Towards a

QUANTUM THEORY OF SPACETIME AND FIELDS

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Meeting on "Noncommutative Geometry and QFT"

IMS, CHENNAI 10-15/01/03

JOINT WORK WITH

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G. PIACITELLI

REFERENCES:

K. FREDENHAGEN, J. ROBERTS, S.D:
PLB '94, CMP '95

D. BAHNS, K. FREDENHAGEN,
G. PIACITELLI, S.D., PLB '02 {hep-th/0201222

— — —, IN PREPARATION

SEE ALSO CONF. AND SCHOOL REPS:

S.D. PROC. ICMP '94 (IP '95),
— AIHP '96
— hep-th/0105251 (KARPACZ)
K. FREDENHAGEN PROC. HESSELBERG MEETING 1999
— SUMMERSCHOOL IN CRETA, 2002

LARGE SCALE STRUCTURE
OF SPACETIME:

Pseudo Riemannian Manifold
locally modelled on the
MINKOWSKI SPACE

SMALL SCALE STRUCTURE?

LOCALIZING AN EVENT WITHIN

$\Delta x \sim \Delta y \sim \Delta z \sim a$

requires (HEISENBERG):

$E \sim \frac{1}{a} \qquad (\hbar = c = G = 1)$

SCHWARZSHILD RADIUS

$R \sim E \sim \frac{1}{a}$

$a \gtrsim R \Rightarrow a \gtrsim 1$

$(a \gtrsim \lambda_p = 1.6 \; 10^{-33} \text{ cm})$

<u>BUT</u>: $\Delta x \sim a$

$\Delta y \sim \Delta z \sim L$

$E \sim \frac{1}{a}$ SPREAD OVER

DISC OF RADIUS L GENERATES

$$\varphi_L(x) \xrightarrow[L \to \infty]{} 0 \quad \text{at any } x!$$

SPACETIME UNCERTAINTY

RELATIONS:

ADMISSIBLE $\Delta x_0 \cdots \Delta x_3$:

IF THE STATE ω DESCRIBES

OUTCOME OF LOCALIZATION OF

AN EVENT WITH UNCERTAINTIES

$\Delta x_0, \ldots \Delta x_3$, E.E.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi\, \omega(T_{\mu\nu})$$

SHOULD GIVE RISE TO <u>NO</u>

TRAPPED SURFACE <u>HIDING THE</u>

<u>EVENT</u>

LEADS TO (DFR, '94, '95):

$$\Delta q_0 (\Delta q_1 + \Delta q_2 + \Delta q_3) \gtrsim \lambda_P^2$$

$$\Delta q_2 \Delta q_3 + \Delta q_3 \Delta q_1 + \Delta q_1 \Delta q_2 \gtrsim \lambda_P^2 .$$

<u>weak form</u>; stronger consequences:

($a \equiv \inf \Delta q_\mu$, $\mu = 0,1,2,3$;
$b = \sup \Delta q_j$, $j = 1,2,3$)

$$a\, b \gtrsim \lambda_P^2$$

(Gys), or in presence of background $g_{\mu\nu}$

$$g_{00}\, a\, b \gtrsim \lambda_P^2 .$$

Will concentrate on the weak form leading to a BASIC MODEL with MINIMAL QUANTUM DEVIATIONS FROM MINKOWSKI SPACE.

BETTER PICTURE OF SPACETIME IN THE SMALL:

$$[q_\mu, q_\nu] = i\, Q_{\mu\nu}$$

$$\Rightarrow \quad \Delta_\omega q_\mu \cdot \Delta_\omega q_\nu \gtrsim \frac{1}{2}\,|\,\omega(Q_{\mu\nu})\,|$$

$Q_{\mu\nu}$ must be s.t. this $\Rightarrow$ STUR

$$\boxed{\text{BASIC MODEL}}$$

$$* \quad [q_\mu, Q] = Q \cdot Q = 0$$

$$[q_0, q_1, q_2, q_3] \equiv \det \begin{pmatrix} q_0 & q_1 & q_2 & q_3 \\ q_0 & q_1 & q_2 & q_3 \\ q_0 & q_1 & q_2 & q_3 \\ q_0 & q_1 & q_2 & q_3 \end{pmatrix}$$

$$= \varepsilon^{\mu\nu\lambda\rho}\, q_\mu q_\nu q_\lambda q_\rho = -\frac{1}{2}\, Q \cdot {*Q}$$

