CHAOS, CONTROL AND SYNCHRONIZATION IN EXCITATORY-INHIBITORY NEURAL NETWORK MODELS

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Chapter 1

Introduction

> Who traces life and seeks to give
> Descriptions of the things that live
> Begins with 'Killing to Dissect'
> He gets the pieces to inspect
> The lifeless limbs beneath his knife
> All parts - but link which gave them life.
> *Goethe*

> The answer to the question of life, the universe
> and everything is twenty-two.
> *Douglas Adams*

Since the development of the electronic computer in the 1940s, the serial processing computational paradigm has successfully held sway. It has developed to the point where it is now ubiquitous. However, there are many tasks which are yet to be successfully tackled computationally. A case in point is the multifarious activities that the human brain performs regularly, including pattern recognition, associative recall, etc. which is extremely difficult, if not impossible to do using traditional computation.

This problem has led to the development of non-standard techniques to tackle situations at which biological information processing systems excel. One of the more successful of such developments aims at "reverse-engineering" the biological apparatus itself to find out why and how it works. The field of neural network models has grown up on the premise that the massively parallel distributed processing and connectionist structure observed in the brain is the key behind its superior performance. By implementing these features in the design of a new class of architectures and algorithms, it is hoped that machines will approach human-like ability in handling real-world situations.

Network models of computation have been enjoying a period of revival for quiet some time now, from the perspective of both theory and applications [85]. These models comprise networks of large numbers of simple processing elements, usually having continuously varying activation values and stochastic threshold dynamics. The activity of these elements, x_i ($i = 1, 2, ..., N$) at some time instant t, are determined by the temporal evolution equation:

$$x_i\ (t) = \mathrm{F}\ (\ \Sigma_j\ W_{ij}\ x_j\ (t-1) - \theta_i\ ),$$

where, θ_i is an internal threshold (usually taken as zero), W_{ij} is the connection weight from element j to element i, and F is a nonlinear activation function. If $W_{ij} > 0$, the synaptic connection between neurons i and j is called *excitatory*; if $W_{ij} < 0$, it is called *inhibitory*. The activation function, F, usually has a sigmoid form, which may be of the following type:

$$\mathrm{F}(z) = \tanh(\frac{z}{a}),$$

a being the slope. For $a = 0$, F is a "hard limiting" or step function,

$$x_i = \mathrm{sgn}\ (\ \Sigma_j\ W_{ij}\ x_j - \theta_i\ ).$$

Different neural network models are specified by

- network topology, i.e. the pattern of connections between the elements comprising the network,
- characteristics of the processing element, e.g. the explicit form of the nonlinear function F, and the value of the threshold, θ,
- learning rule, i.e. the rules for computing the connection weights W_{ij} appropriate for a given task, and,
- updating rule, e.g. the states of the processing elements may be updated in parallel (synchronous updating), sequentially or randomly.

One of the limitations of most network models at present is that they are basically static, i.e., once an equilibrium state is reached, the network remains in that state, until the arrival of new external input [8]. In contrast, real neural networks show a preponderance of dynamical behavior. Once we recall a memory, our minds are not stuck to it, but also recall other associated memories without being prompted by any additional external stimuli. This ability to 'jump' from one memory to another in the absence of appropriate stimuli is one of the hallmarks of the brain. It is an ability which one should try to recreate in a network model if it is ever to come close to human-like performance in intellectual tasks. One of the possible ways of simulating such behavior is through models guided by non-equilibrium dynamics, in particular, chaos. This is because of the much richer dynamical possibilities of such networks, compared to systems governed by convergent dynamics [86, 7].

There is as yet no universally accepted definition of the term "chaos", but the following working definition is adequate for our purpose [188]:

> Chaos is *aperiodic* long-term behavior in *deterministic* systems that exhibit *sensitive dependence* on initial conditions.

