

**Systems Biology Across Scales:  
A Personal View  
XXVII. Waves in Biology:  
Cardiac Arrhythmia**

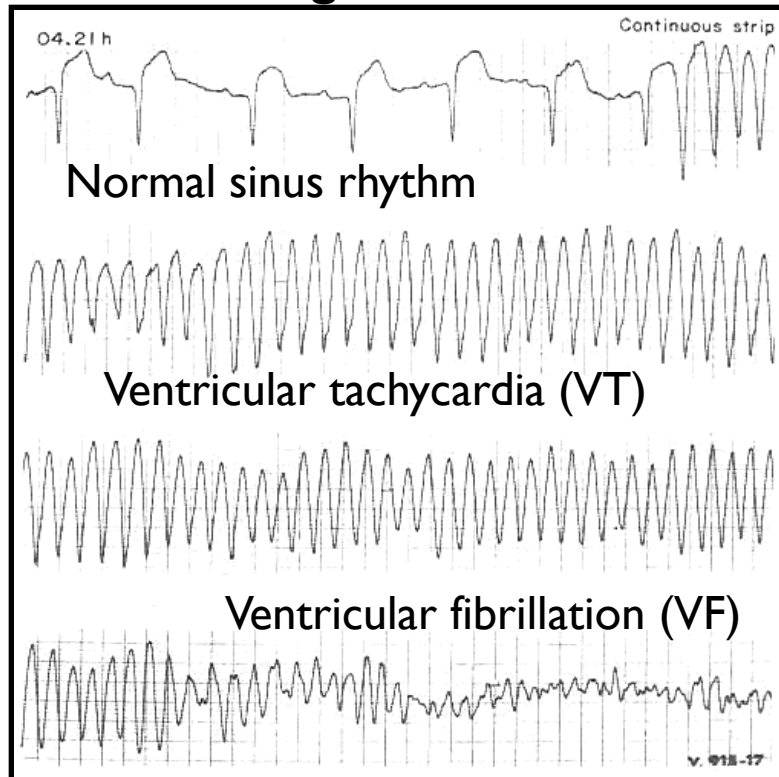
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# The functional importance of biological waves

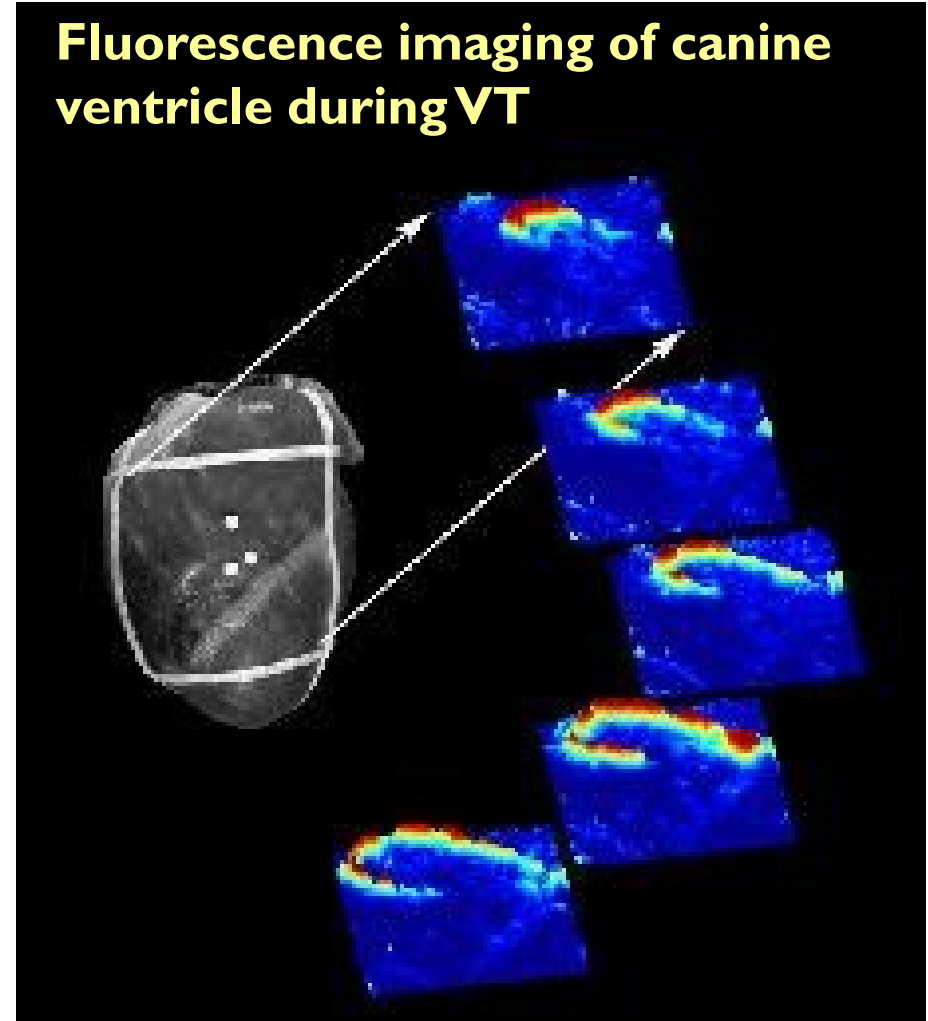
## Spiral Waves $\equiv$ Cardiac Arrhythmias

