Kappa-deformed space-time: Field Theory and Twisted Symmetry

E. Harikumar[1]

School of Physics
University of Hyderabad
Hyderabad

December-2008

[1] harisp@uohyd.ernet.in

Motivations....

- Quantum gravity can be, possibly modeled using non-commutative space-time
- $l_{Planck} = \sqrt{\frac{hG}{c^3}}$ may have a significant role to play in q-gravity.
  (a) String theory models predict existence of minimum length scale
  (b) Area and volume operators in certain loop gravity models have discrete spectra with minimal values. These minimal values are proportional to l_p^2 and lp^3 respectively.
- l_p sets a minimum length scale l_{min} in models describing quantum gravity

Motivations....

- Quantum gravity can be, possibly modeled using non-commutative space-time
- $l_{Planck} = \sqrt{\frac{hG}{c^3}}$ may have a significant role to play in q-gravity.
  (a) String theory models predict existence of minimum length scale
  (b) Area and volume operators in certain loop gravity models have discrete spectra with minimal values. These minimal values are proportional to l_p^2 and lp^3 respectively.
- l_p sets a minimum length scale l_{min} in models describing quantum gravity

Motivations....

- Quantum gravity can be, possibly modeled using non-commutative space-time
- $l_{Planck} = \sqrt{\frac{hG}{c^3}}$ may have a significant role to play in q-gravity.
  (a) String theory models predict existence of minimum length scale
  (b) Area and volume operators in certain loop gravity models have discrete spectra with minimal values. These minimal values are proportional to l_p^2 and lp^3 respectively.
- l_p sets a minimum length scale l_{min} in models describing quantum gravity

- Special Theory of Relativity: Laws of physics must be same in all inertial frames
- If $l_s \geq l_{min}$, $l_{s'} \geq l_{min}$.
- But this is not guaranteed(!) due to Lorentz-Fitsgerald length contraction
- **Modify STR** Space-time structure is governed not only by a fundamental velocity scale c, but also by a fundamental length scale l_p. **D**oubly **S**pecial **R**elativity
- DSR introduces a minimum length scale *without* singling out any preferred frame
- The Energy-Momentum relation get a length scale dependent modification.
- Ex: $E^2 = p^2c^2 + m^2c^4 + \alpha l_p E^3 + \beta l_p^2 E^4 + \ldots\ldots$

- Special Theory of Relativity: Laws of physics must be same in all inertial frames
- If $l_s \geq l_{min}$, $l_{s'} \geq l_{min}$.
- But this is not guaranteed(!) due to Lorentz-Fitsgerald length contraction
- **Modify STR** Space-time structure is governed not only by a fundamental velocity scale c, but also by a fundamental length scale l_p. **D**oubly **S**pecial **R**elativity
- DSR introduces a minimum length scale *without* singling out any preferred frame
- The Energy-Momentum relation get a length scale dependent modification.
- Ex: $E^2 = p^2c^2 + m^2c^4 + \alpha l_p E^3 + \beta l_p^2 E^4 + \ldots\ldots$

- Special Theory of Relativity: Laws of physics must be same in all inertial frames
- If $l_s \geq l_{min}$, $l_{s'} \geq l_{min}$.
- But this is not guaranteed(!) due to Lorentz-Fitsgerald length contraction
- **Modify STR** Space-time structure is governed not only by a fundamental velocity scale c, but also by a fundamental length scale l_p. **D**oubly **S**pecial **R**elativity
- DSR introduces a minimum length scale *without* singling out any preferred frame
- The Energy-Momentum relation get a length scale dependent modification.
- Ex: $E^2 = p^2c^2 + m^2c^4 + \alpha l_p E^3 + \beta l_p^2 E^4 + \ldots\ldots$

- Special Theory of Relativity: Laws of physics must be same in all inertial frames
- If $l_s \geq l_{min}$, $l_{s'} \geq l_{min}$.
- But this is not guaranteed(!) due to Lorentz-Fitsgerald length contraction
- **Modify STR** Space-time structure is governed not only by a fundamental velocity scale c, but also by a fundamental length scale l_p. **D**oubly **S**pecial **R**elativity
- DSR introduces a minimum length scale *without* singling out any preferred frame
- The Energy-Momentum relation get a length scale dependent modification.
- Ex: $E^2 = p^2c^2 + m^2c^4 + \alpha l_p E^3 + \beta l_p^2 E^4 + ...$

- Special Theory of Relativity: Laws of physics must be same in all inertial frames
- If $l_s \geq l_{min}$, $l_{s'} \geq l_{min}$.
- But this is not guaranteed(!) due to Lorentz-Fitsgerald length contraction
- **Modify STR** Space-time structure is governed not only by a fundamental velocity scale c, but also by a fundamental length scale l_p. **D**oubly **S**pecial **R**elativity
- DSR introduces a minimum length scale *without* singling out any preferred frame
- The Energy-Momentum relation get a length scale dependent modification.
- Ex: $E^2 = p^2c^2 + m^2c^4 + \alpha l_p E^3 + \beta l_p^2 E^4 + ...$

- Special Theory of Relativity: Laws of physics must be same in all inertial frames
- If $l_s \geq l_{min}$, $l_{s'} \geq l_{min}$.
- But this is not guaranteed(!) due to Lorentz-Fitsgerald length contraction
- **Modify STR** Space-time structure is governed not only by a fundamental velocity scale c, but also by a fundamental length scale l_p. **D**oubly **S**pecial **R**elativity
- DSR introduces a minimum length scale *without* singling out any preferred frame
- The Energy-Momentum relation get a length scale dependent modification.
- Ex: $E^2 = p^2c^2 + m^2c^4 + \alpha l_p E^3 + \beta l_p^2 E^4 + \ldots\ldots$

- Special Theory of Relativity: Laws of physics must be same in all inertial frames
- If $l_s \geq l_{min}$, $l_{s'} \geq l_{min}$.
- But this is not guaranteed(!) due to Lorentz-Fitsgerald length contraction
- **Modify STR** Space-time structure is governed not only by a fundamental velocity scale c, but also by a fundamental length scale l_p. **D**oubly **S**pecial **R**elativity
- DSR introduces a minimum length scale *without* singling out any preferred frame
- The Energy-Momentum relation get a length scale dependent modification.
- Ex: $E^2 = p^2c^2 + m^2c^4 + \alpha l_p E^3 + \beta l_p^2 E^4 + ...$

Modified dispersion relations

- Many Q-gravity models do give modified Energy-Momentum relations
  - Observations of ultra high energy cosmic ray scattering contradicts standard notions of astroparticle physics.
  - These observations can be explained if the threshold energies required for these processes are not dictated by usual Energy-Momentum relations but by modified ones involving a length scale!
- DSR: Two seemingly different models were constructed recently.
- Are they related? Equivalent?
- IS DSR UNIQUE?
- We will come back to this.

Modified dispersion relations

- Many Q-gravity models do give modified Energy-Momentum relations
- Observations of ultra high energy cosmic ray scattering contradicts standard notions of astroparticle physics.
- These observations can be explained if the threshold energies required for these processes are not dictated by usual Energy-Momentum relations but by modified ones involving a length scale!

- DSR: Two seemingly different models were constructed recently.
- Are they related? Equivalent?
- IS DSR UNIQUE?
- We will come back to this.

Modified dispersion relations

- Many Q-gravity models do give modified Energy-Momentum relations
- Observations of ultra high energy cosmic ray scattering contradicts standard notions of astroparticle physics.
- These observations can be explained if the threshold energies required for these processes are not dictated by usual Energy-Momentum relations but by modified ones involving a length scale!

- DSR: Two seemingly different models were constructed recently.
- Are they related? Equivalent?
- IS DSR UNIQUE?
- We will come back to this.

Modified dispersion relations

    - Many Q-gravity models do give modified Energy-Momentum relations
    - Observations of ultra high energy cosmic ray scattering contradicts standard notions of astroparticle physics.
    - These observations can be explained if the threshold energies required for these processes are not dictated by usual Energy-Momentum relations but by modified ones involving a length scale!
- DSR: Two seemingly different models were constructed recently.
- Are they related? Equivalent?
- IS DSR UNIQUE?
- We will come back to this.

Modified dispersion relations

- Many Q-gravity models do give modified Energy-Momentum relations
- Observations of ultra high energy cosmic ray scattering contradicts standard notions of astroparticle physics.
- These observations can be explained if the threshold energies required for these processes are not dictated by usual Energy-Momentum relations but by modified ones involving a length scale!

- DSR: Two seemingly different models were constructed recently.
- Are they related? Equivalent?
- IS DSR UNIQUE?
- We will come back to this.

Modified dispersion relations

- Many Q-gravity models do give modified Energy-Momentum relations
- Observations of ultra high energy cosmic ray scattering contradicts standard notions of astroparticle physics.
- These observations can be explained if the threshold energies required for these processes are not dictated by usual Energy-Momentum relations but by modified ones involving a length scale!

- DSR: Two seemingly different models were constructed recently.
- Are they related? Equivalent?
- IS DSR UNIQUE?
- We will come back to this.

Modified dispersion relations

- Many Q-gravity models do give modified Energy-Momentum relations
- Observations of ultra high energy cosmic ray scattering contradicts standard notions of astroparticle physics.
- These observations can be explained if the threshold energies required for these processes are not dictated by usual Energy-Momentum relations but by modified ones involving a length scale!

- DSR: Two seemingly different models were constructed recently.
- Are they related? Equivalent?
- IS DSR UNIQUE?
- We will come back to this.

