NON-COMMUTATIVE GEOMETRY ON A TWO-SHEETED SPACE-TIME MATTER FIELDS

(IN COLLABORATION NGUYEN AI VIET)

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- INTRODUCTORY REMARKS
- BASIC ELEMENTS OF CONNES' NCG
- GEOMETRY ON A TWO-SHEETED SPACE-TIME
- GRAVITY
- MATTER FIELDS
- SOME RESULTS; CONCLUSIONS

INTRODUCTORY REMARKS (CONTD)

CONNES, MADORE ET AL ....

• BASIC IDEA

Bypass THE SPECIFICATION OF THE MANIFOLD. GENERALIZATION OF GELFAND'S THEOREM ....

(CLASSICAL TOPOLOGICAL SPACE BASED ON A CONTINUUM CAN BE COMPLETELY RECOVERED BY THE ABELIAN ALGEBRA OF SMOOTH FUNCTIONS)

NATURAL GENERALIZATION — AS STARTING POINT —

NONCOMMUTATIVE, BUT ASSOCIATIVE INVOLUTIVE ALGEBRAS ..

STANDARD OBJECTS OF THE CONVENTIONAL DIFFERENTIAL GEOMETRY IN A PURELY ALGEBRAIC WAY, SETTING UP THE ~~B~~OR A NCG. HOPEFULLY, FROM SUCH GENERAL ALGEBRAS, ONE CAN RECONSTRUCT THE MANIFOLD

(INCIDENCE ALGEBRAS → POSETS (REC. OF MANIFOLD)

MOTIVATION FOR THE PRESENT WORK: SUCCESSES (?) OF STANDARD MODEL IN THIS NEW FRAMEWORK

INTRODUCTORY REMARKS (CONTD)

MAIN MOTIVATION:- STANDARD MODEL IN CURVED SPACE-TIME - A 'PHENOMENOLOGICAL' APPROACH TO INCLUDE GRAVITY. CONTINUATION OF WORK STARTED SOME YEARS AGO WITH LANDI & VIET. TO INCLUDE MATTER FIELD

WHY TWO-SHEETED SPACE-TIME ? CHIRALITY AS FUNDAMENTAL, WE OBSERVE THAT IN SM, LEFT-HANDED FIELDS FORM SU(2) DOUBLETS, RIGHT-HAND ARE SU(2) SINGLETS.

CAN ALSO BE REGARDED AS A DISCRETIZED KALUZA-KLEIN THEORY (INT. CIRCLE REPLACED BY TWO DISCRETE POINTS)

SIMPLIFIED MODEL

NON-COMMUTATIVE GEOMETRY (NCG)

BASICS

LOCAL CONSTRUCTIONS AT A POINT ARE REFORMULATED IN ALGEBRAIC TERMS:

$\mathcal{M} \to \mathcal{A}$: ASSOCIATIVE, INVOLUTIVE ALGEBRA (COMMUTATIVE OR NONCOMMUTATIVE)

ELEMENTS OF THE ALGEBRA $\mathcal{A}$ ARE REPRESENTED AS OPERATORS ON A HILBERT SPACE .

A SYMBOL, δ , $\delta(1)=0$, $\delta(ab)=(\delta a)b+a(\delta b)$,

$\forall a,b \in$, GENERATES A UNIVERSAL DIFFERENTIAL ALGEBRA $\Omega^*(\mathcal{A})$.

$\Omega^0(A)$: ZERO FORMS; $\Omega^1(\mathcal{A})$: ONE-FORM

$$\omega=\sum_i(\delta a_i)\, b_i \; .$$

HIGHER FORMS GENERATED BY REPEATED OF ONE- FORMS AND USING ASSOCIATIVITY, GENERATE A GRADED UNIVERSAL ALGEBRA OF FORMS.

VECTOR SPACES	$\rightarrow$	A- MODULES
(coefficients, real or complex)		(Coefficients belong to the algebra)

TREATING, δ , AS A LINEAR OPERATOR THAT TRANSFORMS

$$\Omega^{P} \rightarrow \Omega^{P+1,}$$

$$\delta(\delta a_{1 \ldots}\delta a_{P}\, b) = \delta a_{1 \ldots}\delta a_{P}\delta b,$$

WE CAN CONVERT THE GRADED ALGEBRA OF FORMS INTO A DIFFERENTIAL ALGEBRA, Ω .

IN PHYSICAL APPLICATIONS, ONE CONSTRUCTS REPRESENTATIONS OF THE DIFFERENTIAL ALGEBRA, $\Omega^{*}(A)$ ON A HILBERT SPACE H,

$$\pi: \quad \Omega^{*}(A) \rightarrow L(H),$$

WHERE L(H) DENOTES THE SPACE OF BOUNDED OPERATORS ON H.

THE LINEAR OPERATOR , δ , IS REPRESENTED BY A SELF-ADJOINT OPERATOR CALLED THE DIRAC OPERATOR, D , ON THE HILBERT SPACE SUCH THAT THE COMMUTATOR [D, a] IS A BOUNDED OPERATOR,$\forall a \in A$,

$$\pi\,(\delta a_{1}\ldots\delta a_{P}\, b) = \Pi\, [D, \pi(a_{i})]\, \pi(b)$$

A, H, AND D FORM A SPECTRAL TRIPLE.

CANONICAL SPECTRAL TRIPLE FOR A PSEUDO-RIEMANNIAN SPIN MANIFOLD:

$$A = C^{\infty}(M, R),$$

$$H = L^2 (M, S),$$

THE SPACE OF SQUARE INTEGRABLE SECTIONS OF A SPIN BUNDLE, AND

$$D = \gamma^{\mu} \nabla_{\mu}$$

TWO SHEETED SPACE-TIME:

MOTIVATION

STUDY STANDARD MODEL IN CURVED SPACE-TIME TO INCLUDE GRAVITY. IT PROVIDES A SIMPLE, STRAIGTFORWARD, BUT A NON-TRIVIAL EXTENSION OF R-GEOMETRY WITHIN THE FRAMEWORK OF NCG. IT MAY BE VIEWED AS KALUZA-KLEIN THEORY WITH INTERNAL SPACE OF TWO DISCRETE POINTS IN THE FIFTH DIMENSION.

