Experimental realization of chaos control by thresholding

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We report the experimental verification of thresholding as a versatile tool for efficient and flexible chaos control. The strategy here simply involves monitoring a single state variable and resetting it when it exceeds a threshold. We demonstrate the success of the technique in rapidly controlling different chaotic electrical circuits, including a hyperchaotic circuit, onto stable fixed points and limit cycles of different periods, by thresholding just one variable. The simplicity of this controller entailing no run-time computation, and the ease and rapidity of switching between different targets it offers, suggests a potent tool for chaos based applications.

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I. INTRODUCTION

Control mechanisms that enable a system to maintain a fixed activity (the ''goal'' or ''target'') even when intrinsically chaotic has many applications in situations ranging from biology to engineering [1,2]. It is thus of considerable interest and potential utility, to devise control algorithms capable of achieving the desired type of behavior in strongly nonlinear systems. In recent years, there has been intense research activity devoted to the design of effective control techniques [1,2]. A large body of work derives from the Ott, Grebogi, and Yorke (OGY) idea [1], which seeks to use small perturbations to place chaotic orbits onto unstable periodic orbits. In this paper, we will experimentally demonstrate an alternate control strategy: the simple and easily implementable threshold mechanism. This strategy does not involve adjusting any parameter in the system, but only involves the occasional resetting of one state variable. We will first introduce the general formalism and then focus on experimental implementation on a range of systems, including the challenging task of controlling a hyperchaotic system [3].

Threshold formalism for multidimensional systems. Consider a general N-dimensional dynamical system, described by the evolution equation $\dot{\mathbf{x}} = F(\mathbf{x};t)$ where $\mathbf{x} \equiv (x_1, x_2, \ldots, x_N)$ are the state variables, and variable x_i is chosen to be monitored and threshold controlled. The prescription for threshold control in this system is as follows: control will be triggered whenever the value of the monitored variable exceeds a critical threshold x^* (i.e., when $x_i > x^*$) and the variable x_i will then be reset to x^* [4–6]. The dynamics continues till the next occurrence of x_i exceeding the threshold, when control resets its value to x^* again.

No run-time knowledge of $F(\mathbf{x})$ is involved, and no computation is needed to obtain the necessary control. The method only involves monitoring a single variable and no parameters are perturbed in the original system. The theoretical basis of the method does not involve stabilizing unstable periodic orbits, but rather involves clipping desired time sequences (symbol sequences in maps) and enforcing a periodicity on the sequence through the thresholding action which acts as a resetting of initial conditions. The effect of this scheme is to limit the dynamic range slightly, i.e., ''snip'' off small portions of the available phase space, and this small controlling action is effective in yielding a range of stable behaviors. In fact, chaos is advantageous here, as it *possesses a rich range of temporal patterns which can be clipped to different behaviors*. This immense variety is not available from thresholding regular systems.

It can be shown analytically for one-dimensional maps and numerically for multidimensional systems that the threshold mechanism yields stable orbits of all orders by simply varying the threshold level [4–6]. But so far there had been no direct experimental verification of this control scheme. To the best of our knowledge, this work is the first such attempt. Now to experimentally demonstrate the range and efficacy of the method, we implement it on three different chaotic electrical circuits, including a hyperchaotic one. The results from our experiments are presented in detail in the sections below.

II. CONTROLLING A CIRCUIT REALIZATION OF NONLINEAR THIRD-ORDER ORDINARY DIFFERENTIAL EQUATIONS

The first experimental setup is a realization of nonlinear third-order ordinary differential equations (ODE), a form known in literature as Jerk equations:

$$\frac{d^3x}{dt^3} + A\frac{d^2x}{dt^2} + \frac{dx}{dt} = G(x), \tag{1}$$

where $G(x)$ is a piecewise linear function: $G(x) = B|x| - C$ with $B = 1.0$, $C = 2.0$, and $A = 0.6$ [7]. The circuit realization of the above uses resistors, capacitors, diodes, and operational amplifiers as shown in Fig. 1. The implementation involves three successive active integrators to generate d^2x/dt^2, dx/dt, and x from d^3z/dt^3, coupled with a nonlinear element that generates $G(x)$ and feeds it back to d^3x/dt^3.

