Remote State Preparation of Arbitrary Two-Qubit State with Unit Success Probability

Ranjana Prakash* and Ajay K. Yadav†

Physics Department, University of Allahabad, Allahabad-211002.

Quantum information processing (QIP) is hard to possible without quantum entanglement [1]. Quantum teleportation (QT) [2] and remote state preparation (RSP) [3-6] are important applications of quantum entanglement. QT is used to transmit an unknown state by a sender to a receiver at distant location. On the other hand, RSP is used to prepare remotely a known state by a sender at the receiver's side. Recently, many RSP schemes are proposed for arbitrary two-qubit state [7-15]. Zha et al. [8] and Wang et al. [9] presented the RSP scheme and the joint RSP (JRSP) scheme of a two-qubit state using a four-qubit cluster state and a six-qubit cluster state, respectively. In both the schemes the probabilities of success can be improved from 1/4 to 1/2, or even to 1 with several special cases. In the present paper we use the scheme proposed by An et al. [6] for the RSP of two-qubit state using four-qubit entangled state and two ancillary qubits with unit success probability without any special cases.

Consider an arbitrary two-qubit state, possessed by Alice, described by

$$|I\rangle = \lambda_0\,|00\rangle + \lambda_1 e^{i\delta_1}\,|01\rangle + \lambda_2 e^{i\delta_2}\,|10\rangle + \lambda_3 e^{i\delta_3}\,|11\rangle, \tag{1}$$

where λ_0, λ_1, λ_2, λ_3 are non-negative real coefficients and $0 \le \delta_1, \delta_2, \delta_3 \le 2\pi$ are the phase angles with the normalization condition $\lambda_0^2+\lambda_1^2+\lambda_2^2+\lambda_3^2=1$. Suppose that given arbitrary two-qubit state is known completely to Alice, but not to Bob. Initially, Alice takes two ancillary qubits $|00\rangle_{12}$, and she shares four-qubit state with Bob as quantum channel given by the expression $|E\rangle_{3456} = (|0000\rangle+|0101\rangle+|1010\rangle+|1111\rangle)_{3456}/2$, where particles (3, 4) are in the possession of Alice and particles (5, 6) belongs to Bob. Now, Alice perform two controlled-NOT gates (CNOT) on the qubits (1, 3) and (2, 4), with 3 and 4 are control qubits and 1 and 2 are targets, respectively. As a result, the four-qubit entangled state $|E\rangle_{3456}$ and the states $|00\rangle_{12}$ become a six-qubit entangled state

$$|E'\rangle_{123456} = \tfrac{1}{2}(|000000\rangle+|010101\rangle+|101010\rangle+|111111\rangle)_{123456} \tag{2}$$

At this stage, Alice measures the particles (1, 2) and (3, 4) in different bases. For the particles (1, 2), the measurement basis is defined as

$$\begin{pmatrix} |\xi_0\rangle_{12} \\ |\xi_1\rangle_{12} \\ |\xi_2\rangle_{12} \\ |\xi_3\rangle_{12} \end{pmatrix} = U \begin{pmatrix} |00\rangle_{12} \\ |01\rangle_{12} \\ |10\rangle_{12} \\ |11\rangle_{12} \end{pmatrix}, \text{with}\, U = \begin{pmatrix} \lambda_0 & \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1 & -\lambda_0 & \lambda_3 & -\lambda_2 \\ \lambda_2 & -\lambda_3 & -\lambda_0 & \lambda_1 \\ \lambda_3 & \lambda_2 & -\lambda_1 & -\lambda_0 \end{pmatrix}, \tag{3}$$

while for the particles (3, 4) measurement basis is

$$\begin{pmatrix} |\zeta_0\rangle_{34} \\ |\zeta_1\rangle_{34} \\ |\zeta_2\rangle_{34} \\ |\zeta_3\rangle_{34} \end{pmatrix} = V \begin{pmatrix} |00\rangle_{34} \\ |01\rangle_{34} \\ |10\rangle_{34} \\ |11\rangle_{34} \end{pmatrix}, \text{with}\, V = \frac{1}{2} \begin{pmatrix} 1 & e^{-i\delta_1} & e^{-i\delta_2} & e^{-i\delta_3} \\ 1 & -e^{-i\delta_1} & e^{-i\delta_2} & -e^{-i\delta_3} \\ 1 & -e^{-i\delta_1} & -e^{-i\delta_2} & e^{-i\delta_3} \\ 1 & e^{-i\delta_1} & -e^{-i\delta_2} & -e^{-i\delta_3} \end{pmatrix}. \tag{4}$$

After the measurements, Alice transmits some classical information about her measurement outcomes to the receiver Bob. Bob then reconstruct the original state on his particles 5 and 6 conditioned on the classical information from Alice.