SPECTRAL TRIPLE

$\mathcal{A}$: $C^\infty(M) \otimes (C \oplus C) = C^\infty(M, C) \oplus C^\infty(M, C)$

$\mathcal{H}$: $L^2(S, M) \oplus L^2(S, M)$

Left & Right square integrable sections of a spinor bundle

D : $\Gamma^M D_M$, $M = 0, 1, 2, 3, 5$

SELF-ADJOINT OPERATOR,

$$\Gamma^\mu = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix} , \quad \Gamma^5 = \begin{pmatrix} 0 & \gamma^5 \\ \gamma^5 & 0 \end{pmatrix}$$

$$D_\mu = \begin{pmatrix} \partial_\mu & 0 \\ 0 & \partial_\mu \end{pmatrix} , \quad D_5 = \sigma \bar{D}_5 ,$$

WHERE

$$\sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \bar{D}_5 = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix}$$

m IS A PARAMETER OF THE DIMENSION OF MASS

DIFFERENTIAL GEOMETRY

0-FORMS:

$$F = f_+ 1 + f_- \underline{r},$$

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \underline{r} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

ACTION OF D ON F

$$DF = [D, F] = 1\text{-FORM} = \Gamma^M (D_M F)$$

GENERAL 1 & 2 FORMS

$$\Gamma^M u_M, \quad \Gamma^M \wedge \Gamma^N P_{MN} \text{ ETC},$$

WHERE u_M & P_{MN} ARE ZERO-FORMS

CARTAN STRUCTURE EQUATIONS

$$T^A = D\Gamma^A - \Gamma^B \wedge \Omega_B^A,$$

$$R^A_B = D\Omega^A_B + \Omega^A_C \wedge \Omega^C_B;$$

Ω^A_B : CONNECTION 1-FORMS

R-GEOMETRY ON A TWO-SHEETED SPACE-TIME

EQUIVALENCE PRINCIPLE EXTENDED TO SUCH A SPACE-TIME REQUIRES A LOCAL ORTHONORMAL BASIS AND GENERALIZED VIELBEINS E^{A}_{M}.

$$\Gamma^{a} = \begin{pmatrix} \gamma^{a} & 0 \\ 0 & \gamma^{a} \end{pmatrix} \; ; \quad \Gamma^{\dot{5}} = \begin{pmatrix} 0 & \gamma^{5} \\ \gamma^{5} & 0 \end{pmatrix}$$

γ^{a}, γ^{5} ARE THE USUAL FLAT DIRAC MATRICES

$$E^{\mu}_{a} = \begin{pmatrix} e^{\mu}_{1a} & 0 \\ 0 & e^{\mu}_{2a} \end{pmatrix} , \quad E^{\mu}_{\dot{5}} = 0$$

$$E^{5}_{a} = - \begin{pmatrix} a_{1a} & 0 \\ 0 & a_{2a} \end{pmatrix} = - A_{a} = - E^{\mu}_{a} A_{\mu}$$

$$E^{5}_{\dot{5}} = \begin{pmatrix} \phi_{1}^{-1} & 0 \\ 0 & \phi_{2}^{-1} \end{pmatrix} = \Phi^{-1}$$

$e^{\mu}_{1a}, e^{\mu}_{2a}$: Vierbeins on the two sheets of S-T

$a_{1,2}$; $\phi_{1,2}$: Vector & Scalar fields

VIELBEINS:

WE ASSSUME A SET OF 0-FORMS, $E^{A}{}_{M}$, AND, $E^{M}{}_{A}$, THAT ARE INVERSES OF EACH OTHER AND RELATE THE TWO FRAMES—LOCALLY FLAT AND CURVED,

$$\Gamma^{A} = \Gamma^{M} E_{M}{}^{A} \quad ; \quad \Gamma^{M} = \Gamma^{A} E_{A}{}^{M}$$

$$E^{A}{}_{M} E^{M}{}_{B} = \delta^{A}{}_{B} \quad ; \quad E^{M}{}_{A} E^{A}{}_{N} = \delta^{M}{}_{N}$$

WITHOUT LOSS OF GENERALITY, WE CAN CHOOSE THEM AS FOLLOWS:

$$E^{\mu}{}_{a} = e^{\mu}{}_{a} I + v^{\mu}{}_{a} r \;\; ; \; E^{a}{}_{\mu} = (e^{a}{}_{\mu} - e^{a}{}_{\nu} z^{\nu}{}_{\rho} v^{\rho}{}_{\mu}) I + e^{a}{}_{\nu} z^{\nu}{}_{\mu} r \; ;$$
$$E^{5}{}_{a} = (a_{\mu+} e^{\mu}{}_{a} + a_{\mu-} v^{\mu}{}_{a}) I + (a_{\mu+} v^{\mu}{}_{a} + a_{\mu-} e^{\mu}{}_{a}) r \; ;$$
$$E^{5}{}_{\mu} = (a_{\mu+} \varphi_{+} + a_{\mu-} \varphi_{-}) I + (a_{\mu+} \varphi_{-} + a_{\mu-} \varphi_{+}) r \; ;$$
$$E^{5}{}_{\xi} = (\varphi_{+} I - \varphi_{-} r) / (\varphi_{+}{}^{2} - \varphi_{-}{}^{2}) \; ;$$
$$E^{5}{}_{5} = \varphi_{+} I + \varphi_{-} r ;$$

THESE VIERBEINS OR METRIC FIELD COMPONENTS CONTAIN THE CUSTOMARY VIERBEINS, $e^{\mu}{}_{a}$, AND, $e^{a}{}_{\mu}$; THE FIELDS, $v^{\mu}{}_{\nu}$ AND $z^{\mu}{}_{\nu}$ TOGETHER WILL RESULT IN MASSIVE TENSOR FIELDS. THEY ARE RELATED BY THE NON-LINEAR EQUATION,

$$v^{\mu}{}_{\nu} + z_{\nu}{}^{\sigma} (\delta_{\sigma}{}^{\mu} - v_{\sigma}{}^{\lambda} v_{\lambda}{}^{\mu}) .$$

$a_{\mu+}$, $a_{\mu-}$ AND φ_{+}, φ_{-} ARE A PAIR OF VECTOR AND SCALAR FIELDS.