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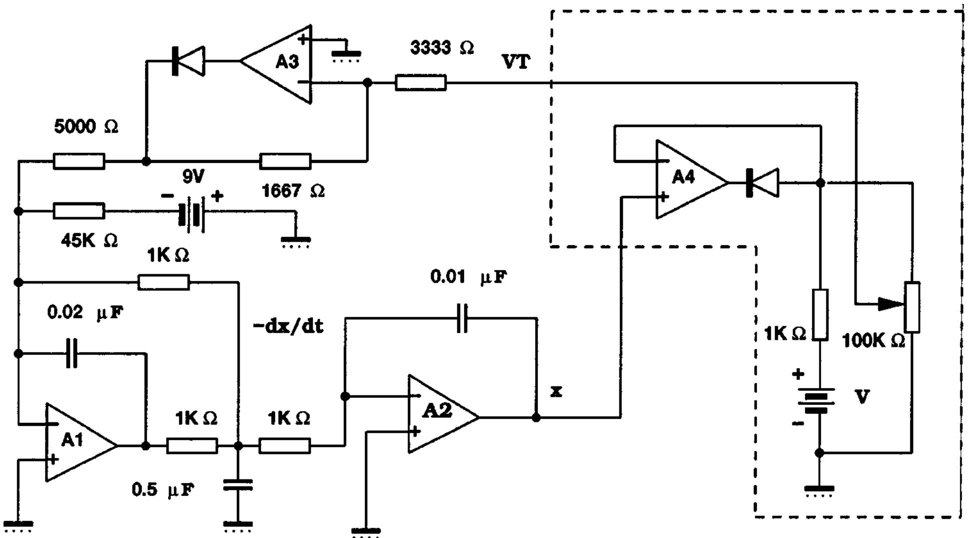


FIG. 1. A general circuit for solving Eq. (1) using a nonlinear feedback element $G(x)=B|x|-C$. The precision clipping control circuit is shown in the dotted box. Here, V_T corresponds to the threshold controlled signal.

Now we implement the threshold mechanism on variable x, i.e., whenever $x>x^*$, x is clipped to x^*. A precision clipping circuit [8] as depicted in the dotted box in Fig. 1 is employed for threshold control. We have chosen component values for the control circuit to be [opamp$=\mu$A741, diode $=$IN4148, load resistor$=1$ kΩ, and threshold reference voltage$=V$, which sets x^*].

Figure 2(a) displays the uncontrolled attractor and Figs. 2(b)–2(d) show some representative results of the threshold action on this chaotic system for a range of threshold values x^* ($x^*<2.4$). It is clear that the mechanism manages to yield cycles of varying periodicities. Further, a detailed comparison shows the *complete agreement between our experimental results and our numerical simulation results*.

So the single threshold variable x has the ability to drag the rest of this three-dimensional system to regular dynamical behavior. The characteristics of the controlled states can be easily varied by just changing the threshold x^* (see Table I). Also note that simply setting the threshold beyond the bounds of the attractor gives back the original dynamics.

The control transience is very short here (typically of the order of 10^{-3} times the controlled cycle). This makes the control practically instantaneous. The underlying reason for this is that the system does not have to be close to any particular unstable fixed point, as in OGY based schemes, before implementing control. Once a specified state variable exceeds the threshold, it is caught immediately in a stable orbit.

The changes in state effected by thresholding, namely, $(x-x^*)$ when $x>x^*$, are typically small (as adjustments are made just after x crosses x^*). Further, for higher periods, the controlling action is infrequent and occurs for short intervals in every controlled cycle. For instance, to control to a 16-cycle with $x^*=2.327$, the thresholding is operational for only ~0.22 msec in an interval of 50 msec.

