NONCOMMUTATIVE PHENOMENOLOGY

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QUANTUM FIELDS ON THE MOYAL PLANE

Quantum field theories constructed on the Moyal plane (noncommutative space-time) exhibit the following features

- Lorentz non-invariance

- CPT violation

- Causality violation

- Twisted statistics

LORENTZ NON-INVARINCE

Lorentz non-invariance can cause the scattering matrix of the theory to be frame dependent.

It leads to many interesting consequences. Eg: Nontrivial corrections to the electric and magnetic properties of elementary or composite particles.

CPT VIOLATION

The θ^{0i} components of the noncommutativity matrix $\theta^{\mu\nu}$ reverse sign under P and CPT. [E. Akofor et al.]

That is

$$\text{P or CPT}: \vec{\theta}^0 \to -\vec{\theta}^0,$$

where $\vec{\theta}^0 = (\theta^{01}, \theta^{02}, \theta^{03})$.

The $\vec{\theta}^0$ contributes to P and thus CPT violation.

It can change the particle anti-particle life times. To order θ^{0i}:

$$\tau_{\text{particle}} - \tau_{\text{antiparticle}} \approx \theta^{0i} P_i^{\text{in}}.$$

This may give rise to K^0 - $\bar{K}^0$ mass difference.

The $g-2$ of μ^+ and μ^- can differ.

Causality violation

We cannot localize spacetime points in these theories. Light cone itself is not sharp.

Acausal propagation in these theories may solve the homogeneity problem of the early universe. (Work in progress.)

Twisted statistics

Statistics of the noncommutative quantum fields are deformed.

Gives rise to Pauli forbidden transitions [Bal et al.] and modified statistical potential. [B. Chakraborty et al.]

PHENOMENOLOGICAL CONSEQUENCES

Here I focus on some phenomenological consequences due to spacetime noncommutativity from particle physics point of view. arXiv:0811.3972 [hep-ph]

- Mass difference in K^0 - $\bar{K}^0$ system.

- Different $g-2$ for μ^+ and μ^-.

- Modified electromagnetic form factors.

We can put bounds on the length scale of spacetime noncommutativity using available experimental data.

NONCOMMUTATIVE K^0 - $\bar{K}^0$ SYSTEM

Mass renormalizations of the CP eigenstates K_S and K_L become $\theta^{\mu\nu}$ dependent.

K_S has two pion states as lowest mass intermediate states.

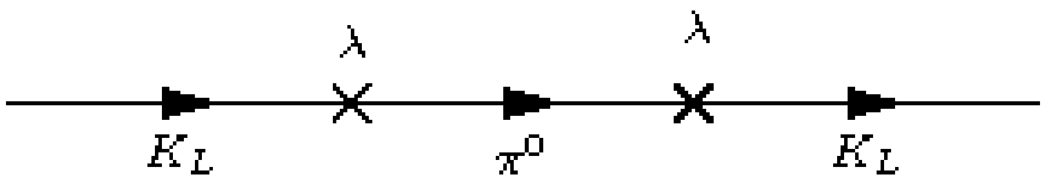
For K_L, the lowest mass intermediate state is a one pion state.

In the mass renormalization of K_S and K_L, the two pion intermediate state of K_S is expected to be smaller than one pion intermediate state of K_L.

K_L should be affected the most by spacetime noncommutativity.

In an arbitrary scattering diagram, the S-operator depends on noncommutativity through the twist factor $\frac{1}{2}\overleftarrow{\partial}_0 \vec{\theta}^0 \cdot \overrightarrow{P}_{\mathrm{in}}$, where $\overleftarrow{\partial}_0$ differentiates the appropriate time argument.

The self-energy diagram for K_L with one pion pole dominance should be affected by this twist factor.



Twisting the mass and decay width of K_L

$$m_L^\theta = m_L \exp\left(\frac{i}{2} m_{K^0} \vec{\theta}^0 . \vec{P}_{\text{in}}\right), \quad \gamma_L^\theta = \gamma_L \exp\left(\frac{i}{2} m_{K^0} \vec{\theta}^0 . \vec{P}_{\text{in}}\right)$$

If we assume that the mass and width differences are arising purely due to noncommutativity, then we have $m_L - m_S = 0$ and $\gamma_L - \gamma_S = 0$ for the commutative case ($\theta^{\mu\nu} = 0$).

In that case $m_L = m_S = m_{K^0}$ and $\gamma_L = \gamma_S = \gamma_{K^0}$.