"Aperiodic long-term behavior" means that there are trajectories which do not settle down to fixed points, periodic orbits, or quasiperiodic orbits as time, $t \to \infty$. Aperiodic behavior is marked by a broad frequency spectrum. "Deterministic" implies that the system has no random/ noisy inputs or parameters. The irregular behavior arises from the system's inherent nonlinearity. "Sensitive dependence on initial conditions" indicates that nearby trajectories separate exponentially fast. As a result, any error in our knowledge of the initial conditions of the system will amplify rapidly, making its behavior effectively unpredictable.

In this thesis, we present some results of theoretical and simulation studies exploring the occurrence and utility of chaotic dynamics in network computation, particularly in the case of networks of nonlinear devices ("neurons") of 2 specific classes: excitatory and inhibitory. Section 1 discusses the biological evidence for chaotic activity in the brain. The necessity of modeling to resolve the controversy of interpreting the biological data is briefly outlined. Even if chaos exists, its role in the functioning of the brain will need to be established. The possible advantages of chaos for information processing is discussed in Section 2. Section 3 contains a brief survey of the work done in the general area of chaotic neural networks. However, so many papers in this area have appeared over the past decade, that any review can at best be partial. We have stressed on work which leads directly to the model network studied in the thesis. We have also tried to trace the "ancestry" of our model. Finally, in Section 4, the scope of the thesis is outlined.

1.1 Neurobiological evidence of chaos

Evidence of deterministic chaos in neuronal systems was found within a very short time of the emergence of the field of chaotic dynamics. Experiments on excitable biological membranes, supplemented by physiologically plausible models of neurons showed that chaos occurs in the presence of periodic stimulation (either chemical or electrical). Hayashi *et al* [79] investigated the nonperiodic behavior in self-sustained oscillation of the internodal cell of *Nitella* under sinusoidal stimulation. The analysis of the nonperiodic oscillations revealed chaotic behavior. Chaotic oscillations was also observed in the molluscan neuron [88]. Another group of researchers [3] studied the self-sustained oscillation of action potentials in a model axon immersed in calcium-deficient sea water, whose dynamics was modeled by the Hodgkin-Huxley equations. The oscillations were analyzed by stroboscopic plots revealing both periodic and chaotic behavior, determined by the amplitude and the frequency of the stimulating current. The results corroborated similar studies carried out previously in squid giant axons (for an overview, see [121] or the article by Aihara and Matsumoto in [87]). The group of Glass and Guevara [72] showed that recurrent inhibition and periodic forcing of neural oscillators can produce chaos and explored its implications in modeling normal and abnormal function in neurophysiology. However, most of the experimental work was done with the membranes in artificial circumstances, the electrical and/or chemical stimulations being far from the physiological state. The occurrence of chaos in equations for membrane excitability even in the absence of stimulation was shown in [34].

After demonstrating chaos in the case of single neurons, the obvious next step was to show it in macroscopic neural assemblages, and in particular, the brain. However, this progression was far from simple. For one thing, the complementary approaches of experimental observations and modeling used successfully in the case of single neuron studies, could not be used in the case of the brain. In the absence of any good model for large-scale brain activity, the evidence for chaos in the brain has to be searched for in such coarse-grained variables as the EEG. One of the complicating factors in studying EEG is the continuous presence of background "spontaneous" neural activity, seemingly random in appearance. One has to therefore devise a test to determine whether this apparent randomness is truly stochastic or owes its origin to deterministic chaos. Analysis of human EEG in various mental states have put forward several candidates for chaotic activity. Evidence of low-dimensional chaos was found in some sleep stages [15] and in 'petit mal' epileptic seizures of small duration [14]. However, any claim based on time-series analysis of EEG data depends on the efficiency of the tests for determining chaotic activity [65, 67, 27, 138]. Such methods usually require large data sets recorded under constant conditions and relatively free of noise. As all three requirements are hard to satisfy in the case of biological data, the detection of low dimensional chaos in brain activity has often been questioned [62]. The dependence of the results of such tests on the brain site at which recording occurs and on the state of the subject (wakeful, resting, or moving) have been shown by Pijn *et al* [145]. Several attempts have been made to devise new tests which will provide unequivocal results when applied to such data sets, including nonlinear forecasting techniques, but as yet no one has come up with an universally accepted method.

