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My thanks to
Prof. K. Srinivas
of the
Institute for Mathematical Sciences
for the invitation extended to speak
before you

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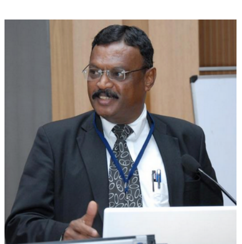

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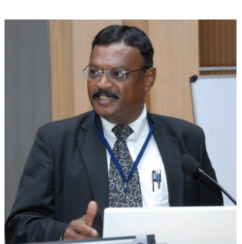

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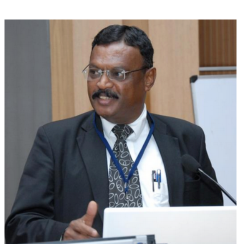

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Norbert Wiener

Born: 26 November 1894 in Columbia, Missouri, USA

Died: 18 March 1964 in Stockholm, Sweden

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Fourier Series

Dirichlet's conditions – General Fourier series – Odd and even functions – Half-range Sine and Cosine series – Complex form of Fourier series – Parseval's identity – Harmonic Analysis.

FourierTransformation

Fourier integral theorem – Fourier transform pair-Sine and Cosine transforms – Properties – Transform of elementary functions – Convolution theorem – Parseval's identity.

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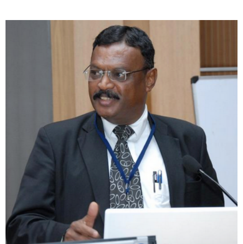

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6. Applications of Fourier series and Transformations

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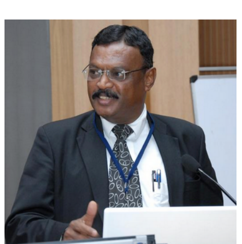

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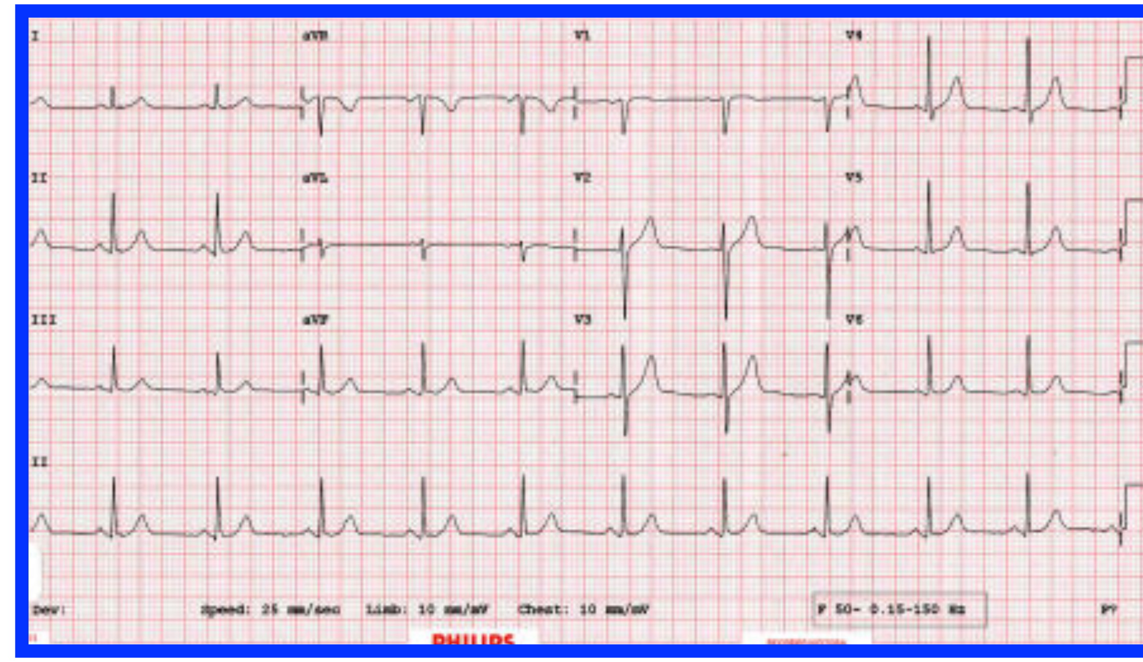

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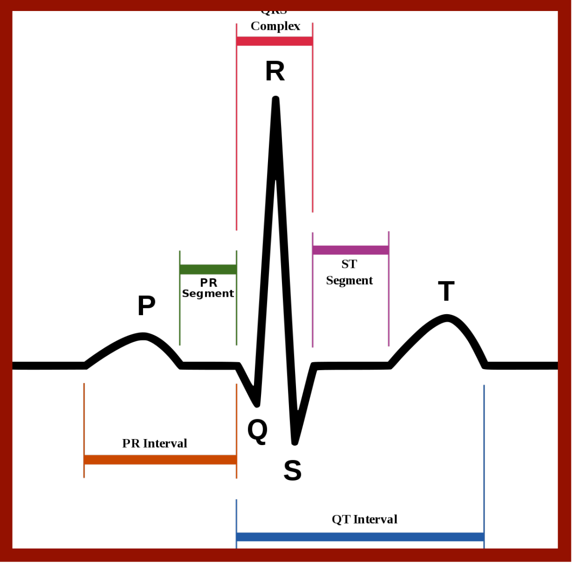

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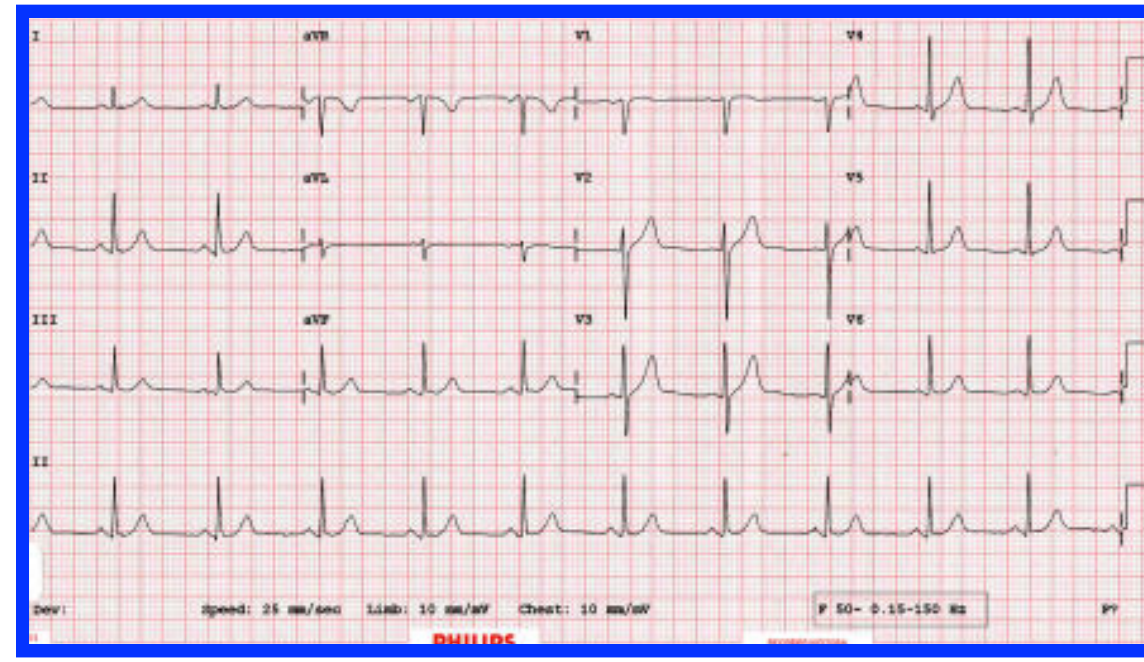

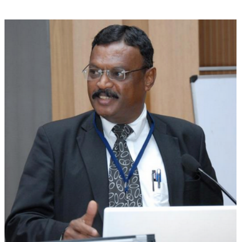

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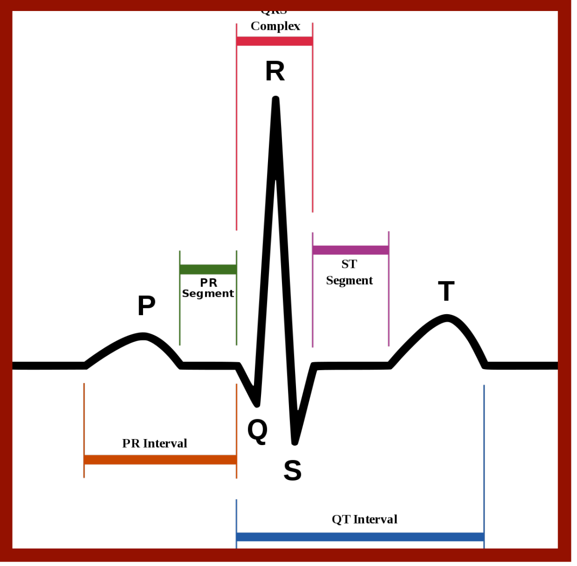

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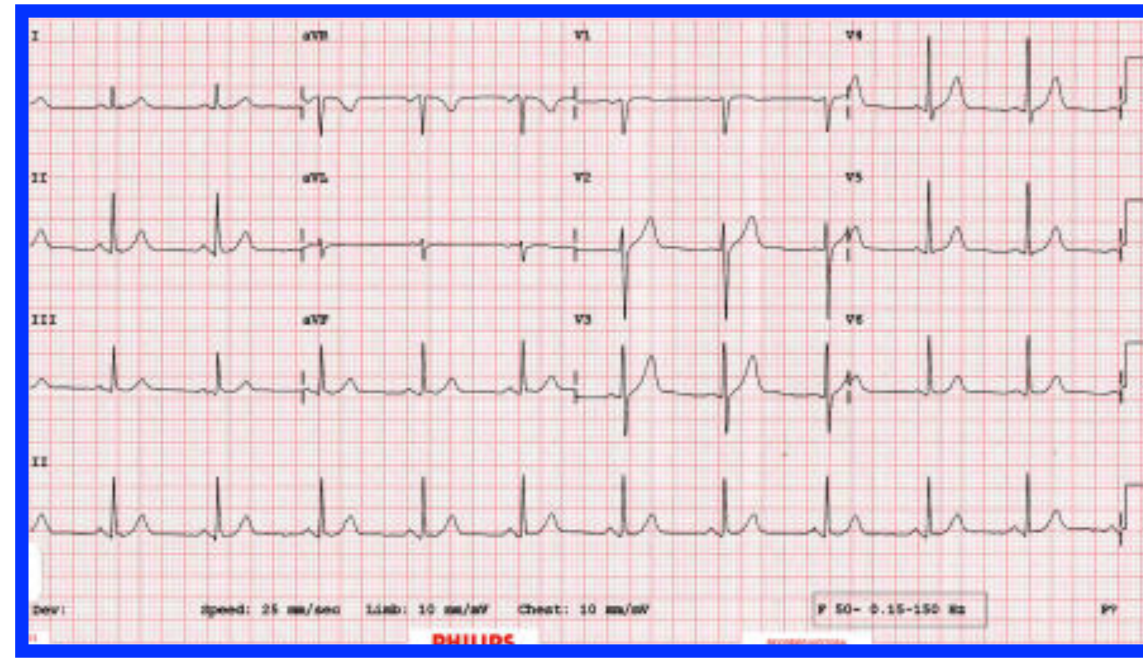

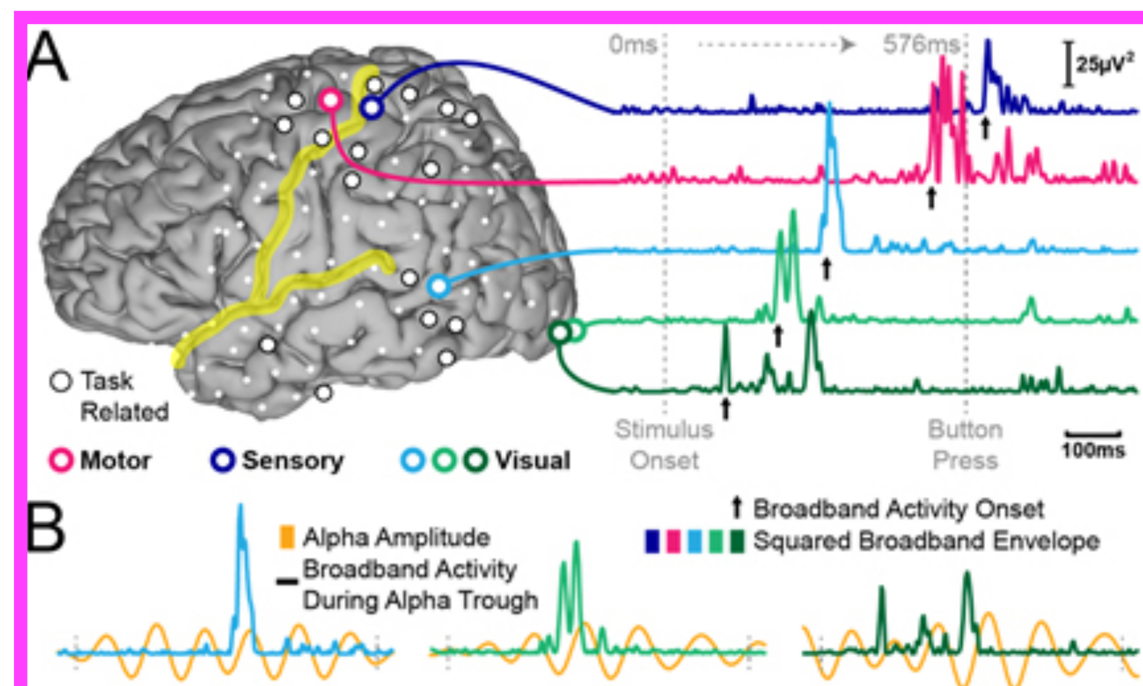

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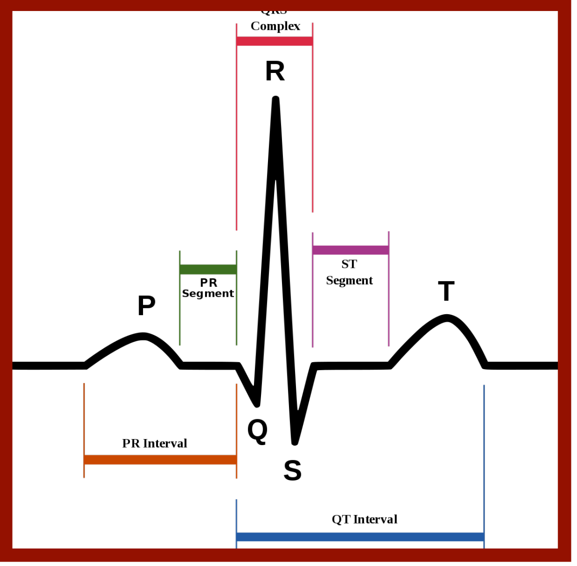