$$** \quad \left(\frac{1}{2}\, Q \cdot {*Q}\right)^2 = I$$

$\vec{e}, \vec{m} \equiv$ el. resp. magn. parts of $\mathcal{Q}$

$$e^2 = m^2$$

$$(\vec{e} \cdot \vec{m})^2 = I$$

define manifold $\Sigma = \Sigma_+ \cup \Sigma_-$

$$\Sigma_\pm \sim SL(2,\mathbb{C}) / \mathrm{diag}$$

$$\sim TS^2$$

BASE : $\vec{e} = \vec{m}$, $e^2 = m^2 = 1$

TANGENT VECTOR : SPEED, ORTH TO $\vec{e} = \vec{m}$, OF BOOST TAKING TO GENERIC PAIR

WEYL REL.

$i\alpha q \quad i\beta q \qquad -\frac{i}{2}\alpha Q \beta$

Regular reps of $[q_\mu, q_\nu] = i Q_{\mu\nu}$

$\equiv$ integrable to unitary reps of Weyl rel.

$\sim$ nondeg. reps of the enveloping

C^*-alg $\mathcal{E}$ $\equiv$

C^* completion of $\{ \mathcal{C}_0(\Sigma, L^1(\mathbb{R}^4))$,

with σ-twisted conv. alg ops at $\sigma \in \Sigma \}$

$$\equiv \mathcal{C}_0(\Sigma, \mathcal{K}) \sim \mathcal{C}_0(\Sigma) \otimes \mathcal{K}.$$

Generated by

$$f(q)\, g(Q),$$

$g \in \mathcal{C}_0(\Sigma)$;

$f \in L^1(\mathbb{R}^4)^\wedge$, $\quad f \equiv (\check{f})^\wedge$

$$f(q) \equiv \int e^{i\alpha q} \check{f}(\alpha)\, d^4\alpha$$

(von Neumann — Weyl – Wigner – Moyal)

$$(a, \Lambda) \equiv L \in P$$

$$q \longrightarrow \Lambda q + a \cdot I$$

induces autom τ_L of $\mathcal{E}$

$$L \longrightarrow \tau_L \in \mathrm{Aut}\,\mathcal{E}$$

action of <u>full</u> Poincaré grp.

(compare QP: Lukierski, Rueyg, Wess, ...)

Pure states of $\mathcal{E}$ generalize points.

OPTIMALLY LOCALIZED STATES ω:

$$(\Delta_\omega q_0)^2 + \ldots + (\Delta_\omega q_3)^2 = \min$$

$$\Updownarrow \quad \omega(g(Q) e^{i\alpha q}) = e^{-\frac{1}{2}|\alpha|^2} \mu(g)$$

μ prob. measure on Σ_1;

if $\sigma \in \Sigma_1$, $[q_\mu, q_\nu] = i\sigma_{\mu\nu} \cdot I$

$\sim$ Schroedinger ops. q_1, q_2; p_1, p_2.

QST seen by optimally loc. states

$\sim \Sigma_1 \times$ PHASE SPACE OF 2-d Q.M.

$\downarrow$ radius λ_p^2

$\downarrow$ effective cells of volume λ_p^4

cf FUZZY PHYS S. MADORE, A.P. BALACHANDRAN

CLASSICAL LIMIT:

$$\mathbb{R}^4 \times S^2 \times \{\pm\}$$

OPTIMALLY LOCALIZED STATES

$$\omega \equiv \mu \circ \eta$$

μ : state of $\mathcal{C}(\Sigma_1) \equiv \mathcal{Z}_1$

η : localisation map $\mathcal{E} \to \mathcal{Z}_1$

$$\eta(f(q)g(Q)) = \int f(\alpha) e^{-\frac{1}{2}|\alpha|^2} d^4\alpha \cdot g|_{\Sigma}.$$

QFT on QST

On $\mathbb{R}^4$:

$$f \in \mathcal{S} \longrightarrow \phi(f) \text{ } \eta \text{ } \Omega$$

unbd op valued distribution

$$\phi \equiv \text{F.T. of } \check{\phi}$$

Def $\phi(q)$ as

$$\phi(q) = \int e^{ikq} \check{\phi}(k) \, d^4k$$

$\phi(q)$ affiliated to $\mathcal{L} \otimes \mathcal{O}$

$\omega \in \mathcal{S}(\mathcal{L})$,

$\langle \omega \times id, \phi(q) \rangle \equiv \phi(\omega) \in \mathcal{O}$
(affiliated to)

replaces "fields at a point."