It has been suggested that the use of chaos control techniques to suppress and enhance aperiodic activity in brain-slice preparations [155] is a clear-cut evidence in favor of the presence of chaotic dynamics in the brain. However, such control methods have been shown to be effective even in non-chaotic systems [37]. The presence of unstable periodic orbits (UPOs) in chaotic trajectories has suggested a new method, relying on the detection of such UPOs in the biological time series. Statistically significant evidence of the existence of UPOs in a crayfish sensory neuron [142] have been reported, with experimental parameters being kept in ranges typically found in the animal's natural environment.

Chaos has also been implicated in certain 'dynamical diseases' - medical problems that have their roots in some underlying dynamical effect [89]. In the neurological context, abnormal oscillations and complex rhythms often pose clinical problems. There may be significant oscillation in a neurological control system that does not normally have a rhythm, e.g., ankle tremor in patients with corticospinal tract disease, various movement disorders like Parkinson's tremors, and abnormal paroxysmal oscillations in the discharge of neurons that occur in many seizures. Otherwise, there may be qualitative changes in the oscillations within an already rhythmic process, resulting in waking abnormality, alerted sleepwake cycles, or rapidly cycling manic depression. Epileptic seizures, which recur in an apparently random manner, may be yet another manifestation of such pathological dynamics. However, in the absence of any reasonable models of such complex neural processes, the role of chaos in these dynamical diseases cannot be established beyond doubt.

1.2 Chaos and information processing

Even if chaos does exist in the brain, the issue of whether it plays any role in the overall cognitive functioning of the brain needs to be looked into. The possible uses of chaos in the biological world has been discussed in [38]. In the context of brain functioning, chaos may have the multiple roles of generating and preserving diversity, maintenance of network activity through disentrainment, dissipation of disturbance and facilitating learning. All these different functions enable the nervous system to be adaptive, and continue to function in the face of an uncertain and unknown environment.

The role of chaos in higher brain functions is discussed in [75]. In the context of information processing, it would appear at first that chaos can play only a negative role. However, the brain is *not* a conventional processor of information, in the sense of classical information theory, as developed by Shannon, Wiener and others. In the case of a highly nonlinear and interconnected system such as the brain, chaotic dynamics might play a counter intuitive role by enhancing the robustness, reliability and overall functionality of neural information processing. The particular case of thalamocortical interactions as a generator of chaotic activity [129] and its possible role for generating self-referential logic and short-term memory, were explored in [130].

Tsuda [192] has suggested several other possible roles of cortical chaos in brain functioning:

- Interpreter for input stimuli via thalamo-cortical interactions.
- Efficient search mechanism in memory.
- Robust information transfer channel for periodically oscillating stimuli.
- Providing dynamic storage of long-term memory.

The work on human EEG analysis has suggested a further possible functional role of chaos [14]. Chaos seems to increase the resonance capacity of the brain, enabling an extremely rich response to an external stimulus, as compared to stable periodic oscillations.