**Arrhythmias:** disturbances in natural rhythm of heart

### ECG recording



### Fluorescence imaging of canine ventricle during VT



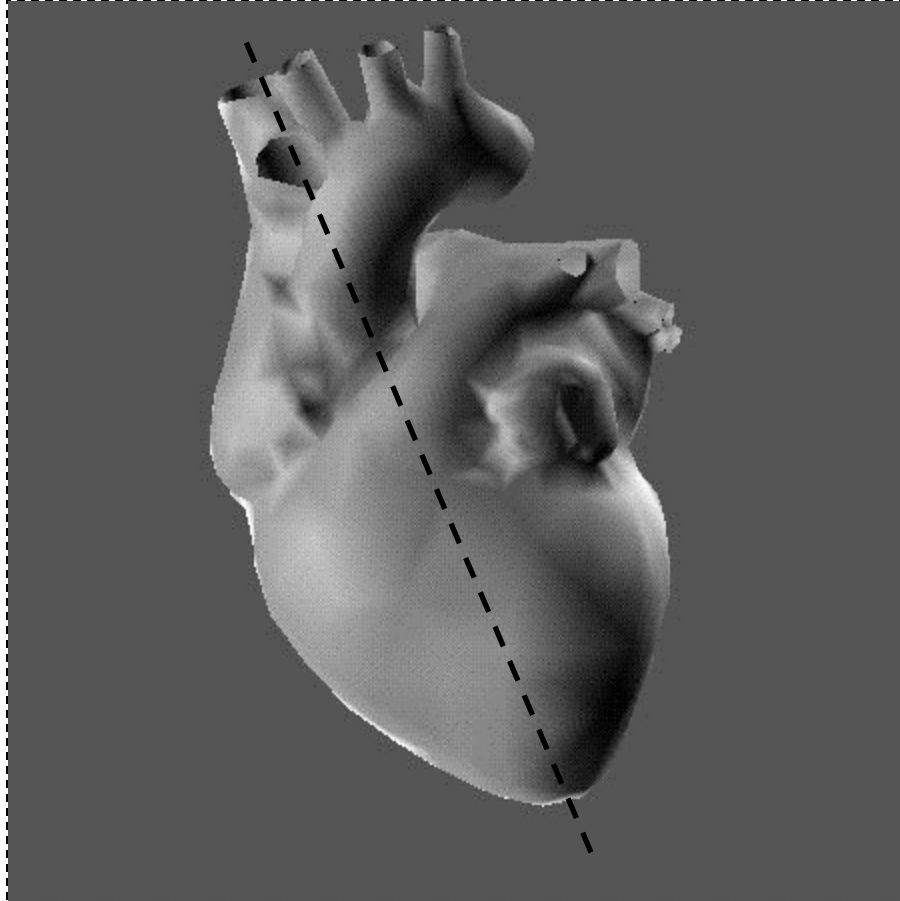
Color proportional to voltage

About half of all cardiac related deaths are due to

# Arrhythmias

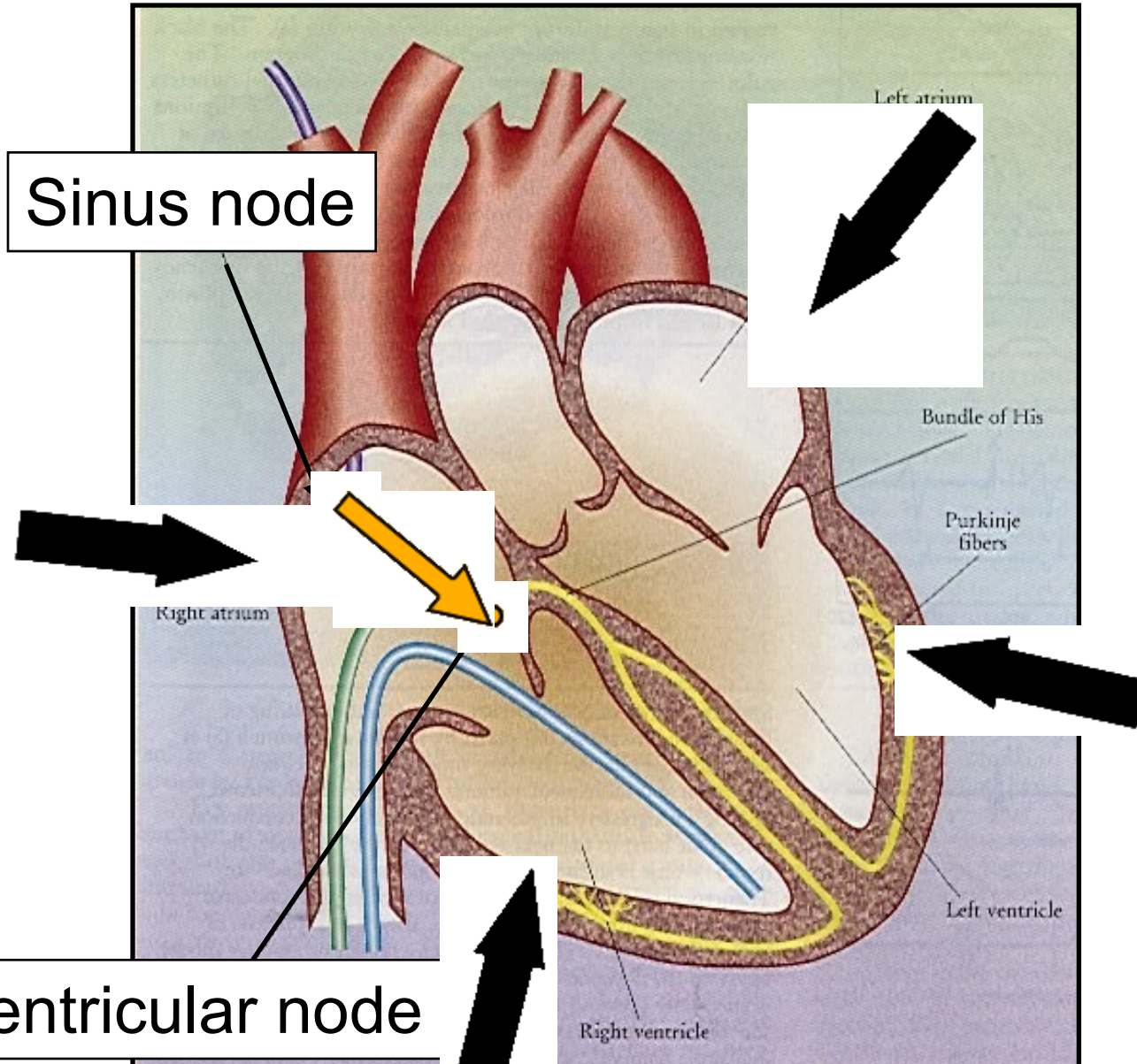
disturbances in the natural rhythm of the heart

# A little anatomy lesson



Take a cross section

# A little anatomy lesson

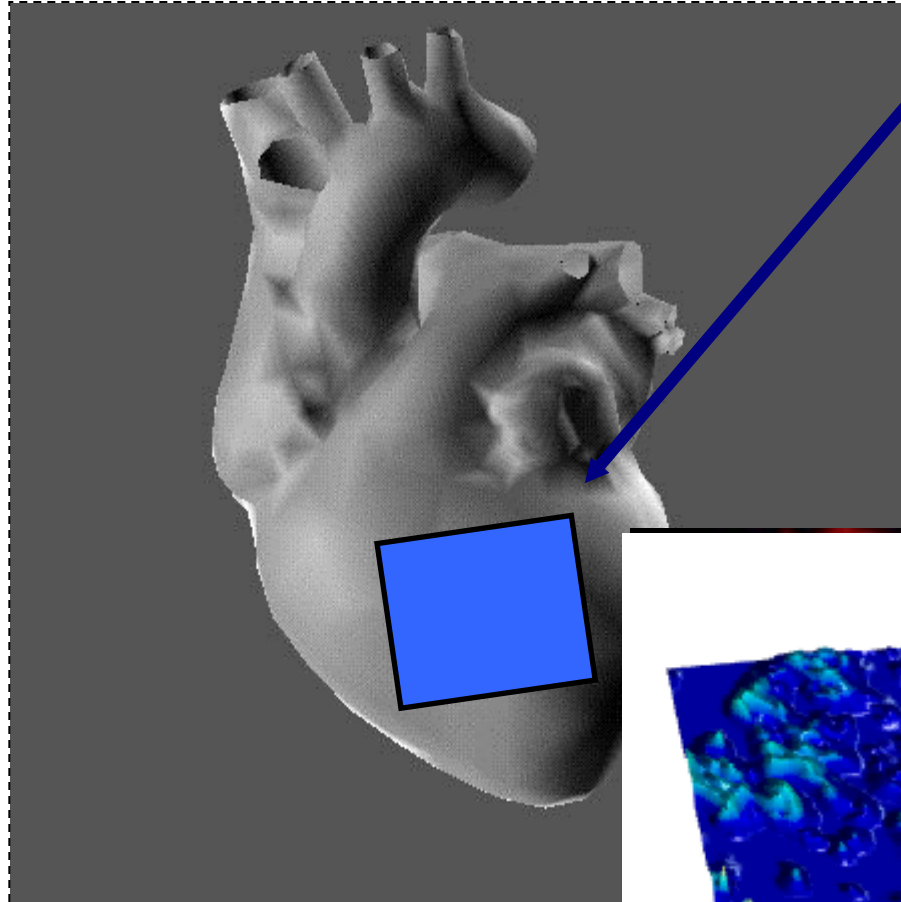


Sinus node

Atrioventricular node

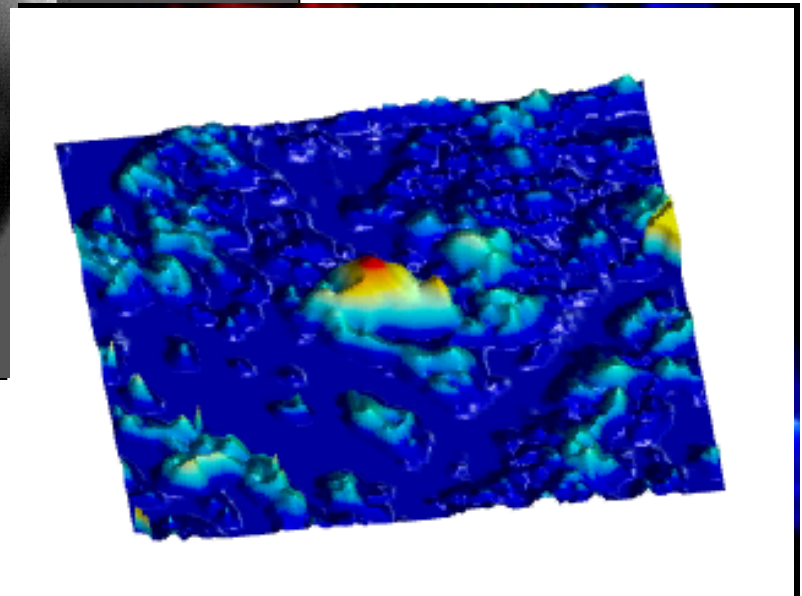
# Ventricular Fibrillation: a deeper look