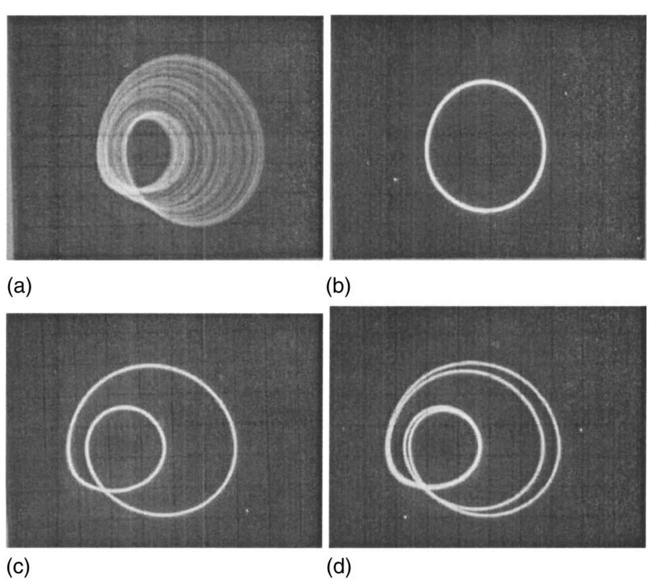


FIG. 2. Attractors in the x-$\dot{x}$ plane: (a) the uncontrolled chaotic system obtained from the circuit realization of Eq. (1) (upper left box); (b) period 1 cycle obtained when $x^*=1$ V (upper right box); (c) period 2 cycle obtained when $x^*=2$ V (lower left box); and (d) period 4 cycle obtained when $x^*=2.1$ V (lower right box).

III. CONTROLLING CHUA'S CIRCUIT

Now we consider a realization of the double scroll chaotic Chua's attractor given by the following set of (rescaled) three coupled ODEs [9]

$$\dot{x}=\alpha[y-x-g(x)], \tag{2}$$

TABLE I. Threshold ranges (in V) vs periodicity of the controlled cycle, for the chaotic system given by Eq. (1).

Threshold for system	Nature of controlled orbit
$x^*<-2.00$	Fixed point
$-2.00<x^*<1.477$	Period 1 cycle
$1.477<x^*<2.242$	Period 2 cycle
$2.242<x^*<2.321$	Period 4 cycle
$2.321<x^*<2.325$	Period 8 cycle
$2.325<x^*<2.331$	Period 16 cycle

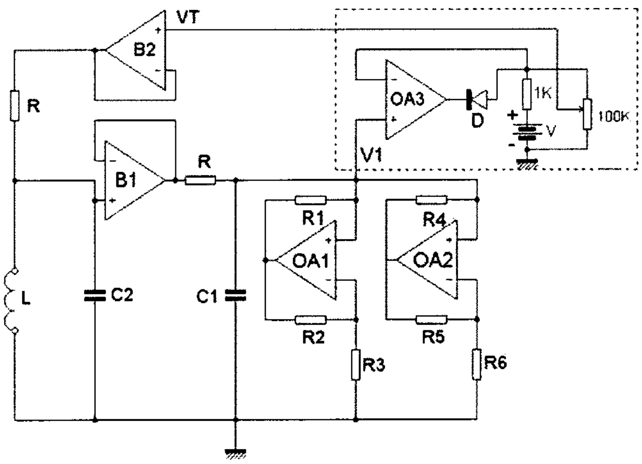


FIG. 3. Chua's chaotic circuit with threshold level controlling circuit (shown in the dotted box). Here, V_T is the threshold controlled signal.

