NEW SYMMETRIES FOR THE ABELIAN TWO-FORM GAUGE THEORY

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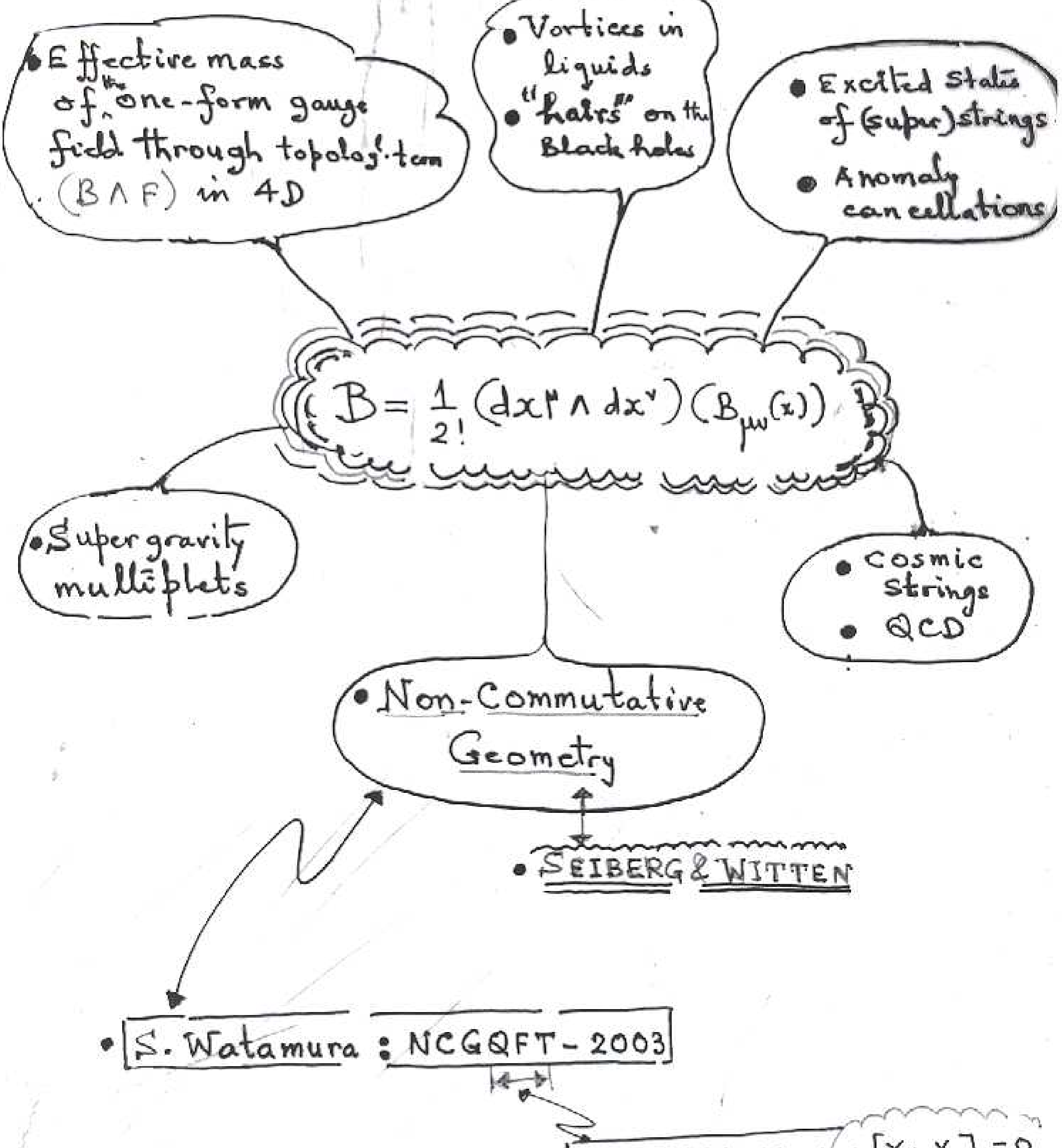
$\boxed{\text{IMSc} - \underbrace{\text{CHENNAI}}_{\Uparrow} : \text{NCGQFT-2003}}$

$\boxed{\text{MADRAS}}$

"RELEVANCE"


• Effective mass of the one-form gauge field through topolog!. term (B ∧ F) in 4D
• Vortices in liquids
• "hairs" on the Black holes
• Excited States of (super) strings
• Anomaly cancellations
$B = \frac{1}{2!} (dx^{\mu} \wedge dx^{\nu}) (B_{\mu\nu}(x))$
• Supergravity multiplets
• Cosmic strings
• QCD
• Non-Commutative Geometry
• SEIBERG & WITTEN
• S. Watamura : NCGQFT-2003
• MY DISCUSSION : $[x_\mu, x_\nu] = 0$

★ COHOMOLOGICAL ASPECTS : 2-FOR[M] A.G.T.

de RHAM C. OPERATORS : (d, δ, Δ)

- $d = dx^\mu \partial_\mu$; $d^2 = 0$; $dx^\mu \wedge dx^\nu = - dx^\nu \wedge dx^\mu$
- $\delta = \pm * d *$, $\delta^2 = 0$ $\left([\Delta, d] = 0 = [\Delta, \delta] \right)$
- $(\delta)\, d$ $\equiv$ (CO-) EXTERIOR DERIVATIVES
- Δ $= (d+\delta)^2 = \{d, \delta\}$: LAPLACIAN OPR.
- $*$ $\equiv$ HODGE DUALITY OPERATION

$\boxed{\text{U. Bruzzo : NCGQFT'0}}$

★ BRST - FORMALISM :- $d \longleftrightarrow Q_B$

Q_B : $\underbrace{\text{BRST}}_{\text{Becchi-Rouet-Stora-Tyutin}}$ - CHARGE ; $Q_B^2 = 0$, $\dot{Q}_B = 0$

- MOTIVATION :

$\left.\begin{array}{l} \delta \rightarrow ? \\ \Delta \rightarrow ? \\ * \rightarrow ? \end{array}\right\}$ WHAT??

