QUANTUM ENTANGLEMENT IN A NONCOMMUTATIVE SYSTEM

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- MOTIVATION FOR STUDYING NC SYSTEMS
  - (i) B.H. / Q.G. (DFR)
  - (ii) QHE
- MOYAL DEFORMATION → A PARTICULAR EXAMPLE
- ISSUES → SPACE-TIME SYMMETRY
  IDENTICAL PARTICLES
  SPIN-STATISTICS

HERE WE WORK WITH MOYAL PLANE

$$[\hat{\bar{x}}_i, \hat{\bar{x}}_j] = i\theta_{ij} = i\theta\epsilon_{ij} \quad ; \; \theta_{0i} = 0$$

ALTERNATIVELY,

$$[\bar{x}_i, \bar{x}_j]_* \equiv \bar{x}_i * \bar{x}_j - \bar{x}_j * \bar{x}_i = i\theta\epsilon_{ij}$$

$$f(\bar{x}) * g(\bar{x}) = f(\bar{x})\, e^{\frac{i}{2}\theta_{ij}\overleftarrow{\partial}_i \overrightarrow{\partial}_j}\, g(\bar{x})$$

MORE FORMALLY,

$$f * g = m_\theta(f \otimes g) = m_0\Big(\underbrace{e^{\frac{i}{2}\theta_{ij}\partial_i \otimes \partial_j}}_{\substack{F_\theta \\ \text{(TWIST OP.)}}} f \otimes g\Big)$$

In R^3, for example,

$$f(x) \to f^R(x) = f(R^{-1}x) \quad ; R \in SO(3)$$

$$\ni \quad (f \cdot g)^R(x) = f^R(x) \cdot g^R(x)$$

BUT IN R^3_*

$$(f * g)^R(\bar{x}) \neq f^R(\bar{x}) * g^R(\bar{x})$$

WE ARE THEREFORE LOSING ENTIRE TENSOR-ALGEBRAIC STRUCTURE!!

HOWEVER, $\exists \; x_i \equiv \frac{1}{2}(\bar{x}^L_i + \bar{x}^R_i)$

s.t. $\quad [x_i, x_j] = 0$

And $\quad J_{ij} = x_i p_j - x_j p_i \;$ satisfy $SO(3)$ algebra

IMP Let $\bar{\xi}^T = (\bar{x}_1, p_1, \bar{x}_2, p_2)$

$\xi^T = (x_1, p_1, x_2, p_2)$

THEN $\quad \bar{\xi}^\alpha \to \xi^\mu = M^\mu{}_\alpha \bar{\xi}^\alpha \quad ; M = \{M^\mu{}_\alpha\} = \begin{pmatrix} 1 & 0 & 0 & \frac{\theta}{2} \\ 0 & 1 & -\frac{\theta}{2} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

IS NOT A CANONICAL TRANSFORMATION

ONE CAN SEE:

SYMPLECTIC MATRIX IS $\quad \bar{\Omega} = M^{-1} \Omega (M^{-1})^T \neq \Omega$

Action of J_{ij} on $(f*g)$

$$J_{ij}(f*g) = (J_{ij}f)*g + f*(J_{ij}g)$$
$$-\frac{1}{2}\Big[\big((p\cdot\theta)_i f*(p_j g) - (p_j f)*((p\cdot\theta)_i g)\big) - i\leftrightarrow j\Big]$$

THIS DEFORMED LEIBNIZ RULE HAS ITS ORIGIN IN HOPF ALGEBRA THEORY OF DRINFELD

GROUP/ALGEBRA ACTION ON V

$$v \to \rho(g)v \quad ; \ g\in G,\ v\in V$$

$\rho \to$ representation

$\downarrow$ GENERALIZE TO ACTION ON $V\otimes W$

$$v\otimes w \to \rho_1(g)v\otimes\rho_2(g)w \equiv \Delta_0(g)(v\otimes w)$$

WE INTRODUCED CO-PRODUCT (Δ_0), WHICH 'SPLITS' $g\in G$ TO ACT ON TENSOR PRODUCTS: $\Delta_0(g) = g\otimes g$

IF V IS ALSO AN ALGEBRA, THEN THE COMPATIBILITY CONDITION IS

$$m_0\big(\Delta_0(g)(v\otimes w)\big) = g\big(m_0(v\otimes w)\big)$$

$\downarrow$ SHOULD BE MODIFIED TO

$$m_\theta\big(\Delta_\theta(g)(v\otimes w)\big) = g\big(m_\theta(v\otimes w)\big) \quad ; \ \Delta_\theta(g) = F_\theta^{-1}\Delta_0(g)F_\theta$$