Under these two sets of bases, i.e., if measurements of the particles (1, 2) in the basis $\{|\xi_0\rangle_{12},|\xi_1\rangle_{12},|\xi_2\rangle_{12},|\xi_3\rangle_{12}\}$ and measurements of the particles (3, 4) in the basis $\{|\zeta_0\rangle_{34},|\zeta_1\rangle_{34},|\zeta_2\rangle_{34},|\zeta_3\rangle_{34}\}$ are carried out independently, the whole quantum system consisting of the six-qubit entangled state (2) can be rewritten as

$$\begin{aligned}|E'\rangle_{123456}=&\frac{1}{4}(\lambda_0|\xi_0\rangle+\lambda_1|\xi_1\rangle+\lambda_2|\xi_2\rangle+\lambda_3|\xi_3\rangle)_{12}(|\zeta_0\rangle+|\zeta_1\rangle+|\zeta_2\rangle+|\zeta_3\rangle)_{34}|00\rangle_{56}\\&+\frac{e^{i\delta_1}}{4}(\lambda_1|\xi_0\rangle-\lambda_0|\xi_1\rangle-\lambda_3|\xi_2\rangle+\lambda_2|\xi_3\rangle)_{12}(|\zeta_0\rangle-|\zeta_1\rangle-|\zeta_2\rangle+|\zeta_3\rangle)_{34}|01\rangle_{56}\\&+\frac{e^{i\delta_2}}{4}(\lambda_2|\xi_0\rangle+\lambda_3|\xi_1\rangle-\lambda_0|\xi_2\rangle-\lambda_1|\xi_3\rangle)_{12}(|\zeta_0\rangle+|\zeta_1\rangle-|\zeta_2\rangle-|\zeta_3\rangle)_{34}|10\rangle_{56}\\&+\frac{e^{i\delta_3}}{4}(\lambda_3|\xi_0\rangle-\lambda_2|\xi_1\rangle+\lambda_1|\xi_2\rangle-\lambda_0|\xi_3\rangle)_{12}(|\zeta_0\rangle-|\zeta_1\rangle+|\zeta_2\rangle-|\zeta_3\rangle)_{34}|11\rangle_{56}.\end{aligned} \quad (5)$$

If the result of Alice's projective measurement of the particles (1, 2) is $|\xi_0\rangle_{12}$, Bob can always prepare the original state with the same probability when any one of the four possible outcomes of Alice's projective measurement of the particles (3, 4) occurs. When Alice's measurement of the particles (1, 2) outcome is $|\xi_1\rangle_{12},|\xi_2\rangle_{12}$ or $|\xi_3\rangle_{12}$ and measurement of the particles (3, 4) outcome is $|\zeta_0\rangle_{34},|\zeta_1\rangle_{34},|\zeta_2\rangle_{34}$ or $|\zeta_3\rangle_{34}$ the remote state preparation cannot be successful. Thus, we can find that Bob can get the original state with the total probability of successful RSP is 25% only.

However, with the strategy of adaptive measurements [6], we can get unit success probability of RSP. 'Adaptive measurements' [6] means sequential measurements that should be performed one by one in such a way that the outcome of a given measurement decides the basis of the next measurement. The first measurement is to be done on the particles (1, 2) in the basis $\{|\xi_0\rangle_{12},|\xi_1\rangle_{12},|\xi_2\rangle_{12},|\xi_3\rangle_{12}\}$, whose outcome is specified by *m* = 0 (1, 2, 3) if $|\xi_0\rangle_{12}(|\xi_1\rangle_{12},|\xi_2\rangle_{12},|\xi_3\rangle_{12})$ is found. Then, depending on the outcome *m*, Alice performs the unitary phase shift operator $\prod^{(m)}$ on the particles (3, 4), which are given by the expressions

$$\prod^{(0)}=I=|00\rangle\langle00|+|01\rangle\langle01|+|10\rangle\langle10|+|11\rangle\langle11|, \quad (6.a)$$

$$\prod^{(1)}=e^{i\delta_1}|00\rangle\langle00|+e^{-i\delta_1}|01\rangle\langle01|+e^{i(\delta_3-\delta_2)}|10\rangle\langle10|+e^{i(\delta_2-\delta_3)}|11\rangle\langle11|, \quad (6.b)$$

$$\prod^{(2)}=e^{i\delta_2}|00\rangle\langle00|+e^{i(\delta_3-\delta_1)}|01\rangle\langle01|+e^{-i\delta_2}|10\rangle\langle10|+e^{i(\delta_1-\delta_3)}|11\rangle\langle11|, \quad (6.c)$$

$$\prod^{(3)}=e^{i\delta_3}|00\rangle\langle00|+e^{i(\delta_2-\delta_1)}|01\rangle\langle01|+e^{i(\delta_1-\delta_2)}|10\rangle\langle10|+e^{-i\delta_3}|11\rangle\langle11|. \quad (6.d)$$

