Control and Synchronization of Chaotic Neurons Under Threshold Activated Coupling

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Abstract. We have studied the spatiotemporal behaviour of threshold coupled chaotic neurons. We observe that the chaos is controlled by threshold activated coupling, and the system yields synchronized temporally periodic states under the threshold response. Varying the frequency of thresholding provides different higher order periodic behaviors, and can serve as a simple mechanism for stabilising a large range of regular temporal patterns in chaotic systems. Further, we have obtained a transition from spatiotemporal chaos to fixed spatiotemporal profiles, by lengthening the relaxation time scale.

Keywords: Chaotic dynamics, Control and Synchronization.

1 Introduction

In the last two decades, many control techniques have been proposed including the OGY [1] for controlling chaos in chaotic dynamical systems. In this paper, we focus on the network of chaotic neuron model that has been proposed in studies of the squid giant axon and the Hodgkin–Huxley equation [2]. We have applied the threshold activated coupling to chaotic neurons to control chaos [3,4,5]. This mechanism works in marked contrast to the OGY method. In the OGY method the chaotic trajectories in the vicinity of unstable fixed points are controlled onto these points. In threshold control, on the other hand, the system does not have to be close to any particular fixed point before implementing the control. Here the trajectory merely has to exceed the prescribed threshold. So the control transience is typically very short. The chaotic neurons are controlled with the threshold activated coupling and spatially synchronized temporal patterns with different periods are obtained. We have investigated the effect of the variation of the relaxation time on the spatiotemporal characteristics of the threshold coupled chaotic neurons. We have also obtained a wide range of stable cyclic behavior of threshold coupled chaotic neurons by simply varying the frequency of control.

The threshold-activated coupling of chaotic systems are relevant for certain mechanical systems like chains of nonlinear springs, as also for some biological

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systems, such as synaptic transmissions among neurons [6]. It is also relevant to population migrations as it is reasonable to model the population of an area (state at a site) as a nonlinear map and when this population exceeds a certain critical amount the "excess" population moves to a neighboring area (site). The threshold mechanism is also reminiscent of the Bak-Tang-Wiesenfeld cellular automata algorithm [7], or the "sandpile" model, which gives rise to self organized criticality (SOC). The model system studied here is however significantly different, the most important difference being that the threshold mechanism now occurs on a *nonlinearly evolving substrate*, i.e. there is an *intrinsic deterministic dynamics* at each site. So the local chaos here is like an *internal* driving or perturbation, as opposed to *external* perturbation/driving in the sandpile model, which is effected by dropping "sand" from outside.

The paper is organized as follows. In Sec.II, a model of chaotic neuron is described. The threshold activated coupling is introduced as a control mechanism for chaotic neurons and the effect of relaxation time scale on the spatiotemporal properties of threshold coupled chaotic neurons is studied. In Sec.III, we have studied the threshold activated coupling at the varying time interval to obtain different temporal patterns of spatially synchronized neurons. The conclusion are presented in the last section.

2 Chaotic Neuron Model

We study the following network of chaotic neuron model proposed by Aihara [2].

$$\begin{aligned} y_{n+1}(i) &= k y_n(i) - \alpha f(y_n(i)) + a, \\ x_{n+1}(i) &= f(y_{n+1}(i)), \end{aligned} \tag{1}$$

where, the sigmod function, $f(x) = 1/[1 + exp(-x/\varepsilon)]$, is the output function of the neuron; and ε is the steepness parameter of the sigmoid function. The internal state of the ith neuron is $y_t(i)$ at time t, $x_t(i)$ is the output of the neuron at time t, k is the decay parameter of the refractoriness, and α is the refractory scaling parameter.

In Fig. 1(a), we have shown the output x of each chaotic neuron for the network size $N = 100$ as a function of parameter α. Other parameters of the map are $k = 0.5$, $a = 1.0$, and $\varepsilon = 0.04$. There are regions in parameter space around $\alpha \sim 1.1$ and ~ 1.4, where the system is chaotic.