CO-PRODUCT ON LIE-ALG. IS DEFORMES AS:

$$\Delta_0(J_{ij}) \to \Delta_\theta(J_{ij}) = F_\theta^{-1}\Delta_0(J_{ij})F_\theta = \underbrace{\Delta_0(J_{ij})}_{J_{ij}\otimes 1 + 1\otimes J_{ij}} - \frac{1}{2}\Big(\big[(p\cdot\theta)_i\otimes p_j - p_j\otimes(p\cdot\theta)_i\big] - i\leftrightarrow j\Big)$$

THIS IS PRECISELY THE DEFORMED LEIBNIZ RULE

CONSEQUENTLY,

(ANTI)-SYMMETRISATION $\longrightarrow$ (TWISTED) (ANTI)-SYMM.

$$\tau_0 (\varphi \otimes \chi) = \chi \otimes \varphi$$

SATISFIES $[\tau_0 , \Delta_0] = 0$ $\longrightarrow$ τ_0 AND THEREFORE STATISTICS IS SUPERSELECTED!

LIKEWISE, DEFORM

$$\tau_0 \to \tau_\theta = F_\theta^{-1} \tau_0 F_\theta \quad ; \quad P_\theta \equiv \frac{1}{2}(\mathbb{1} \pm \tau_\theta)$$

SO THAT,

$$[\tau_\theta , \Delta_\theta] = 0$$ $\longrightarrow$ STATISTICS REMAIN SUPERSELECTED!!

TWISTED BOSONS/FERMIONS SATISFIES DEFORMED CR's

$$a_p a_q = \eta \, e^{i p \wedge q} a_q a_p \quad ; \quad \eta = \pm 1$$
$$p \wedge q = \theta^{\mu\nu} p_\mu q_\nu$$

<u>SOME CONSEQUENCES</u>

- VIOLATION OF PAULI PRINCIPLE
- ALTERS LIFETIMES OF UNSTABLE PARTICLES
- CHANGES ENTANGLEMENT PROPERTIES

$\hookrightarrow$ WE SHOW THIS HERE!!

A BRIEF DIGRESSION ON ENTANGLEMENT

- FIRST INTRODUCED BY EPR (1935) [ONLY $[\hat{x}, \hat{p}] = i\hbar$]
- NOW $\exists$ POSSIBILITIES FOR MANIPULATING QUANTUM INFO'S IN ENTANGLED STATES
  - → QUANTUM TELEPORTATION
  - → CRYPTOGRAPHY
  - → DENSE CODING

IN FACT, VARIOUS PROTOCOLS FOR TELEPORTATION & CRYPTOGRAPHY HAVE BEEN DEVELOPED USING CONTINUOUS VARIABLE ENTANGLED STATES AS RESOURCES.

MOTIVATION FOR THE PRESENT STUDY

TO INVESTIGATE THE IMPACT OF ALTERED SYMPLECTIC STRUCTURE AND TWISTED (ANTI)-SYMMETRISATION ON INFORMATION CAPACITY/ENTROPY ON ENTANGLEMENT

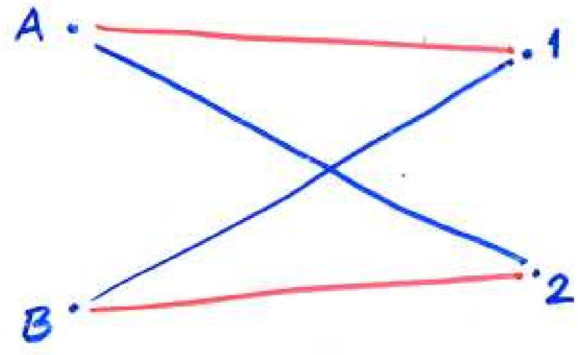
HERE, WE HAVE INITIATED SUCH AN INVESTIGATION AND RESTRICT OURSELVES TO BIPARTITE GAUSSIAN STATES (BOSONIC)

ON SOME OTHER HINTS

(i) ENHANCED PHASE-SPACE UNCERTAINITY

(SEE LATER!)

(ii) HANBURY BROWN & TWISS EFFECT (R. SHREEVASTAVA) (SACHIN ~~[illegible]~~.)