$\delta = \pm * d *$

Long-standing problem

→ WHY ? WHAT ?

→ for 2-form ABELIAN GAUGE-THEORY

1- form Gauge Theory

$\Downarrow$

Non-local & Non-covariant

NEW SYMMETRIES FOR THE ABELIAN TWO-FORM GAUGE THEORY

- BRST - INVARIANT LAGRANGIAN DENSIT

$$Q_B \leftrightarrow d, \qquad \delta \not\equiv Q_{AB}$$

- (co-)BRST - INVARIANT LAG. DENSITY:

$$\begin{tikzcd} Q_B \arrow[r, leftrightarrow] & d \arrow[d, leftrightarrow] & \delta \arrow[r, leftrightarrow] \arrow[d, leftrightarrow] & Q_D \\ & Q_{AD} \arrow[r, dash, dashed] & Q_{AB} & \end{tikzcd}$$

- A BOSONIC SYMMETRY

$$\begin{array}{l} Q_W = \{Q_B, Q_D\} \equiv \{Q_{AD}, Q_{AB}\} \\ \updownarrow \\ \Delta = \{d, \delta\} \equiv d\delta + \delta d \end{array}$$

- A DISCRETE SYMMETRY

$$\Downarrow$$

$$* \Longrightarrow \delta = \underbrace{\pm}_{\longrightarrow (F/B) \text{ OF THEORY}} * d *$$

- (co-)BRST TRANSFORMATIONS & COHOMOLOG

REFERENCES

• [J. Phys. A : Math Gen 33 (2000) 7149]

— E. HARIKUMAR, r.p.m., M. SIVAKUMAR

• Hodge Decomposition Theorem for Abelian Two-Form Gauge Theory

[hep-th/0209136] ↔ r.p.m.

— Abelian two-form gauge theory : Special Features

★ [hep-th/0212240] ↔ r.p.m.

— Gauge transformations, BRST cohomology and Wigner's little group

BRST-INVARIANT $\mathcal{L}$ag. Density: $\mathcal{L}_B$

$$
\begin{aligned}
\mathcal{L}_B = \frac{1}{12} H^{\mu\nu\alpha} H_{\mu\nu\alpha} &+ B^\mu\left(\partial^\sigma B_{\sigma\mu} - \partial_\mu \varphi_1\right) - \frac{1}{2} B^\mu B_\mu \Big\} \\
&- \partial_\mu \bar{\beta}\, \partial^\mu \beta + \left(\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu\right)\left(\partial^\mu C^\nu\right) + \rho\left(\partial \cdot C + \lambda\right) \Big\} \\
&+ \left(\partial \cdot \bar{C} + \rho\right)\lambda
\end{aligned}
$$

- <u>3-form</u>: $H = \frac{1}{3!}\left(dx^\mu \wedge dx^\nu \wedge dx^\sigma\right)\left(H_{\mu\nu\sigma}\right)$

  $H = dB = d\left[\frac{1}{2!}\left(dx^\mu \wedge dx^\nu\right) B_{\mu\nu}\right]$

curvature term $\Rightarrow$ $H_{\mu\nu\sigma} = \partial_\mu B_{\nu\sigma} + \partial_\nu B_{\sigma\mu} + \partial_\sigma B_{\mu\nu}$

- <u>1-form</u>: $\delta B = - * d * \left(\frac{1}{2!} dx^\mu \wedge dx^\nu \, B_{\mu\nu}(x)\right)$

  $= \underbrace{\left(\partial_\mu B^{\mu\nu}\right)}_{\text{Gauge-Fixing term}}\left(dx_\nu\right)$

- B_μ : Auxiliary field : $\Rightarrow B_\mu = \left(\partial^\sigma B_{\sigma\mu} - \partial_\mu \varphi_1\right)$

  φ_1 : Massless scalar field $\Rightarrow \{\Box \varphi_1 = 0\}$

  $\bar{\beta}, \beta$: BOSONIC (ANTI-) GHOST Fields $\left(\bar{\beta}^2 \neq 0, \beta^2 \neq 0\right)$

$\bar{C}_\mu (C_\mu)$: Fermionic Vector (Anti-) ghost fields

$(\bar{C}_\mu)^2 = (C_\mu)^2 = 0,\ C_\mu \bar{C}_\nu + \bar{C}_\nu C_\mu = 0,\ C_\mu C_\nu + C_\nu C_\mu = 0 \ldots)$

$\rho(\lambda)$: Fermionic Scalar (Anti-) ghost aux.^{ry} fields

$(\rho^2 = 0,\ \lambda^2 = 0,\ \rho\lambda + \lambda\rho = 0)$

- $\mathcal{L}$ag. Density RESPECTS (ANTI-) BRST (s_{ab}) s_b OFF-SHELL NILPOTENT SYMMETRIES ($s^2_{(a)b} = 0$)

$$s_b B_{\mu\nu} = (\partial_\mu C_\nu - \partial_\nu C_\mu);\quad s_b C_\mu = \partial_\mu \beta,\quad s_b \bar{C}_\mu = B_\mu$$

$$s_b (B_\mu, \rho, \lambda, \beta) = 0;\quad s_b \varphi_1 = -\lambda;\quad s_b \bar{\beta} = \rho$$

$$s_b (H_{\mu\nu\sigma}) = 0 \Rightarrow s_b (\partial_{[\mu} B_{\nu\sigma]}) = 0$$

$$\Downarrow$$

$s_b \longrightarrow s_{ab}$: $\beta \to \mp i \bar{\beta},\ C_\mu \to \pm i \bar{C}_\mu,\ \rho \to \pm i\lambda$

$\bar{\beta} \to \pm i\beta,\ \bar{C}_\mu \to \pm i C_\mu,\ \lambda \to \pm i \rho$

- GHOST PART OF THE $\mathcal{L}$AG. DENSITY REMAINS INVARIANT. $\Longrightarrow$

- \# ANTI-BRST SYMM.

