I §6 Filters No.10. <u>Germs at a point</u>. Let $\mathfrak{F}$ be the nbhd filter of a point a in a top space X. We call germs w.r.t. $\mathfrak{F}$ as <u>germs at a</u>. Note that there is a single germ of nbhds at a. (More generally, there is a single germ w.r.t. a filter $\mathfrak{F}$ of elts of $\mathfrak{F}$.) Germs of sets locally closed at a are the same as germs of closed sets; indeed, if L is locally closed at a, then $\exists$ V nbhd at a s/t $V \cap L$ is closed in V. Thus germs of locally closed sets at a are closed under finite unions & finite intersections.

Since a belongs to all elts of $\mathfrak{F}$, it follows that $f(a)$ makes sense for any map f with domain an elt of $\mathfrak{F}$. In fact $\tilde{f}(a)$ makes sense for $\tilde{f}$ a germ at a. (Note: $\tilde{f}(a) = \tilde{g}(a)$ does not in general mean $\tilde{f} = \tilde{g}$.) $\tilde{f}(a) =:$ value of the germ $\tilde{f}$ at a.

$X \underset{f'}{\overset{f}{\rightrightarrows}} Y \underset{g'}{\overset{g}{\rightrightarrows}} Z$. Suppose $f(a) = b$. Assume f and f' have the same germ at a, & g, g' have the same germ at b. If f is continuous at a (equivalently, f' is continuous at a), then $g \circ f$ and $g' \circ f'$ have the same germ at a.

I §7 Limits <u>No.1 Limit of a filter</u>. Def: X top space, $\mathfrak{F}$ filter, $x \in X$. x is a <u>limit</u> of $\mathfrak{F}$ (or a <u>limit point</u> of $\mathfrak{F}$) if $\mathfrak{F}$ finer than the nbhd filter at x.
Equivalent terminology: $\mathfrak{F}$ <u>converges</u> to x, is <u>convergent</u> to x.
If $\mathcal{B}$ is a base of $\mathfrak{F}$, we also say $\mathcal{B}$ <u>converges to</u> x (if $\mathfrak{F}$ converges to x)

<u>Propn:</u> A filter base $\mathcal{B}$ converges to x iff every set of a FSN at x contains an element of $\mathcal{B}$. In other words, there are sets of $\mathcal{B}$ as near x as we please.

<u>Remarks:</u> • The finer the filter/the coarser the topology, the more the limits. In the discrete top, the only convergent filters are the trivial ultrafilters (which are the nbhd filters)

• The intersection of a family of filters converging to x also converges to x.

<u>Propn:</u> A filter converges to a point iff every ultrafilter finer than the filter converges to that point.

<u>Caution:</u> In general (if X is not Hausdorff) a filter can have several distinct limit points,