$1, 2 \rightarrow$ DETECTORS

$A, B \rightarrow$ SOURCES VERY FAR FROM EACH OTHER (SAY, DIAMETRICALLY OPPOSITE POINTS OF A STAR)

HERE THE INTENSITIES AT 1 & 2 APPEAR CORRELATED!

INTENSITY CORRELATIONS ARE RELATED TO 4-PT FUNCTIONS:

$$G^{(2)}(x_1, x_2) = \underbrace{G^{(2)}(x_1 - x_2)}_{\text{ASSUMING TRANSLATIONAL INV. (ALSO FOR } \rho)} = \mathrm{tr}\left(\rho\, \Phi^\dagger(x_1)\, \Phi^\dagger(x_2)\, \Phi(x_2)\, \Phi(x_1)\right)$$

DEFINE

$$C(\vec{x}) \equiv C(\vec{x}_1 - \vec{x}_2) \equiv \frac{G^{(2)}(x)}{G^{(1)}(x_1)\, G^{(2)}(x_2)} \quad ; \; G^{(1)}(x_1) = \mathrm{tr}\left(\rho\, \Phi^\dagger(x_1) \Phi(x_1)\right)$$

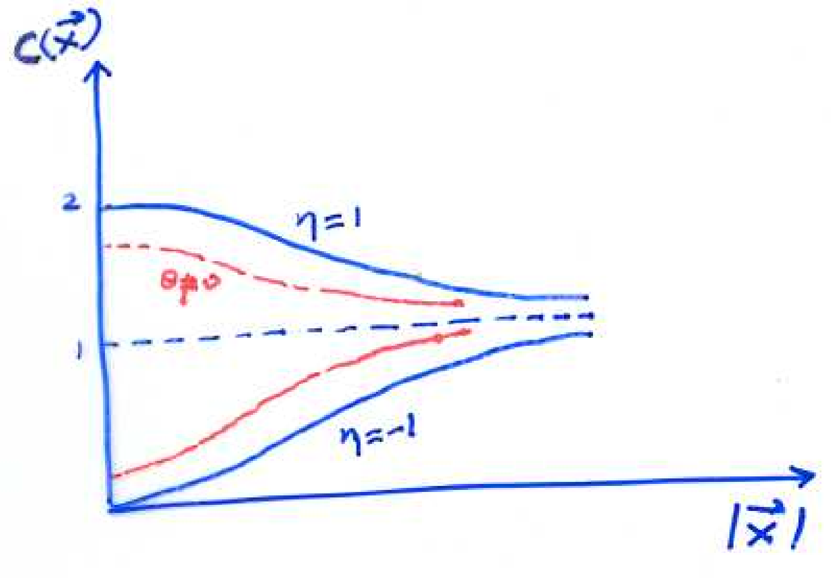
FOR AVERAGE NUMBER OF PARTICLES WITH MOM. $\vec{p}$

$$N_p \propto e^{-\alpha(\vec{p}-\vec{p}_0)^2}$$

$$C(\vec{x}) = 1 + \eta \cdot \frac{4\alpha^2}{4\alpha^2+\vec{\theta}^2}\, e^{-\frac{4\alpha^2\vec{x}^2+(\vec{x}\cdot\vec{\theta})^2}{2\alpha(4\alpha^2+\vec{\theta}^2)}} \cdot e^{-\frac{4\alpha\vec{x}\cdot(\vec{p}_0\times\vec{\theta})}{(4\alpha^2+\vec{\theta}^2)}}$$

$$e^{-2\alpha\left[\frac{\vec{\theta}^2\vec{p}_0^2-(\vec{\theta}\cdot\vec{p}_0)^2}{4\alpha^2+\vec{\theta}^2}\right]}$$

$\theta^{0i} = 0$; $\vec{\theta} = \{\theta_i \equiv \epsilon_{ijk}\,\theta_{jk}\}$



NC REDUCES HBT (ANTI) CORRELATIONS!

$C \simeq 1 \longrightarrow$ DISTINGUISHABLE PARTICLES?

REDUCED CORRELATION $\longrightarrow$ ENTANGLEMENT IS REDUCED ?

