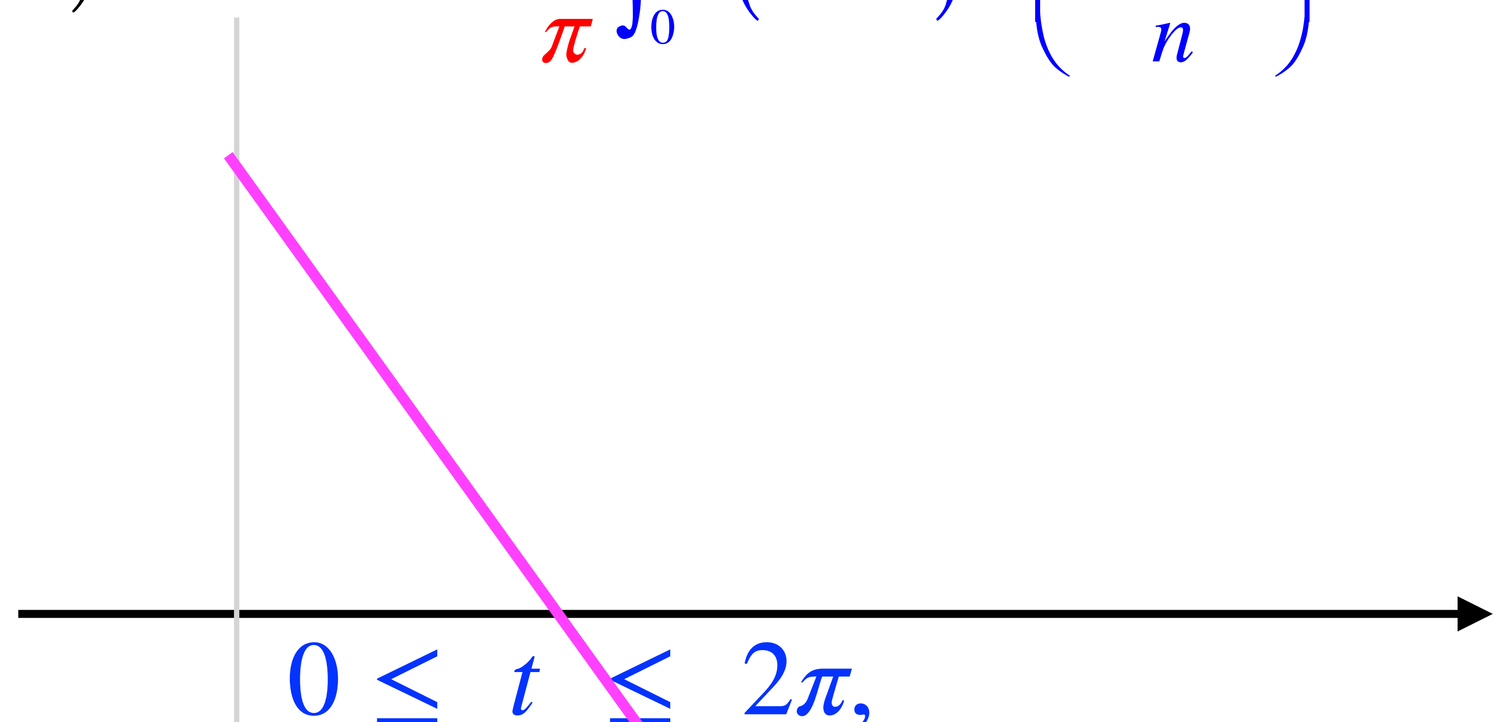

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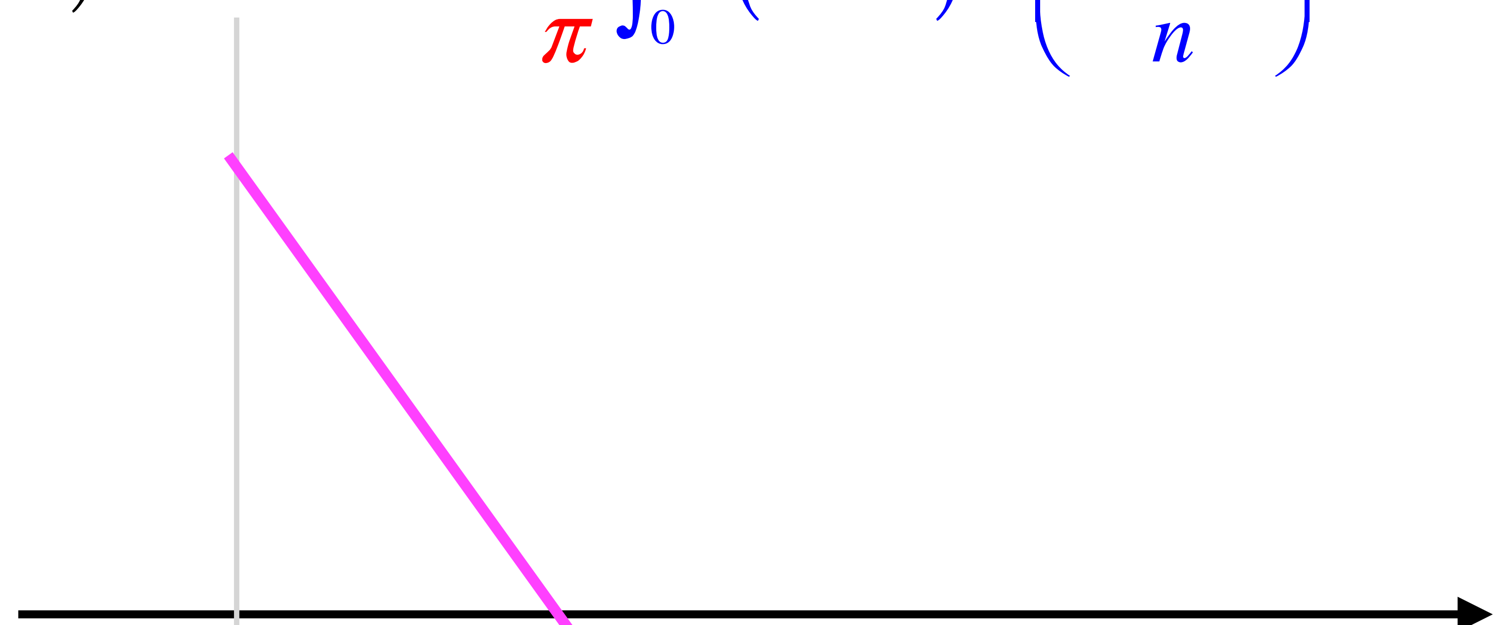
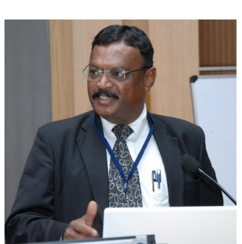
Continue and complete



$0 \leq \ t \ \leq \ 2\pi,$



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Continue and complete

$$b_n=\frac{1}{\pi}\int_0^{2\pi}f(t).\sin nt\ dt=\frac{1}{\pi}\int_0^{2\pi}(\pi-t).\sin nt\ dt$$

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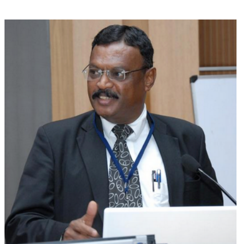

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Fourier series for functions with point of discontinuity

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Expansion 3 *Find the Fourier series of the function*

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$$f(t)=\begin{cases}0 \text{ if } & -\pi\le t\le 0,\\ 1 \text{ if } & 0\le t\le \pi\end{cases}$$

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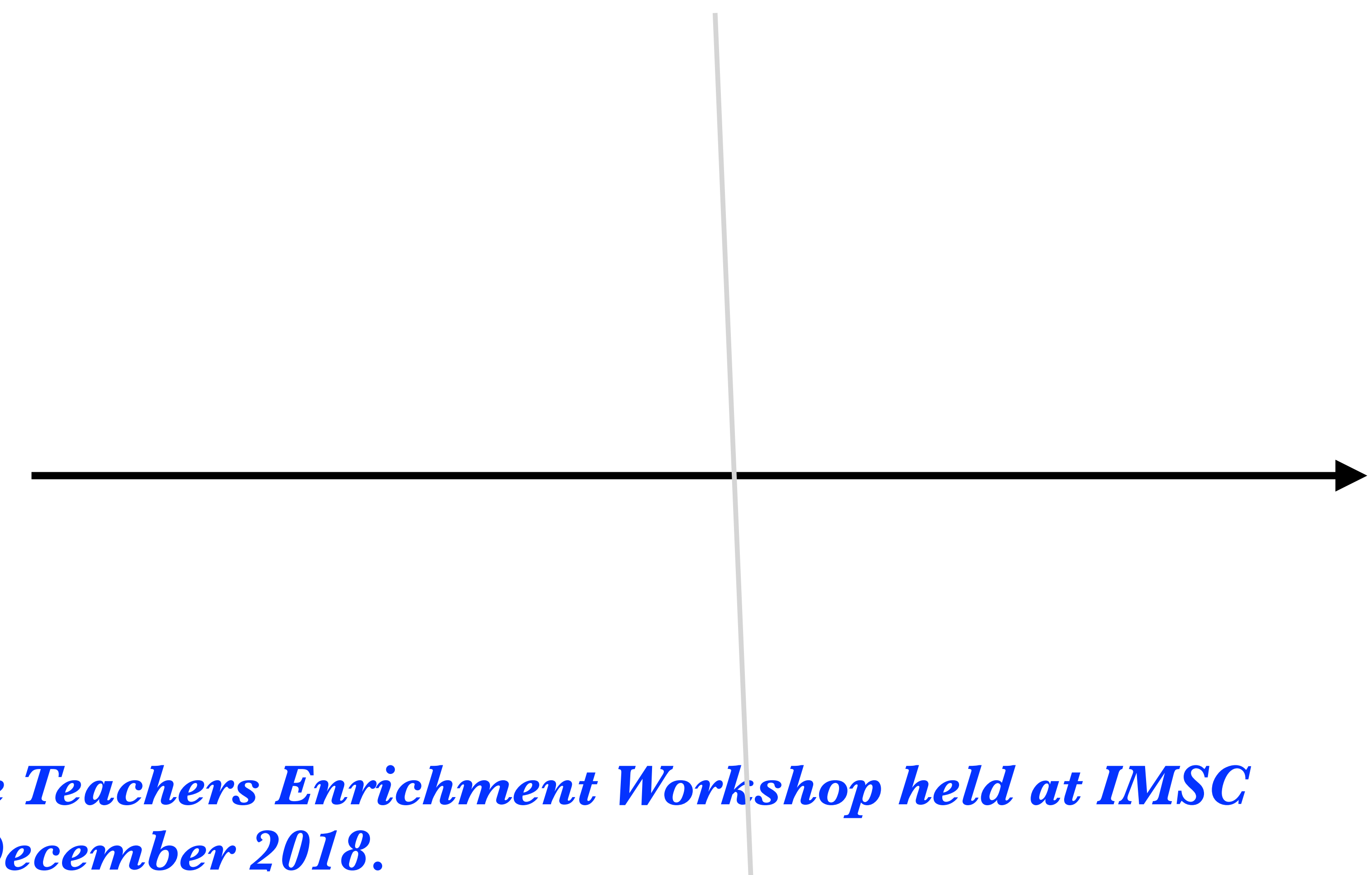

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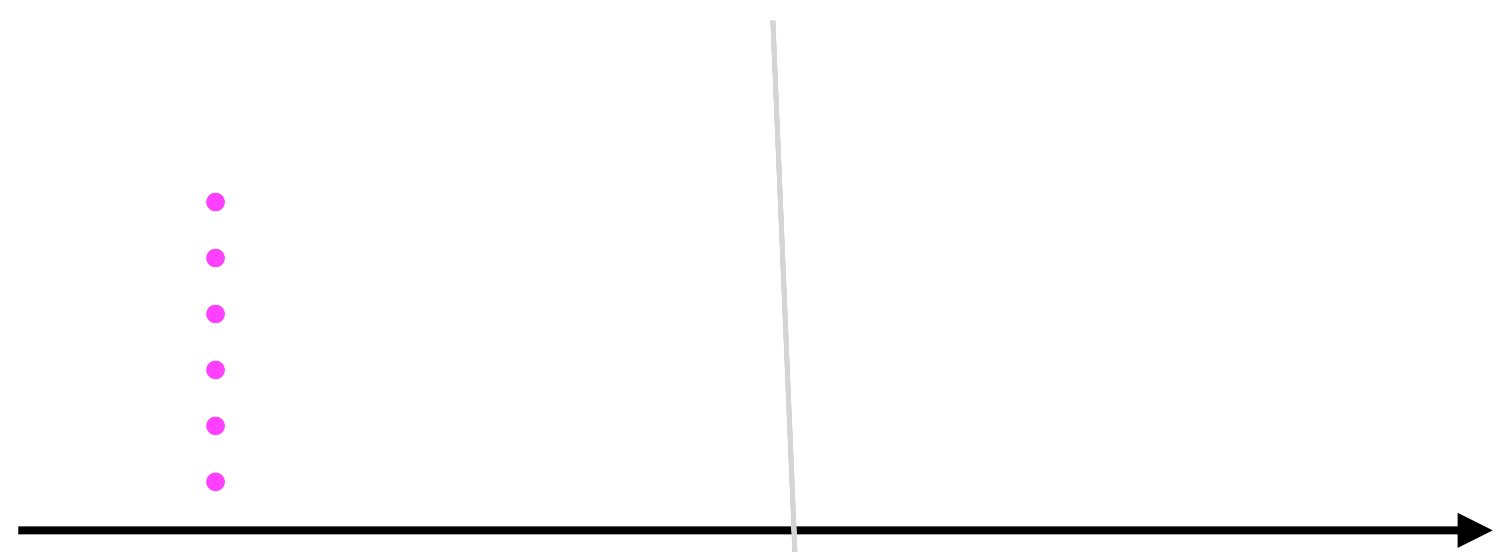

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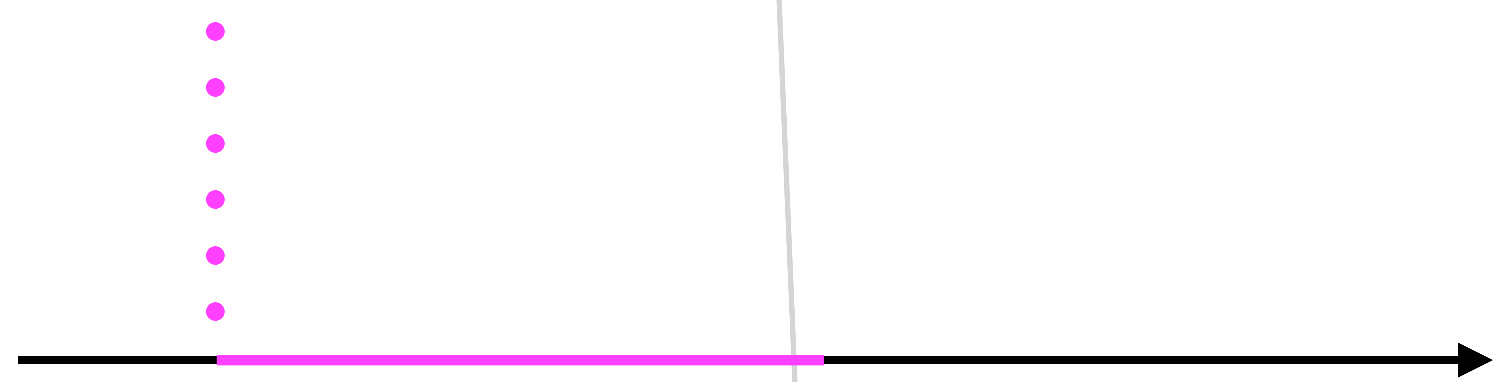

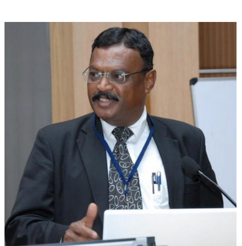

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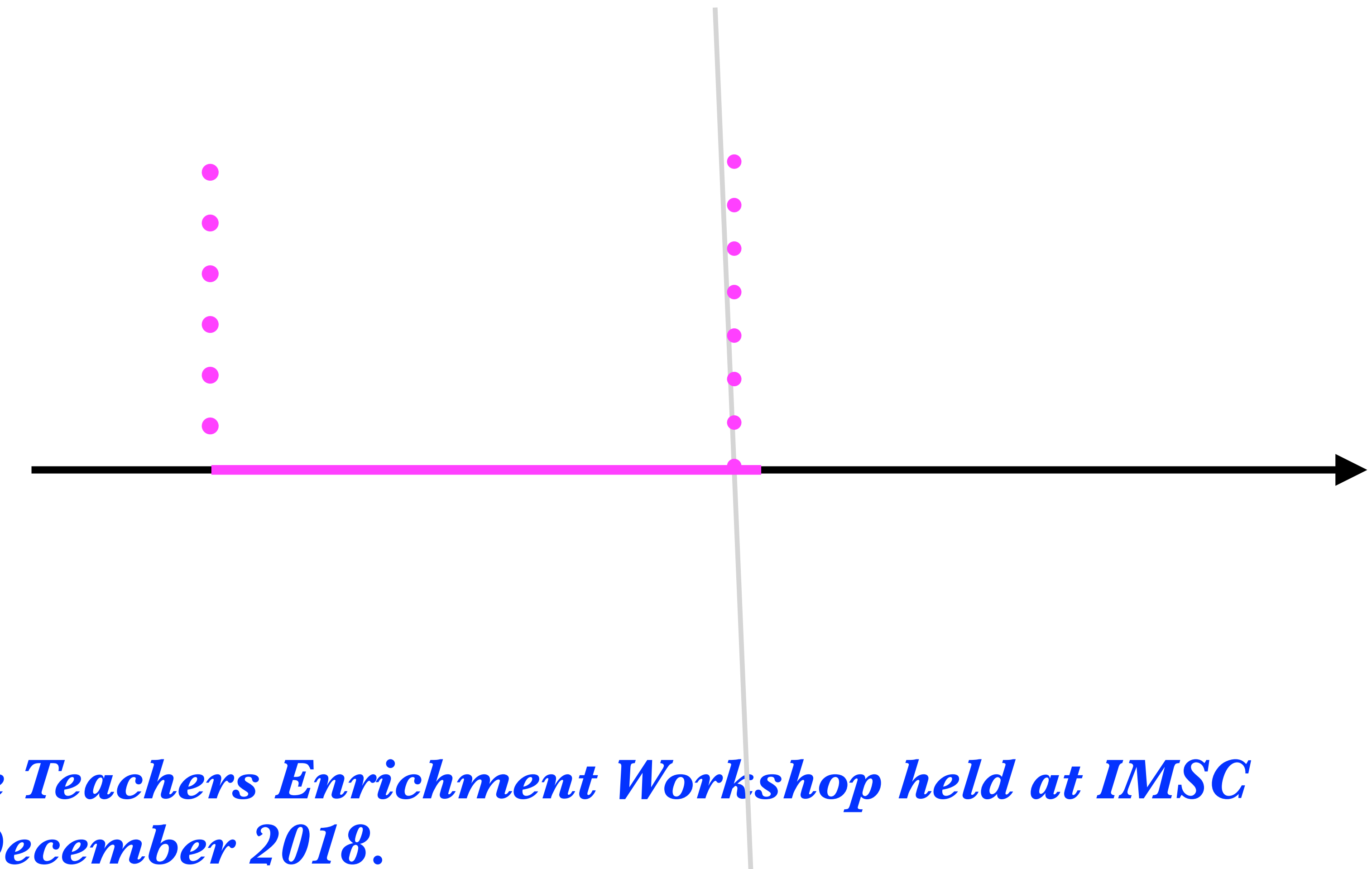

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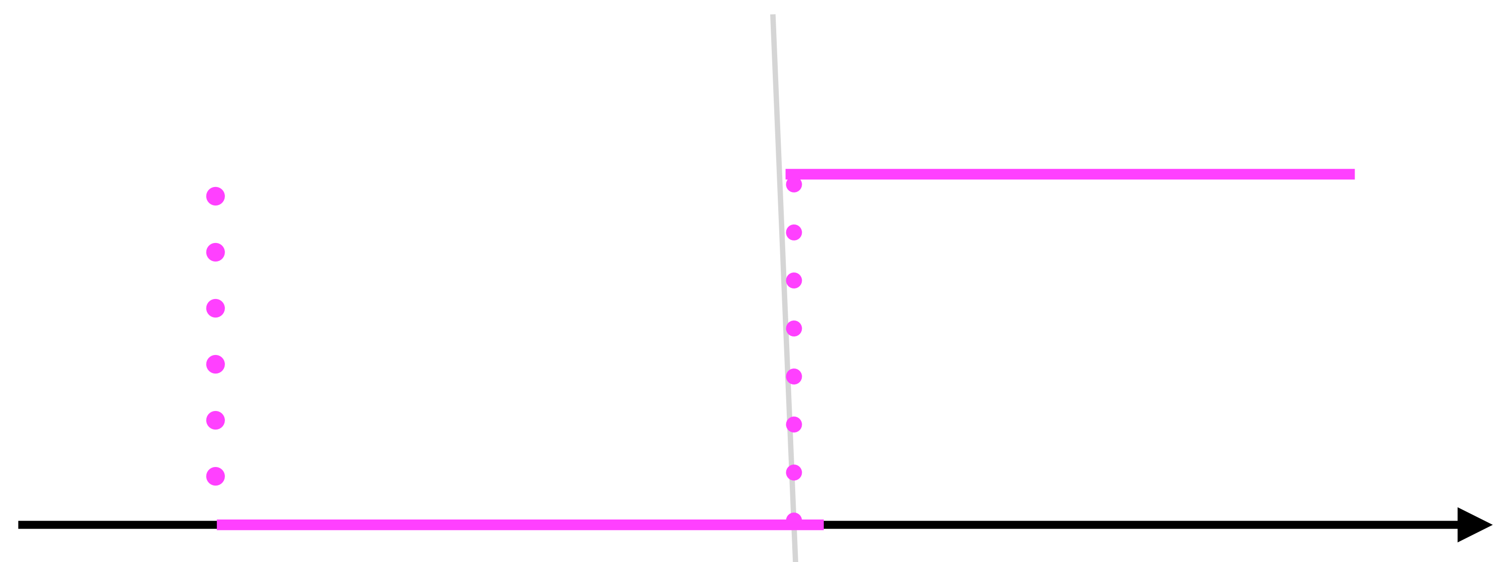

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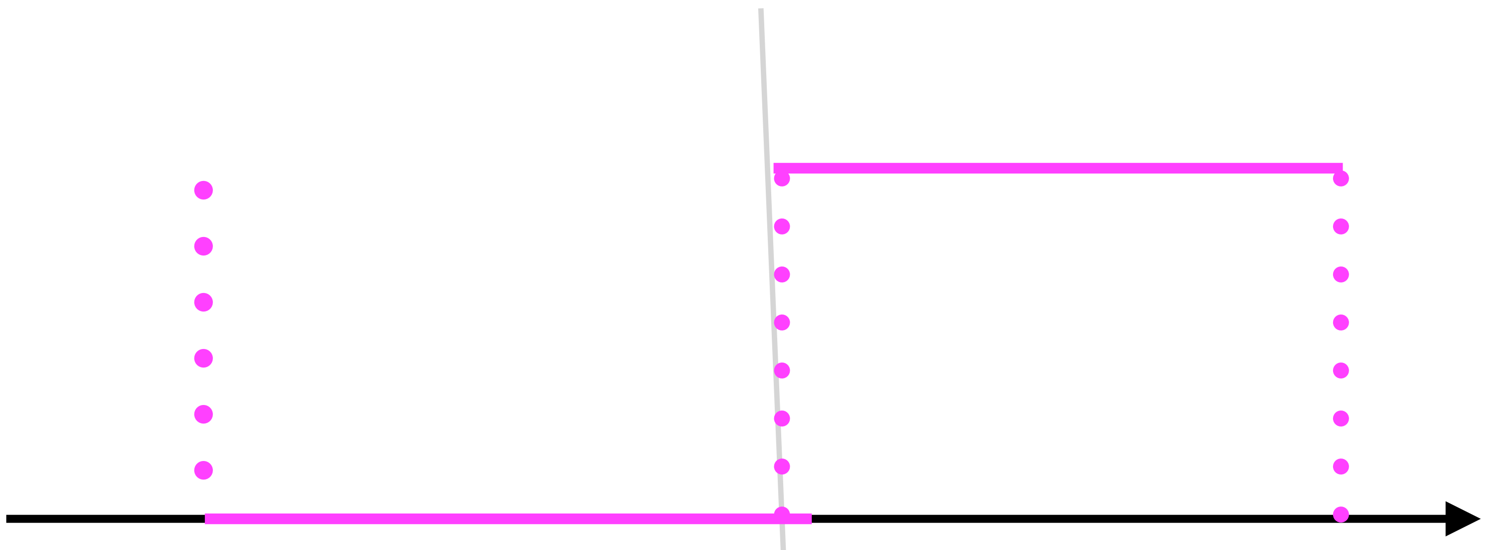

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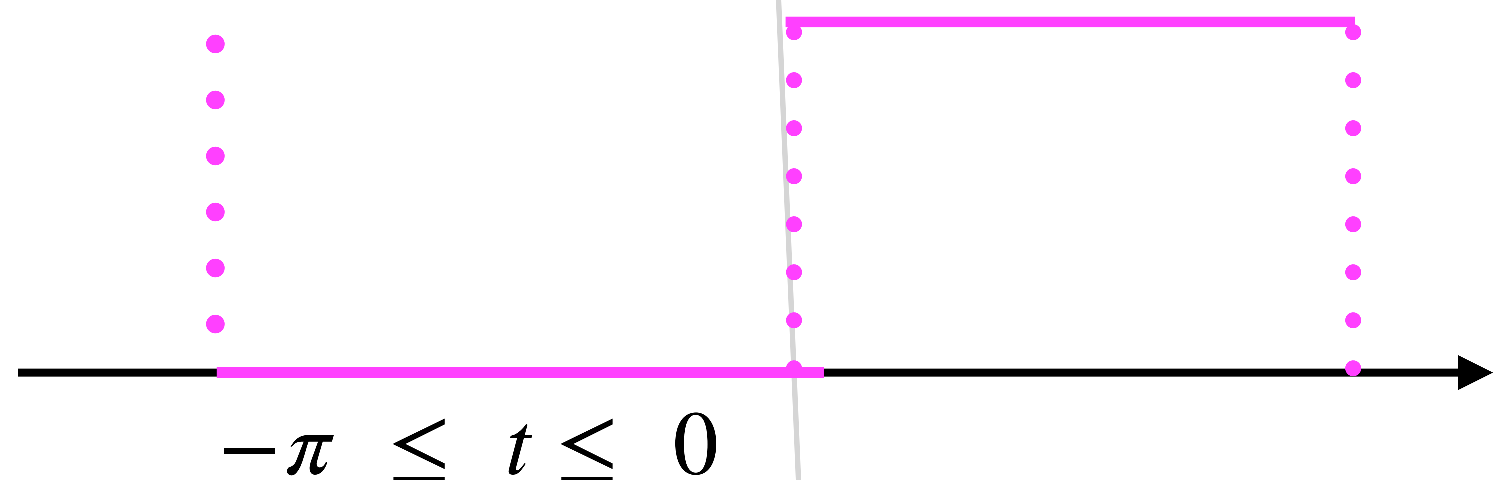