2.1 Threshold Mechanism

Now, on this nonlinear network a threshold activated coupling is incorporated [3,4,5]. The coupling is triggered when a site in the network exceeds the critical value y^* i.e. when a certain site $y_n(i) > y^*$. The super critical site then relaxes (or "topples") by transporting the excess $\delta = (y_n(i) - y^*)$ equally to its two neighbors:

$$y_n(i) \to y^*$$

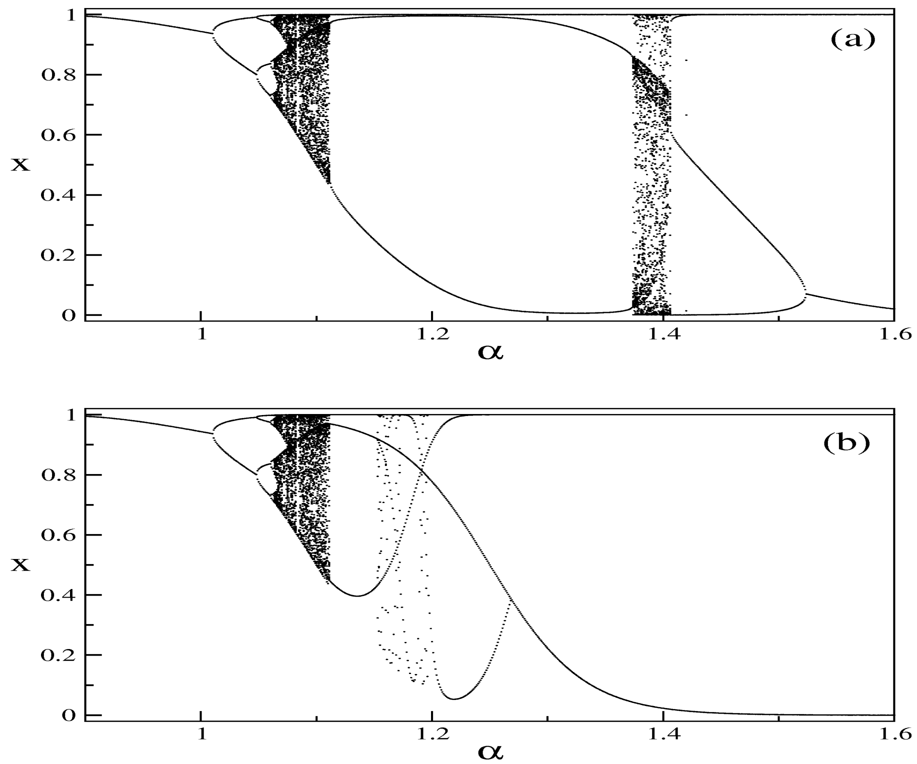

Fig. 1. The output of each neuron x as a function of bifurcation parameter α for $N = 100$ uncoupled neurons (a) and threshold $y^* = 0.5$ coupled neurons (b)

$$y_n(i-1) \rightarrow y_n(i-1) + \delta/2$$
$$y_n(i+1) \rightarrow y_n(i+1) + \delta/2 \quad (2)$$

The process above occurs in parallel, i.e. all supercritical sites at any instant relax simultaneously, according to Eqs. 2, and this constitutes one relaxation time step. After r such relaxation steps, the system undergoes the next chaotic update. In some sense then, time n associated with the chaotic dynamics is measured in units of r. The relaxation of a site may initiate an avalanche of relaxation activity, as neighboring sites can exceed the threshold limit after receiving the excess from a supercritical site, thus starting a domino effect. This induces a bi-directional transport to the two boundaries of the array. These boundaries are open so that the "excess" may be transported out of the system.

The spatiotemporal behavior of the threshold coupled chaotic systems under different threshold levels has been investigated both numerically and analytically, specifically, for the case of networks of chaotic logistic maps [3,4,5]. There exist many *phases* in threshold space ($0 < x^* < 1$), i.e. for $x^* \leq 3/4$ the dynamics goes to a fixed point. When $0.75 < x^* < 1.0$, the dynamics is attracted to *cycles*