DSR and k-deformed space-time

- There are certain q-gravity models whose low energy limit shows modified Energy-Momentum relations as in DSR.
- These q-gravity models with $\Lambda > 0$ (and goes over to $\Lambda = 0$ limit smoothly) are shown to have deformed de Sitter group as the symmetry group. The deformation parameter q here is related to l_p as in $q = l_p\sqrt{\Lambda}$.
- In the $\Lambda \to 0$ limit, the symmetry group reduces to k-Poincare group and NOT Poincare group.

- Algebraic structure governing the deformation of Energy-Momentum relation in these models at Planck scale is k-Poincare algebra

DSR and k-deformed space-time

- There are certain q-gravity models whose low energy limit shows modified Energy-Momentum relations as in DSR.
- These q-gravity models with $\Lambda > 0$ (and goes over to $\Lambda = 0$ limit smoothly) are shown to have deformed de Sitter group as the symmetry group. The deformation parameter q here is related to l_p as in $q = l_p\sqrt{\Lambda}$.
- In the $\Lambda \to 0$ limit, the symmetry group reduces to k-Poincare group and NOT Poincare group.

- Algebraic structure governing the deformation of Energy-Momentum relation in these models at Planck scale is k-Poincare algebra

DSR and k-deformed space-time

- There are certain q-gravity models whose low energy limit shows modified Energy-Momentum relations as in DSR.
- These q-gravity models with $\Lambda > 0$ (and goes over to $\Lambda = 0$ limit smoothly) are shown to have deformed de Sitter group as the symmetry group. The deformation parameter q here is related to l_p as in $q = l_p\sqrt{\Lambda}$.
- In the $\Lambda \to 0$ limit, the symmetry group reduces to k-Poincare group and NOT Poincare group.

- Algebraic structure governing the deformation of Energy-Momentum relation in these models at Planck scale is k-Poincare algebra

DSR and k-deformed space-time

- There are certain q-gravity models whose low energy limit shows modified Energy-Momentum relations as in DSR.
- These q-gravity models with $\Lambda > 0$ (and goes over to $\Lambda = 0$ limit smoothly) are shown to have deformed de Sitter group as the symmetry group. The deformation parameter q here is related to l_p as in $q = l_p\sqrt{\Lambda}$.
- In the $\Lambda \to 0$ limit, the symmetry group reduces to k-Poincare group and NOT Poincare group.

- Algebraic structure governing the deformation of Energy-Momentum relation in these models at Planck scale is k-Poincare algebra

Moyal space: summary of essential results

- Generic NC spaces are defined with co-ordinates obeying

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{k^2}\Theta_{\mu\nu}(k\hat{x})$$

- $\Theta_{\mu\nu}(k\hat{x}) = \theta^0_{\mu\nu} + \theta_{\mu\nu}{}^{\lambda}\hat{x}_\lambda + \theta_{\mu\nu}{}^{\lambda\sigma}\hat{x}_\lambda\hat{x}_\sigma + \ldots\ldots\ldots$

- Moyal space is the one where $\theta_{\mu\nu}{}^{\lambda}, \theta_{\mu\nu}{}^{\lambda\sigma}, \ldots..$ all are set to *ZERO*.

$$[\hat{X}_\mu, \hat{X}_\nu] = i\theta_{\mu\nu}$$

- Weyl-Moyal map:

$$\hat{f} = \int dk dx f(x) e^{ik\cdot(\hat{X}-x)}$$

Moyal space: summary of essential results

- Generic NC spaces are defined with co-ordinates obeying

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{k^2}\Theta_{\mu\nu}(k\hat{x})$$

- $\Theta_{\mu\nu}(k\hat{x}) = \theta^0_{\mu\nu} + \theta_{\mu\nu}{}^{\lambda}\hat{x}_\lambda + \theta_{\mu\nu}{}^{\lambda\sigma}\hat{x}_\lambda\hat{x}_\sigma + ...$
- Moyal space is the one where $\theta_{\mu\nu}{}^{\lambda}, \theta_{\mu\nu}{}^{\lambda\sigma}, ...$ all are set to *ZERO*.

$$[\hat{X}_\mu, \hat{X}_\nu] = i\theta_{\mu\nu}$$

- Weyl-Moyal map:

$$\hat{f} = \int dkdx f(x) e^{ik\cdot(\hat{X}-x)}$$

Moyal space: summary of essential results

- Generic NC spaces are defined with co-ordinates obeying

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{k^2}\Theta_{\mu\nu}(k\hat{x})$$

- $\Theta_{\mu\nu}(k\hat{x}) = \theta^0_{\mu\nu} + \theta_{\mu\nu}{}^{\lambda}\hat{x}_\lambda + \theta_{\mu\nu}{}^{\lambda\sigma}\hat{x}_\lambda\hat{x}_\sigma + \ldots\ldots\ldots$

- Moyal space is the one where $\theta_{\mu\nu}{}^{\lambda}, \theta_{\mu\nu}{}^{\lambda\sigma}, \ldots..$ all are set to *ZERO*.

$$[\hat{X}_\mu, \hat{X}_\nu] = i\theta_{\mu\nu}$$

- Weyl-Moyal map:

$$\hat{f} = \int dk dx f(x) e^{ik\cdot(\hat{X}-x)}$$

Moyal space: summary of essential results

- Generic NC spaces are defined with co-ordinates obeying

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{k^2}\Theta_{\mu\nu}(k\hat{x})$$

- $\Theta_{\mu\nu}(k\hat{x}) = \theta^0_{\mu\nu} + \theta_{\mu\nu}{}^{\lambda}\hat{x}_\lambda + \theta_{\mu\nu}{}^{\lambda\sigma}\hat{x}_\lambda\hat{x}_\sigma + \ldots\ldots\ldots$

- Moyal space is the one where $\theta_{\mu\nu}{}^{\lambda}, \theta_{\mu\nu}{}^{\lambda\sigma}, \ldots\ldots$ all are set to *ZERO*.

$$[\hat{X}_\mu, \hat{X}_\nu] = i\theta_{\mu\nu}$$

- Weyl-Moyal map:

$$\hat{f} = \int dkdx f(x) e^{ik\cdot(\hat{X}-x)}$$

Moyal space:......

- $\hat{f}\hat{g}$ induces a modified product rule:
  Moyal Star Product $f * g$
- $$f * g = f(x) e^{\frac{i}{2}\partial^x_\mu \theta^{\mu\nu} \partial^y_\nu} g(y)|_{x=y}$$
- 1. $*$ product is associative
  2. $\int dx f * g = \int dx f g$
  3. $\int dx(f * g * h) = \int dx(g * h * f) = \int dx(h * f * g)$
  4. $(f * g)^{cc} = g^{cc} * f^{cc}$
     - Quadratic part of the NC action is same as the commutative one
       Propagator is not modified: no change in dispersion relations
       Interactions are modified

Moyal space:......

- $\hat{f}\hat{g}$ induces a modified product rule: Moyal Star Product $f * g$
- $$f * g = f(x) e^{\frac{i}{2}\partial^x_\mu \theta^{\mu\nu} \partial^y_\nu} g(y)|_{x=y}$$
- 1. $*$ product is associative
  2. $\int dx f * g = \int dx f g$
  3. $\int dx (f * g * h) = \int dx (g * h * f) = \int dx (h * f * g)$
  4. $(f * g)^{cc} = g^{cc} * f^{cc}$
  - Quadratic part of the NC action is same as the commutative one

    Propagator is not modified: no change in dispersion relations

    Interactions are modified

Moyal space:......

- $\hat{f}\hat{g}$ induces a modified product rule:
  Moyal Star Product $f * g$
- $$f * g = f(x) e^{\frac{i}{2}\partial^x_\mu \theta^{\mu\nu} \partial^y_\nu} g(y)|_{x=y}$$
- 1. $*$ product is associative
  2. $\int dx f * g = \int dx f g$
  3. $\int dx (f * g * h) = \int dx (g * h * f) = \int dx (h * f * g)$
  4. $(f * g)^{cc} = g^{cc} * f^{cc}$
     - Quadratic part of the NC action is same as the commutative one
       Propagator is not modified: no change in dispersion relations
       Interactions are modified

Moyal space:......

- $\hat{f}\hat{g}$ induces a modified product rule:
  Moyal Star Product $f * g$
- $$f * g = f(x) e^{\frac{i}{2}\partial^x_\mu \theta^{\mu\nu} \partial^y_\nu} g(y)|_{x=y}$$
- 1. $*$ product is associative
  2. $\int dx f * g = \int dx f g$
  3. $\int dx (f * g * h) = \int dx (g * h * f) = \int dx (h * f * g)$
  4. $(f * g)^{cc} = g^{cc} * f^{cc}$

  - Quadratic part of the NC action is same as the commutative one
    Propagator is not modified: no change in dispersion relations
    Interactions are modified

Twisted symmetry

- $[\hat{X}_\mu, \hat{X}_\nu] = i\theta_{\mu\nu}$ breaks the Lorentz invariance of Moyal plane.
- The notion of fields transforming under representations of Poincare group is in trouble -Can not view field quanta as particles with definite spin and mass

Twisted symmetry

- $[\hat{X}_\mu, \hat{X}_\nu] = i\theta_{\mu\nu}$ breaks the Lorentz invariance of Moyal plane.
- The notion of fields transforming under representations of Poincare group is in trouble -Can not view field quanta as particles with definite spin and mass

- in commutative space $f_{\rho\sigma} = (x_\rho x_\sigma)$ transform as a rank-2 tensor,

$$M_{\mu\nu} f_{\rho\sigma} = i(f_{\mu\sigma}\eta_{\nu\rho} - f_{\nu\sigma}\eta_{\mu\rho} + f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\mu}\eta_{\nu\sigma})$$

- Chaichian and co workers showed that the symmetry algebra of Moyal spacetime is realised by the *twisted* Poincare-Hopf algebra and not by the Poincare algebra
- $f_{\rho\sigma} = \frac{1}{2}(x_\rho * x_\sigma + x_\sigma * x_\rho)$ transform as a rank-2 tensor under twisted action, i.e.,

$$M^t_{\mu\nu} f_{\rho\sigma} = i(f_{\mu\sigma}\eta_{\nu\rho} - f_{\nu\sigma}\eta_{\mu\rho} + f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\mu}\eta_{\nu\sigma}).$$

- $$M^t_{\mu\nu}([x_\rho, x_\sigma]_*) = 0 = M^t_{\mu\nu}\theta_{\rho\sigma}$$

.