However, the most well-known work to obtain empirical physiological evidence for the possible relevance of chaos to brain function is probably that of W. J. Freeman. Through his work on the large-scale collective behavior of neurons in the perception of olfactory stimuli [51], [177], [52], a concrete link between chaos and cognition has been built up through a successful combination of biological experiments and computational modeling. Olfactory stimuli are detected by receptor neurons in the nasal passage. The number of receptors excited by a smell is a measure of the intensity of the stimulus, while the spatial pattern of activated receptors is dependent on the nature of the scent. On trapping molecules carrying specific odors these receptors fire action potentials which are transmitted to the olfactory bulb in the cortex. The bulb then transmits signals to the olfactory cortex which, in its turn, sends information to many regions of the brain. The test animals used by Freeman were trained to recognize several different odors and were then subjected to smells, both familiar and unfamiliar. EEG data was recorded by a grid-like array of electrodes placed over the surface of the olfactory bulb. Most of the time, the EEGs showed irregular oscillations. However, when an animal inhaled, a "burst" occurred in each EEG tracing as all the waves in the array became more regular for a brief period until the animal exhaled. These waves, named gamma waves, had a higher amplitude and frequency than usual and varied in frequency from 20 - 90 Hz, mostly occurring in the neighborhood of 40 Hz. Each set of burst recordings had a common carrier waveform, although the average amplitude of the different recordings varied widely. It was inferred that, as the carrier waveform changed during each inhalation, even for the same stimulus, the information about a particular scent was not encoded in the shape of the waveform but rather, in the spatial pattern of the carrier-wave amplitudes across the bulb, which remained invariant over trials. On plotting the different amplitudes of the carrier waves in different regions of a surface representing the locations in the grid-like array over the olfactory bulb from which they were obtained, a contour diagram was produced. This remained the same for a specific scent throughout the testing period. However, if the reinforcement associated with a scent was altered, then the amplitude contour map representing it also changed. This indicated that the olfactory bulb is involved in the assigning of meaning to stimuli. One of the early pointers to chaotic activity was the aperiodicity of the common carrier wave in the bulb both during and between bursts. Another clue was the sudden transitions of neuronal networks in the bulb and the cortex from a nonburst to a active, bursting state. These factors prompted the development of a model for the olfactory system having cells in a network connected by both excitatory and inhibitory synapses. Computer simulations of the model showed that it recreated all the observed behavior of the olfactory system and, thus, was an accurate representation of it. The network was then made to produce EEGs of extended bursts and of inter burst activity for a longer period than is possible in actual EEGs. The attractor [1] of the underlying dynamics, reconstructed from the EEG data using the delay-coordinate technique was found to be chaotic in nature. The primary findings suggested that a separate chaotic attractor is maintained for each stimulus and the act of perception consists of a transition of the system from the domain of influence of one attractor to another. Later findings led Freeman to hypothesize that each brain area (rather than each stimulus) has a chaotic attractor. A specific sensory stimulus drives the system into a localized region within the attractor, which can be identified with the appearance of specific spatial patterns of carrier waveform amplitudes, associated with a specific stimulus. Further, the attractors themselves would have to change as a result of new experience and continued development of the brain.

This description of non equilibrium nervous activity has suggested several possible functions of chaos:

- providing rapid and unbiased access to a number of possible attractors, one of which is selected dependent upon the stimulus,
- acting as a "novelty filter" by failing to converge to any of the existing attractors in the presence of a significant but unidentified stimulus, and,
- allowing the system to escape from the existing set of attractors and add a new response to a novel stimulus under reinforcement

Thus, according to this picture, chaotic activity is fundamental to the general process of perception.

1.3 A brief survey of chaotic neural network models

Extremely simplified models of neurons connected in a network via suitable connection weights were known to implement various logical functions since the 1940s [124]. The subject received fresh impetus a decade and half ago due to some breakthroughs, e.g., the identification of a class of globally connected network models with 'spin glass' models [126] of condensed matter physics by Hopfield [90] (see also [159]). These developments were however restricted to networks subject to equilibrium dynamics. Such systems converge to a time-independent solution (a "fixed-point" attractor) after starting off from some initial condition. On the other hand, the brain never settles down to a steady state but appears to exhibit a rich variety of non-periodic behavior.

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[1] *Attractor* of a dynamical system is a set to which all neighboring trajectories converge. Stable fixed points and stable limit cycles are examples. An attractor that exhibits sensitive dependence on initial conditions is a chaotic attractor.

The development of nonlinear dynamical systems theory - in particular, the discovery of "deterministic chaos" in extremely simple systems - has furnished the theoretical tools necessary for analyzing non-equilibrium network dynamics. Neurobiological studies indicating the presence of chaotic dynamics in the brain and its possible role in biological information processing has provided further motivation. Thus, the ability to design networks with aperiodic behavior promises to add a new dimension to our understanding of how the brain works.

