DIMENSIONAL REDUCTION, MONOPOLES AND DYNAMICAL SYMMETRY BREAKING

BRIAN DOLAN
DUBLIN INSTITUTE FOR ADVANCED STUDIES
NATIONAL UNIVERSITY OF IRELAND, MAYNOOTH

RICHARD SZABO
HERIOT-WATT UNIVERSITY, EDINBURGH

OUTLINE

- EQUIVARIANT DIMENSIONAL REDUCTION ($M \times S^2$)
  - GAUGE FIELDS $U(k) \to U(k_0) \times U(k_1) \times \ldots \times U(k_m)$
  - SPINOR FIELDS AND YUKAWA COUPLINGS
  - DYNAMICAL SYMMETRY BREAKING
- FUZZY S^2
  - SPINOR FIELDS, UNIVERSAL DIRAC OPERATOR

EQUIVARIANT DIMENSIONAL REDUCTION

$X = M \times S^2$: GAUGE GROUP $G = U(k)$

$\nwarrow$ d DIMENSIONAL

$S^2 \cong SU(2)/U(1)$, $SU(2)$ ACTS TRIVIALLY ON M

$$\begin{array}{ccc} \mathbb{C}^k & \to & E \\ & & \downarrow \\ & & M \end{array}$$

INDUCE A BUNDLE $B = SU(2) \times_{U(1)} E$ OVER X

$g \in SU(2)$, $e \in E$, $h \in U(1)$: $h(g,e) = (gh^{-1}, he)$

$$\begin{array}{ccc} E & \xrightarrow{\text{INDUCE}} & B \\ \downarrow & & \downarrow \\ M & \xleftarrow[\text{RESTRICT}]{} & X \end{array}$$

- B : $SU(2)$ EQUIVARIANT BUNDLE OVER X.
- E : $U(1)$ " " " M

$\Rightarrow$ $E = \bigoplus_p E_{(p)}$ $p \in \mathbb{Z}$ LABELS IRREPS. OF $U(1)$ i.e. CHARGE

$\mathbb{C}^k$ IS A REPRESENTATION SPACE FOR $SU(2)$ $\left(\begin{smallmatrix}\text{DIM}\\ m+1\end{smallmatrix}\right)$

$$\mathbb{C}^k = \bigoplus_{i=0}^{m} \mathbb{C}^{k_i} \qquad \sum_{i=0}^{m} k_i = \cancel{m}\, k$$

$$\underline{v} = \begin{pmatrix} \underline{v}_0 \\ \underline{v}_1 \\ \vdots \\ \underline{v}_m \end{pmatrix} \qquad \underline{v} \in \mathbb{C}^k, \quad \underline{v}_i \in \mathbb{C}^{k_i}$$

$$\mathcal{B} = \bigoplus_{i=0}^{m} B_i \quad , \quad B_i = E_{(p_i)} \times \mathcal{L}^{p_i}$$

LINE BUNDLE OVER S^2, CHERN NUMBER p_i

$$\begin{array}{ccc} \mathbb{C} \to \mathcal{L}^{p_i} & \qquad & \mathbb{C}^{k_i} \to E_{(p_i)} = E_i \\ \downarrow & & \downarrow \\ S^2 & & M \end{array}$$

SU(2) GENERATORS : $\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $\tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$[\tau_3, \tau_\pm] = \pm 2 \tau_\pm$$

$$\tau_\pm : E_{(p)} \to E_{(p\pm 2)}$$

$$\tau_\pm : E_{(p_i)} \times \mathcal{L}^{p_i} \to E_{(p_i \pm 2)} \times \mathcal{L}^{p_i \pm 2}$$

$\tau_\pm$ ACTION DETERMINED BY A <u>CHAIN</u> :

$$\boxed{0 \to \underset{U(k_m)}{B_m} \xrightarrow{\Phi_m} \underset{U(k_{m-1})}{B_{m-1}} \xrightarrow{\Phi_{m-1}} \cdots \xrightarrow{\Phi_1} \underset{U(k_1)}{B_1} \xrightarrow{\Phi_0} \underset{U(k_0)}{B_0} \to 0}$$