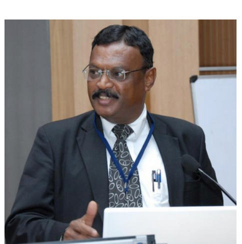

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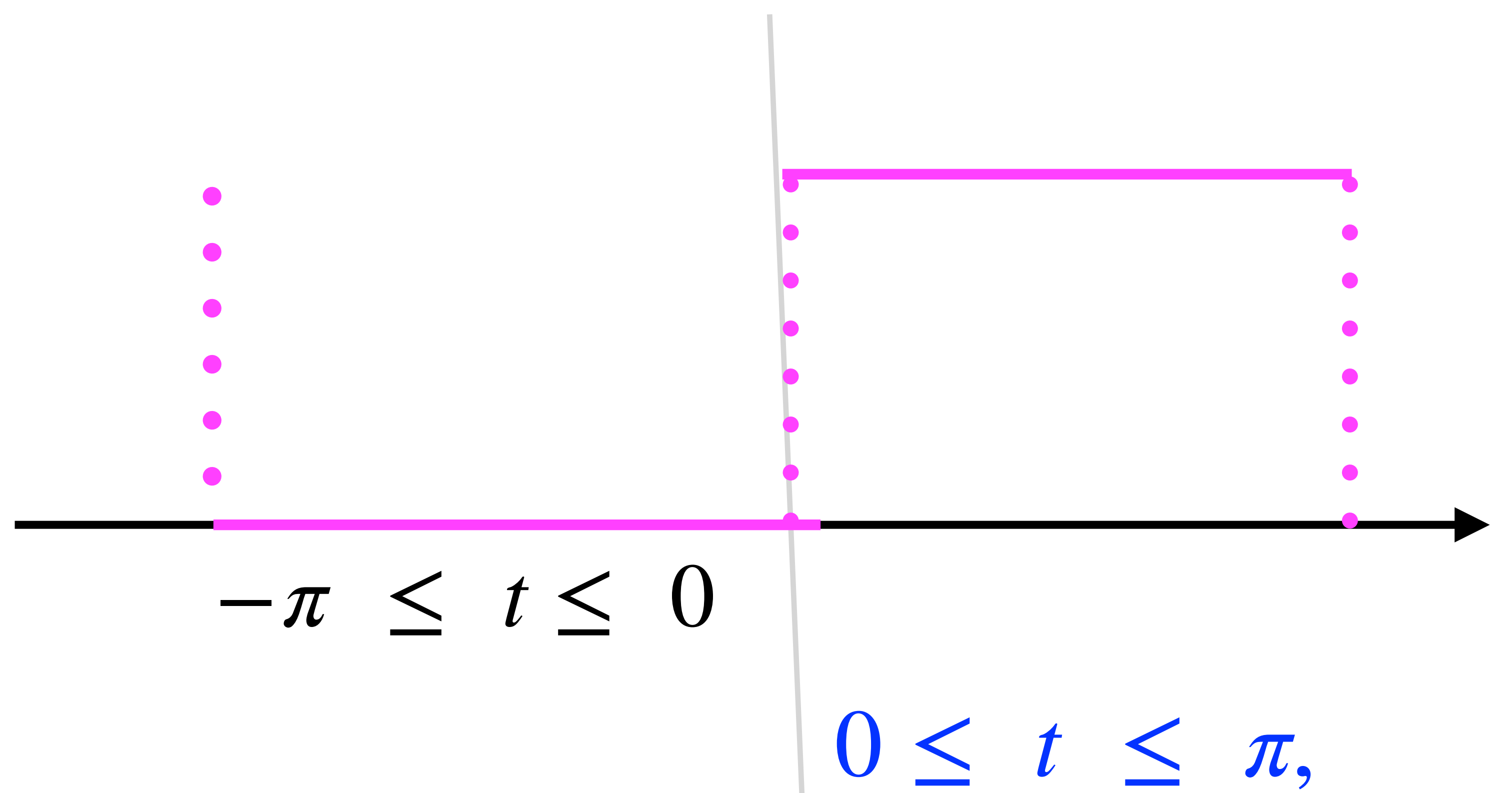

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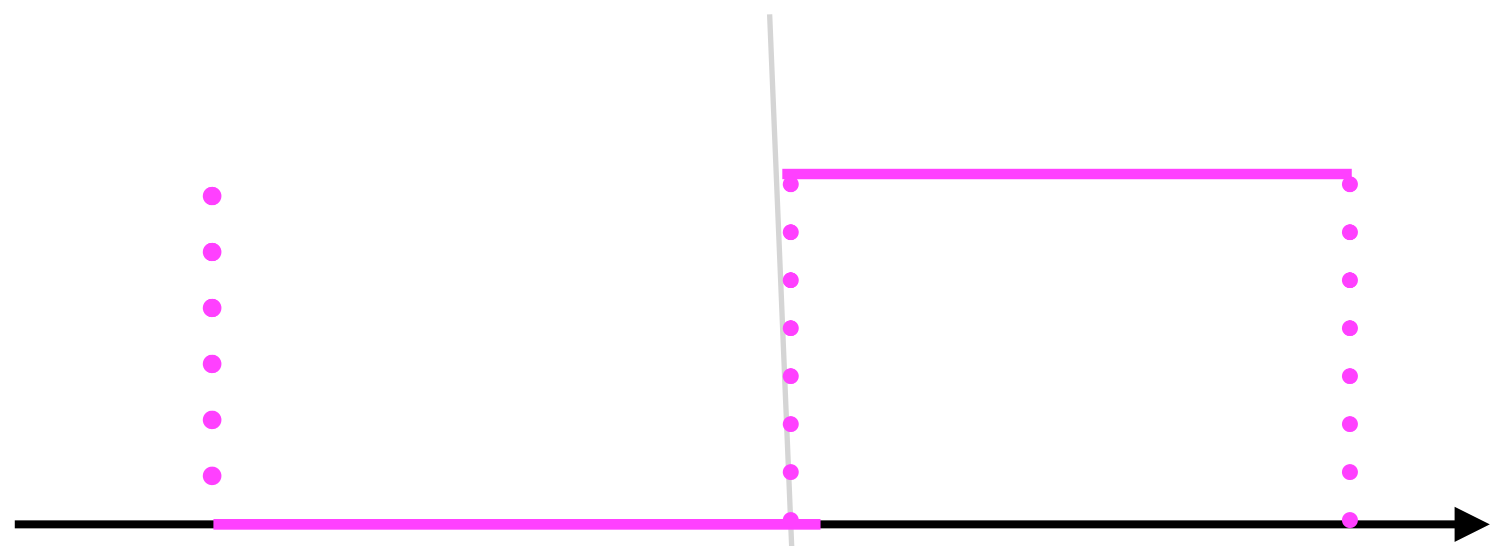

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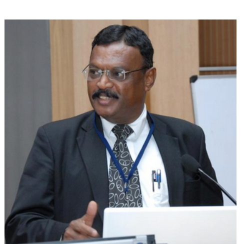

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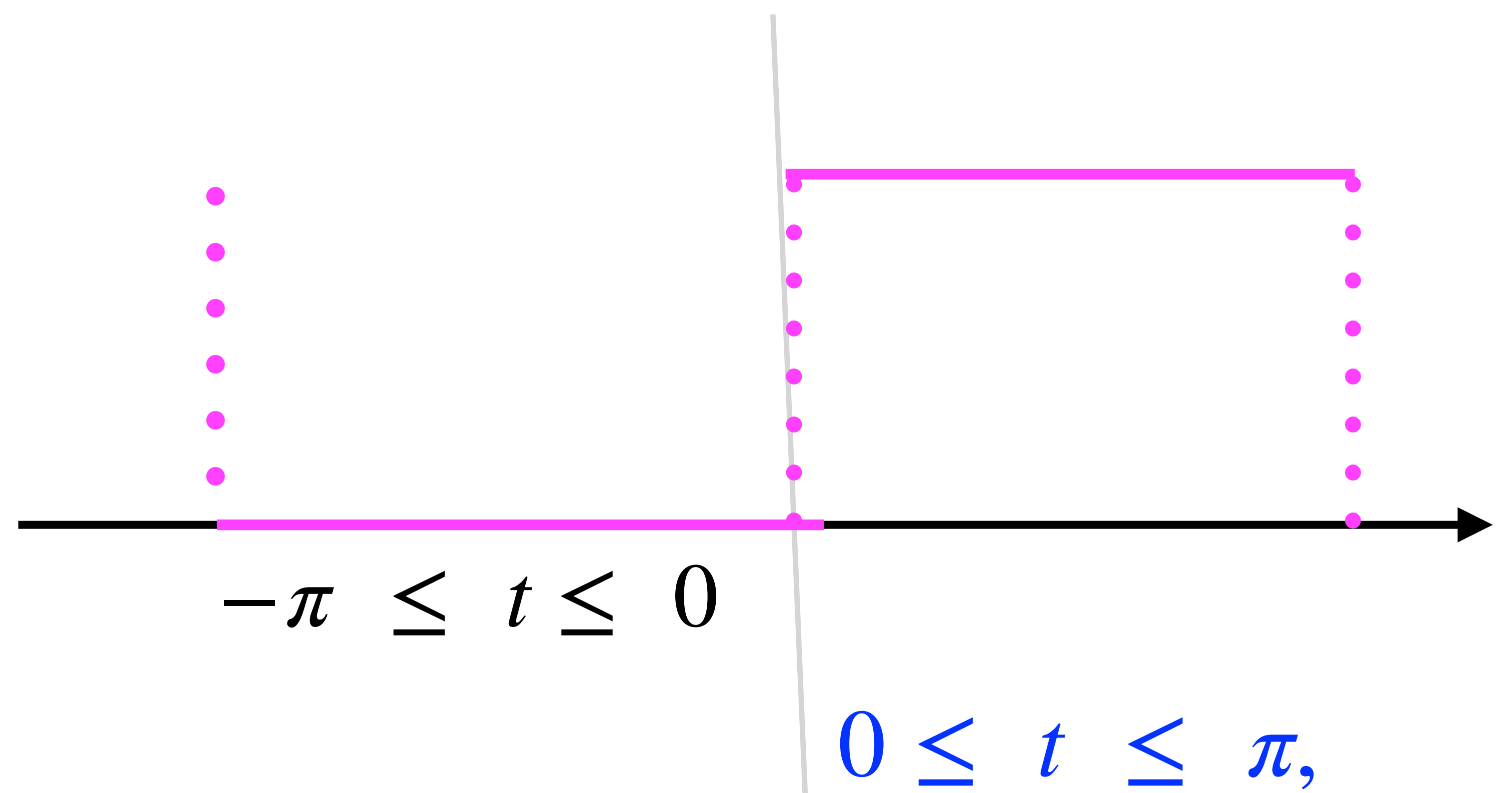

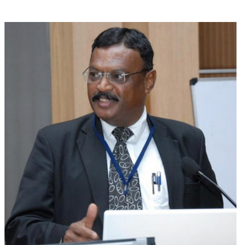

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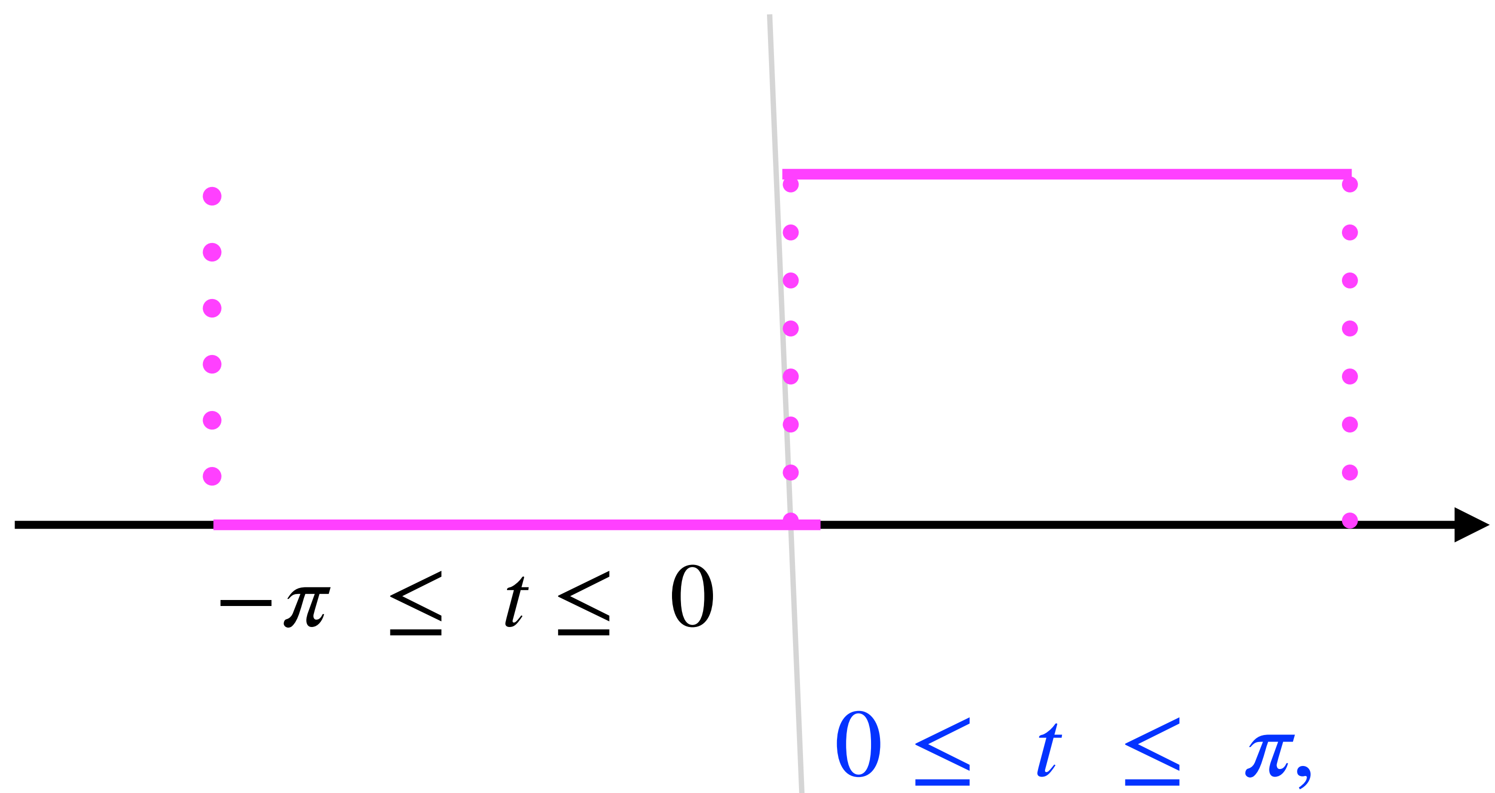

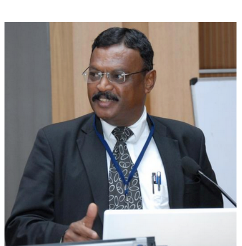

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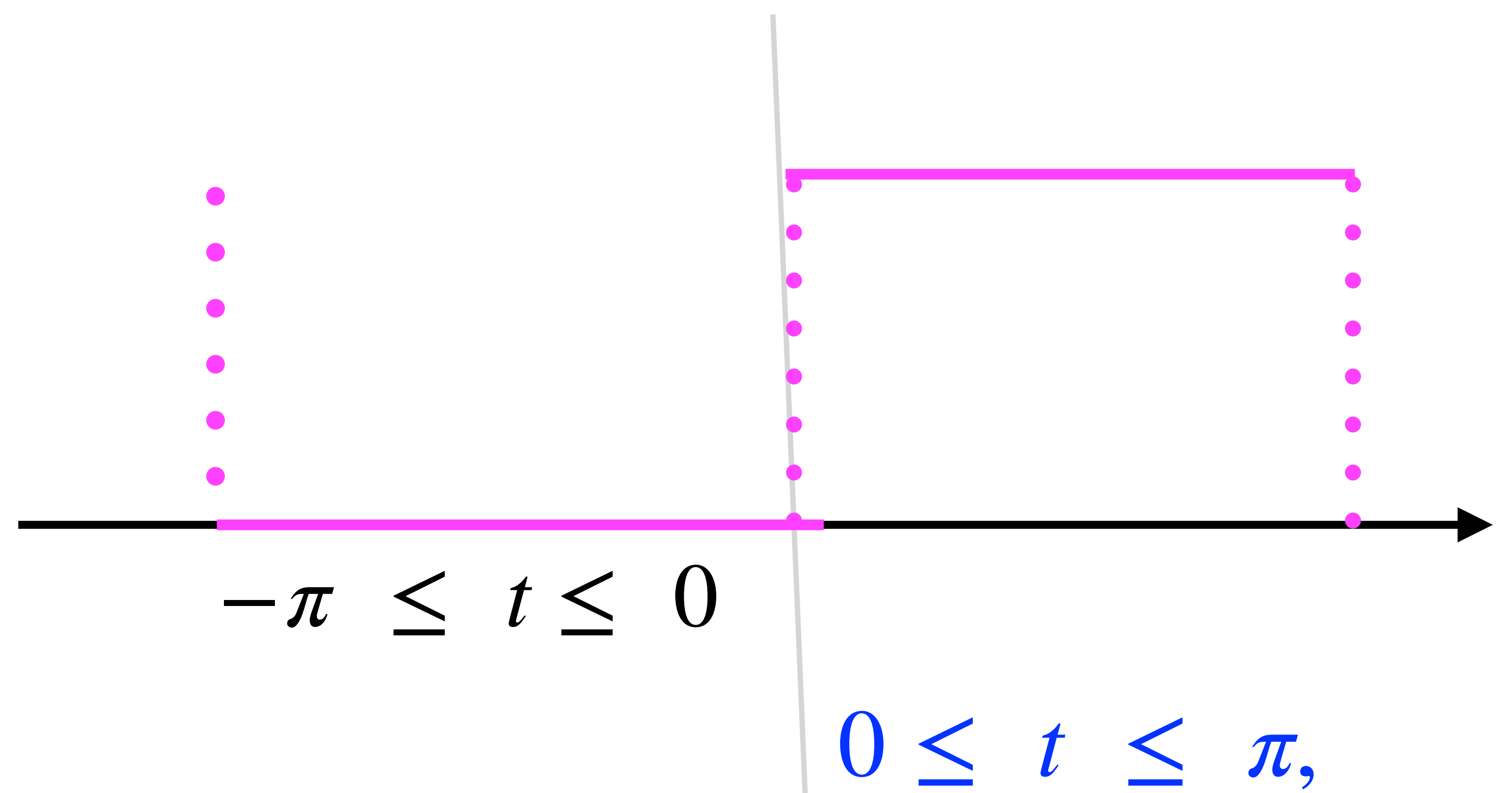

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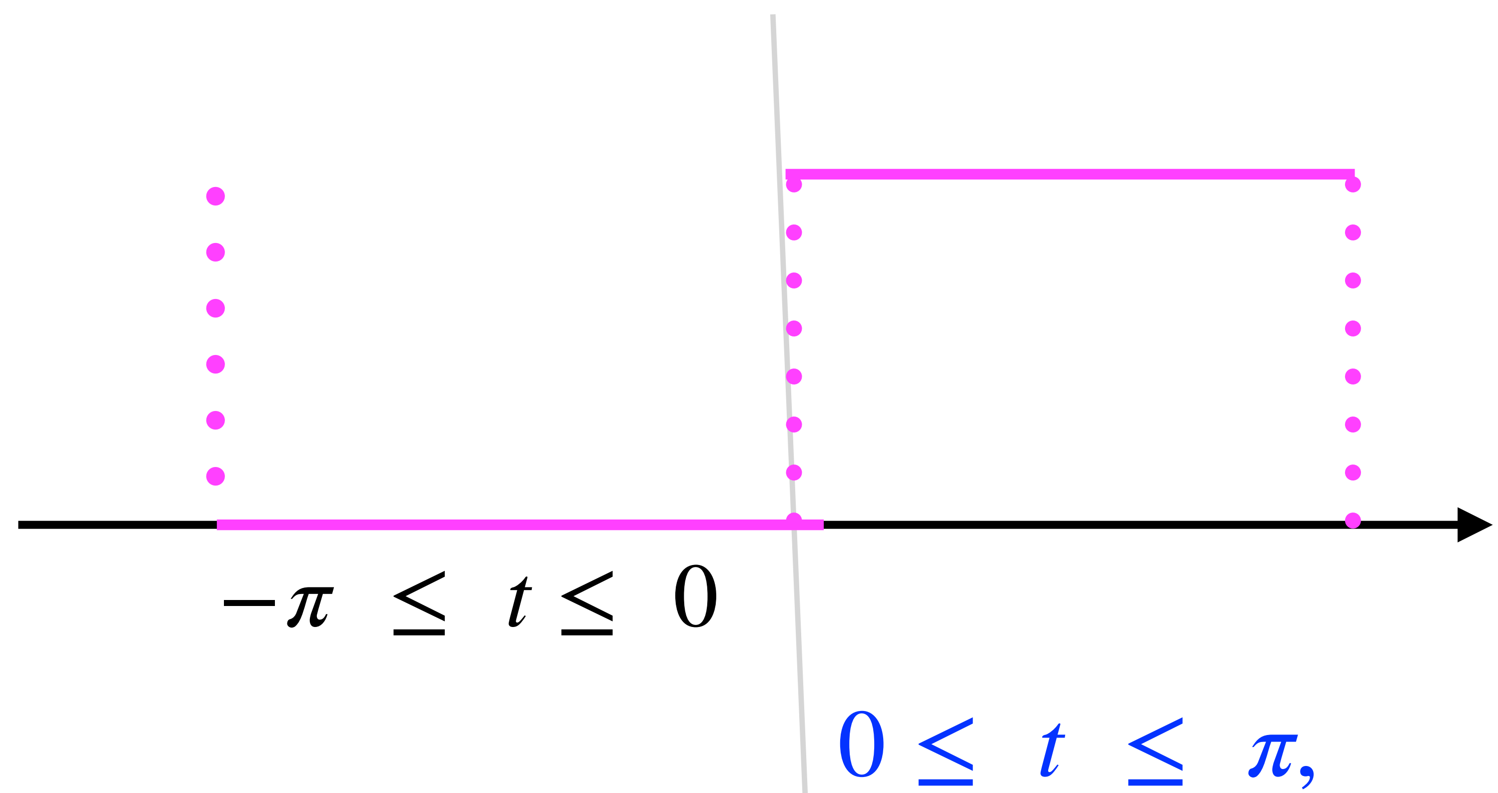

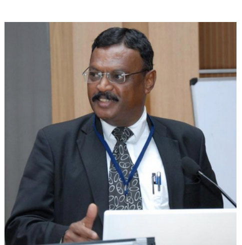

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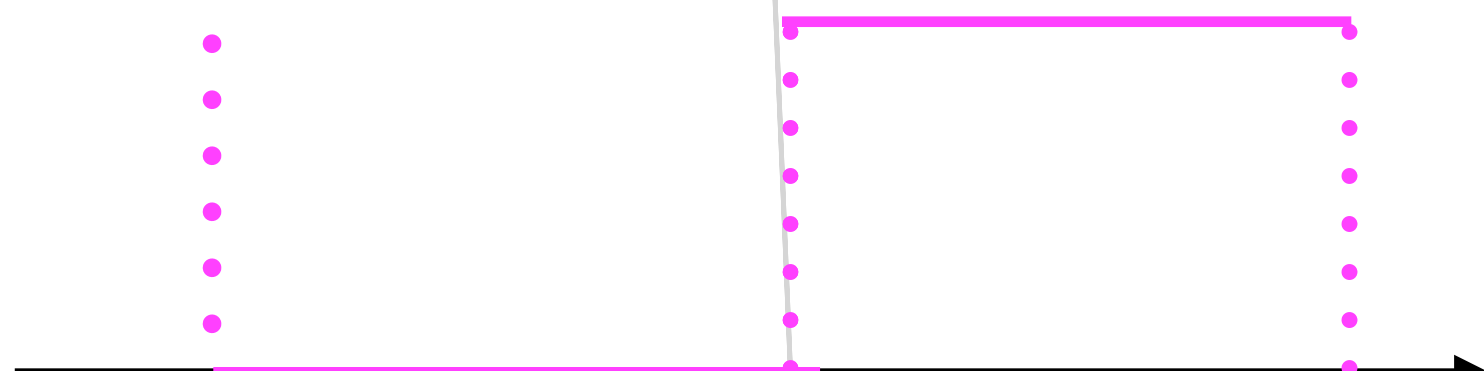

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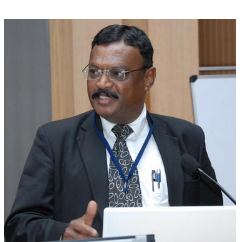