Several efforts in designing and applying chaotic neural networks have been reported. One of the first such studies was on a continuous time randomly connected network [108], whose individual elements are inherently stable. The high dimensionality of this example precludes a theoretical understanding. However, numerical studies showed the occurrence of chaos.

Chaotic dynamics in a globally connected network with Gaussian distribution of connection weights was theoretically established by Sompolinsky *et al* [180] in the thermodynamic limit (i.e., $N \rightarrow \infty$, $N=$ the number of neurons). This was extended to networks with variable connectivity in [49], which showed through numerical simulations that the connectivity is not a determinant parameter for the behavior of such nets.

Aihara *et al* [4] developed a neural network model where each neuron behaved as a chaotic system, due to the introduction of delayed interactions [127]. This model was shown to be effective in solving optimization problems [35], as the chaotic behavior could be used as to perform a deterministic version of *simulated annealing* [103]. The network can also be used for image segmentation [77] and associative recall [1].

Recurrent neural networks with a single hidden layer have been proposed as a realization of one-dimensional maps of an interval onto itself which show chaotic behavior [9]. By encoding images as strings and relating them to stable limit cycles of a chaotic map [10], such networks have been used to store and retrieve images.

Destexhe [44] has studied two-dimensional networks of excitatory and inhibitory neurons which evolve in continuous time, with time-delayed interactions. As the number of neurons, connectivity and synaptic weights are varied, the model exhibits a transition from spatially uniform oscillations to spatiotemporal chaos. Several properties of the spatiotemporal phase and the information transport in such a system was studied. Hayashi [80] has also examined a continuous-time evolving network of excitatory and inhibitory neurons, with the aim of implementing a model of dynamical associative memory.

Associative recall through non-equilibrium dynamics has also been explored (in the discrete time evolution context) by Thomas *et al* [191], who use a network with randomly connected excitatory neurons with an inhibitory interneuron that adjusts their threshold, and Yamakawa *et al* [205].

Wang [197, 24] has studied the simple system of an excitatory and inhibitory neuron evolving in discrete time to analytically establish the occurrence of chaos. A similar system has been studied in [149] to see how onset of chaos occurs as a function of stimulus intensity. Variants of the model have been used for demonstration of chaos control methods in the neural context [179, 186].

Coupled map lattices [99] [100] which are networks of chaotic systems coupled to each other, either locally or globally, share several common features with neural networks. The relation of a globally coupled map lattice to neural network models has been explored in [101]. Such systems have, in fact, been proposed as plausible models of neural computation for performing optimization [94] and associative recall [96].

The Hopfield model, which shows only equilibrium dynamics, has been shown to exhibit chaos when a nonlinear self-feedback term is introduced [33]. The chaotic dynamics prevents the network from staying at a local minima indefinitely and therefore, the system can be used to solve combinatorial optimization problems.

Other network models which show chaos include higher order networks [196], 'dynamical perceptron' [102], stochastic dynamics networks [57] [192] and networks of coupled Hindmarsh-Rose neurons [74].

In the present thesis, a particularly simple model comprising excitatory and inhibitory neurons, which are updated in parallel after discrete time intervals, is considered. The 'simplicity' of this model has enabled a detailed theoretical understanding of its behavior, which could not be achieved in many of the aforementioned models, owing to their relative complexity. The origin of this model may be traced back to the work of other investigators as follows:

- Wilson and Cowan [204] derived coupled nonlinear differential equations for the dynamics of spatially localized populations containing both *excitatory and inhibitory model neurons.* The model showed simple and multiple hysteresis phenomena as well as limit cycles. However, it evolves in continuous time, as a result of which the original 2-variable autonomous system is not capable of exhibiting chaos. As pointed out by Choi and Huberman [36] and explored in depth later by Wang and Blum [198], the discretisation of time can often lead to qualitatively different behavior in a network model.

- Amari [5] used an *additive model* of a neuron to study the dynamics of randomly connected neuron-like elements.