\# $s_{ab} (H_{\mu\nu\sigma}) = 0$

- GHOST - SCALE SYMM. TRANSFORMS.

$$\beta \to e^{2\Sigma}\beta\,,\quad C_\mu \to e^{\Sigma}C_\mu\,,\quad \rho \to e^{-\Sigma}\rho$$

$$\bar\beta \to e^{-2\Sigma}\bar\beta\,,\quad \bar C_\mu \to e^{-\Sigma}\bar C_\mu\,,\quad \lambda \to e^{+\Sigma}\lambda$$

- GENERATORS OF THE ABOVE SYMMETRY TRANSFORMATIONS:

Q_B, Q_{AB}, Q_g (GHOST CHARGE)

- ALGEBRA: $Q_B^2 = 0$, $Q_{AB}^2 = 0$, $\{Q_B, Q_{AB}\} = 0$

$$i[Q_g, Q_B] = +Q_B\,,\quad i[Q_g, Q_{AB}] = -Q_{AB}$$

$$\Updownarrow$$

$$d^2 = 0\,,\quad \delta^2 = 0\,,\quad \{d, \delta\} \neq 0 = \Delta$$

$$d f_n \sim \tilde f_{n+1}\,,\quad \delta f_n \sim \tilde{\tilde f}_{n-1}$$

Given:
$(iQ_g|\psi\rangle_n = n|\psi\rangle_n)$

$$i Q_g \underbrace{Q_B|\psi_n\rangle} = (n+1)\underbrace{Q_B|\psi\rangle_n}$$

$$i Q_g Q_{AB}|\psi\rangle_n = (n-1)\, Q_{AB}|\psi\rangle_n$$

$$\Downarrow$$

$$Q_{AB} \neq \delta$$

$\Delta = ?\ \ \delta = ?$

(CO-) BRST INVARIANT $\mathcal{L}$AG. DENSITY

$$\mathcal{L}_D = \frac{1}{2}\mathcal{B}^\mu \mathcal{B}_\mu - \mathcal{B}^\mu\left(\frac{1}{2}\varepsilon_{\mu\nu\lambda\zeta}\partial^\nu B^{\lambda\zeta} - \partial_\mu \varphi_2\right) - \frac{1}{2}B^\mu B_\mu + B^\mu\left(\partial^\sigma B_{\sigma\mu} - \partial_\mu \varphi_1\right) - \partial_\mu \bar\beta\, \partial^\mu \beta + \rho(\partial\cdot C + \lambda) + (\partial\cdot\bar C + \rho)\lambda + (\partial_\mu \bar C_\nu - \partial_\nu \bar C_\mu)(\partial^\mu C^\nu)$$

\# AUXILIARY FIELDS $\mathcal{B}_\mu$ & φ_2 HAVE BEEN INTRODUCED TO LINEARIZE THE K.E. TERM $\left(\frac{1}{12}H^{\mu\nu\sigma}H_{\mu\nu\sigma}\right)$ OF THE $\mathcal{L}$AG. DENSITY.

E. O. M. $\Box \varphi_2 = 0, \quad \mathcal{B}_\mu = \frac{1}{2}\varepsilon_{\mu\nu\lambda\zeta}\partial^\nu B^{\lambda\zeta} - \partial_\mu \varphi_2$

DUAL (CO-) BRST SYMM. TRANSFORM^c:

$s_d B_{\mu\nu} = \varepsilon_{\mu\nu\kappa\zeta}\partial^\kappa \bar C^\zeta, \quad s_d \bar C_\mu = -\partial_\mu \bar\beta, \quad s_d C_\mu = \mathcal{B}_\mu$

$s_d \beta = \lambda, \quad s_d \varphi_2 = -\rho, \quad s_d(\bar\beta, \lambda, \rho, \varphi_1, B_\mu, \mathcal{B}_\mu) = 0$

CONTRAST $\quad \boxed{s_d(\partial^\sigma B_{\sigma\mu}) = 0}$

$s_b B_{\mu\nu} = (\partial_\mu C_\nu - \partial_\nu C_\mu), \quad s_b C_\mu = \partial_\mu \beta, \quad s_b \bar C_\mu = B_\mu, \quad s_b \bar\beta = \rho$

$s_b \varphi_1 = -\lambda, \quad s_b(\beta, \lambda, \rho, \varphi_2, B_\mu, \mathcal{B}_\mu) = 0, \quad \boxed{s_b(H_{\mu\nu\sigma}) = 0}$

- $S_d \longrightarrow S_{ad}$ (Anti-co-BRST symm. transf)

$$\Uparrow$$

$$\beta \to \mp i\bar{\beta}, \quad C_\mu \to \pm i\bar{C}_\mu, \quad \rho \to \pm i\lambda$$
$$\bar{\beta} \to \pm i\beta, \quad \bar{C}_\mu \to \pm iC_\mu, \quad \lambda \to \pm i\rho$$

★ GENERATORS OF SYMMETRY TRANSF^{NS}:

$$Q_D^2 = 0, \quad Q_{AD}^2 = 0, \quad \{Q_D, Q_{AD}\} = 0$$

- EXTENDED ALGEBRA:

$$Q_B^2 = 0 = Q_D^2 = Q_{AB}^2 = Q_{AD}^2 \; ; \{Q_B, Q_{AB}\} = \{Q_D, Q_{AD}\} = 0$$
$$\{Q_B, Q_D\} = Q_W = \{Q_{AD}, Q_{AB}\}, \; [Q_W, Q_r] = 0$$
$$(r = B, AB, D, AD, g)$$
$$i[Q_g, Q_B] = +Q_B, \; i[Q_g, Q_D] = -Q_D, \quad \{Q_D, Q_{AB}\} = 0$$
$$i[Q_g, Q_{AD}] = +Q_{AD}, \; i[Q_g, Q_{AB}] = -Q_{AB}, \quad \{Q_B, Q_{AD}\} = 0$$

$$\Updownarrow$$

$$\boxed{d^2 = \delta^2 = 0, \; \{d, \delta\} = \Delta, \; [\Delta, d] = [\Delta, \delta] = 0}$$

