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Asynchronous updating of threshold-coupled chaotic neurons

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Abstract. We study a network of chaotic model neurons incorporating threshold-activated coupling. We obtain a wide range of spatiotemporal patterns under varying degrees of asynchronicity in the evolution of the neuronal components. For instance, we find that sequential updating of threshold-coupled chaotic neurons can yield dynamical switching of the individual neurons between two states. So varying the asynchronicity in the updating scheme can serve as a control mechanism to extract different responses, and this can have possible applications in computation and information processing.

Keywords. Asynchronous updating; chaos control.

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1. Introduction

Studies of dynamical systems most commonly use parallel (or synchronous) updating schemes, where the individual local maps are iterated forward simultaneously. Here we focus on an alternate updating scheme, namely we consider *asynchronous evolution*, where the *updates are not concurrent* [1–4].

In certain situations asynchronous updating can be closer to physical reality than synchronous evolution [1]. For instance, there exist specific physical situations where an extended system is comprised of a collection of elemental dynamical units which evolve asynchronously, as there is no global clock, like in neuromorphology where the neurons, neuronal groups and functional layers of the brain, are believed to be usually asynchronous [5]. Interestingly, it has been shown that the asynchronous updating often leads to more ordered behaviour than simultaneous updating [3,4].

In this paper we study a network of chaotic model neurons, incorporating *threshold-activated coupling*, and investigate the range of pattern formation obtained under varying degrees of synchronicity in its dynamical updating scheme. The details of the model and results are described in the sections below.

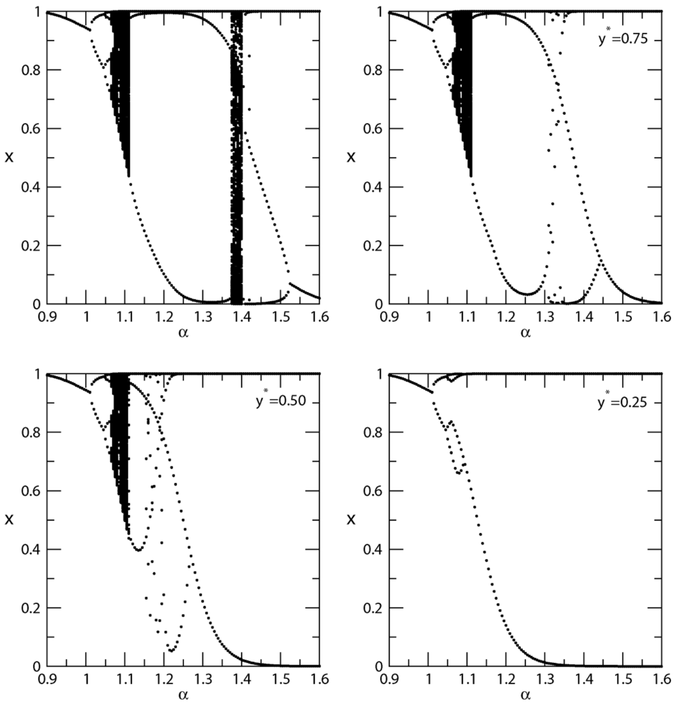

Figure 1. Bifurcation diagrams of the single chaotic neuron map with no thresholding (top left), and under different values of threshold $y^* = 0.75$ (top right), 0.5 (bottom left) and 0.25 (bottom right).

2. Adaptive dynamics of the chaotic neuron

We use the chaotic model neuron, proposed by Aihara *et al* [6], as the building block of our network. The dynamics of the neuron is given in discrete time n by the map:

$$\begin{aligned} y_{n+1} &= ky_n - \alpha f(y_n) + a, \\ x_{n+1} &= f(y_{n+1}), \end{aligned} \tag{1}$$

where $f(x) = 1/[1 + \exp(-x/\varepsilon)]$. The internal state of the neuron is y_n and the output of the neuron is x_n at time n. The decay parameter of the refractoriness is k, and α is the refractory scaling parameter. The sigmoid function, $f(\cdot)$, is the output function of the neuron, and ϵ is the steepness parameter of the sigmoid function.