Then to the lowest order in $\vec{\theta}^0$:

$$\Delta m \simeq \frac{\gamma_{K^0}}{2} \left(\frac{m_{K^0}}{2} \vec{\theta}^0 . \vec{P}_{\text{in}}\right), \quad \Delta\gamma \simeq 2 m_{K^0} \left(\frac{m_{K^0}}{2} \vec{\theta}^0 . \vec{P}_{\text{in}}\right)$$

To the lowest order in $\vec{\theta}^0$

$$r_{K^0} \equiv \frac{|m_{K^0} - m_{\bar{K}^0}|}{m_{K^0}} \simeq \delta_\perp (m_{K^0}\vec{\theta}^0.\vec{P}_{in})\sqrt{1+\tan^2(\phi_{SW})}.$$

The CPT figure of merit r_{K^0} from KTeV experiment $r_{K^0} \equiv \frac{|m_{K^0} - m_{\bar{K}^0}|}{m_{K^0}} \lesssim 10^{-18}$ gives a bound for the noncommutativity length scale

$$\sqrt{\theta} \lesssim 10^{-32}\text{m}.$$

This corresponds to a lower bound for energy E associated with spacetime noncommutativity: $E \gtrsim 10^{16}$ GeV.

NONCOMMUTATIVITY BOUND FROM MUON $g-2$

CPT violation measurements on the $g-2$ of positive and negative muons may give a bound on spacetme noncommutativity.

In a non-abelian gauge theory the S-operator

$$S_{(\theta)} \neq S_{(\theta=0)}$$

This can affect the lepton-photon vertex function through the contribution from hadronic (and thus non-abelian) loops.

The S-operator $S_{(\theta)}$ depends only on θ^{0i},

$$S_{(\theta)} = S_{\theta^{0i}}$$

Charge conjugation C and time reversal T on $S_{(\theta)}$ do not affect θ^{0i}.

Parity P changes its sign. Nonzero θ^{0i} contributes to P and thus CPT violation.

The appearance of the term $\vec{\theta}^0 \cdot \overrightarrow{P}_{\rm in}$ suggests that to the leading order in $\theta^{\mu\nu}$:

$$\text{amount of CPT violation} \approx O(\vec{\theta}^0 \cdot \overrightarrow{P}_{\rm in}).$$

From the CERN experiments, the standard CPT figure of merit

$$r_g^\mu \equiv \frac{|g_\mu^+ - g_\mu^-|}{g_\mu^{avg}} \lesssim 10^{-8}$$

To the leading order in $\theta^{\mu\nu}$,

$$r_g^\mu \approx m_\mu^{avg}\, \vec{\theta}^0 \cdot \overrightarrow{P}_{\rm in}$$

We get the maximum bound when $\vec{\theta}^0$ and $\overrightarrow{P}_{\rm in}$ are parallel. In that case

$$|\vec{\theta}^0| \lesssim 10^{-8}/(m_\mu \gamma)^2 v$$

where γ (= 29.3) is the relativistic factor.

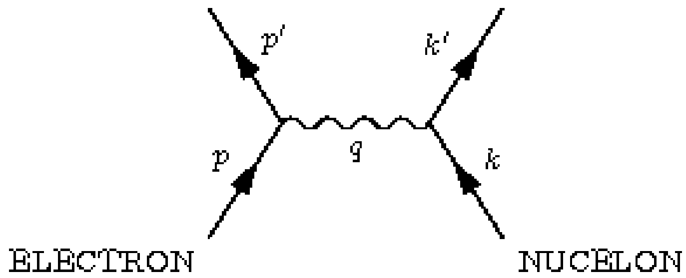
This equation gives an upper bound for the length scale of noncommutativity

$$\sqrt{\theta} \lesssim 10^{-20}\text{m}.$$

This corresponds to a lower bound for the energy scale $E \gtrsim 10^4$ GeV.

NONCOMMUTATIVE ELECTROMAGNETIC FROM FACTORS

Elastic electron - nucleon scattering at the tree level is affected by spacetime noncommutativity.



$$\langle p', k' | S^{(2)} | p, k \rangle = \langle p', k' | \mathbf{T} \Big(\frac{(-i)^2}{2!} \int d^4x d^4y \ j_\mu^{p,n}(x) A^\mu(x) j_\mu^e(y) A^\mu(y) \Big) | p, k \rangle$$

In an abelian gauge theory such as QED,

$$S_{(\theta)} = S_{(\theta=0)} = S$$

The noncommutative version of the above matrix element is

$${}_{(\theta)}\langle p', k'|S^{(2)}_{(\theta)}|p, k\rangle_{(\theta)} = {}_{(\theta)}\langle p', k'|S^{(2)}|p, k\rangle_{(\theta)}$$

The incident- and outgoing state vectors are twisted. In terms of the untwisted (commutative) operators

$$\begin{aligned} |p,\ k\rangle_{(\theta)} &= e^{\frac{i}{2}p\wedge k} c_N^\dagger(p) c_e^\dagger(k)|0\rangle = e^{\frac{i}{2}p\wedge k}|p,\ k\rangle, \\ {}_{(\theta)}\langle p',\ k'| &= e^{-\frac{i}{2}p'\wedge k'}\langle 0|c_N(p')c_e(k') = e^{-\frac{i}{2}p'\wedge k'}\langle p',\ k'|, \end{aligned}$$

where $p \wedge k = p_\mu \theta^{\mu\nu} k_\nu$.