Inject voltage sensitive dye



Fluorescence  
imaging with  
CCD camera

**VF: Spiral breakup**

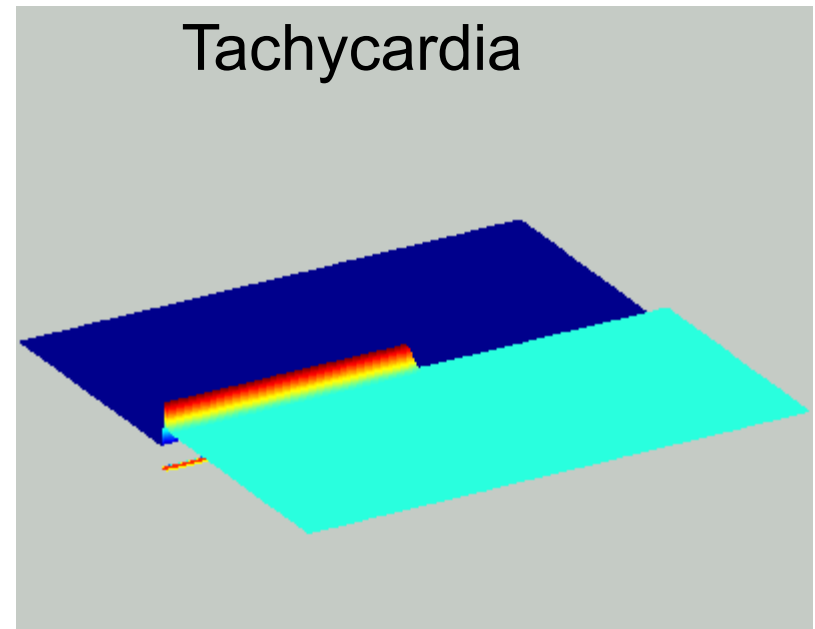
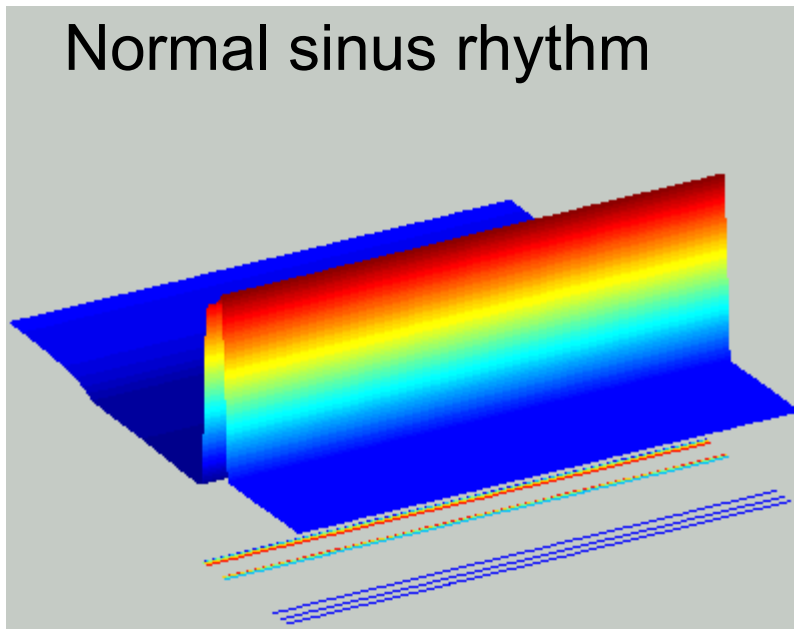


**surface of canine ventricle**

**Color proportional to voltage**

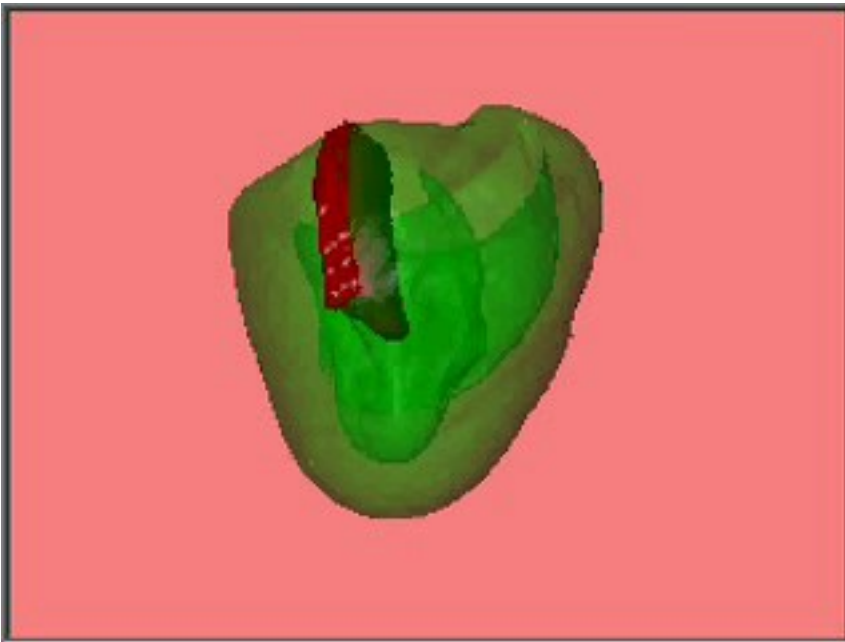
# VT = Spiral /Pinned Rotating Wave

- Underlying cause of VF: formation and subsequent breakup of spiral waves
- Spiral waves: self-sustaining excitations of cardiac tissue
- Leads to tachycardia : abnormally rapid heart beat

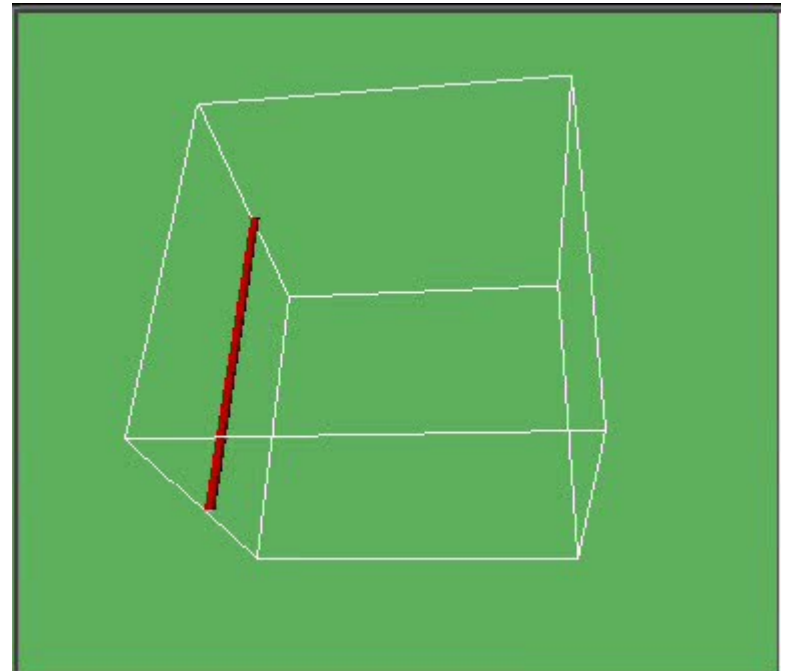


# VF = Spatiotemporal Chaos

- If tachycardia is not terminated, a spiral wave may break up into multiple spiral waves: fibrillation
- Self-sustaining activity: only terminated by external intervention



Movie:AV Panfilov



Movie:AV Panfilov



# How to model excitation in cardiac tissue ?

**For spatially extended systems → reaction diffusion eqn:**

$$\frac{\partial V}{\partial t} = -\frac{I_{ion}}{C_m} + D\Delta V$$

*Intracellular communication via gap junctions*

$V$	: transmembrane potential
$I_{ion}$	: ionic currents
$C_m$	: membrane capacitance
$D$	: diffusion coefficient

Depending on level of biological realism required, different models for ionic currents, e.g.:

- Panfilov Model (2 variables) – phenomenological
- Luo-Rudy I Model (8 variables) – based on Hodgkin-Huxley

# How to model excitation in cardiac tissue?

The simplest model that shows spiral wave breakup

## ***Panfilov Model***

$$\dot{e} = D\Delta e - f(e) - g$$

$$\dot{g} = \varepsilon(e, g) \cdot (k \cdot e - g)$$

$$f(e) = \begin{cases} C_1 & e < e_1 \\ -C_2 \cdot e + a & e_1 \leq e \leq e_2 \\ C_3(e - 1) & e > e_2 \end{cases}$$