After this, she measure her particles (3, 4) in the basis $\{|\zeta_0\rangle_{34},|\zeta_1\rangle_{34},|\zeta_2\rangle_{34},|\zeta_3\rangle_{34}\}$. Now, Alice broadcasts 4 bits of classical information to Bob for identifying her sixteen possible measurement outcomes in the following way:

'00' ('00', '01', '10' or '11'), '01' ('00', '01', '10' or '11'),
'10' ('00', '01', '10' or '11'), '11' ('00', '01', '10' or '11'),

if she found

$|\xi_0\rangle_{12}\prod^{(0)}(|\zeta_0\rangle_{34},|\zeta_1\rangle_{34},|\zeta_2\rangle_{34}$ or $|\zeta_3\rangle_{34})$, $|\xi_1\rangle_{12}\prod^{(1)}(|\zeta_0\rangle_{34},|\zeta_1\rangle_{34},|\zeta_2\rangle_{34}$ or $|\zeta_3\rangle_{34})$,
$|\xi_2\rangle_{12}\prod^{(2)}(|\zeta_0\rangle_{34},|\zeta_1\rangle_{34},|\zeta_2\rangle_{34}$ or $|\zeta_3\rangle_{34})$, $|\xi_3\rangle_{12}\prod^{(3)}(|\zeta_0\rangle_{34},|\zeta_1\rangle_{34},|\zeta_2\rangle_{34}$ or $|\zeta_3\rangle_{34})$,

respectively. On the basis of Alice's classical information, Bob performs suitable unitary operation on his particles (5, 6) to prepare the required state (1).

For example, let Alice's measurement outcome is $|\xi_2\rangle_{12}$, i.e., *m* = 2, the unmeasured particles 3, 4, 5 and 6 collapse into the (unnormalized) state,

$$|\Psi_2\rangle_{3456} = \frac{1}{4}(\lambda_2 |0000\rangle - \lambda_3 e^{i\delta_1} |0101\rangle - \lambda_0 e^{i\delta_2} |1010\rangle + \lambda_1 e^{i\delta_3} |1111\rangle)_{3456}.$$

In this case, Alice does not measure the particles (3, 4) immediately but applies to it a unitary phase-shift operator, $\prod^{(2)}$ given by Eq. (6.c), thus transferring $|\Psi_2\rangle_{3456}$ to

$$|\Psi_2'\rangle_{3456} = \frac{1}{4}(\lambda_2 e^{i\delta_2} |0000\rangle - \lambda_3 e^{i\delta_3} |0101\rangle - \lambda_0 |1010\rangle + \lambda_1 e^{i\delta_1} |1111\rangle)_{3456}.$$

Only after that operation does Alice measure the particles (3, 4) in the basis $\{|\zeta_0\rangle_{34}, |\zeta_1\rangle_{34}, |\zeta_2\rangle_{34}, |\zeta_3\rangle_{34}\}$. Expressed in terms of $\{|\zeta_0\rangle_{34}, |\zeta_1\rangle_{34}, |\zeta_2\rangle_{34}, |\zeta_3\rangle_{34}\}$, the above state reads

$$\begin{aligned} |\Psi_2'\rangle_{3456} = \frac{1}{4}[(&|\zeta_0\rangle_{34}(-\lambda_0 |10\rangle + \lambda_1 e^{i\delta_1} |11\rangle + \lambda_2 e^{i\delta_2} |00\rangle - \lambda_3 e^{i\delta_3} |01\rangle)_{56} \\ &+ |\zeta_1\rangle_{34}(-\lambda_0 |10\rangle - \lambda_1 e^{i\delta_1} |11\rangle + \lambda_2 e^{i\delta_2} |00\rangle + \lambda_3 e^{i\delta_3} |01\rangle)_{56} \\ &+ |\zeta_2\rangle_{34}(\lambda_0 |10\rangle + \lambda_1 e^{i\delta_1} |11\rangle + \lambda_2 e^{i\delta_2} |00\rangle + \lambda_3 e^{i\delta_3} |01\rangle)_{56} \\ &+ |\zeta_3\rangle_{34}(\lambda_0 |10\rangle - \lambda_1 e^{i\delta_1} |11\rangle + \lambda_2 e^{i\delta_2} |00\rangle - \lambda_3 e^{i\delta_3} |01\rangle)_{56})]. \end{aligned}$$

Corresponding to Alice's measurement outcome $|\zeta_n\rangle_{34}$, $n \in (0, 1, 2, 3)$, recovery operators are $\sigma_{x,5}\sigma_{z,5} \otimes \sigma_{z,6}$, $\sigma_{x,5}\sigma_{z,5} \otimes I_6$, $\sigma_{x,5} \otimes I_6$ and $\sigma_{x,5} \otimes \sigma_{z,6}$, respectively. Results are summarized in the following table.