$$\dot{y} = x - y + z, \tag{3}$$

$$\dot{z} = -\beta y, \tag{4}$$

where $\alpha = 10$ and $\beta = 14.87$ and the piecewise linear function $g(x) = bx + 1/2(a-b)(|x+1| - |x-1|)$ with $a = -1.27$ and $b = -0.68$. The corresponding circuit component values are [$L = 18$ mH, $R = 1710$ Ω, $C_1 = 10$ nF, $C_2 = 100$ nF, $R_1 = 220$ Ω, $R_2 = 220$ Ω, $R_3 = 2.2$ kΩ, $R_4 = 22$ kΩ, $R_5 = 22$ kΩ, $R_3 = 3.3$ kΩ, $D =$ IN4148, $B_1, B_2 =$ Buffers, OA1–0A3: opamp μA741]. Note that the circuit of Fig. 3 is the ring structure configuration of the classic Chua's circuit [9,10]. The uncontrolled attractor from this system is displayed in Fig. 4(a).

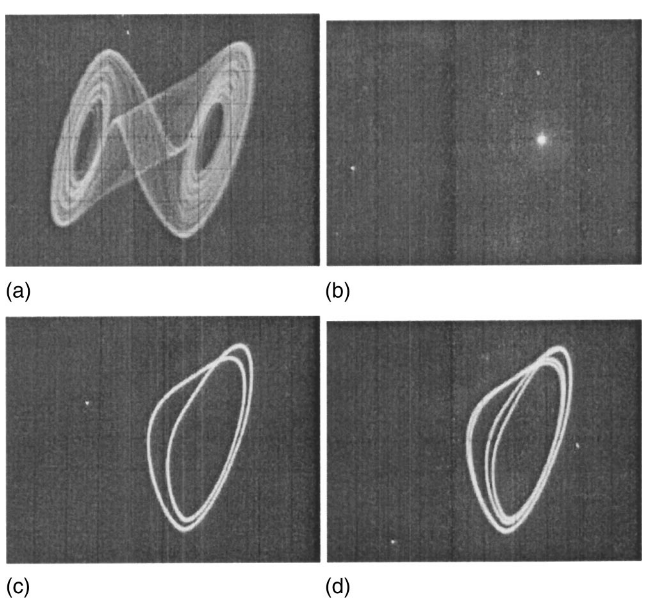


FIG. 4. Attractors in the V_1-V_2 plane, corresponding to the x-y plane of Eqs. (2)–(4). (a) Uncontrolled chaotic attractor (upper left box); (b) fixed point obtained when $x^* = 1.8$ V (upper right box); (c) period 2 cycle obtained when $x^* = 2.7$ V (lower left box); and (d) period 4 cycle obtained when $x^* = 2.71$ V (lower right box).

TABLE II. Threshold ranges (in V) vs periodicity of the controlled cycle, for the chaotic system given by Eqs. (2–4).

Threshold for system	Nature of controlled orbit
$x^* < 1.84375$	Fixed point
$1.84375 < x^* < 2.235$	Period 1 cycle
$2.235 < x^* < 2.258$	Period 2 cycle
$2.258 < x^* < 2.264$	Period 4 cycle
$2.264 < x^* < 2.265$	Period 8 cycle
$2.265 < x^* < 2.2653$	Period 16 cycle

Now we implement an even more minimal thresholding. Instead of demanding that the x variable be reset to x^* if it exceeds x^*, we only demand this in Eq. (3). This has very easy implementation, as it avoids modifying the value of x in the nonlinear element $g(x)$, which is harder to do. So then all we do is to implement $\dot{y} = x^* - y + z$ instead of Eq. (3), when $x > x^*$, and there is no controlling action if $x \leqslant x^*$. In the circuit, the voltage V_T corresponds to x^*. The resulting controlled orbits with respect to threshold x^* is given in Figs. 4(b)–4(d) ($x^* < 2.7$). So the threshold control works on the system rapidly and can control to a wide range of temporal behaviors (see Table II).