On this nonlinear function a threshold activated response is incorporated [7,8], namely, if the internal state of the neuron takes value more than the threshold y^*, it is relaxed to y^*.

$$y_n \to y^* \quad \text{if } y_n > y^*. \tag{2}$$

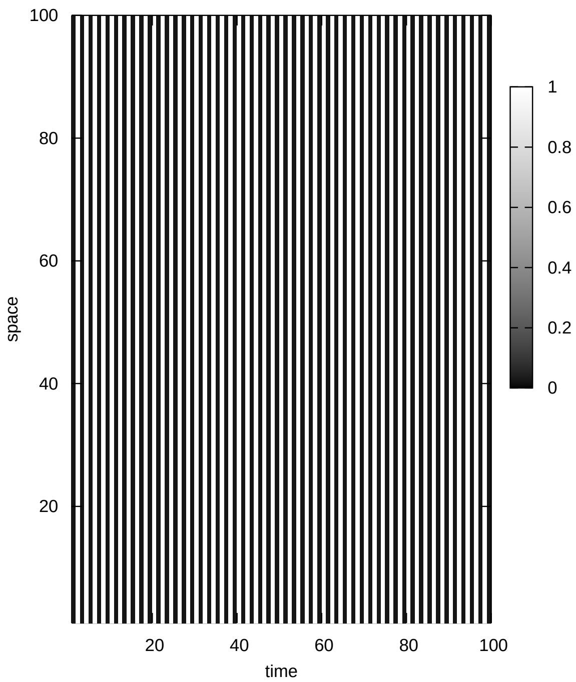

Figure 2. Space-time density plots of an array of threshold-coupled neurons (with $\alpha = 1.4$) for synchronous updating, i.e. with $n_{\rm sync} = N = 100$ (i.e. $p_{\rm sync} = 1$). Here threshold value $y^* = 0.5$, size $N = 100$ and relaxation time $r = 1000$. All sites are synchronized and take values 0 and 1 alternatively.

The bifurcation diagrams of the uncontrolled neuron map and the threshold-controlled neuron map for threshold values $y^* = 0.75$, 0.5 and 0.25 are shown in figure 1. There are two chaotic bands in the uncontrolled neuron map. Under large threshold values, the chaotic dynamics around $\alpha = 1.4$ is transformed to a fixed point, while for lower values of threshold both chaotic regions disappear and one obtains complete regularity.

3. Threshold coupled neuron maps

Now we study a network of such model neurons. Using label i as the site/node index in the lattice/network, with $i \in [1, N]$, where N is the size of the system, the local dynamics is given specifically as follows:

$$\begin{aligned} y_{n+1}(i) &= ky_n(i) - \alpha f(y_n(i)) + a, \\ x_{n+1}(i) &= f(y_{n+1}(i)), \end{aligned} \tag{3}$$

where $f(x) = 1/[1 + \exp(-x/\varepsilon)]$.

On this network of local neuronal dynamics a threshold activated coupling is incorporated [7,8]. The coupling is triggered when the internal state of neuron

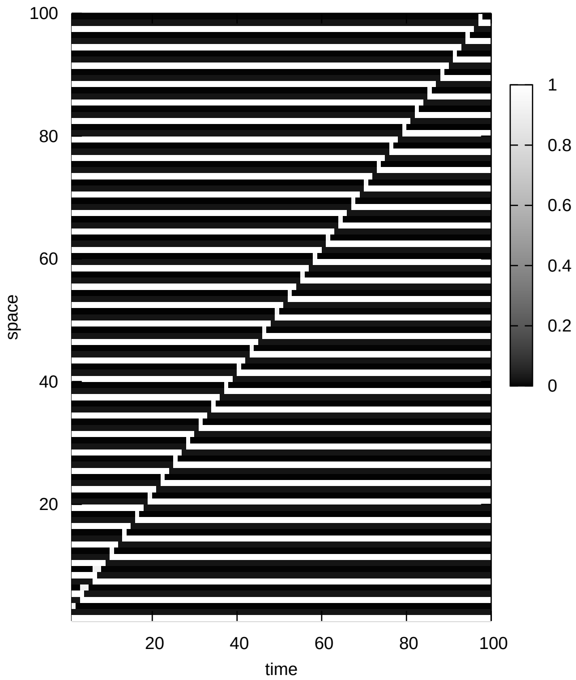

Figure 3. Space-time density plots of an array of threshold-coupled neurons for sequential updating with $n_{\rm sync} = 1$ (i.e. $p_{\rm sync} = 0.01$). Here threshold value $y^* = 0.5$, size $N = 100$ and relaxation time $r = 1000$. Alternate sites shifts from 0 to 1 (or in reverse order) at different times.