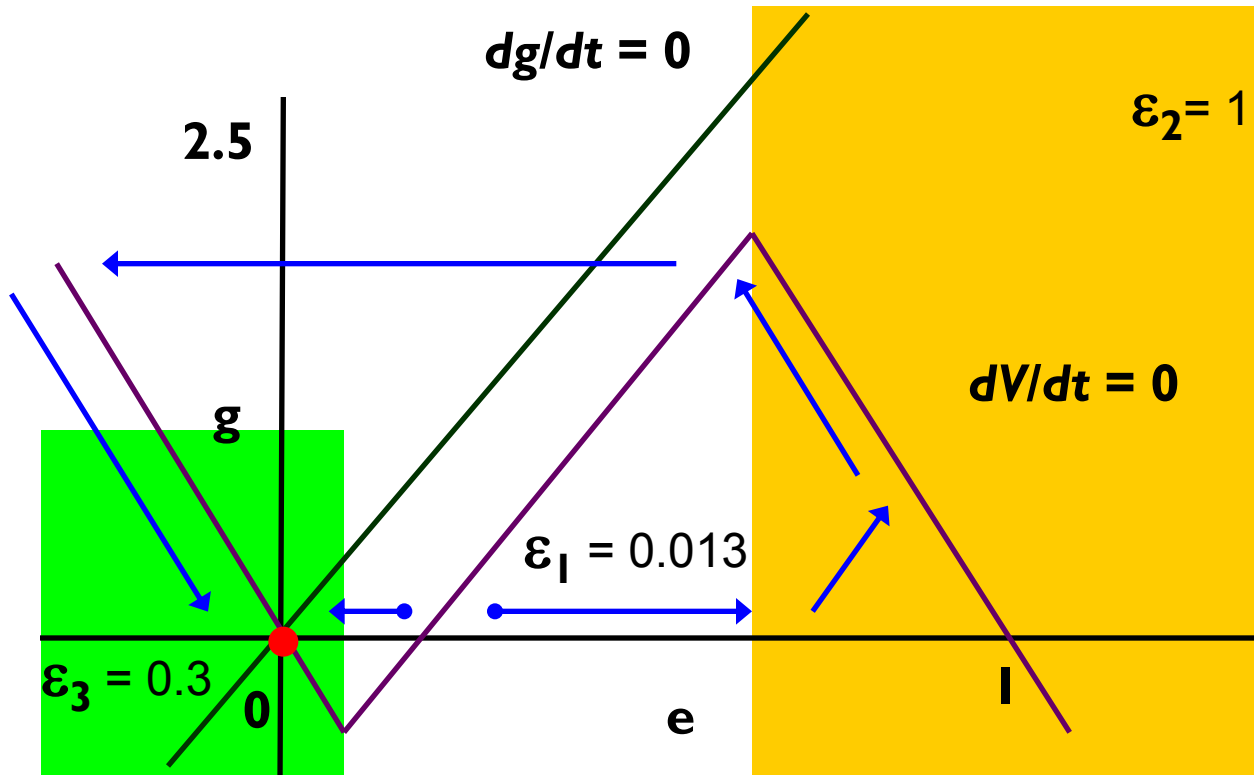
$$\varepsilon(e) = \begin{cases} \varepsilon_1 & e < e_2 \\ \varepsilon_2 & e \geq e_2 \\ \varepsilon_3 & e \leq e_1, g < g_1 \end{cases}$$

*A piecewise linear variation of Fitzhugh-Nagumo model*

Other more complicated non-ionic models with increasing realism

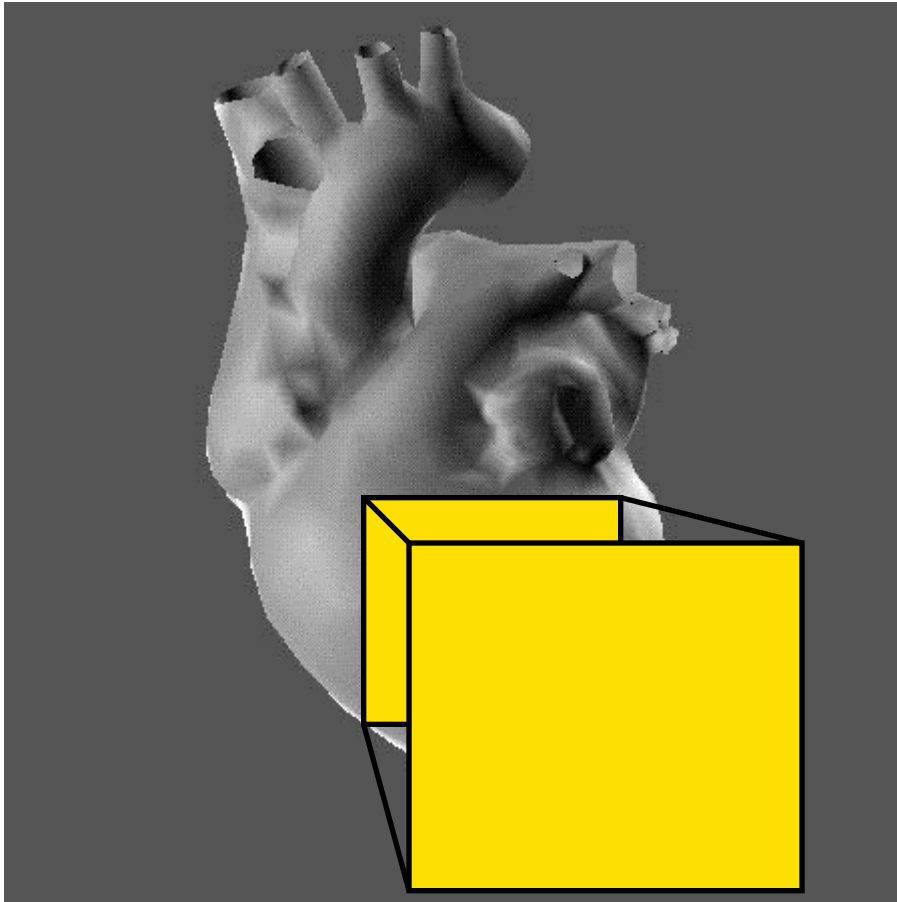
- Karma Model (2 variables) & Fenton-Karma model (3 variables)
- Aliev-Panfilov model (2 variables)

# Dynamics of the Panfilov Model



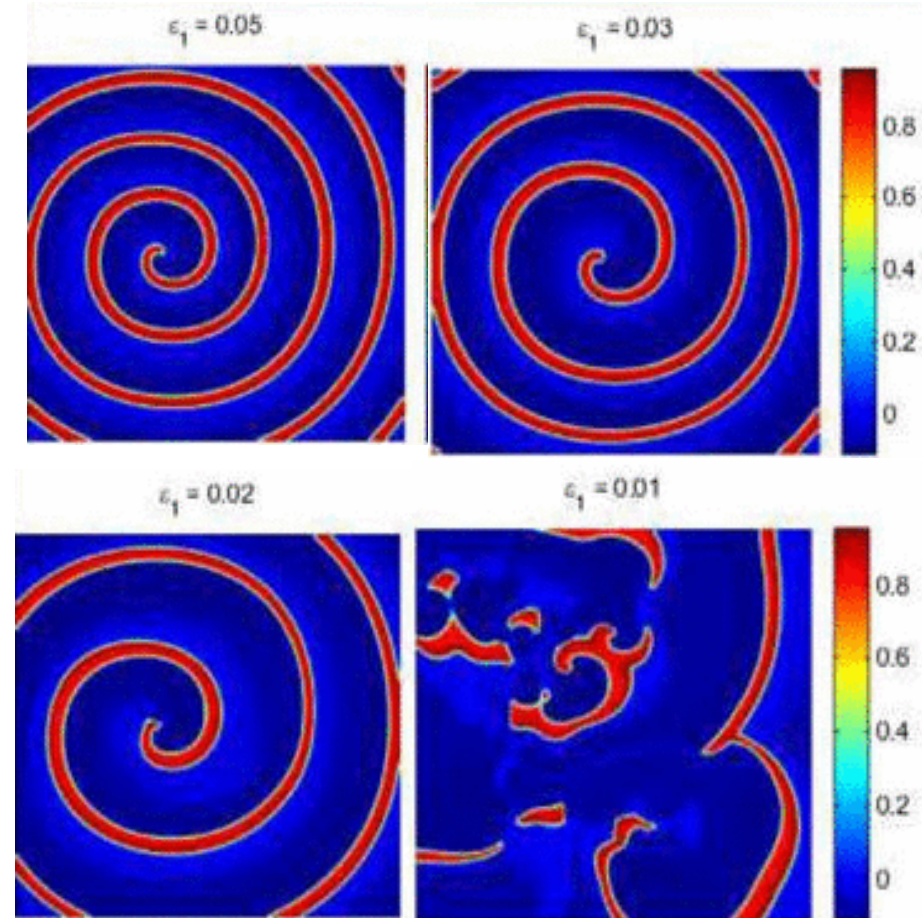
- Quiescent state:  $V = 0, g = 0$  (only stable fixed point)
- Stimulation below threshold: decays to quiescent state
- Stimulation above threshold : action potential

# Spiral Turbulence in Panfilov model



As  $\varepsilon_1$  decreases, the pitch of the spiral decreases ... ultimately leading to spiral breakup.