WE CALCULATE THE METRICS ,

$$G^{MN} = \begin{bmatrix} G^{\mu\nu} & G^{\mu 5} \\ G^{5\mu} & G^{55} \end{bmatrix} \quad G_{MN} = \begin{bmatrix} G_{\mu\nu} & G_{\mu 5} \\ G_{5\mu} & G_{55} \end{bmatrix}$$

IN TERMS OF THE ABOVE COMPONENT FIELDS. THUS WE HAVE IN GENERAL SIX INDEPENDENT FIELDS, A PAIR OF TENSORS, A PAIR OF VECTORS AND A PAIR OF SCALARS.

~~[illegible]~~, IN R-GEOMETRY, METRIC COMPATIBILITY AND VANISHIN TORSION DETERMINES THE CONNECTION COEFFICIENTS IN TERMS OF THE METRICFIELDS OR VIERBEINS. WE WOULD LIKE TO EXTEND THIS TO THE TWO-SHEETED GEOMETRY.

GRAVITY SECTOR
(CONNECTION ONE-FORMS $\Omega^A{}_B$; TORSION T^A)

R-GEOMETRY : $T^A = 0$
TOO RESTRICTIVE FOR OUR PURPOSES

A SET OF MINIMAL CONDITIONS:

- SPACE-TIME TORSION FREE CONDITION

$$T_{aBC} = 0$$

- METRIC COMPATIBILITY CONDITION

$$\Omega_{AB} = - \Omega_{BA}$$

- A SUPPLEMENTARY CONDITION

$$\Omega_{AB\dot{5}} = 0$$

THESE CONDITIONS ARE SUFFICIENT TO EXPRESS THE NON-VANISHING CONNECTION COEFFICIENTS AND TORSION COMPONENTS IN TERMS OF METRIC FIELDS.
ALSO, IMPORTANTLY, PROVIDE LAGRANGIANS WITH PROPER KINETIC TERMS FOR ALL THE FIELDS

NON-VANISHING $\Omega_B{}^A$, T^A

Ω_{abc} , $\Omega_{a\dot{5}b}$, $\Omega_{\dot{5}ab} = -\Omega_{a\dot{5}b}$

EXPRESSED IN TERMS OF METRIC FIELDS, THEY ARE GIVEN BY

$$\Omega_{abc} = -\frac{1}{2}\Big(\frac{1}{2} E^\mu_b E^\nu_c\big[(D_\mu E_{\nu a} - D_\nu E_{\mu a}) - (A_\mu D_5 E_{\nu a} - A_\nu D_5 E_{\mu a})\big] - (a \leftrightarrow b) + (a \leftrightarrow c)\Big)$$

$$\Omega_{a\dot{5}b} = -\frac{1}{2}(D_5 E_{\mu a}) E^5_{\dot{5}} (\tilde{E}^\mu_b + E^\mu_b) = -\Omega_{\dot{5}ab}$$

NON-VANISHING T^A

$T_{\dot{5}bc}$, $T_{\dot{5}b\dot{5}}$, $T_{\dot{5}\dot{5}\dot{5}}$

$$T_{\dot{5}bc} = \frac{1}{2} E^\mu_b E^\nu_c \big[(D_\mu A_\nu - D_\nu A_\mu)\Phi + (A_\nu D_5 (A_\mu \Phi) - A_\mu D_5 (A_\nu \Phi))\big]$$

$$T_{\dot{5}b\dot{5}} = -\frac{1}{2} E^5_{\dot{5}} \Big[\frac{1}{2}(\tilde{E}^\mu_b + E^\mu_b)(D_\mu E^{\dot{5}}_5 - D_5 E^{\dot{5}}_\mu) + m(\tilde{E}^5_b - E^5_b)(E^{\dot{5}}_5 + \tilde{E}^{\dot{5}}_5)\Big]$$

$$T_{\dot{5}\dot{5}\dot{5}} = m \tilde{E}^5_{\dot{5}} E^5_{\dot{5}} (E^{\dot{5}}_5 + \tilde{E}^{\dot{5}}_5)$$

WITH THE CONNECTIONS KNOWN, WE CAN CALCULATE THE RIEMANN TENSOR AND THE SCALAR CURVATURE.

THEN, THE EINSTEIN-HILBERT-CARTAN LAGRANGIAN:

$$L_{EHC} = \mathrm{Tr}\,(< R_{AB}, \Gamma^{A} \wedge \Gamma^{B} >) + \mathrm{Tr} < T^{A}, T_{A} >$$

COMMENT :

THE FIRST TERM INCLUDES THE USUAL RIEMANN SCALAR CURVATURE LEADING TO THE EINSTEIN-HILBERT ACTION. IT ALSO INCLUDES SEVERAL INTERACTION TERMS BETWEEN THE FIELDS, BUT LACKS THE PROPER KINETIC TERMS FOR THEM. THUS THEIR PHYSICAL INTERPRETATION REMAINS UN CLEAR.

THE SECOND TERM ARISING FROM THE NON-VANISHING COMPONENTS OF TORSION PROVIDES THE REQUIRED KINETIC TERMS FOR REMAINING FIELDS. THUS WE HAVE A FULL AND SATISFACTORY ACTION KEEPING ALL THE FIELDS ON AN EQUAL BASIS.