IV. CONTROLLING HYPERCHAOS

Now we demonstrate the method on a hyperchaotic electrical circuit. This constitutes a stringent test of the control method since the system posseses more than one positive Lyapunov exponent, and so more than one unstable eigendirection has to be reigned in by thresholding a single variable. In particular, we consider the realization of four coupled nonlinear (rescaled) ordinary differential equations of the form

$$\dot{x}_1 = (k-2)x_1 - x_2 - G(x_1 - x_3), \tag{5}$$

$$\dot{x}_2 = (k-1)x_1 - x_2, \tag{6}$$

$$\dot{x}_3 = -x_4 + G(x_1 - x_3), \tag{7}$$

$$\dot{x}_4 = \beta x_3, \tag{8}$$

where

$$G(x_1 - x_3) = \tfrac{1}{2} b[|x_1 - x_3 - 1| + (x_1 - x_3 - 1)]$$

with $k = 3.85$, $b = 88$, and $\beta = 18$ [11]. The circuit realization of the above is displayed in Fig. 5, with component values [$L = 18$ mH, $C_2 = 68$ nF, $R = 1.8$ kΩ, $C = 68$ nF, $R_1 = 2.8$ kΩ, $R_2 = 1$ kΩ, and $D_1 =$ IN4148]. Figure 6(a) displays the (uncontrolled) hyperchaotic attractor resulting from this circuit, and it is characterized by two maximal positive Lyapunov exponents $\lambda_1 = 0.13$ and $\lambda_2 = 0.05$.

Again we implement a *partial* thresholding on variable x_3: whenever $x_3 > x^*$ in the system, $G(x_1 - x_3)$ in Eq. (5)

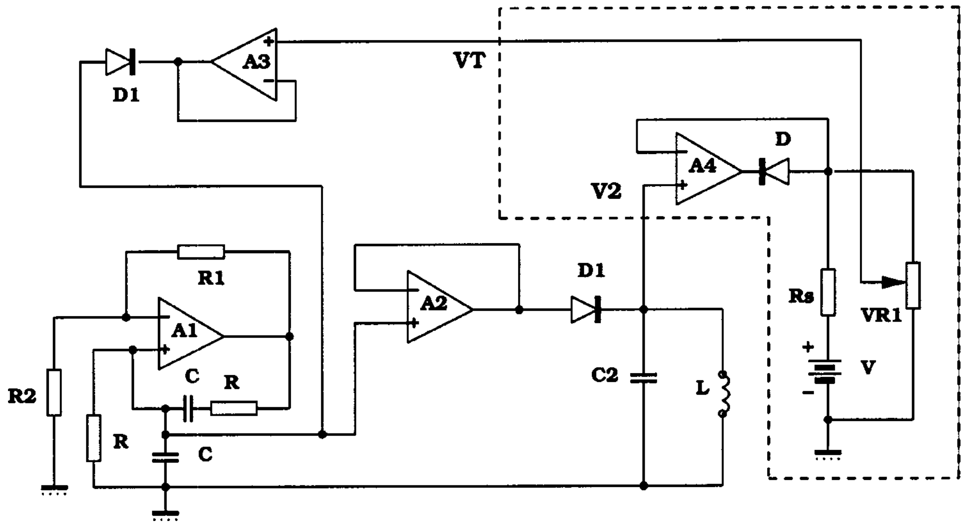


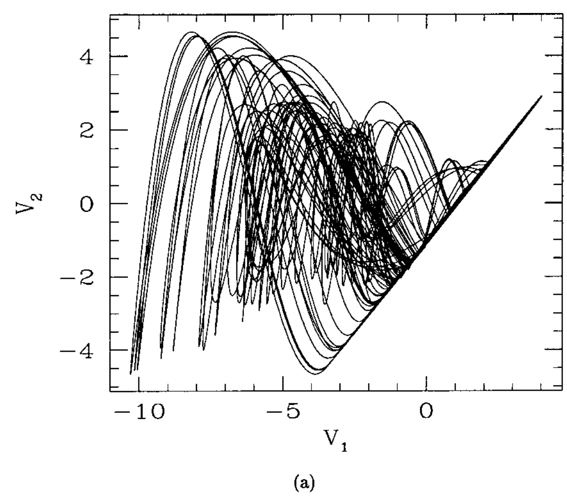
FIG. 5. Circuit implementation of Eqs. (5)–(8), with the precision clipping control circuit in the dotted box. V_T is the threshold controlled signal.