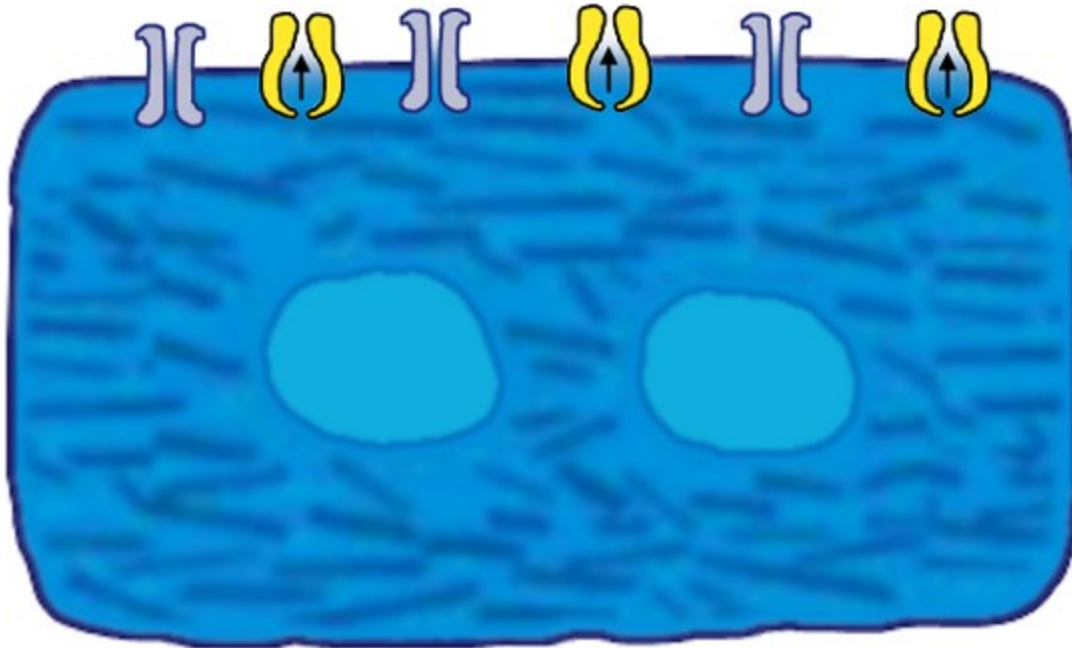
Pseudo-color plots of  $V$  at various values of  $\varepsilon_1$  ( $\varepsilon_3 = 0.3$ )



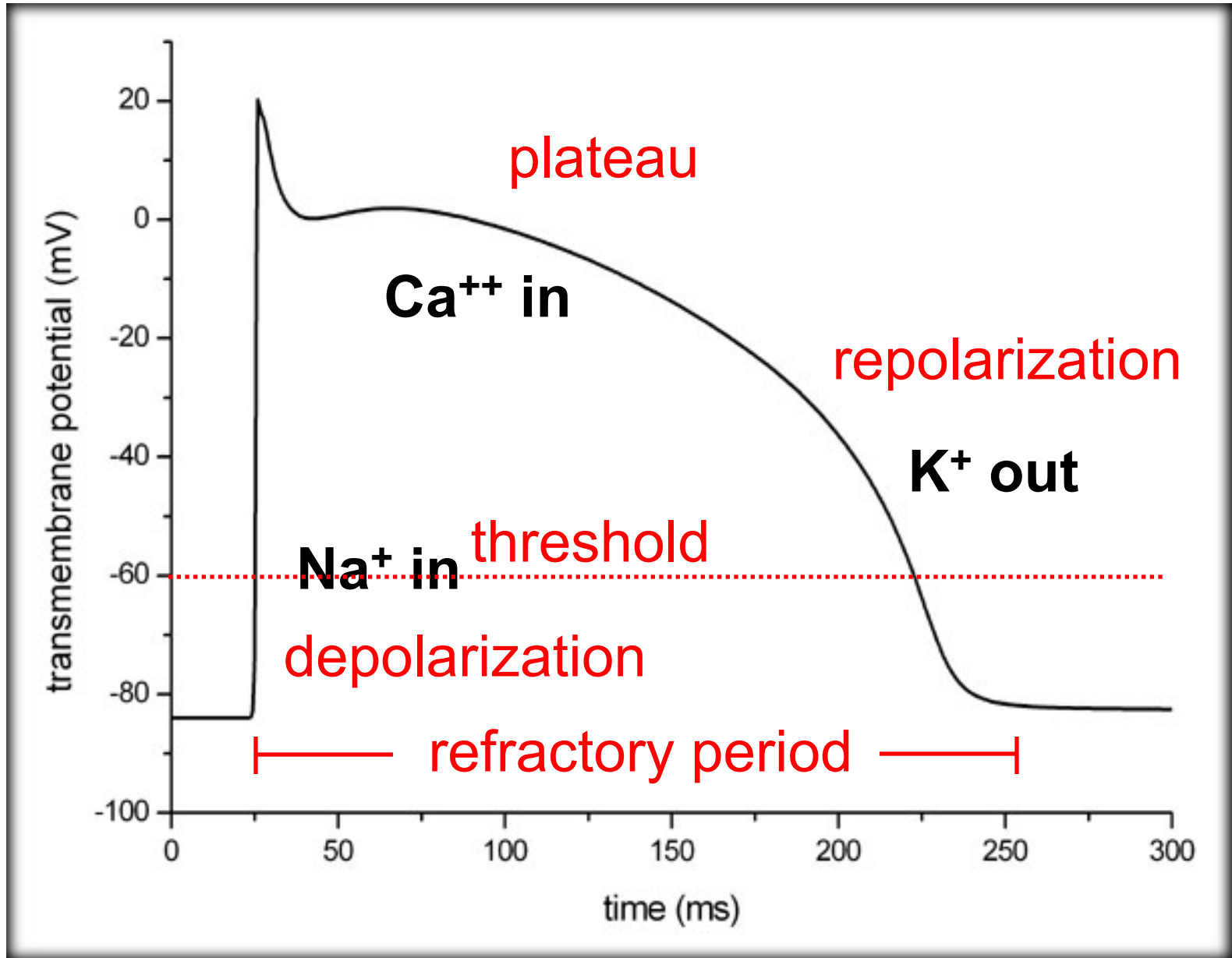
# The single cardiac cell

*ion pumps*

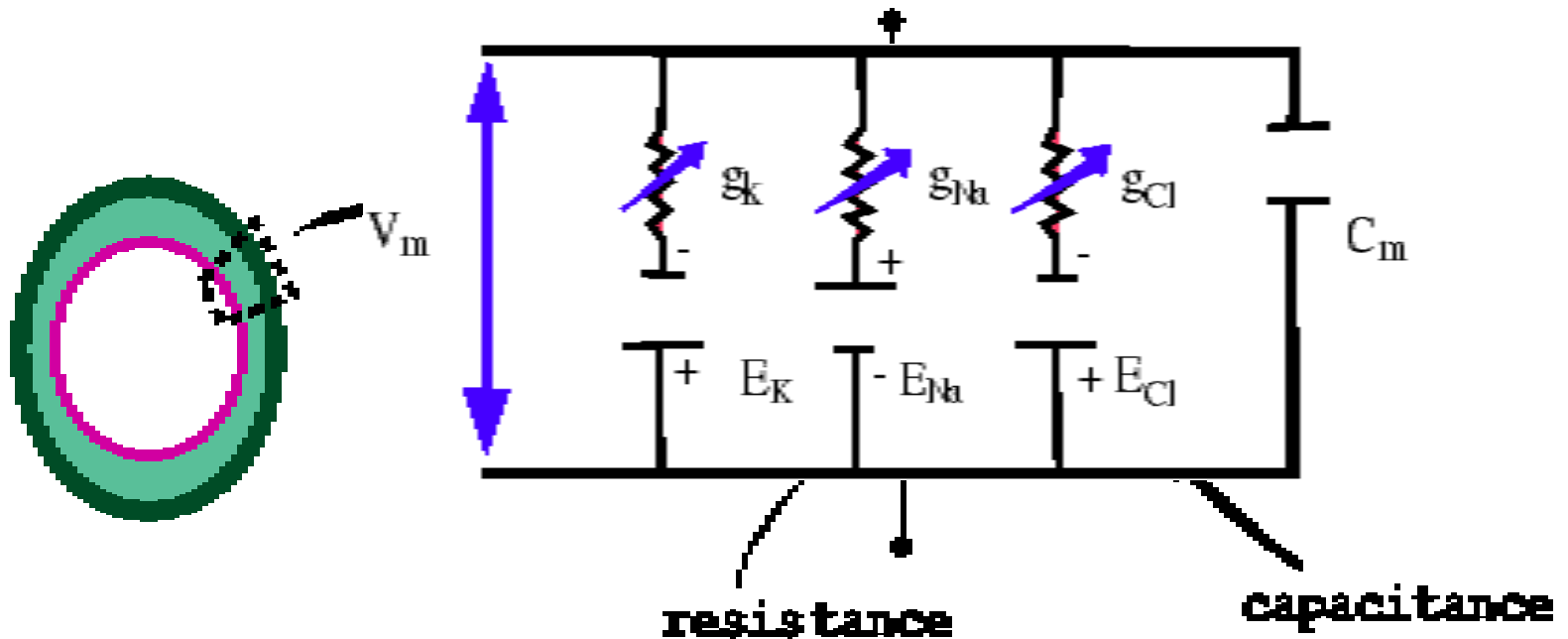
ion transport against the  
*concentration gradient*