- in commutative space $f_{\rho\sigma} = (x_\rho x_\sigma)$ transform as a rank-2 tensor,

$$M_{\mu\nu} f_{\rho\sigma} = i(f_{\mu\sigma}\eta_{\nu\rho} - f_{\nu\sigma}\eta_{\mu\rho} + f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\mu}\eta_{\nu\sigma})$$

- Chaichian and co workers showed that the symmetry algebra of Moyal spacetime is realised by the *twisted* Poincare-Hopf algebra and not by the Poincare algebra
- $f_{\rho\sigma} = \frac{1}{2}(x_\rho * x_\sigma + x_\sigma * x_\rho)$ transform as a rank-2 tensor under twisted action, i.e.,

$$M^t_{\mu\nu} f_{\rho\sigma} = i(f_{\mu\sigma}\eta_{\nu\rho} - f_{\nu\sigma}\eta_{\mu\rho} + f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\mu}\eta_{\nu\sigma}).$$

- $$M^t_{\mu\nu}([x_\rho, x_\sigma]_*) = 0 = M^t_{\mu\nu}\theta_{\rho\sigma}$$

.

- in commutative space $f_{\rho\sigma} = (x_\rho x_\sigma)$ transform as a rank-2 tensor,

$$M_{\mu\nu} f_{\rho\sigma} = i(f_{\mu\sigma}\eta_{\nu\rho} - f_{\nu\sigma}\eta_{\mu\rho} + f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\mu}\eta_{\nu\sigma})$$

- Chaichian and co workers showed that the symmetry algebra of Moyal spacetime is realised by the *twisted* Poincare-Hopf algebra and not by the Poincare algebra
- $f_{\rho\sigma} = \frac{1}{2}(x_\rho * x_\sigma + x_\sigma * x_\rho)$ transform as a rank-2 tensor under twisted action, i.e.,

$$M^t_{\mu\nu} f_{\rho\sigma} = i(f_{\mu\sigma}\eta_{\nu\rho} - f_{\nu\sigma}\eta_{\mu\rho} + f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\mu}\eta_{\nu\sigma}).$$

-

$$M^t_{\mu\nu}([x_\rho, x_\sigma]_*) = 0 = M^t_{\mu\nu}\theta_{\rho\sigma}$$

.

- in commutative space $f_{\rho\sigma} = (x_\rho x_\sigma)$ transform as a rank-2 tensor,

$$M_{\mu\nu} f_{\rho\sigma} = i(f_{\mu\sigma}\eta_{\nu\rho} - f_{\nu\sigma}\eta_{\mu\rho} + f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\mu}\eta_{\nu\sigma})$$

- Chaichian and co workers showed that the symmetry algebra of Moyal spacetime is realised by the *twisted* Poincare-Hopf algebra and not by the Poincare algebra
- $f_{\rho\sigma} = \frac{1}{2}(x_\rho * x_\sigma + x_\sigma * x_\rho)$ transform as a rank-2 tensor under twisted action, i.e.,

$$M^t_{\mu\nu} f_{\rho\sigma} = i(f_{\mu\sigma}\eta_{\nu\rho} - f_{\nu\sigma}\eta_{\mu\rho} + f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\nu}\eta_{\mu\sigma} - f_{\rho\mu}\eta_{\nu\sigma}).$$

- $$M^t_{\mu\nu}([x_\rho, x_\sigma]_*) = 0 = M^t_{\mu\nu}\theta_{\rho\sigma}$$

.

- Attempts to construct NC gravity by demanding a compatibility between $*$ product and the action of deformed generators led to the twisted Leibnitz rule for the symmetry generators.

$$\begin{array}{ccc} \alpha \otimes \beta & \longrightarrow & (\rho \otimes \rho)\Delta(g)\alpha \otimes \beta \\ m \downarrow & & \downarrow m \\ m(\alpha \otimes \beta) & \longrightarrow & \rho(g)m(\alpha \otimes \beta) \end{array}$$

- It was argued that the twisted Hopf structure of the symmetries have interesting implications in field theory
- We study the k-Poincare algebra which is the symmetry algebra of k-deformed spacetime, construction of field theory on k-spacetime and some of the interesting properties of this theory.

- Attempts to construct NC gravity by demanding a compatibility between $*$ product and the action of deformed generators led to the twisted Leibnitz rule for the symmetry generators.

$$\begin{array}{ccc} \alpha \otimes \beta & \longrightarrow & (\rho \otimes \rho)\Delta(g)\alpha \otimes \beta \\ m \downarrow & & \downarrow m \\ m(\alpha \otimes \beta) & \longrightarrow & \rho(g)m(\alpha \otimes \beta) \end{array}$$

- It was argued that the twisted Hopf structure of the symmetries have interesting implications in field theory
- We study the k-Poincare algebra which is the symmetry algebra of k-deformed spacetime, construction of field theory on k-spacetime and some of the interesting properties of this theory.

- Attempts to construct NC gravity by demanding a compatibility between $*$ product and the action of deformed generators led to the twisted Leibnitz rule for the symmetry generators.

$$\begin{array}{ccc} \alpha \otimes \beta & \longrightarrow & (\rho \otimes \rho)\Delta(g)\alpha \otimes \beta \\ m \downarrow & & \downarrow m \\ m(\alpha \otimes \beta) & \longrightarrow & \rho(g)m(\alpha \otimes \beta) \end{array}$$

- It was argued that the twisted Hopf structure of the symmetries have interesting implications in field theory
- We study the k-Poincare algebra which is the symmetry algebra of k-deformed spacetime, construction of field theory on k-spacetime and some of the interesting properties of this theory.

k-spacetime

- Generic NC spaces are defined with co-ordinates obeying

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{k^2}\Theta_{\mu\nu}(k\hat{x})$$

where $\Theta_{\mu\nu}(k\hat{x}) = \theta^0_{\mu\nu} + \theta_{\mu\nu}{}^{\lambda}\hat{x}_\lambda + \theta_{\mu\nu}{}^{\lambda\sigma}\hat{x}_\lambda\hat{x}_\sigma + \ldots\ldots\ldots$

- with $\theta^0_{\mu\nu} = 0, \theta_{\mu\nu}{}^{\lambda\sigma} = 0, \ldots$
- Only non-vanishing term $\theta_{\mu\nu}{}^{\lambda}$
- Thus we have $[\hat{x}_\mu, \hat{x}_\nu] = iC^\lambda_{\mu\nu}\hat{x}_\lambda$ Lie algebraic type NC
- choice: $C^\lambda_{\mu\nu} = a_\mu\delta_{\nu\lambda} - a_\nu\delta_{\mu\lambda}, a_\mu, \mu = 0, 1, \ldots., n-1$ are real
- choice: $a_0 = a = \frac{1}{k}$, $a_i = 0, i = 1, 2, \ldots\ldots, n-1$

k-spacetime

- Generic NC spaces are defined with co-ordinates obeying

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{k^2}\Theta_{\mu\nu}(k\hat{x})$$

where $\Theta_{\mu\nu}(k\hat{x}) = \theta^0_{\mu\nu} + \theta_{\mu\nu}{}^{\lambda}\hat{x}_\lambda + \theta_{\mu\nu}{}^{\lambda\sigma}\hat{x}_\lambda\hat{x}_\sigma + \ldots\ldots\ldots$

- with $\theta^0_{\mu\nu} = 0, \theta_{\mu\nu}{}^{\lambda\sigma} = 0, \ldots$
- Only non-vanishing term $\theta_{\mu\nu}{}^{\lambda}$
- Thus we have $[\hat{x}_\mu, \hat{x}_\nu] = iC^\lambda_{\mu\nu}\hat{x}_\lambda$ Lie algebraic type NC
- choice: $C^\lambda_{\mu\nu} = a_\mu\delta_{\nu\lambda} - a_\nu\delta_{\mu\lambda}, a_\mu, \mu = 0, 1, ..., n-1$ are real
- choice: $a_0 = a = \frac{1}{k}$, $a_i = 0, i = 1, 2, ..., n-1$

k-spacetime

- Generic NC spaces are defined with co-ordinates obeying

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{k^2}\Theta_{\mu\nu}(k\hat{x})$$

where $\Theta_{\mu\nu}(k\hat{x}) = \theta^0_{\mu\nu} + \theta_{\mu\nu}{}^{\lambda}\hat{x}_\lambda + \theta_{\mu\nu}{}^{\lambda\sigma}\hat{x}_\lambda\hat{x}_\sigma + \dots\dots\dots$