becomes $G(x_1-x^*)$, i.e., we have $\dot{x}_1=(k-2)x_1-x_2-G(x_1-x^*)$, while Eqs. (6)–(8) are unchanged. When $x_3 \leqslant x^*$, there is no action at all. A precision clipping circuit [8] as depicted in the dotted box in Fig. 5 is employed for the above scheme, which is even simpler to implement than thresholding x_3 throughout the system. We have chosen component values for the control circuit to be [opamp $=\mu$A741, diode $(D)=$IN4148 or IN34A, series resistor $Rs=1\ \mathrm{k}\Omega$ and threshold reference voltage$=V$, which sets the x^*].

Both our experiments and our numerical simulations (which are in complete agreement) show that this scheme successfully yields regular stable cycles under a very wide range of thresholds. A representative example with threshold set at 0 V is displayed in Fig. 6(b), which shows the controlled cycle in the V_1-V_2 plane, which corresponds to the rescaled x_1-x_3 plane of Eqs. (5)–(8).

So it is evident that a *single thresholded variable has the ability to clip the full four-dimensional hyperchaotic system to regular dynamical behavior* (see Figs. 7 and 8 for some examples of the geometries of the controlled orbits). Thus, the period and geometry of the controlled states can be easily varied by setting x^* in different windows. For instance, thresholding at 0 V yields a 1 T attractor (with respect to the x_1 variable), while thresholding at 0.3 V yields period 3 T, 0.32 V yields period 8 T, 0.33 V yields period 5 T, and 0.35 V yields period 13 T.

Note that this technique has a certain similarity with impulse methods [13], in that they are both stroboscopic in operation and act only on state variables, not on parameters. The difference lies primarily in that thresholding acts *only* when the system is above threshold and thus can be very infrequent. Impulse methods, on the other hand, act at fixed intervals. Further, the control action here is a simple resetting of one variable, while the periodic pulse method involves an additive (negative or positive) or multiplicative pulse to one or more variables. It also appears that pulse methods need to implement more controlling action than thresholding. For instance, in the example in Ref. [14] a three-dimensional (3D) system needs pulsing on two variables for control, while here even a four-dimensional hyperchaotic system needs only one variable to be thresholded. A further advantage of the thresholding method is that exact analytical results are available for thresholding 1D chaotic maps [5], and these indicate a theoretical basis for the success of the method.



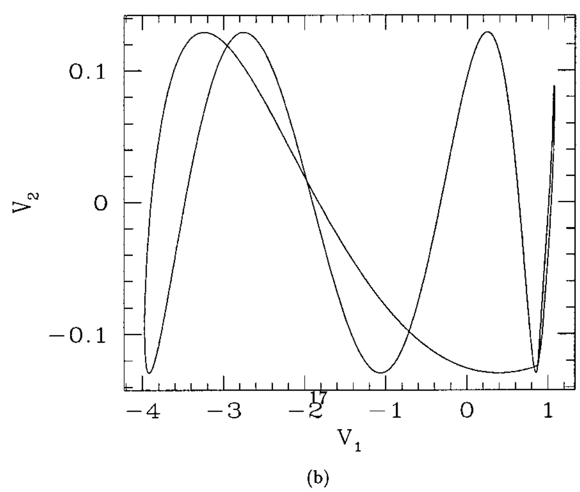


FIG. 6. (a) Uncontrolled hyperchaotic attractor, (b) controlled attractor for threshold$=0$ V, in the V_1-V_2 plane, corresponding to the x_1-x_3 plane of Eqs. (5)–(8).