FREE FIELDS OK

(GAUSSIAN DECAY OF $[\phi(\omega), \phi(\omega_a)]$
IN SPACELIKE DIR. IF ω OPTIMALLY l.c.)

INTERACTING QFT:

PROBLEMS WITH!

- NON UNIQUENESS OF EXT. OF THE CLASSICAL PRESCRIPTIONS
- BREAKDOWN OF RELATIVISTIC INV.
- " OF CAUSALITY

(- - - - -)

ROUTE I (DFR '94, '95)

$$\int_{q_0=t} d^3q \, f(q)$$ has well defined meaning as a gen. lin map

but requires $\int d\sigma$ undefined or rest. to Σ_1 : breaks Lorentz

$$H_I(t) = \int_{\Sigma_1} d\sigma \int h_I(q) \, d^3q$$

$$\text{e.g.} = \int_{\Sigma_1} d\sigma \int_{x_0=t} d^3x \, (\phi *_\sigma \cdots *_\sigma \phi)(x)$$

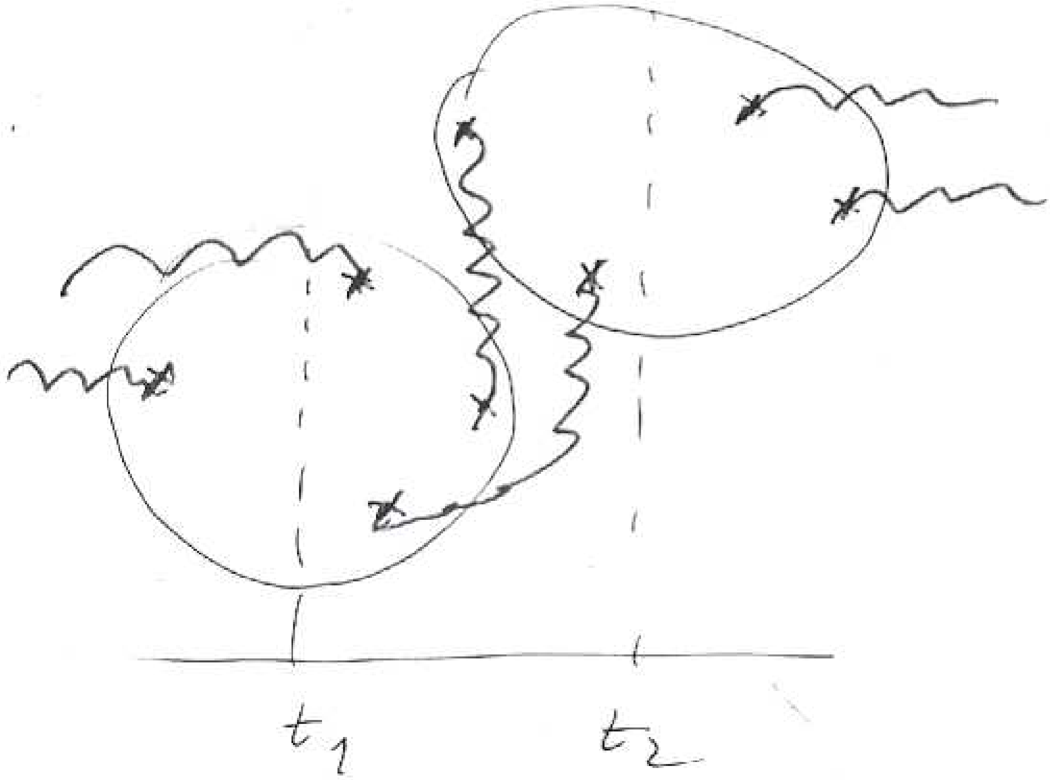
FORMALLY SELF-ADJOINT : <u>NO</u>

PROBLEM WITH UNITARITY

(cf BDFP for explicit comps.)
LIAO, SIBOLD
FEYNMAN RULES REPLACED BY:

$$S = T \exp i \int_{-\infty}^{\infty} H_I(t)\, dt$$

$$H_I(t) = \int G(t, x_1, \dots, x_4) :\phi(x_1) \dots \phi(x_4): d^4x_1 \dots d^4x_4$$



MUST TIME ORDER $t_1, t_2, \dots$

$\boxed{\text{NOT}}$ THE TIME ARGUMENTS OF THE FIELD OPERATORS! THAT WOULD BE ALLOWED IF H_I WERE <u>LOCAL</u>

E.G. IF WE <u>COULD</u> WRITE

$$(\phi * \phi)(x) = e^{\frac{i}{2} Q_{\mu\nu} \frac{\partial}{\partial x_\mu} \frac{\partial}{\partial x_\nu}} \phi(x)\phi(y) \Big|$$

NON UNITARITY!