- with $\theta^0_{\mu\nu} = 0, \theta_{\mu\nu}{}^{\lambda\sigma} = 0, \dots$
- Only non-vanishing term $\theta_{\mu\nu}{}^{\lambda}$
- Thus we have $[\hat{x}_\mu, \hat{x}_\nu] = iC^\lambda_{\mu\nu}\hat{x}_\lambda$ Lie algebraic type NC
- choice: $C^\lambda_{\mu\nu} = a_\mu\delta_{\nu\lambda} - a_\nu\delta_{\mu\lambda}, a_\mu, \mu = 0, 1, ..., n-1$ are real
- choice: $a_0 = a = \frac{1}{k}$, $a_i = 0, i = 1, 2, ..., n-1$

k-spacetime

- Generic NC spaces are defined with co-ordinates obeying

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{k^2}\Theta_{\mu\nu}(k\hat{x})$$

where $\Theta_{\mu\nu}(k\hat{x}) = \theta^0_{\mu\nu} + \theta_{\mu\nu}{}^{\lambda}\hat{x}_\lambda + \theta_{\mu\nu}{}^{\lambda\sigma}\hat{x}_\lambda\hat{x}_\sigma + \ldots\ldots\ldots$

- with $\theta^0_{\mu\nu} = 0, \theta_{\mu\nu}{}^{\lambda\sigma} = 0, \ldots$
- Only non-vanishing term $\theta_{\mu\nu}{}^{\lambda}$
- Thus we have $[\hat{x}_\mu, \hat{x}_\nu] = iC^\lambda_{\mu\nu}\hat{x}_\lambda$ Lie algebraic type NC
- choice: $C^\lambda_{\mu\nu} = a_\mu\delta_{\nu\lambda} - a_\nu\delta_{\mu\lambda}, a_\mu, \mu = 0, 1, \ldots., n-1$ are real
- choice: $a_0 = a = \frac{1}{k}$, $a_i = 0, i = 1, 2, \ldots\ldots, n-1$

k-spacetime

- Generic NC spaces are defined with co-ordinates obeying

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{k^2}\Theta_{\mu\nu}(k\hat{x})$$

where $\Theta_{\mu\nu}(k\hat{x}) = \theta^0_{\mu\nu} + \theta_{\mu\nu}{}^{\lambda}\hat{x}_\lambda + \theta_{\mu\nu}{}^{\lambda\sigma}\hat{x}_\lambda\hat{x}_\sigma + ...$

- with $\theta^0_{\mu\nu} = 0, \theta_{\mu\nu}{}^{\lambda\sigma} = 0, ...$
- Only non-vanishing term $\theta_{\mu\nu}{}^{\lambda}$
- Thus we have $[\hat{x}_\mu, \hat{x}_\nu] = iC^\lambda_{\mu\nu}\hat{x}_\lambda$ Lie algebraic type NC
- choice: $C^\lambda_{\mu\nu} = a_\mu\delta_{\nu\lambda} - a_\nu\delta_{\mu\lambda}, a_\mu, \mu = 0, 1, ..., n-1$ are real
- choice: $a_0 = a = \frac{1}{k}$, $a_i = 0, i = 1, 2, ..., n-1$

k-spacetime

- Generic NC spaces are defined with co-ordinates obeying

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{k^2}\Theta_{\mu\nu}(k\hat{x})$$

where $\Theta_{\mu\nu}(k\hat{x}) = \theta^0_{\mu\nu} + \theta_{\mu\nu}{}^{\lambda}\hat{x}_\lambda + \theta_{\mu\nu}{}^{\lambda\sigma}\hat{x}_\lambda\hat{x}_\sigma + \ldots\ldots\ldots$

- with $\theta^0_{\mu\nu} = 0, \theta_{\mu\nu}{}^{\lambda\sigma} = 0, \ldots$
- Only non-vanishing term $\theta_{\mu\nu}{}^{\lambda}$
- Thus we have $[\hat{x}_\mu, \hat{x}_\nu] = iC^\lambda_{\mu\nu}\hat{x}_\lambda$ Lie algebraic type NC
- choice: $C^\lambda_{\mu\nu} = a_\mu\delta_{\nu\lambda} - a_\nu\delta_{\mu\lambda}, a_\mu, \mu = 0, 1, \ldots., n-1$ are real
- choice: $a_0 = a = \frac{1}{k},\ a_i = 0, i = 1, 2, \ldots\ldots, n-1$

k-spacetime.......

- k-spacetime co-ordinates satisfy:

$$[\hat{x}_i, \hat{x}_j] = 0, \quad [\hat{x}_0, \hat{x}_i] = ia\hat{x}_i$$

- The symmetry algebra of this spacetime is k-Poincare algebra

$$[M_{\mu\nu}, M_{\alpha\beta}] = i(\eta_{\mu\beta}M_{\nu\alpha} - \eta_{\mu\alpha}M_{\nu\beta} + \eta_{\nu\alpha}M_{\mu\beta} - \eta_{\nu\beta}M_{\mu\alpha})$$

$$[M_i, P_\mu] = i\epsilon_{i\mu j}P_j, \quad [P_\mu, P_\nu] = 0, \quad [N_i, P_0] = iP_i$$

$$[N_i, P_j] = i\delta_{ij}\left(\frac{1}{2a}(1 - e^{-2aP_0}) + \frac{a}{2}\vec{P}^2\right) - iaP_iP_j$$

with Casimir $m^2 = (\frac{2}{a}sinh(\frac{aP_o}{2}))^2 - \vec{P}^2 e^{aP_0}$

k-spacetime.......

- k-spacetime co-ordinates satisfy:

$$[\hat{x}_i, \hat{x}_j] = 0, \quad [\hat{x}_0, \hat{x}_i] = ia\hat{x}_i$$

- The symmetry algebra of this spacetime is k-Poincare algebra

$$[M_{\mu\nu}, M_{\alpha\beta}] = i(\eta_{\mu\beta}M_{\nu\alpha} - \eta_{\mu\alpha}M_{\nu\beta} + \eta_{\nu\alpha}M_{\mu\beta} - \eta_{\nu\beta}M_{\mu\alpha})$$

$$[M_i, P_\mu] = i\epsilon_{i\mu j}P_j, \quad [P_\mu, P_\nu] = 0, \quad [N_i, P_0] = iP_i$$

$$[N_i, P_j] = i\delta_{ij}\left(\frac{1}{2a}(1 - e^{-2aP_0}) + \frac{a}{2}\vec{P}^2\right) - iaP_iP_j$$

with Casimir $m^2 = (\frac{2}{a}sinh(\frac{aP_o}{2}))^2 - \vec{P}^2 e^{aP_0}$

- Another k-deformed algebra studied is one where

$$[N_i, P_j] = i(\delta_{ij} P_0 - a P_i P_j), \qquad [N_i, P_0] = i(1 - a P_0) P_i$$

with Casimir $M^2 = (P_0^2 - \vec{P}^2)(1 - a P_0)^{-2}$

- The two DSR models constructed have energy-momentum relations given by these two Casimirs respectively.
- The non-commutative structure of the underlying space-time of both DSRs are same, showing the equivalence of the physical models.

- Another k-deformed algebra studied is one where

$$[N_i, P_j] = i(\delta_{ij} P_0 - a P_i P_j), \qquad [N_i, P_0] = i(1 - a P_0) P_i$$

  with Casimir $M^2 = (P_0^2 - \vec{P}^2)(1 - a P_0)^{-2}$

- The two DSR models constructed have energy-momentum relations given by these two Casimirs respectively.
- The non-commutative structure of the underlying space-time of both DSRs are same, showing the equivalence of the physical models.

- Another k-deformed algebra studied is one where

  $$[N_i, P_j] = i(\delta_{ij}P_0 - aP_iP_j), \qquad [N_i, P_0] = i(1 - aP_0)P_i$$

  with Casimir $M^2 = (P_0^2 - \vec{P}^2)(1 - aP_0)^{-2}$

- The two DSR models constructed have energy-momentum relations given by these two Casimirs respectively.
- The non-commutative structure of the underlying space-time of both DSRs are same, showing the equivalence of the physical models.

Symmetry algebra of k-spacetime

- There are different approaches to construct field theory on k-spacetime.
- Using fields which are functions of $\hat{x}_\mu$ and defining the action which is invariant under k-Poincare algebra.
- Map kappa-spacetime co-ordinates and their functions to commutative ones and work with these commutative functions.
  - We take the second approach

Symmetry algebra of k-spacetime

- There are different approaches to construct field theory on k-spacetime.
- Using fields which are functions of $\hat{x}_\mu$ and defining the action which is invariant under k-Poincare algebra.
- Map kappa-spacetime co-ordinates and their functions to commutative ones and work with these commutative functions.
  - We take the second approach

Symmetry algebra of k-spacetime

- There are different approaches to construct field theory on k-spacetime.
- Using fields which are functions of $\hat{x}_\mu$ and defining the action which is invariant under k-Poincare algebra.
- Map kappa-spacetime co-ordinates and their functions to commutative ones and work with these commutative functions.
  - We take the second approach

Symmetry algebra of k-spacetime

- There are different approaches to construct field theory on k-spacetime.
- Using fields which are functions of $\hat{x}_\mu$ and defining the action which is invariant under k-Poincare algebra.
- Map kappa-spacetime co-ordinates and their functions to commutative ones and work with these commutative functions.
    - We take the second approach

Symmetry......