at certain site in the lattice exceeds the critical value y^*, i.e. $y_n(i) > y^*$. The supercritical site then relaxes (or 'topples') by transporting the excess $\delta = (y_n(i) - y^*)$ to its neighbour:

$$y_n(i) \to y^*,$$
$$y_n(i+1) \to y_n(i+1) + \delta. \tag{4}$$

The above process occurs in parallel, i.e. all supercritical sites at any instant relax simultaneously, according to eqs (4), and this constitutes one relaxation time step. After r such relaxation steps, the system undergoes the next chaotic update. In some sense then, time n associated with the chaotic dynamics is measured in units of r. The relaxation of a site may initiate an avalanche of relaxation activity, as neighbouring sites can exceed the threshold limit after receiving the excess from a supercritical site, thus starting a domino effect. This induces a uni-directional transport to the open boundary of the array. The 'excess' is transported out of the system. After r relaxation steps the next dynamical update of the sites occurs.

The dynamical outcome of the system crucially depends on the relaxation time r, i.e. on the time-scales for autonomously updating each site and propagating the threshold-activated coupling between sites. When $r \to \infty$, the system is fully relaxed before the subsequent dynamical update and so the time-scales of the two

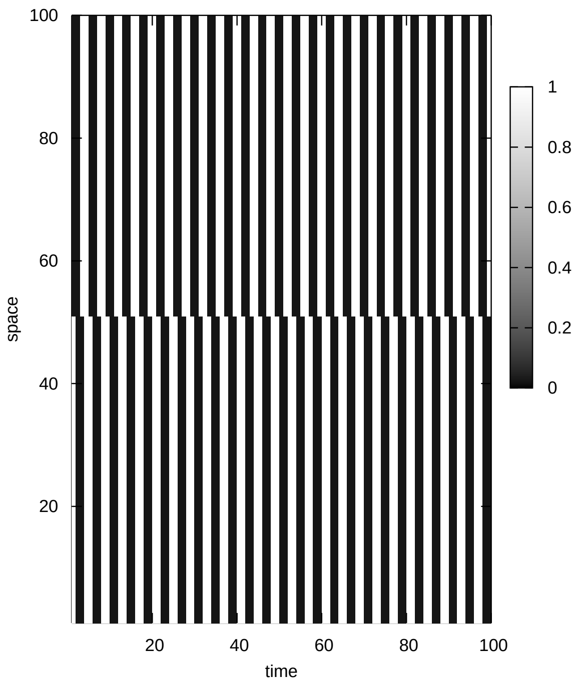

Figure 4. Space-time density plots of an array of threshold-coupled neurons for sequential updating with $n_{\rm sync} = N/2 = 50$ (i.e. $p_{\rm sync} = 0.5$). Here threshold value $y^* = 0.5$, size $N = 100$ and relaxation time $r = 1000$.

processes, the intrinsic chaotic dynamics of each site and the threshold-activated relaxation, are separable. At the other end of the spectrum is the limit of very small r where the local dynamics and the coupling take place simultaneously. The system is driven to spatiotemporal chaos for the short relaxation time [9].

In our previous study on the effect of asynchronous updating on a system with short relaxation time (i.e. very small r), it has been shown that the asynchronous updating induces more order in the system [4]. In the present study, we focus on the asynchronous updating of threshold coupled systems at the other limit, namely, $r \to \infty$. We show in the following section, that asynchronous evolution of threshold-coupled chaotic neurons, yields a rich variety of spatiotemporal patterns and binary sequences.

4. Spatiotemporal patterns obtained under varying asynchronicity in the updating

In order to effectively study the influence of varying degrees of synchronicity we investigate the following dynamics: We break the network into subsets, and update the sites belonging to each subset simultaneously, while updating the different subsets sequentially. We denote the number of sites (nodes) updated together as $n_{\rm sync}$.