# Excitation of a cardiac cell: The Action Potential



The ionically detailed models are based on the Hodgkin-Huxley formalism



$$C_m \frac{dV}{dt} = -g_{Na}(V - V_{Na}) - g_K(V - V_K) - g_r(V - V_r) + I_{app}$$

# How to model excitation in cardiac tissue?

## Luo-Rudy I model (1991)

$$\frac{\partial V}{\partial t} = -\frac{I_{ion}}{C_m} + D\Delta V$$

The LR-I ionic current term:

$$I_{ion} = I_{Na} + I_{si} + I_K + I_{Kl} + I_{Kp} + I_b$$

### **Inward Currents:**

Fast inward sodium current :  $I_{Na} = g_{Na} m^3 h j (V - E_{Na})$

Slow inward calcium current :  $I_{si} = g_{si} d f (V - E_{si})$ ,

*Intra-cellular calcium enters the scene:*  $E_{si} = 7.7 - 13.03 \ln(Ca)$

Calcium density evolves as

$$\frac{dCa}{dt} = -10^{-4} I_{si} + 0.07(10^{-4} - Ca)$$

### **Outward Currents:**

Time-dep outward potassium current :  $I_K = g_K X X_i (V - E_K)$

Time-indep outward potassium current :  $I_{Kl} = g_{Kl} K_{l\infty} (V - E_{Kl})$

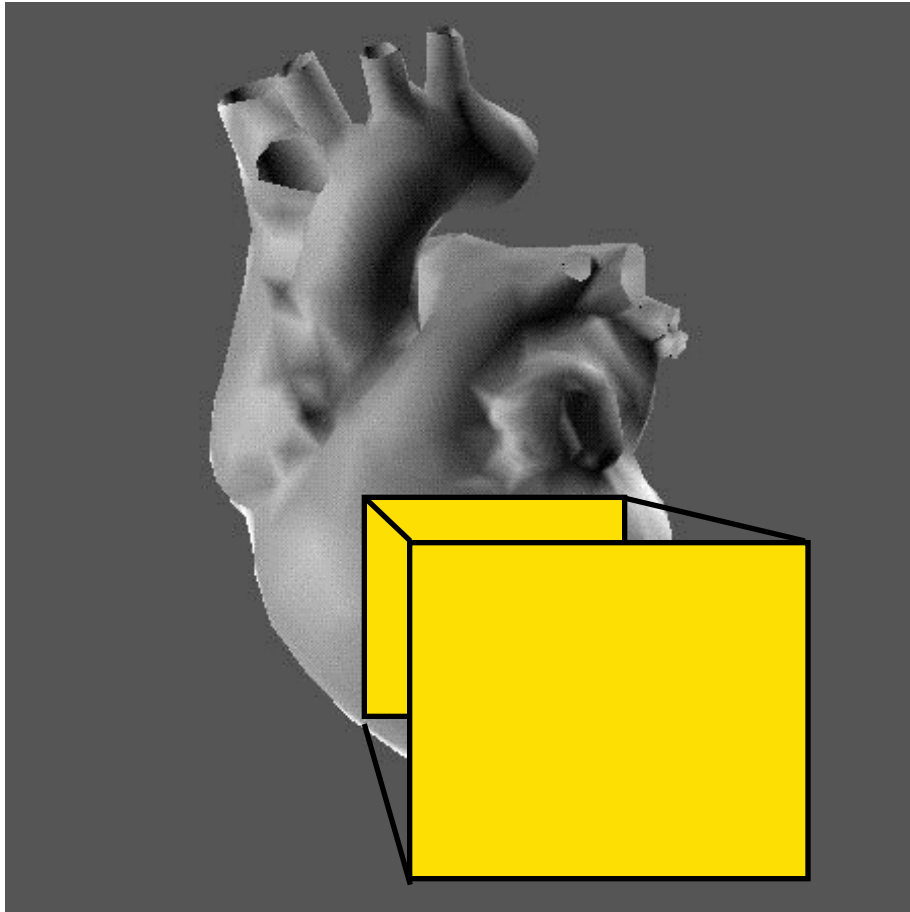
Plateau outward potassium current :  $I_{Kp} = g_{Kp} K_p (V - E_{Kp})$