- We derive the action of Lorentz algebra on k-spacetime co-ordinates and also obtain their derivative operators.
- These operators satisfy usual Poincare algebra relations, but have modified Casimirs
- We obtain different possible invariant actions for scalar theory.
- We derive the modified Leibnitz rule(twisted co-products) of these generators and compatible flip operator.
- Using this, we derive the deformed commutation rules between $A, A^\dagger$/twisted statistics.

Symmetry......

- We derive the action of Lorentz algebra on k-spacetime co-ordinates and also obtain their derivative operators.
- These operators satisfy usual Poincare algebra relations, but have modified Casimirs
- We obtain different possible invariant actions for scalar theory.
- We derive the modified Leibnitz rule(twisted co-products) of these generators and compatible flip operator.
- Using this, we derive the deformed commutation rules between $A, A^\dagger$/twisted statistics.

Symmetry......

- We derive the action of Lorentz algebra on k-spacetime co-ordinates and also obtain their derivative operators.
- These operators satisfy usual Poincare algebra relations, but have modified Casimirs
- We obtain different possible invariant actions for scalar theory.
- We derive the modified Leibnitz rule(twisted co-products) of these generators and compatible flip operator.
- Using this, we derive the deformed commutation rules between $A, A^\dagger$/twisted statistics.

Symmetry......

- We derive the action of Lorentz algebra on k-spacetime co-ordinates and also obtain their derivative operators.
- These operators satisfy usual Poincare algebra relations, but have modified Casimirs
- We obtain different possible invariant actions for scalar theory.
- We derive the modified Leibnitz rule(twisted co-products) of these generators and compatible flip operator.
- Using this, we derive the deformed commutation rules between $A, A^\dagger$/twisted statistics.

Symmetry......

- We derive the action of Lorentz algebra on k-spacetime co-ordinates and also obtain their derivative operators.
- These operators satisfy usual Poincare algebra relations, but have modified Casimirs
- We obtain different possible invariant actions for scalar theory.
- We derive the modified Leibnitz rule(twisted co-products) of these generators and compatible flip operator.
- Using this, we derive the deformed commutation rules between $A, A^\dagger$/twisted statistics.

K-spacetime, ordering, Leibnitz rules

- We have $[\hat{x}_0, \hat{x}_i] = ia\hat{x}_i, \quad [\hat{x}_i, \hat{x}_j] = 0$
- $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$ This defines a unique mapping of functions on k-spacetime to that on commutative space time

$$F(\hat{x}_\varphi)|0 >= F_\varphi(x)$$

- Any $M(\hat{x})$ can be expanded as a power series in $\hat{x}_\mu$. $M(\hat{x})$ can be written as LC of monomials of $\hat{x}_0, \hat{x}_1, ..., \hat{x}_{n-1}$ with $m_0, m_1, ...m_{n-1}$ as powers and polynomials of lower order $P(\hat{x})$. Thus

$$[M(\hat{x}) - P(\hat{x})]\,|0 >= M(x)$$

- Natural ordering:$\hat{x}_0$ to the right/left of $\hat{x}_i$
  $\hat{x}_0$ and $\hat{x}_i$ treated symmetrically.

K-spacetime, ordering, Leibnitz rules

- We have $[\hat{x}_0, \hat{x}_i] = ia\hat{x}_i, \quad [\hat{x}_i, \hat{x}_j] = 0$
- $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$ This defines a unique mapping of functions on k-spacetime to that on commutative space time

$$F(\hat{x}_\varphi)|0 >= F_\varphi(x)$$

- Any $M(\hat{x})$ can be expanded as a power series in $\hat{x}_\mu$. $M(\hat{x})$ can be written as LC of monomials of $\hat{x}_0, \hat{x}_1, ..., \hat{x}_{n-1}$ with $m_0, m_1, ...m_{n-1}$ as powers and polynomials of lower order $P(\hat{x})$. Thus

$$\left[M(\hat{x}) - P(\hat{x})\right]|0 >= M(x)$$

- Natural ordering:$\hat{x}_0$ to the right/left of $\hat{x}_i$
  $\hat{x}_0$ and $\hat{x}_i$ treated symmetrically.

K-spacetime, ordering, Leibnitz rules

- We have $[\hat{x}_0, \hat{x}_i] = ia\hat{x}_i, \quad [\hat{x}_i, \hat{x}_j] = 0$
- $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$ This defines a unique mapping of functions on k-spacetime to that on commutative space time

$$F(\hat{x}_\varphi)|0 >= F_\varphi(x)$$

- Any $M(\hat{x})$ can be expanded as a power series in $\hat{x}_\mu$. $M(\hat{x})$ can be written as LC of monomials of $\hat{x}_0, \hat{x}_1, ..., \hat{x}_{n-1}$ with $m_0, m_1, ...m_{n-1}$ as powers and polynomials of lower order $P(\hat{x})$. Thus

$$[M(\hat{x}) - P(\hat{x})] \, |0 >= M(x)$$

- Natural ordering:$\hat{x}_0$ to the right/left of $\hat{x}_i$ $\hat{x}_0$ and $\hat{x}_i$ treated symmetrically.

K-spacetime, ordering, Leibnitz rules

- We have $[\hat{x}_0, \hat{x}_i] = ia\hat{x}_i, \quad [\hat{x}_i, \hat{x}_j] = 0$
- $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$ This defines a unique mapping of functions on k-spacetime to that on commutative space time

$$F(\hat{x}_\varphi)|0 >= F_\varphi(x)$$

- Any $M(\hat{x})$ can be expanded as a power series in $\hat{x}_\mu$. $M(\hat{x})$ can be written as LC of monomials of $\hat{x}_0, \hat{x}_1, ..., \hat{x}_{n-1}$ with $m_0, m_1, ...m_{n-1}$ as powers and polynomials of lower order $P(\hat{x})$. Thus

$$[M(\hat{x}) - P(\hat{x})] \, |0 >= M(x)$$

- Natural ordering:$\hat{x}_0$ to the right/left of $\hat{x}_i$ $\hat{x}_0$ and $\hat{x}_i$ treated symmetrically.

K-spacetime, ordering, Leibnitz rules

- Imposing

$$[\partial_i, \hat{x}_j] = \delta_{ij}\varphi(A), \quad [\partial_i, \hat{x}_0] = ia\partial_i\gamma(A)$$

$$[\partial_0, \hat{x}_i] = 0, \quad [\partial_0, \hat{x}_0] = 1,$$

  with $A = ia\partial_0$, we get from $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$

- $$\hat{x}_i = x_i \varphi(A)$$

$$\hat{x}_0 = x_0 \psi(A) + ia\partial_i\gamma(A)$$

- from the commutators we get $\frac{\varphi'}{\varphi}\psi = \gamma - 1$
  ($\varphi(0) = 1, \psi(0) = 1, \gamma(0) = \varphi'(0) + 1)$
- Leibnitz rule for ∂_i is modified

$$\Delta_\varphi(\partial_i) = \partial_i^x \frac{\varphi(A_x + A_y)}{\varphi(A_x)} + \partial_i^y \frac{\varphi(A_x + A_y)}{\varphi(A_y)}$$

- $$\Delta_\varphi(\partial_0) = \partial_0 \otimes I + I \otimes \partial_0 = \partial_0^x I^y + I^x \partial_0^y$$

K-spacetime, ordering, Leibnitz rules

- Imposing

$$[\partial_i, \hat{x}_j] = \delta_{ij}\varphi(A), \quad [\partial_i, \hat{x}_0] = ia\partial_i\gamma(A)$$

$$[\partial_0, \hat{x}_i] = 0, \quad [\partial_0, \hat{x}_0] = 1,$$

  with $A = ia\partial_0$, we get from $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$

-

$$\hat{x}_i = x_i\varphi(A)$$

$$\hat{x}_0 = x_0\psi(A) + ia\partial_i\gamma(A)$$

- from the commutators we get $\frac{\varphi'}{\varphi}\psi = \gamma - 1$
  ($\varphi(0) = 1, \psi(0) = 1, \gamma(0) = \varphi'(0) + 1)$
- Leibnitz rule for ∂_i is modified

$$\Delta_\varphi(\partial_i) = \partial_i^x \frac{\varphi(A_x + A_y)}{\varphi(A_x)} + \partial_i^y \frac{\varphi(A_x + A_y)}{\varphi(A_y)}$$

- $$\Delta_\varphi(\partial_0) = \partial_0 \otimes I + I \otimes \partial_0 = \partial_0^x I^y + I^x \partial_0^y$$

K-spacetime, ordering, Leibniz rules

- Imposing

$$[\partial_i, \hat{x}_j] = \delta_{ij}\varphi(A), \quad [\partial_i, \hat{x}_0] = ia\partial_i\gamma(A)$$
$$[\partial_0, \hat{x}_i] = 0, \quad [\partial_0, \hat{x}_0] = 1,$$

  with $A = ia\partial_0$, we get from $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$

- $$\hat{x}_i = x_i\varphi(A)$$
$$\hat{x}_0 = x_0\psi(A) + ia\partial_i\gamma(A)$$

- from the commutators we get $\frac{\varphi'}{\varphi}\psi = \gamma - 1$
  ($\varphi(0) = 1, \psi(0) = 1, \gamma(0) = \varphi'(0) + 1)$
- Leibniz rule for ∂_i is modified

$$\Delta_\varphi(\partial_i) = \partial_i^x \frac{\varphi(A_x + A_y)}{\varphi(A_x)} + \partial_i^y \frac{\varphi(A_x + A_y)}{\varphi(A_y)}$$

- $$\Delta_\varphi(\partial_0) = \partial_0 \otimes I + I \otimes \partial_0 = \partial_0^x I^y + I^x \partial_0^y$$