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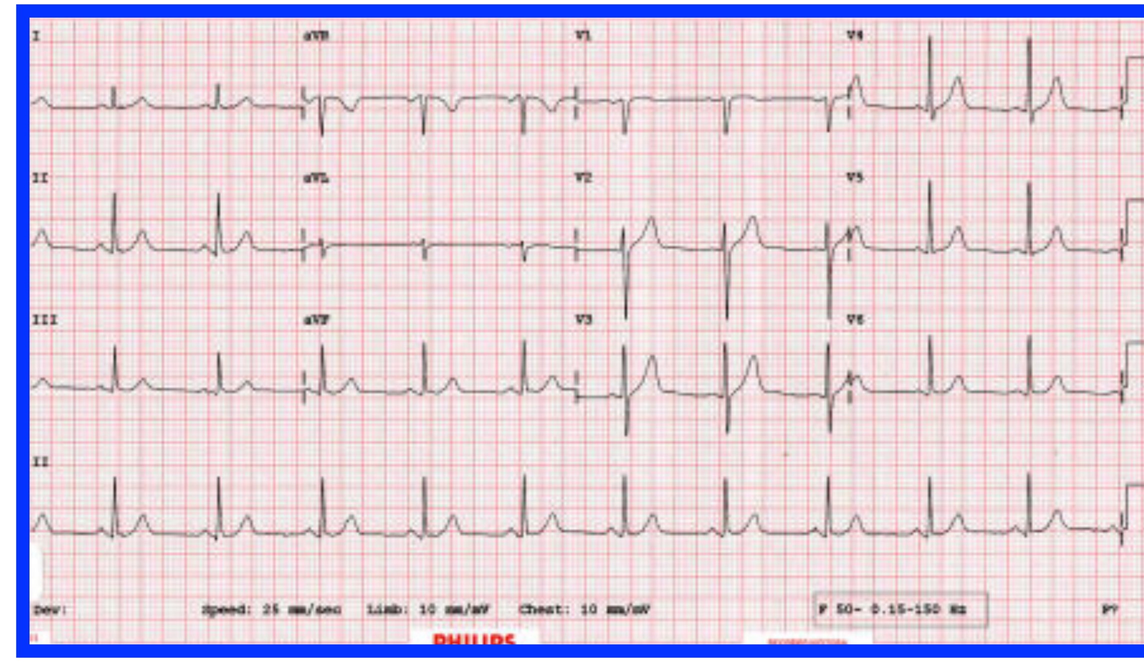
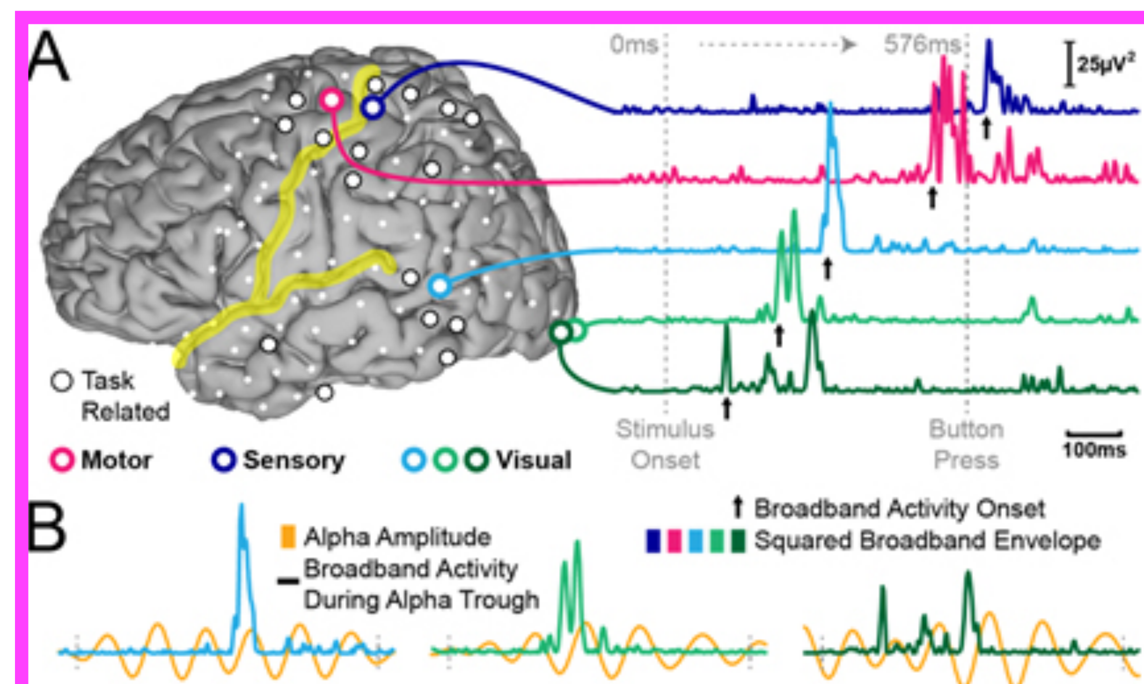
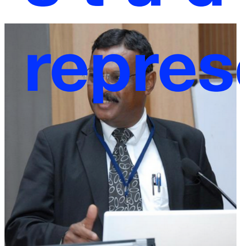


If we look at each of the curve we can notice a periodicity in some interval. Periodic functions play an important role in the communication theory.

One of the most important applications of periodic functions is in the study of electromagnetic radiation. Such diverse forms of energy as cosmic rays, X rays, ultraviolet light, infrared radiation, visible light, radar, radio waves, and microwaves all have one property in common: they are all periodic functions. Their properties and behaviour can be studied by drawing graphical representations.





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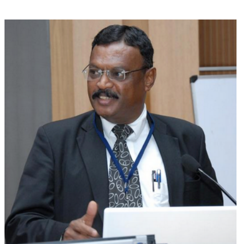

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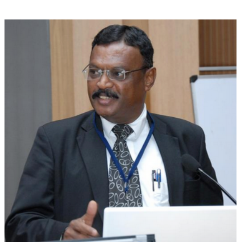

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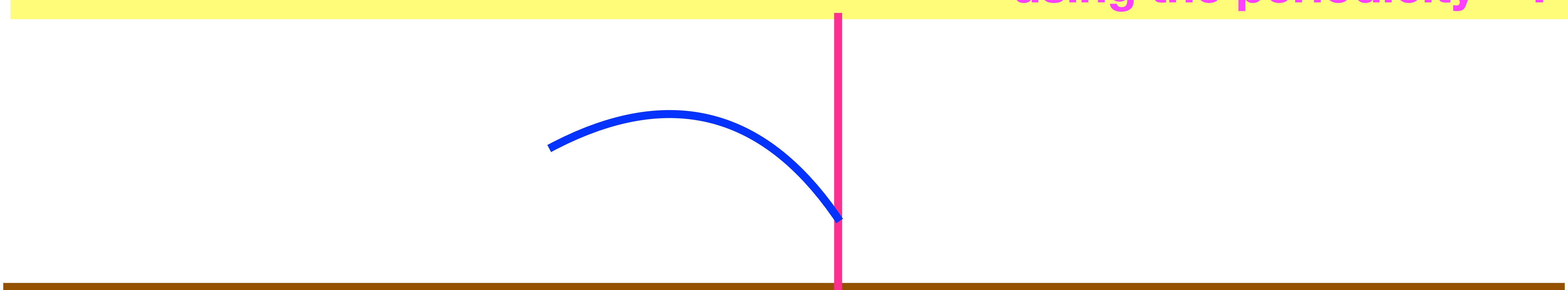

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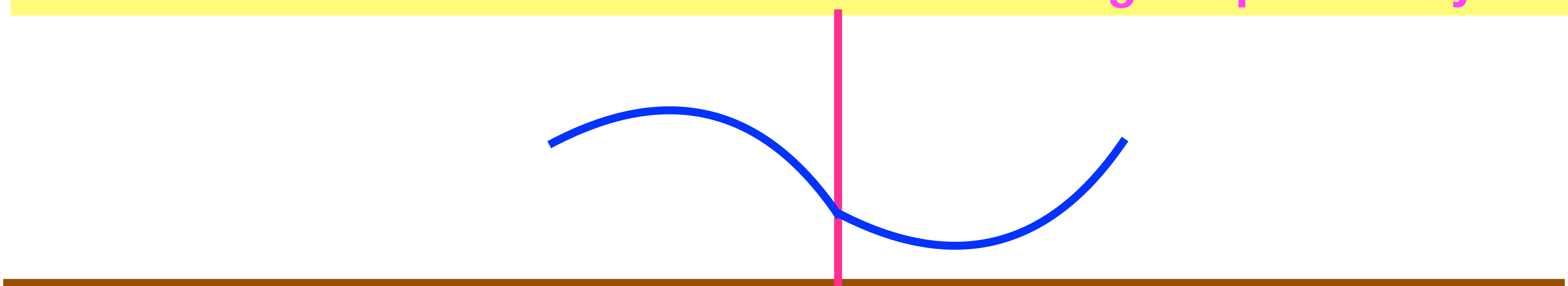

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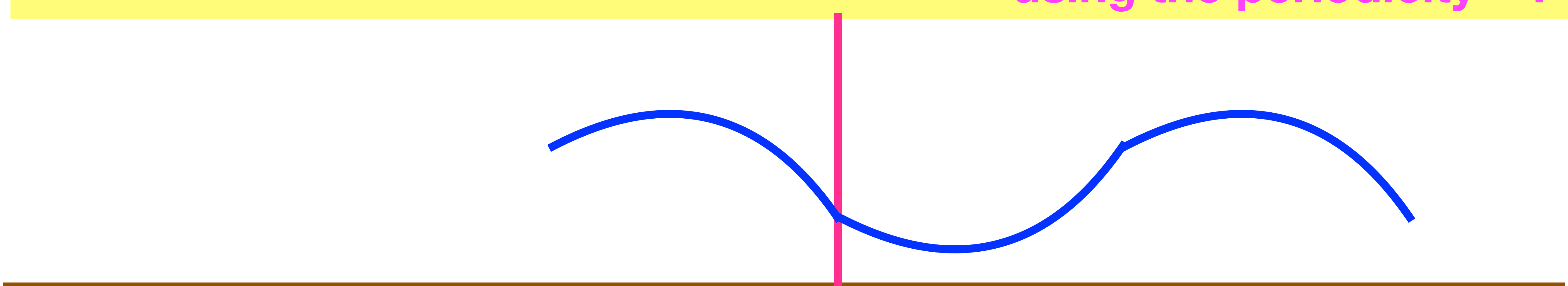

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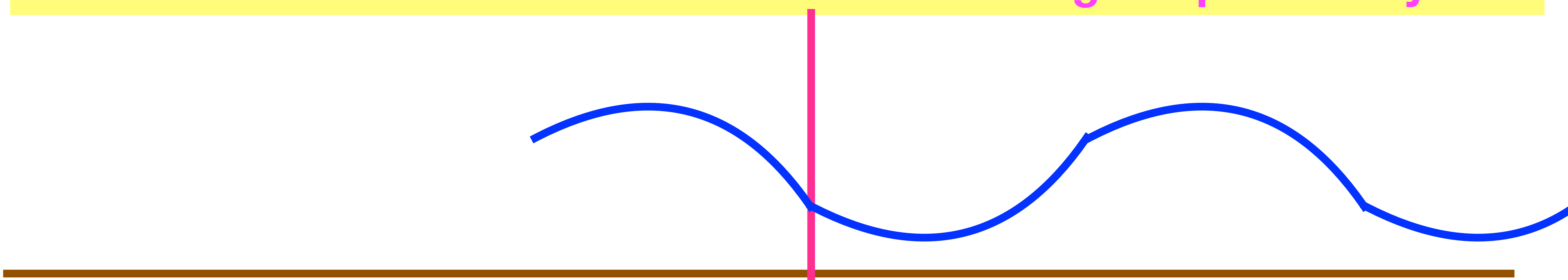

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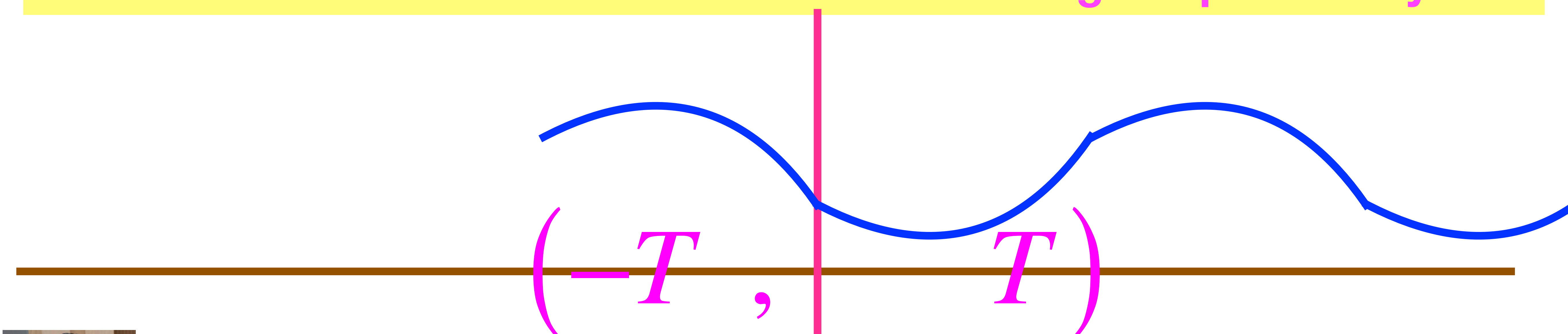

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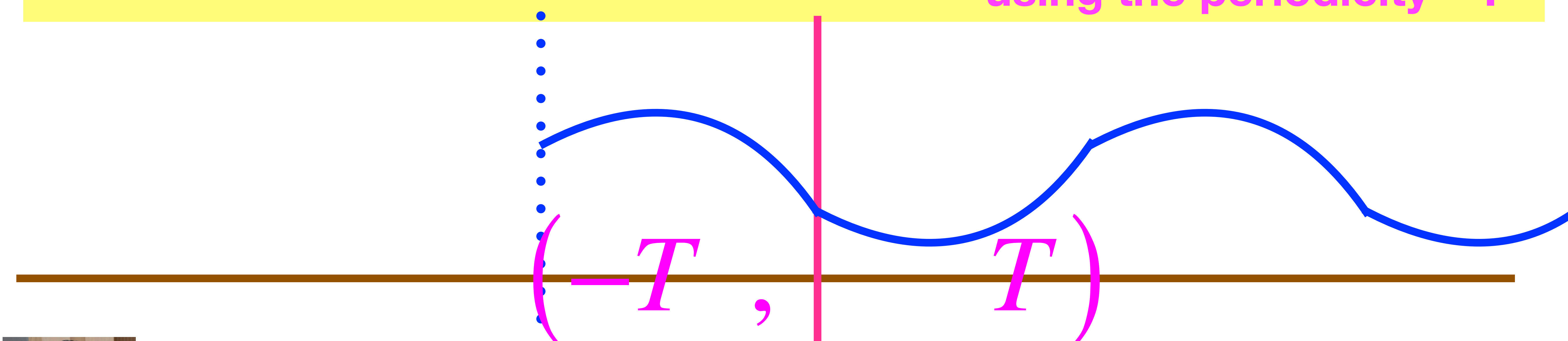

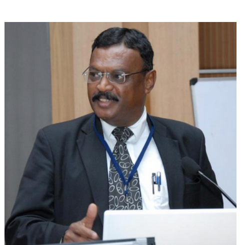

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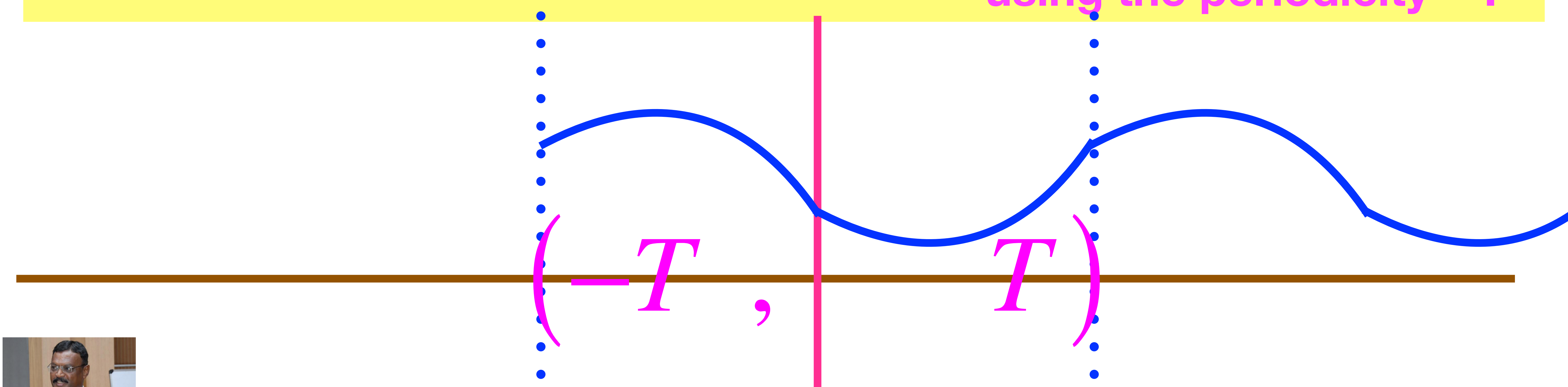