- Q_W IS THE GENERATOR OF A BOSONIC SYMMETRY TRANSFORMATION ($\underline{s_W^2 \neq 0}$) ($s_W = \{s_B, s_D\}$)

$$\# \; s_W B_{\mu\nu} = \partial_\mu \mathcal{B}_\nu - \partial_\nu \mathcal{B}_\mu + \varepsilon_{\mu\nu\kappa\varsigma} \partial^\kappa B^\varsigma \; ; \quad s_W C_\mu \overset{\Downarrow}{=} \partial_\mu \lambda$$

$$s_W \bar{C}_\mu = \overset{\Downarrow}{-} \partial_\mu \rho , \quad s_W (\varphi_1, \varphi_2, B_\mu, \mathcal{B}_\mu, \rho, \lambda, \beta, \bar{\beta}) = 0$$

\# GHOST FIELDS EITHER DO NOT TRANSFORM OR TRANSFORM AT THE MOST UPTO A VECTOR GAUGE TRANSFORMATION.

- GHOST \# CONSIDERATION : ($i\, Q_g |\psi\rangle_n = n\, |\psi\rangle_n$)

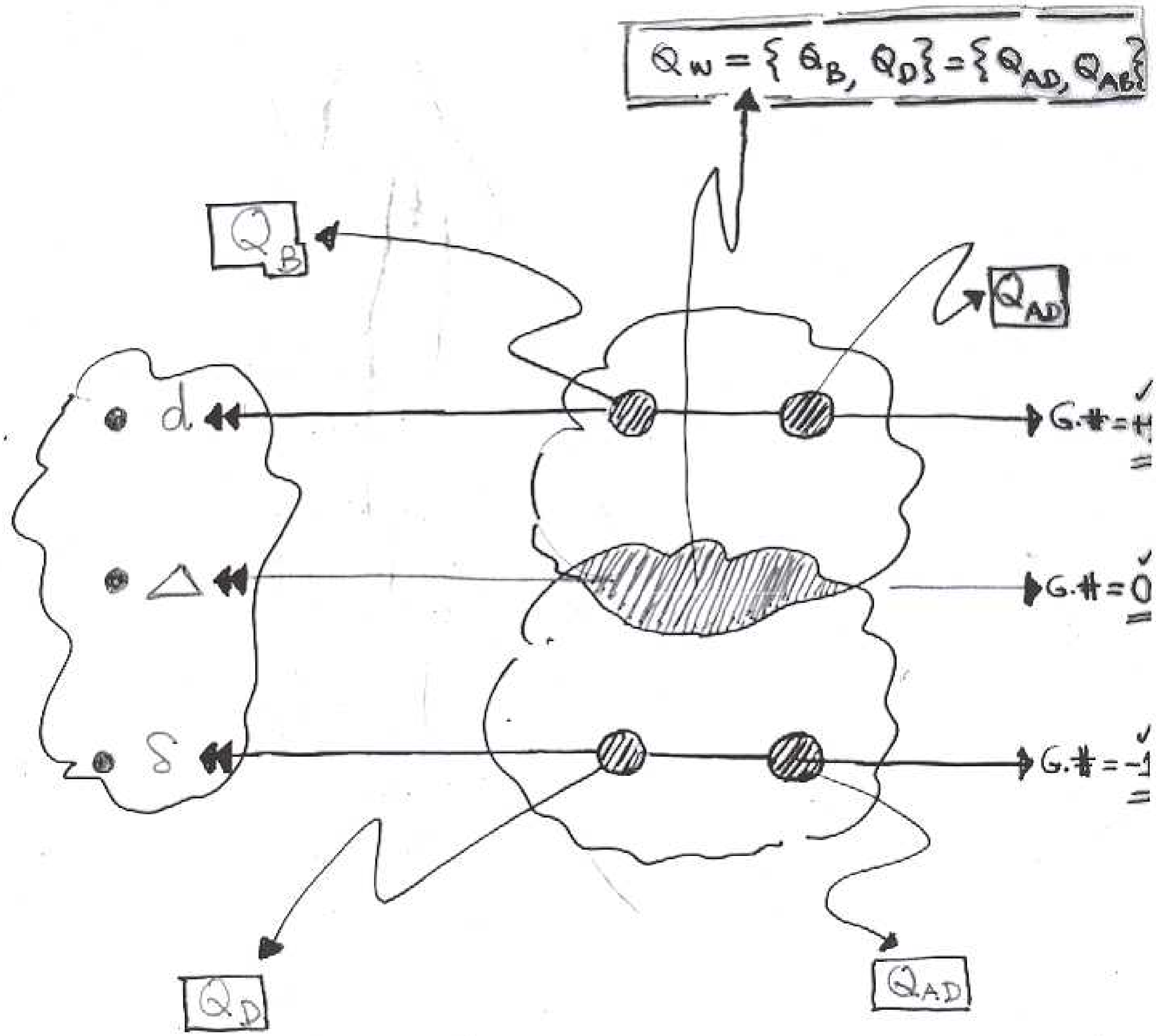
$$i\, Q_g \underbrace{Q_{B(AD)} |\psi\rangle_n} = (n+1) \underbrace{Q_{B(AD)} |\psi\rangle_n}$$

$$i\, Q_g \underbrace{Q_{D(AB)} |\psi\rangle_n} = (n-1) \underbrace{Q_{D(AB)} |\psi\rangle_n}$$

$$i\, Q_g \big(\underbrace{Q_W \quad |\psi\rangle_n}\big) = n \, \big(\underbrace{Q_W |\psi\rangle_n}\big)$$

$$\{Q_B, Q_D\} \equiv \{Q_{AB}, Q_{AD}\}$$

- MAPPING : $Q_{B(AD)} \rightarrow d$; $Q_{D(AB)} \rightarrow \delta$, $Q_W = \Delta$



$2 \rightarrow 1$: MAPPING

$$(Q_B, Q_{AD}) \rightarrow d$$

$$(Q_D, Q_{AB}) \rightarrow \delta$$

$$\{Q_B, Q_D\} = \{Q_{AD}, Q_{AB}\} \rightarrow \Delta = d\delta + \delta d$$

- TWO-TO-ONE : MAPPING ??