K-spacetime, ordering, Leibnitz rules

- Imposing

$$[\partial_i, \hat{x}_j] = \delta_{ij}\varphi(A), \quad [\partial_i, \hat{x}_0] = ia\partial_i\gamma(A)$$

$$[\partial_0, \hat{x}_i] = 0, \quad [\partial_0, \hat{x}_0] = 1,$$

  with $A = ia\partial_0$, we get from $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$

-

$$\hat{x}_i = x_i \varphi(A)$$

$$\hat{x}_0 = x_0 \psi(A) + ia\partial_i \gamma(A)$$

- from the commutators we get $\frac{\varphi'}{\varphi}\psi = \gamma - 1$
  ($\varphi(0) = 1, \psi(0) = 1, \gamma(0) = \varphi'(0) + 1$)
- Leibnitz rule for ∂_i is modified

$$\Delta_\varphi(\partial_i) = \partial_i^x \frac{\varphi(A_x + A_y)}{\varphi(A_x)} + \partial_i^y \frac{\varphi(A_x + A_y)}{\varphi(A_y)}$$

- $\Delta_\varphi(\partial_0) = \partial_0 \otimes I + I \otimes \partial_0 = \partial_0^x I^y + I^x \partial_0^y$

K-spacetime, ordering, Leibnitz rules

- Imposing

$$[\partial_i, \hat{x}_j] = \delta_{ij}\varphi(A), \quad [\partial_i, \hat{x}_0] = ia\partial_i\gamma(A)$$

$$[\partial_0, \hat{x}_i] = 0, \quad [\partial_0, \hat{x}_0] = 1,$$

  with $A = ia\partial_0$, we get from $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$

- $$\hat{x}_i = x_i\varphi(A)$$

$$\hat{x}_0 = x_0\psi(A) + ia\partial_i\gamma(A)$$

- from the commutators we get $\frac{\varphi'}{\varphi}\psi = \gamma - 1$
  ($\varphi(0) = 1, \psi(0) = 1, \gamma(0) = \varphi'(0) + 1)$
- Leibnitz rule for ∂_i is modified

$$\Delta_\varphi(\partial_i) = \partial_i^x \frac{\varphi(A_x + A_y)}{\varphi(A_x)} + \partial_i^y \frac{\varphi(A_x + A_y)}{\varphi(A_y)}$$

- $$\Delta_\varphi(\partial_0) = \partial_0 \otimes I + I \otimes \partial_0 = \partial_0^x I^y + I^x \partial_0^y$$

k-Poincare algebra, Casimir and Dispersion relation

- No modification in the Lorentz algebra
- Demand $M_{\mu\nu}$ and $\hat{x}_\mu$ close linearly, satisfy Jacobi identity, smooth commutative limit
- $$[M_{i0}, \hat{x}_0] = \hat{x}_i + iaM_{i0}$$
  $$[M_{i0}, \hat{x}_j] = -\delta_{ij}\hat{x}_0 - iaM_{ij}$$
- Leibnitz rule

$$\Delta_\varphi(M_{ij}) = M_{ij} \otimes I + I \otimes M_{ij}$$

$$\Delta_\varphi(M_{i0}) = M_{i0} \otimes I + e^A \otimes M_{i0} + ia\partial_j \frac{1}{\varphi(A)} \otimes M_{ij}$$

k-Poincare algebra, Casimir and Dispersion relation

- No modification in the Lorentz algebra
- Demand $M_{\mu\nu}$ and $\hat{x}_\mu$ close linearly, satisfy Jacobi identity, smooth commutative limit
-

$$[M_{i0}, \hat{x}_0] = \hat{x}_i + iaM_{i0}$$

$$[M_{i0}, \hat{x}_j] = -\delta_{ij}\hat{x}_0 - iaM_{ij}$$

- Leibnitz rule

$$\Delta_\varphi(M_{ij}) = M_{ij} \otimes I + I \otimes M_{ij}$$

$$\Delta_\varphi(M_{i0}) = M_{i0} \otimes I + e^A \otimes M_{i0} + ia\partial_j \frac{1}{\varphi(A)} \otimes M_{ij}$$

k-Poincare algebra, Casimir and Dispersion relation

- No modification in the Lorentz algebra
- Demand $M_{\mu\nu}$ and $\hat{x}_\mu$ close linearly, satisfy Jacobi identity, smooth commutative limit
- $$[M_{i0}, \hat{x}_0] = \hat{x}_i + iaM_{i0}$$
  $$[M_{i0}, \hat{x}_j] = -\delta_{ij}\hat{x}_0 - iaM_{ij}$$
- Leibniz rule

$$\Delta_\varphi(M_{ij}) = M_{ij} \otimes I + I \otimes M_{ij}$$

$$\Delta_\varphi(M_{i0}) = M_{i0} \otimes I + e^A \otimes M_{i0} + ia\partial_j \frac{1}{\varphi(A)} \otimes M_{ij}$$

k-Poincare algebra, Casimir and Dispersion relation

- No modification in the Lorentz algebra
- Demand $M_{\mu\nu}$ and $\hat{x}_\mu$ close linearly, satisfy Jacobi identity, smooth commutative limit
- $$[M_{i0}, \hat{x}_0] = \hat{x}_i + iaM_{i0}$$
  $$[M_{i0}, \hat{x}_j] = -\delta_{ij}\hat{x}_0 - iaM_{ij}$$
- Leibnitz rule

$$\Delta_\varphi(M_{ij}) = M_{ij} \otimes I + I \otimes M_{ij}$$

$$\Delta_\varphi(M_{i0}) = M_{i0} \otimes I + e^A \otimes M_{i0} + ia\partial_j \frac{1}{\varphi(A)} \otimes M_{ij}$$

k-Poincare algebra, Casimir and Dispersion relation

- Enlarge the algebra:

$$[M_{\mu\nu}, D_\lambda] = \delta_{\nu\lambda} D_\mu - \delta_{\mu\lambda} D_\nu$$

$$[D_\mu, D_\nu] = 0$$

$$[D_\mu, \hat{x}_\nu] = \delta_{\mu\nu} \sqrt{1 - a^2 D_\alpha D_\alpha} + i a_0 (\delta_{\mu 0} D_\nu - \delta_{\mu\nu} D_0)$$

$$D_0 = -i\partial_0 \frac{sinhA}{A} - ia \triangle \frac{e^{-A}}{2\varphi^2}; \qquad D_i = \partial_i \frac{e^{-A}}{\varphi}$$

-

$$[M_{\mu\nu}, \Box] = 0, [\Box, \hat{x}_\mu] = 2D_\mu$$

$$\Box = \triangle \frac{e^{-A}}{\varphi^2} + 2\partial_0^2 (1 - coshA) A^2$$

k-Poincare algebra, Casimir and Dispersion relation

- Enlarge the algebra:

$$[M_{\mu\nu}, D_\lambda] = \delta_{\nu\lambda} D_\mu - \delta_{\mu\lambda} D_\nu$$

$$[D_\mu, D_\nu] = 0$$

$$[D_\mu, \hat{x}_\nu] = \delta_{\mu\nu}\sqrt{1 - a^2 D_\alpha D_\alpha} + ia_0(\delta_{\mu 0} D_\nu - \delta_{\mu\nu} D_0)$$

$$D_0 = -i\partial_0 \frac{sinhA}{A} - ia \triangle \frac{e^{-A}}{2\varphi^2}; \qquad D_i = \partial_i \frac{e^{-A}}{\varphi}$$

-

$$[M_{\mu\nu}, \Box] = 0, [\Box, \hat{x}_\mu] = 2D_\mu$$

$$\Box = \triangle \frac{e^{-A}}{\varphi^2} + 2\partial_0^2(1 - coshA)A^2$$

k-Poincare algebra, Casimir and Dispersion relation

- The Casimir

$$D_\mu D_\mu = \Box(1 - \frac{a^2}{4}\Box) \quad \text{quartic}$$

- $\Box$ is quadratic in space derivatives.
- $(\Box(1 - \frac{a^2}{4}\Box) - m^2)\Phi(x) = 0$

$$\mathcal{A} - m^2 - \frac{a^2}{4}\mathcal{A}^2 = 0$$

with $\mathcal{A} = \frac{a^2}{4} Sinh^2(\frac{ap_0}{2}) - p_i^2 \frac{e^{-aP_0}}{\varphi^2(ap_0)}$

- $(\Box - m^2)\Phi(x) = 0$

$$\frac{4}{a^2} Sinh^2(\frac{ap_0}{2}) - p_i^2 \frac{e^{-ap_0}}{\varphi(ap_0)^2} - m^2 = 0$$

k-Poincare algebra, Casimir and Dispersion relation

- The Casimir

$$D_\mu D_\mu = \Box(1 - \frac{a^2}{4}\Box) \quad \text{quartic}$$

- $\Box$ is quadratic in space derivatives.
- $(\Box(1 - \frac{a^2}{4}\Box) - m^2)\Phi(x) = 0$

$$\mathcal{A} - m^2 - \frac{a^2}{4}\mathcal{A}^2 = 0$$

with $\mathcal{A} = \frac{a^2}{4}Sinh^2(\frac{ap_0}{2}) - p_i^2 \frac{e^{-aP_0}}{\varphi^2(ap_0)}$