NOTE (DFR '94, '95):

- IF $\mathcal{H}(x)$ IS THE FREE HAMILTONIAN DENSITY AND H THE FREE HAMILTONIAN,

$$H = \int_{x_0 = t} \mathcal{H}(x_0, \vec{x})\, d^3x$$

BUT ALSO

$$H = \int_{q_0 = t} \mathcal{H}(q)\, d^3q$$

UNAFFECTED BY Q.S.T.

- EXPLICIT FORM OF NONLOCAL KERNEL IN CASE OF $:\phi^3:$ GIVEN BY:

$$\int_{q_0 = t} f_1(q) f_2(q) f_3(q)\, d^3q = \int d^4a\, d^4b \int_{x_0 = t + \lambda_P (b-a)_0} f_1(x + \lambda_P a) f_2(x + \lambda_P b) f_3(x)\, S(a,b)\, d^3x$$

$$S(a,b) = -\frac{1}{2\pi^4} \left(\frac{\sin \gamma_+(a,b)}{\gamma_+(a,b)} + \frac{\sin \gamma_-(a,b)}{\gamma_-(a,b)} \right),$$

$$\gamma_\pm(a,b) = 2 \| a_0 \vec{b} - b_0 \vec{a} \pm \vec{a} \times \vec{b} \|.$$

ROUTE II (BDFP to appear)

$$\phi(x_1)\dots\phi(x_n) \rightsquigarrow \phi(q)\phi(q')\dots\phi(q^{(n)})$$

$q, q', \dots$ indep. variables:

$$q_\mu \otimes I \otimes I \otimes \dots = q^1_\mu$$
$$I \otimes q_\mu \otimes I \otimes \dots = q^2_\mu$$
$$\text{- - - -} \qquad \text{- -}$$
$$I \otimes I \otimes \dots \otimes q_\mu = q^n_\mu .$$

$$q^{ij} \equiv q^i - q^j$$

$$[q^{ij}_\mu, q^{ij}_\nu] \neq 0$$

IF WE IDENTIFY THE CENTRES, SAME C.R. AS q_μ!

$\mathcal{E}$ is a $\mathcal{Z}$ - bimodule, $\mathcal{Z} =$
$= \mathcal{Z}(M(\mathcal{E})) = \mathcal{C}_B(\Sigma)$:

$$\mathcal{E} \otimes_{\mathcal{Z}} \dots \otimes_{\mathcal{Z}} \mathcal{E}$$

- $[q^{ij}_\mu, q^{ij}_\nu] = 2i\,Q_{\mu\nu}$

$$\bar{q}_\mu \equiv \frac{1}{n}\sum_j q^j_\mu \quad \text{barycenter}$$

- $[\bar{q}_\mu, q^{ij}_\nu] = 0$ strong sense

von Neumann uniqueness at each $\sigma \in \Sigma$

$$\Rightarrow \quad \bar{q}_\mu\, q^{ij}_\nu \cong \bar{q}_\mu \otimes_Z q_\nu$$

defines a natural map:

$$\bar{q} \longrightarrow \frac{1}{\sqrt{n}}\, q \otimes I \otimes I \cdots$$

$$q^i - q^j \longrightarrow I \otimes (q^i - q^j)$$

$$\mathcal{E}^{\otimes_Z n} \longrightarrow \mathcal{E}^{\otimes_Z (n+1)} \quad \text{(into)}$$

COINCIDENT PTS: $q^{ij} = 0$ VIOLATES C.R.

$$\left[\frac{1}{\sqrt{2}}\, q^{ij}_\mu, \frac{1}{\sqrt{2}}\, q^{ij}_\nu\right] = i\,Q_{\mu\nu}$$

COMPOSE THE MAP ABOVE

$$\mathcal{E}^{\otimes_Z n} \longrightarrow \mathcal{E}^{\otimes_Z (n+1)}$$

WITH $\quad id \otimes \eta \otimes \cdots \otimes \eta$

$\eta : \mathcal{E} \longrightarrow \mathcal{Z}_1$ optimal loc. map

gives

$$\begin{array}{ccc} \mathcal{E}^{\otimes_Z n} & \xrightarrow{E} & \mathcal{E}_1 \\ \text{quotient} \downarrow & \nearrow & \\ \mathcal{E}_1^{\otimes_Z n} & & \end{array}$$