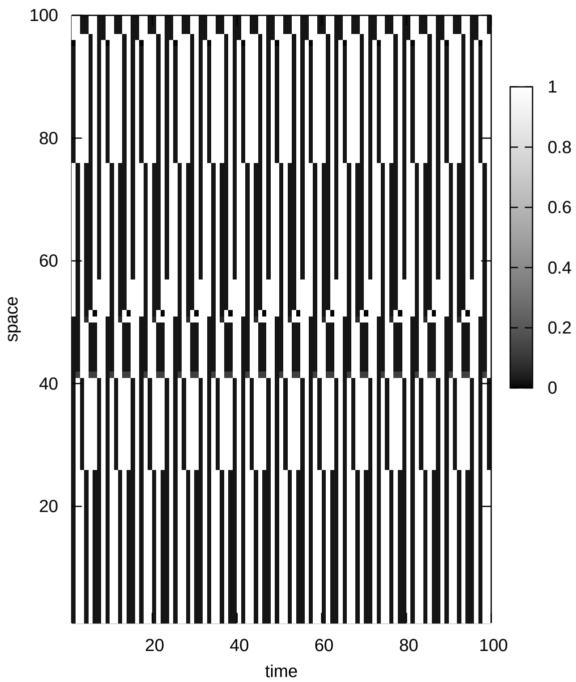

Figure 5. Space-time density plots of an array of threshold-coupled neurons for sequential updating with $n_{\rm sync} = \frac{3}{4}N = 75$ (i.e. $p_{\rm sync} = 0.75$). Here threshold value $y^* = 0.5$, size $N = 100$ and relaxation time $r = 1000$.

So the fraction $p_{\rm sync} = n_{\rm sync}/N$ serves as an effective parameter for synchronicity. The limiting case of $p_{\rm sync} = n_{\rm sync}/N = 1$, namely $n_{\rm sync} = N$, corresponds to the usual parallel updates. On the other hand, $n_{\rm sync} = 1/N$, which tends to 0 as $N \to \infty$ corresponds to the completely asynchronous update. So, as $n_{\rm sync}/N$ takes values from 0 to 1 (i.e. $n_{\rm sync}$ takes values 1 to N), one has decreasing degrees of synchronicity in the evolution. Here we study *sequential* asynchronous updating, where the subsets consist of contiguous elements. That is, the first subset consists of elements $i = 1, \ldots, n_{\rm sync}$, the second subset consists of elements $i = n_{\rm sync} + 1, \ldots, 2n_{\rm sync.}$, etc.

In our system, the local parameter values are in the chaotic regime: $k = 0.5$, $\alpha = 1.4$, $a = 1.0$ and $\varepsilon = 0.04$. Note again that the relaxation time r is large here, i.e., we update the chaotic neurons asynchronously and allow the whole network to relax by threshold-activated transport, for sufficiently long time.

First we study the network of threshold-coupled chaotic neurons under synchronous updating, namely the $p_{\rm sync} = 1$ limit. For $y^* = 0.5$, this yields a state where all neurons are *synchronized* and evolve in period 2 cycles, with the sites taking values 0 and 1 alternatively. In figure 2 the space-time density plot of $x_n(i)$ for this case is shown.

As one varies $p_{\rm sync}$, we find that sequential updating yields *different regular spatiotemporal patterns depending upon the degree of synchronicity.* For instance, in the limiting case of sequential updating with $n_{\rm sync} = 1$, where one site at a time is updated down the chain ('typewriter mode'), the state of the sites shift from value 1 to 0 (or the reverse). This binary switching is clearly evident in the space-time density plot of figure 3.

When we update more than one site at a time (i.e. $n_{\rm sync} > 1$) the pattern of the output $x_n(i)$ changes with the degree of synchronicity $p_{\rm sync} = n_{\rm sync}/N$. Representative examples of this behaviour, for $p_{\rm sync} = 0.5$ and 0.75, are shown in figures 4 and 5.

These results are independent of the threshold value for $y^* < 0.8$. Further, note that for $y^* \sim 0.9$, the network of threshold-coupled neurons is chaotic under synchronous updating, while sequential updating induces order in the system.

5. Conclusion

In this work we have shown how coupled chaotic neuron maps under different updating schemes give wide-ranging spatiotemporal patterns. So varying the updating rules provides us with a library of binary patterns, with the degree of synchronicity $p_{\rm sync}$ having a one-to-one correspondence with a particular pattern. This variety of controlled switching and shifting binary sequences might prove useful for information processing applications [10]. Further changing updating rules may also serve as a control strategy for extracting different controlled spatiotemporal patterns from extended complex systems [11].

Note that one can get different spatiotemporal patterns of 0's and 1's in a network of threshold-coupled chaotic systems by varying the thresholding level at different network sites. However, we have shown here that one can also get the desired patterns under fixed threshold levels, by just varying the degree of synchronicity in the updating. In certain electronic devices it is much easier to change the degree of synchronicity of the sequential updating schemes, rather than manipulate the individual levels [12]. Our results are thus very relevant in such situations.

Lastly, since various dynamical binary responses are generated by threshold-controlled chaotic neural networks under sequential updating, it may be possible to use such networks to obtain different logic operations as a function of time, without changing the system parameters. Thus this system has potential to be used as dynamical switching gates, which have direct bearing on the construction of reconfigurable computer architectures [13].

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