$$\mathcal{L}_{R1} = (16\pi G_N)^{-1}(D\Omega_{ab})_{cd+}\eta^{ad}\eta^{bc}$$

$$\begin{aligned}
\mathcal{L}_{R1} \;=\; & (64\pi G_N)^{-1}(e_{d\tau} + s_{d\lambda}v^{\lambda}_{\tau})[(2X^{ab[\rho,\tau]}\eta^{cd} - X^{bc[\rho,\tau]}\eta^{ad})\partial_\rho(X^{[\mu,\nu]}_{bc}P_{a\mu\nu} - Y^{[\mu,\nu]}Q_{a\mu\nu}) \\
& -(2Y^{ab[\rho,\tau]}\eta^{cd} - Y^{bc[\rho,\tau]}\eta^{ad})\partial_\rho(X^{[\mu,\nu]}_{bc}Q_{a\mu\nu} + Y^{[\mu,\nu]}_{bc}P_{a\mu\nu})] \\
& +(64\pi G_N)^{-1}(X^{bc[\rho,\tau]}X^{[\mu,\nu]}_{bc}\eta^{ad} + 2X^{ab[\rho,\tau]}X^{d\;[\mu,\nu]}_{\;b})(-P_{a\mu\nu}\partial_\rho(e_{d\tau} + s_{d\lambda}v^{\lambda}_{\tau}) \\
& +Q_{a\mu\nu}\partial_\rho s_{d\tau}) + (64\pi G_N)^{-1}(X^{bc[\rho,\tau]}Y^{[\mu,\nu]}_{bc}\eta^{ad} + 2X^{ab[\rho,\tau]}Y^{d\;[\mu,\nu]}_{\;b} \\
& +Y^{bc[\rho,\tau]}X^{[\mu,\nu]}_{bc}\eta^{ad} + 2Y^{ab[\rho,\tau]}X^{d\;[\mu,\nu]}_{\;b})(P_{a\mu\nu}\partial_\rho s_{d\tau} + Q_{a\mu\nu}\partial_\rho(e_{d\tau} + s_{d\lambda}v^{\lambda}_{\tau})) \\
& (64\pi G_N)^{-1}(Y^{bc[\rho,\tau]}Y^{[\mu,\nu]}_{bc}\eta^{ad} + 2Y^{ab[\rho,\tau]}Y^{d\;[\mu,\nu]}_{\;b}) \\
& (P_{a\mu\nu}\partial_\rho(e_{d\tau} + s_{d\lambda}v^{\lambda}_{\tau}) + Q_{a\mu\nu}\partial_\rho s_{d\tau}) \\
& +m(32\pi G_N)^{-1}a_{\tau-}[((X^{bc[\rho,\tau]}X^{[\mu,\nu]}_{bc} - Y^{bc[\rho,\tau]}Y^{[\mu,\nu]}_{bc})\eta^{ad} - 2(X^{ab[\rho,\tau]}X^{d\;[\mu,\nu]}_{\;b} \\
& -Y^{ab[\rho,\tau]}X^{d\;[\mu,\nu]}_{-b}))(P_{a\mu\nu}s_{d\rho} + Q_{a\mu\nu}(e_{d\rho} + s_{d\lambda}v^{\lambda}_{\rho})) \\
& +((Y^{bc[\rho,\tau]}X^{[\mu,\nu]}_{bc} - X^{bc[\rho,\tau]}Y^{[\mu,\nu]}_{bc})\eta^{ad} + 2(Y^{ab[\rho,\tau]}X^{d\;[\mu,\nu]}_{\;b} \\
& +X^{ab[\rho,\tau]}X^{d\;[\mu,\nu]}_{\;b}))(P_{a\mu\nu}(e_{d\rho} + s_{d\lambda}v^{\lambda}_{\rho}) + Q_{a\mu\nu}s_{d\rho})].
\end{aligned} \tag{A.3.5}$$

$$\begin{aligned}
\mathcal{L}_{R2} \;=\; & (16\pi G_N)^{-1}((D\Gamma_a)_{ec+}(D\Gamma_b)_{fd+} + (D\Gamma_a)_{ec-}(D\Gamma_b)_{fd-}) \\
& (\eta^{ab}\eta^{cd}\eta^{ef} + 3\eta^{ad}\eta^{be}\eta^{cf} + \eta^{ac}\eta^{bd}\eta^{ef}) \\
=\; & (64\pi G_N)^{-1}(\eta^{ab}X^{cd[\mu,\nu]}X^{[\rho,\tau]}_{cd} - 3X^{bc[\mu,\nu]}X^{a\;[\rho,\tau]}_{\;c} + X^{ac[\mu,\nu]}X^{b\;[\rho,\tau]}_{\;c}) \\
& [(P_{a\mu\nu}P_{b\rho\tau} - Q_{a\mu\nu}Q_{b\rho\tau})] \\
& +(64\pi G_N)^{-1}(\eta^{ab}Y^{cd[\mu,\nu]}Y^{[\rho,\tau]}_{cd} - 3Y^{bc[\mu,\nu]}Y^{a\;[\rho,\tau]}_{\;b} + Y^{ac[\mu,\nu]}Y^{b\;[\rho,\tau]}_{\;c}) \\
& [-P_{a\mu\nu}P_{b\rho\tau} + Q_{a\mu\nu}Q_{b\rho\tau}] \\
& +(64\pi G_N)^{-1}(-\eta^{ab}(X^{cd[\mu,\nu]}Y^{[\rho,\tau]}_{cd} - X^{cd[\rho,\tau]}Y^{[\mu,\nu]}_{cd}) + 2(X^{bc[\mu,\nu]}Y^{a\;[\rho,\tau]}_{\;c} \\
& -X^{bc[\rho,\tau]}Y^{a\;[\mu,\nu]}_{\;c}))[P_{a\mu\nu}Q_{b\rho\tau} - Q_{a\mu\nu}P_{b\rho\tau}],
\end{aligned} \tag{A.3.6}$$

$$\begin{aligned}
\mathcal{L}_{R3} \;=\; & (8\pi G_N)^{-1}[(D\Omega_{a\dot{5}})_{\dot{5}d+}\eta^{ad} + 4((D\Gamma_a)_{\dot{5}d+}(D\Gamma_b)_{\dot{5}c+} \\
& +(D\Gamma_a)_{\dot{5}d-}(D\Gamma_b)_{\dot{5}c-})(\eta^{ad}\eta^{bc} - \eta^{ac}\eta^{bd})] \\
=\; & -m^2(4\pi G_N)^{-1}(\phi_+^2 + \phi_-^2)^{-2}[(\phi_+^2 - \phi_-^2)(s^{\mu\nu}s_{\mu\nu} - s^{\mu}_{\mu}s^{\nu}_{\nu}) \\
& +\phi_+(\phi_+(v^{\mu\nu}s_{\mu\rho}v^{\rho\tau}s_{\tau\nu} - v^{\mu\nu}s_{\mu\nu}) + \phi_- s^{\mu\nu}s^{\rho}_{\mu}s_{\nu\rho})].
\end{aligned} \tag{A.3.7}$$