SEPARABILITY CRITERIA FOR A BIPARTITE SYSTEM

$(\theta = 0)$

CONSIDER THE BIPARTITE BOSONIC STATE

$$\Psi(p_1, p_2) = N\left(\psi_1(p_1) \otimes \psi_2(p_2) + \psi_2(p_1) \otimes \psi_1(p_2)\right)$$

LET $\hat{\xi}^T = (\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2)$ (IN ORDER) BE THE PHASE SP. OPERATORS

$$\{\hat{\xi}^\mu \hat{\xi}^\nu\} \rightarrow \{\hat{\xi}'^\mu \hat{\xi}'^\nu\} = \{S^\mu{}_\alpha S^\nu{}_\beta \hat{\xi}^\alpha \hat{\xi}^\beta\} = S\{\hat{\xi}^\alpha \hat{\xi}^\beta\} S^T$$

$$S \in Sp(4, \mathbb{R})$$

$$\frac{1}{2}\{\hat{\xi}^\mu, \hat{\xi}^\nu\} + \frac{1}{2}[\hat{\xi}^\mu, \hat{\xi}^\nu] = \underbrace{\hat{V}}_{\text{OP. VAR. MATRIX}} + i\Omega$$

$\hookrightarrow$ SYMPL. MATRIX (C-NO. ENTRIES)

IF 'ρ' BE THE DENSITY MATRIX,

$$V = tr(\rho \hat{V}) = \begin{pmatrix} \langle \hat{x}^2 \rangle & \frac{1}{2}\langle\{x, p\}\rangle \\ \frac{1}{2}\langle\{x, p\}\rangle & \langle \hat{p}^2 \rangle \end{pmatrix}$$

IS THE VARIANCE MATRIX, IF $\langle \hat{x} \rangle = \langle \hat{p} \rangle = 0$

[OTHERWISE, $V = \{ v^{\mu\nu} \equiv \frac{1}{2}\langle\{\hat{\xi}^\mu, \hat{\xi}^\nu\}\rangle - \langle\hat{\xi}^\mu\rangle\langle\hat{\xi}^\nu\rangle \}$]

HAS TRANSLATIONAL INV. IN PHASE SP.

PHYSICALITY STATEMENT

$$\det V \geq \frac{1}{4}$$

(SYMPL. INV. STATEMENT OF HEISENBERG'S UNCERTINITY REL.

$$\Delta x \Delta p \geq \frac{1}{2}$$

BY WILLIAMSON'S TH.

$\exists\ S \in Sp(4,\mathbb{R})$ s.t.

$$V \rightarrow V' = SVS^T = \mathrm{diag}(\nu/2, \nu/2)$$

PHYSICALITY COND. $\nu^2 \geq 1$

CAN BE GENERALISED TO n-MODE SYSTEM:

$$V \rightarrow V' = SVS^T = \mathrm{diag}(\nu_1/2, \underbrace{\nu_2/2, \ldots, \nu_n/2, \nu_1/2, \ldots, \nu_n/2}_{})$$

SYMPLECTIC SPECTRUM
(AT LEAST DOUBLY DEG.)

THE SYMPLECTIC SPEC. CAN BE OBTAINED AS ORDINARY EIGEN-VALUES OF $|2i\Omega V|$.

PHYSICALITY COND.

$$\nu_i^2 \geq 1 \quad \forall i$$

NOW THE ORDINARY EIGEN-VALUES OF $(V' + i\Omega)$ SATISFY

$$\frac{1}{2}(\nu_i \pm 1) \geq 0$$

$$\Rightarrow \boxed{V + i\Omega \geq 0}$$

$Sp(4,\mathbb{R})$ INVARIANT STATEMENT

PERES-HORODECKI PPT CRITERION

$$V \longrightarrow \tilde{V} = \Lambda V \Lambda \quad ; \quad \Lambda = \mathrm{diag}(1,1,1,-1)$$

FOR ANY 2-MODE GAUSSIAN STATES, THE SEPARABILITY CRITERION IS

$$\tilde{\nu}_{min}^{2} \geq 1$$

OTHERWISE, IT IS ENTANGLED!