$\mathcal{E}_1 \begin{cases} = \text{restr to } \Sigma_1 \\ = \mathcal{E}/_{N_1} \\ N_1 = \text{fn zero on } \Sigma_1 \end{cases}$

C.E. leaving $\frac{1}{m}\sum q^j$ alone

evaluating on each $f\left(\frac{1}{\sqrt{2}}(q^i - q^j)\right)$

the optimally localizing map η

$$:\phi^m(q):_Q \equiv$$

$$\equiv E\left(:\phi(q_1) \ldots \phi(q_m):\right)$$

$$\eta : \mathcal{E} \longrightarrow \mathcal{Z}_1$$

is characterized by

$$\eta(q_\mu) = 0$$

$$\eta\Big(\sum_\mu q_\mu^2\Big) = 2\cdot I$$

IMMEDIATELY CHECKED FOR

$$\eta \otimes \eta \quad , \quad \frac{1}{\sqrt{2}}(q \otimes I - I \otimes q)$$

INCIDENTALLY NOTE THAT THE (EUCL., 4-D) DISTANCE IN QST IS THE OPERATOR d: $d^2 =$

$$= |q - q'|^2 \equiv \sum_{\mu=0}^{3} (q_\mu \otimes I - I \otimes q_\mu)^2$$

SINCE $\frac{1}{\sqrt{2}}(q - q') \cong q$,

$$|q - q'|^2 \geq 4, \quad d \geq 2\lambda_P,$$

WHAT ABOUT

$$\inf |\vec{q} - \vec{q}'| \quad ?$$

$$\frac{1}{\sqrt{2}}(\vec{q} - \vec{q}') \cong \vec{q}$$

$$(\inf |\vec{q}|)^2 = \inf_\omega \omega(q_1^2 + q_2^2 + q_3^2)$$

$$\geq \omega(q_2^2) + 2\,\Delta q_1 \cdot \Delta q_3$$

$$\geq \omega(q_2^2) + |m_2|$$

since choice of axis 2 was arbitrary,

$$(\inf |\vec{q}|)^2 \geq |\vec{m}| = \|\sigma\|$$

and inf has to occur for $\sigma \in \Sigma_1$.

Choose $\vec{n}_2 = \vec{m}$: $[q_2, q_1] = [q_2, q_3] = 0$, $[q_3, q_1] = -iI$; can make choice of ω s.t. $\Delta q_1 = \Delta q_3 = \frac{1}{\sqrt{2}}$, $\Delta q_2 < \varepsilon$:

$\Rightarrow$

$$\boxed{\inf |\vec{q} - \vec{q}'| = \sqrt{2} \cdot \lambda_P}$$

$$H_I(t) \equiv \int_{q_0=t}' d^3q \;\; \lambda :\phi^n(q):_Q$$

a-priori an op. valued fn on Σ_1, is <u>constant</u> on Σ_1; no ad-hoc integration on 0 is needed.

$\lambda \to \lambda(t)$ (e.g. $\lambda e^{-\varepsilon t^2}$)

$$S_\lambda \equiv T \exp i \int_{-\infty}^{\infty} \lambda(t) H_I(t)\, dt$$

Then the GELL'MANN-LOW EXPANSION

$$S_\lambda \Big/ \langle S_\lambda \rangle_0$$

if finite at each order of p.e., i.e. NO UV divergence.

BUT : $:\;:_Q$ does not leave the FREE HAMILTONIAN UNCHANGED.

$$\int_{q_0=t} d^3q \; \lambda :\phi^n(q):_Q =$$

$$= \int_{x_0=t} d^3x \; H^I_{eff}(x) \, ;$$

$$H^I_{eff}(x) =$$

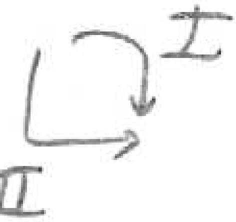
$$= \int_{\mathbb{R}^{4n}} d^4x_1 \ldots d^4x_n \; K_n(x-x_1, \ldots, x-x_n) \; \lambda :\phi(x_1) \ldots \phi(x_n): \, ,$$

$$K_n(y_1, \ldots, y_n) =$$

$$= C_n \, e^{-\frac{1}{2} \sum_{\mu, j} (y_j^\mu)^2} \, \delta^{(4)}\Big(\sum_{j=1}^{n} y_j\Big).$$

(BDFP, to appear soon)