$p_i = m - 2i$

CURVATURE

$x^{\hat\mu}, x^{\hat\nu}$: CO-ORDINATES ON X, METRIC $\hat{g}_{\hat\mu\hat\nu}$

x^{μ}, x^{ν} : CO-ORDINATES ON M, $g_{\mu\nu}$

$y, \bar y$: CO-ORDINATES ON S^2, $g_{y\bar y} = \dfrac{4R^2}{(1+y\bar y)^2}$

$$\beta = \frac{2\,dy}{(1+y\bar y)}, \quad \bar\beta = \frac{2\,d\bar y}{(1+y\bar y)}$$: ONE-FORMS ON S^2

MONOPOLE POTENTIAL: $a_p = \dfrac{p}{2(1+y\bar y)}(\bar y\,dy - y\,d\bar y)$

$$f_p = -\frac{p}{(1+y\bar y)^2}\, dy \wedge d\bar y$$

$$\frac{i}{2\pi}\int_{S^2} f_p = p$$: FIRST CHERN NUMBER FOR $\mathcal{L}^p$

$U(k)$ GAUGE POTENTIAL ON X, $\mathcal{A}\ (= -\mathcal{A}^\dagger)$
SPLITS INTO $k_i \times k_j$ BLOCKS

$$\boxed{\mathcal{A} = A(x) + a(y) + \Phi(x)\,\bar\beta(y) - \Phi^\dagger(x)\,\beta(y)}$$

$$A = \sum_{i=0}^{m} A^i, \quad a = \sum_{i=0}^{m} a_{m-2i}$$

$$\Phi = \begin{pmatrix} 0 & \phi_1 & 0 & \cdots & 0 \\ 0 & 0 & \phi_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ & & & & \phi_m \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

$k \times k$ MATRIX

ϕ_i : $k_{i-1} \times k_i$

$U(\phi_i)$ GAUGE POTENTIAL

$$\mathcal{A}^{i,i} = A^i + a_{m-2i}$$

$$\mathcal{A}^{i,i+1} = \phi_{i+1}(x)\,\bar{\beta}(y)$$

FIELD STRENGTH ON X: $\mathcal{F} = d\mathcal{A} + \mathcal{A}\wedge\mathcal{A}$

$$\boxed{\begin{aligned}\mathcal{F} = F + f &+ [\Phi, \Phi^+]\,\beta\wedge\bar{\beta} \\ &+ D\Phi\wedge\bar{\beta} - D\Phi^+\wedge\beta\end{aligned}}$$

$$D\Phi = d\Phi + [A, \Phi]$$

$$F = \sum_{i=0}^{m} F^i \qquad f = \sum_{i=0}^{m} f_{m-2i}$$

$$\mathcal{F}^{i,i}_{\mu\nu} = F^i_{\mu\nu}(x)$$

$$\mathcal{F}^{i,i}_{y\bar{y}} = -\frac{1}{(1+y\bar{y})^2}\left(m-2i+4\phi_i^+\phi_i - 4\phi_{i+1}\phi_{i+1}^+\right)$$

$$\mathcal{F}^{i,i+1}_{\mu\bar{y}} = \frac{2}{(1+y\bar{y})}\,D_\mu\phi_{i+1}(x) = -\left(\mathcal{F}^{i+1,i}_{\mu y}\right)^+$$

$$D\phi_{i+1} = d\phi_{i+1} + A^i\phi_{i+1} - \phi_{i+1}A^{i+1}$$

ACTION ON X:

$$\hat{S} = -\frac{1}{4\tilde{g}^2} \int d^d x \, dy \, d\bar{y} \sqrt{g} \, \mathrm{tr}_k \, \mathcal{F}_{\hat{\mu}\hat{\nu}} \mathcal{F}^{\hat{\mu}\hat{\nu}}$$

$\tilde{g}$: $SU(k)$ GAUGE COUPLING (OVERALL $U(1)$ DROPS OUT)