$$\mathcal{L}_{R1} = (16\pi G_N)^{-1}(D\Omega_{ab})_{cd+}\eta^{ad}\eta^{bc}$$

$$\begin{aligned}
\mathcal{L}_{R1} \;=\;& (64\pi G_N)^{-1}(e_{d\tau} + s_{d\lambda}v^{\lambda}_{\tau})[(2X^{ab[\rho,\tau]}\eta^{cd} - X^{bc[\rho,\tau]}\eta^{ad})\partial_\rho(X^{[\mu,\nu]}_{bc}P_{a\mu\nu} - Y^{[\mu,\nu]}Q_{a\mu\nu}) \\
& -(2Y^{ab[\rho,\tau]}\eta^{cd} - Y^{bc[\rho,\tau]}\eta^{ad})\partial_\rho(X^{[\mu,\nu]}_{bc}Q_{a\mu\nu} + Y^{[\mu,\nu]}_{bc}P_{a\mu\nu})] \\
& +(64\pi G_N)^{-1}(X^{bc[\rho,\tau]}X^{[\mu,\nu]}_{bc}\eta^{ad} + 2X^{ab[\rho,\tau]}X^{d\ [\mu,\nu]}_{\ b})(-P_{a\mu\nu}\partial_\rho(e_{d\tau} + s_{d\lambda}v^{\lambda}_{\tau}) \\
& +Q_{a\mu\nu}\partial_\rho s_{d\tau}) + (64\pi G_N)^{-1}(X^{bc[\rho,\tau]}Y^{[\mu,\nu]}_{bc}\eta^{ad} + 2X^{ab[\rho,\tau]}Y^{d\ [\mu,\nu]}_{\ b} \\
& +Y^{bc[\rho,\tau]}X^{[\mu,\nu]}_{bc}\eta^{ad} + 2Y^{ab[\rho,\tau]}X^{d\ [\mu,\nu]}_{\ b})(P_{a\mu\nu}\partial_\rho s_{d\tau} + Q_{a\mu\nu}\partial_\rho(e_{d\tau} + s_{d\lambda}v^{\lambda}_{\tau})) \\
& (64\pi G_N)^{-1}(Y^{bc[\rho,\tau]}Y^{[\mu,\nu]}_{bc}\eta^{ad} + 2Y^{ab[\rho,\tau]}Y^{d\ [\mu,\nu]}_{\ b}) \\
& (P_{a\mu\nu}\partial_\rho(e_{d\tau} + s_{d\lambda}v^{\lambda}_{\tau}) + Q_{a\mu\nu}\partial_\rho s_{d\tau}) \\
& +m(32\pi G_N)^{-1}a_{\tau-}[((X^{bc[\rho,\tau]}X^{[\mu,\nu]}_{bc} - Y^{bc[\rho,\tau]}Y^{[\mu,\nu]}_{bc})\eta^{ad} - 2(X^{ab[\rho,\tau]}X^{d\ [\mu,\nu]}_{\ b} \\
& -Y^{ab[\rho,\tau]}X^{d\ [\mu,\nu]}_{-b}))(P_{a\mu\nu}s_{d\rho} + Q_{a\mu\nu}(e_{d\rho} + s_{d\lambda}v^{\lambda}_{\rho})) \\
& +((Y^{bc[\rho,\tau]}X^{[\mu,\nu]}_{bc} - X^{bc[\rho,\tau]}Y^{[\mu,\nu]}_{bc})\eta^{ad} + 2(Y^{ab[\rho,\tau]}X^{d\ [\mu,\nu]}_{\ b} \\
& +X^{ab[\rho,\tau]}X^{d\ [\mu,\nu]}_{\ b}))(P_{a\mu\nu}(e_{d\rho} + s_{d\lambda}v^{\lambda}_{\rho}) + Q_{a\mu\nu}s_{d\rho})].
\end{aligned} \tag{A.3.5}$$

$$\begin{aligned}
\mathcal{L}_{R2} \;=\;& (16\pi G_N)^{-1}((D\Gamma_a)_{ec+}(D\Gamma_b)_{fd+} + (D\Gamma_a)_{ec-}(D\Gamma_b)_{fd-}) \\
& (\eta^{ab}\eta^{cd}\eta^{ef} + 3\eta^{ad}\eta^{bc}\eta^{cf} + \eta^{ac}\eta^{bd}\eta^{ef}) \\
=\;& (64\pi G_N)^{-1}(\eta^{ab}X^{cd[\mu,\nu]}X^{[\rho,\tau]}_{cd} - 3X^{bc[\mu,\nu]}X^{a\ [\rho,\tau]}_{\ c} + X^{ac[\mu,\nu]}X^{b\ [\rho,\tau]}_{\ c}) \\
& [(P_{a\mu\nu}P_{b\rho\tau} - Q_{a\mu\nu}Q_{b\rho\tau})] \\
& +(64\pi G_N)^{-1}(\eta^{ab}Y^{cd[\mu,\nu]}Y^{[\rho,\tau]}_{cd} - 3Y^{bc[\mu,\nu]}Y^{a\ [\rho,\tau]}_{\ b} + Y^{ac[\mu,\nu]}Y^{b\ [\rho,\tau]}_{\ c}) \\
& [-P_{a\mu\nu}P_{b\rho\tau} + Q_{a\mu\nu}Q_{b\rho\tau}] \\
& +(64\pi G_N)^{-1}(-\eta^{ab}(X^{cd[\mu,\nu]}Y^{[\rho,\tau]}_{cd} - X^{cd[\rho,\tau]}Y^{[\mu,\nu]}_{cd}) + 2(X^{bc[\mu,\nu]}Y^{a\ [\rho,\tau]}_{\ c} \\
& -X^{bc[\rho,\tau]}Y^{a\ [\mu,\nu]}_{\ c}))[P_{a\mu\nu}Q_{b\rho\tau} - Q_{a\mu\nu}P_{b\rho\tau}],
\end{aligned} \tag{A.3.6}$$