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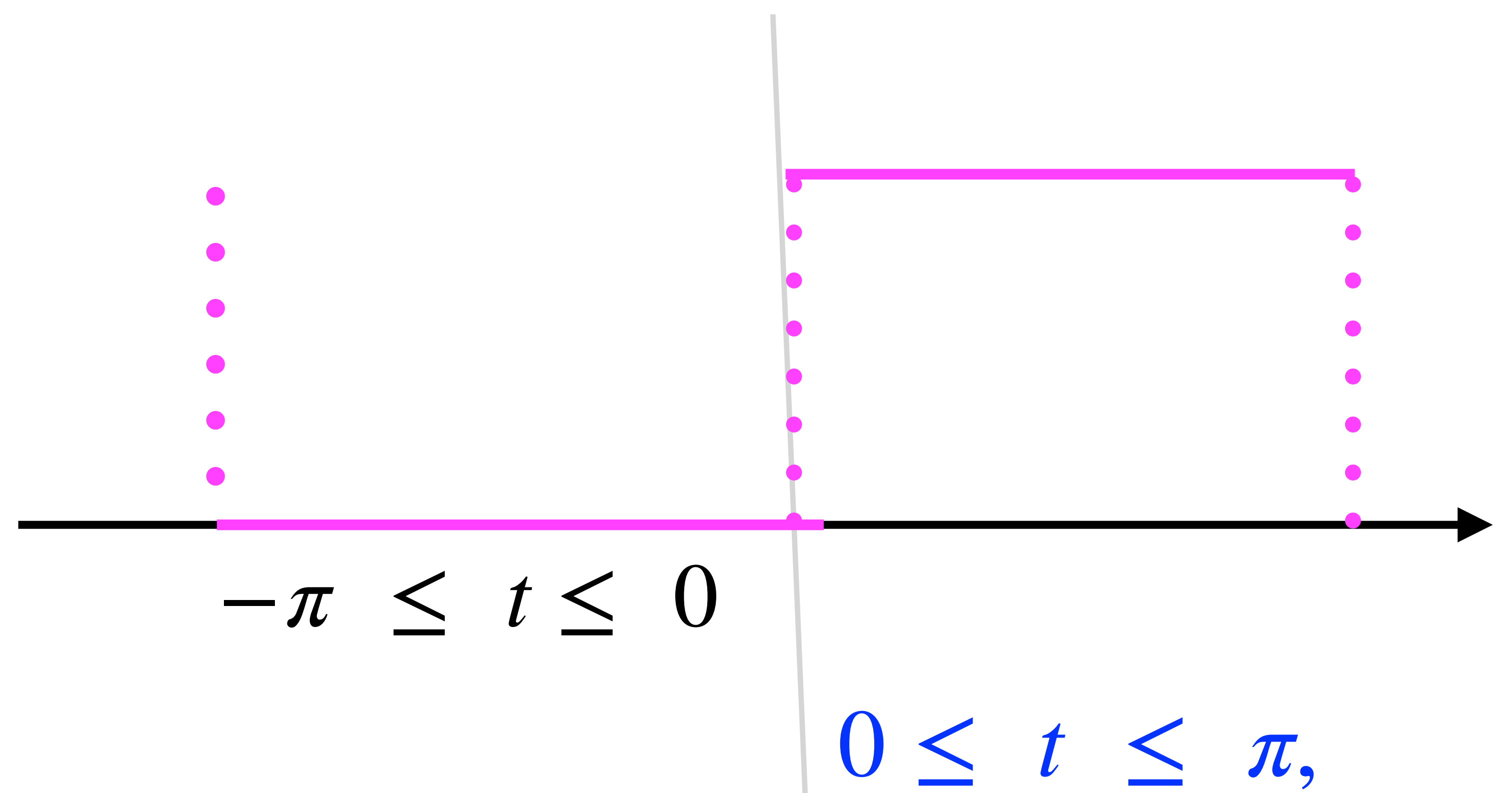

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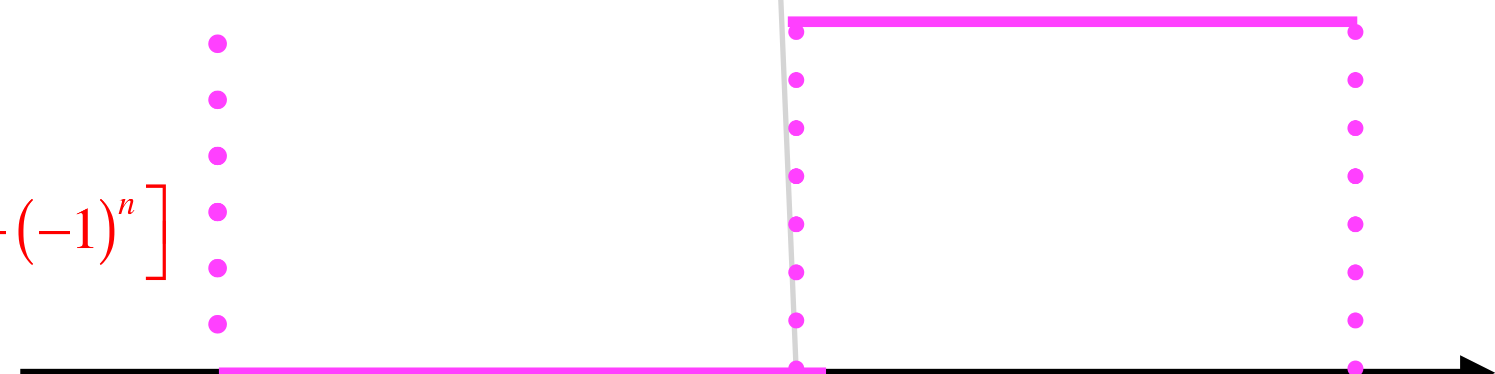

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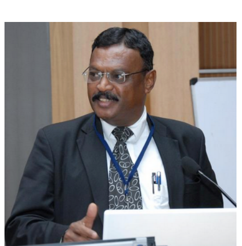

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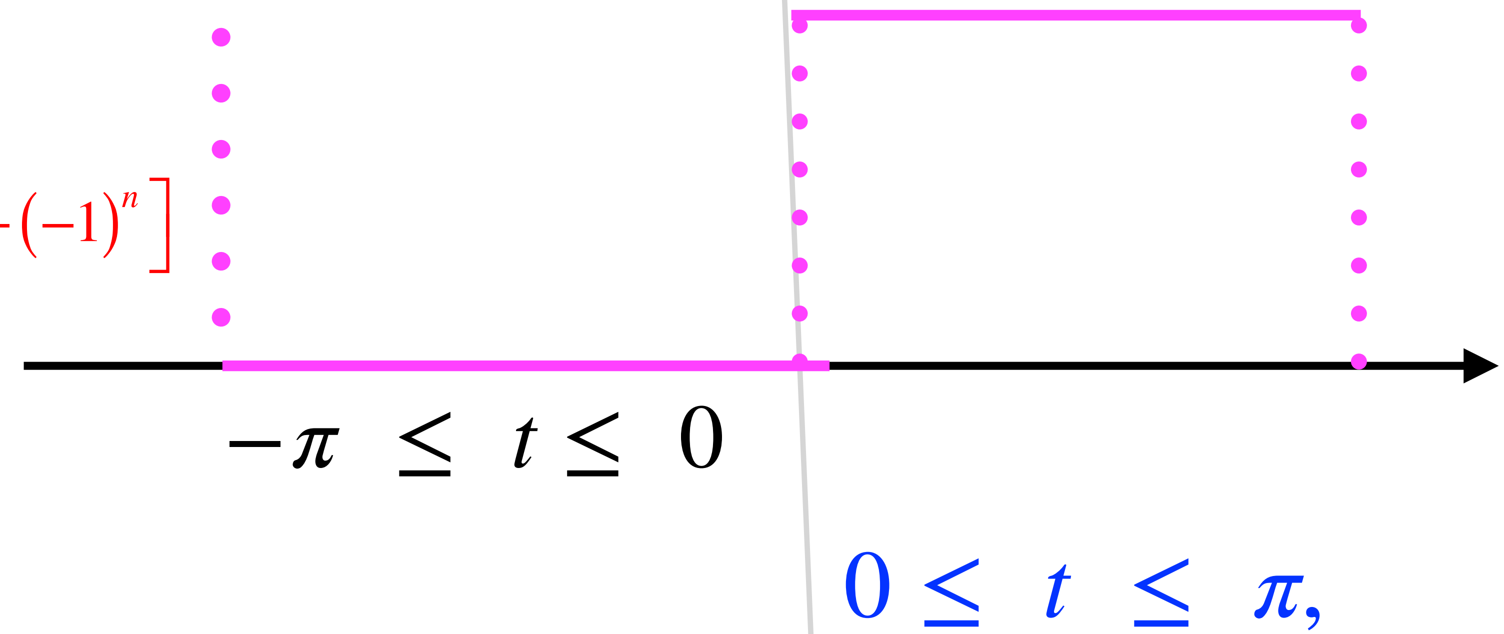
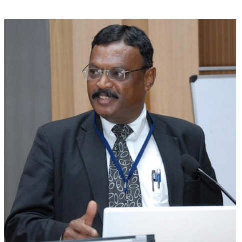
$$=\begin{cases}\dfrac{2}{n\pi} & \text{if } n \text{ is odd}\\ 0 & \text{if } n \text{ is even}\end{cases}$$





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Let us now expand the same function in a different period interval say

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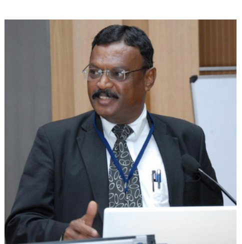

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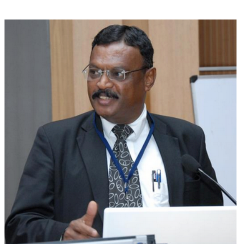

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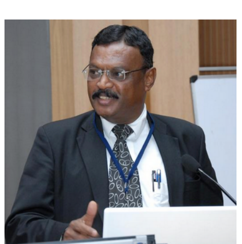

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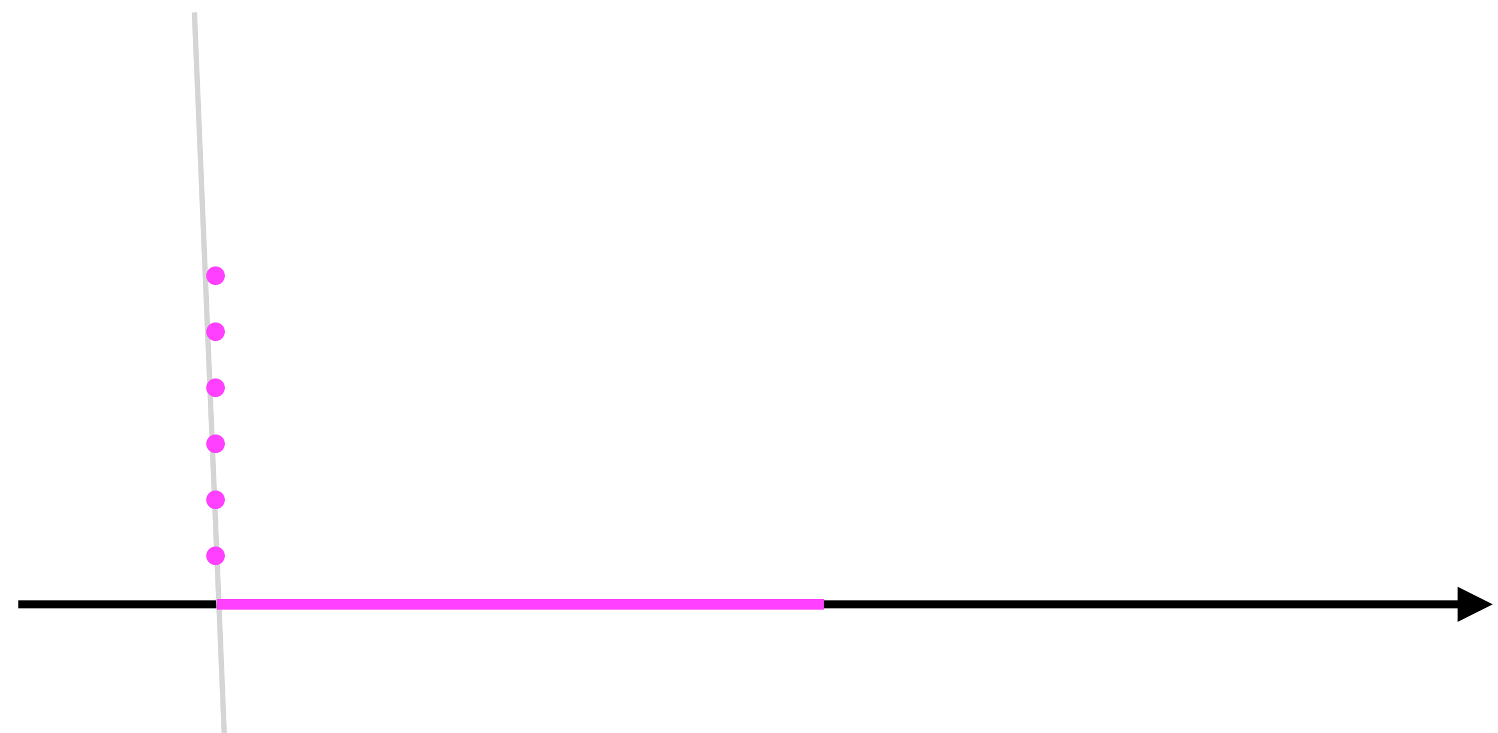

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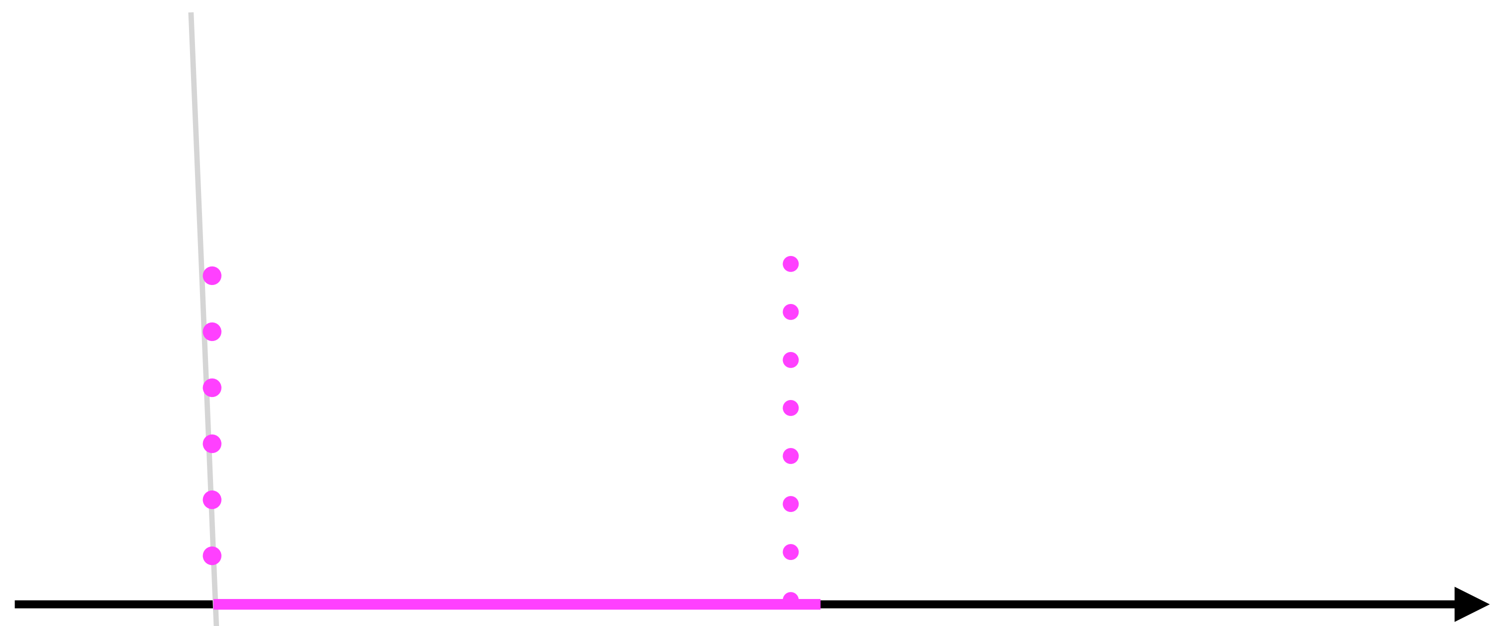

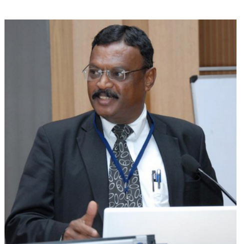

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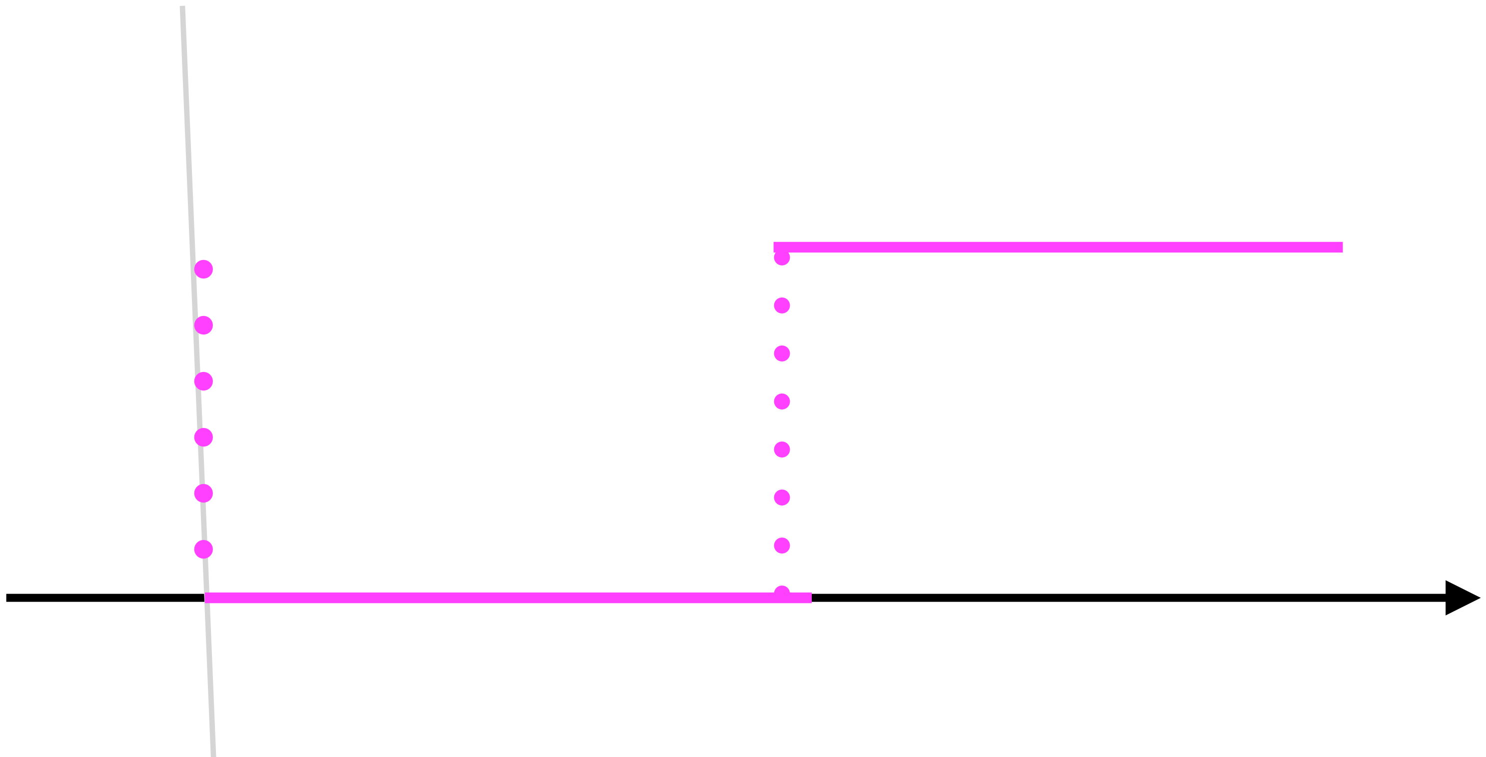

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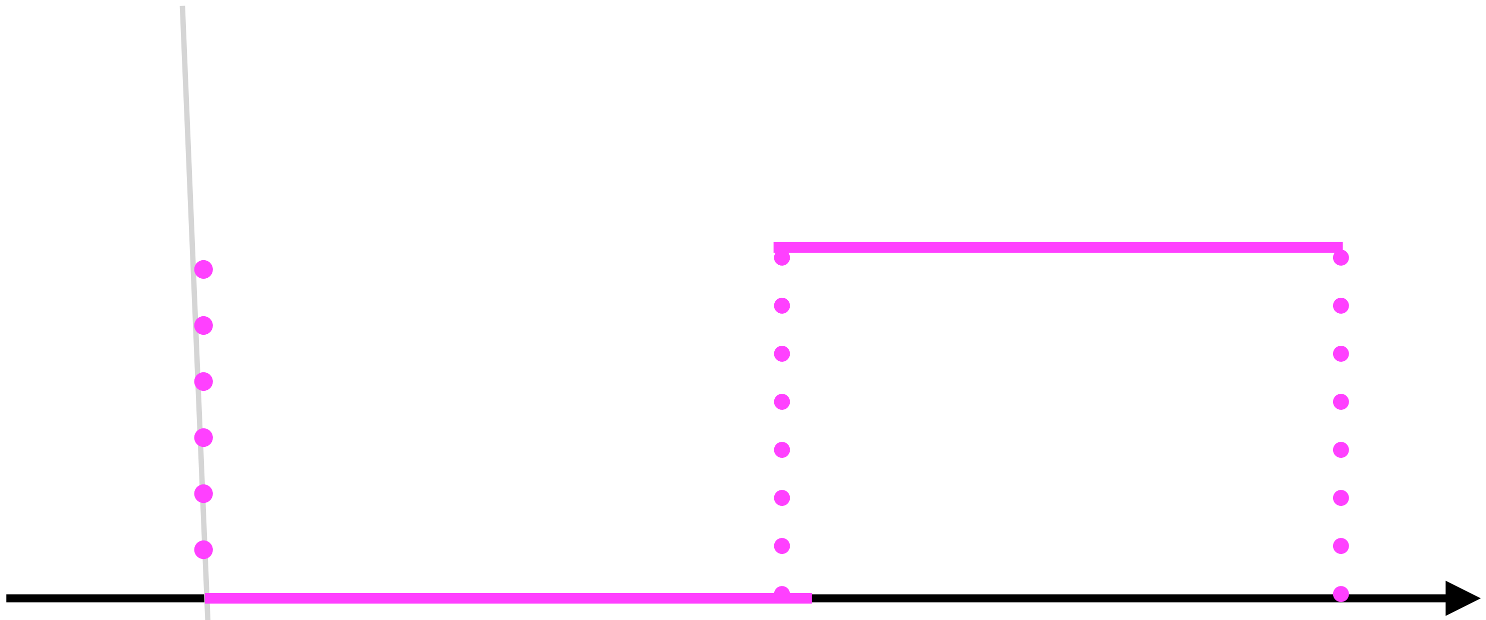

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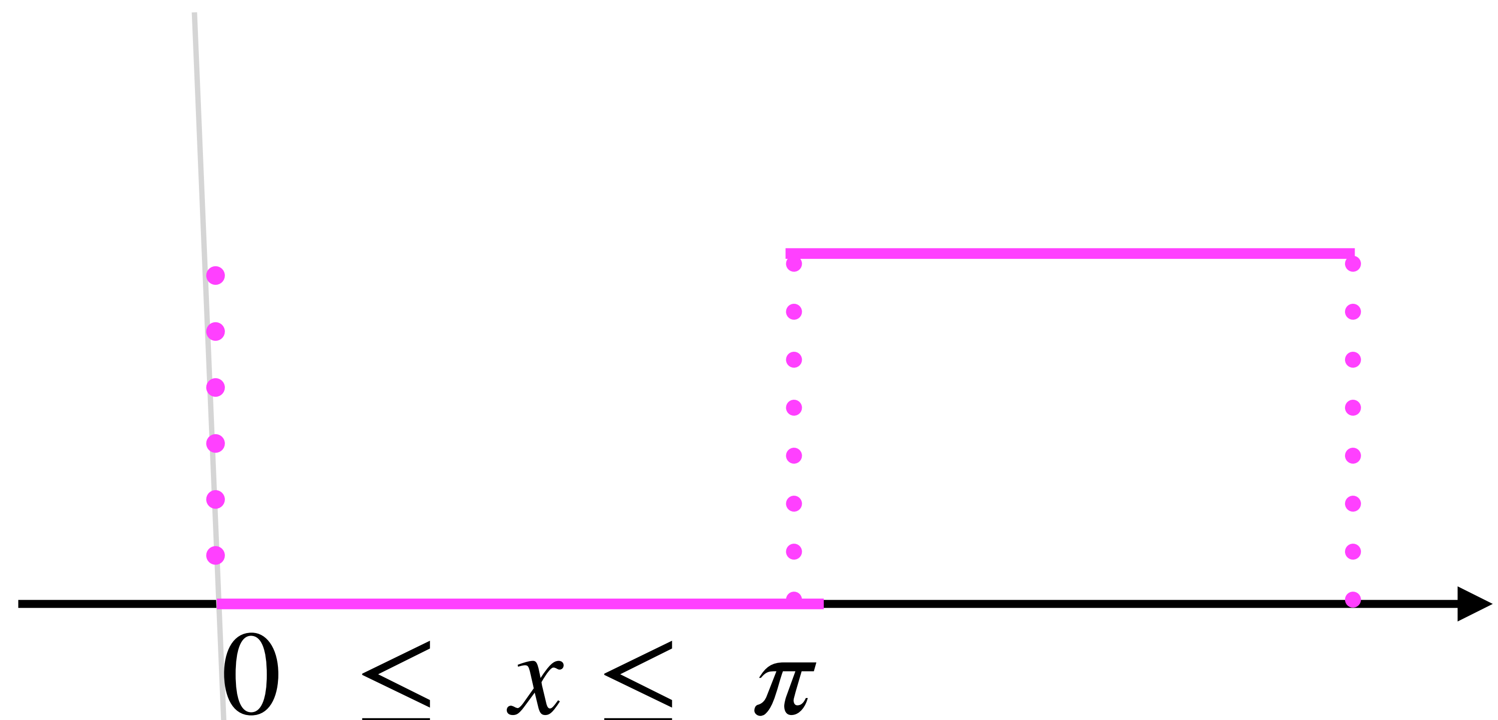

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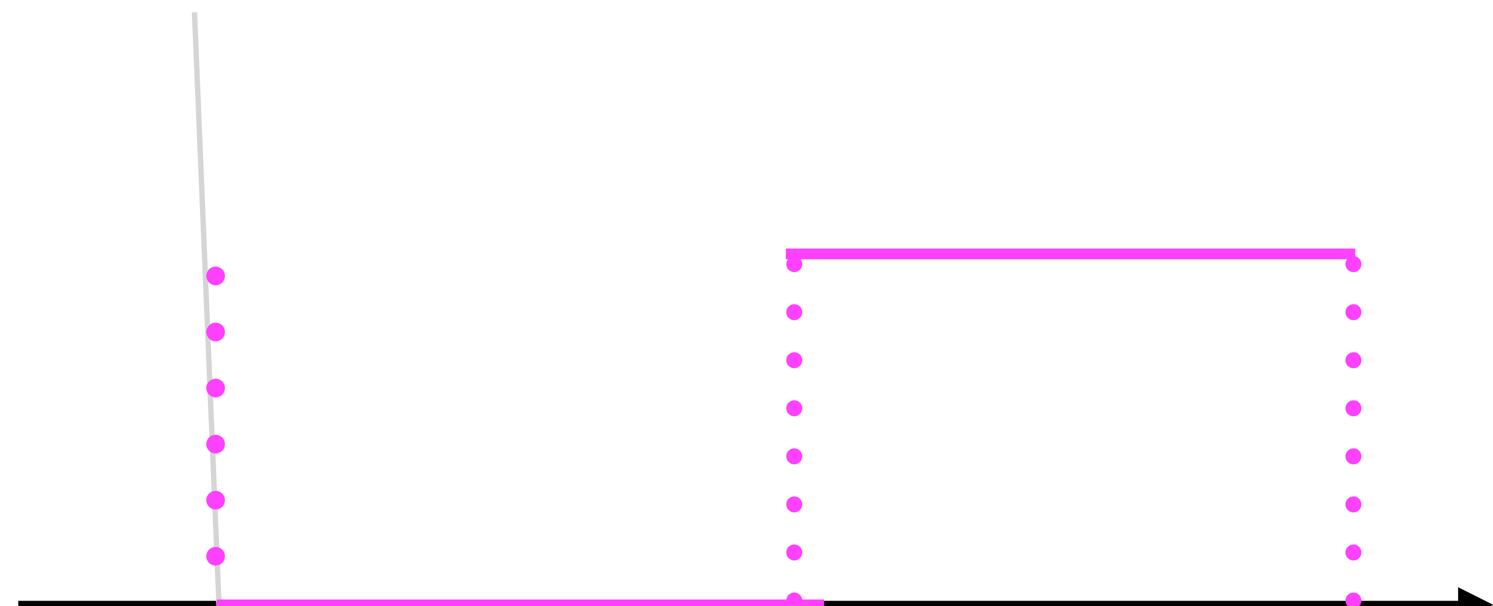