— EXPLAINED FOR 2D (1+1) FREE ABELIAN ONE-FORM GAUGE THEORY IN THE GEOMET[RICAL] SUPERFIELD APPROACH TO BRST FORMALIS[M]

— r.p.m: J. Phys. A: Math Gen 35 (2002) 3711

- THE ANALOGUE OF * ??

DISCRETE TRANSFORMATIONS

1)
$$\left.\begin{array}{l} B_{\mu\nu} \to \mp \frac{i}{2}\, \varepsilon_{\mu\nu\kappa\varsigma} B^{\kappa\varsigma} \\ \varphi_1 \to \pm i\, \varphi_2, \quad \mathcal{B}_\mu \to \mp i\, B_\mu \\ \varphi_2 \to \mp i\, \varphi_1, \quad B_\mu \to \pm i\, \mathcal{B}_\mu \end{array}\right\} \Rightarrow$$

BOSONIC PART OF THE LAGN DENSITY REMAI[NS] INVARIANT

$$\left.\begin{array}{l} \mathcal{L}_B^{(0)} = \frac{1}{2} \mathcal{B}^\mu \mathcal{B}_\mu - \mathcal{B}^\mu \left(\frac{1}{2} \varepsilon_{\mu\nu\lambda\varsigma} B^{\lambda\varsigma} - \partial_\mu \varphi_2\right) \\ \quad + B^\mu \left(\partial^\sigma B_{\sigma\mu} - \partial_\mu \varphi_1\right) - \frac{1}{2} B_\mu B^\mu \end{array}\right\} \Rightarrow \widetilde{\mathcal{L}_B^{(0)}}$$

2)
$$\begin{array}{lll} C_\mu \to \pm i \bar{C}_\mu, & \beta \to \mp i \bar{\beta}, & \rho \to \pm i \lambda \\ \bar{C}_\mu \to \pm i C_\mu, & \bar{\beta} \to \pm i \beta, & \lambda \to \pm i \rho \end{array}$$

$$\left.\begin{array}{l} \mathcal{L}_G^{(0)} = (\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu)(\partial^\mu C^\nu) - \partial_\mu \bar{\beta}\, \partial^\mu \beta \\ \quad + \rho(\partial \cdot C + \lambda) + (\partial \cdot \bar{C} + \rho)\lambda \end{array}\right\} \Rightarrow \widetilde{\mathcal{L}_G^{(0)}}$$

- THE TOTAL LAG$^{N.}$ DENSITY

$$\mathcal{L}_D = \mathcal{L}_B^{(0)} + \mathcal{L}_G^{(0)} \xrightarrow{\;*\;} \mathcal{L}_D$$

UNDER ① + ② SYMMETRY TRANSF$^{NS.}$

★ ON ANY GENERIC FIELD $\Phi(x)$, IT CAN BE CHECKED THAT:

$$s_D\, \Phi(x) = \pm * s_B * \Phi(x)$$

$$\boxed{\delta = \pm * d *}$$

$$\Phi(x) \equiv B_{\mu\nu},\ \varphi_1,\ \varphi_2,\ B_\mu,\ \mathcal{B}_\mu,\ C_\mu,\ \bar{C}_\mu,\ \beta,\ \bar{\beta},\ \rho,\ \lambda$$

↳ Deser, Gomberoff, Henneaux, Teitelb
Phys. Lett. 400B (9

- THE SIGNS ($\pm$) IN THE ABOVE ARE DICTATE[D] BY TWO SUCCESSIVE OPERATIONS OF ①+②:

$$*\,(*\,\Phi(x)) = \pm\, \Phi(x)$$

4D-2-form Gauge Theory $\Big\} \Rightarrow$

$$*\,(*B) = +B \Rightarrow \beta, \bar{\beta}, \varphi_1, \varphi_2, B_{\mu\nu}, B_\mu, \ldots$$

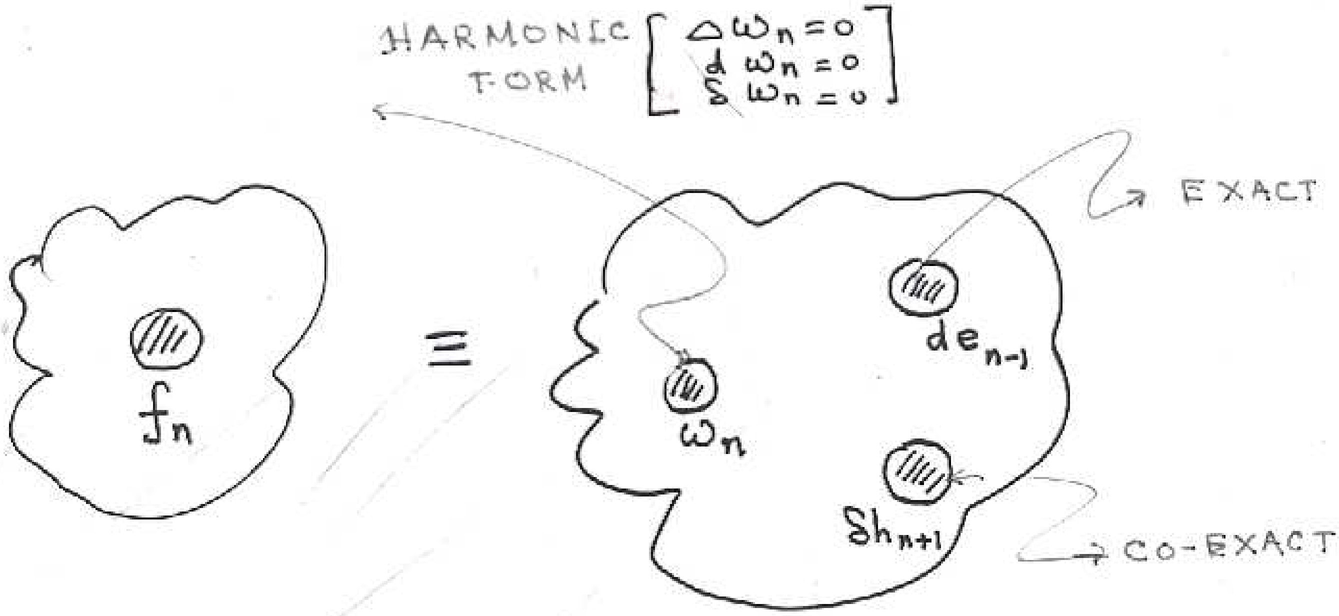
$$*\,(*F) = -F \Rightarrow \rho, \lambda, C_\mu, \bar{C}_\mu$$