$$\begin{aligned}
\mathcal{L}_{R3} \;=\;& (8\pi G_N)^{-1}[(D\Omega_{a\dot{5}})_{\dot{5}d+}\eta^{ad} + 4((D\Gamma_a)_{\dot{5}d+}(D\Gamma_b)_{\dot{5}c+} \\
& +(D\Gamma_a)_{\dot{5}d-}(D\Gamma_b)_{\dot{5}c-})(\eta^{ad}\eta^{bc} - \eta^{ac}\eta^{bd})] \\
=\;& -m^2(4\pi G_N)^{-1}(\phi_+^2 + \phi_-^2)^{-2}[(\phi_+^2 - \phi_-^2)(s^{\mu\nu}s_{\mu\nu} - s^{\mu}_{\mu}s^{\nu}_{\nu}) \\
& +\phi_+(\phi_+(v^{\mu\nu}s_{\mu\rho}v^{\rho\tau}s_{\tau\nu} - v^{\mu\nu}s_{\mu\nu}) + \phi_-s^{\mu\nu}s^{\rho}_{\mu}s_{\nu\rho})].
\end{aligned} \tag{A.3.7}$$

$$\mathcal{L}_{T1} = (4g_V^2)^{-1}[(T_{\dot{5}bc})_+(T^{\dot{5}bc})_+ - (T_{\dot{5}bc})_-(T^{\dot{5}bc})_-] \,, \tag{A.4.3}$$

$$\begin{aligned}\mathcal{L}_{T1} = & -(4g_V^2)^{-1}(X_{bc}^{[\mu,\nu]}X^{bc[\rho,\tau]} + Y_{bc}^{[\mu,\nu]}Y^{bc[\rho,\tau]})[(\phi_+^2 + \phi_-^2)(\tfrac{1}{4}a_{\mu\nu+}a_{\rho\tau_+} + \tfrac{1}{4}a_{\mu\nu-}a_{\rho\tau-} \\ & +m^2 a_{\mu-}a_{\nu+}a_{\rho-}a_{\tau+}) + m a_{\rho-}a_{\tau+}(2a_{\mu\nu+}\phi_+\phi_- + a_{\mu\nu-}(\phi_+^2 - \phi_-^2))] \\ & +m(g_V^2)^{-1}X_{bc}^{[\mu,\nu]}Y^{bc[\rho,\tau]}[(\phi_+^2 - \phi_-^2)(a_{\mu\nu+}a_{\tau-}a_{\rho+} + a_{\nu+}a_{\mu-}a_{\rho\tau+}) \\ & +2\phi_+\phi_-(a_{\mu\nu-}a_{\tau+}a_{\rho-} + a_{\rho\tau-}a_{\nu-}a_{\mu+})],\end{aligned} \tag{A.4.4}$$

$$\mathcal{L}_{T2} = (2g_V^2)^{-1}[(T_{\dot{5}\dot{5}c})_+(T^{\dot{5}\dot{5}c})_+ - (T_{\dot{5}\dot{5}c})_-(T^{\dot{5}\dot{5}c})_-], \tag{A.4.5}$$

$$\begin{aligned}\mathcal{L}_{T2} = & -(2g_V^2)^{-1}(\phi_+^2 + \phi_-^2)^{-1}[\tfrac{1}{4}g^{\mu\nu}(\partial_\mu\phi_+\partial_\nu\phi_+ + \partial_\mu\phi_-\partial_\nu\phi_-) + m^2(a_-^\mu a_{\mu-}(4 - 3\phi_+^2) \\ & +\phi_-^2 a_+^\mu a_{\mu+} - 2\phi_+\phi_- a_{\mu-}a_+^\mu + 4a_{\mu+}v_\rho^\mu(v^{\rho\lambda}a_{\lambda+} - a_+^\rho\phi_+\phi_- + 2a_-^\rho - a_-^\rho\phi_+^2)) \\ & +m(a_-^\mu\phi_+ + 2a_{\rho+}v^{\rho\mu}\phi_+ - a_+^\mu\phi_-)\partial_\mu\phi_-]\end{aligned} \tag{A.4.6}$$

$$\mathcal{L}_{T3} = -(4g_V^2)^{-1}(T_{\dot{5}\dot{5}\dot{5}})_+T_+^{\dot{5}\dot{5}\dot{5}} = -\frac{m^2}{g_V^2k^2}\phi_+^2(\phi_+^2 + \phi_-^2)^{-2}. \tag{A.4.7}$$

RESULTS; DISCUSSION

GRAVITY SECTOR

THE FULL ACTION THUS CONTAINS SIX INDEPENDENT FIELDS:

- $(e^{\mu}{}_{a,} (v^{\mu}{}_{a,} z^{a}{}_{\mu}))$, $(a_{\mu+,} a_{\mu\_})$, $(\varphi_{+,} \varphi_{-})$.

  Metric Vector Scalar

ONE COMPONENT OF EACH PAIR HAS ZERO MASS, THE OTHER IS MASSIVE.

- SPECIAL CASE

IF THE COMPONENTS ON THE SECOND SHEET ARE IDENTICAL TO THE ONES ON THE FIRST SHEET, WE REPRODUCE EXACTLY THE KALUZA-KLEIN ZERO MODE THEORY.

- TORSION $T = 0$ IN ALL ITS COMPONENTS

(NATURAL GENERALIZATION OF R-GEOMETRY)

THE FIELDS ARE NOT ALL INDEPENDENT
$v_a{}^{\mu} = \beta e_a{}^{\mu}$, $a_{\mu+} = \alpha\, a_{\mu-}$, $\varphi_+ = \varphi_-$
WHERE, α & β, ARE ARBITRAY FUNCTIONS.