- Hopfield [91] used *graded-response* (i.e., continuous valued) neurons in a globally coupled network to study the process of associative recall of patterns previously stored in the network, extending his work on a similar model comprising binary state neurons [90]. As the connections are symmetric (i.e., $W_{ij} = W_{ji}$), the network always converges to an equilibrium state. Chaotic activity, which is a non-equilibrium state, is therefore absent in this model. The network evolves in continuous time with asynchronous updating of neurons.

- Little [113] described a network of binary neural elements, similar to the one used by Hopfield, but the neurons being *updated in parallel.*

- Marcus and Westervelt [117] studied a *discrete time* version of the Hopfield model referred above. The stability of the equilibrium states of this 'iterated map neural network' was guaranteed by having a symmetric connection weight matrix.

- Wang's model [197] involves only a pair of excitatory and inhibitory neurons coupled to each other, with the dynamics evolving in discrete time. It is a very simple model which can show chaotic behavior. It is the closest relative of the network model that will be described in the thesis. Note that, one needs to impose severe restrictions on the model of Wang, as compared to the one required by the proposed one, to make it analytically tractable. These restrictions prevented a full exploration of various interesting features of the system, even in the limited case of an asymmetric, sigmoid activation function.

1.4 Scope of the thesis

The present thesis reports some results of investigation on the behavior of simple excitatory-inhibitory network models. Almost throughout, a strict form of Dale's hypothesis (i.e., a neuron has exclusively excitatory or inhibitory synaptic connections) is assumed. The resultant discrete-time dynamics (with synchronous or parallel updating of the neural elements) has shown a variety of interesting features. The underlying motivation is to look at the simplest neural module capable of showing chaotic behavior and to use the knowledge gained from studying this system to obtain a broader understanding of the possible relevance of chaotic dynamics to brain functioning.

In Chapter 2, we introduce and analyze the basic module of the excitatory-inhibitory neural network, which is the main topic of our investigation [169, 170]. As a first step towards understanding the behavior of such a network, the intrinsic properties of an excitatory-inhibitory pair is studied in detail. If $x(n)$ and $y(n)$ represent the activation states of the excitatory and inhibitory elements at the nth time instant, respectively, then the discrete-time evolution equations are:

$$x(n+1) = \mathbf{F}_{\mu_1}(\ a\,x(n)\ -\ b\,y(n)\ + I(n)),\ y(n+1) = \mathbf{F}_{\mu_2}(\ c\,x(n)\ -\ d\,y(n)\ + I'(n))$$

where $\mathbf{F}_{\mu}(z)$ can be a asymmetric/ anti-symmetric, sigmoidal/ 'piecewise linear' activation function with parameter μ, (a, b, c, d) are the self- and interconnection weights for the excitatory and inhibitory elements and I, I' denote magnitude of external stimuli. The model is analytically treated under the restrictive assumption of $b/a = d/c = k$ (say). For $k = 1$, the 2-dimensional system reduces to an equivalent 1-dimensional system with the corresponding variable being $z = x - y$ (absorbing a

and b in μ_1 and μ_2 respectively), defined in the interval [-1,1]. For sigmoidal activation functions, this system exhibits a period-doubling route to chaos for sigmoidal activation function. The piecewise linear activation function also leads to chaotic behavior following a route similar to that of the "tent" map, defined over the unit interval [0,1] as:

$$\begin{aligned} x(n+1) &= ax(n), \text{ for } 0 < x(n) < 0.5, \\ &= a(1-x(n)), \text{ for } 0.5 < x(n) < 1, \end{aligned}$$

where $a \in [0,2]$ is a parameter.

The presence of chaos can be analytically demonstrated for a range of values of (μ_1, μ_2). For $k \leq 1$ the chaos is symmetry-broken. There are two chaotic intervals corresponding to $I_1 : [0,1]$ and $I_2 : [0,-1]$ which are disconnected, i.e., if $z(0) \epsilon I_1$, then $z(n) \epsilon I_1 \ \forall \ n$ necessarily and similarly for I_2. However, for $k > 1$ the symmetry is restored and a trajectory starting from any initial z can go to both I_1 and I_2. The introduction of a threshold, θ, in this picture enables one to go from chaos to order through the variation of θ. The concept of a dynamical threshold is motivated by the existence of refractory period in biological neural network. A detailed study of the dependence of chaotic activity on the magnitude of threshold has been done.