- IN EXPLICIT TERMS

$$s_D(B) = + * s_B * (B)$$

$$s_D(F) = - * s_B * (F)$$

4D FREE ABELIAN 2-FORM GAUGE THEORY IS A FIELD THEORETIC MODEL FOR THE HODGE THEORY



$$f_n = \omega_n + d e_{n-1} + \delta h_{n+1}$$

- HODGE DECOMPOSITION THEOREM (HDT)

- HDT IN Q. HILBERT SPACE OF STATES: $i\,Q_g\,|\psi\rangle_n = n\,|\psi\rangle_n$

$$|\psi\rangle_n = |\omega\rangle_n + Q_B\,|\chi\rangle_{n-1} + Q_D\,|\theta\rangle_{n+1}$$

$$\equiv |\omega\rangle_n + Q_{AD}\,|\chi\rangle_{n-1} + Q_{AB}\,|\theta\rangle_{n+1}$$

- AESTHETIC REASONS (MAXIMAL SYMMETR

$$|phys\rangle \equiv |\,\text{HARMONIC STATES}\rangle \equiv |\omega\rangle_n$$

DEF^N

$$\boxed{Q_B\,|\omega\rangle_n = 0, \quad Q_D\,|\omega\rangle_n = 0, \quad Q_W\,|\omega\rangle_n = 0}$$

r.p.m: hep-th/0212240

★ RESTRICTIONS ON THE PHYSICAL VACUUM & PHYSICAL STATES OF THE THEORY:

$$Q_B\,|vac\rangle = 0, \quad Q_D\,|vac\rangle = 0, \quad Q_W\,|vac\rangle = 0$$

$$\left\{ \begin{array}{l} b_{\mu\nu}(k)\,|vac\rangle = 0, \quad C_\mu(k)\,|vac\rangle = 0, \quad \bar{C}_\mu(k)\,|vac\rangle = 0 \\ \bar{b}(k)\,|vac\rangle = 0, \quad f_1(k)\,|vac\rangle = 0, \quad f_2(k)\,|vac\rangle = 0 \end{array} \right.$$

ANNIHILATION OPERATORS

- PHYSICAL STATES $|phys\rangle$

$Q_B|phys\rangle = 0 \Rightarrow$ [1st-CLASS-CONSTRAINT OF THE GAUGE-THEORY] $|phys\rangle = 0$

$Q_D|phys\rangle = 0 \Rightarrow$ [DUAL-VERSION OF THE ABOVE CONSTRAINT] $|phys\rangle = 0$

$Q_W|phys\rangle = 0 \Rightarrow$ [CONSISTENT WITH THE ABOVE TWO]

$\Rightarrow |phys\rangle = 0$

★ # ON-SHELL NILPOTENT (CO-)BRST INVARIANT LAGRANGIAN DENSITY:

$$
\begin{aligned}
\mathcal{L}^{(0)}_{BD} = {} & \frac{1}{2}\left(\partial^\kappa B_{\kappa\mu} - \partial_\mu \varphi_1\right)\left(\partial_\nu B^{\nu\mu} - \partial^\mu \varphi_1\right) \\
& - \frac{1}{2}\left(\frac{1}{2}\varepsilon_{\mu\nu\kappa\zeta}\,\partial^\nu B^{\kappa\zeta} - \partial_\mu \varphi_2\right)\left(\frac{1}{2}\varepsilon^{\mu\xi\sigma\eta}\,\partial_\xi B_{\sigma\eta} - \partial^\mu \varphi_2\right) \\
& - \partial_\mu \bar{\beta}\,\partial^\mu \beta + \left(\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu\right)\left(\partial^\mu C^\nu\right) - \frac{1}{2}\left(\partial\cdot\bar{C}\right)\left(\partial\cdot C\right)
\end{aligned}
$$

- EQUATIONS OF MOTION:

$$\Box\varphi_1 = \Box\varphi_2 = \Box\beta = \Box\bar{\beta} = \Box B_{\mu\nu} = 0$$

$$\Box C_\mu = \frac{3}{2}\,\partial_\mu(\partial\cdot C)\;; \qquad \Box \bar{C}_\mu = \frac{3}{2}\,\partial_\mu(\partial\cdot\bar{C})$$

$\Uparrow$

NO AUXILIARY FIELDS

- TRANSLATION SUBGROUP $T(2)$ OF THE WIGNER'S LITTLE GROUP: → Weinberg (1964)

$$\Updownarrow$$

(DUAL-) GAUGE TRANSFORMATIONS

$$\Updownarrow$$

(CO-) BRST TRANSFORMATIONS

r.p.m. : $\boxed{\text{hep-th/0212240}}$

- RESTRICTIONS:

$$\boxed{C^0(k) = C^3(k)}\,; \quad \boxed{\bar{C}^0(k) = \bar{C}^3(k)}$$

$$\left.\begin{aligned} k^\mu &= (\omega, 0, 0, \omega) \\ k_\mu &= (\omega, 0, 0, -\omega) \end{aligned}\right\} \boxed{k^2 = 0}$$

$$\partial \cdot C \longrightarrow \boxed{k \cdot C = 0}\,; \quad \partial \cdot \bar{C} \longrightarrow \boxed{k \cdot \bar{C} = 0}$$