MATTER FIELDS

THE GAUGE SECTOR

TWO ABELIAN GAUGE FIELDS WITH TWO HIGGS SCALAR FIELDS AS PART OF A GENERALIZED 1-FORM:

$$\mathcal{B} = \Gamma^M B_M \equiv \Gamma^\mu B_\mu + \mathbf{F}^5 B_5$$

$$B_\mu = b_{\mu+} \mathbb{1} + b_{\mu-} \underline{r}$$

$$B_5 = h_+ \mathbb{1} + h_- \underline{r}$$

CURVATURE TWO-FORM

$$\mathcal{G} = \mathcal{D}\mathcal{B} + \mathcal{B}\wedge\mathcal{B}$$

$$\mathcal{L}_G = \frac{1}{2g^2}\langle G, G\rangle$$
$$= -\frac{1}{g^2}\left(\tilde{G}_{ab}G^{ab} + 2\tilde{G}_{a\dot{5}}G^{a\dot{5}} + \frac{1}{2}\tilde{G}_{\dot{5}\dot{5}}G^{\dot{5}\dot{5}}\right)$$

G_{ab}, $G_{a\dot{5}}$, $G_{\dot{5}\dot{5}}$ ARE EXPRESSED IN TERMS OF METRIC FIELDS

A.5 Gauge Lagrangian

The Lagrangian terms in Eqn.(6.8) can be calculated as follows

$$\mathcal{L}_{G1} = -\frac{1}{4g^2}(g^{\mu\rho} + g^{\sigma\lambda} v^\mu_\sigma v^\rho_\lambda)(g^{\nu\tau} + g^{\sigma\lambda} v^\nu_\sigma v^\tau_\lambda)(b_{\mu\nu+} b_{\rho\tau+} + b_{\mu\nu-} b_{\rho\tau-}) \quad (A.5.1)$$

$$\begin{aligned}\mathcal{L}_{G2} = \; & -\tfrac{1}{g^2}(\phi_+^2 + \phi_-^2)^{-1}(g^{\mu\nu}\mathcal{D}_\mu \eta \mathcal{D}_\nu \bar{\eta} \\ & +2ig^\mu \sigma v^\nu_\sigma b_{\mu+}((\bar{\eta} - m)\mathcal{D}_\mu \eta - (\eta - m)\mathcal{D}_\mu \bar{\eta}) \\ & +4g^{\sigma\tau} v^\mu_\sigma v^\nu_\tau b_{\mu+} b_{\nu+}(\eta - m)(\bar{\eta} - m))\end{aligned} \quad (A.5.2)$$

$$\mathcal{L}_{G3} = \frac{1}{2g^2}(\phi_+^2 + \phi_-)^{-1}(\bar{\eta}\eta - m^2)^2. \quad (A.5.3)$$

$$\begin{aligned}\mathcal{L}_{G4} = \; & \tfrac{1}{g^2}(X^{[\mu,\nu]}_{ab} X^{ab[\rho,\tau]} + Y^{[\mu,\nu]}_{ab} Y^{ab[\rho,\tau]}) \\ & [((b_{\mu\nu+} - ib_{\mu\nu-})a_\rho(\mathcal{D}_\tau \eta + 2i(\eta - m)b_{\tau-}) + c.c) \\ & +(\tfrac{1}{2}a_\mu a^*_\rho(\mathcal{D}_\nu \bar{\eta}(\mathcal{D}_\tau \eta + 2i(\eta - m)b_{\tau-}) + 4b_{\nu-}(\eta - m)(\bar{\eta} - m)) + c.c)] \\ & -\tfrac{1}{2g^2}(\phi_+^2 + \phi_-^2)^{-1} g^{\mu\nu}[(X_\mu + X^*_\mu)(Y_\nu + Y^*_\nu) + (X_\mu - X^*_\mu + 4imv^\rho_\mu b_{\rho+})(Z_\nu - Z^*_\nu) \\ & +\tfrac{1}{2}(Y_\mu + Y^*_\mu)(Y_\nu + Y^*_\nu) - \tfrac{1}{2}(Z_\mu - Z^*_\mu)(Z_\nu - Z^*_\nu)]\end{aligned} \quad (A.5.4)$$

vhere

$$\begin{aligned} a_\mu &= a_{\mu+} + ia_{\mu-} \\ X_\mu &= (\mathcal{D}_\mu - 2iv^\nu_\mu b_{\nu+})\eta \\ Y_\mu &= (a_\mu + iv^\nu_\mu a_\nu)(\eta - \bar{\eta})(\eta - m) \\ Z_\mu &= (a_\mu - iv^\nu_\mu a_\nu)(\eta + m)(\eta + \bar{\eta} - 2m), \end{aligned} \quad (A.5.5)$$

:.c denotes the complex conjugates.

COMMENTS

$$\mathcal{L}_G = \mathcal{L}_{G1} + \mathcal{L}_{G2} + \mathcal{L}_{G3} + \mathcal{L}_{G4}$$

$\mathcal{L}_{G1}$: KINETIC TERMS OF THE VECTOR GAUGE FIELDS $b_{\mu+}$, $b_{\mu-}$

$\mathcal{L}_{G2}$: KINETIC TERMS OF THE HIGGS FIELDS

$\mathcal{L}_{G3}$: QUARTIC HIGGS POTENTIAL (CORRECT FORM FOR SSB)

$\mathcal{L}_{G4}$: INTERACTION TERMS WITH THE SCALAR AND VECTOR COMPONENTS OF THE GRAVITY SECTOR

FERMIONIC SECTOR

TWO CHIRAL SPINOR FIELDS ψ_L, ψ_R ON THE TWO SHEETS OF SPACE-TIME

$$\mathcal{L}_F = i\,\bar{\Psi}\,\Gamma^A\left(E_A^M(D_M + iB_M) + \frac{i}{4}\,\Gamma^B\Gamma^C\Omega_{BCA}\right)\Psi$$

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad \psi_{\substack{L \\ R}} = \frac{1 \pm \gamma_5}{2}\,\psi$$

AS IN THE GAUGE SECTOR, WE CAN WRITE

$$\mathcal{L}_F = \mathcal{L}_{F1} + \mathcal{L}_{F2} + \mathcal{L}_{F3} + \mathcal{L}_{F4}$$

$\mathcal{L}_{F1}$: GENERALIZATION OF THE FERMIONIC LAGRANGIAN TO A TWO-SHEETED SPACE-TIME

OTHER TERMS INVOLVE SCALAR & VECTOR COMPONENTS OF THE METRIC TENSOR

RESULTS SHOWN IN THE NEXT TRANSP.