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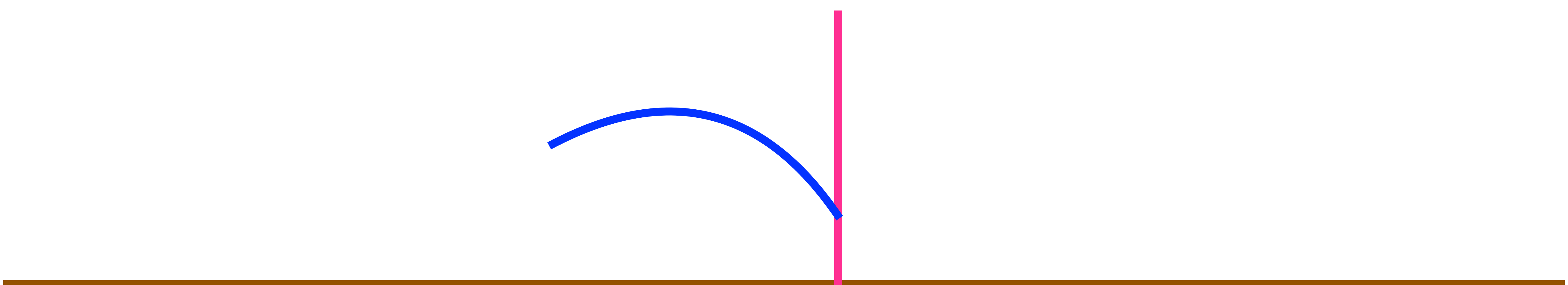

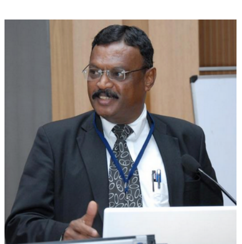

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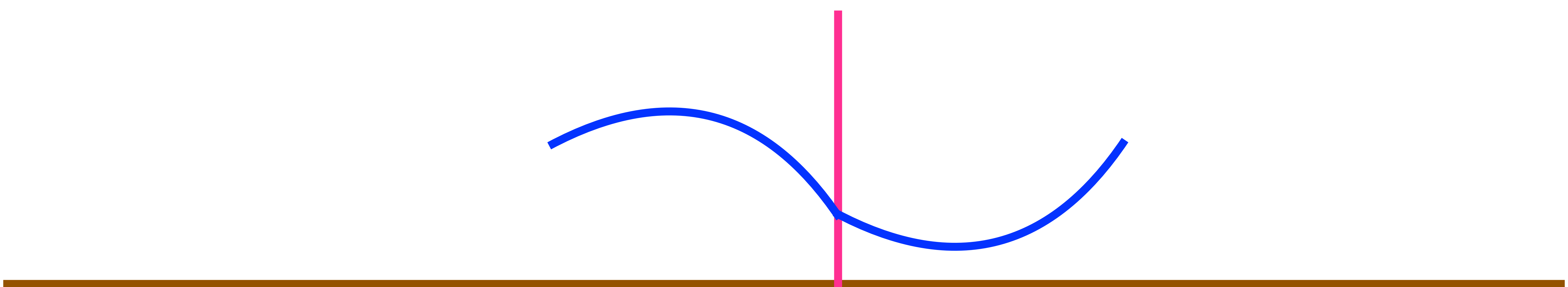

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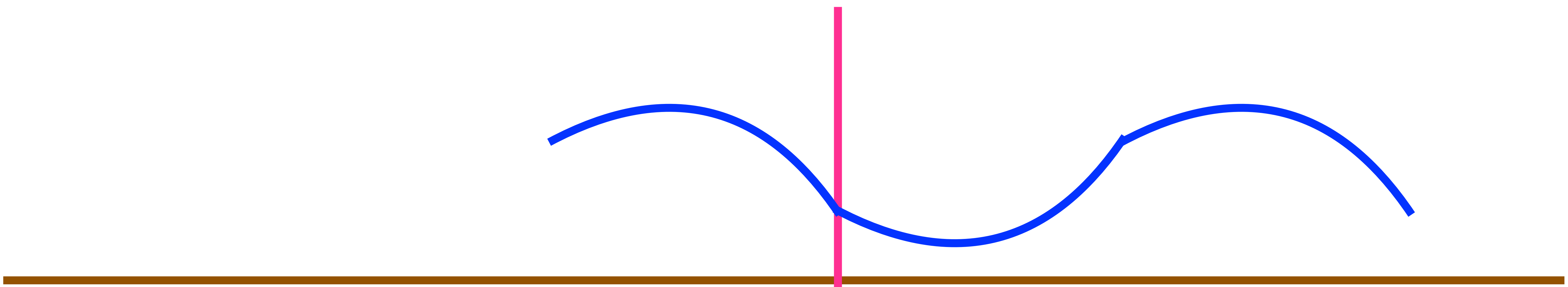

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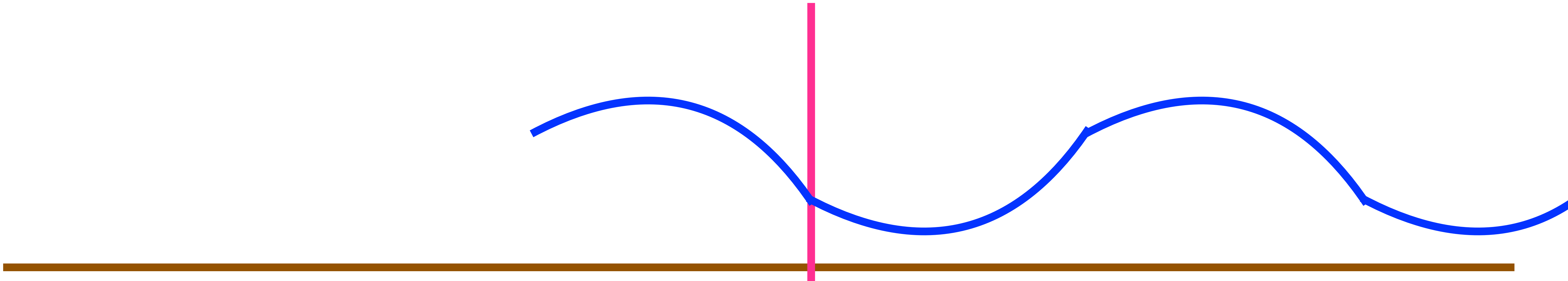

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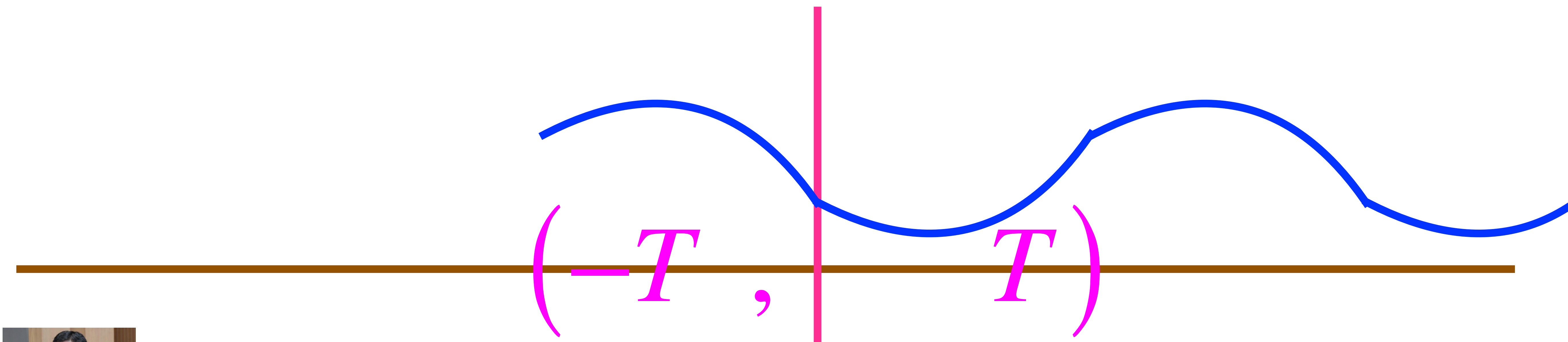

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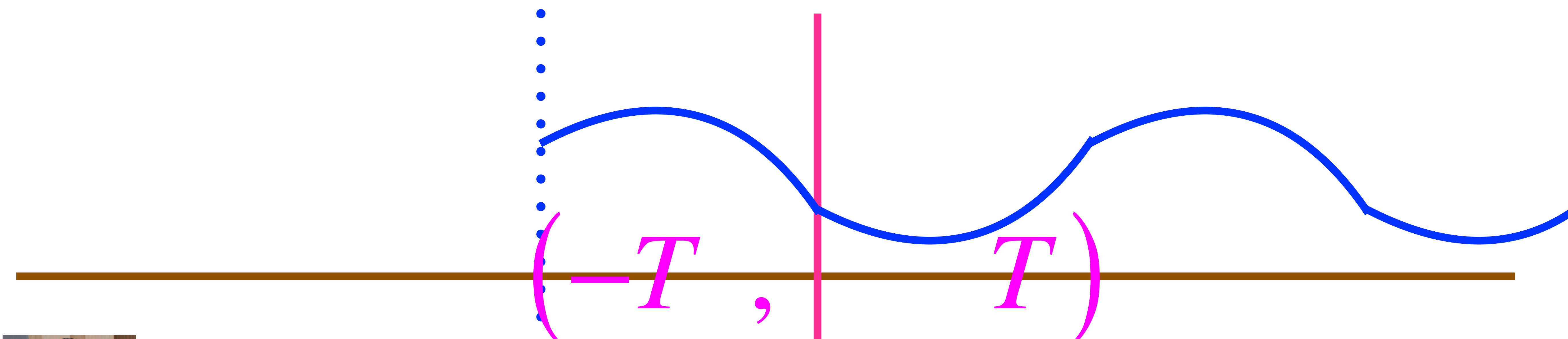

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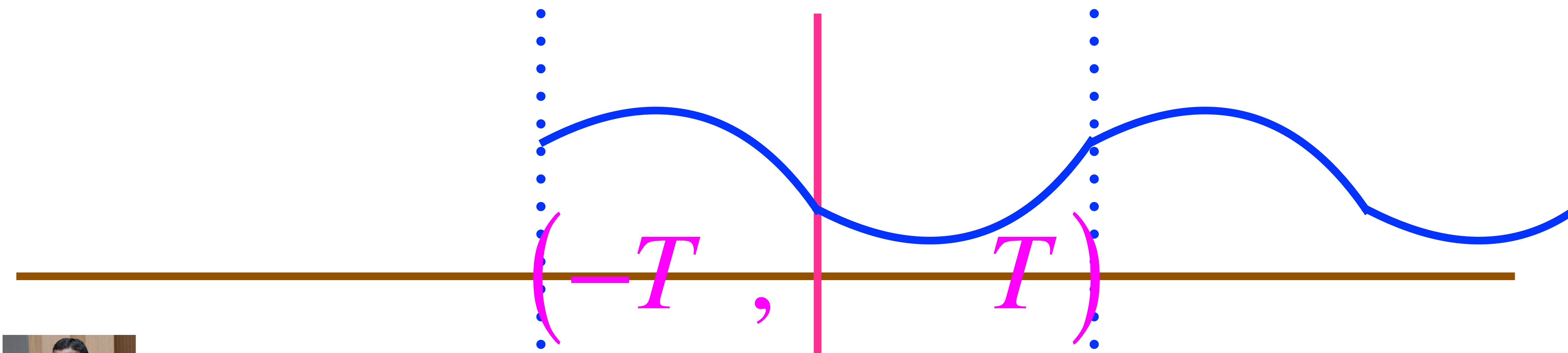
$(-T \;, \; T)$

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Properties of periodic functions

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$$If, f(t) = f(t+T) \ \ \forall \ t \ \ then, f(t+nT) \ = \ f(t) \ \ \forall \ n \in Z$$

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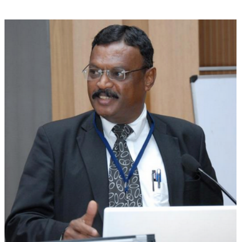
$$h(t) \;\; = \;\; \alpha \, f(t) + \beta \; g(t) \; also \; has \; period \; T$$



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In fact the set of all periodic functions with the same period , say T , form an abelian group under addition and closed w.r.to the scalar multiplication.

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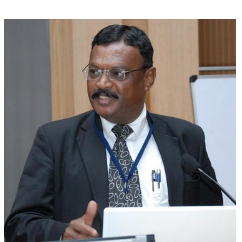
A sinusoid has 3 basic properties:

i. **Amplitude** - height of wave

ii. **Frequency** = 1/T [Hz]

iii. **Phase** - tells you where the peak is (needs a reference)



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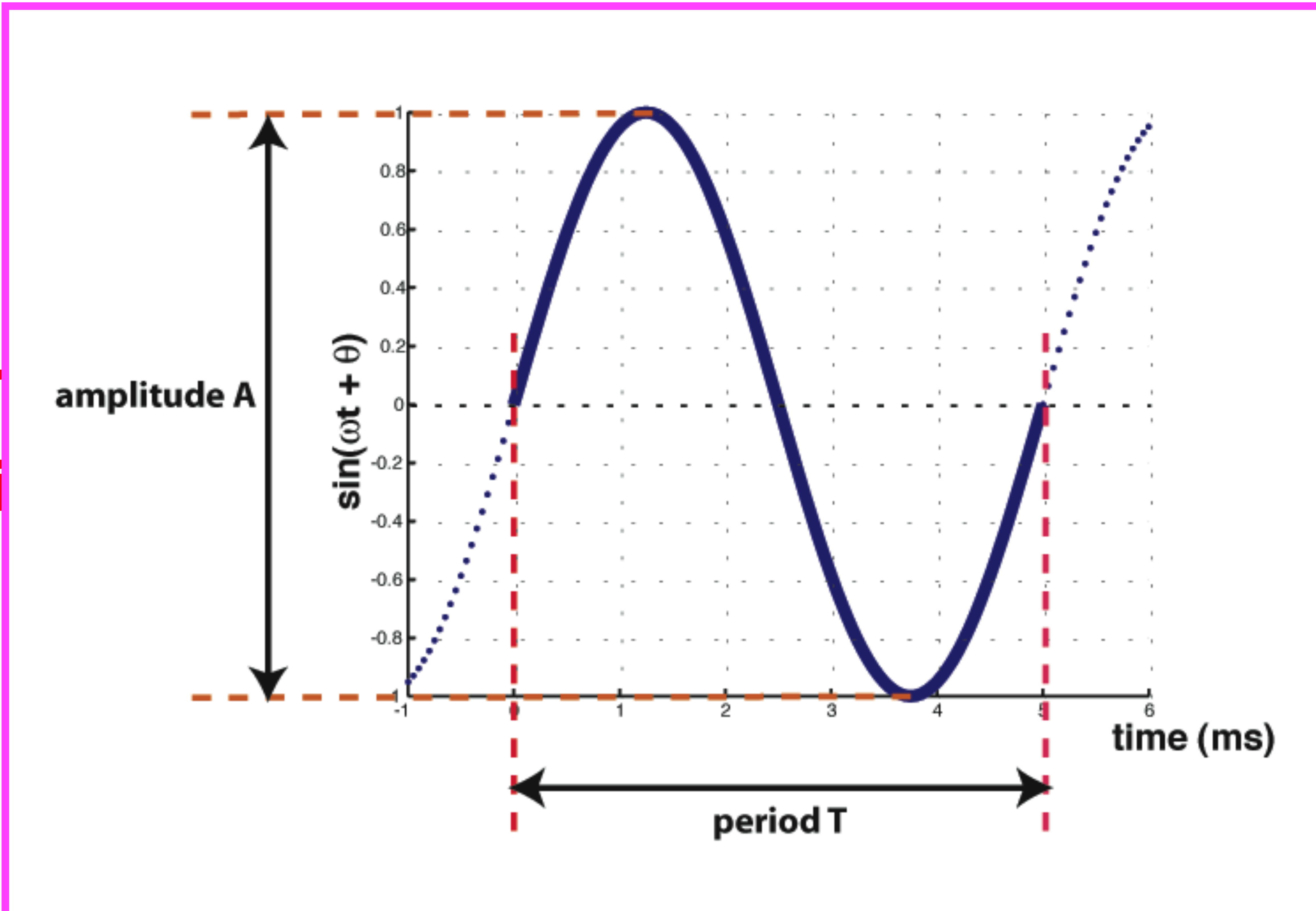
[Trigonometry review](#) ⇒ Sinusoids (e.g. tones)

i.

ii.

iii



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Lecture delivered during the Teachers Enrichment Workshop held at IMSC between 26th November to 1st December 2018.