- FOR THE ABOVE CHOICES

$$k^2 = 0 \quad \Box C_\mu = 0, \quad \Box \bar{C}_\mu = 0 \quad \to k^2 = 0$$

$$\partial \cdot C = 0 \qquad \partial \cdot \bar{C} = 0$$

NORMAL MODE EXPANSION IN TERMS OF CREATION & ANNIHILATION OPERATORS:

$$B_{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^{3/2}(2k_0)^{3/2}} \left[b^{\dagger}_{\mu\nu}(k)\, e^{ik\cdot x} + b_{\mu\nu}(k)\, e^{-ik\cdot x} \right]$$

$$C_\mu(x) = \int \frac{d^3k}{(2\pi)^{3/2}(2k_0)^{3/2}} \left[C^{\dagger}_{\mu}(k)\, e^{ik\cdot x} + C_{\mu}(k)\, e^{-ik\cdot x} \right]$$

$$\overline{C}_\mu(x) = \int \frac{d^3k}{(2\pi)^{3/2}(2k_0)^{3/2}} \left[\overline{C}^{\dagger}_{\mu}(k)\, e^{ik\cdot x} + \overline{C}_{\mu}(k)\, e^{-ik\cdot x} \right]$$

$$\beta(x) = \int \frac{d^3k}{(2\pi)^{3/2}(2k_0)^{3/2}} \left[b^{\dagger}(k)\, e^{+ik\cdot x} + b(k)\, e^{-ik\cdot x} \right]$$

$$\overline{\beta}(x) = \int \frac{d^3k}{(2\pi)^{3/2}(2k_0)^{3/2}} \left[\overline{b}^{\dagger}(k)\, e^{ik\cdot x} + \overline{b}(k)\, e^{-ik\cdot x} \right]$$

$$\varphi_1(x) = \int \frac{d^3k}{(2\pi)^{3/2}(2k^0)^{3/2}} \left[f^{\dagger}_1(k)\, e^{ik\cdot x} + f_1(k)\, e^{-ik\cdot x} \right]$$

$$\varphi_2(x) = \int \frac{d^3k}{(2\pi)^{3/2}(2k^0)^{3/2}} \left[f^{\dagger}_2(k)\, e^{ik\cdot x} + f_2(k)\, e^{-ik\cdot x} \right]$$

- A SINGLE PARTICLE QUANTUM STATE (SPQS) FOR A <u>2-FORM</u> <u>GAUGE</u> <u>FIELD</u> WITH POLARISATION TENSOR ($e^{\mu\nu}(k)$)

$$e^{\mu\nu}(k)\, b^{+}_{\mu\nu}(k)\, |vac\rangle \equiv \| \tilde{e}, vac \rangle\rangle \equiv \| SPQS \rangle\rangle$$

$\Uparrow$

CREATION FROM VACUUM

- PHYSICALITY CONDITION W.R.T. Q_B

$$Q_B \| \tilde{e}, vac \rangle\rangle = 0 \Rightarrow Q_B\, e^{\mu\nu} b^{+}_{\mu\nu} |vac\rangle = 0$$

$$[Q_B, e^{\mu\nu} b^{+}_{\mu\nu}(k)]\, |vac\rangle = 0 \qquad \because \underbrace{Q_B |vac\rangle = 0}_{\text{Def}^{n}}$$

★ $s_B B_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu \equiv +i[Q_B, B_{\mu\nu}(x)]$

$\Downarrow$

INSERT NORMAL MODE EXPANSION

$\Downarrow$

$$[Q_B, b^{+}_{\mu\nu}(k)] = (k_\mu C^{+}_\nu - k_\nu C^{+}_\mu)$$

$$[Q_B, b_{\mu\nu}(k)] = -(k_\mu C_\nu - k_\nu C_\mu)$$

$$\Longrightarrow \quad [Q_B, e^{\mu\nu} b^{\dagger}_{\mu\nu}] |vac\rangle \cong 2 e^{\mu\nu} k_\mu \underbrace{C^{\dagger}_{\nu}(k) |vac\rangle}_{\neq \text{ NOT-ZERO}}$$

$$Q_B \| \tilde{e}, vac \rangle\rangle = 0 \Rightarrow \underbrace{k_\mu e^{\mu\nu} = 0 \equiv - e^{\mu\nu} k_\nu}_{\Downarrow \\ \text{"TRANSVERSALITY" COND}^{n}}$$

- FOR A "LONGITUDINAL" OR "SCALAR" 2-FORM GAUGE FIELD ($k_\mu e^{\mu\nu} = - e^{\mu\nu} k_\nu \neq 0$)

$\Downarrow$

NO-GHOST THEOREM

$\Uparrow$

$(\cdots)\, C^{\dagger}_{\mu} |vac\rangle = Q_B \| \tilde{e}, vac \rangle\rangle$: BRST-EXACT STATE

$\Downarrow$

NOT A PHYSICAL STATE

★ SIMILARILY, CO-BRST CHARGE PRODUCES:

$$Q_D \| SPQS \rangle\rangle = 0 \Longrightarrow \varepsilon_{\mu\nu\eta\xi} k^\eta e^{\mu\nu} (\bar{C}_\xi)^{\dagger} |vac\rangle = 0$$

$$\boxed{\varepsilon_{\mu\nu\eta\xi} e^{\mu\nu} k^\eta = 0} \quad \because (\bar{C}_\mu)^{\dagger} |vac\rangle \neq 0$$

$\longrightarrow$ DUAL-TRANSVERSALITY CONDITION

→ WHERE WE HAVE USED:

$$S_D B_{\mu\nu} = \varepsilon_{\mu\nu\eta\varsigma}\, \partial^\eta \bar{C}^\varsigma \equiv +i[Q_D, B_{\mu\nu}(x)]$$

$$\Downarrow$$

MODE EXPANSION YIELDS

$$[Q_D, b^\dagger_{\mu\nu}] = \varepsilon_{\mu\nu\eta\varsigma}\, k^\eta (\bar{C}_\varsigma)^\dagger$$

$$[Q_D, b_{\mu\nu}] = -\varepsilon_{\mu\nu\eta\varsigma}\, k^\eta (\bar{C}_\varsigma)$$