IT IS MEASURED THROUGH LOGARITHMIC NEGATIVITY

$$E = \max\left[0, -\log_2(\tilde{\nu}_{min})\right]$$

SINGLE PARTICLE GAUSSIAN STATES IN MOM. SP.

$$\psi(\vec{p};\vec{a},\vec{p}_0,\alpha) = \frac{1}{\sqrt{\pi\alpha}}\, e^{-\frac{1}{2\alpha}\left(\vec{p}-\frac{\vec{p}_0}{2}\right)^2 - i\vec{p}\cdot\vec{b}}$$

$$b_i = a_i + \frac{\theta}{2}\epsilon_{ij} p_{0j}$$

USE $\hat{x}_i = i\partial_{p_i} - \frac{\theta}{2}\epsilon_{ij} p_j$

THEN $\langle \hat{\vec{x}} \rangle_\psi = \vec{a}$; $\langle \hat{\vec{p}} \rangle_\psi = \frac{\vec{p}_0}{2}$

$$V^1_{NC} = \begin{pmatrix} \frac{\theta^2\alpha}{8}+\frac{1}{2\alpha} & 0 & 0 & -\frac{\theta\alpha}{4} \\ 0 & \frac{\theta^2\alpha}{8}+\frac{1}{2\alpha} & \frac{\theta\alpha}{4} & 0 \\ 0 & \frac{\theta\alpha}{4} & \frac{\alpha}{2} & 0 \\ -\frac{\theta\alpha}{4} & 0 & 0 & \alpha/2 \end{pmatrix}$$

JUST READ-OFF

$$\Delta\hat{x}_1 = \Delta\hat{x}_2 = \sqrt{\frac{\theta^2\alpha}{8}+\frac{1}{2\alpha}} \quad ; \quad \Delta\hat{p}_1 = \Delta\hat{p}_2 = \sqrt{\frac{\alpha}{2}}$$

THE VARIOUS UNCERTAINITY REL. ARE

$$\Delta\hat{x}_1\,\Delta\hat{x}_2 = \frac{\theta^2\alpha}{8}+\frac{1}{2\alpha} \quad ; \quad \Delta\hat{x}_1\Delta\hat{p}_1 = \Delta\hat{x}_2\Delta\hat{p}_2 = \sqrt{\frac{\theta^2\alpha^2}{16}+\frac{1}{4}}$$

$$\Rightarrow \left(\Delta\hat{x}_1\Delta\hat{x}_2\right)_{min} = \frac{\theta}{2} \rightarrow \text{AT } \alpha=\alpha_0=\frac{2}{\theta}$$

$$\Delta\hat{x}_1\Delta\hat{p}_1\Big|_{\alpha=\alpha_0} = \Delta\hat{x}_2\Delta\hat{p}_2\Big|_{\alpha=\alpha_0} = \frac{1}{\sqrt{2}} > \frac{1}{2}$$

THE UNCERTAINITIES ARE ENHANCED FOR $\theta \neq 0$
$\Rightarrow$ REDUCTION OF ENTROPY & ENTANGLEMENT!

EFFECTIVE COMM. VARIANCE MATRIX

$$V_C^1 \equiv M V_{NC}^1 M^T = V_{NC}^1\Big|_{\theta=0} = \mathrm{diag}\left(\frac{1}{2\alpha}, \frac{1}{2\alpha}, \frac{\alpha}{2}, \frac{\alpha}{2}\right)$$

SO THAT,

$$\Delta\hat{x}_1 \Delta\hat{x}_2 = \frac{1}{2\alpha} \quad ; \quad \Delta\hat{x}_1 \Delta\hat{p}_1 = \Delta\hat{x}_2 \Delta\hat{p}_2 = \frac{1}{2}$$

$$\Delta\hat{x}_1 \Delta\hat{x}_2\Big|_{min} = 0 \quad \text{AT} \quad \alpha \to \infty$$

ONE THEREFORE DOES NOT SEE ANY EFFECT OF NC-TY IN V_C^1 - AT THE SINGLE PARTICLE LEVEL, IF COMM. VARIANCE MATRIX IS CONSIDERED.

HOWEVER, FOR TWO PARTICLES

$$V_C^2(\theta) \equiv M^{(2)} V_{NC}^{(2)} M^{(2)^T} \neq V_{NC}^2\Big|_{\theta=0}$$

$$M^{(2)} = \begin{pmatrix} M \otimes \mathbb{1} & 0 \\ 0 & \mathbb{1} \otimes M \end{pmatrix}$$

WILL HAVE θ-DEPENDENCE

- HERE, WE SHALL BE WORKING WITH $V_C^2(\theta)$, AS WE CAN DIRECTLY USE WILLIAMSON'S THEOREM.

- WE FIND THAT THIS θ-DEP. WILL BE RESPONSIBLE FOR MODIFYING PERES-HORODECKI CRITERION.