The S-matrix element becomes

$$
\begin{aligned}
i\mathcal{M}_{(\theta)} &= e^{-\frac{i}{2}p'\wedge k'} e^{\frac{i}{2}p\wedge k}\; j_\mu^{p,n}(p',p)\left(\frac{ig^{\mu\nu}}{q^2}\right) j_\nu^{e}(k',k) \\
&= \frac{-ig_{\mu\nu}e^{\frac{i}{2}(p+k)\wedge q}}{q^2}\Big[ie\bar{u}(k')\gamma^\nu u(k)\Big]\Big[-ie\bar{N}(p')\Gamma^\mu(p',p)N(p)\Big]
\end{aligned}
$$

The vertex function Γ^μ contains all the details of the internal structure of the nucleon.

We infer that the additional $\theta^{\mu\nu}$ dependent factor represents the noncommutative modification of the internal structure of the nucleon.

The nucleon charge-current density is effectively modified in the noncommutative case.

It takes the form

$$j_\mu^{p,n(\theta)}(k,p,q) = ie\ e^{\frac{i}{2}(p+k)\wedge q}\bar{N}(p')\left[\gamma_\mu F_1^{p,n}(q^2) + \frac{\kappa^{p,n}}{2m^{p,n}}\sigma_{\mu\nu}q_\nu F_2^{p,n}(q^2)\right]N(p)$$

This shows that the electromagnetic form factors are modified.

$$F_{1,2}^{p,n(\theta)}(k,p,q) = e^{\frac{i}{2}(p+k)\wedge q}F_{1,2}^{p,n}(q^2)$$

In the noncommutative case, the spatial distributions of charge and magnetic moment of the nucleon (Sachs form factors), $G^{(\theta)}_{E_{p,n}}$ and $G^{(\theta)}_{M_{p,n}}$ respectively are,

$$\begin{aligned} G^{(\theta)}_{E_{p,n}}(k,p,q) &= e^{\frac{i}{2}(p+k)\wedge q}\left(F_1^{p,n}(q^2) - \frac{q^2}{4m^2_{p,n}}F_2^{p,n}(q^2)\right) \\ G^{(\theta)}_{M_{p,n}}(k,p,q) &= e^{\frac{i}{2}(p+k)\wedge q}\left(F_1^{p,n}(q^2) + F_2^{p,n}(q^2)\right) \end{aligned}$$

They are now functions of four-momenta and direction-dependent, unlike the commutative case.

Possible experimental signals due to these effects should be explored further.

Noncommutative CMB

CMB radiation can carry the signature of physics of small scale - noncommutative spacetime.

The scalar field driving the inflation is noncommutative

$$\phi_\theta = \phi_0 \, e^{\frac{1}{2}\overleftarrow{\partial} \wedge P}.$$

This gives a noncommutative power spectrum for the primordial scalar field

$$P_{\Phi_\theta}(\mathbf{k}) = P_{\Phi_0}(k) \, \cosh(H\vec{\theta}^0 \cdot \mathbf{k}).$$

This in turn makes the angular correlation for two point temperature fluctuations $C_l = \frac{1}{2l+1}\sum_m \langle a_{lm} a^*_{lm}\rangle$ to become noncommutative.

$$\langle a_{lm} a^*_{l'm'}\rangle_\theta = \frac{2}{\pi}\int dk \sum_{l''=0,\ l'':\text{even}}^{\infty} (i)^{l-l'}(-1)^m(2l''+1)\, k^2 \Delta_l(k)\Delta_{l'}(k) P_{\Phi_0}(k) i_{l''}(\theta k H)$$
$$\times\sqrt{(2l+1)(2l'+1)}\begin{pmatrix} l & l' & l'' \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} l & l' & l'' \\ -m & m' & 0 \end{pmatrix}.$$

On fitting the CMB data we get

$$\sqrt{\theta} \lesssim 1.36\times 10^{-19}\ \text{m}.$$

This corresponds to a lower bound for energy scale: $E \gtrsim 10^3$ GeV.

Conclusions

- Spacetime noncommutativity parameter is constrained using available experimental data.

(i) $K^0 - \bar{K}^0$ system $\rightarrow E \gtrsim 10^{16}$ GeV

(ii) Muon $g-2$ difference $\rightarrow E \gtrsim 10^{4}$ GeV

(iii) CMB data $\rightarrow E \gtrsim 10^{3}$ GeV

- Electromagnetic form factors and the distributions of charge and magnetization of the nucleon are modified.