Find the period of the function

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Find the period of the function $f(t) = cos\left(\frac{t}{4}\right) + cos\left(\frac{t}{3}\right)$

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$$f(t+T)=f\left(t\right)\Rightarrow cos\left(\frac{t+T}{4}\right)+cos\left(\frac{t+T}{3}\right)=cos\left(\frac{t}{4}\right)+cos\left(\frac{t}{3}\right)$$

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$$cos\left(\frac{t+T}{4}\right) = cos\left(\frac{t}{4}\right)$$

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$$cos\left(\frac{t+T}{4}\right) = cos\left(\frac{t}{4}\right) \quad \Rightarrow \frac{T}{4} = 2n\pi \Rightarrow \boxed{T = 8n\pi}$$

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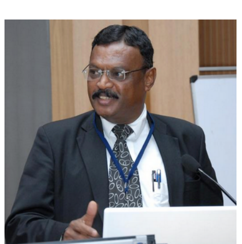
$$l.c.m(6,8) = 24$$



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Find the period of the function $f(t) = \cos\left(\frac{t}{4}\right) + \cos\left(\frac{t}{3}\right)$

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$$l.c.m(6,8) = 24$$

$$\therefore \text{The required period is } \boxed{T = 24\pi}$$

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Conditions for periodicity of the function

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Conditions for periodicity of the function

$$f(t) = cos(\lambda_1 t) + cos(\lambda_2 t)$$

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Conditions for periodicity of the function

$$f(t)=cos(\lambda_1 t)+cos(\lambda_2 t)$$

$$f(t+T)=f(t)\Rightarrow cos(\lambda_1[t+T])+cos(\lambda_2[t+T])=cos(\lambda_1 t)+cos(\lambda_2 t)$$

$$cos(\lambda_1[t+T])=cos(\lambda_1 t)\Rightarrow \lambda_1 T=2m\pi$$

Conditions for periodicity of the function

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$$f(t+T) = f(t) \Rightarrow cos(\lambda_1 [t+T]) + cos(\lambda_2 [t+T]) = cos(\lambda_1 t) + cos(\lambda_2 t)$$

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$$cos(\lambda_2[t+T]) = cos(\lambda_2 t) \Rightarrow \lambda_2 T = 2n\pi$$

$$\boxed{\therefore \frac{\lambda_1}{\lambda_2} = \frac{m}{n}\,(rational)}$$

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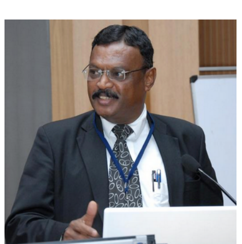
$$The\ function, g(t) = cos(7t) + cos(2+\pi)t$$



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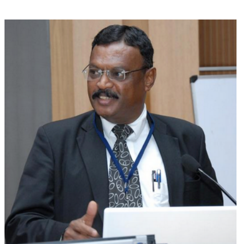
$$f(t+T)=f\left(t\right)\Rightarrow cos\left(\lambda_1\left[t+T\right]\right)+cos\left(\lambda_2\left[t+T\right]\right)=cos\left(\lambda_1 t\right)+cos\left(\lambda_2 t\right)$$

$$cos\left(\lambda_1\left[t+T\right]\right)=cos\left(\lambda_1 t\right)\Rightarrow \lambda_1 T=2m\pi$$

$$cos\left(\lambda_2\left[t+T\right]\right)=cos\left(\lambda_2 t\right)\Rightarrow \lambda_2 T=2n\pi$$

$$\boxed{\therefore \frac{\lambda_1}{\lambda_2}=\frac{m}{n}\,(rational)}$$

$The\ function, g\left(t\right)=cos\left(7t\right)+cos\left(2+\pi\right)t$ **is not periodic**



Conditions for periodicity of the function

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$$cos(\lambda_2 [t+T]) = cos(\lambda_2 t) \Rightarrow \lambda_2 T = 2n\pi$$

$$\boxed{\therefore \frac{\lambda_1}{\lambda_2} = \frac{m}{n} (rational)}$$

$$The\ function, g(t) = cos(7t) + cos(2+\pi)t$$ **is not periodic**

$$\because \frac{\lambda_1}{\lambda_2} = \frac{7}{2+\pi} (irrational)$$

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Consider the following functions and its graphs

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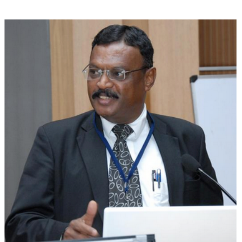
$$f(t) = 1$$



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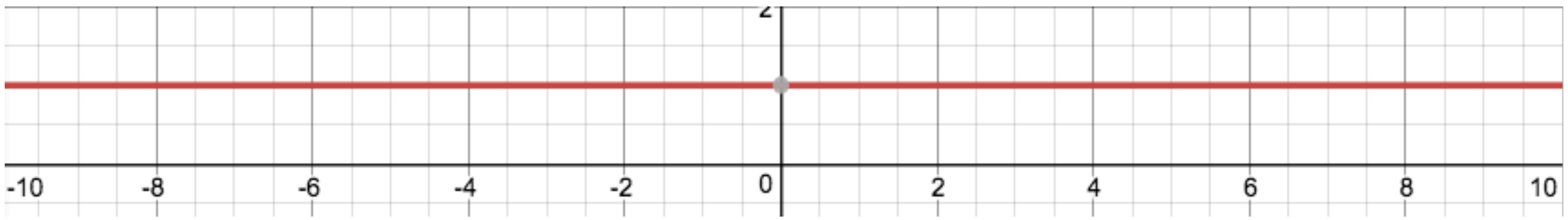

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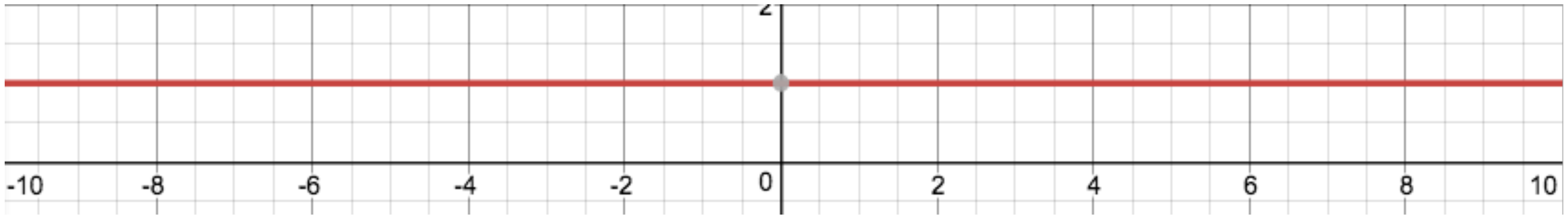
$f(t) \;=\; 1$



Period 0

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Consider the following functions and its graphs

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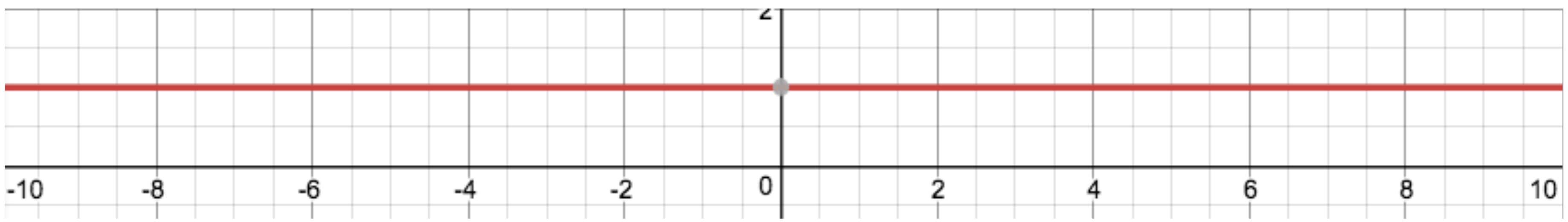

Period 0

$f(t) = sint$

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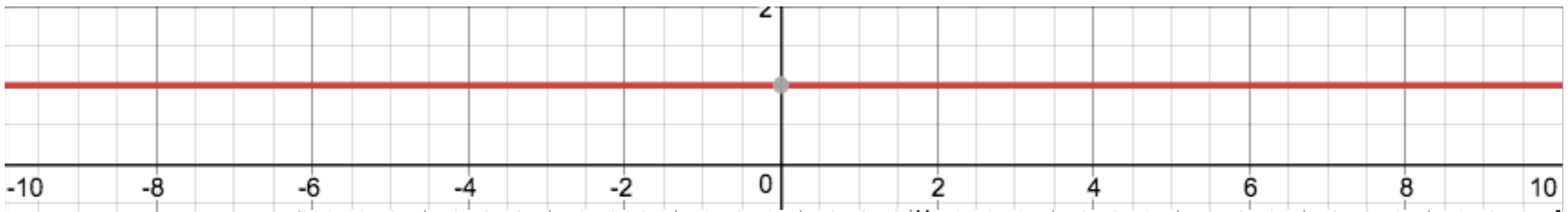

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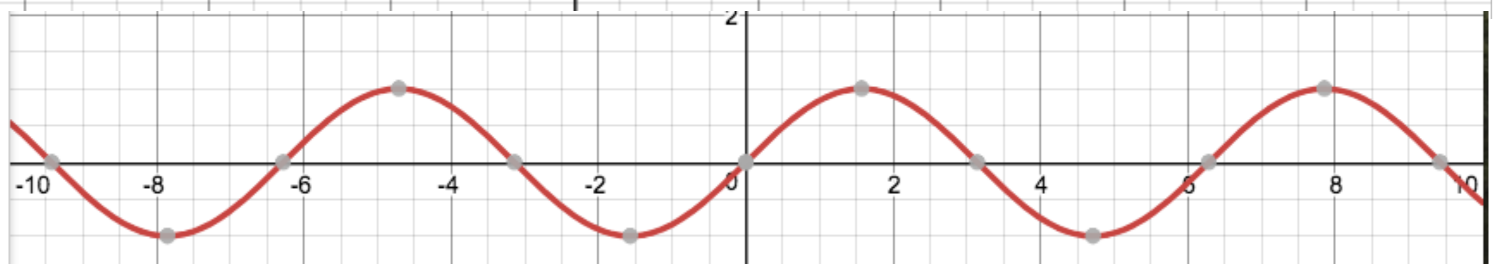

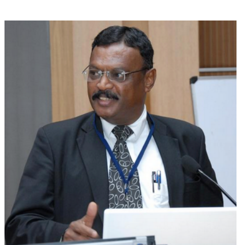

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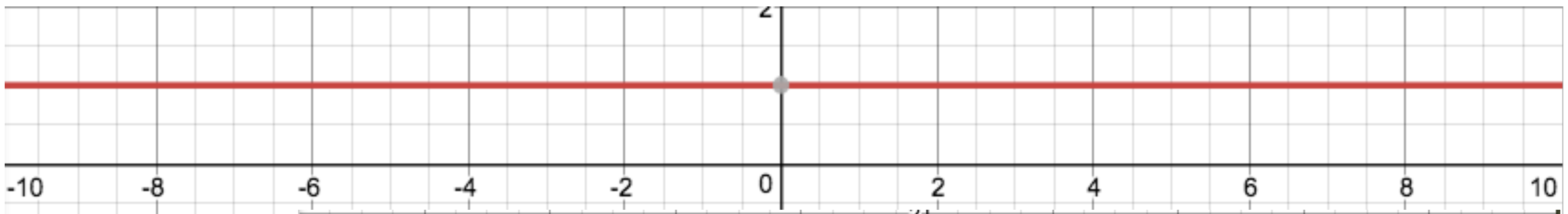

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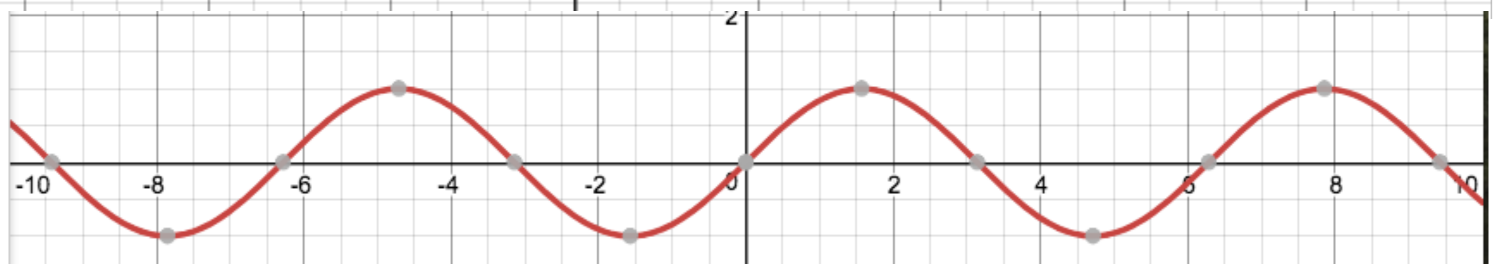
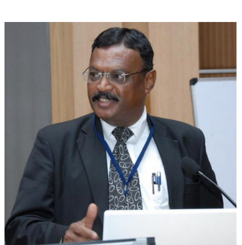


Period 2π



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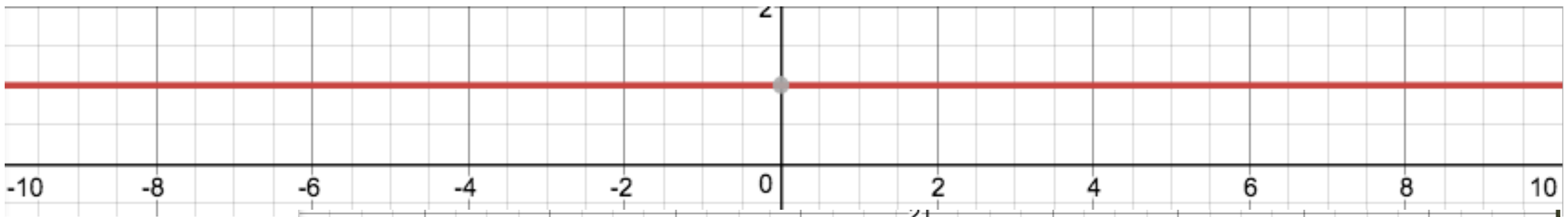

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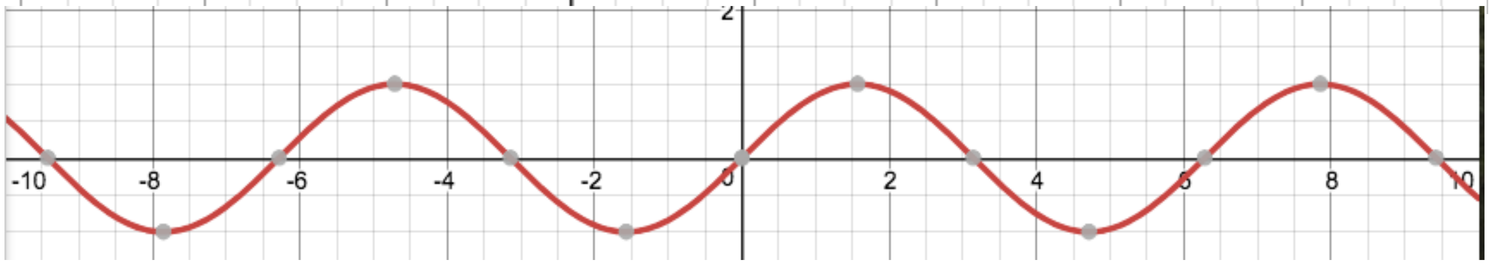