INTEGRATE OVER S^2 → ACTION ON M:

$$S = \int d^d x \sqrt{g} \left\{ -\frac{1}{g^2} F_{\mu\nu} F^{\mu\nu} + (D_\mu \Phi)^+ D_\mu \Phi + V(\Phi) \right\}$$

$$V(\Phi) = \frac{g^2}{2} \left(\frac{1}{4 g^2 R^2} \Sigma - [\Phi, \Phi^+] \right)^2$$

$$\Sigma = \begin{pmatrix} m \, 1_{k_0} & & & \\ & (m-2) \, 1_{k_1} & & 0 \\ & 0 & \ddots & \\ & & & -m \, 1_{k_m} \end{pmatrix}$$

$$g^2 = \frac{\tilde{g}^2}{4\pi R^2}$$

d-DIMENSIONAL GAUGE COUPLING

$V(\Phi)$ CAN LEAD TO SYMMETRY BREAKING:

$$\frac{\delta V}{\delta \Phi} = 0 \Rightarrow [\Phi, [\Phi, \Phi^+]] = \frac{1}{4 g^2 R^2} [\Phi, \Sigma]$$

$$\boxed{[\Phi, \Phi^+] = \frac{1}{4 g^2 R^2} \Sigma}$$

e.g. $k_0 = k_1 = \dots = k_m = n$; $k = n(m+1)$

$$SU(k) \to SU(n)^{m+1} \times U(1)^m$$

$$\Phi^0 = \begin{pmatrix} 0 & \phi_1^0 & 0 & \cdots & 0 \\ 0 & 0 & \phi_2^0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \phi_m^0 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \qquad \phi_i^0 = \frac{\sqrt{i(m-i+1)}}{2gR}\, 1_n$$

BREAKS $SU(n)^{m+1} \times U(1)^m \to SU(n)$

$m = 1$, $\phi_1 = \dfrac{1}{2gR} + h$, $h = h^+$ n^2 PHYSICAL HIGGS BOSONS.

$$SU(2n) \to SU(n) \times SU(n) \times U(1) \xrightarrow{\Phi^0} SU(n)_{DIAG.}$$

n^2 GAUGE BOSONS BECOME MASSIVE, $M_W^2 = 1/2R^2$

HIGGS MASS $M_h^2 = 1/R^2$

FERMIONS

ON S^2 EIGENSPINORS:

$$\not{D}^{(p)}_{S^2}\, \chi_{(j,p)} = \pm \mu_{jp}\, \chi_{(j,p)}$$

$$\boxed{\mu_{jp} = \frac{1}{R}\sqrt{\left(j+\frac{1+p}{2}\right)\left(j+\frac{1-p}{2}\right)}}$$

DEGENERACY $2j+1$

$$\chi_{(j,p)} = \begin{pmatrix} \chi^{+}_{(j,p)} \\ \pm \chi^{-}_{(j,p)} \end{pmatrix} \qquad j \geqslant \frac{1+|p|}{2}$$

$\chi^{\pm}_{(j,p)}$ SECTION OF $\mathcal{L}^{p\pm 1}$

$j = \frac{|p|}{2} - \frac{1}{2} \quad \Rightarrow \quad |p|$ ZERO MODES, $|p| \geqslant 1$

$p \geqslant 1$: p NEGATIVE CHIRALITY MODES $\chi^{-}_{(p)}$

$p \leq -1$: $|p|$ POSITIVE CHIRALITY MODES $\chi^{+}_{(p)}$

$$\boxed{\text{INDEX}\left(\not{D}^{(p)}\right) = -p}$$

ON X, $\{\Gamma^{\hat{\mu}}, \Gamma^{\hat{\nu}}\} = -2\,\hat{g}^{\hat{\mu}\hat{\nu}}\,\mathbb{1}$

$$\gamma^{y} = -\frac{(1+y\bar{y})}{2R}\,\sigma_+ \qquad \gamma^{\bar{y}} = \frac{(1+y\bar{y})}{2R}\,\sigma_-$$

$$\Gamma^{\mu} = \gamma^{\mu} \otimes \mathbb{1}_2, \quad \Gamma^{y} = -\gamma \otimes \gamma^{y}, \quad \Gamma^{\bar{y}} = \gamma \otimes \gamma^{\bar{y}}$$

CHIRALITY OPERATOR ON M, $\gamma^2 = 1$

$\Gamma^{\hat{\mu}}$: $2^{d/2+1} \times 2^{d/2+1}$ MATRICES

Ψ ON X IN FUNDAMENTAL OF $SU(R)$:

$$\Psi^{\pm} = \tfrac{1}{2}(1 \pm \sigma_3)\,\Psi^{\pm}, \qquad \Psi^{\pm} = \begin{pmatrix} \Psi_0^{\pm} \\ \vdots \\ \Psi_m^{\pm} \end{pmatrix}$$

$$\Psi_i^{+} = \sum_{j=\frac{|p_i|+1}{2}}^{\infty} \sum_{r=0}^{2j} \psi_{(j,p_i)r}(x)\, \chi^{+}_{(j,p_i)r}(y)$$