Some Simple Commutative Examples

1. 2-Particles on $\mathbb{R}^1$

$$\psi_1(p; p_0, \alpha) = \frac{1}{(\pi\alpha)^{1/4}} e^{-\frac{1}{2\alpha}\left(p - \frac{p_0}{2}\right)^2}$$

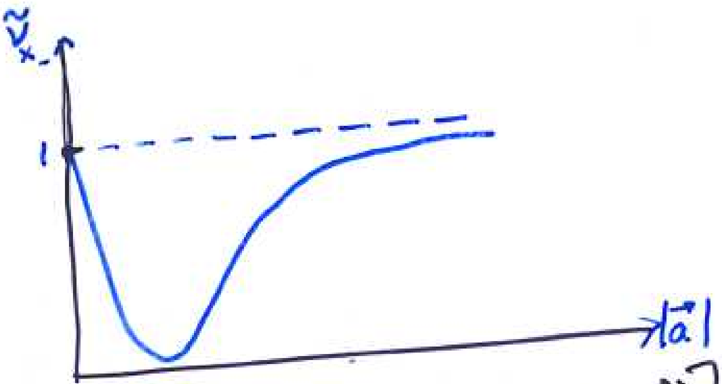
$$\psi_2(p; p_0, \beta) = \psi_1(p; -p_0, \beta)$$

THE SYMMETRIC BOSONIC STATE CAN BE SHOWN TO BE SEPARABLE ONLY WHEN $p_0 = 0$ & $\alpha = \beta$ BY COMPUTING $\tilde{V}$ AND PERES-HORODECKI CRITERION.

2. 2-Particles on $\mathbb{R}^2$

$$\psi_1(\vec{p}_1) = \frac{1}{\sqrt{\pi\alpha}} \exp\left(-\frac{\vec{p}_1^{\,2}}{2\alpha} - i\vec{p}_1 \cdot \vec{a}_1\right)$$

$$\psi_2(\vec{p}_2) = \frac{1}{\sqrt{\pi\alpha}} \exp\left(-\frac{\vec{p}_2^{\,2}}{2\alpha} + i\vec{p}_2 \cdot \vec{a}_2\right)$$



$$\Psi(\vec{p}) = N\left[\psi_1(\vec{p}_1) \otimes \psi_2(\vec{p}_2) + \psi_2(\vec{p}_1) \otimes \psi_1(\vec{p}_2)\right]$$

CAN BE SHOWN TO BE ENTANGLED AT ALL POINTS $\in (0, \infty)$ AND SEPARABLE AT $|\vec{a}| \to 0$ or ∞.

FINALLY TO 2-PARTICLES IN R^2_*

IDENTIFICATION OF APPROPRIATE PH-SP. OP.

THE NAIVE 2-PARTICLE PHASE-SP. OPERATORS HAVE TO BE MODIFIED BY "CONJUGATION"

$$F_\theta^{-1}(\xi^\mu \otimes \mathbb{1})F_\theta \quad \underset{=F_\theta^{-2}\tau_0}{\xleftrightarrow{\tau_\theta = F_\theta^{-1}\tau_0 F_\theta}} \quad F_\theta^{-1}(\mathbb{1}\otimes \xi^\mu)F_\theta$$

2-PARTICLE "TWISTED" SYMMETRIC WAVE FUNCTION

$$\boxed{\Psi = N\left(\psi_1 \otimes \psi_2 + F_\theta^{-2}(\psi_2 \otimes \psi_1)\right)}$$

OBTAINED BY "TWISTED" $P_\theta = \frac{1}{2}(\mathbb{1}\otimes\mathbb{1} + \tau_\theta)$

ACTION ON $(\psi_1 \otimes \psi_2)$.

STRATEGY

- THE "EFFECTIVE" COMMUTATIVE VARIANCE MATRIX IS OBTAINED

$$V^2_{eff}(\theta) = \begin{pmatrix} M\otimes\mathbb{1} & 0 \\ 0 & \mathbb{1}\otimes M \end{pmatrix} V^2_{NC}(\theta) \begin{pmatrix} M^T\otimes\mathbb{1} & 0 \\ 0 & \mathbb{1}\otimes M^T \end{pmatrix}$$

- DEDUCE THE θ-ANALOG OF THE PHYSICALITY COND

$$(\nu_-^{NC})^2 \geq \left(\nu_-^{NC}(\theta)_{min}\right)^2$$

$\| \min\left((\nu_{x-}^{NC})^2, (\nu_{y-}^{NC})^2\right)$

$\downarrow$ LOWEST SYMPLECTIC EIGEN-VALUE WHEN $\Delta x_1, \Delta x_2$ IS MINIMIZED.