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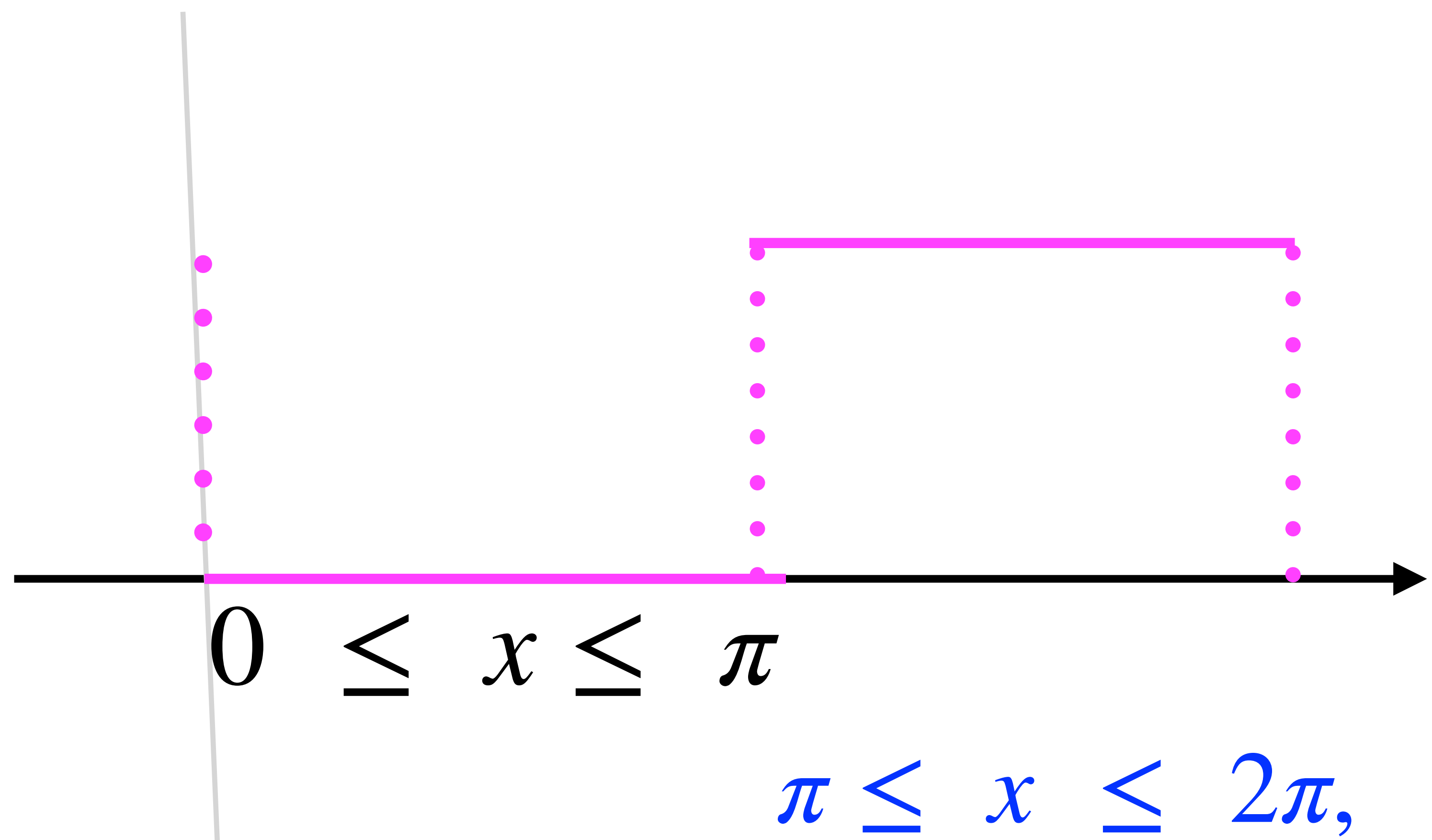

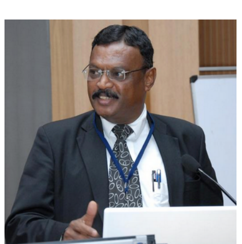

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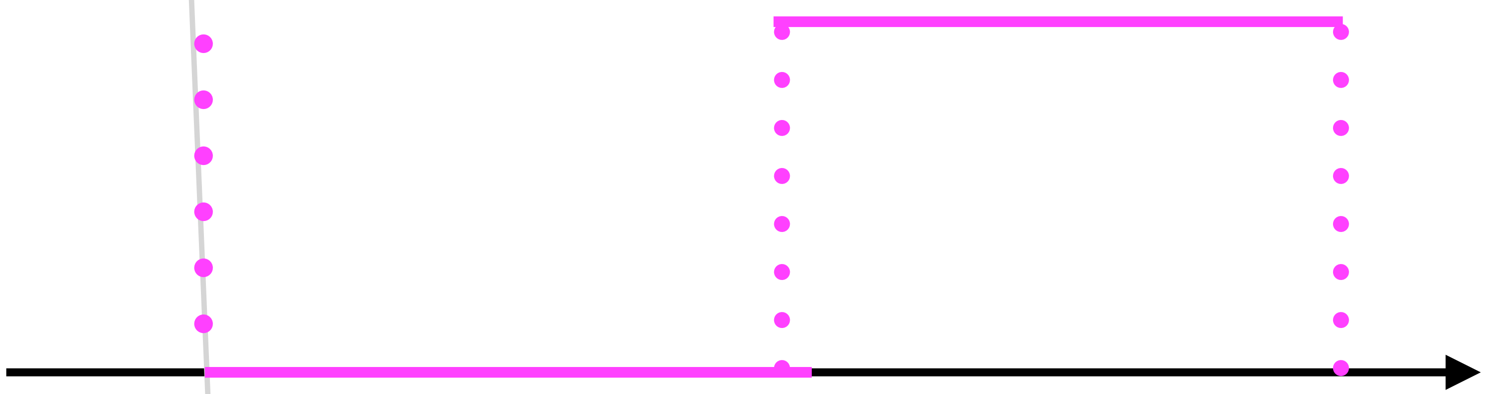

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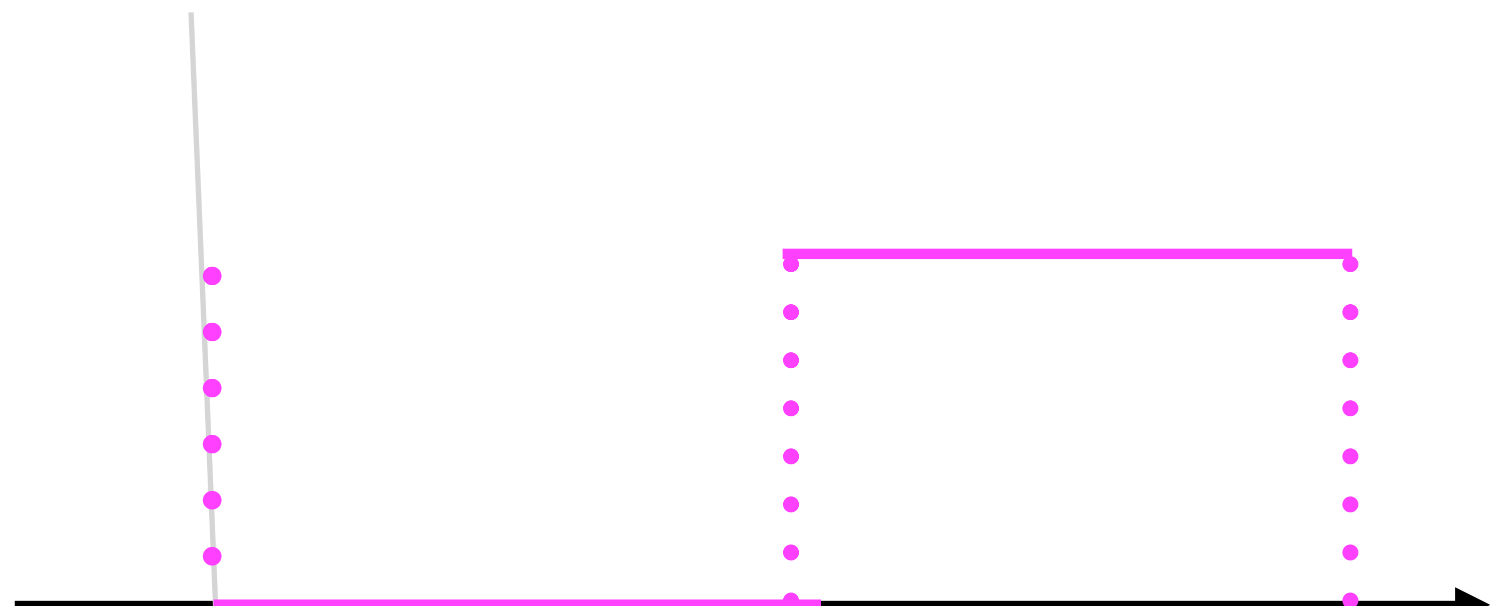

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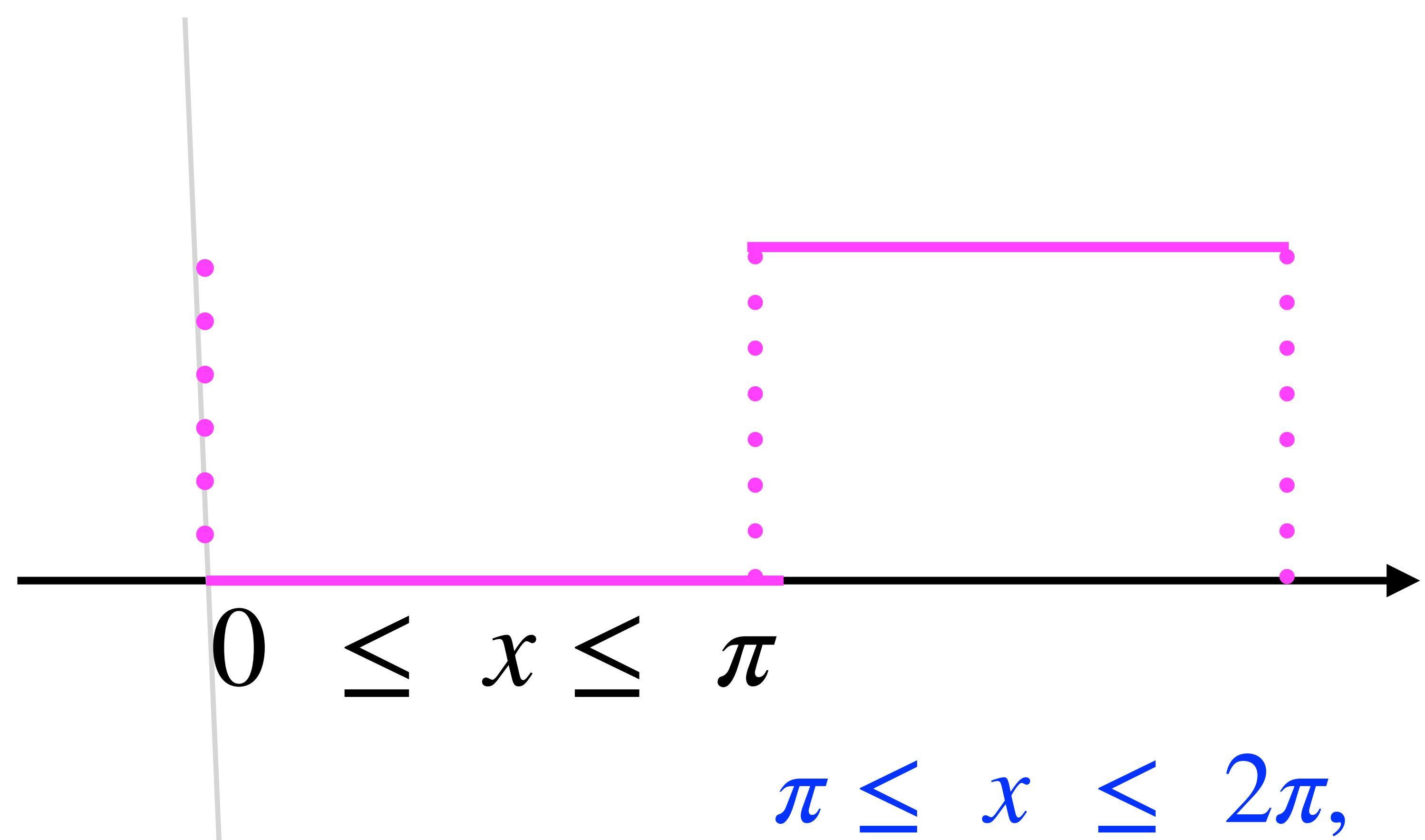

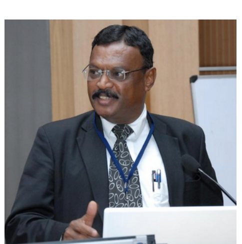

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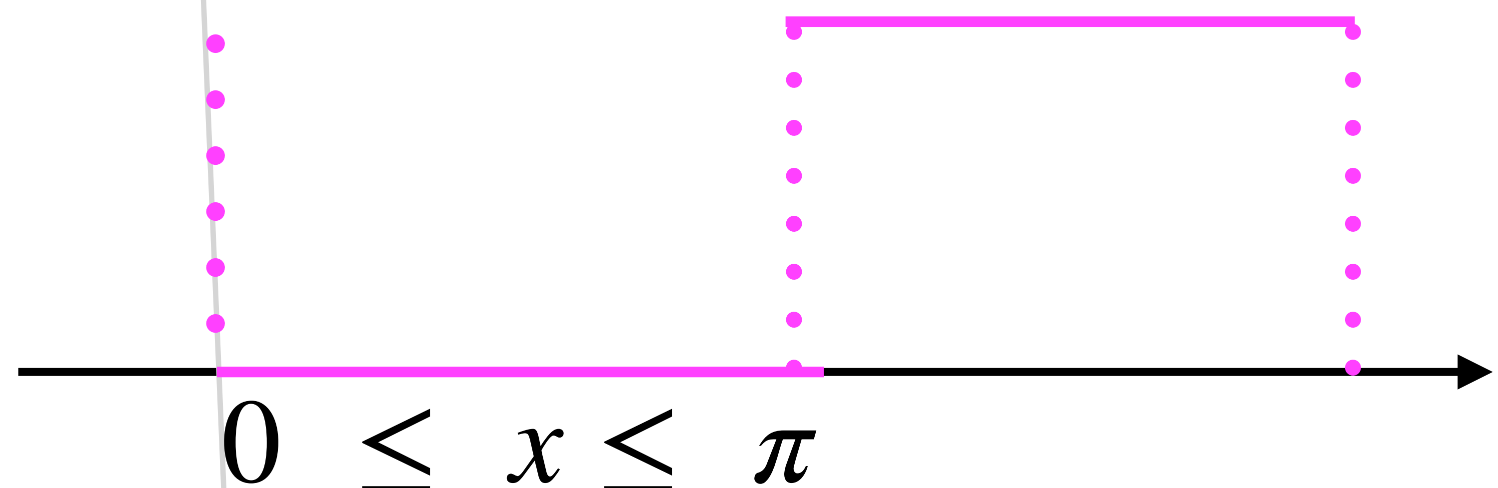

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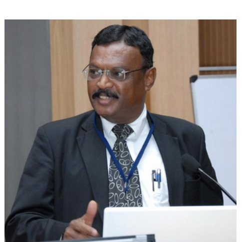

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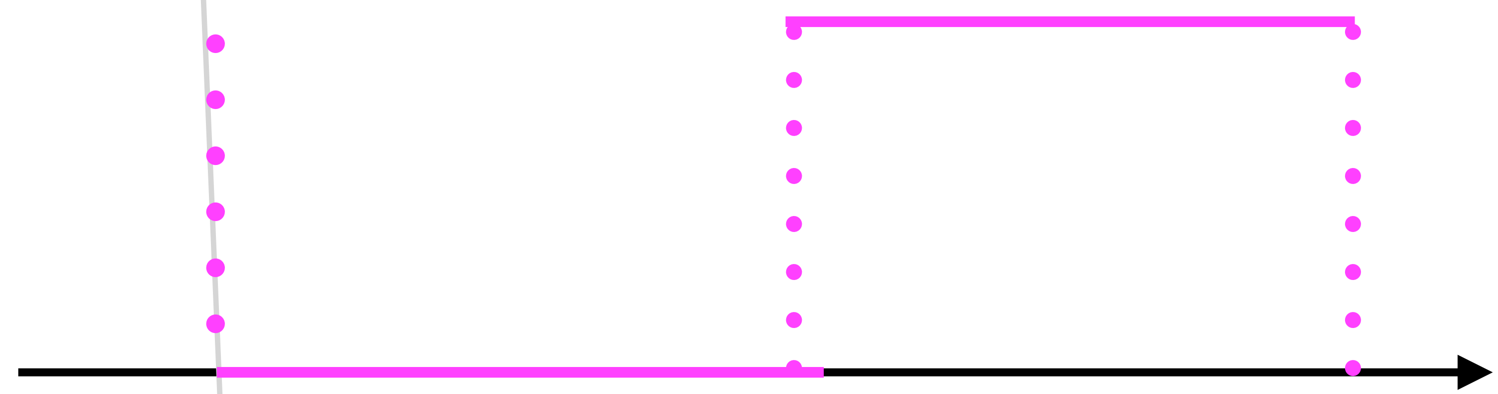

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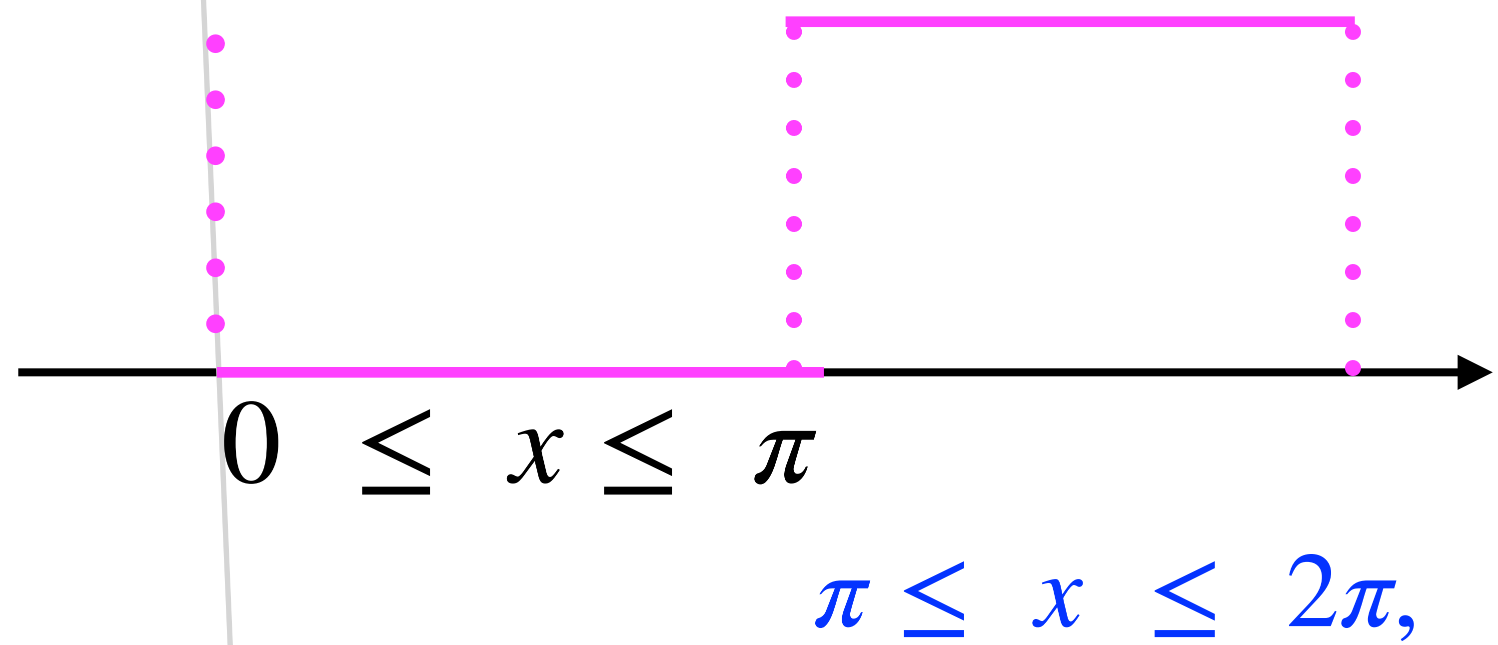

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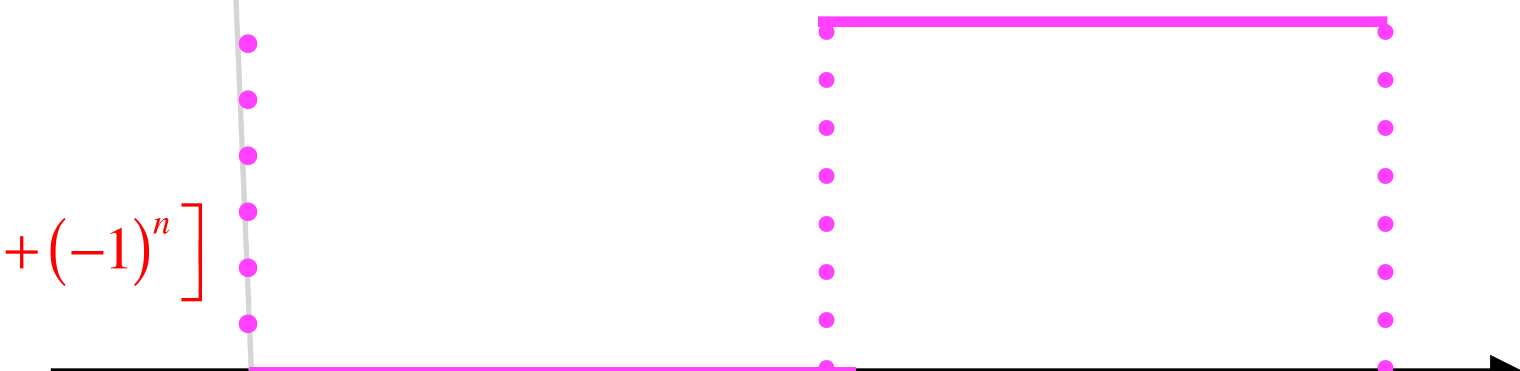
$$= \frac{1}{\pi}\int_\pi^{2\pi} \text{Sin } nt\, dt = -\frac{1}{\pi}\left.\left(\frac{\cos nt}{n}\right)\right|_{x=\pi}^{x=2\pi} = \frac{1}{n\pi}\left[-1 + (-1)^n\right]$$



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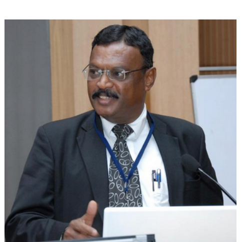

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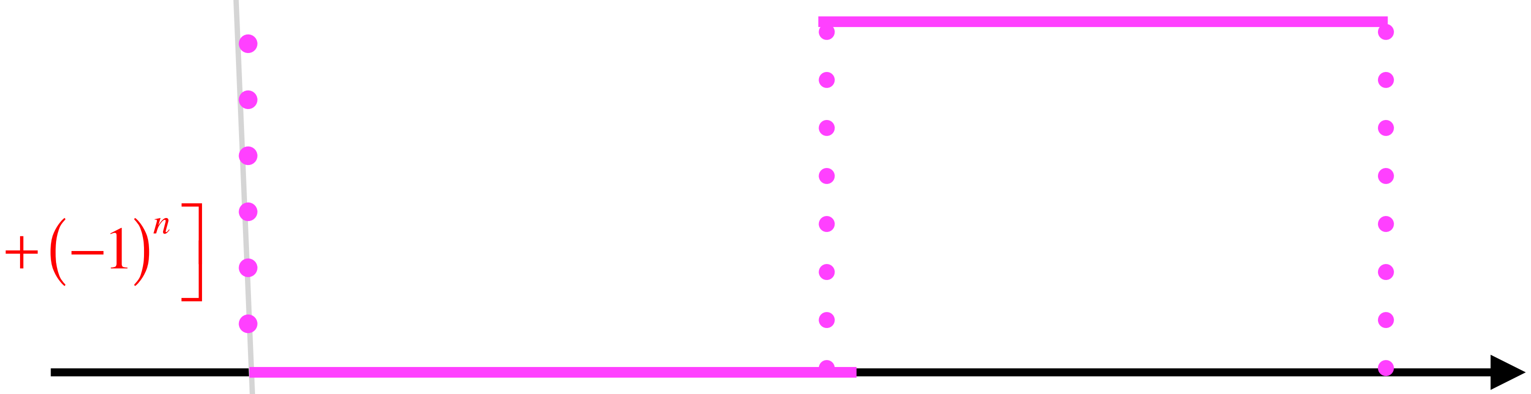
$$=\begin{cases}-\dfrac{2}{n\pi} & \text{if } n \text{ is odd}\\ 0 & \text{if } n \text{ is even}\end{cases}$$