Background current :  $I_b = g_b (V - E_b)$

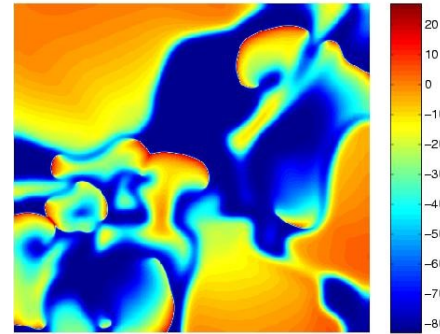


90 mm x 90 mm

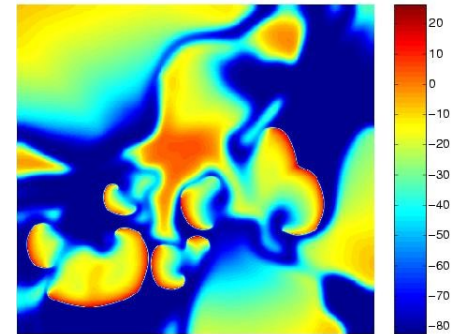
# Spiral Chaos in the Luo-Rudy model



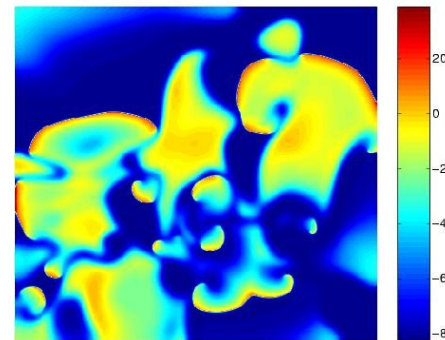
T = 30 ms



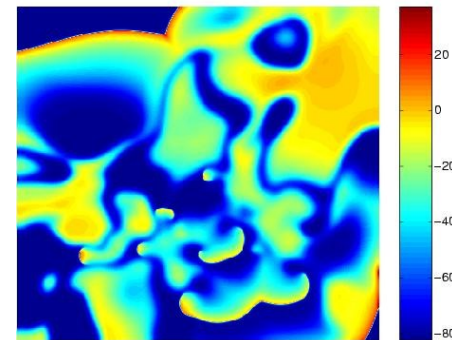
T = 90 ms



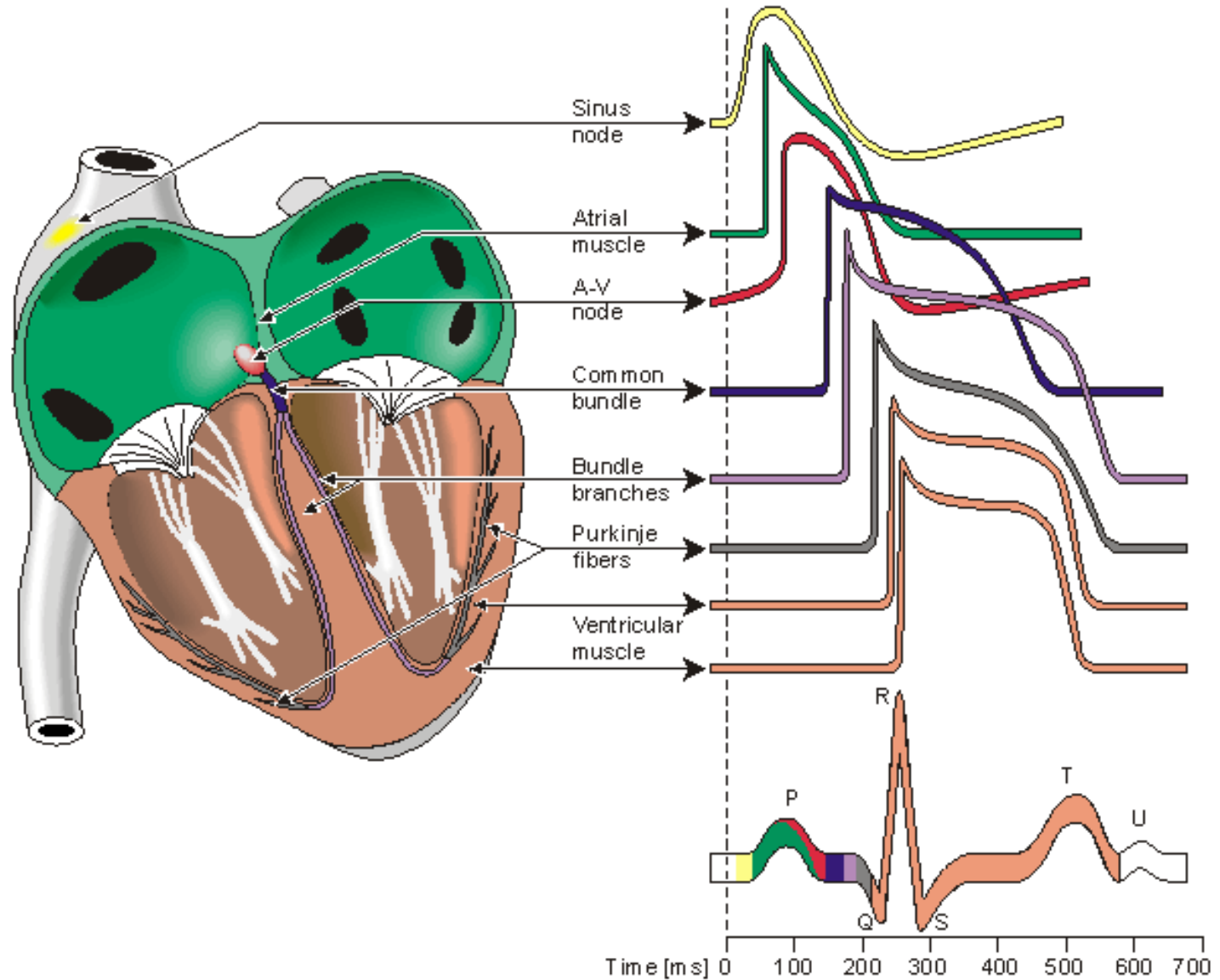
T = 150 ms



T = 210 ms



# Cardiac action potential and ECG



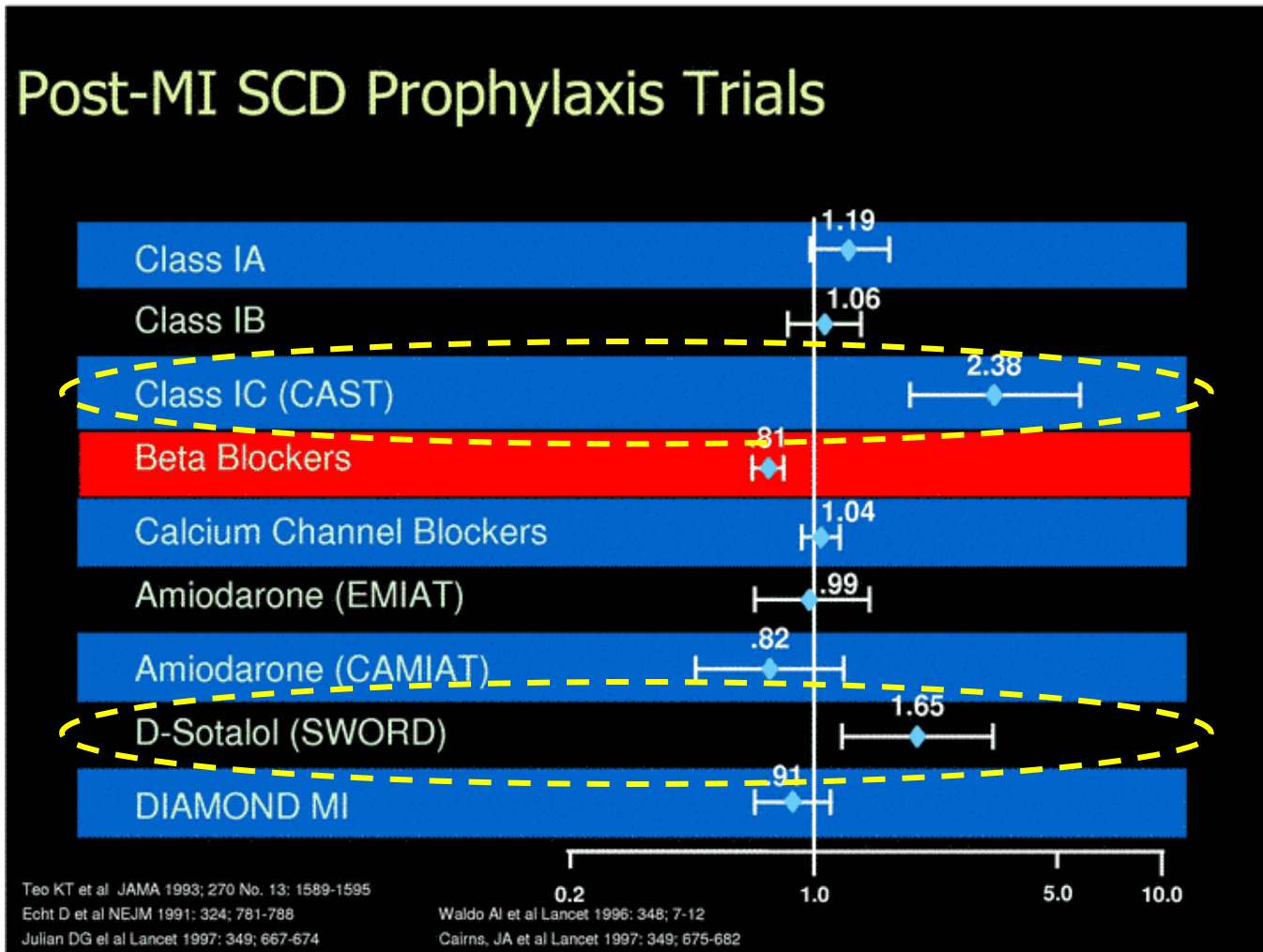
**Are there any remedies against cardiac arrhythmia ?**

# Classes of Anti-arrhythmic drugs

Singh Vaughan Williams classification (1970)

- **Class I** agents interfere with the  $\text{Na}^+$  channel.
- **Class II** agents are anti-sympathetic nervous system agents, mostly beta blockers
- **Class III** agents affect  $\text{K}^+$  efflux.
- **Class IV** agents affect  $\text{Ca}^+$  channels and the AV node.
- **Class V** agents work by other or unknown mechanisms.

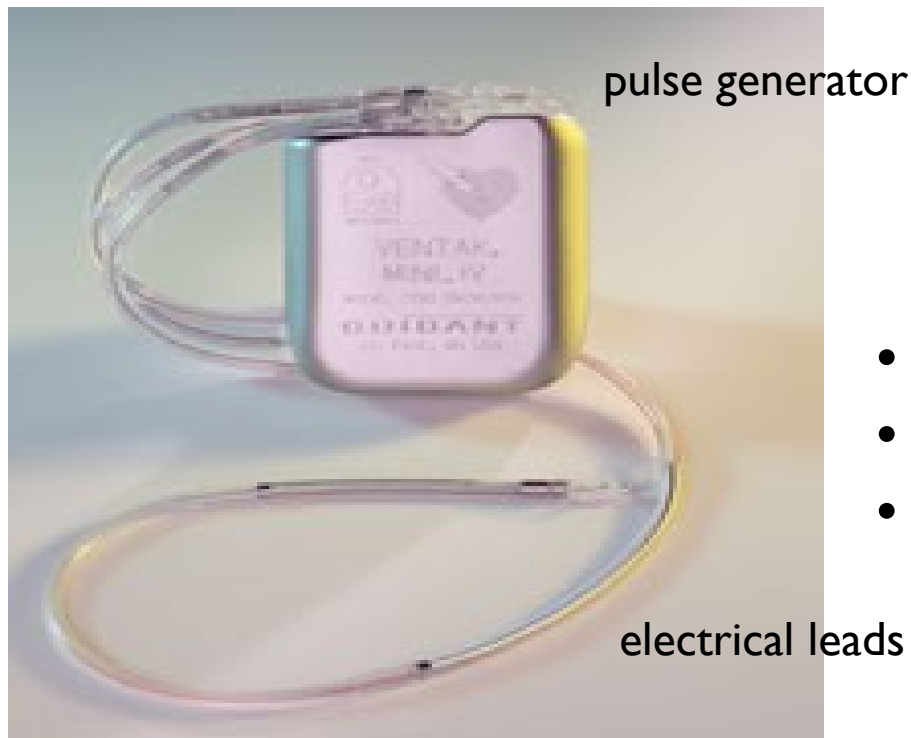
# Problem with the Pharmaceutical Approach



Drugs developed to prevent cardiac arrest killed even more people !