DIRAC ON M

$$\Psi_i^{-} = \sum_{j=\frac{|p_i|+1}{2}}^{\infty} \sum_{r=0}^{2j} \tilde{\psi}_{(j,p_i)r}(x)\, \chi^{-}_{(j,p_i)r}(y)$$

ZERO MODE CONTRIBUTIONS:

$p_i \geq 1$, $\displaystyle\sum_{r=0}^{p_i-1} \tilde{\psi}_{(p_i)}(x)\chi^{-}_{(p_i)r}$ TO Ψ^{-}

$p_i \leq -1$ $\displaystyle\sum_{r=0}^{-p_i-1} \psi_{(p_i)}(x)\, \chi^{+}_{(p_i)r}$

DIRAC ACTION ON X:

$$\hat{S}_D = \int_X d^{d+2}x \sqrt{g}\; \Psi^\dagger \hat{\not{D}}(\mathcal{A}) \Psi$$

$\mathcal{A}$ CONTRIBUTES A TERM:

$$\Psi^\dagger \left(\Phi\, \gamma \otimes \sigma_- + \Phi^\dagger \gamma \otimes \sigma_+ \right) \Psi$$

$$= \left((\Psi^+)^\dagger , (\Psi^-)^\dagger \right) \begin{pmatrix} \Phi^\dagger \gamma \Psi^- \\ \Phi \gamma \Psi^+ \end{pmatrix}$$

INTEGRATE OVER S^2:

$$S_D = S_{D_0} + \sum_{i=0}^{m} S_D^{(p_i)}$$

$$\boxed{\begin{aligned} S_{D_0} = \int_M d^dx \sqrt{g} \Big\{ &\sum_{i=0}^{m_-} \sum_{r=0}^{p_i-1} \tilde{\psi}^\dagger_{(p_i)r} \not{D}_M \tilde{\psi}_{(p_i)r} \\ &+ \sum_{i=m_+}^{m} \sum_{r=0}^{|p_i|-1} \psi^\dagger_{(p_i)r} \not{D}_M \psi_{(p_i)r} \Big\} \\ \frac{g}{2} \int_M d^dx \sqrt{g}\, &\underbrace{\left\{ \phi^\dagger_{m_+} \psi^\dagger_{(-1)} \gamma \tilde{\psi}_{(1)} + \phi_{m_+} \tilde{\psi}^\dagger_{(1)} \gamma \psi_{(-1)} \right\}}_{\text{ONLY PRESENT FOR ODD } m} \\ m_+ = \left\lceil \frac{m+1}{2} \right\rceil , \quad &m_- = \left\lfloor \frac{m-1}{2} \right\rfloor \end{aligned}}$$

- FOR m ODD, ZERO MODES GIVE YUKAWA COUPLINGS TO $\psi_{(-1)}$ AND $\tilde{\psi}_{(1)}$ ON M.

NON-ZERO MODES:

$$S_D^{(p_i)} = \int_M d^dx \sqrt{g} \sum_{j=j_{min}}^{\infty} \sum_{r=0}^{2j}$$

$$\times \Big\{ \psi^{\dagger}_{(j,p_i)r} \left(\not{D}_M + \mu_{j,p_i} \gamma \right) \psi_{(j,p_i)r} + \tilde{\psi}^{\dagger}_{(j,p_i)r} \left(\not{D}_M + \mu_{j,p_i} \gamma \right) \tilde{\psi}_{(j,p_i)r} + \frac{g}{2} \left(\psi^{\dagger}_{(j,p_i)r} \phi_i^{+} \gamma \tilde{\psi}_{(j,p_i+2)r} + \text{h.c.} \right) \Big\}$$

- $\phi_i^{\circ} = v_i$, MASS MATRIX

$$\left(\psi^{\dagger}_{(j,p_i)r} , \tilde{\psi}^{\dagger}_{(j,p_i+2)r} \right) \begin{pmatrix} \mu_{j,p_i} & g\bar{v}_i/2 \\ g v_i/2 & \mu_{j,p_i+2} \end{pmatrix} \begin{pmatrix} \gamma \psi_{(j,p_i)r} \\ \gamma \tilde{\psi}_{(j,p_i+2)r} \end{pmatrix}$$

$$\boxed{\mu_{\pm} = \frac{\mu_{j,p_i} + \mu_{j,p_i+2} \pm \sqrt{(\mu_{j,p_i} - \mu_{j,p_i+2})^2 + g^2 |v|^2}}{2}}$$