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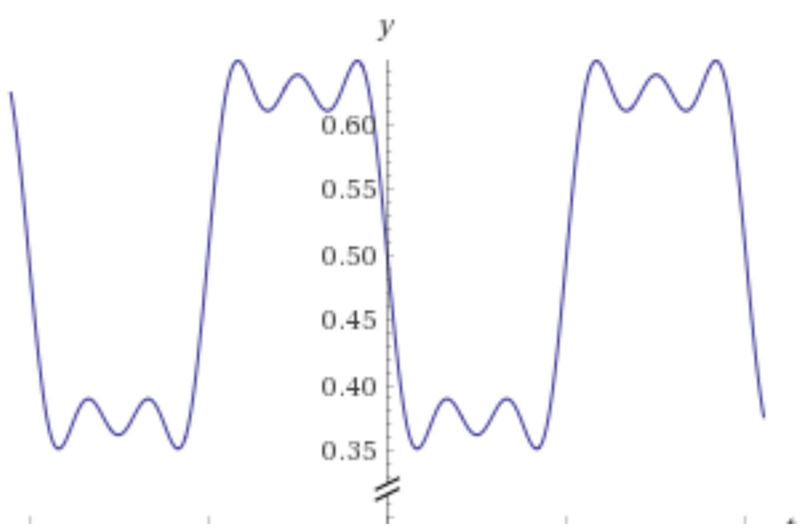

(t from −6.6 to 6.6)

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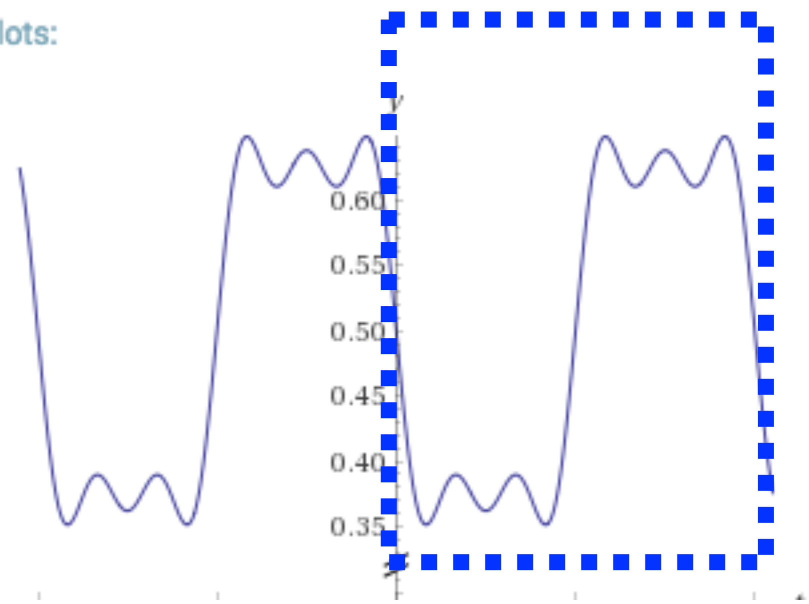

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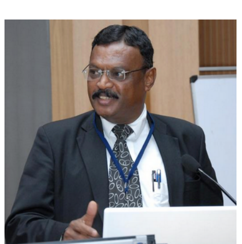

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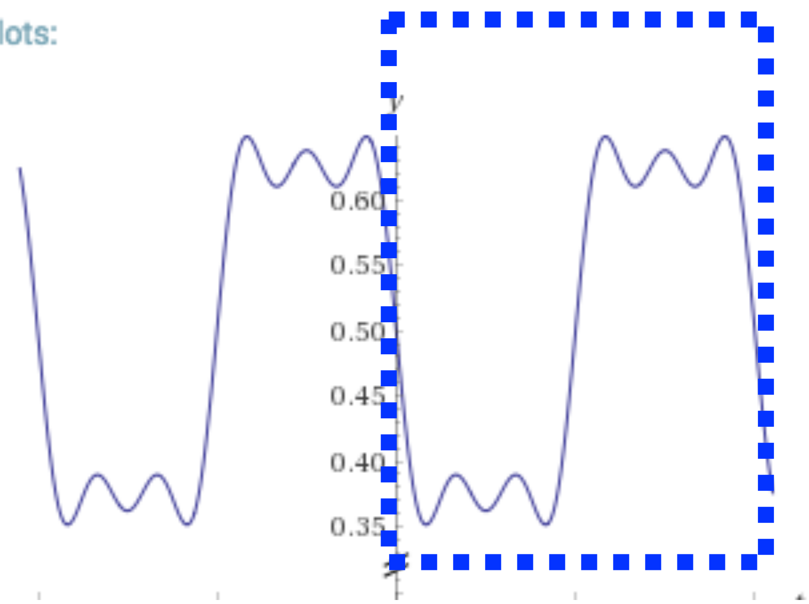

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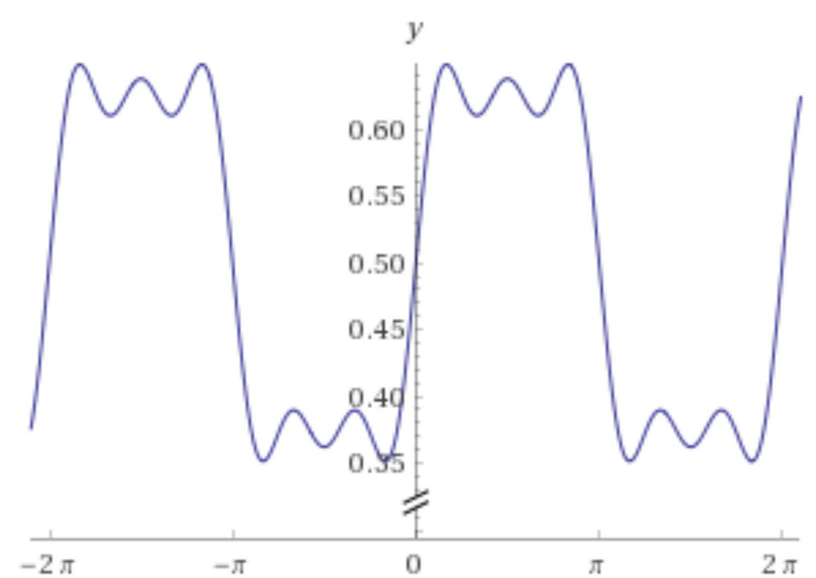

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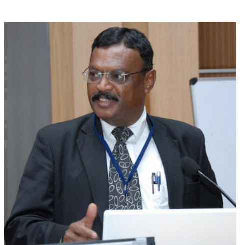

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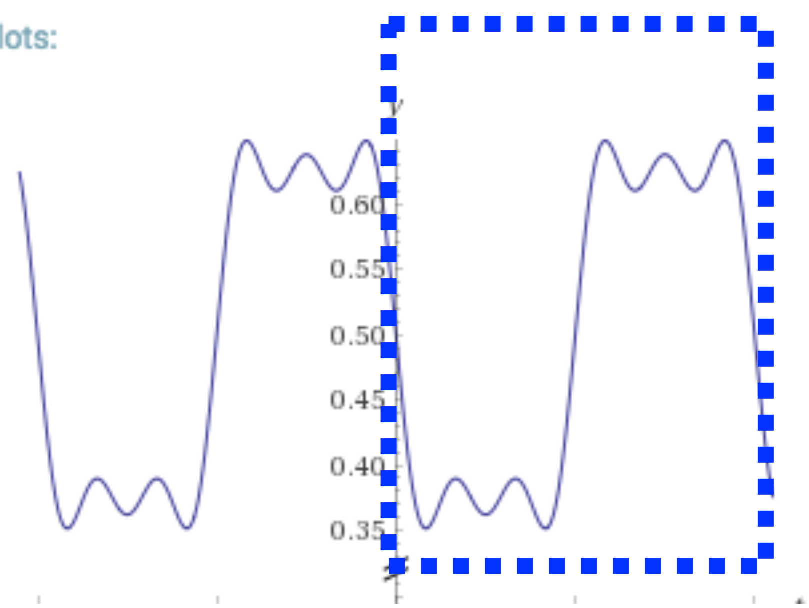

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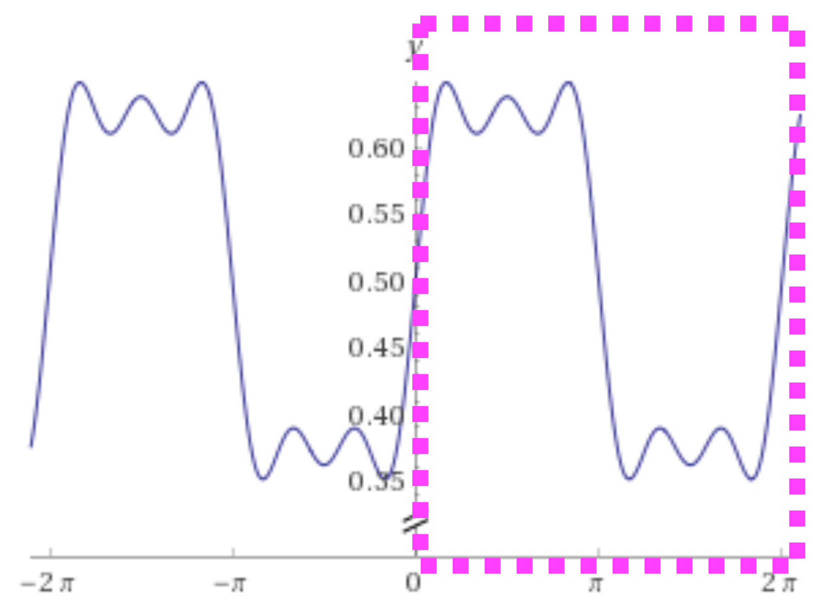

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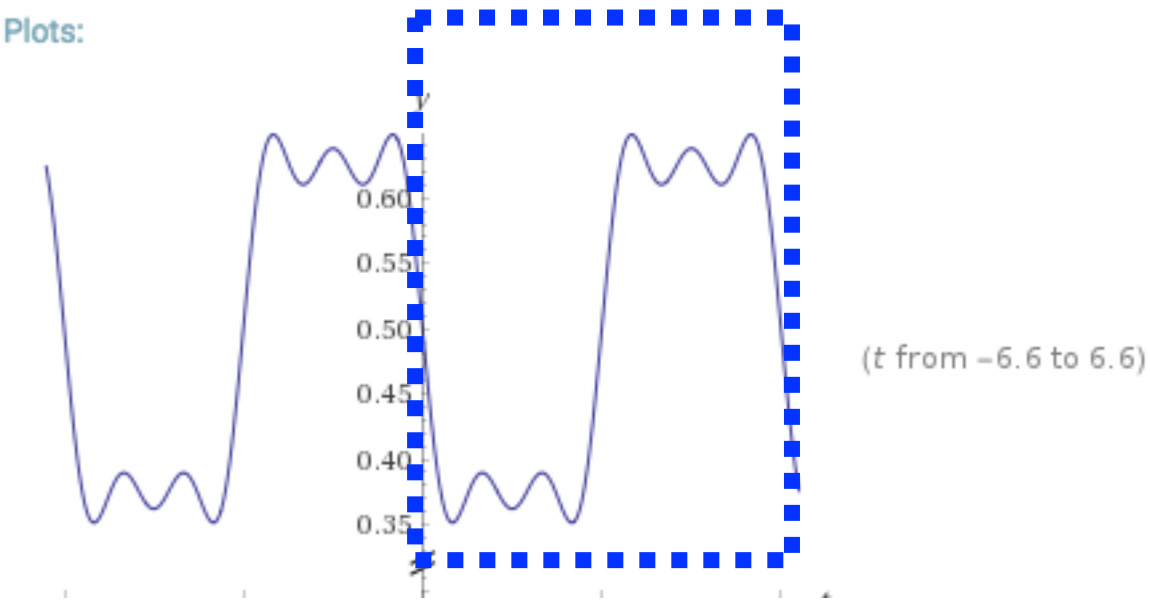

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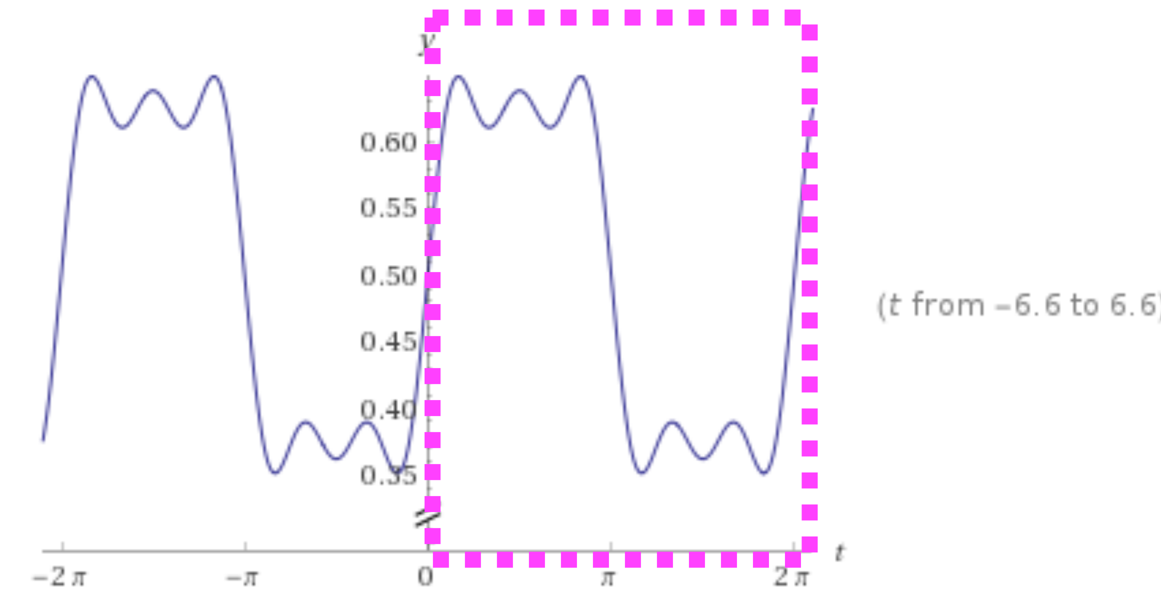

Observe that the Fourier representation of the given function differs with an interval.

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As an exercise expand the function f(t) as a Fourier series in the interval

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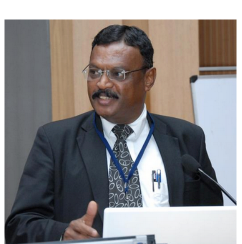
$$0 \le t \le \pi$$

Expansion 4 Find the Fourier series of the function

$$0 \leq t \leq \pi$$



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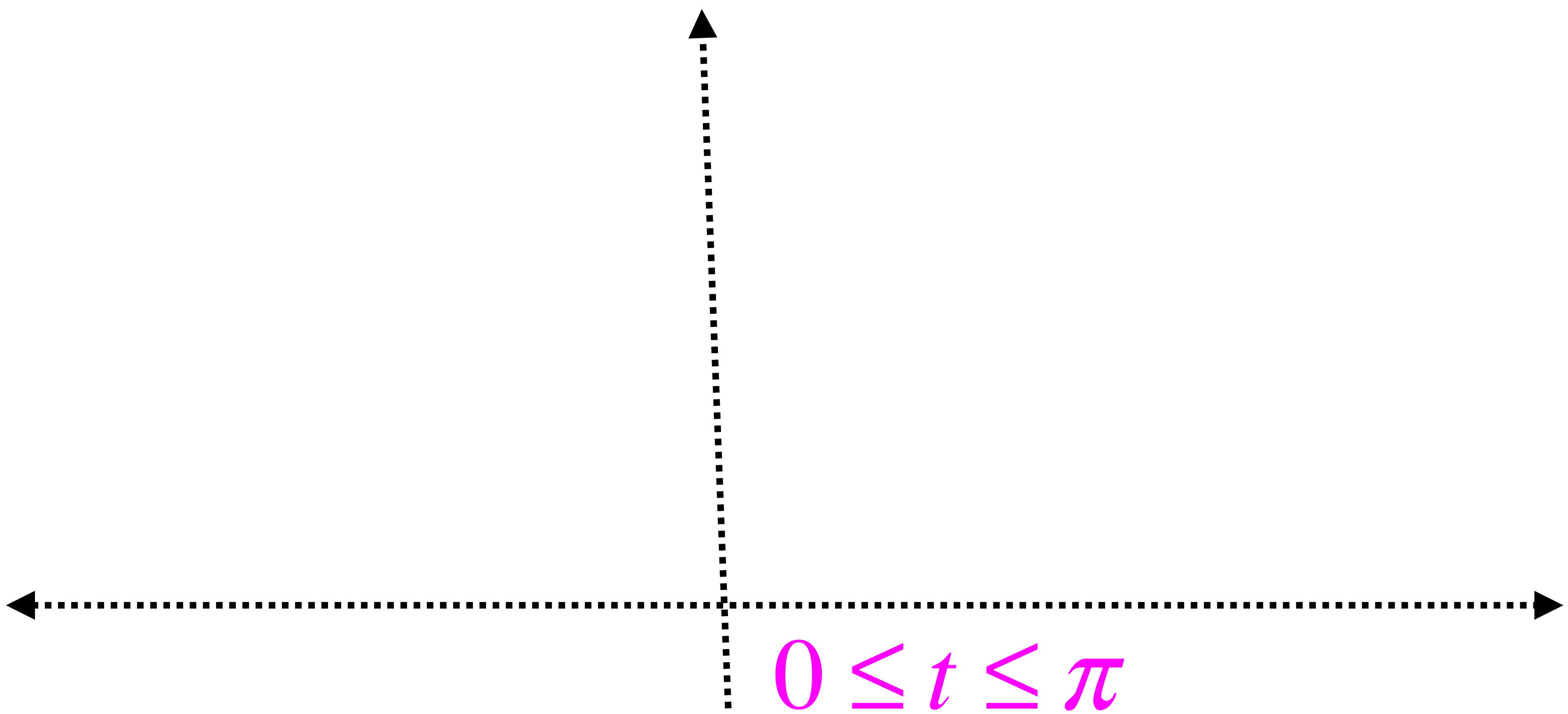

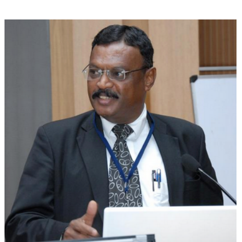

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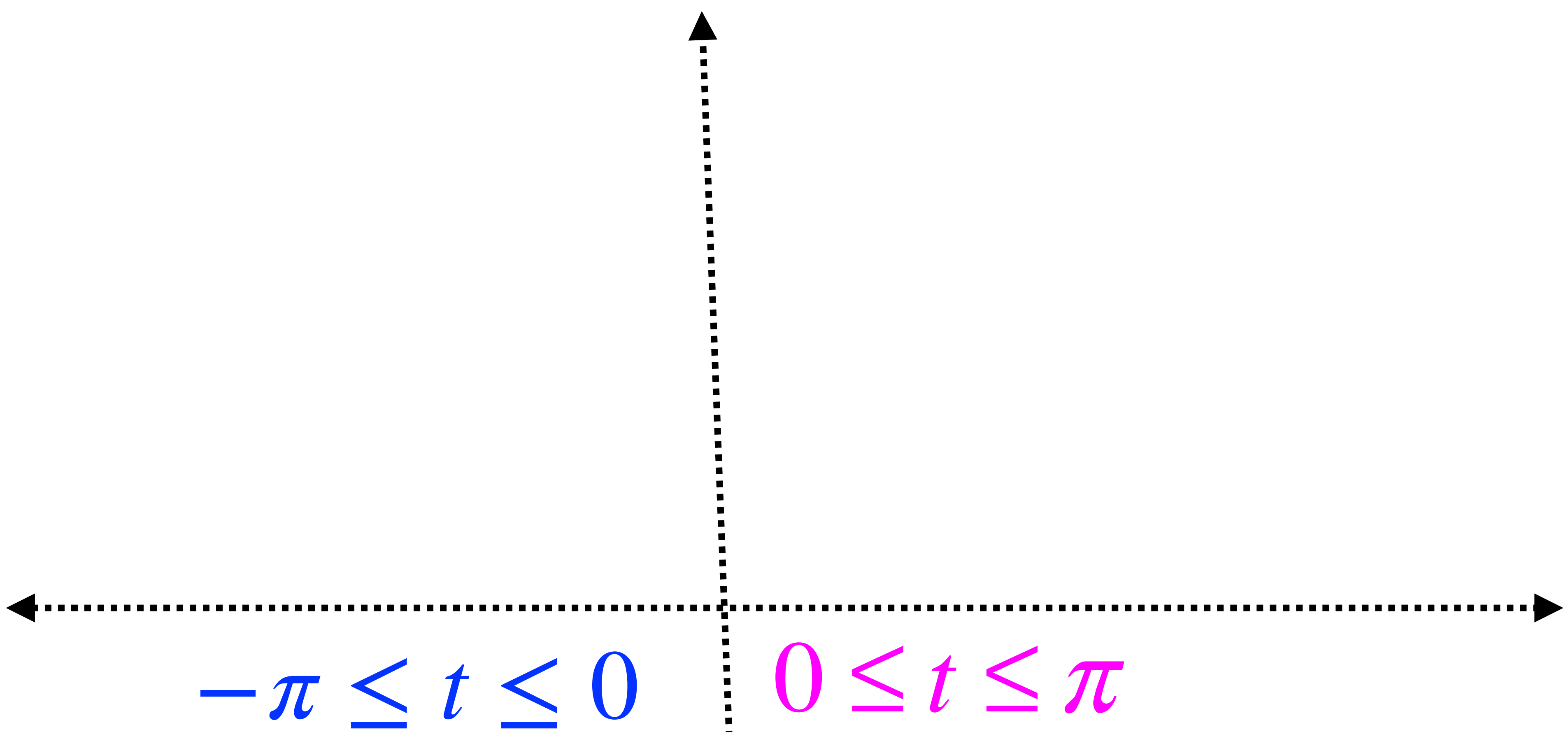

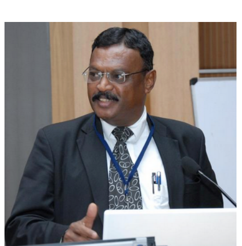

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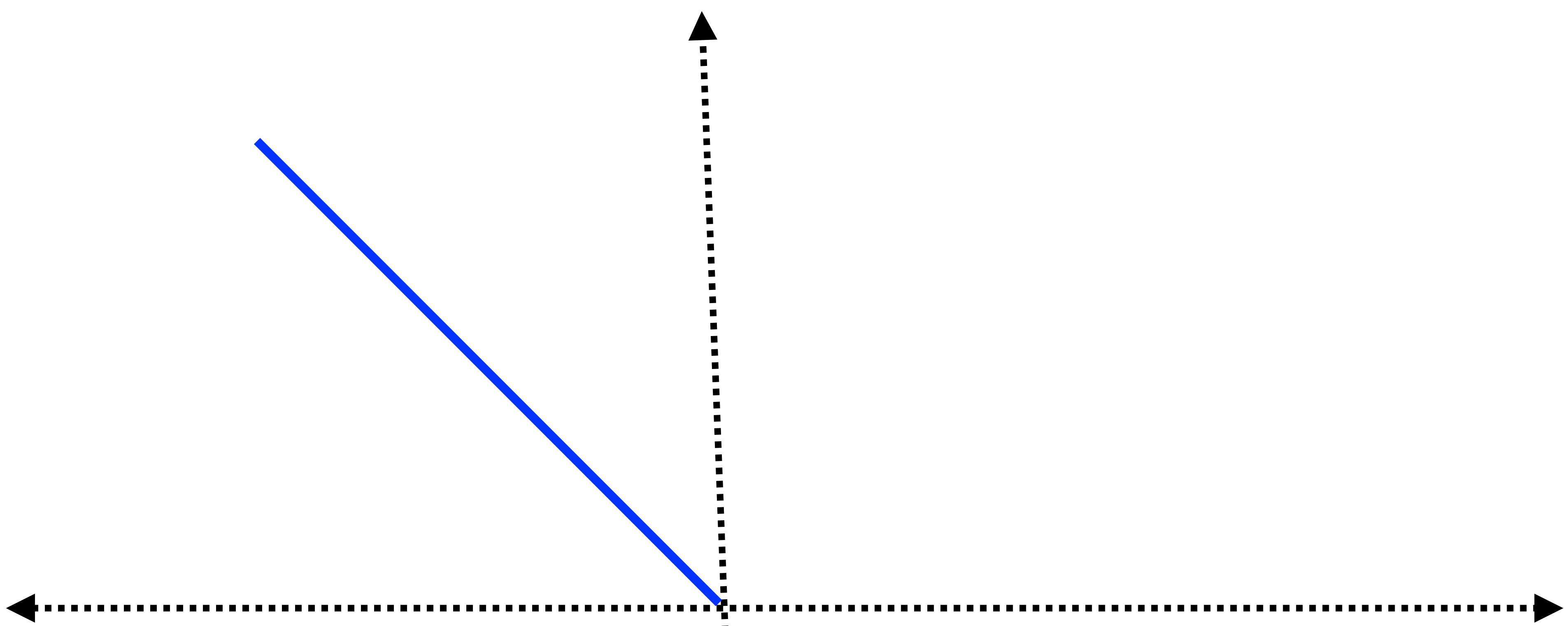

$-\pi \le t \le 0$ $\quad$ $0 \le t \le \pi$

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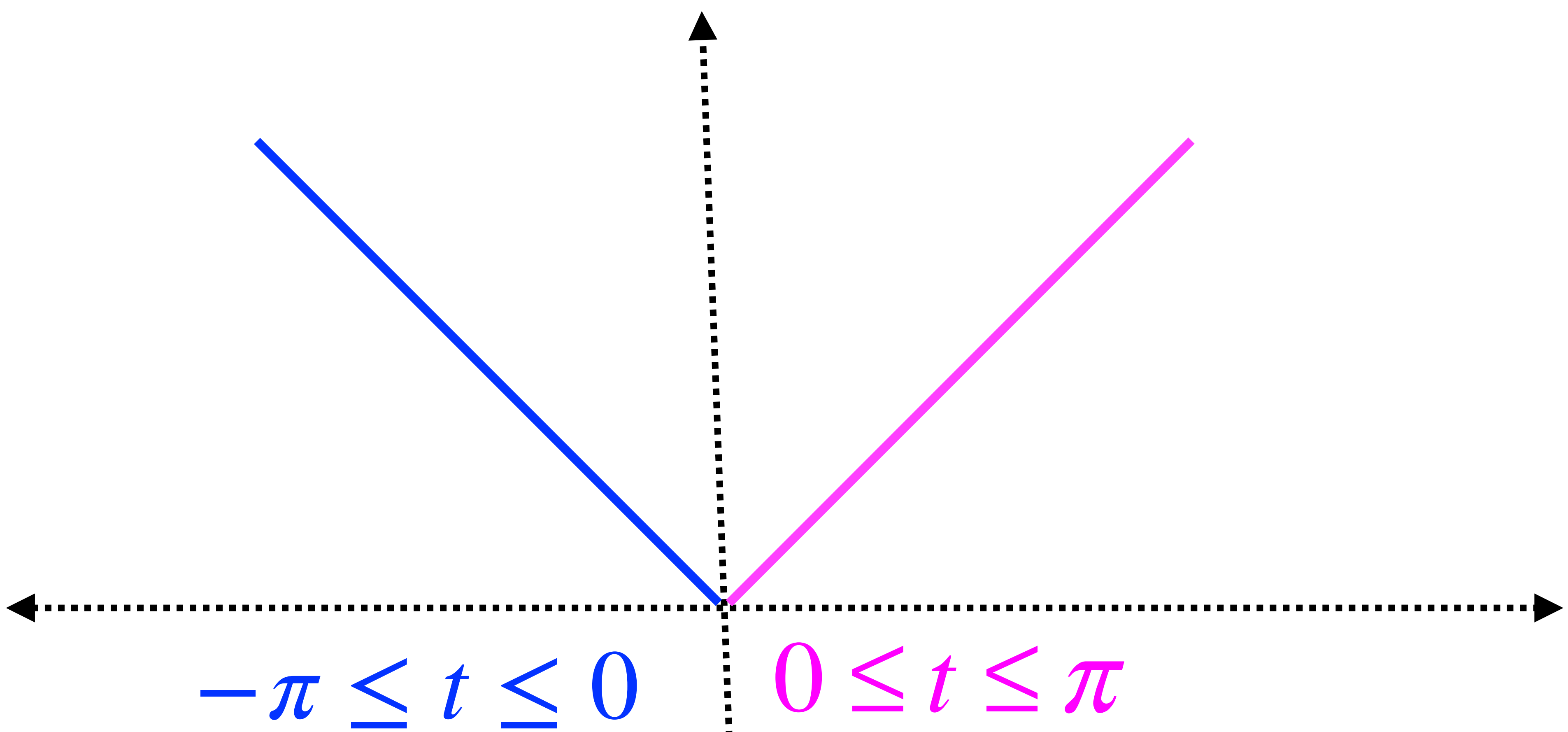