In Chapter 3, the nonlinear resonance phenomenon exhibited by a chaotic neural pair, on stimulation with weak periodic signal, is studied [174]. This is remarkably similar to "stochastic resonance" (SR) seen in non-deterministic systems. SR is a recently observed nonlinear phenomena in noisy systems, whereby the noise helps in amplifying a sub threshold signal (which would have been otherwise undetected) when the signal frequency is close to a critical value. As the output of a chaotic process is indistinguishable from that of a noisy system, the question of whether a similar process occurs in the former case is studied in this chapter.

Before looking at the behavior of the excitatory-inhibitory neural pair, we study a simpler model for analytical convenience. The model chosen is a anti-symmetric piecewise linear map defined in the interval [-1,1]. The behavior of the system is controlled by a parameter, a $(0 < a < 4)$. Onset of chaos is seen to occur at $a = 1$. The chaos is symmetry broken, i.e., the system is restricted to either of the two sub-intervals (0,1] and (0,-1], depending on initial condition. Symmetry is restored at $a = 2$.

To observe SR, the value of a is kept close to 2, and then modulated sinusoidally with amplitude δ and frequency ω. The response of the system shows a non-monotonic behavior as ω is varied, attaining a peak value at ω_c, a "critical frequency", which depends on a_0 and δ - a clear signature of a SR-type phenomenon. Some analytical calculations have also been done - in particular obtaining the invariant probability density and the dominant time scale of the time-varying processes. The implication of the above study is that chaotic neural networks can amplify weak signals in a noisy background, thus enhancing its information processing capabilities. We have also studied kinetic aspects, such as hysteresis, of the above model.

The one-dimensional map equivalent to the excitatory- inhibitory neural model exhibits SR-type behavior similar to that reported above for the anti-symmetric chaotic map. The dependence of such 'resonance' on the relative magnitudes of μ_1 and μ_2 and other parameters has been studied. Analytical results have been obtained in the case of piecewise linear activation function. For sigmoidal function, numerical studies have been done.

Having studied in Chapter 2 the existence of chaotic activity in our proposed model, in Chapter 4 we proceed to control it [167, 166, 168]. With this objective, piecewise linear maps are studied as approximations of excitatory-inhibitory neural pairs, for analytical convenience. In principle, any piecewise linear map can be represented by a recurrent neural network with properly connected excitatory and inhibitory elements having appropriate thresholds and linear activation functions with a cutoff.

The "tent" map, defined above, is subjected to dynamical variable feedback control. In this method, a particular interval $I : (x_0 - \delta, x_0 + \delta)$ of the unit interval is chosen and whenever $x(n+1) \epsilon I$, a feedback of magnitude $k \; |x(n+1) - x_0|$ is applied to the system (k is a constant). The dependence of control on the parameter set (x_0, δ, k) is studied in detail and a geometrical understanding of the control process is obtained.

Control is also achieved through the use of a dynamical threshold (as outlined in Chapter 2). This is same as subjecting the model to small-amplitude periodic perturbations. Various periodic cycles can be stabilized by periodically varying the threshold. A numerical study of the control method has been done.

The chaos control method is then implemented on a network model comprising N excitatory and N inhibitory elements. The model exhibits both periodic and chaotic behavior, depending on parameter values. Numerical simulations are carried out for N=3, the number being kept low for ease of visual representation. The control process made the chaotic trajectories converge to any one of a large number of possible unstable periodic attractors. The potentially high storage capacity, as well as, the extremely rapid speed of convergence, are notable features of the network. The model also attempts to explain the occurrence of olfactory hallucinations in certain types of epileptic seizures.