$\mu_{j,p} \sim 1/R \qquad v \sim 1/gR \qquad \Rightarrow \mu_{\pm} \sim 1/R$

- CHOOSE $\Psi = \begin{pmatrix} \Psi^{+} \\ \Psi^{-} \end{pmatrix}$ TO BE WEYL ON X

$\Rightarrow \Psi^{+} \sim \psi^{+} \chi^{+} , \quad \Psi^{-} \sim \tilde{\psi}^{-} \chi^{-}$

WITH ψ^{+}, $\tilde{\psi}^{-}$ WEYL ON M $\qquad (\psi^{-} = \tilde{\psi}^{+} = 0)$

IF $g_{\mu\nu}$ IS LORENTZIAN $\Rightarrow$ NO DIRECT MASS TERM, ONLY YUKAWA TERMS!

FUZZY SPHERE (MADORE, 1992)

$$S^2 \approx \mathbb{C}P^1 : \begin{pmatrix} z^1 \\ z^2 \end{pmatrix} \in \mathbb{C}^2 , \begin{pmatrix} z^1 \\ z^2 \end{pmatrix} \approx \lambda \begin{pmatrix} z^1 \\ z^2 \end{pmatrix} : S^2$$

$\forall \; \lambda \in \mathbb{C} \setminus \{0\}$

CHOOSE λ s.t. $z^\dagger z = 1 \Rightarrow S^3 \subset \mathbb{C}^2$

$$\begin{pmatrix} z^1 \\ z^2 \end{pmatrix} \approx e^{i\delta} \begin{pmatrix} z^1 \\ z^2 \end{pmatrix} \Rightarrow S^3/_{U(1)} \approx SU(2)/_{U(1)} \approx S^2$$

FUNCTION
$$f_L = \sum_{l=0}^{L} \sum_{m=-l}^{l} f^m_l \, Y^l_m(\theta, \phi)$$
$$= \underbrace{f^{\beta_1 \cdots \beta_L}_{\alpha_1 \cdots \alpha_L}}_{(L+1)^2 \text{ CO-EFFICIENTS}} \bar{z}_{\beta_1} \cdots \bar{z}_{\beta_L} z^{\alpha_1} \cdots z^{\alpha_L}$$

$$\boxed{\left[\frac{\partial}{\partial \bar{z}_\beta}, \bar{z}_\alpha \right] = \delta_\alpha{}^\beta \xrightarrow{\sigma} [a^\beta, a^+_\alpha] = \delta_\alpha{}^\beta}$$

FOCK SPACE : $a^\beta |0\rangle = 0$

MATRIX $(L+1) \times (L+1)$
$$\hat{f}_L = f^{\beta_1 \cdots \beta_L}_{\alpha_1 \cdots \alpha_L} a^+_{\beta_1} \cdots a^+_{\beta_L} |0\rangle\langle 0| a^{\alpha_1} \cdots a^{\alpha_L}$$

$$\boxed{\begin{array}{ll} \bar{z}_\alpha \to (a^+_\alpha)^L , & \frac{\partial}{\partial \bar{z}_\alpha} \to (a^\alpha)^L \\ z^\alpha \to (a^\alpha)^R , & \frac{\partial}{\partial z^\alpha} \to (a^+_\alpha)^R \end{array}}$$

MATRIX MULTIPLICATION $\hat{f}_L \cdot \hat{g}_L \xrightarrow{\sigma^{-1}} f_L * g_L$ STAR PRODUCT

$$[\hat{f}_L, \hat{g}_L] \neq 0 \Rightarrow [f_L, g_L]_* \neq 0$$

FUZZY DIRAC OPERATOR

$$\sigma_\pm = \tfrac{1}{2}(\sigma_1 \pm i\sigma_2), \qquad \sigma_- |\Omega\rangle = 0 \qquad |\Omega\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