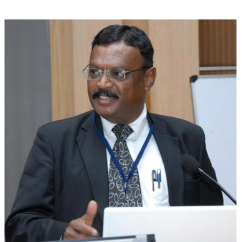

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$$= \frac{1}{\pi}\int_0^{\pi}(t).d\left(\frac{\sin nt}{n}\right)dt = \frac{1}{\pi}\left\{t.\frac{\sin nt}{n} - \left(-\frac{\cos nt}{n^2}\right)\right\}\Bigg|_{t=0}^{t=\pi}$$

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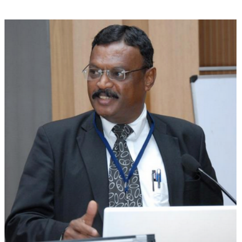
$$That\ is,\ f(t)=\frac{\pi}{2}-\frac{2}{\pi}\sum_{n=1}^{\infty}\left\{\frac{\cos(2n-1)t}{(2n-1)^2}\right\}$$



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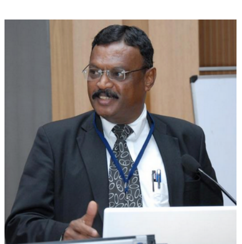

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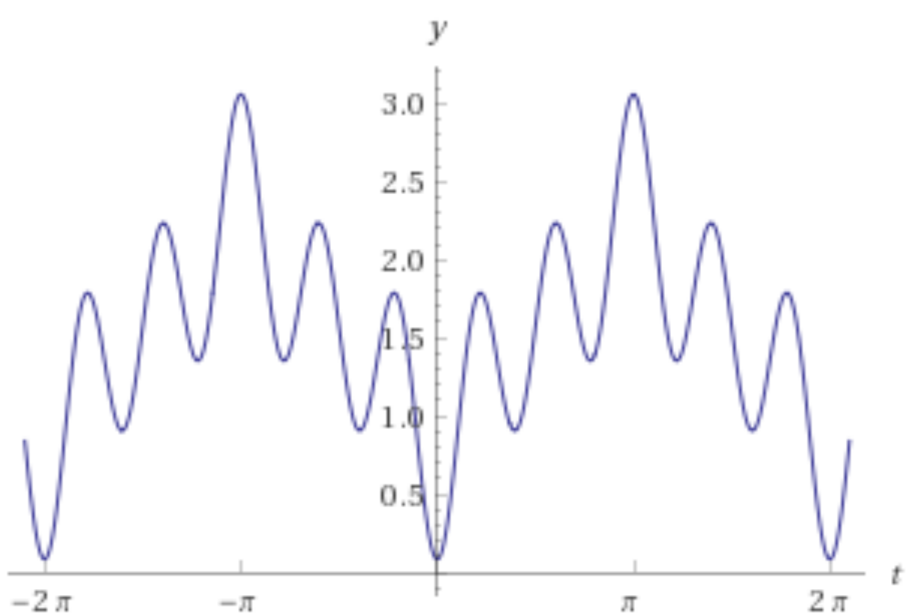
Input interpretation:

plot	$\frac{\pi}{2}-\frac{2}{\pi}\left(\cos(t)+\frac{1}{3}\cos(3t)+\cos(5t)\right)$

Plots:



(t from −6.6 to 6.6)

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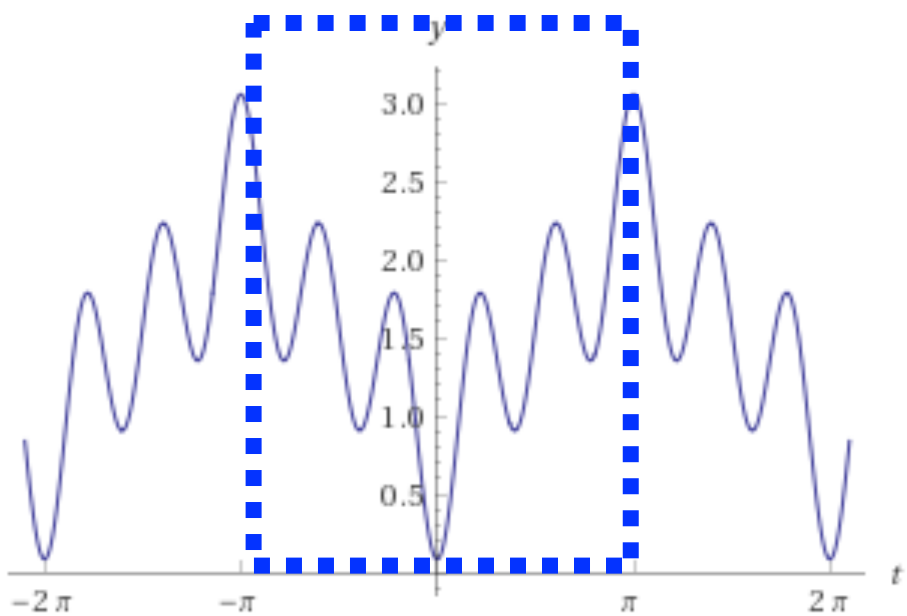

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Expansion 5 *Find the Fourier series of the function*

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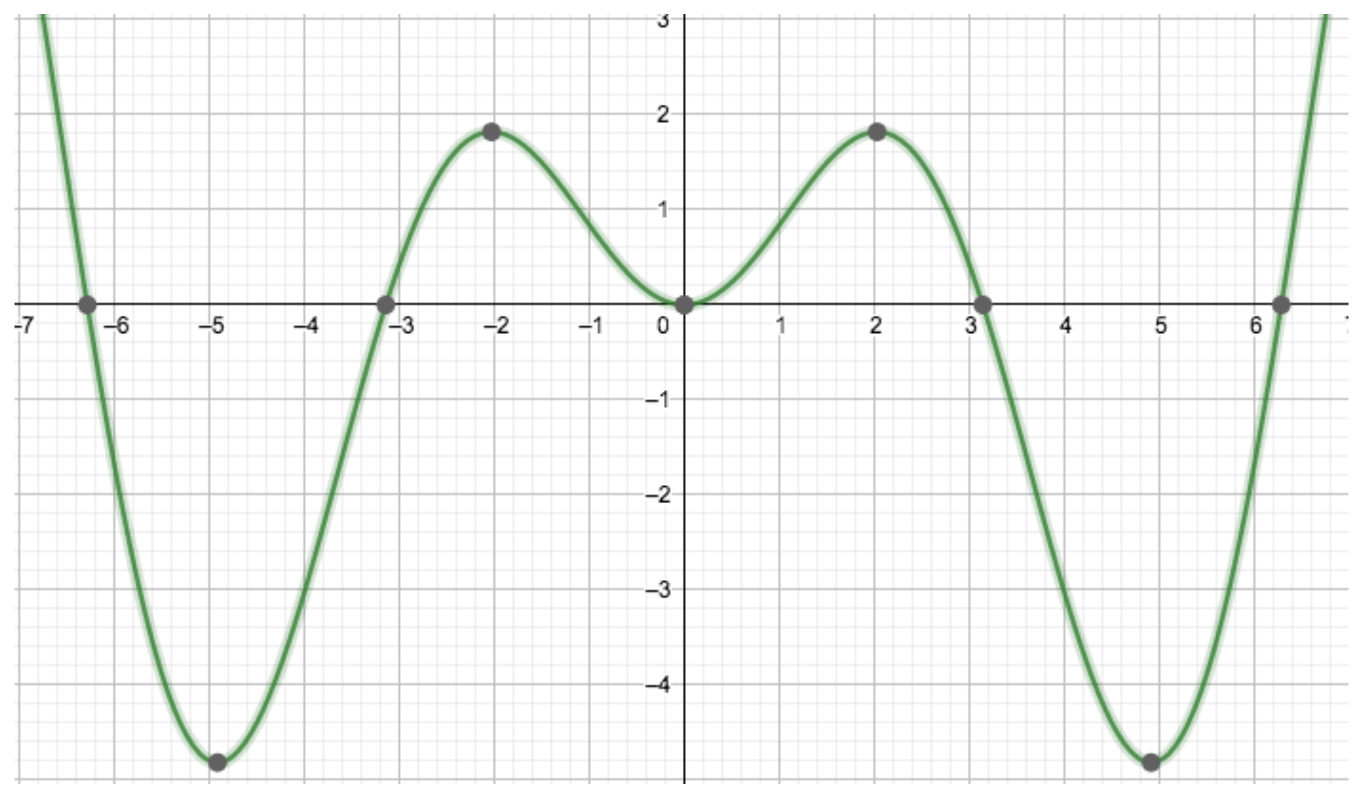

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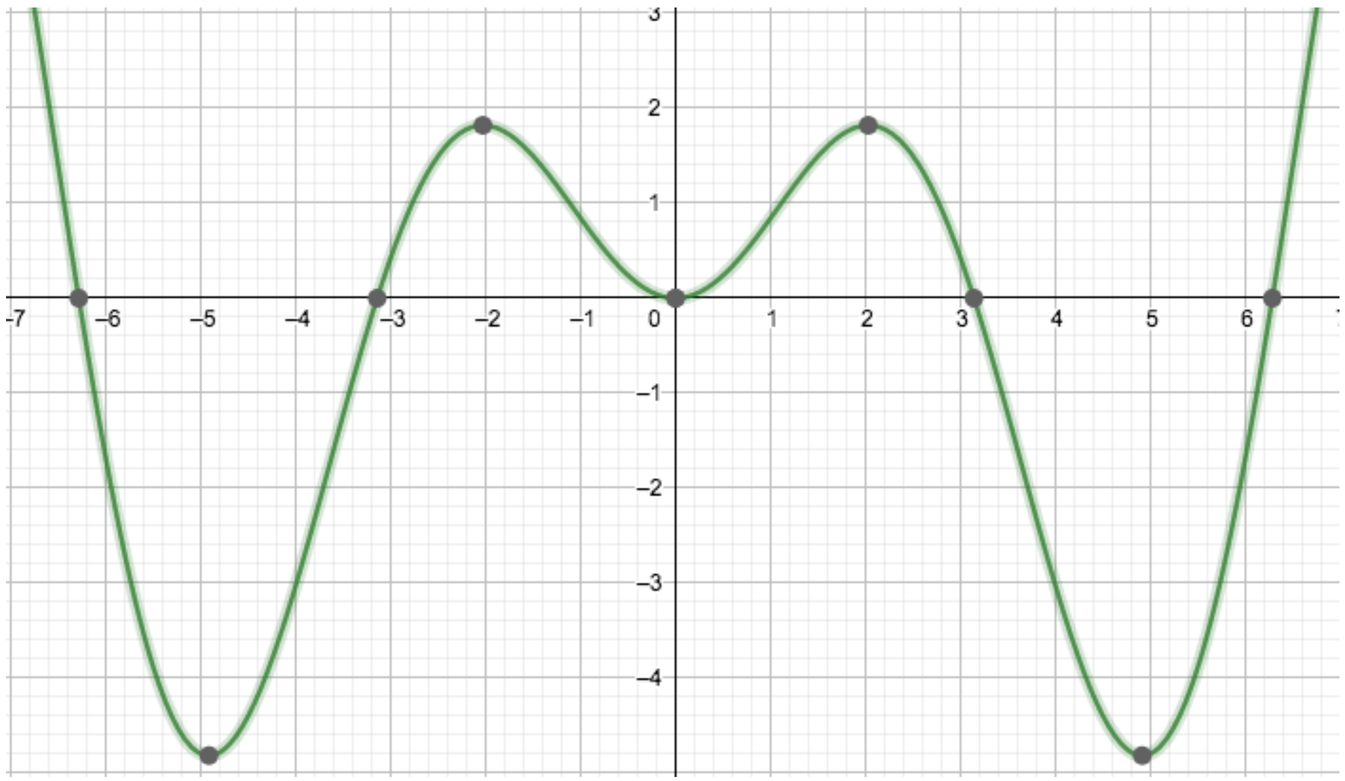

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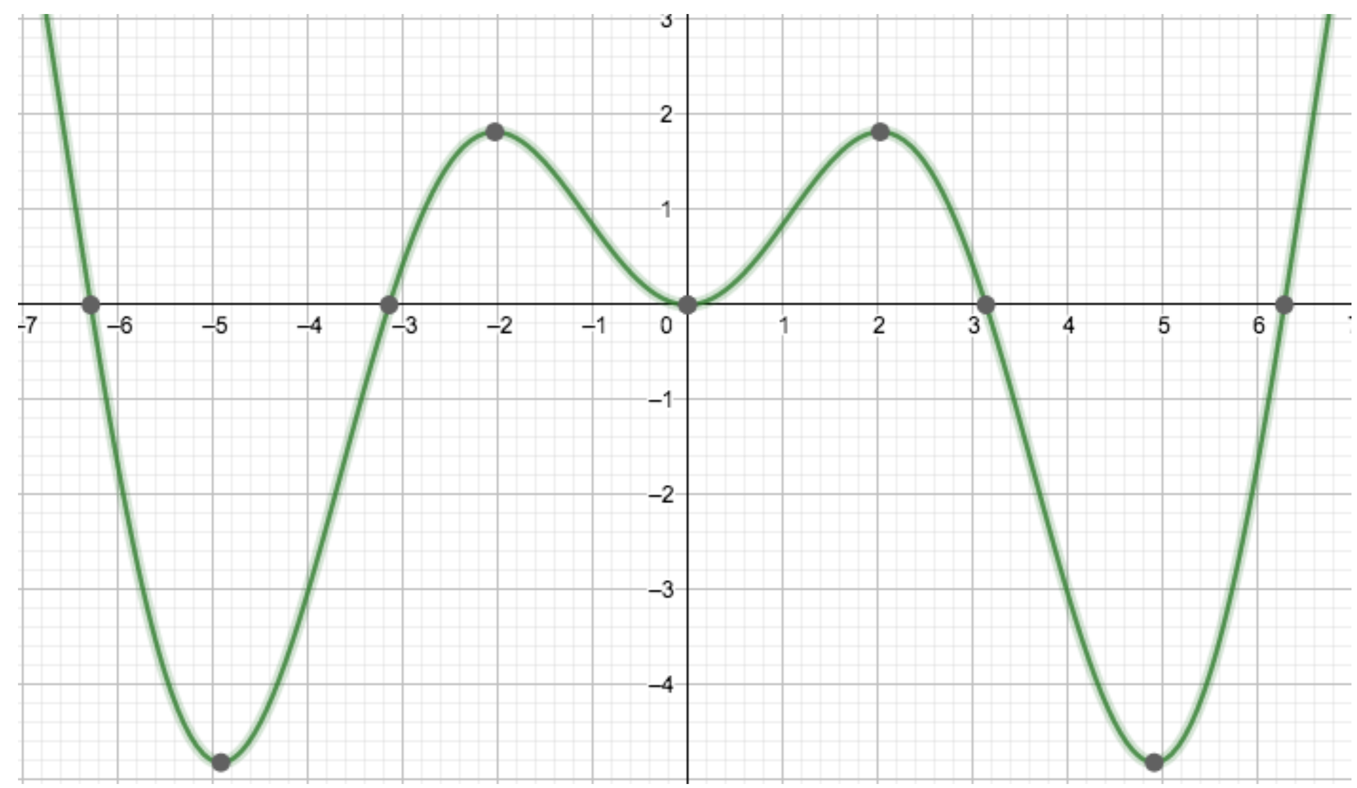
a_0
a_n
a_1
b_n
b_1



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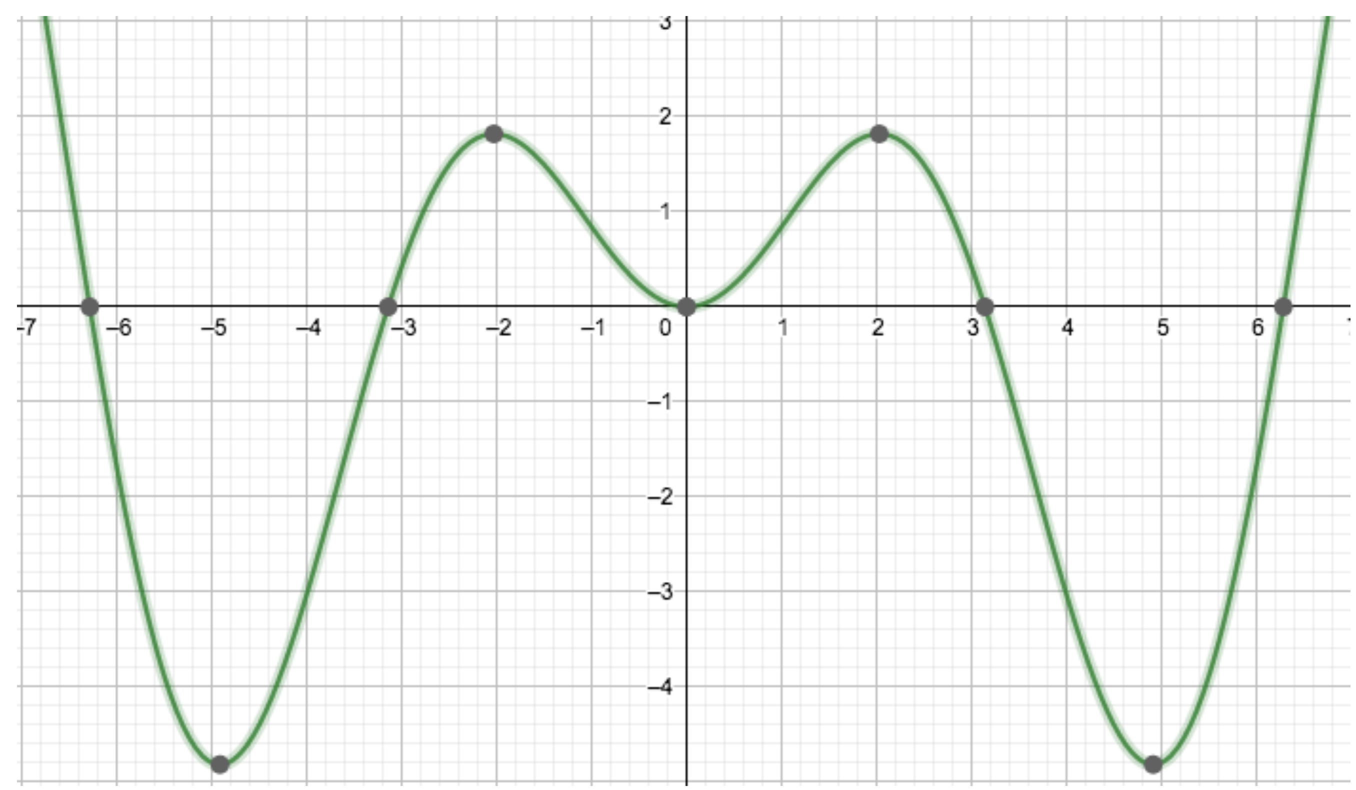

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b_n	$\frac{1}{\pi}\int_0^{2\pi} t.\sin t.\sin nt\,dt$	0
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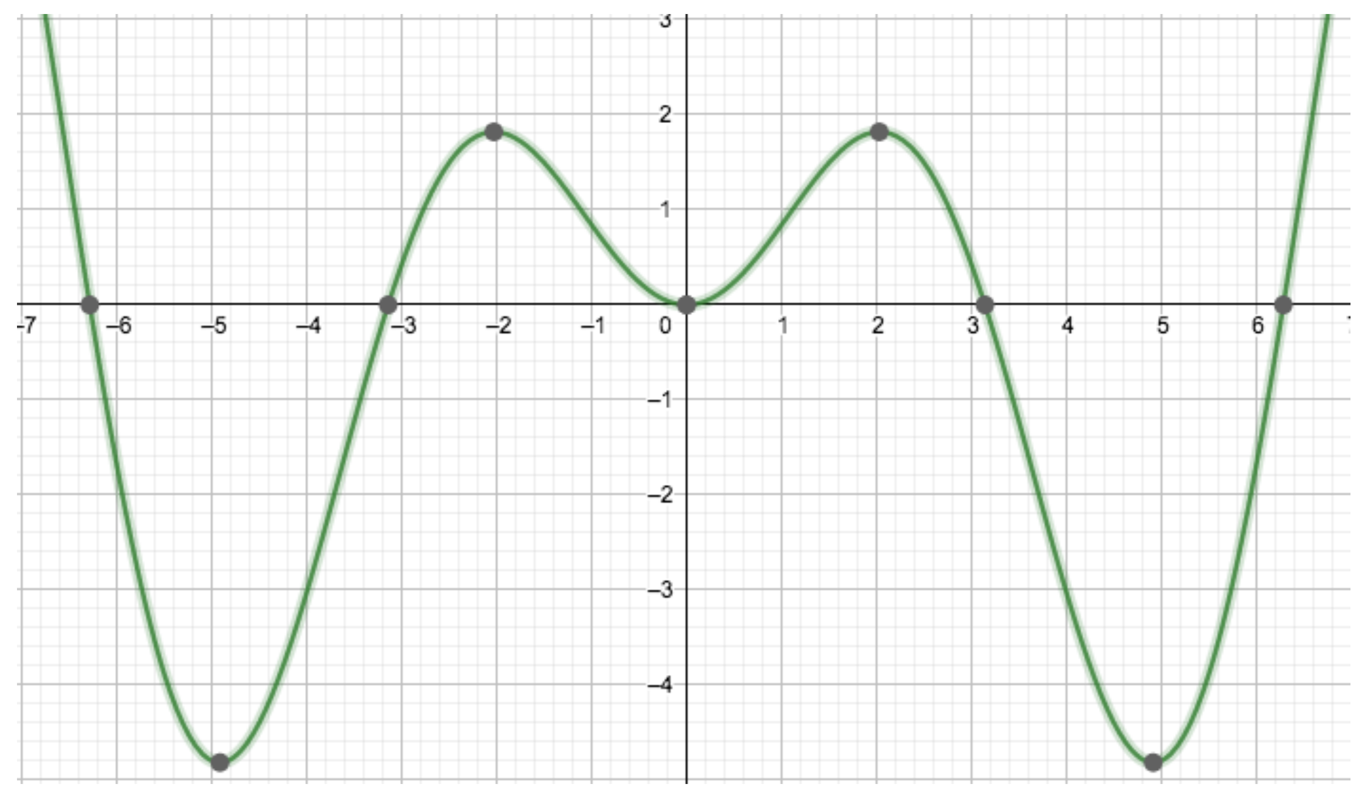

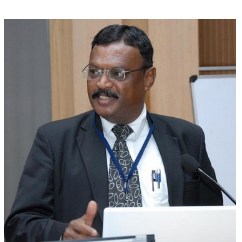

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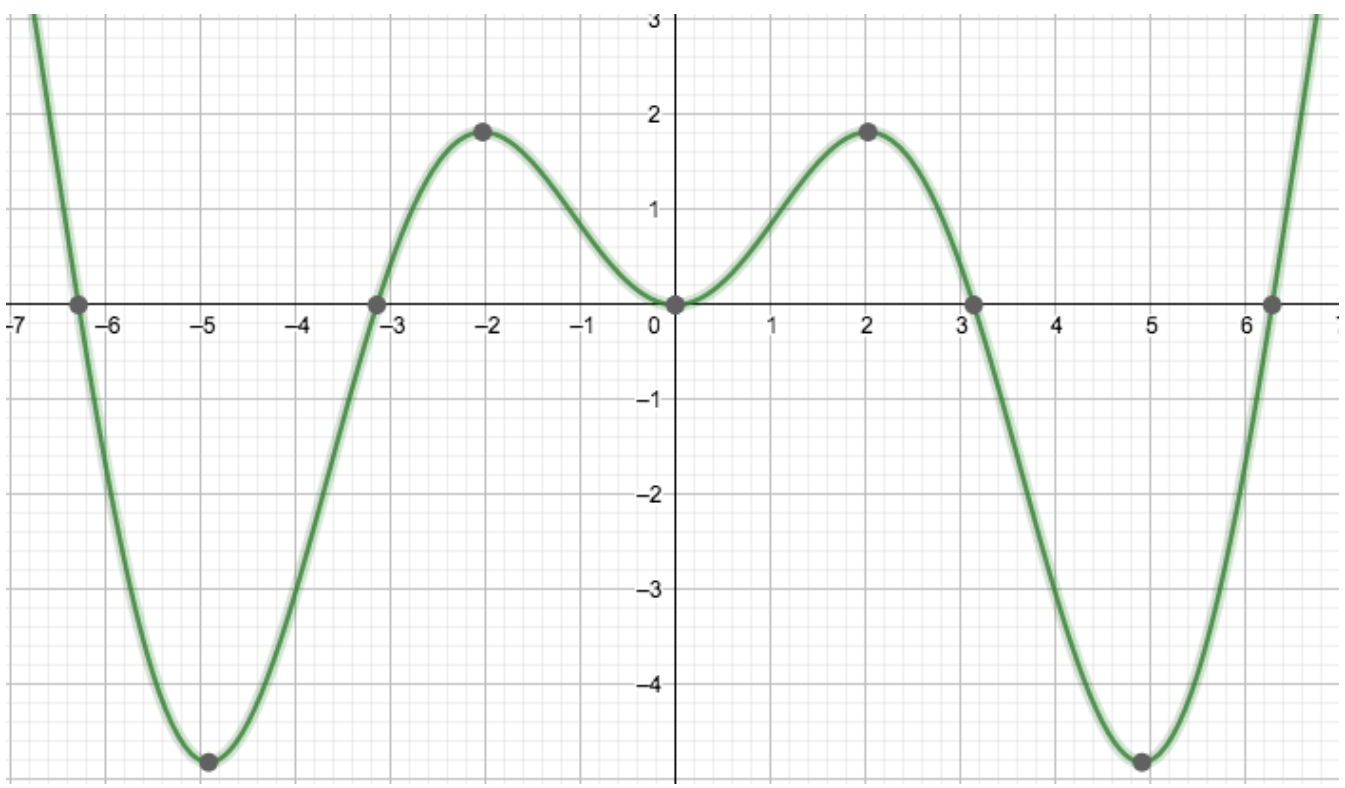

$$\therefore t.\sin t=-1+\pi \sin t-\frac{1}{2}\cos t+2\sum_{n=2}^{\infty}\left(\frac{\cos nt}{n^2-1}\right)$$