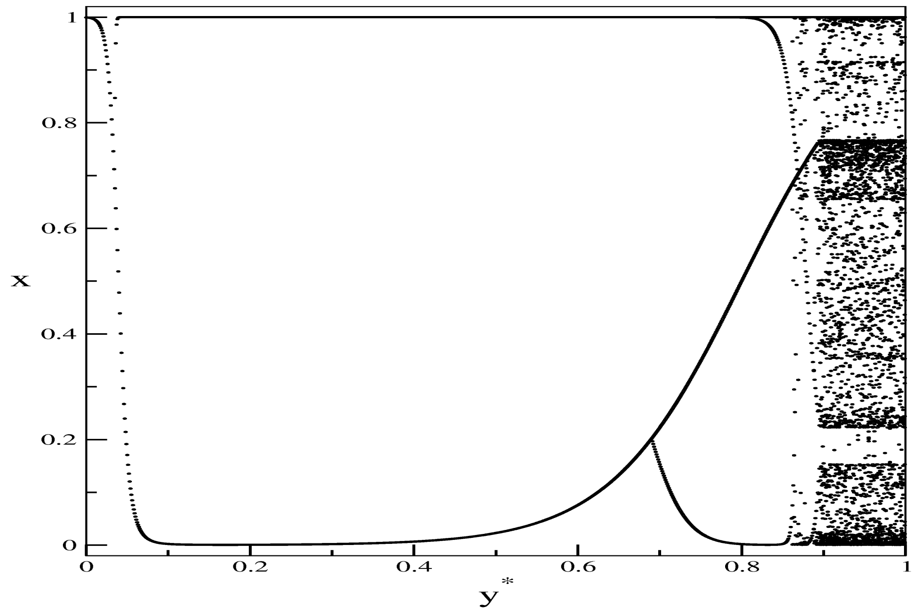

Fig. 2. The output of threshold coupled neurons x as a function of threshold y^* for the fixed value of $\alpha = 1.4$

whose periodicities depend on the value of x^*. By tuning the threshold level x^* one thus obtain spatiotemporal cycles of different orders.

In Fig. 1(b), we have shown the output x of each chaotic neuron as a function of parameter α with a threshold $y^* = 0.5$. With the threshold activated coupling, the chaos in the chaotic neurons is controlled around $\alpha \sim 1.4$. In Fig. 2, we have shown the output x of each chaotic neuron as a function of threshold y^* for the fixed value of $\alpha = 1.4$. There are two period–doubling bifurcations near $y^* \sim 0$ and ~ 0.7.

For $\alpha = 1.4$, we get controlled period-2 output dynamics of the network with the threshold values $y^* < 0.7$. The network of chaotic neurons is synchronized with period two and all sites are taking values close to 0 and 1 alternatively for $y^* < 0.7$ and $\alpha = 1.4$, where the system is chaotic in the absence of threshold activated coupling. In Fig. 3, the space-time density plot of $x_n(i)$ is shown for fixed values of $\alpha = 1.4$, $y^* = 0.5$ and $N = 100$ after the transient dynamics.

2.2 Relaxation Timescale

Note however, that the dynamical outcome crucially depends the relaxation time r, i.e. on the timescales for autonomously updating each site and propagating the threshold-activated coupling between sites. For sufficiently large value of relaxation time, i.e. in the limit $r \to \infty$, the system is fully relaxed (subcritical) before the subsequent dynamical update. So the time scales of the two