- IT TURNS OUT THAT $\alpha_{min}(b_1)$ IS A ~~SLOW~~ WEAK DEPENDENCE ON b_1.

- NOW APPLY PERES-HORODECKI CRITERION

(i) CONSTRUCT PARTIAL-TRANSPOSED VARIANCE MATRIX:

$$V \to \tilde{V} = \Lambda V \Lambda$$

$$\Lambda = diag(1,1,1,1,1,1,-1,-1)$$

(ii) OBTAIN: $\tilde{\nu}_-^{NC} = \min\left(\tilde{\nu}_{x-}^{NC}, \tilde{\nu}_{y-}^{NC}\right)$

- ENTANGLEMEN CRITERION IS NOW

$$(\tilde{\nu}_-^{NC})^2 < (\nu_-^{NC}(\theta))^2_{min}$$

$$E^{NC} = \max\left[0, -\tfrac{1}{2}\log_2(\tilde{\nu}_-)^2\right] \quad ; \tilde{\nu}_-^2 = (\tilde{\nu}_-^{NC})^2 - (\nu_-^{NC}(\theta))^2_{min} + 1$$

- RESTRICT HERE TO $\vec{F}_0 = 0$, $\vec{b} = (b_1, 0)$.

CONCLUSIONS

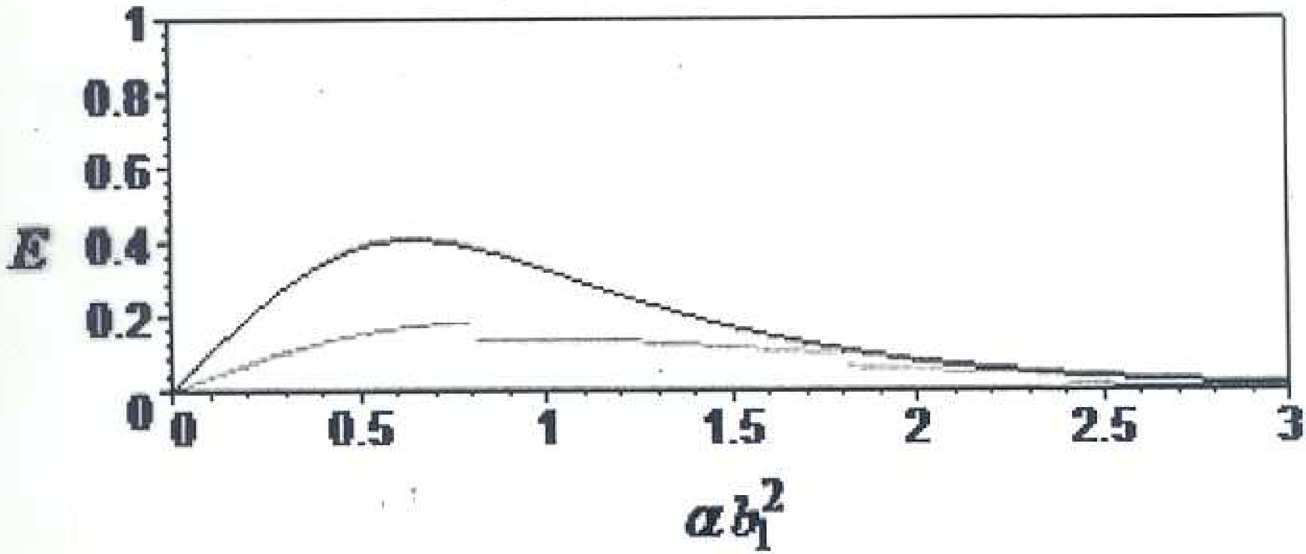
1) ENTANGLEMENT IS REDUCED FOR $\theta \neq 0$.
   → MOST PROMINENT WHEN $\alpha b_1^2 \sim \mathcal{O}(1)$
   → ASYMPTOTICALLY SEPARABLE. (ALSO FOR $\theta = 0$)
   → PRESUMABLY CORRELATED TO ENHANCED PHASE-SPACE UNCERTAINITIES.
   → IN FACT, THE CRITERION IS ITSELF MODIFIED.

2) RELATION WITH GRAVITY INDUCED (PENROSE) DECOHERENCE ?