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Allowing the limit as t tends to " pi " we get,

$$\therefore t.\sin t = -1 + \pi \sin t - \frac{1}{2}\cos t + 2\sum_{n=2}^{\infty}\left(\frac{\cos nt}{n^2 - 1}\right)$$

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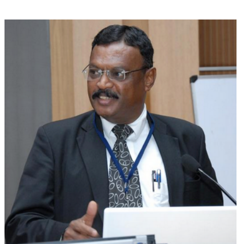
$$0 = -1 + \frac{1}{2} + 2\sum_{n=2}^{\infty}\frac{(-1)^n}{n^2 - 1}$$



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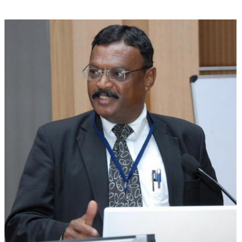

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That is, $\frac{1}{1.3} - \frac{1}{2.4} + \frac{1}{3.4} - \frac{1}{4.5} + \ldots = \frac{1}{4}$

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Expansion 6 *Prove that the Fourier series of the function*

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and hence show that,

$$\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\frac{1}{7.9}+\ldots=\frac{1}{4}(\pi-2)$$

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Suppose f(x) is a periodic function defined on an interval

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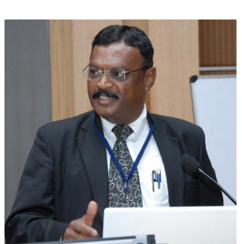

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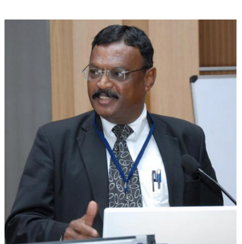

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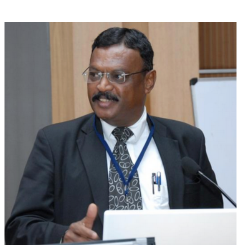

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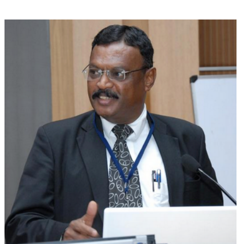

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$$If\ x = \alpha, then\ w = \frac{\pi\alpha}{c} := \beta;\ \ x = \alpha + 2c, then\ w = \frac{\pi(\alpha+2c)}{c} := \beta + 2\pi$$

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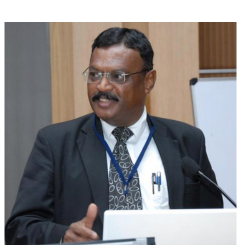

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$$Therefore,\ \ F(w) = \frac{a_0}{2} + \sum_{1}^{\infty} a_n \cos nw + \sum_{1}^{\infty} b_n \sin nw$$

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$$b_n = \frac{1}{c}\int_{\beta}^{\beta+2\pi} f(w)\,.\,Sin(nw)\ dw = \frac{1}{c}\int_{\alpha}^{\alpha+2c} f(w)\,.\,Sin\left(\frac{n\pi x}{c}\right)\ dx$$

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ODD and Even functions

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- **Extending ,a given function in a half interval, to a full interval as an odd function or even function we get a Fourier series respectively consists of Sine series and cosine series only.**

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Half range series

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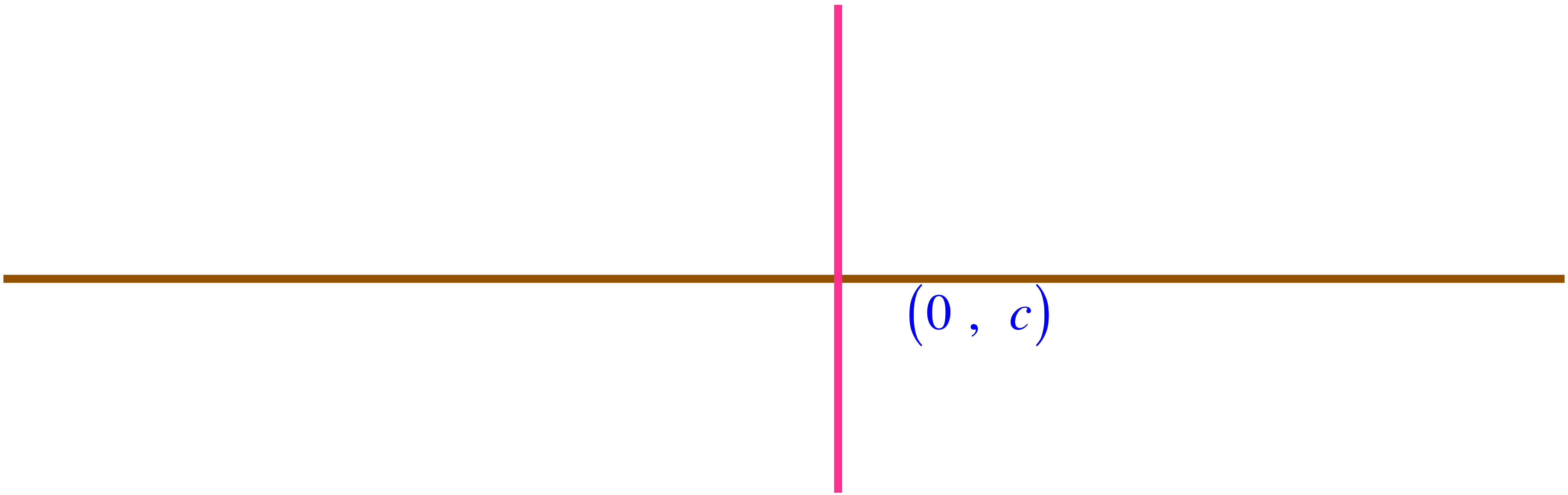

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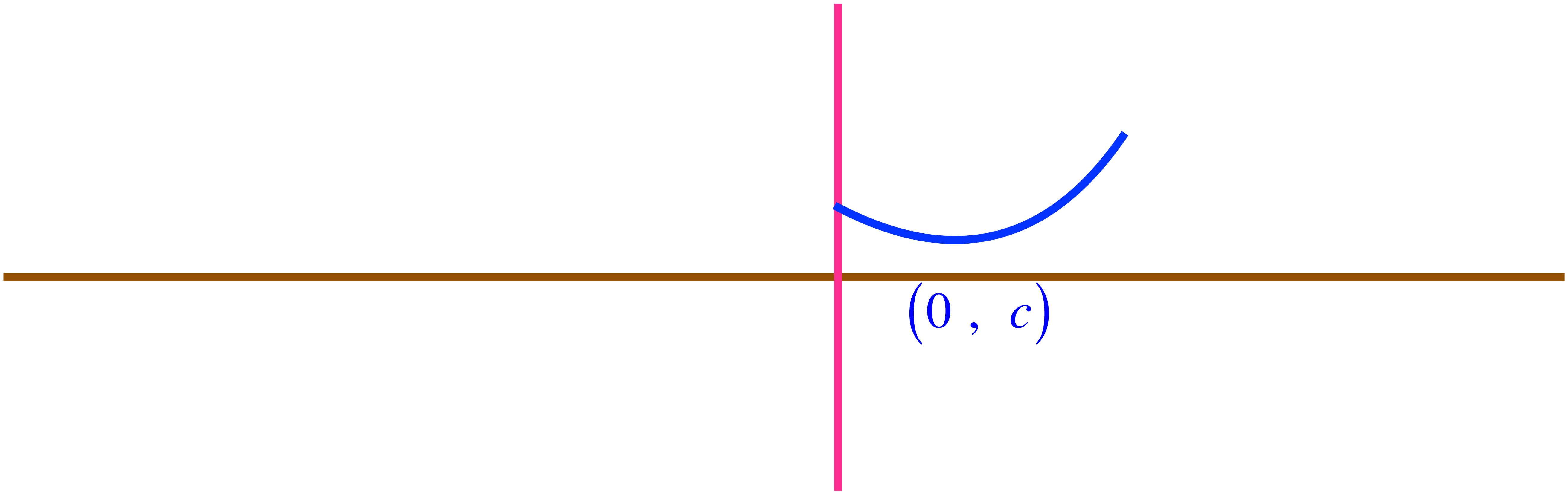

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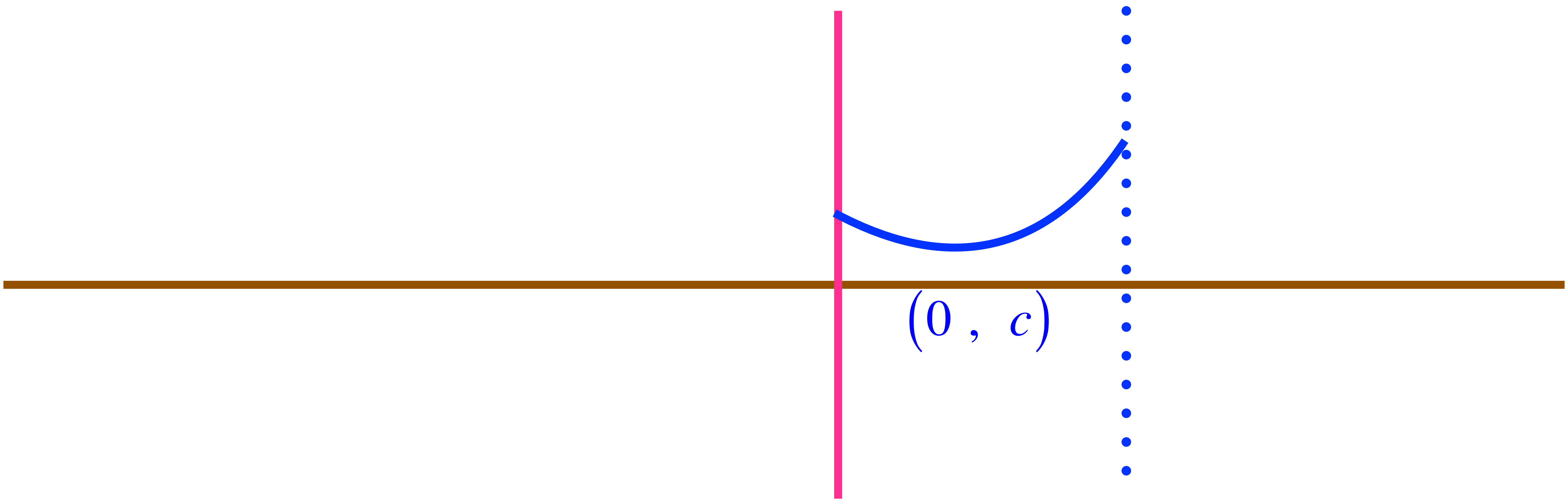

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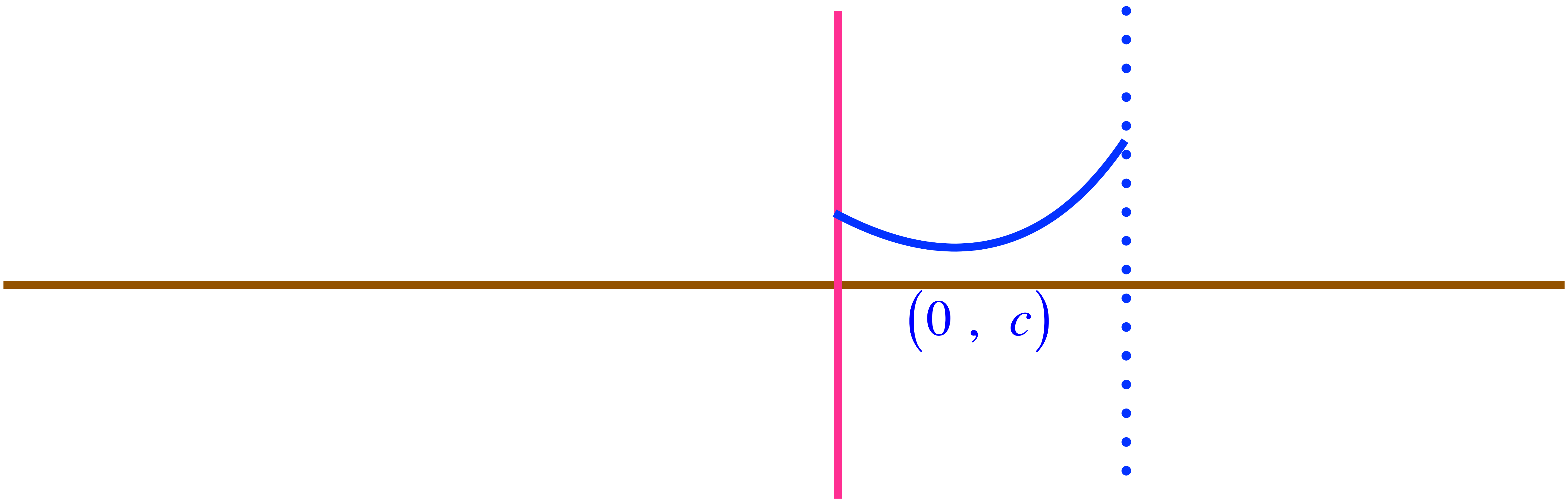

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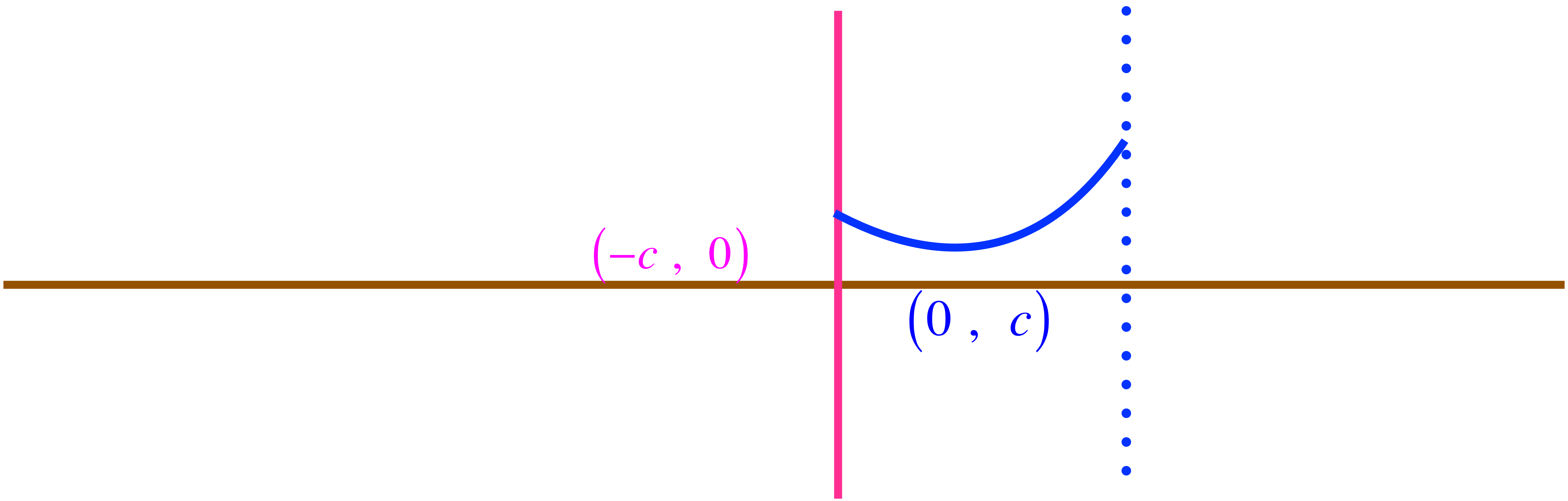

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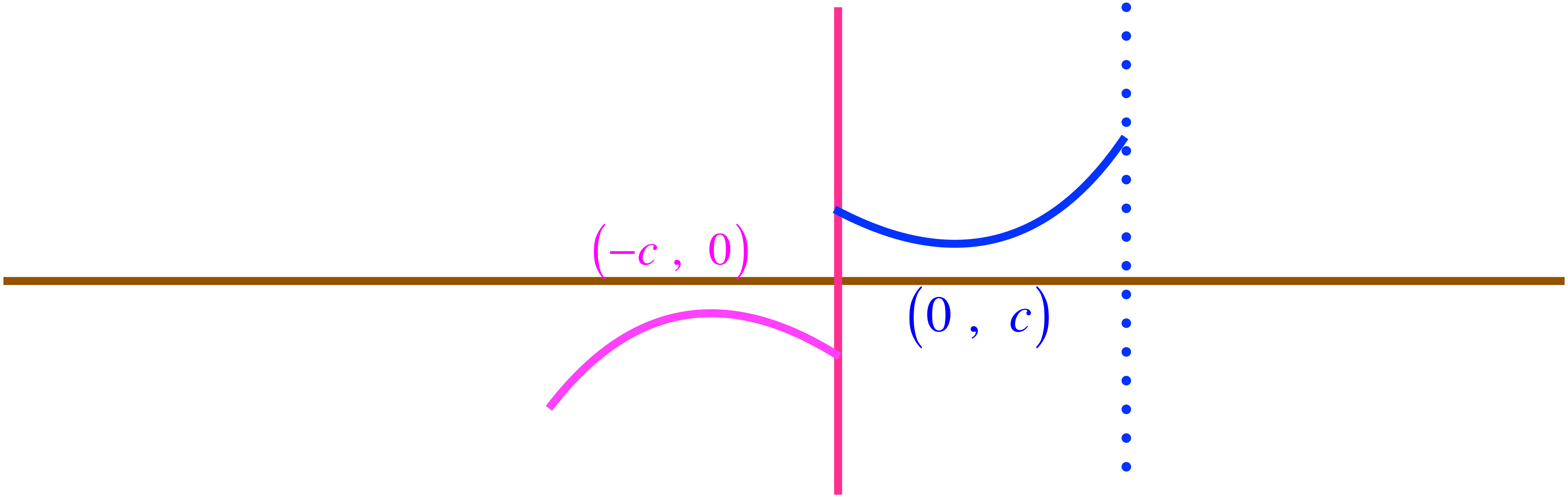

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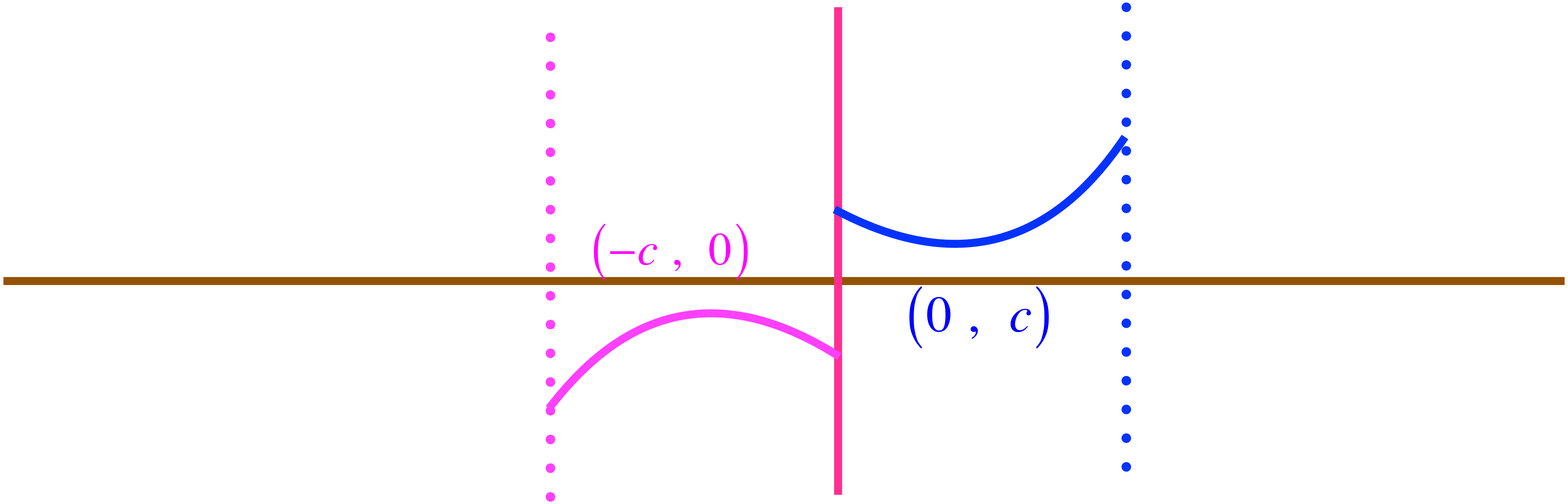

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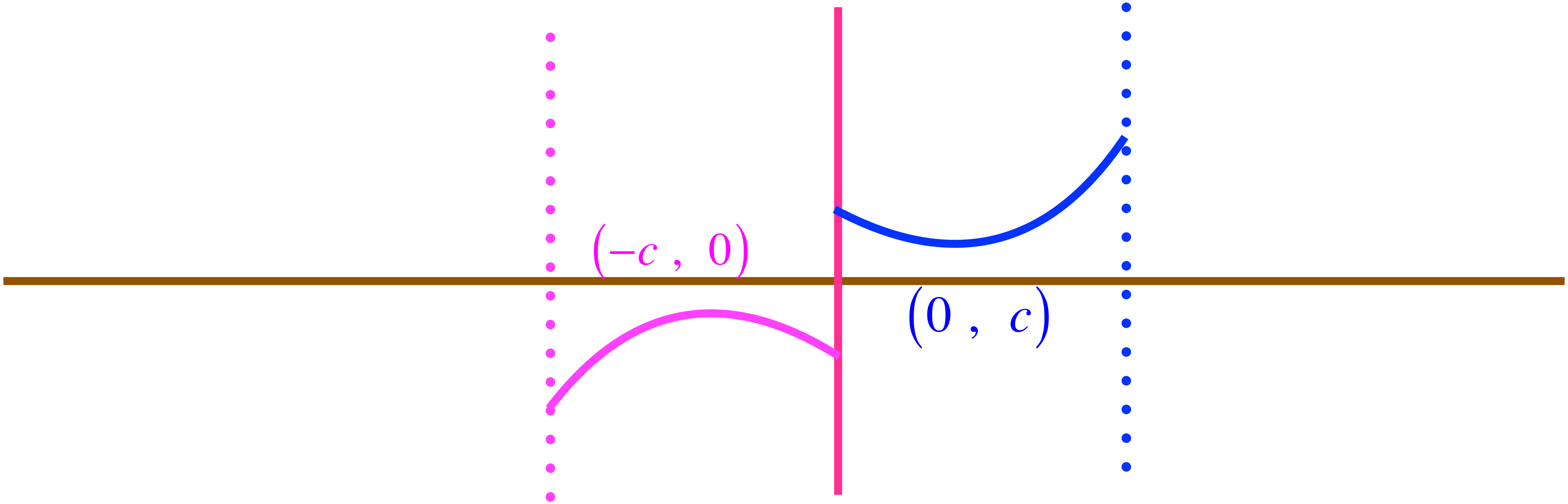

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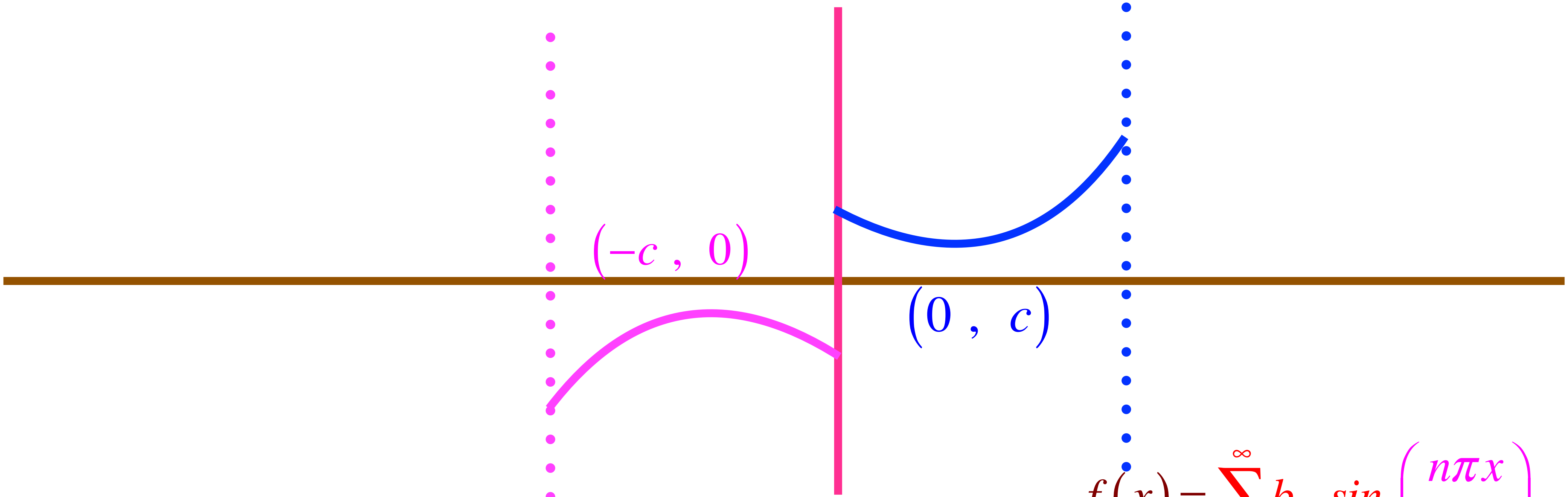

$$f(x) = \sum_{1}^{\infty} b_n \ sin\left(\frac{n\pi x}{c}\right)$$

$$with \ b_n = \frac{2}{c}\int_0^c f(t)\sin\left(\frac{n\pi x}{c}\right) dt$$

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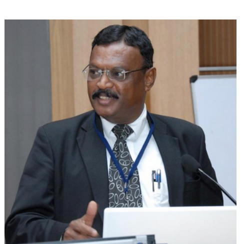