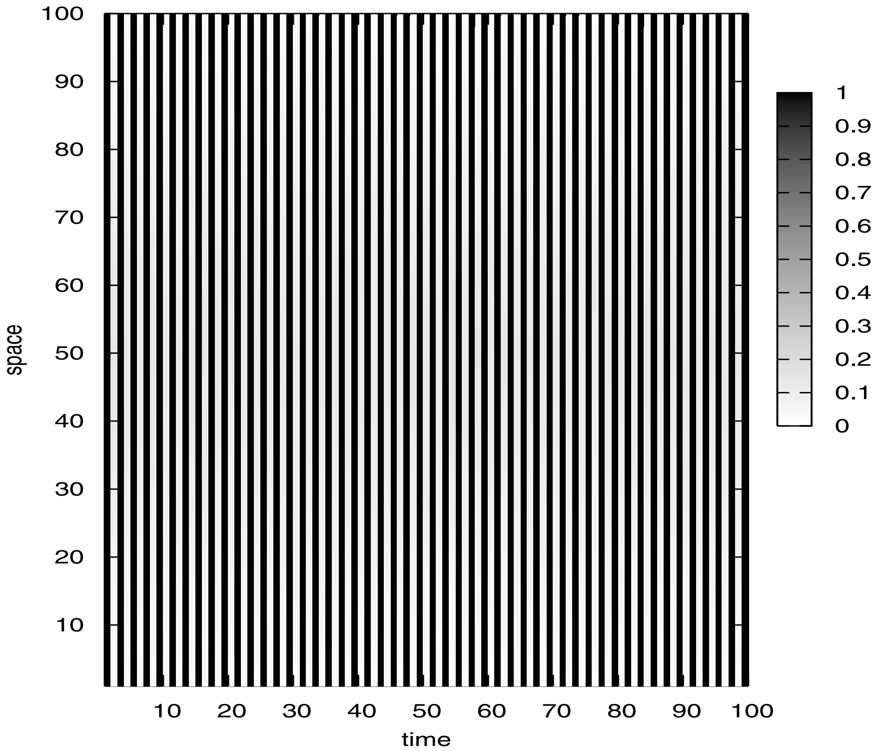

Fig. 3. Space-time density plots of an array of threshold-coupled chaotic neurons, with threshold value $y^* = 0.5$, size $N = 100$, and relaxation time $r = 1000$. The x-axis denotes the time and the y-axis denotes the site index.

processes, the intrinsic chaotic dynamics of each site and the threshold-activated relaxation, are separable. Here the relaxation mechanism is much faster than the chaotic evolution, enabling the system to relax completely before the next chaotic iteration. This scenario is similar to the SOC model, where the driving force (perturbation) is very dilute, e.g. in the sandpile model the avalanche of activity, initiated by an external "sand grain" dropped on the pile, ceases before the next "sand grain" perturbs the pile.

At the other end of the spectrum is the limit of very small r where the local dynamics and the coupling take place simultaneously. It is evident that lowering r essentially allows us to move from a picture of separated relaxation processes to one where the relaxation processes can overlap, and disturbances do not die out before the subsequent chaotic update. It was observed in [8] that for short relaxation times the system is driven to spatiotemporal chaos. This is due to the fast driving rate of the system which does not provide enough time to spatially distribute the perturbations and allow the excess to propagate to the open boundaries. However large r gives the system enough time to relax, and allows the excess to be transported out of the system through the open ends. So for