- $(\Box - m^2)\Phi(x) = 0$

$$\frac{4}{a^2}Sinh^2(\frac{ap_0}{2}) - p_i^2 \frac{e^{-ap_0}}{\varphi(ap_0)^2} - m^2 = 0$$

k-Poincare algebra, Casimir and Dispersion relation

- The Casimir

$$D_\mu D_\mu = \Box(1 - \frac{a^2}{4}\Box) \quad \text{quartic}$$

- $\Box$ is quadratic in space derivatives.
- $(\Box(1 - \frac{a^2}{4}\Box) - m^2)\Phi(x) = 0$

$$\mathcal{A} - m^2 - \frac{a^2}{4}\mathcal{A}^2 = 0$$

with $\mathcal{A} = \frac{a^2}{4} Sinh^2(\frac{ap_0}{2}) - p_i^2 \frac{e^{-aP_0}}{\varphi^2(ap_0)}$

- $(\Box - m^2)\Phi(x) = 0$

$$\frac{4}{a^2} Sinh^2(\frac{ap_0}{2}) - p_i^2 \frac{e^{-ap_0}}{\varphi(ap_0)^2} - m^2 = 0$$

k-Poincare algebra, Casimir and Dispersion relation

- The Casimir

$$D_\mu D_\mu = \Box(1 - \frac{a^2}{4}\Box) \quad \text{quartic}$$

- $\Box$ is quadratic in space derivatives.
- $(\Box(1 - \frac{a^2}{4}\Box) - m^2)\Phi(x) = 0$

$$\mathcal{A} - m^2 - \frac{a^2}{4}\mathcal{A}^2 = 0$$

with $\mathcal{A} = \frac{a^2}{4}Sinh^2(\frac{ap_0}{2}) - p_i^2\frac{e^{-aP_0}}{\varphi^2(ap_0)}$

- $(\Box - m^2)\Phi(x) = 0$

$$\frac{4}{a^2}Sinh^2(\frac{ap_0}{2}) - p_i^2\frac{e^{-ap_0}}{\varphi(ap_0)^2} - m^2 = 0$$

Star Product

- $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$

$$F_\varphi(\hat{x}_\varphi) G_\varphi(\hat{x}_\varphi)|0>= F_\varphi *_\varphi G_\varphi$$

- $(f *_\varphi g)(x) = m_0[e^{x_i(\Delta_\varphi - \Delta_0)\partial_i} f(u)g(v)]|_{u=t=x_i}$
  Δ_φ is the twisted co-product of ∂_i
-
$$\mathcal{F}_\varphi = e^{N_x ln \frac{\varphi(A_x+A_y)}{\varphi(A_x)} + N_y(A_x + ln \frac{\varphi(A_x+A_y)}{\varphi(A_y)})}$$

- Twist element $\mathcal{F}_\Lambda = e^{-\Lambda N \otimes A + (1-\Lambda) A \otimes N}, \quad \Lambda = 1, 0$ for L/R ordering. Here $N = x_i \partial_i$.

Star Product

- $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$

$$F_\varphi(\hat{x}_\varphi)G_\varphi(\hat{x}_\varphi)|0 >= F_\varphi *_\varphi G_\varphi$$

- $(f *_\varphi g)(x) = m_0[e^{x_i(\Delta_\varphi - \Delta_0)\partial_i} f(u)g(v)]|_{u=t=x_i}$
  Δ_φ is the twisted co-product of ∂_i
-
$$\mathcal{F}_\varphi = e^{N_x ln\frac{\varphi(A_x+A_y)}{\varphi(A_x)} + N_y(A_x + ln\frac{\varphi(A_x+A_y)}{\varphi(A_y)})}$$
- Twist element $\mathcal{F}_\Lambda = e^{-\Lambda N \otimes A + (1-\Lambda) A \otimes N}, \quad \Lambda = 1, 0$ for L/R ordering. Here $N = x_i \partial_i$.

Star Product

- $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$

$$F_\varphi(\hat{x}_\varphi) G_\varphi(\hat{x}_\varphi)|0> = F_\varphi *_\varphi G_\varphi$$

- $(f *_\varphi g)(x) = m_0[e^{x_i(\Delta_\varphi - \Delta_0)\partial_i} f(u)g(v)]|_{u=t=x_i}$
  Δ_φ is the twisted co-product of ∂_i
-

$$\mathcal{F}_\varphi = e^{N_x ln \frac{\varphi(A_x+A_y)}{\varphi(A_x)} + N_y(A_x + ln \frac{\varphi(A_x+A_y)}{\varphi(A_y)})}$$

- Twist element $\mathcal{F}_\Lambda = e^{-\Lambda N \otimes A + (1-\Lambda) A \otimes N}, \quad \Lambda = 1, 0$ for L/R ordering. Here $N = x_i \partial_i$.

Star Product

- $\hat{x}_\mu = x_\alpha \Phi_{\alpha\mu}(\partial)$

$$F_\varphi(\hat{x}_\varphi)G_\varphi(\hat{x}_\varphi)|0>= F_\varphi *_\varphi G_\varphi$$

- $(f *_\varphi g)(x) = m_0[e^{x_i(\Delta_\varphi - \Delta_0)\partial_i} f(u)g(v)]|_{u=t=x_i}$
  Δ_φ is the twisted co-product of ∂_i
-

$$\mathcal{F}_\varphi = e^{N_x ln \frac{\varphi(A_x+A_y)}{\varphi(A_x)} + N_y(A_x + ln \frac{\varphi(A_x+A_y)}{\varphi(A_y)})}$$

- Twist element $\mathcal{F}_\Lambda = e^{-\Lambda N\otimes A+(1-\Lambda)A\otimes N}, \quad \Lambda = 1, 0$ for L/R ordering. Here $N = x_i\partial_i$.

Twisted Flip Operator

- (anti)Symmetric states of the physical Hilbert space are projected from the tensor product state

$$\frac{1}{2}(1 \pm \tau_0)(f \otimes g) = \frac{1}{2}(f \otimes g \pm g \otimes f).$$

- $g : f \otimes g = (D \otimes D) \vartriangle (g) f \otimes g$, $g \in$ symm. algebra
- $[\vartriangle (g), \tau_0] = 0$
- for the NC case $[\Delta_\varphi, \tau_\varphi] = 0$

$$\Delta_\varphi = \mathcal{F}_\varphi^{-1} \Delta_0 \mathcal{F}_\varphi$$

$$\tau_\varphi = \mathcal{F}_\varphi^{-1} \tau_0 \mathcal{F}_\varphi = e^{i(x_i P_i \otimes A - A \otimes x_i P_i)} \tau_0$$

Twisted Flip Operator

- (anti)Symmetric states of the physical Hilbert space are projected from the tensor product state

$$\frac{1}{2}(1 \pm \tau_0)(f \otimes g) = \frac{1}{2}(f \otimes g \pm g \otimes f).$$

- $g : f \otimes g = (D \otimes D) \triangle (g) f \otimes g$, $g \in$ symm. algebra
- $[\triangle (g), \tau_0] = 0$
- for the NC case $[\Delta_\varphi, \tau_\varphi] = 0$

$$\Delta_\varphi = \mathcal{F}_\varphi^{-1} \Delta_0 \mathcal{F}_\varphi$$

$$\tau_\varphi = \mathcal{F}_\varphi^{-1} \tau_0 \mathcal{F}_\varphi = e^{i(x_i P_i \otimes A - A \otimes x_i P_i)} \tau_0$$

Twisted Flip Operator

- (anti)Symmetric states of the physical Hilbert space are projected from the tensor product state

$$\frac{1}{2}(1 \pm \tau_0)(f \otimes g) = \frac{1}{2}(f \otimes g \pm g \otimes f).$$

- $g : f \otimes g = (D \otimes D) \triangle (g) f \otimes g$, $g \in$ symm. algebra
- $[\triangle (g), \tau_0] = 0$
- for the NC case $[\Delta_\varphi, \tau_\varphi] = 0$

$$\Delta_\varphi = \mathcal{F}_\varphi^{-1} \Delta_0 \mathcal{F}_\varphi$$

$$\tau_\varphi = \mathcal{F}_\varphi^{-1} \tau_0 \mathcal{F}_\varphi = e^{i(x_i P_i \otimes A - A \otimes x_i P_i)} \tau_0$$

Twisted Flip Operator

- (anti)Symmetric states of the physical Hilbert space are projected from the tensor product state

$$\frac{1}{2}(1 \pm \tau_0)(f \otimes g) = \frac{1}{2}(f \otimes g \pm g \otimes f).$$

- $g : f \otimes g = (D \otimes D) \vartriangle (g) f \otimes g$, $g \in$ symm. algebra
- $[\vartriangle (g), \tau_0] = 0$
- for the NC case $[\Delta_\varphi, \tau_\varphi] = 0$

$$\vartriangle_\varphi = \mathcal{F}_\varphi^{-1} \Delta_0 \mathcal{F}_\varphi$$

$$\tau_\varphi = \mathcal{F}_\varphi^{-1} \tau_0 \mathcal{F}_\varphi = e^{i(x_i P_i \otimes A - A \otimes x_i P_i)} \tau_0$$

Twisted Flip Operator

- for bosons $f \otimes g = \tau_{\varphi}(f \otimes g)$
- $\phi(x) \otimes \phi(y) - e^{-(A\otimes N - N\otimes A)}\phi(y)\phi(x) = 0$
- $(\Box - m^2)\Phi(x) = 0$ with $\varphi = e^{-\frac{A}{2}} = e^{-\frac{ia\partial_0}{2}}$ is

$$\left[\partial_i^2 + \frac{4}{a^2} Sinh^2(\frac{ia\partial_0}{2}) - m^2\right]\Phi = 0$$

- $\Phi(x) = \int \frac{d^4p}{2\Omega_k(p)}\left[A(\omega_k, \vec{p})e^{-ip\cdot x} + A^{\dagger}(\omega_k, \vec{p})e^{ip\cdot x}\right]$
  $A^{\dagger}(\pm\omega_k, \vec{p}) = A^{\dagger}(\mp\omega_k, \vec{p})$.