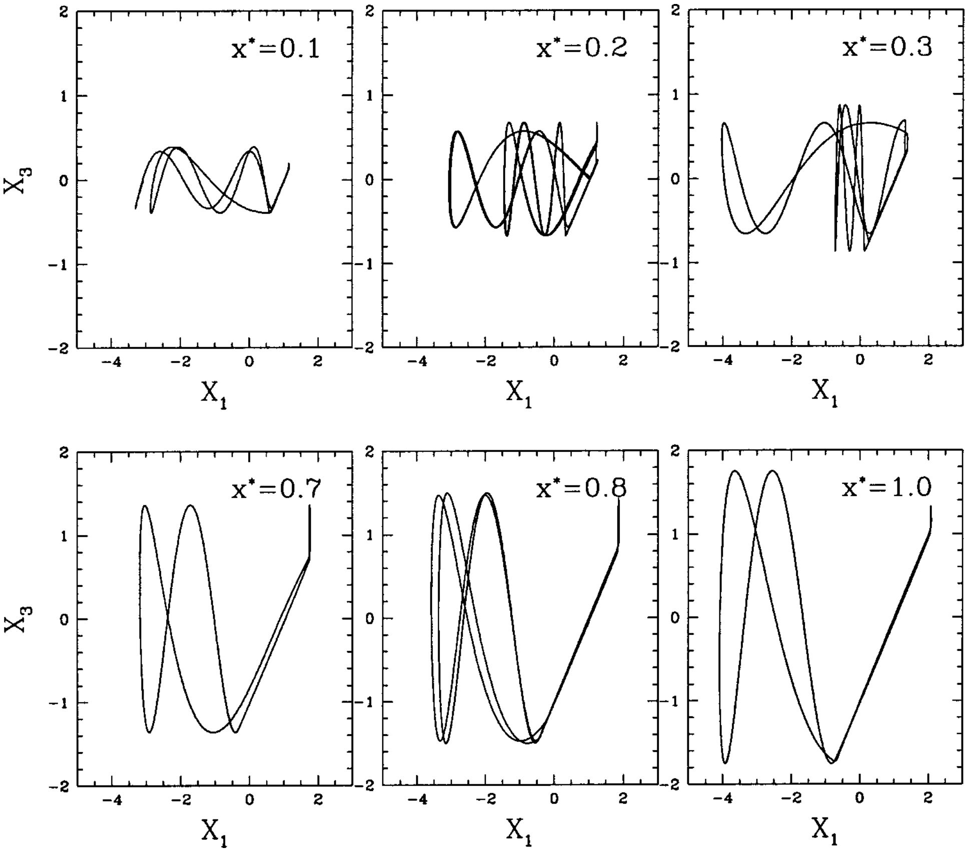


FIG. 7. Controlled attractors in the x_1-x_3 plane, obtained from the hyperchaotic system by thresholding the x_3 variable in Eq. (5) with threshold values: (i) $x^*=0.1$, (ii) $x^*=0.2$, (iii) $x^*=0.3$, (iv) $x^*=0.7$, (v) $x^*=0.8$, and (vi) $x^*=1.0$.