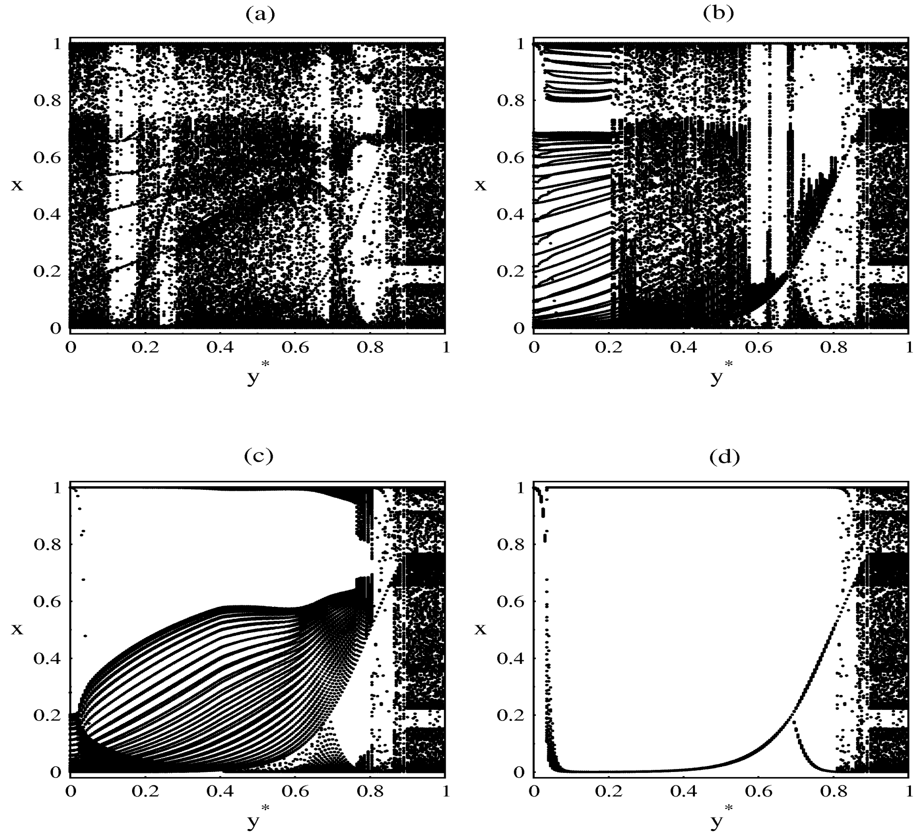

Fig. 4. Bifurcation diagram of the output state of each neuron with respect to threshold y^* for an array of threshold–coupled chaotic neurons. Size N=50 and relaxation times are (a) $r = 1$, (b) $r = 50$, (c) $r = 100$, and (d) $r = 1000$.

large r the system displays very regular behavior for a large range of threshold values.

In Fig. 4, the bifurcation diagram of the output state of each neuron is shown as a function of threshold y^* for different values of relaxation time r. Fig. 5 shows a example of dynamical transition of threshold coupled chaotic neurons from a fixed temporal behavior to spatiotemporal chaos as r becomes smaller. There is a transition from the spatiotemporal fixed point to spatiotemporal chaos as we decrease the relaxation time. These transitions result from the competition between the rate of intrinsic driving arising from the local chaotic dynamics and the time required to propagate the threshold–activated coupling.

3 Thresholding at Varying Interval

Now, we implement the threshold mechanism at varying intervals n_c, with $1 < n_c \leq 20$, i.e. the thresholding frequency ranges from once every two iterates of

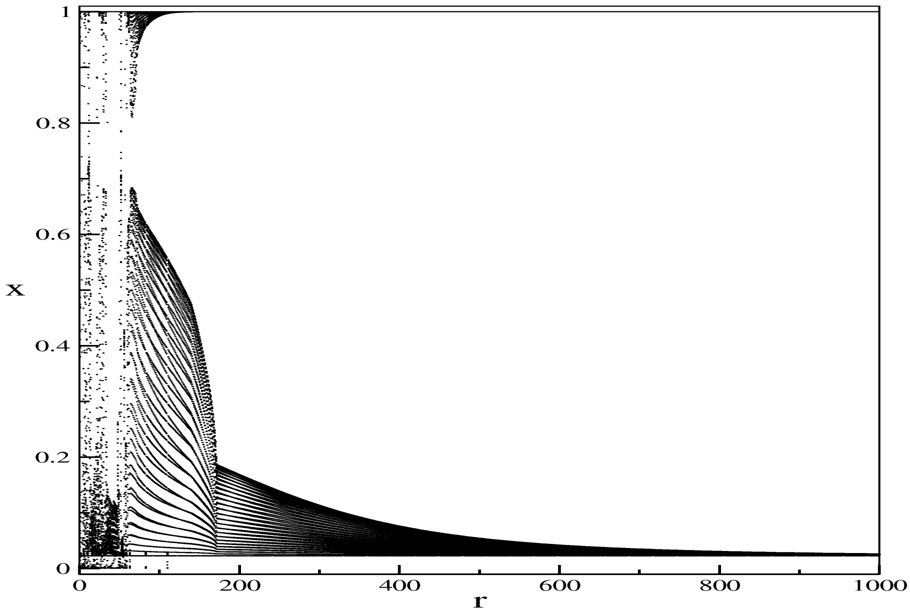

Fig. 5. Bifurcation diagram of the output state x of each neuron with respect to the relaxation time r for an array of threshold–coupled chaotic neurons with threshold $y^* = 0.5$ and system size $N = 50$

the chaotic neuron maps to once every 20 iterates. We find that for all n_c in this range the chaotic neuron maps gets controlled onto an exact and stable orbit of periodicity $p \geq n_c$, where p is an integer multiple of n_c [9].

In Fig. 6 the temporal behavior of threshold coupled chaotic neurons is shown for thresholding at different intervals n_c. It is clear evident that thresholding at different frequencies yields different temporal periods. For instance one obtain temporal periods $15, 5, 7$, and 18 with $n_c = 3, 5, 7$ and 9 respectively. In fact it can serve as a control parameter for selecting a different cyclic behaviors from the chaotic neuron dynamics. This has particular utility in obtaining higher order periods, which are difficult to obtain otherwise.