  $p_0^{\pm} = \pm\omega_k(p) = \pm\frac{2}{a}sinh^{-1}(\frac{a}{2}\sqrt{p_i^2 + m^2})$,

  $\Omega_k(p) = \frac{1}{a}Sinh(a\omega_k(p))$

Twisted Flip Operator

- for bosons $f \otimes g = \tau_{\varphi}(f \otimes g)$
- $\phi(x) \otimes \phi(y) - e^{-(A \otimes N - N \otimes A)} \phi(y)\phi(x) = 0$
- $(\Box - m^2)\Phi(x) = 0$ with $\varphi = e^{-\frac{A}{2}} = e^{-\frac{ia\partial_0}{2}}$ is

$$\left[\partial_i^2 + \frac{4}{a^2} Sinh^2(\frac{ia\partial_0}{2}) - m^2\right] \Phi = 0$$

- $\Phi(x) = \int \frac{d^4p}{2\Omega_k(p)} \left[A(\omega_k, \vec{p})e^{-ip\cdot x} + A^\dagger(\omega_k, \vec{p})e^{ip\cdot x}\right]$
  $A^\dagger(\pm\omega_k, \vec{p}) = A^\dagger(\mp\omega_k, \vec{p})$.

$p_0^{\pm} = \pm\omega_k(p) = \pm\frac{2}{a} sinh^{-1}(\frac{a}{2}\sqrt{p_i^2 + m^2})$,

$\Omega_k(p) = \frac{1}{a} Sinh(a\omega_k(p))$

Twisted Flip Operator

- for bosons $f \otimes g = \tau_\varphi (f \otimes g)$
- $\phi(x) \otimes \phi(y) - e^{-(A \otimes N - N \otimes A)} \phi(y)\phi(x) = 0$
- $(\Box - m^2)\Phi(x) = 0$ with $\varphi = e^{-\frac{A}{2}} = e^{-\frac{ia\partial_0}{2}}$ is

$$\left[\partial_i^2 + \frac{4}{a^2} Sinh^2 (\frac{ia\partial_0}{2}) - m^2\right] \Phi = 0$$

- $\Phi(x) = \int \frac{d^4p}{2\Omega_k(p)} \left[A(\omega_k, \vec{p}) e^{-ip\cdot x} + A^\dagger(\omega_k, \vec{p}) e^{ip\cdot x}\right]$
  $A^\dagger(\pm\omega_k, \vec{p}) = A^\dagger(\mp\omega_k, \vec{p})$.

$p_0^\pm = \pm\omega_k(p) = \pm\frac{2}{a} sinh^{-1}(\frac{a}{2}\sqrt{p_i^2 + m^2})$,

$\Omega_k(p) = \frac{1}{a} Sinh(a\omega_k(p))$

Twisted Flip Operator

- for bosons $f \otimes g = \tau_\varphi(f \otimes g)$
- $\phi(x) \otimes \phi(y) - e^{-(A\otimes N - N\otimes A)}\phi(y)\phi(x) = 0$
- $(\Box - m^2)\Phi(x) = 0$ with $\varphi = e^{-\frac{A}{2}} = e^{-\frac{ia\partial_0}{2}}$ is

$$\left[\partial_i^2 + \frac{4}{a^2} Sinh^2(\frac{ia\partial_0}{2}) - m^2\right]\Phi = 0$$

- $\Phi(x) = \int \frac{d^4p}{2\Omega_k(p)}\left[A(\omega_k, \vec{p})e^{-ip\cdot x} + A^\dagger(\omega_k, \vec{p})e^{ip\cdot x}\right]$
  $A^\dagger(\pm\omega_k, \vec{p}) = A^\dagger(\mp\omega_k, \vec{p})$.

  $p_0^\pm = \pm\omega_k(p) = \pm\frac{2}{a} sinh^{-1}(\frac{a}{2}\sqrt{p_i^2 + m^2})$,

  $\Omega_k(p) = \frac{1}{a} Sinh(a\omega_k(p))$

Twisted commutators

- $$A^\dagger(p)A(q) - e^{-a(q_0\partial_{p_i}p_i + \partial_{q_i}q_i p_0)}A(q)A^\dagger(p) = -\delta^3(\vec{p}-\vec{q})$$

  $$A^\dagger(p_0,\vec{p})A^\dagger(q_0,\vec{q}) - e^{-a(-q_0\partial_{p_i}p_i + \partial_{q_i}q_i p_0)}A^\dagger(q_0,\vec{q})A^\dagger(p_0,\vec{p}) = 0$$

  $$A(p_0,\vec{p})A(q_0,\vec{q}) - e^{-a(q_0\partial_{p_i}p_i - \partial_{q_i}q_i p_0)}A(q_0,\vec{q})A(p_0,\vec{p}) = 0$$

  p_0, q_0 as given above

Deformed Product

- $$A(p)\circ A(q) = e^{-\frac{3a}{2}(p_0-q_0)}A(p_0, e^{\frac{aq_0}{2}}\vec{p})A(q_0, e^{-\frac{ap_0}{2}}\vec{q})$$

$$A^\dagger(p)\circ A^\dagger(q) = e^{\frac{3a}{2}(p_0-q_0)}A^\dagger(p_0, e^{-\frac{aq_0}{2}}\vec{p})A(q_0, e^{\frac{ap_0}{2}}\vec{q})$$

$$A^\dagger(p)\circ A(q) = e^{\frac{3a}{2}(p_0+q_0)}A^\dagger(p_0, e^{\frac{aq_0}{2}}\vec{p})\circ A(q_0, e^{\frac{ap_0}{2}}\vec{q})$$

$$A(p)\circ A^\dagger(q) = e^{-\frac{3a}{2}(p_0+q_0)}A(p_0, e^{-\frac{aq_0}{2}}\vec{p})\circ A^\dagger(q_0, e^{-\frac{ap_0}{2}}\vec{q}).$$

- Using this, we can re-express commutators as in the commutative case

$$[A(p_0,\vec{p}), A(q_0,\vec{q})]_\circ = 0, \quad [A^\dagger(p_0,\vec{p}), A^\dagger(q_0,\vec{q})]_\circ = 0,$$

$$[A(p_0,\vec{p}), A^\dagger(q_0,\vec{q})]_\circ = \delta^3(\vec{p}-\vec{q})$$

Deformed Product

- $$A(p)\circ A(q) = e^{-\frac{3a}{2}(p_0-q_0)}A(p_0, e^{\frac{aq_0}{2}}\vec{p})A(q_0, e^{-\frac{ap_0}{2}}\vec{q})$$
$$A^\dagger(p)\circ A^\dagger(q) = e^{\frac{3a}{2}(p_0-q_0)}A^\dagger(p_0, e^{-\frac{aq_0}{2}}\vec{p})A(q_0, e^{\frac{ap_0}{2}}\vec{q})$$
$$A^\dagger(p)\circ A(q) = e^{\frac{3a}{2}(p_0+q_0)}A^\dagger(p_0, e^{\frac{aq_0}{2}}\vec{p})\circ A(q_0, e^{\frac{ap_0}{2}}\vec{q})$$
$$A(p)\circ A^\dagger(q) = e^{-\frac{3a}{2}(p_0+q_0)}A(p_0, e^{-\frac{aq_0}{2}}\vec{p})\circ A^\dagger(q_0, e^{-\frac{ap_0}{2}}\vec{q}).$$
- Using this, we can re-express commutators as in the commutative case
$$[A(p_0,\vec{p}), A(q_0,\vec{q})]_\circ = 0, \quad [A^\dagger(p_0,\vec{p}), A^\dagger(q_0,\vec{q})]_\circ = 0,$$
$$[A(p_0,\vec{p}), A^\dagger(q_0,\vec{q})]_\circ = \delta^3(\vec{p}-\vec{q})$$

Conclusion

- We have obtained the twisted co-product for the symmetry algebra of kappa-space time.
- Using the casimirs, we have shown that more than one invariant action for scalar field is possible (having correct commutative limit).
- Flip operator compatible with the twisted co-product is derived.
- Twisted commutators between creation and annihilation operators are obtained.

In collaboration with: S. Meljanac, D. Meljanac, K. S. Gupta,T. R. Govindarajan

Phys.Rev.D77:105010,2008

Conclusion

- We have obtained the twisted co-product for the symmetry algebra of kappa-space time.
- Using the casimirs, we have shown that more than one invariant action for scalar field is possible (having correct commutative limit).
- Flip operator compatible with the twisted co-product is derived.
- Twisted commutators between creation and annihilation operators are obtained.

In collaboration with: S. Meljanac, D. Meljanac, K. S. Gupta,T. R. Govindarajan

Phys.Rev.D77:105010,2008

Conclusion

- We have obtained the twisted co-product for the symmetry algebra of kappa-space time.
- Using the casimirs, we have shown that more than one invariant action for scalar field is possible (having correct commutative limit).
- Flip operator compatible with the twisted co-product is derived.
- Twisted commutators between creation and annihilation operators are obtained.

In collaboration with: S. Meljanac, D. Meljanac, K. S. Gupta,T. R. Govindarajan

Phys.Rev.D77:105010,2008