CLIFFORD VACUUM

DIRAC SPINOR : $\chi = \chi^- |\Omega\rangle + \chi^+ \sigma_+ |\Omega\rangle$

$\chi^\pm$ SECTIONS OF $\mathcal{L}^{\pm 1}$ ($\mathcal{L}^{p \pm 1}$ FOR SPIN$_c$)

$$\not{D}_{S^2} = \sigma^+ \bar{\nabla} + \sigma^- \nabla \qquad \begin{array}{l} \bar{\nabla} : \chi^- \to \chi^+ \\ \nabla : \chi^+ \to \chi^- \end{array}$$

FUZZY SPINORS (GROSSE + PREŠNAJDER)

$$\chi^\pm \to \hat{\chi}^\pm \begin{cases} L \times (L+1) \text{ MATRIX} \\ (L+1) \times L \text{ MATRIX} \end{cases} \quad L = j_{MAX} + 1/2$$

$$\bar{\nabla} \to \bar{K} = (a^\alpha)^L (a^\beta)^R \varepsilon_{\alpha\beta} \qquad \bar{K} : \hat{\chi}^- \to \hat{\chi}^+$$

$$\nabla \to K = (a^+_\alpha)^L (a^+_\beta)^R \varepsilon^{\alpha\beta} \qquad K : \hat{\chi}^+ \to \hat{\chi}^-$$

$$\not{D} = \sigma^+ \bar{K} + \sigma^- K$$

INCLUDING MONOPOLES

$$\hat{\chi}^+ : (L - p/2) \times (L + 1 + p/2)$$

$$\hat{\chi}^- : (L + 1 - p/2) \times (L + p/2)$$

$$\boxed{\not{D} = \sigma^+ \bar{K} + \sigma^- K}$$

UNIVERSAL!

ZERO MODES :

$$\hat{\chi}^+_{\alpha_1 \cdots \alpha_{|p|-1}} = a^+_{\alpha_1} \cdots a^+_{\alpha_{|p|-1}} |0\rangle\langle 0| , \; p \le -1$$

$L_{MIN} = \frac{|p|-1}{2}$

$$\hat{\chi}^{-\, \alpha_1 \cdots \alpha_{p-1}} = |0\rangle\langle 0| a^{\alpha_1} \cdots a^{\alpha_{p-1}} , \; p \ge 1$$

m ODD, $\quad m = 2m_+ - 1$, ZERO MODES, Ψ_0

$$\Psi_0^{\pm} = \bigoplus_{i=0}^{m} \Psi^{\pm}_{(\phi_i)}$$

$$\Psi_0 = \begin{pmatrix} \Psi_0^+ \\ \Psi_0^- \end{pmatrix} = \left(\begin{array}{c} 0 \\ \vdots \\ 0 \\ \hline \Psi^+_{(-1)} \\ \vdots \\ \Psi^+_{(-m)} \\ \hline \Psi^{-}_{(m)} \\ \vdots \\ \Psi^-_{(1)} \\ \hline 0 \\ \vdots \\ 0 \end{array}\right) \begin{matrix} \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} m_+ \\ \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} m_+ \\ \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} m_+ \\ \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} m_+ \end{matrix}$$

$$(\Phi^\dagger \sigma^+ + \Phi \sigma^-)\Psi_0 = \left(\begin{array}{c} 0 \\ \hline \phi_1^\dagger \Psi^-_{(m)} \\ \vdots \\ \phi^\dagger_{m_+} \Psi^-_{(1)} \\ \hline 0 \\ \vdots \\ 0 \\ \hline 0 \\ \vdots \\ 0 \\ \hline \phi_{m_+} \Psi^+_{(-1)} \\ \vdots \\ \phi_m \Psi^+_{(-m)} \\ \hline 0 \end{array}\right) \begin{matrix} \} 1 \\ \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} m_+ \\ \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} m_+ - 1 \\ \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} m_+ - 1 \\ \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} m_+ \\ \} 1 \end{matrix}$$

$$\Phi = \begin{pmatrix} 0 & \phi_1 & 0 & \cdots & 0 \\ 0 & 0 & \phi_2 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & & \phi_m \\ 0 & 0 & \cdots & & 0 \end{pmatrix}$$

CONCLUSIONS + OUTLOOK

- EQUIVARIANT DIMENSIONAL REDUCTION
  - DYNAMICAL SYMMETRY BREAKING
  - ZERO MODES OF ϕ ACQUIRE YUKAWA COUPLINGS

- OTHER COSETS, K/H $\left(\mathbb{C}P^N_F\right)$