# Controlling chaos in biology

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**We describe various techniques to control the dynamics of strongly nonlinear systems. These procedures are remarkably successful in stabilizing regular dynamic behaviours, as well as in directing chaotic trajectories rapidly to a desired state. Further, we highlight some interesting and potentially important applications to biological systems.**

A variety of physical and biological systems are well modelled by coupled nonlinear equations[1]. In most cases such systems are capable of displaying several types of dynamical behaviour: fixed points, limit cycles, bi-stability, birhythmicity or chaos, for instance. Typically the nature of the motion depends on the value of one or more parameters. In real systems these may be quantities such as electric fields, temperature, pressure gradient, pH, molarity or kinetic rates. Generically the nature of the dynamics is governed by these parameters, and one can obtain a wide repertoire of dynamical patterns by tuning them[1].

Now these parameters can change (as a result of fluctuations in the environment, for instance) and this can push the system to drastically different kinds of dynamic behaviour. Thus it is of considerable interest to develop mechanisms of self-regulation or control in systems intrinsically capable of very complicated dynamics, so that it is guaranteed to maintain a fixed activity (the 'goal') even when subject to environmental perturbations.

The need for control mechanisms in order that a system is guaranteed to maintain fixed activity even when subject to environmental fluctuations has long been discussed in biology[2]. Situations wherein this is thought to play a role, include pupillary servomechanism[3], biological thermostats and regulation of cell reactions[2]. Although the details of the control mechanism operating in a given situation may be system-specific, from a theoretical standpoint it is important to study the general principles by which systems can be brought back to a desired state by self-regulation. The concepts developed rigorously through the study of model systems can then provide a framework for understanding the more complex mechanisms by which biophysical processes maintain a steady state.

## Adaptive control algorithm

An adaptive control algorithm was recently proposed by Huberman and Lumer[4] and developed and extended by Sinha *et al.*[5,6]. It was demonstrated that the algorithm was a powerful and robust tool for regulating multi-dimensional, multiparameter, strongly nonlinear systems. The procedure *utilizes an error signal proportional to the difference between the goal output and the actual output of the system.* The error signal drives the evolution of the parameters which re-adjust so as to reduce the error to zero.

A general $N$-dimensional dynamical system is described by the evolution equation:

$$\dot{\mathbf{X}} \equiv \frac{d\mathbf{X}}{dt} = F(\mathbf{X}; \mu; t), \quad (1)$$

where $\mathbf{X} \equiv (X_1, X_2, \ldots X_N)$ are the state variables and $\mu = (\mu_1, \mu_2, \ldots \mu_M)$ are the parameters whose values determine the nature of the dynamics. The prescription for adpative control is through the additional dynamics:

$$\dot{\mu} = \varepsilon(X - X_s), \quad (2)$$

where $X_s$ is the desired steady state value and $\varepsilon$ indicates the 'stiffness of control'. For one-dimensional systems there is no ambiguity in eq. (2), but in higher dimensional cases $X$ can in principle be any one of the dynamical variables characterizing the system.

This technique is very effective in bringing the system back to its original dynamical state after a sudden perturbation in the system parameters changes its dynamical behaviour drastically. For instance, when a parameter is perturbed, driving the system from a fixed point into the chaotic regime (say by changing the parameter instantaneously by an amount $\delta$ – a 'shock'), this control mechanism is capable of pulling the system rapidly back to the initial state. See Figure 1 for an example of the control dynamics in a complex (high dimensional, multiparameter) nonlinear system of biological relevance.

The scheme is called 'adaptive' as in this procedure the parameters (which determine the nature of the dynamics) *self-adjust* or *adapt* themselves to yield the desired dynamics. It is also sometimes called 'dynamic feedback control' in the literature. Since this adaptive principle is remarkably robust and efficient in generic nonlinear systems, it is of considerable utility in a large variety of phenomena, ranging from biological units to control engineering.