Home-work 6

on lecture dated 10/10/09

1. Let $\{v_1, \cdots, v_n\}$ be a basis for a vector space V and let $w_1, \cdots, w_n$ be any n vectors in any vector space W.

   (a) Show that there exists a unique $T \in L(V, W)$ such that $Tv_i = w_i \; \forall i$.

   (b) Show that the T of (a) above is 1-1 if and only if $\{w_1, \cdots, w_n\}$ is linearly independent.

   (c) Show that the T of (a) above is onto if and only if $sp(\{w_1, \cdots, w_n\}) = W$

2. If $T \in L(V, W)$, verify that $ker\ T$ is a subspace of V and that $ran\ T$ is a subspace of W.

3. For $T \in L(V, W)$, prove that T is 1-1 $\Leftrightarrow ker\ T = \{0\}$

4. For $T \in L(V, W)$, prove that $T0 = 0$ - where of course the 0 on the left (resp., right) is the null vector in V (resp., W).

5. Verify that $L(V, W)$ is a vector space with respect to the vector operations defined by

$$(S + T)v = Sv + Tv \ , \ (\alpha T)v = \alpha Tv, \ \forall v \in V$$

6. For $S, T, U \in L(V)$, with multiplication defined by $(ST)v = S(Tv)$, verify that

$$(ST)U = S(TU), \alpha(ST) = (\alpha S)T, (S+T)U = SU+TU, U(S+T) = US+UT.$$