Period 2π

$f(t) = sin2t$

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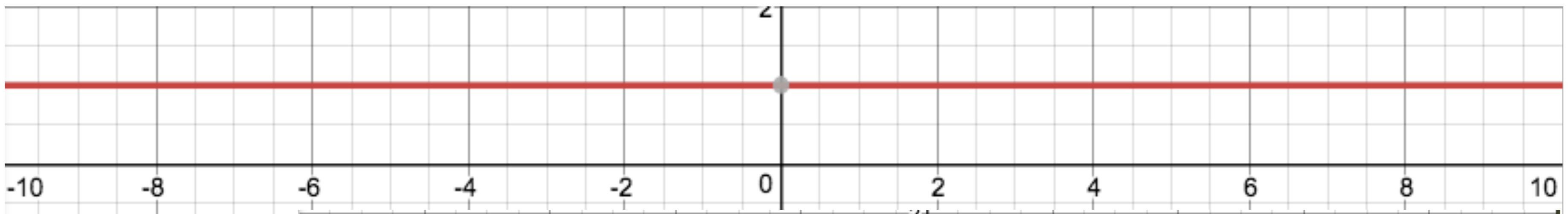

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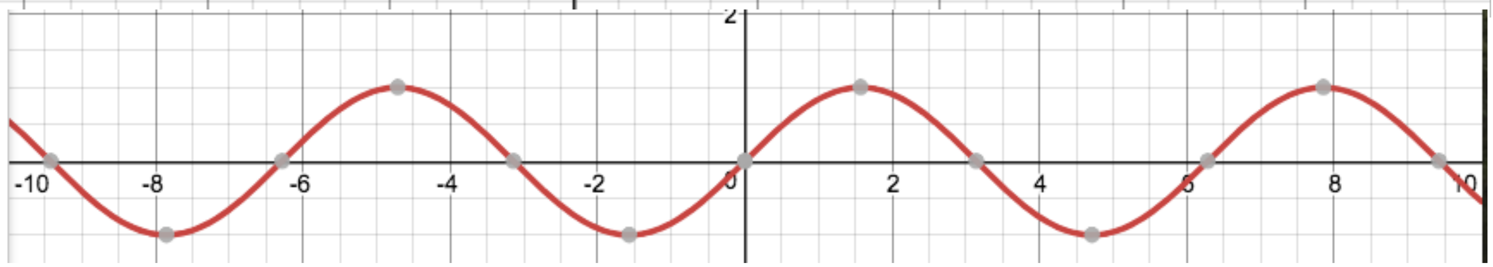

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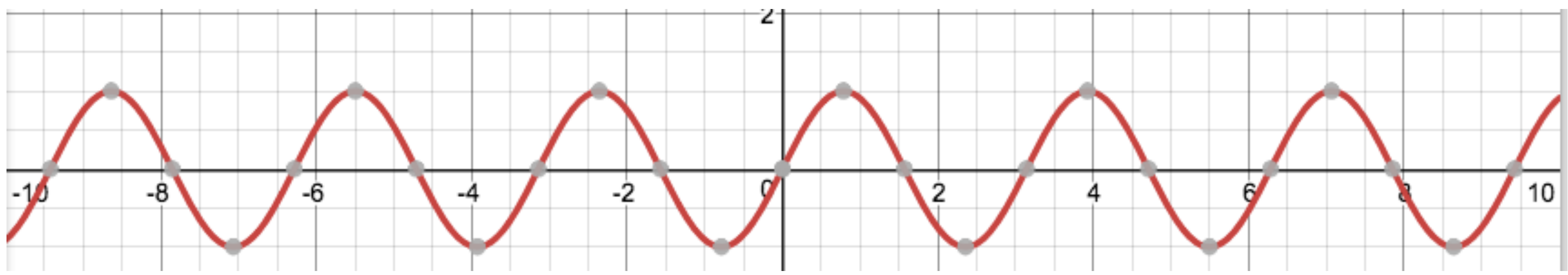

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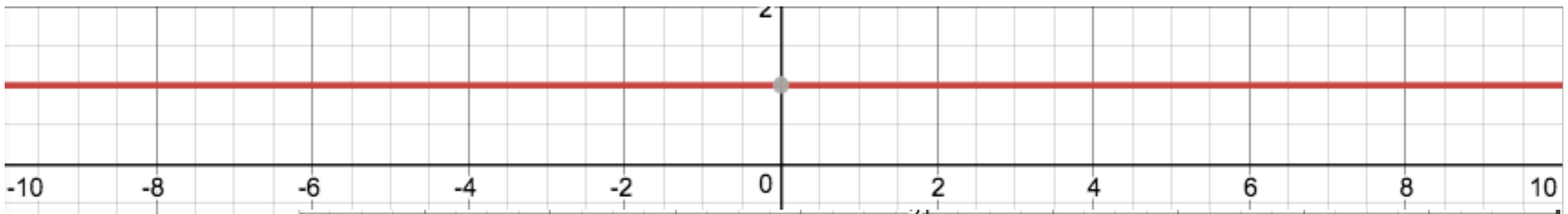

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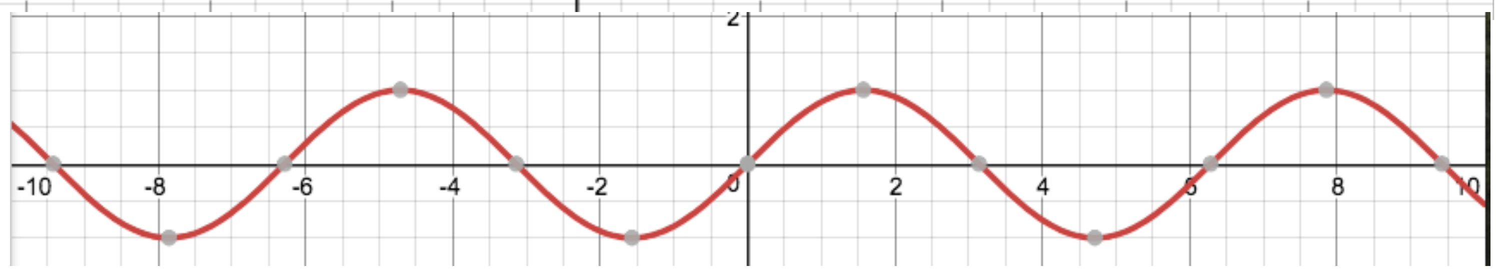

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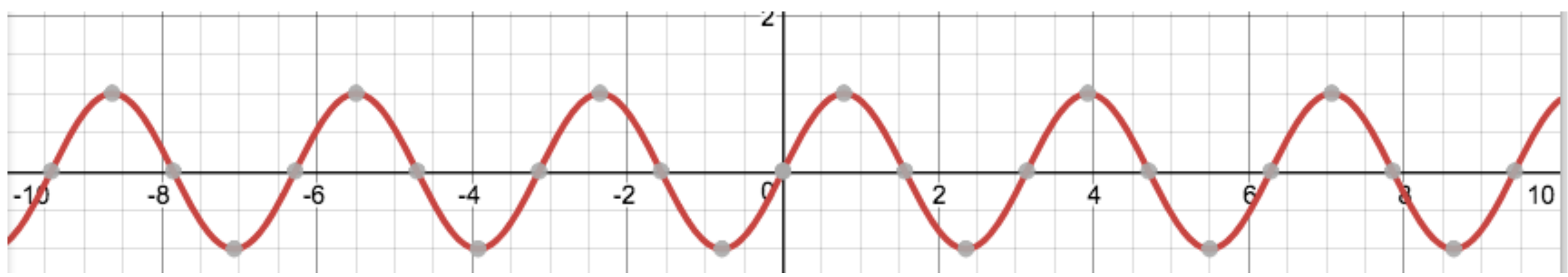

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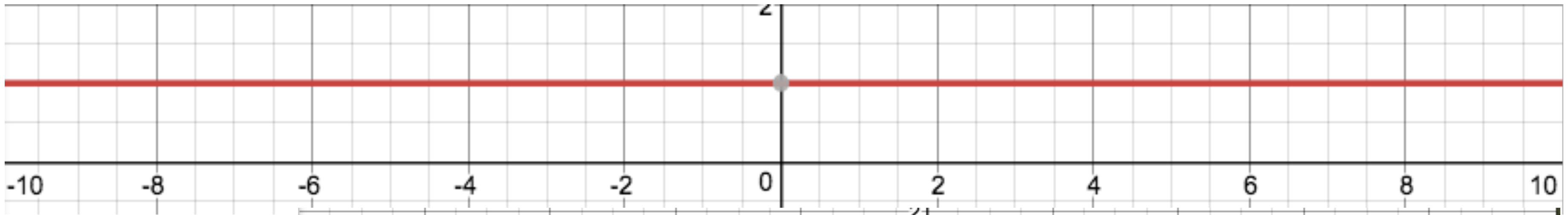

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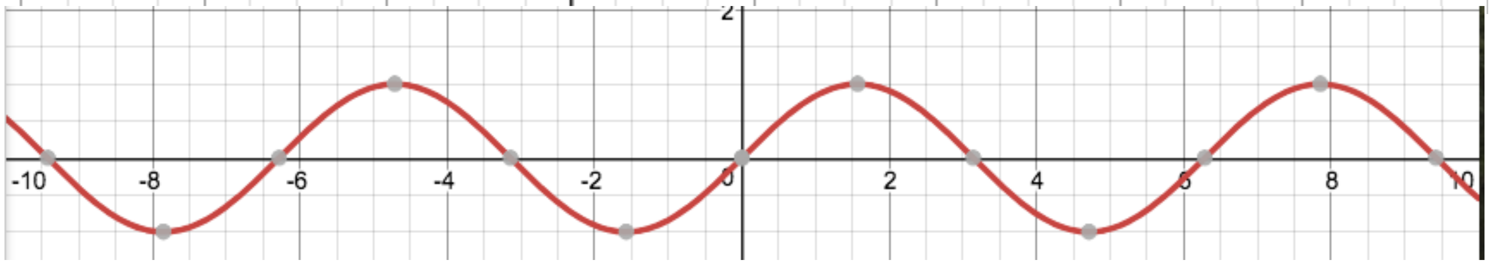
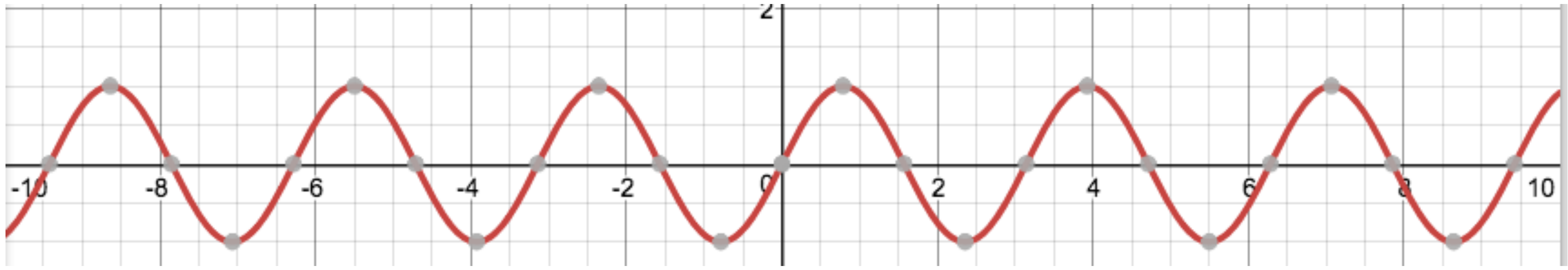


Period 2π

$f(t) = sin2t$



Period π

$f(t) = sin3t$

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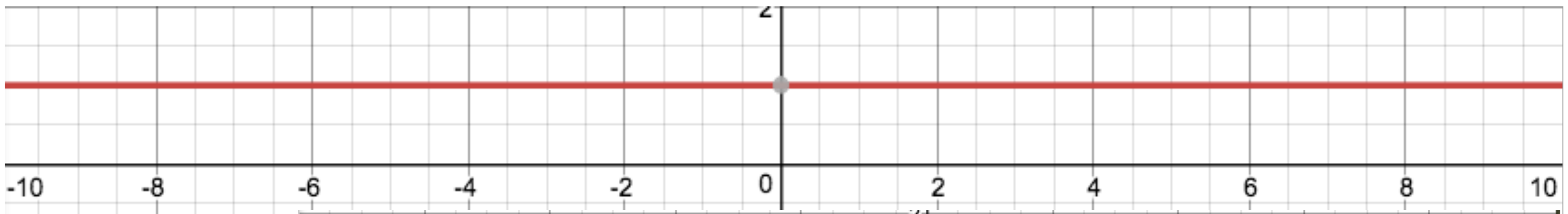
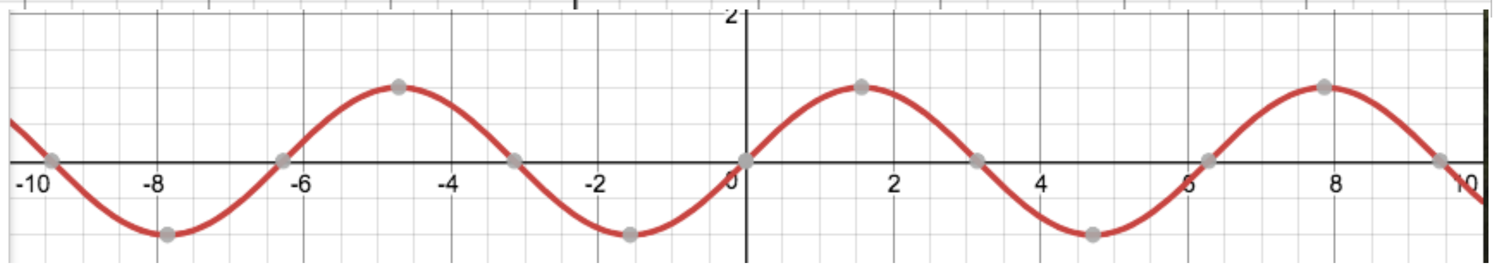
Consider the following functions and its graphs