In Chapter 5, the collective dynamics and synchronization in assemblies of coupled chaotic neural pairs is studied [176, 174, 171]. In the brain, synchronization of neural assemblies seems to be employed in "visual binding". The question of what happens if competitive synchronizing interactions occur among different neural assemblies is the motivation to study such interaction among coupled chaotic systems.

Synchronization can be achieved through both unidirectional and bidirectional coupling among chaotic elements. In the case of unidirectional coupling, an n-dimensional autonomous system is divided into two parts, a 'driving' and a 'responding' subsystem. A replica of the 'responding' subsystem is then created and driven with the $\mathbf{x}_d$ variables of the original system. The two systems will synchronize only if the 'conditional Lyapunov exponents' of the 'responding' subsystem are all negative. For bidirectional coupling, the coupling magnitude should be at least greater than half the magnitude of the Lyapunov exponent of the uncoupled system, for synchronization to occur.

Synchronization of chaotic activity among small assemblies of coupled neural pairs has been numerically studied. In the case of bidirectional coupling of two neural pairs, on-off intermittency phenomena is observed. For three neural pairs (A,B,C) coupled to each other, such that (A,B) and (B,C) are connected, but not (A,C), we find synchronization to occur between (A,C), although neither (A,B) nor (B,C) synchronize. We call this 'mediated synchronization', as B appears to be mediating the synchronization process, without itself taking part in it. For unidirectional coupling, a particularly interesting feature is the effect of competitive interactions on the synchronization dynamics. To obtain a theoretical understanding of some of the numerical observations, we study the effect of two 'driving' systems driving a single 'responding' system, each system being governed by the well-known Lorenz equations. One of the variables (y) of the 'responding' system is defined in terms of the corresponding variables of the two 'driving' systems as:

$$y = ay_1 + (1-a)y_2$$

where a is the 'competition' parameter. For a=0 or 1, the conventional Lorenz attractor is obtained. However, as $a \rightarrow 0.5$, the resultant chaotic attractor is found to be qualitatively different and more complex than the conventional Lorenz attractor. The results of linear stability analysis suggests that the trajectory of the system moves among the stable and unstable manifolds of a large number of unstable fixed points. This is responsible for the observed complicated dynamics of the system. We define a 'desynchronization parameter', δ and have obtained a scaling relation of δ with a.

In Chapter 6, we have used excitatory-inhibitory networks to study certain problems of early vision, the stage of visual processing at which the primitive features of an image are extracted. Two models have been studied: a two-layer network for segmentation (in particular, object-background discrimination) and a three-layer network for adaptive smoothing and edge detection.

The segmentation network model consists of a layer of excitatory and a layer of inhibitory neurons coupled to each other. On presenting the network with a noisy image, the object and the background portions are found to have different dynamical behavior, enabling segmentation to be done [172].

In the three-layer network model, contrast enhancement, followed by edge detection, is studied. A layered network with a sigmoidal activation function is found to give high contrast when a gray-level image is processed through it. The interaction between excitatory and inhibitory neurons results in a filtering process whereby edges of the image are obtained. This has motivated the designing of a model of retinal processing. The model consists of an input layer of excitatory neurons (analogous to the photoreceptor layer in the retina), followed by a layer of coupled pairs of inhibitory and excitatory neurons (analogous to the horizontal and bipolar cell layers of the retina, respectively). The lateral connections among the inhibitory neurons and the inter-layer connections between all three layers allows the local gradient of the image to be computed, the information being fed back to the input layer. An iterative process is then used to adaptively smooth the image. From the resulting enhanced image, one can obtain the edges, either by employing an additional pair of excitatory-inhibitory neuronal layer, or with a conventional gradient thresholding technique. The proposed model has been implemented on different types of images, and its performance compared with some existing models for image enhancement and edge-extraction (namely, Perona-Malik diffusion method and the Canny operator). Although the model compares favorably with some standard methods of edge detection, its main contribution lies in the area of adaptive smoothing.

A concluding summary with an outlook on further work that can be done extending the aforesaid ideas is presented in Chapter 7.