PHYSICAL IMPLICATIONS

THE MATHEMATICAL FORMALISM PROVIDES ACTION FUNCTIONALS. THE FIELDS, HOWEVER, DO NOT NECESSARILY HAVE CORRECT DIMENSIONS RESCALING USING THE DIMENSIONAL PARAMETERS IS NECESSARY.

THREE DIMENSIONAL PARAMETERS

G_N = THE NEWTONIAN CONSTANT

g_V = A NEW GRAVITIONAL CONSTANT ASSOCIATED WITH TORSION

m = The Parmeter of dim. mass ASSOCIATED WITH ∂_μ IN THE FIFTH DIMENSION

g = DIMENSIONLESS GAUGE COUPLING

THESE ARE THE ONLY PARAMETERS

WE LOOK AT THE KINETIC TERMS OF EACH FIELD. BY FIXING THE STANDARD COEFFICIENTS OF THESE KINETIC TERMS, WE DETERMINE THE VARIOUS COUPLINGS AND MASSES IN TERMS OF THE FOUR PARAMETERS G_N, g_V, m and g.

REDEFINING AND RESCALING, WE DEFINE THE PHYSICAL FIELDS AS

$$b_{-\mu} \leftrightarrow i\, b_{-\mu}$$

$$\eta \leftrightarrow g\eta = h_+ + m + h_-$$

$$\bar{\eta} \leftrightarrow g\bar{\eta} = h_+ + m - h_-$$

$$v^{\mu\nu} \rightarrow \sqrt{\pi G_N}\, v^{\mu\nu}$$

(LEADS TO FIERZ-PAULI LAGRANGIAN FOR A SPIN-2 TENSOR MESON OF mass m)

$$\phi_+ \rightarrow \exp(2g_v\sigma) \;; \quad \phi_- \rightarrow 2g_v\phi_-$$

$$\left(\mathcal{L}_\phi = -\frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) - \frac{1}{2}(\partial_\mu\phi_-)(\partial^\mu\phi_-) - m^2(g_v)^{-1}\phi_-^2 - m^2(g_v)^{-1} \right)$$

MASSLESS BRANS-DICKE SCALAR FIELD σ AND THE MASSIVE ϕ_-. A COSMOLOGICAL CONSTANT TERM m^2/g_v

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$$a_{\mu\pm} \rightarrow g_v\, a_{\mu\pm}$$

$$\left(\mathcal{L}_a = -\frac{1}{4}\left(a_+^{\mu\nu} a_{\mu\nu+} + a_-^{\mu\nu} a_{-\mu\nu} + 2m^2 a_-^{\mu} a_{-\mu} \right) \right.$$

(massless $a_{+\mu}$, massive $a_{-\mu}$ WITH MASS $m/\sqrt{2}$)

THE GAUGE SECTOR

$$\mathcal{L}_{b\eta} = -\frac{1}{4} b_{\mu\nu+} b_{+}^{\mu\nu} - \frac{1}{4} b_{\mu\nu-} b_{-}^{\mu\nu}$$
$$-(\phi_+^2+\phi_-^2)^{-1}(D_\mu\bar{\eta})(D^\mu\eta) - \frac{g^2}{2}(\phi_+^2+\phi_-^2)\left(\bar{\eta}\eta - \left(\frac{m}{g}\right)^2\right)^2$$

SSB $\rightarrow$ MASSLESS $b_{\mu\nu+}$; MASSIVE $b_{-\mu}$ WITH <u>2m</u> AS THE MASS. SURVIVING HIGGS MASS <u>$\sqrt{2}m$</u>

CURRENTS COUPLED TO $b_{\pm\mu}$

$b_{+\mu}$: $J_+^\mu = -g\bar{\psi}\gamma^\mu\psi$; $J_{5+}^\mu = -ig\sqrt{\pi G_N}\,\bar{\psi}\gamma^\nu\gamma^5 v_\nu^\mu\psi$

$b_{-\mu}$: $J_-^\mu = g\sqrt{\pi G_N}\,\bar{\psi}\gamma^\nu v_\nu^\mu\psi$; $J_{5-}^\mu = -ig\bar{\psi}\gamma^\mu\gamma^5\psi$

PARITY VIOLATING INTERACTIONS DUE TO EXTENDED GRAVITY

$a_{+\mu}$ $\quad J_+^\mu = -m\pi G_N g_v\,\bar{\psi}\gamma^a Y_{ab}^{[\nu,\mu]} S_\nu^b \psi_- \cdots$

$\quad J_{5+}^\mu = i m\pi G_N g_v\,\bar{\psi}\gamma^a\gamma^5 X_{-ab}^{[\nu,\mu]} S_\nu^b\psi \cdots$

$a_{-\mu}$ HAS BOTH γ^μ & γ^5 COUPLINGS

SUMMARY AND CONCLUSIONS

- THE REDEFINITIONS OF THE FIELDS LEADS TO A SUBALGEBRA OF $\mathcal{A} = C^\infty(M,\mathbb{C}) \oplus C^\infty(M,\mathbb{C})$ TO AN ALGEBRA WHERE THE TWO FUNCTIONS ON THE TWO SHEETS ARE COMPLEX CONJUGATES OF EACH OTHER.

  THIS IS DICTATED BY PHYSICAL REQUIREMENTS

- MINIMAL SOLUTION OF CARTAN-MAURER STRUCTURE EQUATIONS LEADING TO THE DETERMINATION OF METRIC, CONNECTION AND TORSION IN TERMS OF GENERALIZED VIELBEINS

- FROM THE POINT OF VIEW OF PHYSICS, A RICH AND COMPLEX STRUCTURE -
  - NEW INTERACTIONS OF MATTER FIELDS WITH MASSIVE TENSOR, VECTOR AND SCALAR COMPONENTS OF THE METRIC
  - PARITY VIOLATING INTERACTIONS COULD BE CP VIOLATING IN THE CONTEXT OF THE FULL STANDARD MODEL IN THE EARLY UNIVERSE
  - GRAVITATIONAL SECTOR HAS A SCALAR FIELD WITH A SCALAR POTENTIAL AND A COSMOLOGICAL CONSTANT TERM.

- SOFTENING OF DIVERGENCES - CORELATED INTERACTIONS