$f(t) = 1$



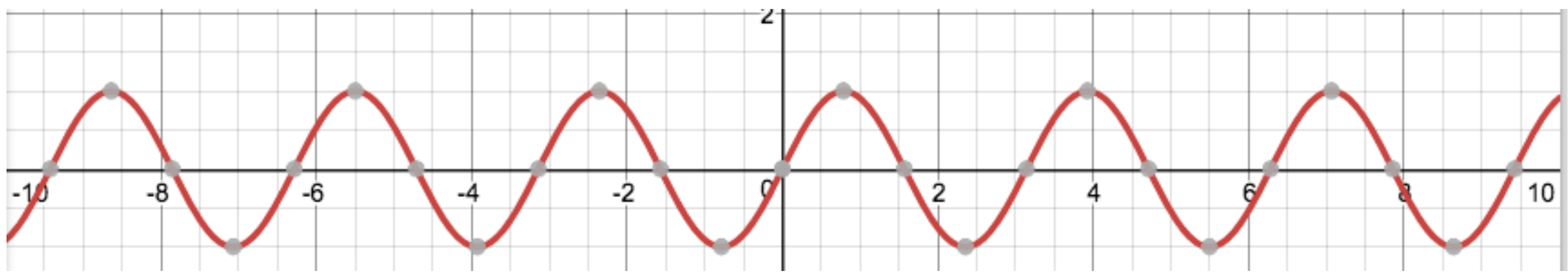
Period 0

$f(t) = sint$



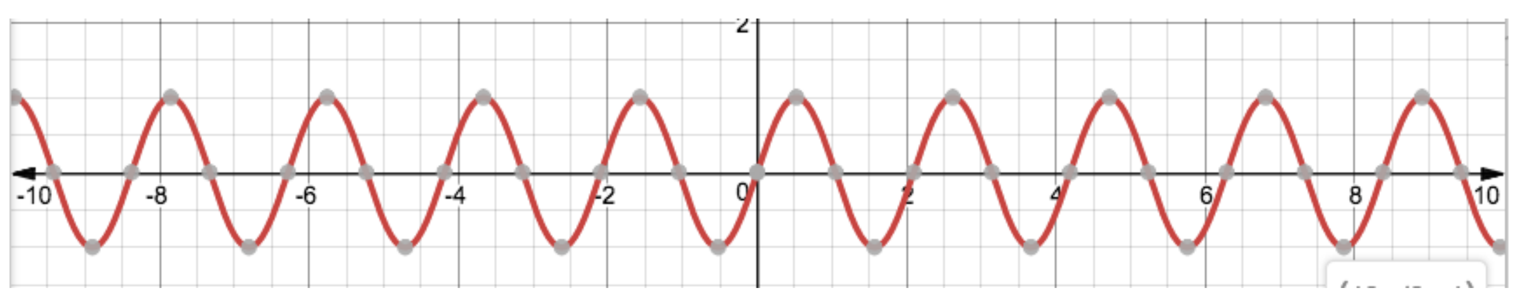
Period 2π

$f(t) = sin2t$



Period π

$f(t) = sin3t$



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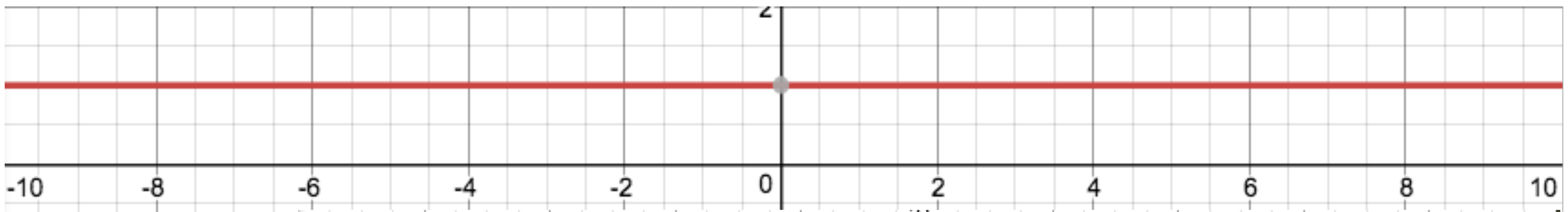
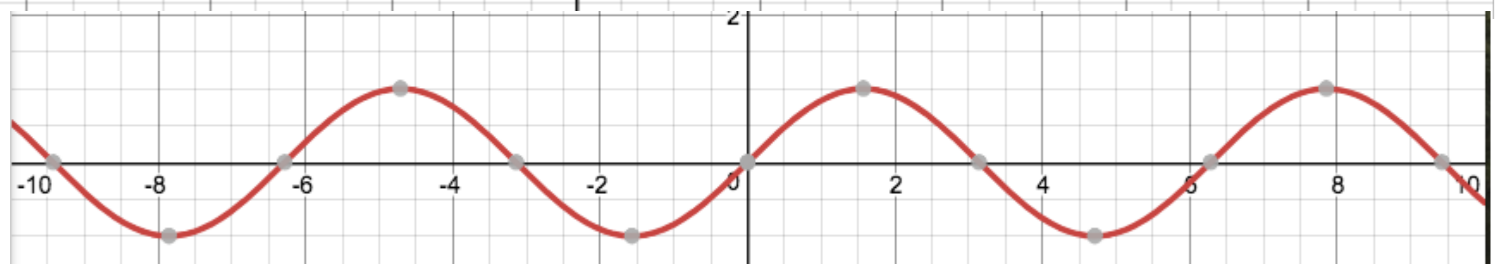
Consider the following functions and its graphs

$f(t) = 1$



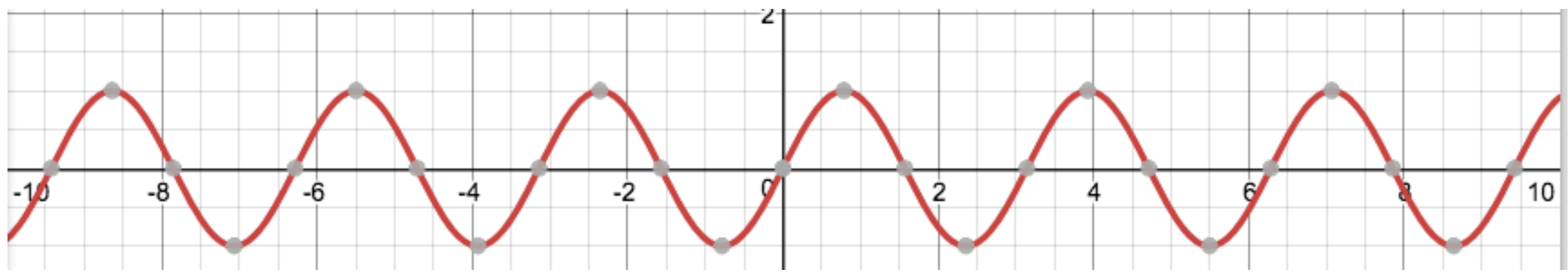
Period 0

$f(t) = sin t$



Period 2π

$f(t) = sin 2t$



Period π

$f(t) = sin 3t$

Period $\dfrac{\pi}{3}$

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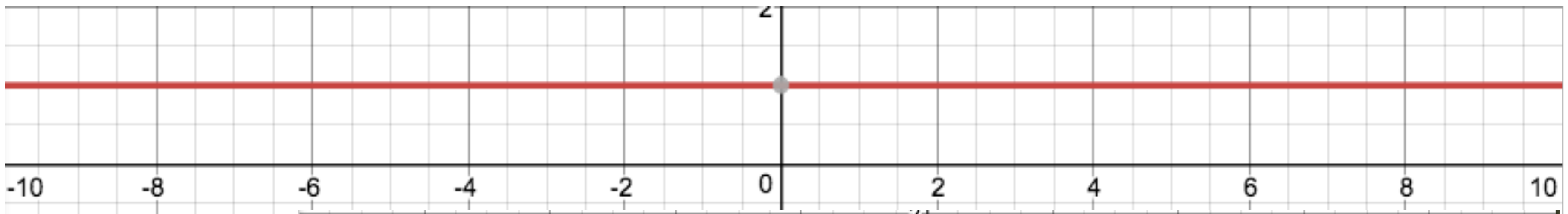
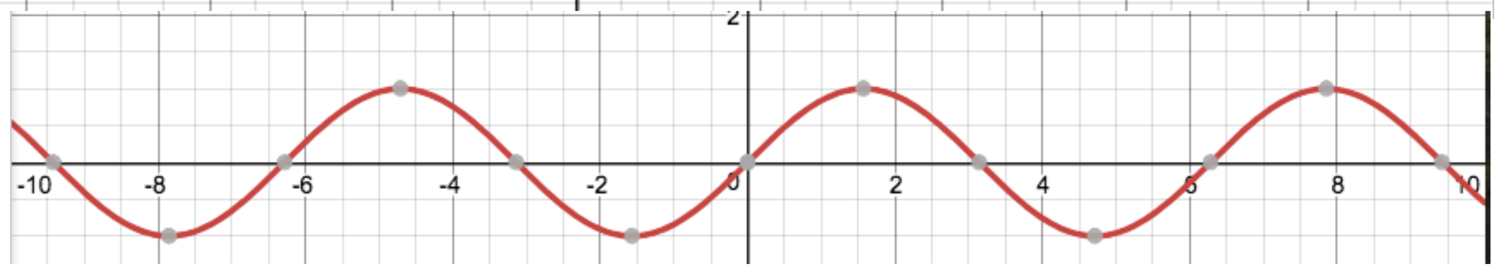
Consider the following functions and its graphs

$f(t) = 1$



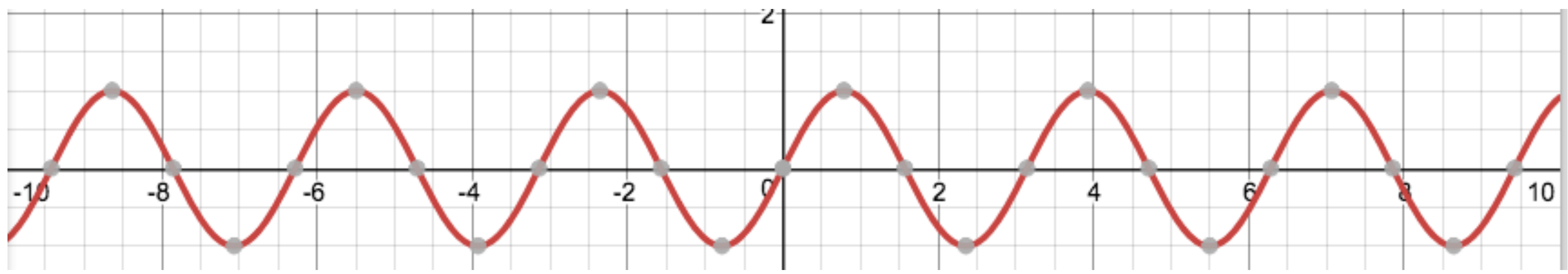
Period 0

$f(t) = sint$



Period 2π

$f(t) = sin2t$

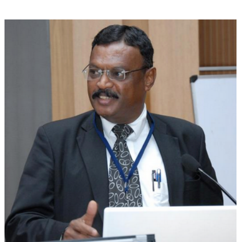


Period π

$f(t) = sin3t$

Period $\dfrac{\pi}{3}$

$f(t) = sin4t$



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Consider the following functions and its graphs

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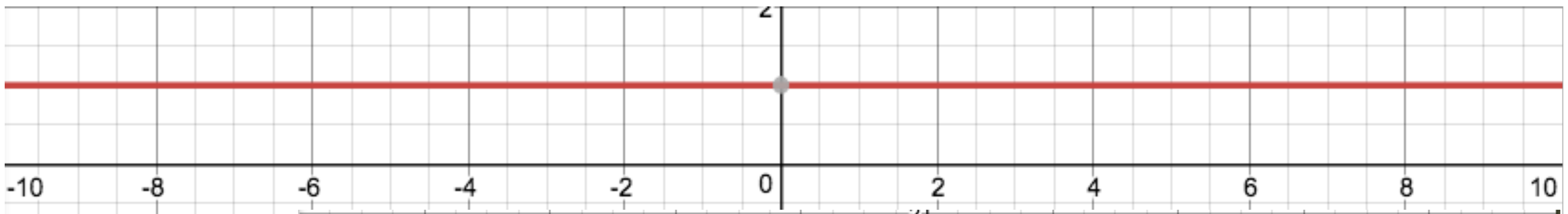
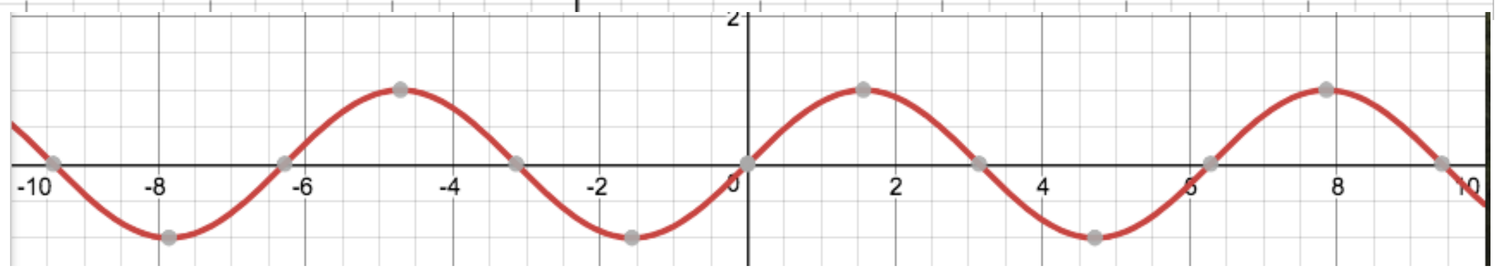


Period 0

$f(t) = sin t$



Period 2π

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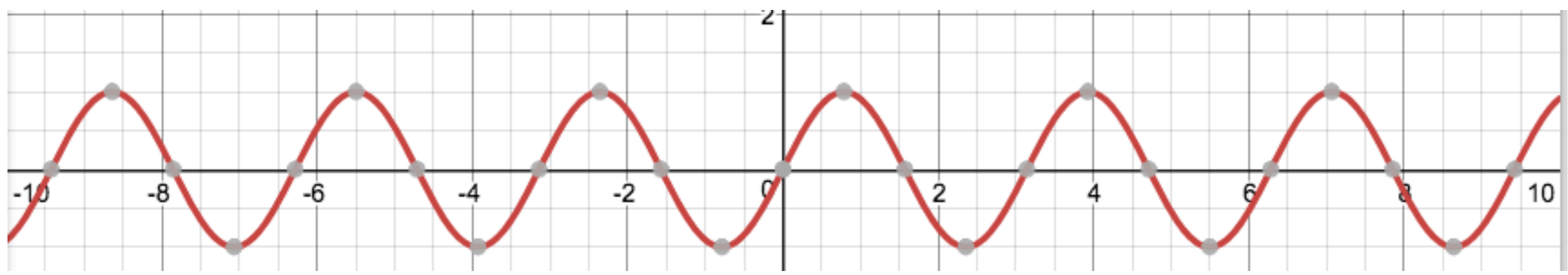
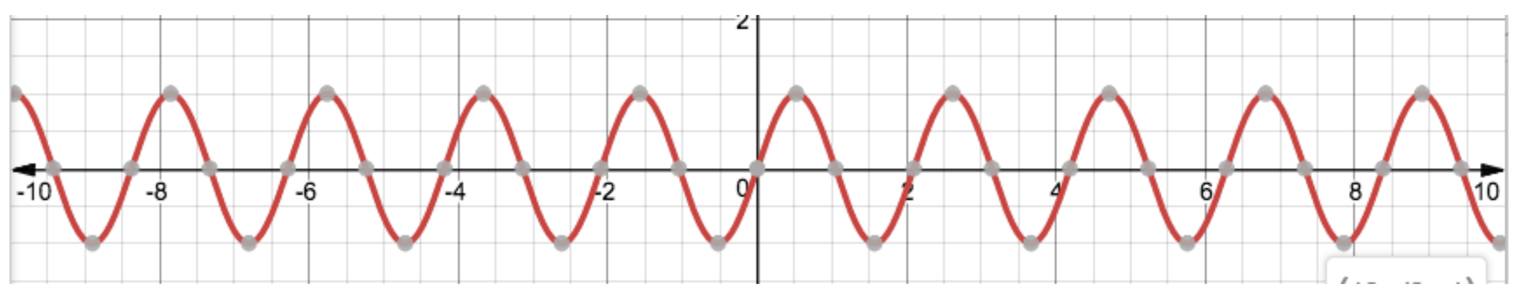


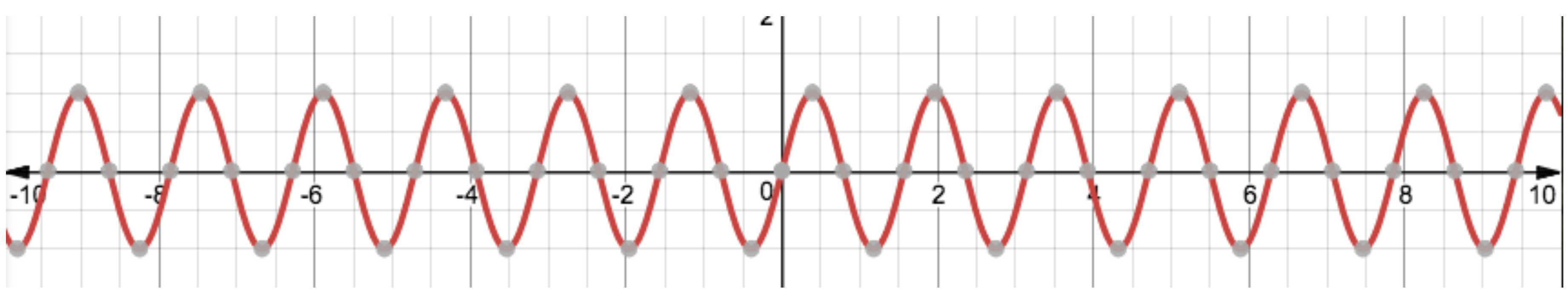
Period π

$f(t) = sin 3t$



Period $\dfrac{\pi}{3}$

$f(t) = sin 4t$



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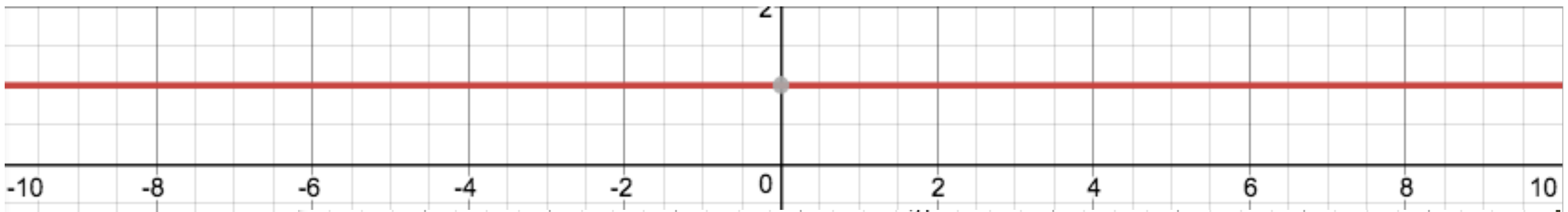