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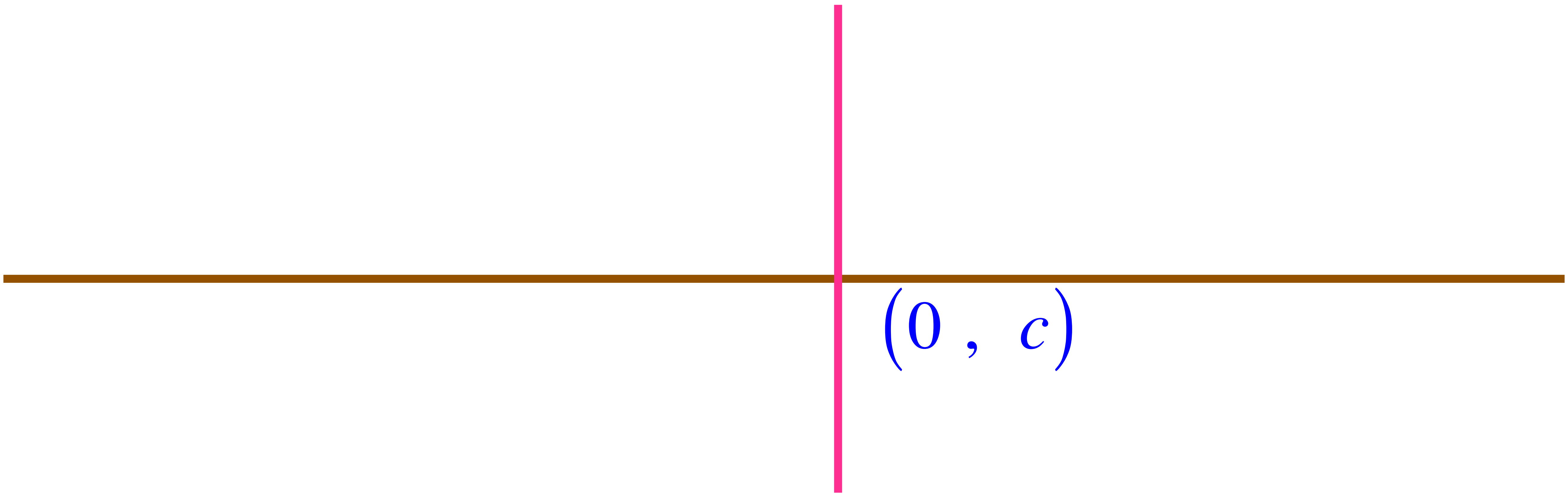

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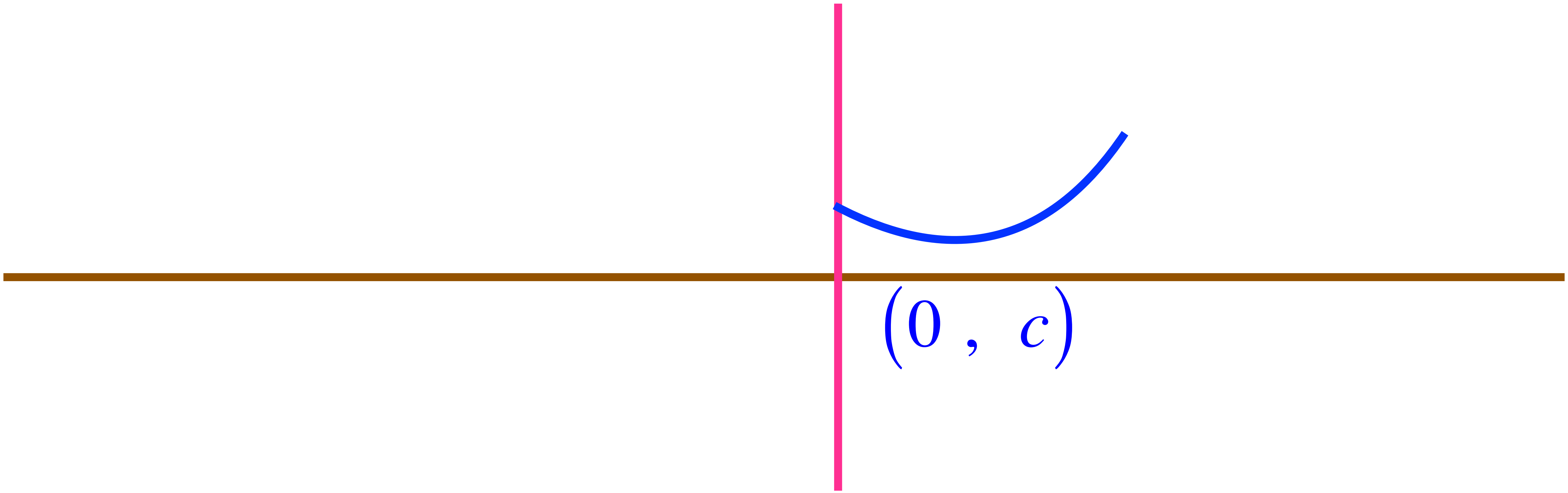

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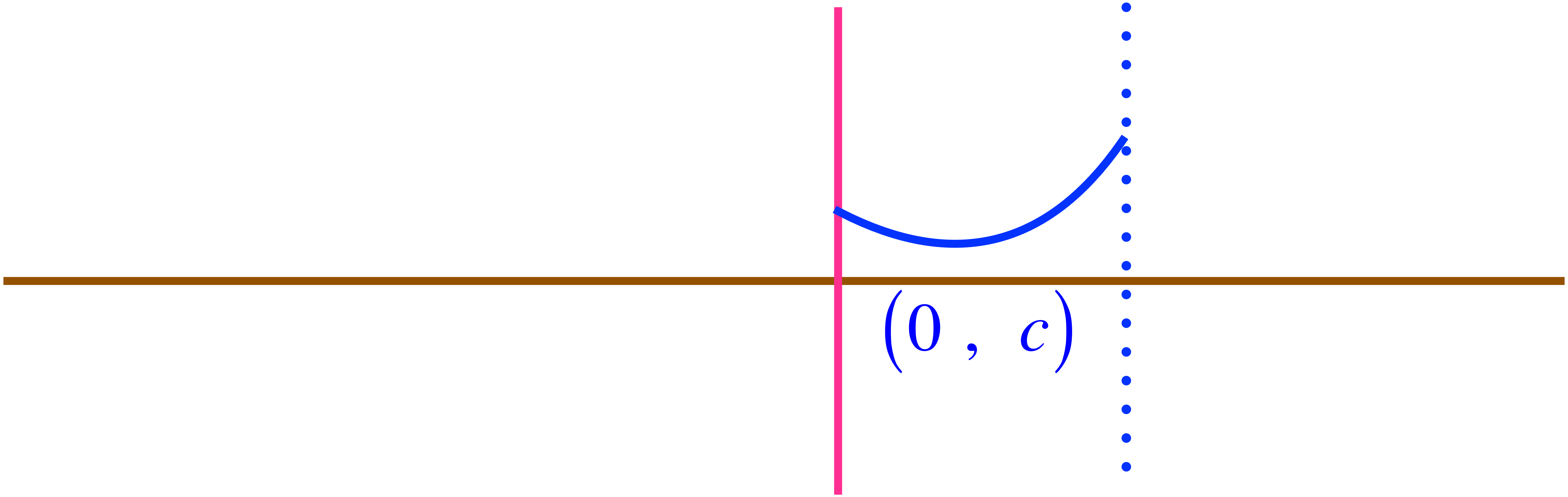

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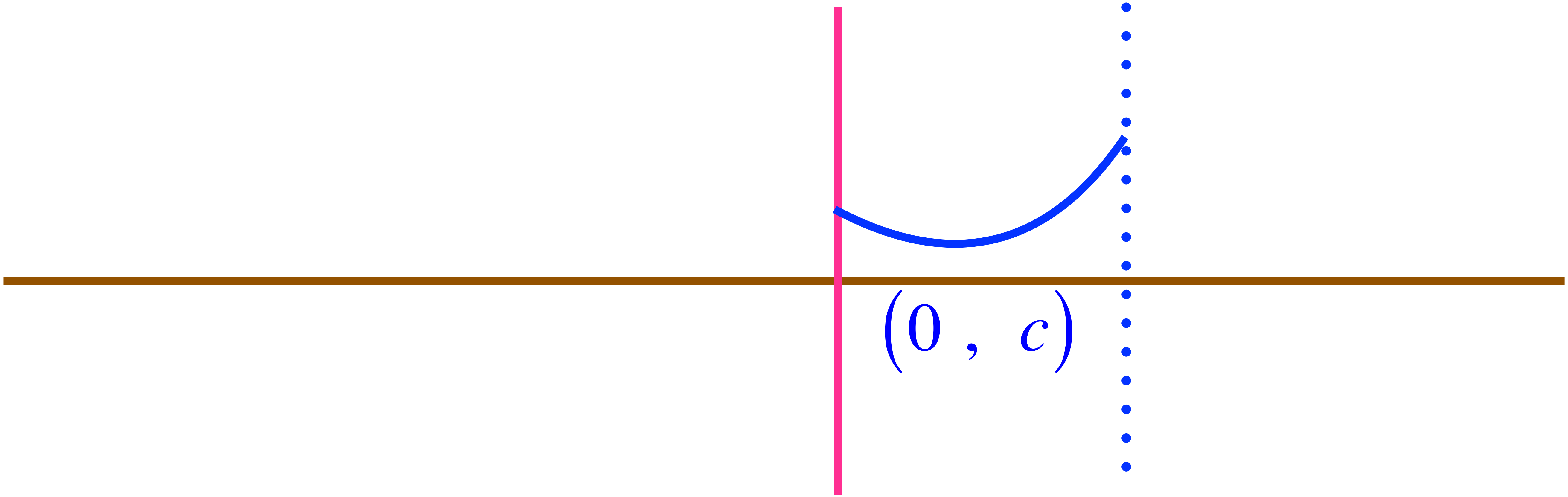

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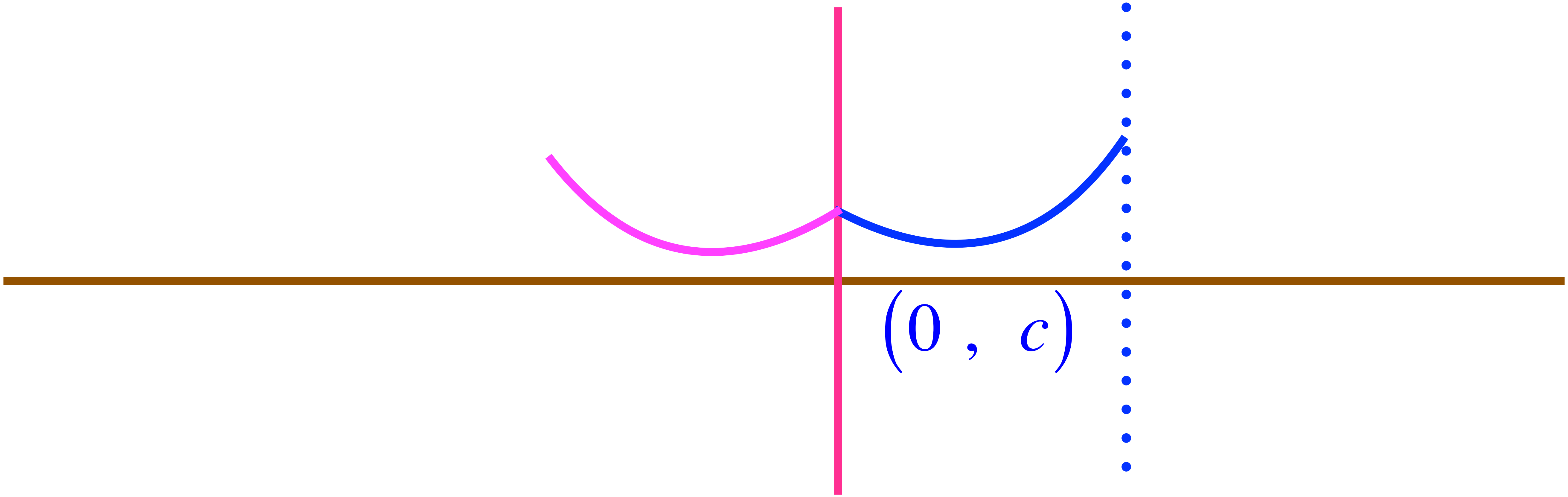

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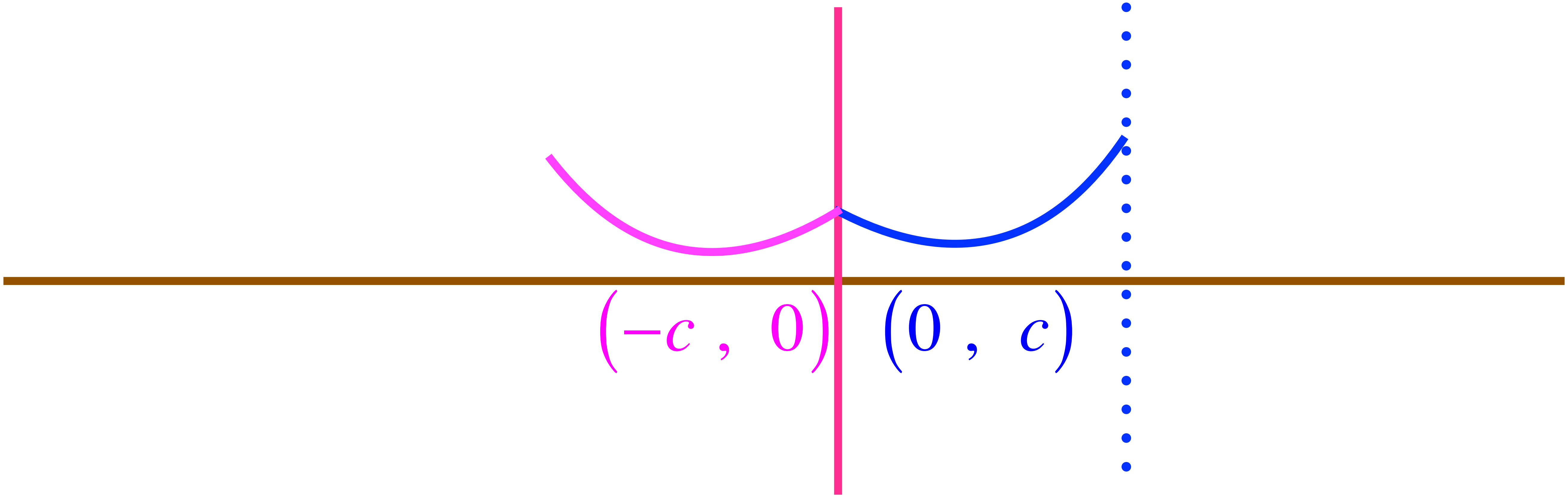

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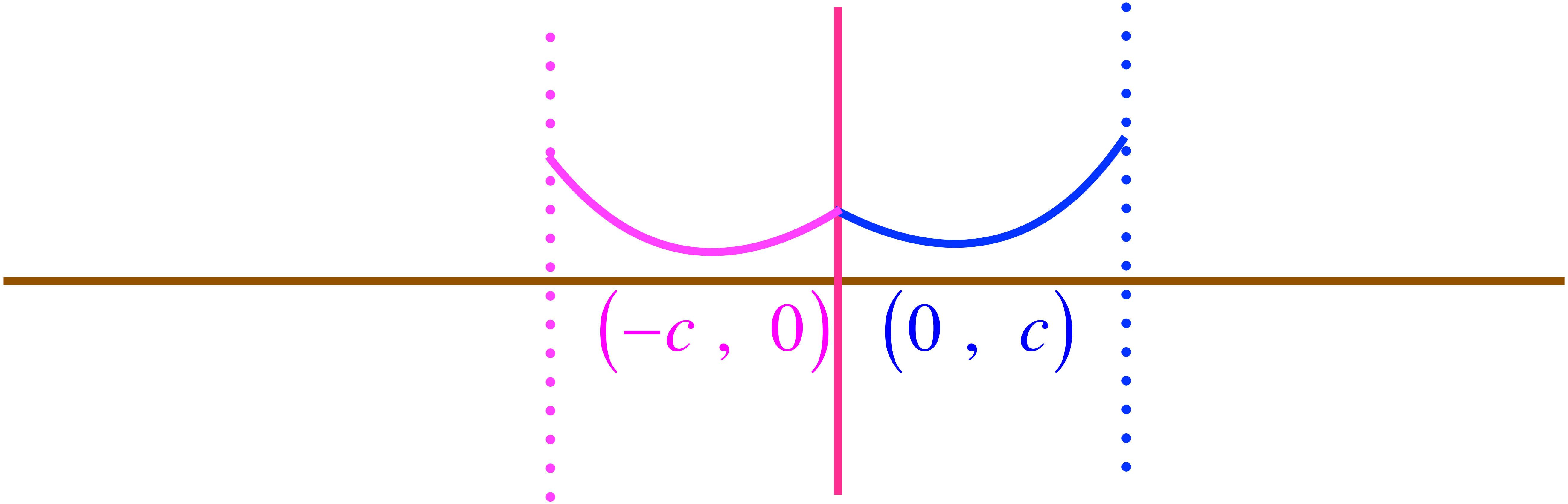

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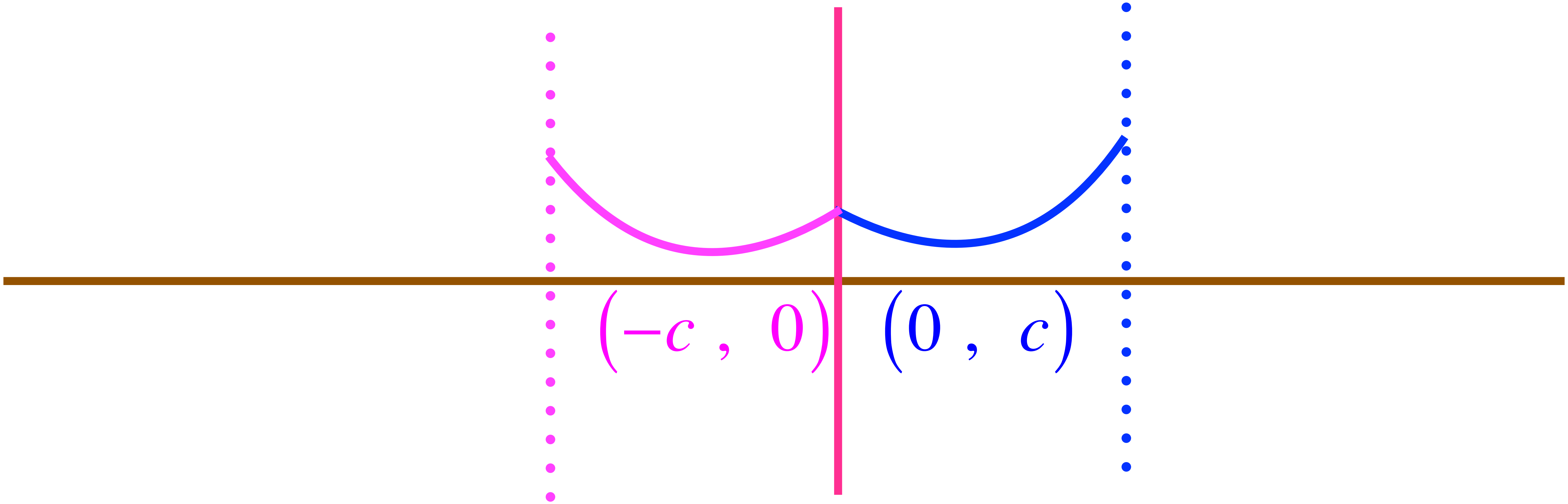

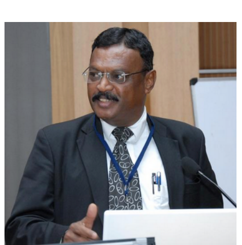

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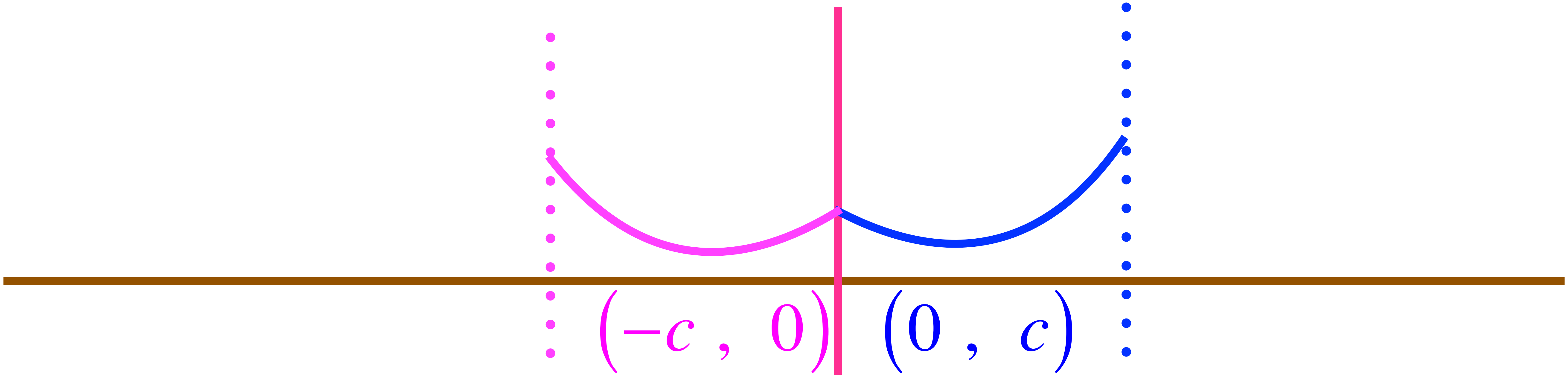
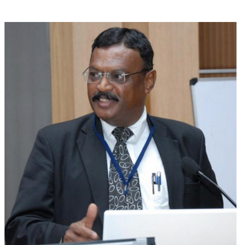


$$f(x)=\frac{a_0}{2}+\sum_{1}^{\infty} a_n \cos\left(\frac{n\pi x}{c}\right),\ with$$

$$a_0=\frac{2}{c}\int_0^c f(t)\,dt,\quad a_n=\frac{2}{c}\int_0^c f(t)\cos\left(\frac{n\pi x}{c}\right)dt$$



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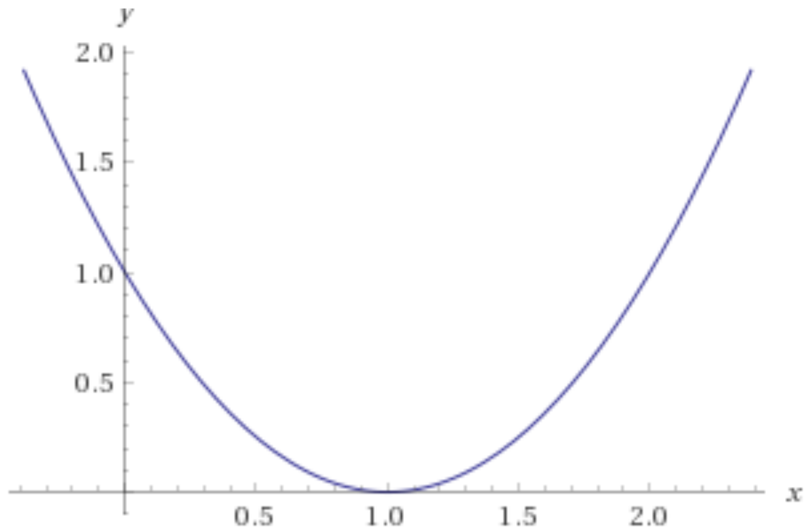
Find the Fourier Sine and Cosine series of the function



$$f(x) = (x-1)^2, \ x \in (0,1)$$

Find the Fourier Sine series of the function

$$f(x) = x(\pi - x), \ x \in [0,\pi]$$

and hence show that, $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n^4}\right) = \frac{\pi^4}{90}$ & $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(2n-1)^6} = \frac{\pi^6}{960}$

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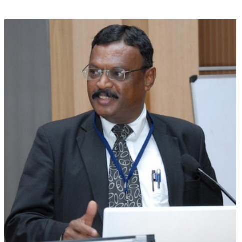
Parseval's formula



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Parseval's formula

$$We\,have,\ f(x) = \frac{a_0}{2} + \sum_{1}^{\infty} a_n \cos\left(\frac{n\pi x}{c}\right) + \sum_{1}^{\infty} b_n \sin\left(\frac{n\pi x}{c}\right)$$

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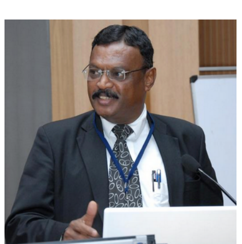
$$Hence,\left[f(x)\right]^2=\frac{a_0}{2}f(x)+\sum_{1}^{\infty}a_n\;f(x)cos\left(\frac{n\pi x}{c}\right)+\sum_{1}^{\infty}b_n f(x)\;sin\left(\frac{n\pi x}{c}\right)$$



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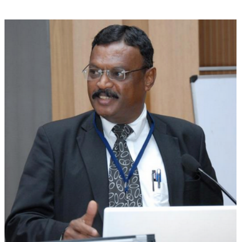
$$\therefore \int_{-c}^{c}\left(f(x)\right)^2 dx=\frac{a_0}{2}\int_{-c}^{c} f(x)dx+\sum_{1}^{\infty} a_n\int_{-c}^{c} f(x) cos\left(\frac{n\pi x}{c}\right)dx+\sum_{1}^{\infty} b_n\int_{-c}^{c} f(x) sin\left(\frac{n\pi x}{c}\right)d$$



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$$\boxed{\therefore\int_{-c}^{c}(f(x))^2\,dx=\frac{c{a_0}^2}{2}+\sum_{1}^{\infty}c{a_n}^2+\sum_{1}^{\infty}c{b_n}^2=c\left\{\frac{{a_0}^2}{2}+\sum_{1}^{\infty}\left({a_n}^2+{b_n}^2\right)\right\}}$$

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The energy of the signal given by the function f(x) is

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$$Energy\{f(x)\}=\left(\frac{1}{c}\int_{-c}^{c}(f(x))^2\,dx\right)^{\frac{1}{2}}=\sqrt{\left\{\frac{a_0{}^2}{2}+\sum_{1}^{\infty}\left(a_n{}^2+b_n{}^2\right)\right\}}$$

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The root mean square value of f(x) defined in an interv
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$$RMS\{f(x)\} = \left(\frac{1}{b-a} \int_{a}^{b} \left|f(x)\right|^2 dx \right)^{\frac{1}{2}}$$

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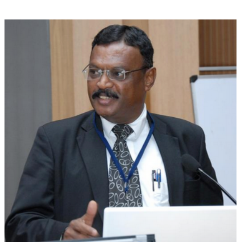
Using the mean value we can compute the the Fourier coefficients as follows:



Using the mean value we can compute the the Fourier coefficients as follows:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\ dt = 2\times\frac{1}{2\pi}\int_0^{2\pi} f(t)\ dt$$

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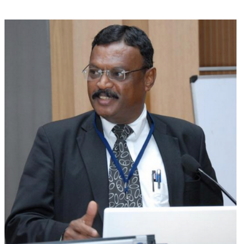

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