Chapter I. Topological Structures Summary

<u>Defn:</u> A collection of open sets is a <u>sub base</u> if every open set is a union of finite intersections of elts from the collection.

The topology may be recovered from ~~the set~~ a subbase. Any collection of subsets may form the sub base of a topology: There is no axiom that needs to be satisfied.

The topology may be recovered from any of the following: a base, a subbase, Nbhds of all points, a fundamental system of nbhds at every point.

Axioms of topology O_I, O_{II}. Axioms for a base: B_I

Axioms for a sub-base: None. Axioms for nbhds: V_I, V_{II}, V_{III}, V_{IV}

Axioms for fundamental systems of nbhds: FSN_{II}, FSN_{III}, FSN_{IV}

A topology may be specified by specifying any of these subject to the axioms.

- Arbitrary union of LF family of closed sets is closed
- $\overset{\circ}{\overline{\complement A}} = \complement \bar{A}$.
- Interior, Frontier, & Exterior (of any set) form a partition of X
- $\overset{\circ}{\overline{A \cap B}} = \mathring{A} \cap \mathring{B}$ $\qquad \overline{A \cup B} = \bar{A} \cup \bar{B}$
- If A is open then $A \cap \bar{B} \subseteq \overline{A \cap B}$.