- FOR "$\varepsilon_{\mu\nu\eta\varsigma} k^\eta e^{\mu\nu} \neq 0$", WE HAVE:

$$\underbrace{(\ldots\ldots)(\bar{C}^\varsigma)^\dagger |vac\rangle}_{} = Q_D \left(e^{\mu\nu}(k)\, b^\dagger_{\mu\nu}(k)\right)|vac\rangle$$

$$\equiv Q_D \| SPQS\rangle\rangle \text{ : BRST CO-EXACT STATE}$$

$$\Downarrow$$

CO-BRST exact STATE

$$\Downarrow$$

$$\boxed{\text{TRIVIAL STATE : } \underline{\underline{\text{NOT}}} \text{ PHYSICAL}}$$

⟸ w.r.t.
(i) BRST-COHOMOLOGY
&
(ii) H.D.T.

- ULTIMATELY, WE HAVE:

$$\left.\begin{array}{l} Q_B \| SPQS\rangle\rangle = 0 \\ Q_D \| SPQS\rangle\rangle = 0 \\ Q_W \| SPQS\rangle\rangle = 0 \end{array}\right\} \Longrightarrow$$

(i) TRANSVERSALITY $(k_\mu e^{\mu\nu} = -e^{\mu\nu} k_\nu$
&
(ii) MASSLESSNESS
$(\Box B_{\mu\nu} = 0 \Rightarrow k^2 = 0)$

- BRST TRANSFORMED (ie gauge trasf^{d}) $\|SPQS\rangle\rangle$ IS GIVEN BY:

$$e^{\mu\nu}(k) \rightarrow e'^{\mu\nu}_{(B)}(k) = e^{\mu\nu}(k) + (k^\mu c^{\dagger\nu} - k^\nu c^{\dagger\mu})$$

$$\Rightarrow \|SPQS\rangle\rangle \rightarrow \|SPQS\rangle\rangle + \text{BRST-EXACT STATE}$$

- MATHEMATICALLY:

$$\underbrace{\left[e^{\mu\nu}(k) + (k^\mu c^{\dagger\nu} - k^\nu c^{\dagger\mu})\right] b^\dagger_{\mu\nu}(k)\,|vac\rangle}_{\|SPQS\rangle\rangle'_{(B)}}$$

$$= \|\tilde{e}, vac\rangle\rangle + Q_B\left(-2\,c^{\mu\dagger}\bar{c}^\dagger_\mu(k)\right)|vac\rangle$$

$$= \|SPQS\rangle\rangle + \text{BRST-EXACT STATE}$$

PHYSICAL TRIVIAL STATE

$\Downarrow$

★ # $\|SPQS\rangle\rangle'^{(g)}_{(B)}$ AND $\|SPQS\rangle\rangle$ BELONG TO THE SAME COHOMOLOGY CLASS W.R.T. "Q_B" ($Q_B^2 = 0$, $\dot{Q}_B = 0$)

Landi-NCGQFT '03

$\Downarrow$

$$Q_B\|SPQS\rangle\rangle'^{(g)}_{(B)} = Q_B\|SPQS\rangle\rangle = 0$$

$\Uparrow$

PHYSICALITY CRITERIA

- SIMILARLY CO-BRST (ie. dual-gauge) TRANSFORMS STATES, WITH THE FOLLOWING CHANGES

$$e^{\mu\nu}(k) \longrightarrow e'^{\mu\nu}_{(dB)}(k) = e^{\mu\nu}(k) + \varepsilon^{\mu\nu\sigma\varsigma}\, k_\sigma \bar{c}^\dagger_\varsigma$$

CAN BE EXPRESSED AS:

$$||SPQS\rangle\rangle \longrightarrow ||SPQS\rangle\rangle'_{(dB)} = ||SPQS\rangle\rangle + Q_D(-2\bar{c}^\dagger_\mu c^\dagger_\mu)|va$$

★ $||SPQS\rangle\rangle^{(dg)}_{(dB)}$ AND $||SPQS\rangle\rangle$ DIFFER BY A BRST CO-EXACT STATE. THUS, THEY BELONG TO THE SAME co-COHOMOLOGY-CLASS W.R.T. Q_D (ie. $Q_D^2 = 0$, $\dot{Q}_D = 0$).

⇓

$\boxed{\text{FREE 2-FORM ABELIAN GAUGE THEORY}}$

A FIELD THEORETIC MODEL FOR THE HODGE THEORY IN 4D SPACE-TIME

⇑

J. Phys. A: Math Gen (2000)

A QUASI-TOPOLOGICAL FIELD THEORY

(hep-th/0209136)

INTERESTING CONNECTION WITH 4D WIGNER'S LITTLE GP.

(hep-th/0212240)

2D (non-) ABELIAN GAUGE THEORIES

Int. J. Mod. Phys. A 15 (2000)
J. Phys. A: Math Gen 33 (2000)
→ Abelian Gauge Theory

J. Phys. A: Math Gen 34 (2001)
Mod. Phys. Lett. A 14 (1999)
→ non-Abelian Gauge Theory

Phys. Lett. B521 (2001)
J. Phys. A: Math Gen 35 (2002) 3711
Mod. Phys. Lett A17 (2002)
J. Phys. A: Math Gen 35 (2002) 6919
J. Phys. A: Math Gen 35 (2002) 8817
→ Superfield Approach → Geometrical Origin & Mappings

Mod. Phys. Lett A15 (2000)
Mod. Phys. Lett. A 16 (2001)
→ Interacting Theory (DIRAC FIELDS + U(1) GAUGE FIELD)

hep-th/0205135: Cohomological Aspects of Gauge Theories: Superfied formalism
(Review)

4D - FREE 2-FORM GAUGE THEORY

J. Phys. A: Math Gen. 33 (2000) E. Harikumar, r.p.m and M. Sivakumar

hep-th/0209136
→ r.p.m.