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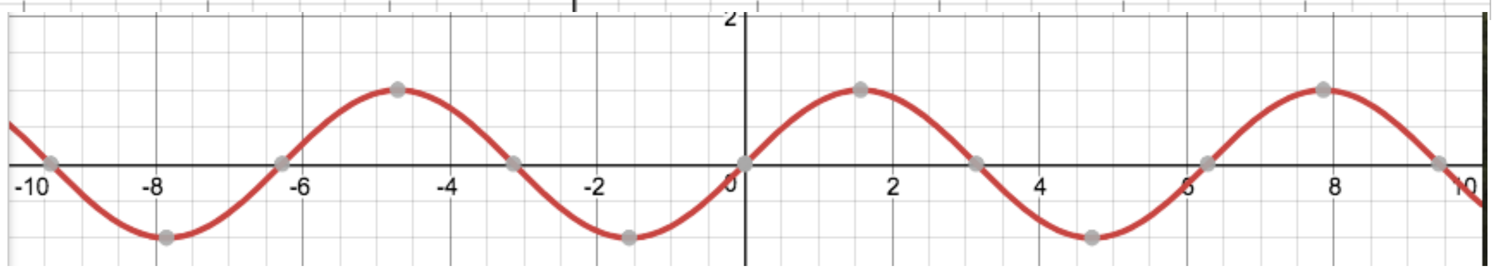

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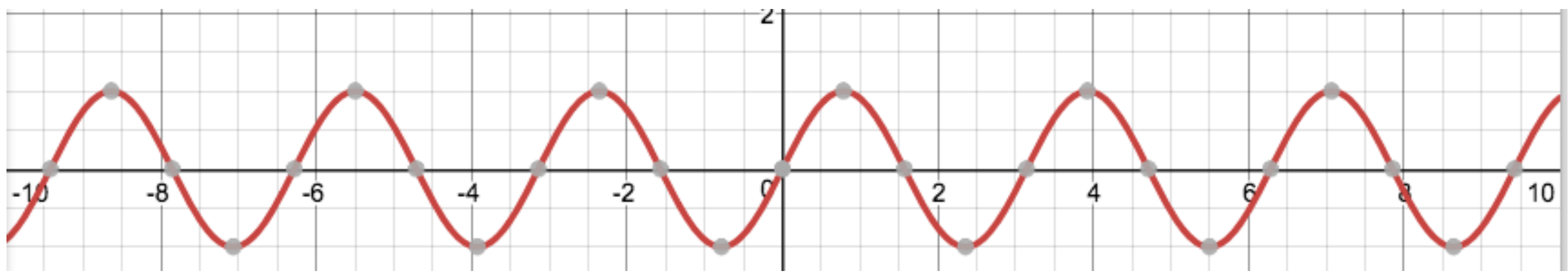

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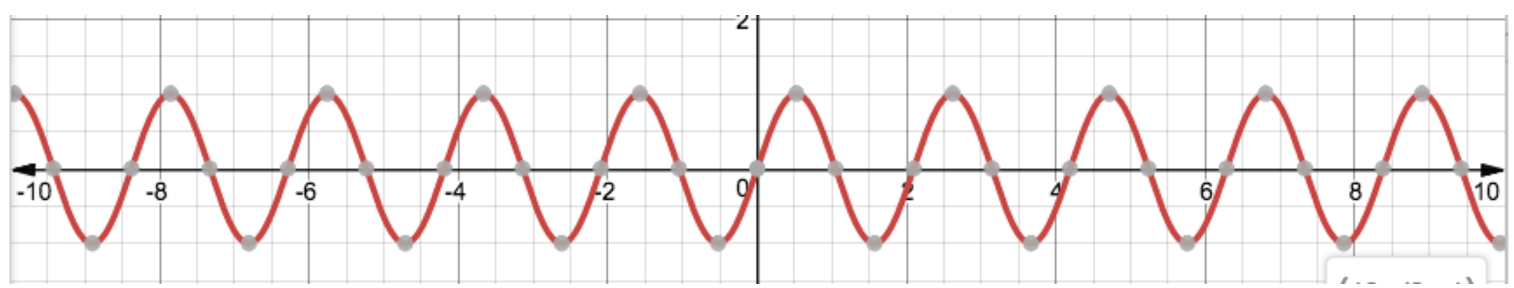

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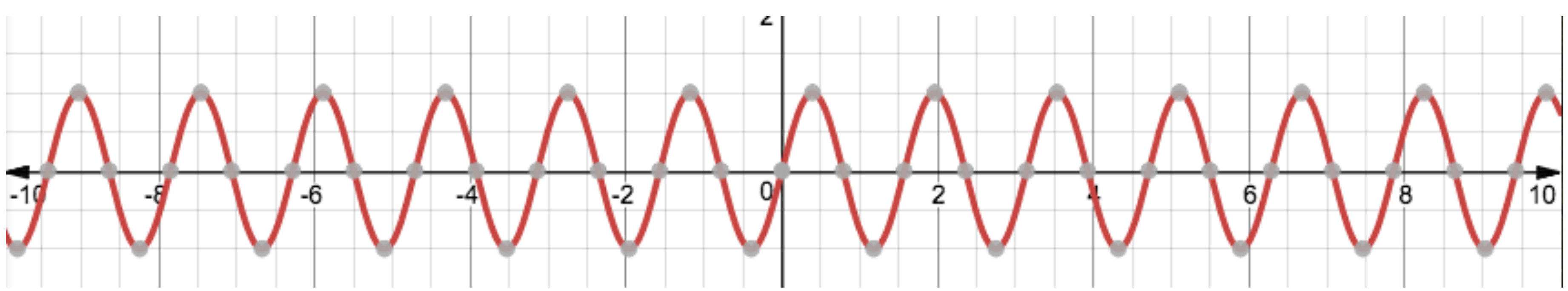

Period $\dfrac{\pi}{4}$

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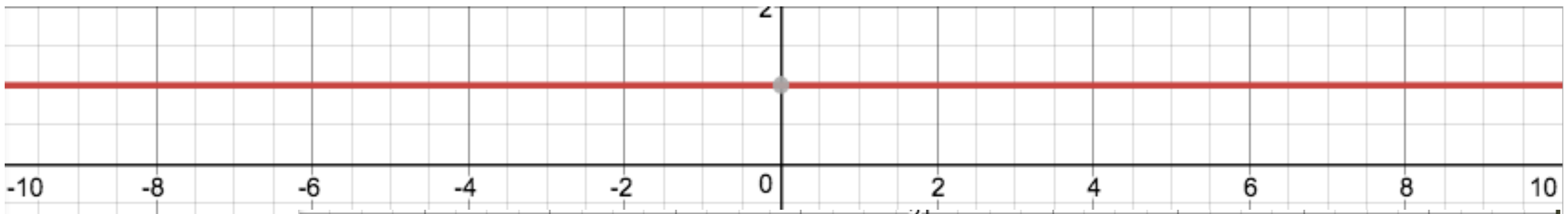
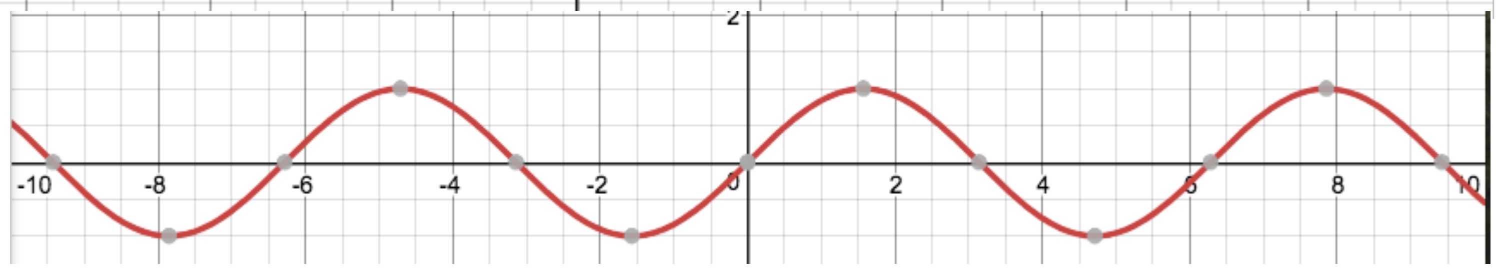


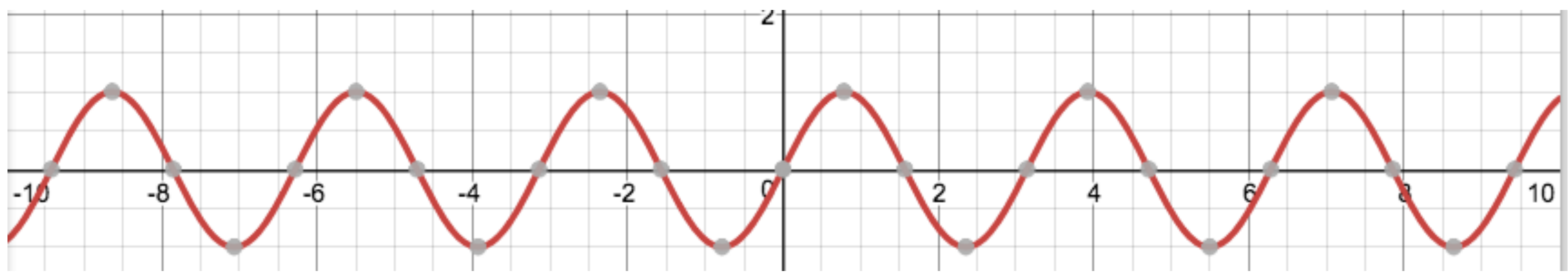
Period 0

$f(t) = sint$



Period 2π

$f(t) = sin2t$

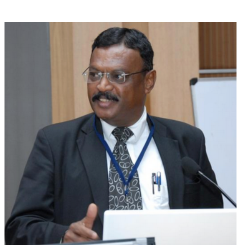


Period π

$f(t) = sin3t$

Period $\dfrac{\pi}{3}$

$f(t) = si$



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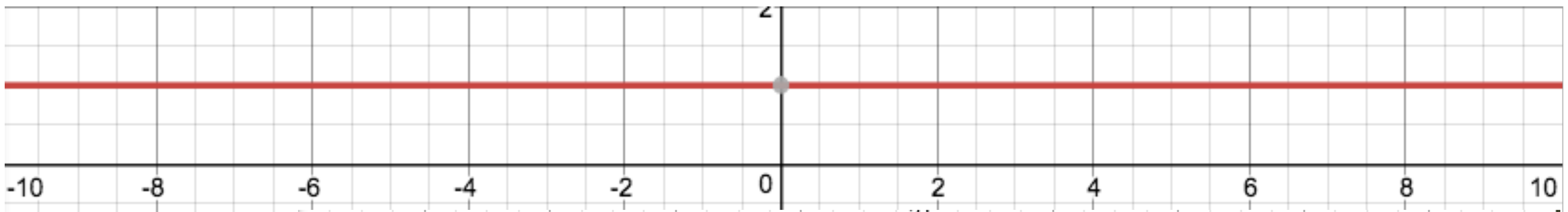

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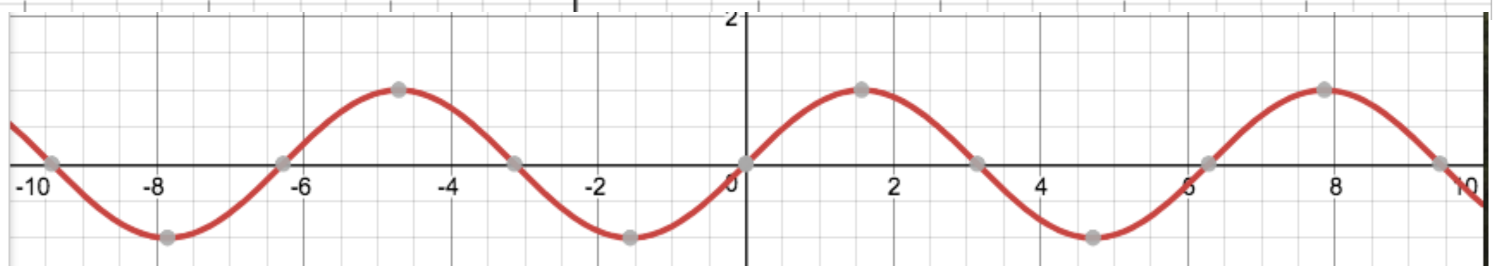

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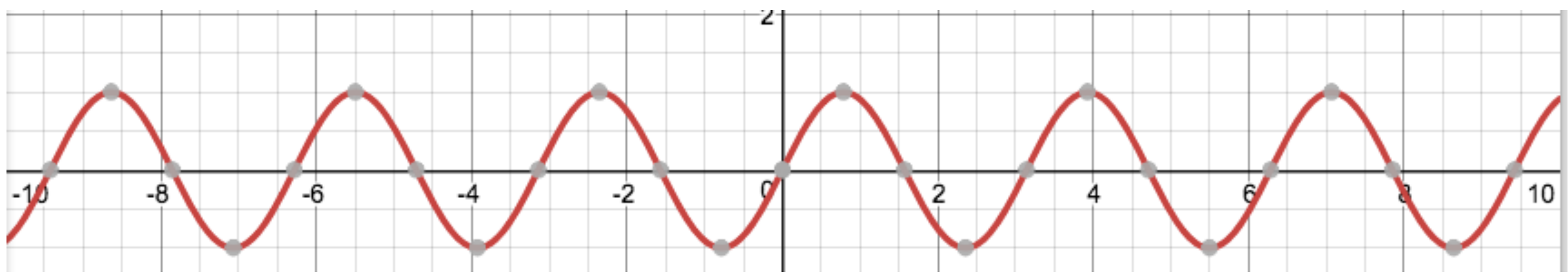

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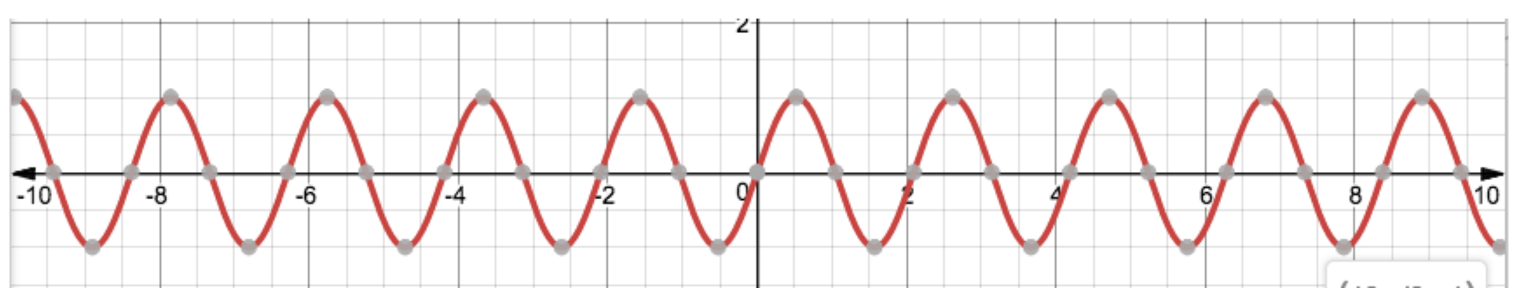
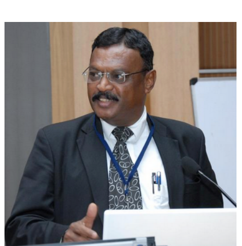