However: both



— VIOLATE LORENTZ

— NON LOCAL

III YANG FELDMAN EQ:

$$\phi(q) = \phi^{in}(q) + \lambda \Delta^{ret} \times :\phi^3(q):$$

PARTIAL WICK PRODUCTS, LORENZ COV. PRES.
BUT BREAKS DOWN ELSEWHERE: ASYMPT. STATES?
(BDFP WORK IN PROGRESS)

IV GAUGE THEORY ON

QST (WESS, collabor. ZUMINO, LUKIERSKI collabor.)
(DF in preparation)

"Classical" diff. calculus on $\mathcal{E}$:

$$d^{cl} A = \frac{\partial}{\partial a_\mu} \tau_a(A)\Big|_{a=0} \, da_\mu$$

UNIVERSAL DIFF. CALC.:

$$dA = I \otimes A - A \otimes I$$

(cf A. CONNES "Noncommutative Geometry").

METRIC ?

CODIFFERENTIAL δ ?

ANSWER (FOR FLAT CASE

$$g = \mathrm{diag}(+,-,-,-)),$$

WHICH YIELDS BACK TO CLASSICAL YANG – MILLS EQ

(NONCOMMUTATIVE GAUGE:

$$U(1) \longrightarrow \mathcal{U}(\mathcal{C}_0(\mathbb{R}) + \mathbb{C} I)$$

ON QST:

$$U(1) \longrightarrow \mathcal{U}(\mathcal{E} + \mathbb{C} \cdot I) \equiv G =$$

$$= \prod \times \mathcal{G}$$

$\mathcal{G}(\sigma) = \{$unitaries of the form $I + T$, T compact$\}$, $\sigma \in \Sigma$;)

ESSENTIAL INGREDIENT:

$$\mathrm{Ad}\,(Q^{-1}q)_\mu = \frac{\partial}{\partial x_\mu}$$

(Q^{-1} def by QUANTUM CONDITIONS).

GAUGE TR:

$$q \longrightarrow U q U^{-1} \neq q\,!$$

J. WESS et al:

$$q_{cov} = q + \langle A, dq \rangle$$

allows to construct gauge inv quantities.

PROB. OBSERVABLE TESTS?
(contact with work by
AMELINO-CAMELIA et al ?)
WESS et al
CHAICHIAN et al; MORITA

HOW COULD THESE RELATIONS REDUCE POSSIBLY TO THE ACWARD REL.

$$[q_\mu, q_\nu] = i\, g_{00}^{-1}\, Q_{\mu\nu}$$

IN A QUASI STATIONARY, SEMI CLASSICAL APPROXIMATION?

AN ACWARD ARGUMENT IN FAVOUR: REPLACE THE NON IN. g_{00}^{-1} BY THE <u>INVARIANT</u> SCALAR CURVATURE R:

$$[q_\mu, q_\nu] = i\alpha R\, Q_{\mu\nu}$$

$Q_{\mu\nu}$ as in the basic model. EE $\Rightarrow$

$$[q_\mu, q_\nu] = -8\pi i \alpha\, g^{\lambda\rho} T_{\lambda\rho}\, Q_{\mu\nu}$$

APPROXIMATION: REPLACE R.H.S. BY EXPECTATION IN A SHARPLY LOCALIZED STATE ω, AND $g^{\lambda\rho}$ BY CLASSICAL

$$[g_\mu, g_\nu] = -8\pi i \alpha\, \theta_{\mu\nu}\, g^{\lambda\rho}\, \omega(T_{\lambda\rho})$$

(2) SPACELIKE TO THE SHARP LOCALIZATION REGION

$$\omega(T_{\lambda\rho}) = \omega_0(T_{\lambda\rho})$$

(2) ASSUME

$$(\Omega, T_{\lambda\rho}\Omega) = \Lambda\, \delta_{\lambda 0}\, \delta_{\rho 0}$$

STATIONARY APPROX FOR $g^{\lambda\rho}$:

$$g^{0j} = g^{j0} = 0 \qquad j = 1,2,3$$

THEN $g^{00} = (g_{00})^{-1}$

AND OUR APPROX. QUANTUM CONDITIONS BECOME:

$$[g_\mu, g_\nu] = -8\pi i \alpha \Lambda g_{00}^{-1} Q_{\mu\nu}$$

HOWEVER THE DEPENDENCE UPON $J_{\mu\nu}$ OF $[q_\mu, q_\nu]$ MIGHT HAVE PHYSICALLY MEANINGFUL CONSEQUENCES:

— INFLATION ?

— COSMOLOGICAL CONSTANT $\neq 0$?

— DARK MATTER ?