4 Conclusion

We have studied the spatiotemporal behaviour of threshold coupled chaotic neurons. We observe that the chaos is controlled by threshold activated coupling, and the system yields synchronized temporally periodic states under the threshold response. Varying the frequency of thresholding provides different higher order periodic behaviors, and can serve as a simple mechanism for stabilising a large range of regular temporal patterns in chaotic systems. Further, we have obtained a transition from spatiotemporal chaos to fixed spatiotemporal profiles, by lengthening the relaxation time scale.

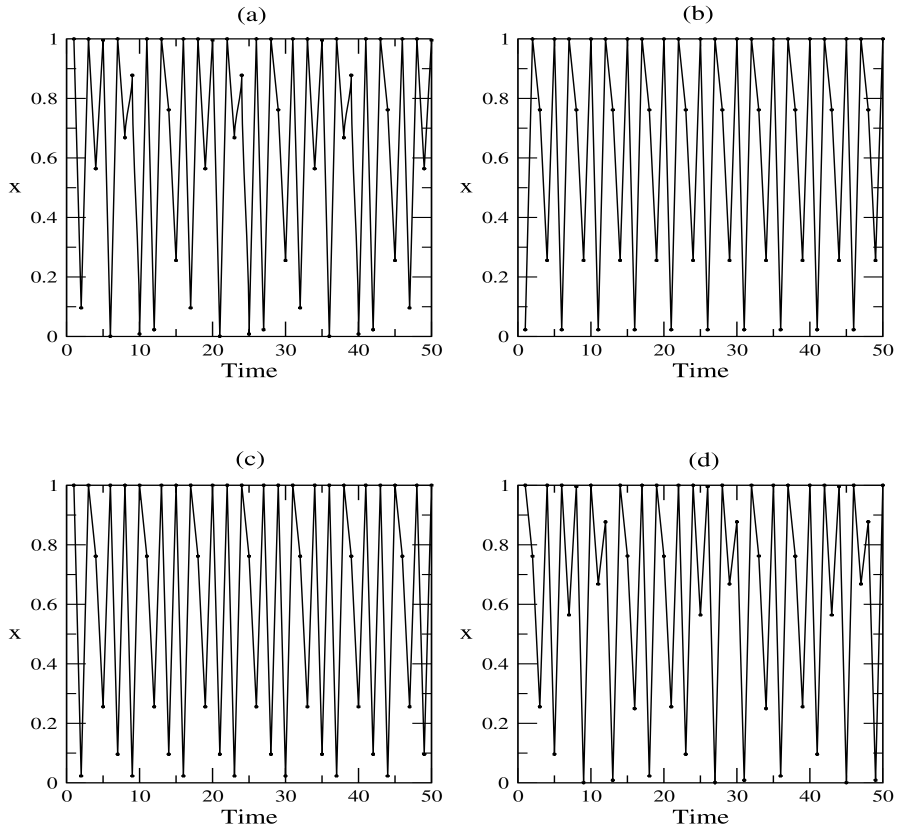

Fig. 6. The temporal behavior of an array of spatially synchronized threshold-coupled logistic maps, with threshold value $y^* = 0.5$, size $N = 100$, and relaxation time $r = 10000$ for (a) $n_c = 3$, (b) $n_c = 5$, (c) $n_c = 7$, and (d) $n_c = 9$. The spatially synchronized neurons have temporal periods $15, 5, 7$, and 18 for $n_c = 3, 5, 7$ and 9 respectively.

Threshold-coupled chaotic systems have the capacity to directly and flexibly implement fundamental logic and arithmetic operations [10]. Such extended dynamical systems offer much scope of parallelism, allowing rapid solutions of certain problems utilizing the collective responses and collective properties of the system [11,12]. So the varied temporal and spatial responses of the array of model neurons studied here, can also be potentially harnessed to accomplish different computational tasks, and the system may be used for information processing [13].

Note that in terms of practical implementation, the threshold mechanism has been implemented in electrical circuits [14]. Parallel distributed processing with spatio–temporal chaos, on the basis of a model of chaotic neural networks, has also been proposed [13], and a mixed analog/digital chaotic neuro–computer prototype system has been constructed for quadratic assignment problems (QAPs)

[15]. So our model system of threshold-coupled neurons, which combines threshold mechanisms and neuronal units, should be readily realizable with electrical circuits.

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