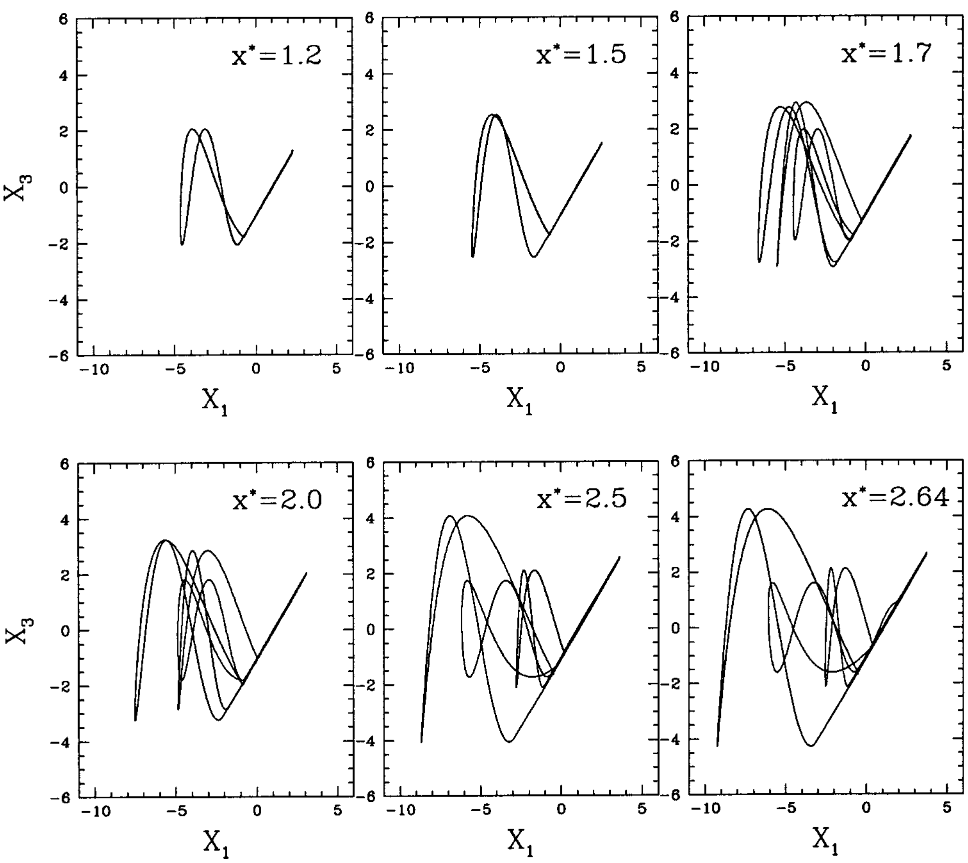


FIG. 8. Controlled attractors in the x_1-x_3 plane, obtained from the hyperchaotic system by thresholding the x_3 variable in Eq. (5), with threshold values: (i) $x^*=1.2$, (ii) $x^*=1.5$, (iii) $x^*=1.7$, (iv) $x^*=2.0$, (v) $x^*=2.5$, and (vi) $x^*=2.84$.

Lastly, note a few limitations of this method. In most systems, very high-order periods are usually obtained in narrow windows of threshold values. So these targets are quite susceptible to noise, and consequently they are harder to obtain, as one needs very accurate threshold level determination. Also, high thresholds acting at the edges of attractors are less robust and more susceptible to fluctuations.

Further, while threshold control will always yield some regular orbit, it is not clear at the outset the full range of dynamic behaviors that can be obtained by thresholding. So one needs an initial exploratory run over threshold parameter space to map out the dynamic possibilities for different thresholds. Such a run clearly lays out the scope of the threshold mechanism in a specific system. The more complex is the time series of a system, the greater is the diversity of controlled orbits, e.g., in the hyperchaotic example above, the variety of orbits that may be obtained is very wide indeed. As noted before, chaos, especially hyperchaos, is particularly interesting in this context, as it possesses a rich range of temporal patterns which can be clipped to many different behaviors.

V. CONCLUSIONS

In summary, it is clearly evident from these experiments that the technique is powerful, efficient, and robust, and we have applied it successfully to obtain a wide range of regular behaviors. The method involves no adjustment of parameters, but merely the manipulation of *one* state variable, even in hyperchaotic systems possessing more than one unstable eigendirection. A significant motivation in verifying the efficacy of this strategy in experiments was the possible applications of such a scheme to technical applications such as chaos computing [12] and communications [15]. Such applications require swift control with no run time computations, i.e., a nonfeedback control which can be employed as a look-up table [16]. This is exactly what thresholding offers. Further, the controller is very simple and flexible, and this has clear cost benefits in any attempts to exploit the richness of chaos.

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[16] This method may be better suited for optical and electrical systems which lend themselves easily to limiters, while it may be harder to implement limiters in chemical and mechanical systems.