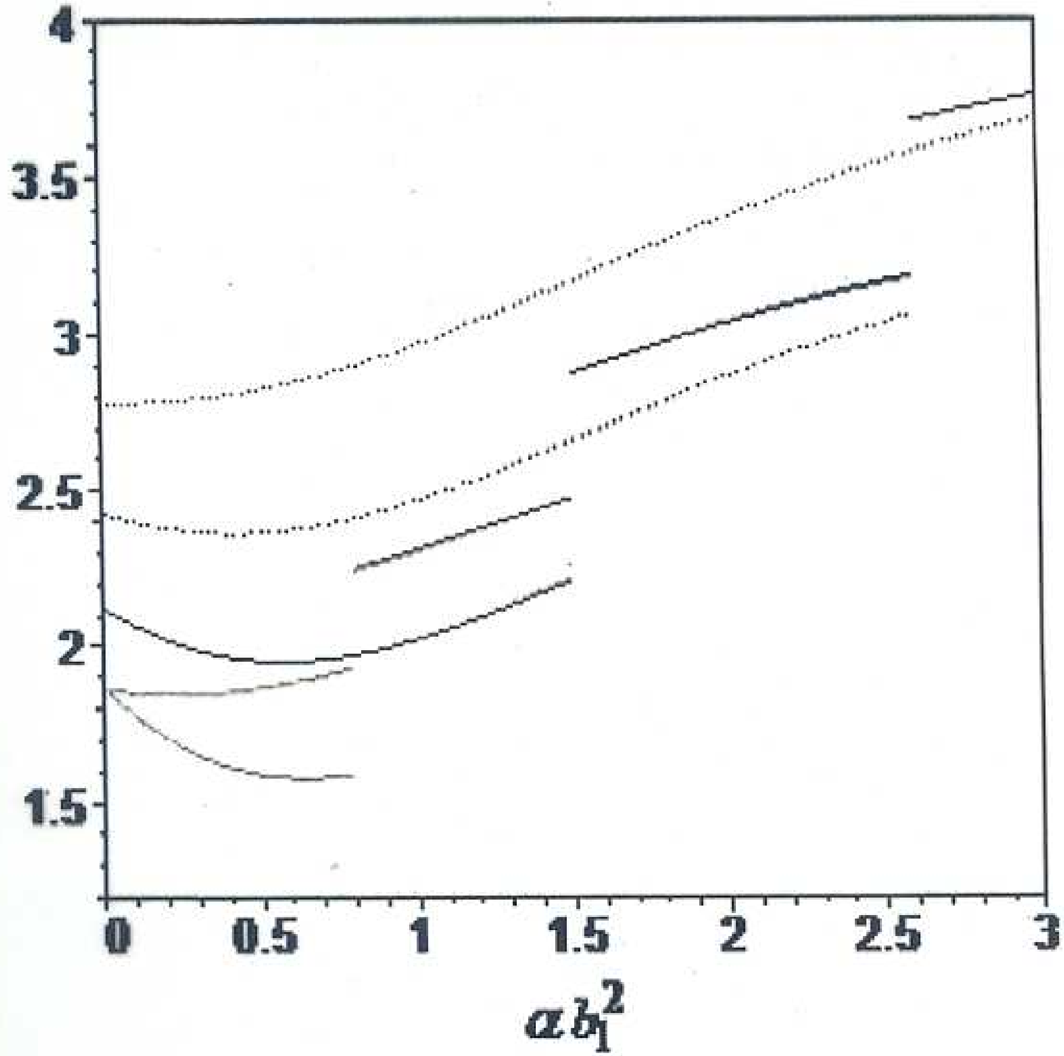
3) HERE WE ARE ONLY INTERESTED IN STUDYING INFORMATION CONTENT OF BIPARTITE NC SYSTEM.
   → NOT REGARDED AS RESOURCE FOR PERFORMING TELEPORTATION, DENSE CODING

4) MAY BE RELEVANT FOR 2D SYSTEM, HAVING EFFECTIVE NC.
   e.g. QHE, GRAPHENE ETC.

ALSO IN HAWKING RADIATION, WHICH CAN BE REGARDED AS QUANTUM ENTANGLEMENT EFFECTS



1
0.8
0.6
E
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
αb_1^2



4
3.5
3
2.5
2
1.5
0
0.5
1
1.5
2
2.5
3
αb_1^2