In conclusion, we prepare an arbitrary two-qubit state remotely via four-qubit entangled state with unit success probability. For the improvement of the success probability two ancillary qubits and the adaptive measurement techniques are used. Since after performing two CNOT gates the combined state of particles 1, 2, 3, 4, 5 and 6 becomes six-qubit entangled state (2). So, one can say that our present scheme is the RSP of an arbitrary two-qubit state via six-qubit entangled state with unit success probability.

Table 1. Remote state preparation of an arbitrary two-qubit state

	FMB(1, 2) $\|\xi_0\rangle$ & UPSO(3, 4) $\prod^{(0)}$		FMB(1, 2) $\|\xi_2\rangle$ & UPSO(3, 4) $\prod^{(2)}$	
SMB(3, 4)	CI	UT(5, 6)	CI	UT(5, 6)
$\|\zeta_0\rangle$	0000	$I \otimes I$	1000	$\sigma_x\sigma_z \otimes \sigma_z$
$\|\zeta_1\rangle$	0001	$I \otimes \sigma_z$	1001	$\sigma_x\sigma_z \otimes I$
$\|\zeta_2\rangle$	0010	$\sigma_Z \otimes \sigma_z$	1010	$\sigma_x \otimes I$
$\|\zeta_3\rangle$	0011	$\sigma_Z \otimes I$	1011	$\sigma_x \otimes \sigma_z$
	FMB(1, 2) $\|\xi_1\rangle$ & UPSO(3, 4) $\prod^{(1)}$		FMB(1, 2) $\|\xi_3\rangle$ & UPSO(3, 4) $\prod^{(3)}$	
SMB(3, 4)	CI	UT(5, 6)	CI	UT(5, 6)
$\|\zeta_0\rangle$	0100	$I \otimes \sigma_x\sigma_z$	1100	$\sigma_x\sigma_z \otimes \sigma_x$
$\|\zeta_1\rangle$	0101	$I \otimes \sigma_x$	1101	$\sigma_x\sigma_z \otimes \sigma_x\sigma_z$
$\|\zeta_2\rangle$	0110	$\sigma_z \otimes \sigma_x$	1110	$\sigma_x \otimes \sigma_x\sigma_z$
$\|\zeta_3\rangle$	0111	$\sigma_z \otimes \sigma_x\sigma_z$	1111	$\sigma_x \otimes \sigma_x$

FMB: Alice's first measurement basis; UPSO: unitary phase-shift operator; SMB: Alice's second measurement basis; CI: classical information announces by Alice and UT: unitary transformation perform by Bob.

References:

[1] M.A. Nielsen and I.L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press) 2000.

[2] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres and W.K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).

[3] A.K. Pati, Phys. Rev. A **63**, 014302 (2000).

[4] H.K. Lo, Phys. Rev. A **62**, 012313 (2000).

[5] C.H. Bennett, D.P. DiVincenzo, P.W. Shor, J.A. Smolin, B.M. Trehal and W.K. Wootters, Phys. Rev. Lett. **87**, 077902 (2001).

[6] N.B. An, T.B. Cao, V.D. Nung and J. Kim, Adv. Nat. Sci.: Nanosci. Nanotechnol. **2**, 035009 (2010).

[7] J.M. Liu, X.L. Feng and C.H. Oh, Eur. Phys. Lett. **87**, 30006 (2009).

[8] X.W. Zha and H.Y. Song, Opt. Commun. **284**, 1472 (2011).

[9] D. Wang, X.W. Zha and Q. Lan, Opt. Commun. **284**, 5853 (2011).

[10] N.B. An, J. Phys. B: At. Mol. Opt. Phys. **42**, 125501 (2009).

[11] N.B. An, Opt. Commun. **283**, 4113 (2010).

[12] H.H. Liu, L.Y. Cheng, X.Q Shao, L.L. Sun, S. Zhang and K.H. Yeon, Int. J. Theor. Phys. **50**, 3023 (2011).

[13] X.Q. Xiao, J.M. Liu and G.H. Zeng, J. Phys. B **44**, 075501 (2011).

[14] Z.Y. Wang, Commun. Theor. Phys. **55**, 244 (2011).

[15] Z.Y. Wang, Quant. Inform. Process. **11**, 1585 (2012).