Period $\dfrac{\pi}{3}$

$f(t) = si$

$$f(t) = 1 + 2sint - sin2t + \frac{2}{3}sin3t - \frac{1}{2}sin4t$$



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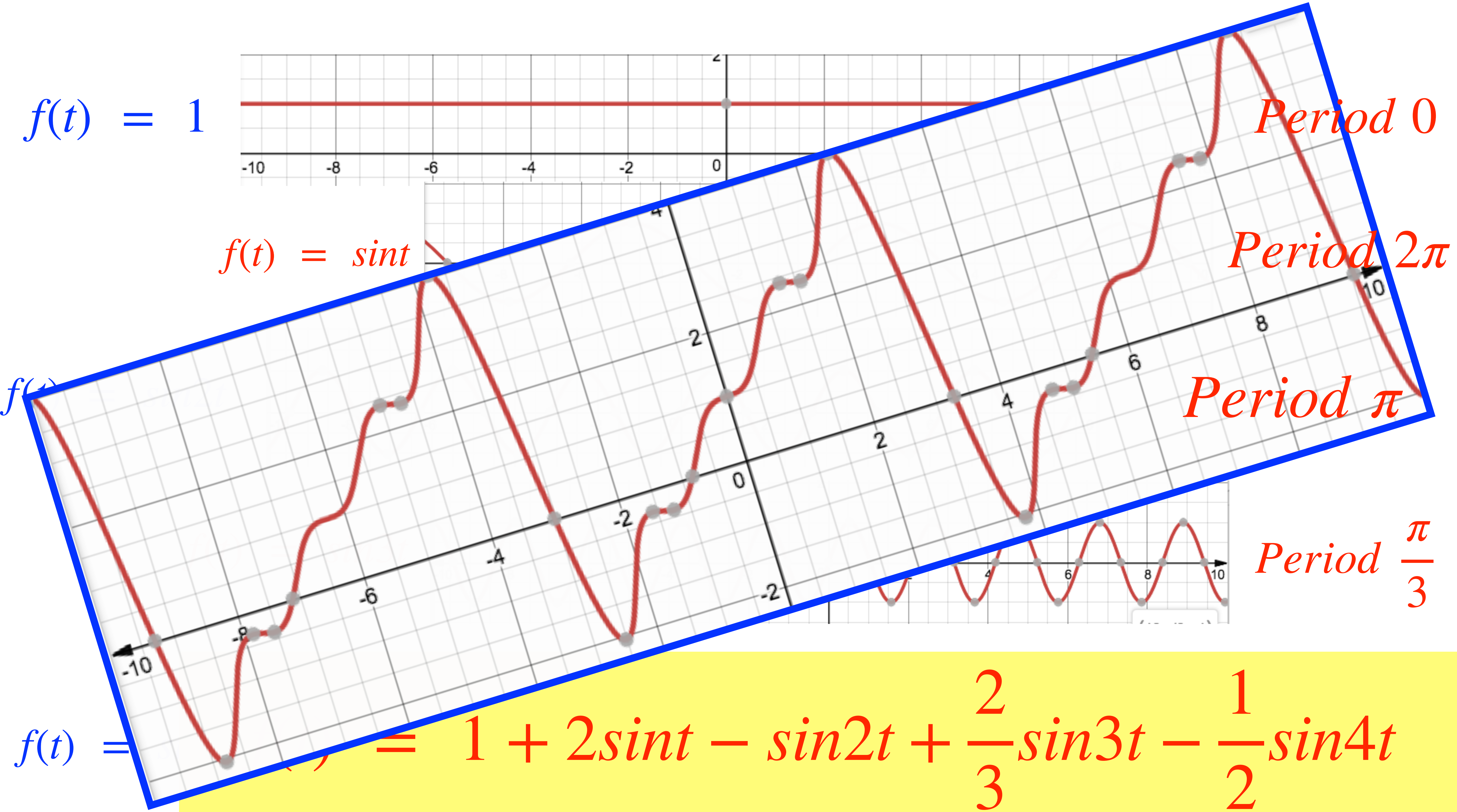

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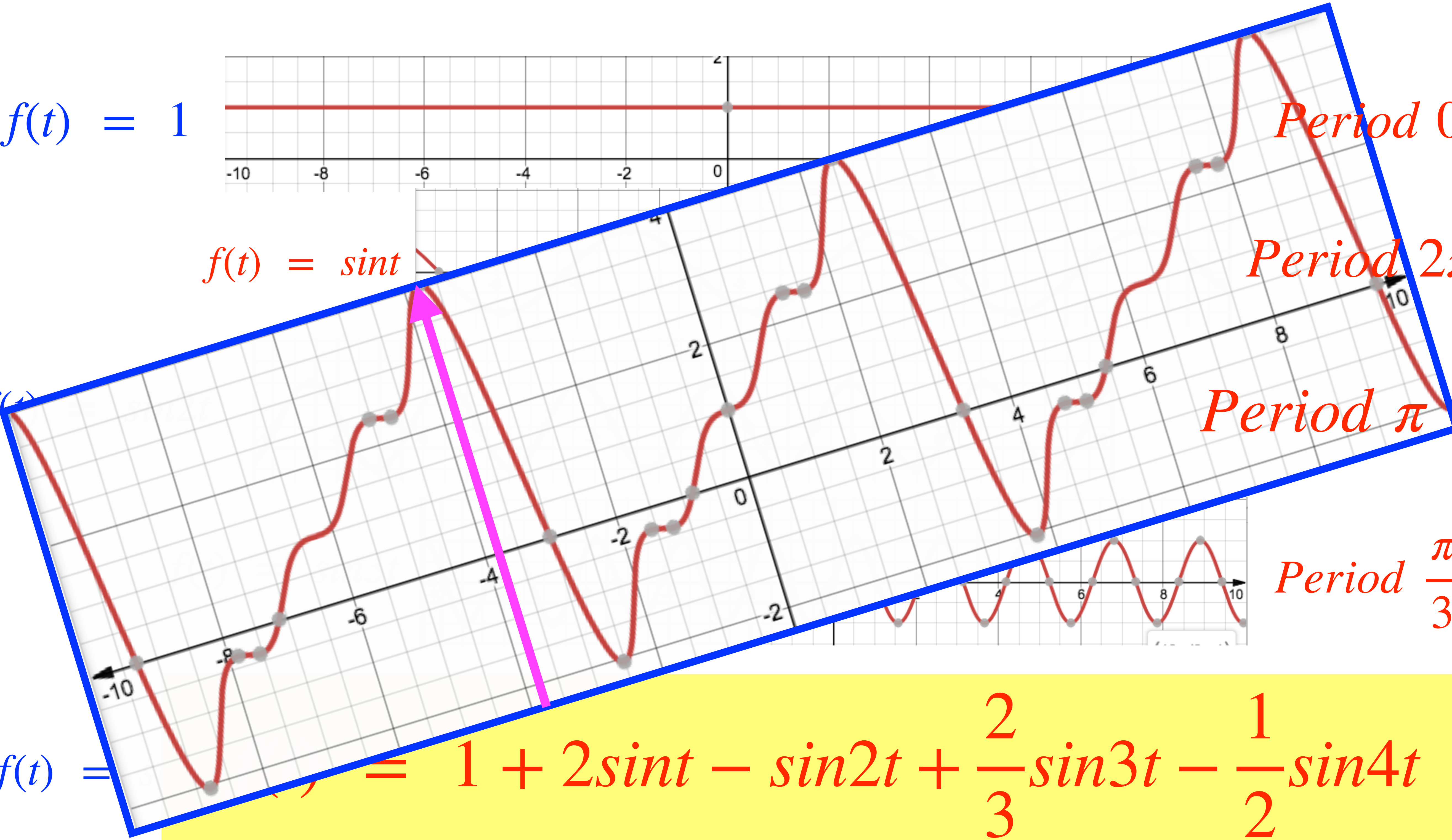

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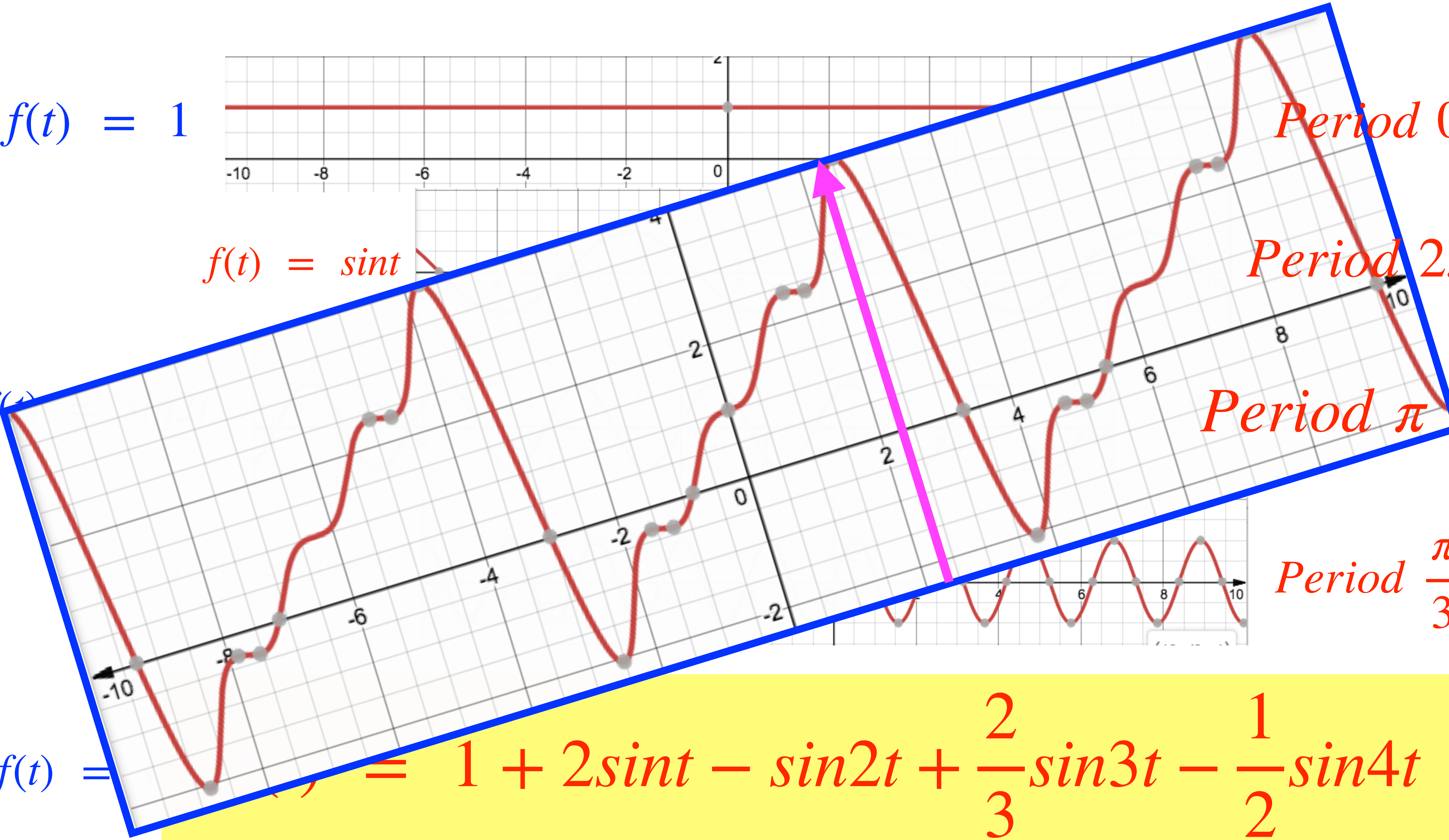

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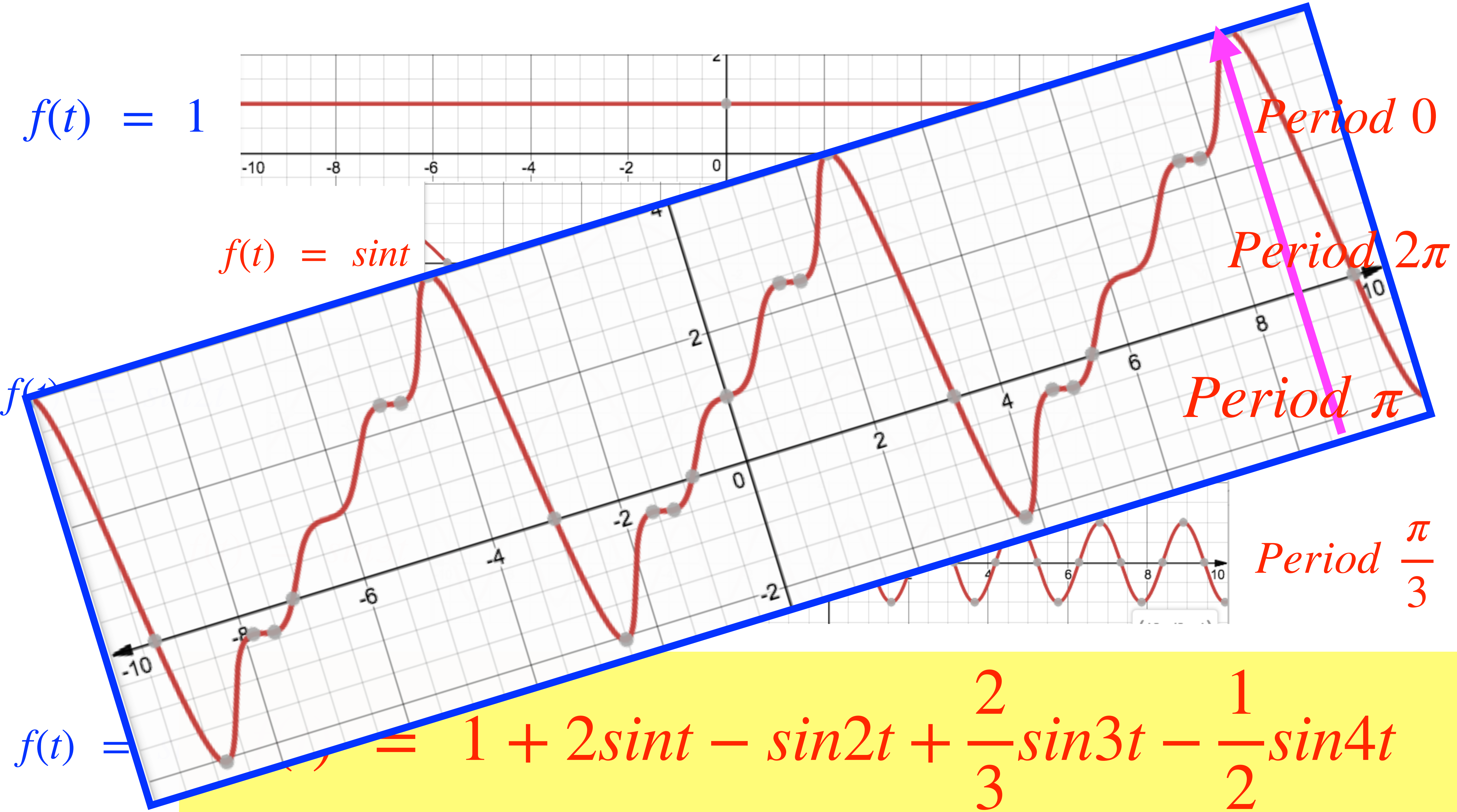

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$$\textit{What is the period of } \sum_{k=1}^{n} \alpha_k f_k(t)?, \textit{ if } \alpha_k \in \Re \textit{ and the period of } f_k(t) \textit{ is } T_k$$

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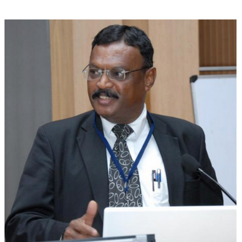

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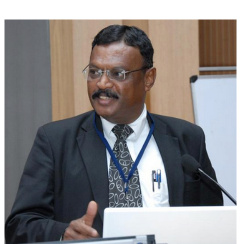
$$(-\pi, \pi) \; or \quad or \; (0,2\pi) \; or \; (\alpha, \alpha + 2\pi)$$



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Fourier series for function with period 2π

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