# Electrical therapy with ICDs

## Implantable Cardioverter-Defibrillator



- constantly monitors heart rhythm.
- detects arrhythmia.
- delivers *programmed* treatment.

### Variety of possible treatments:

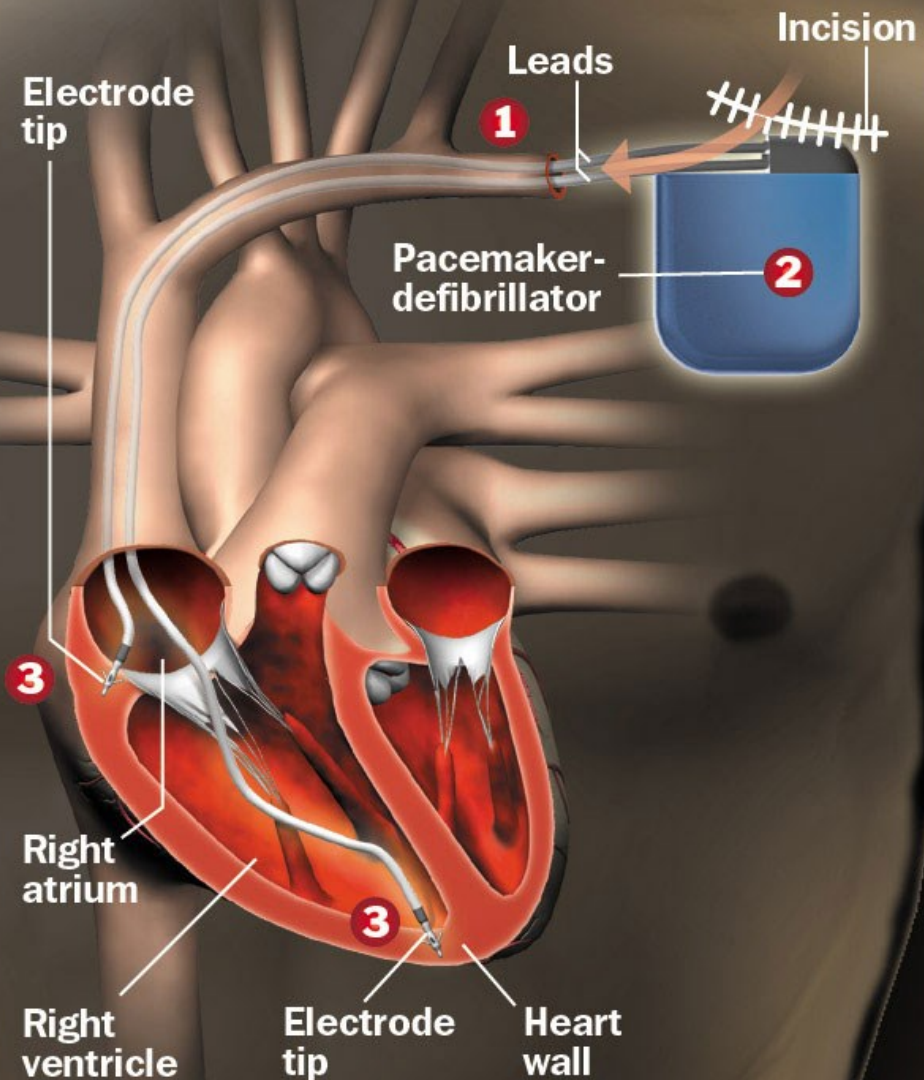
- Pacing: deliver a sequence of low-amplitude pulses.
- Cardioversion: a mild shock (if pacing fails in terminating VT).
- Defibrillation: large shock to terminate VF.

## ... And How It Got There

**1** Doctors made a small incision near the collar bone and threaded two thin wire leads through a vein into the heart

**2** The pacemaker-defibrillator was implanted under the skin, connected to the leads and tested

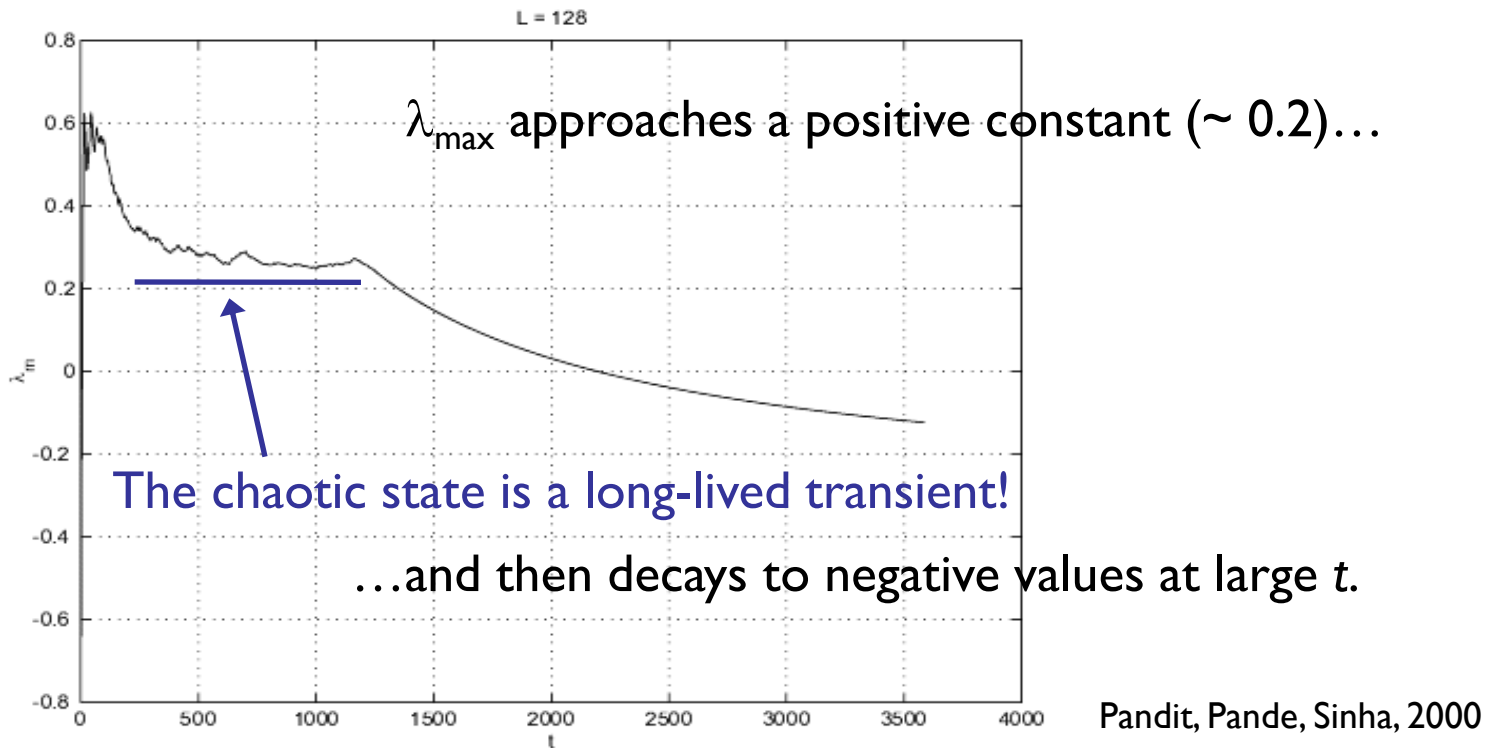
**3** Electrodes embedded in the heart muscle will monitor the heart-beat and correct any irregularity with either small pulses or a shock of electricity





# The transience of patterns

The largest Lyapunov exponent ( $\lambda_{\max}$ ) measures the degree of chaotic activity as a function of time  $t$



But so what ? Why care ?

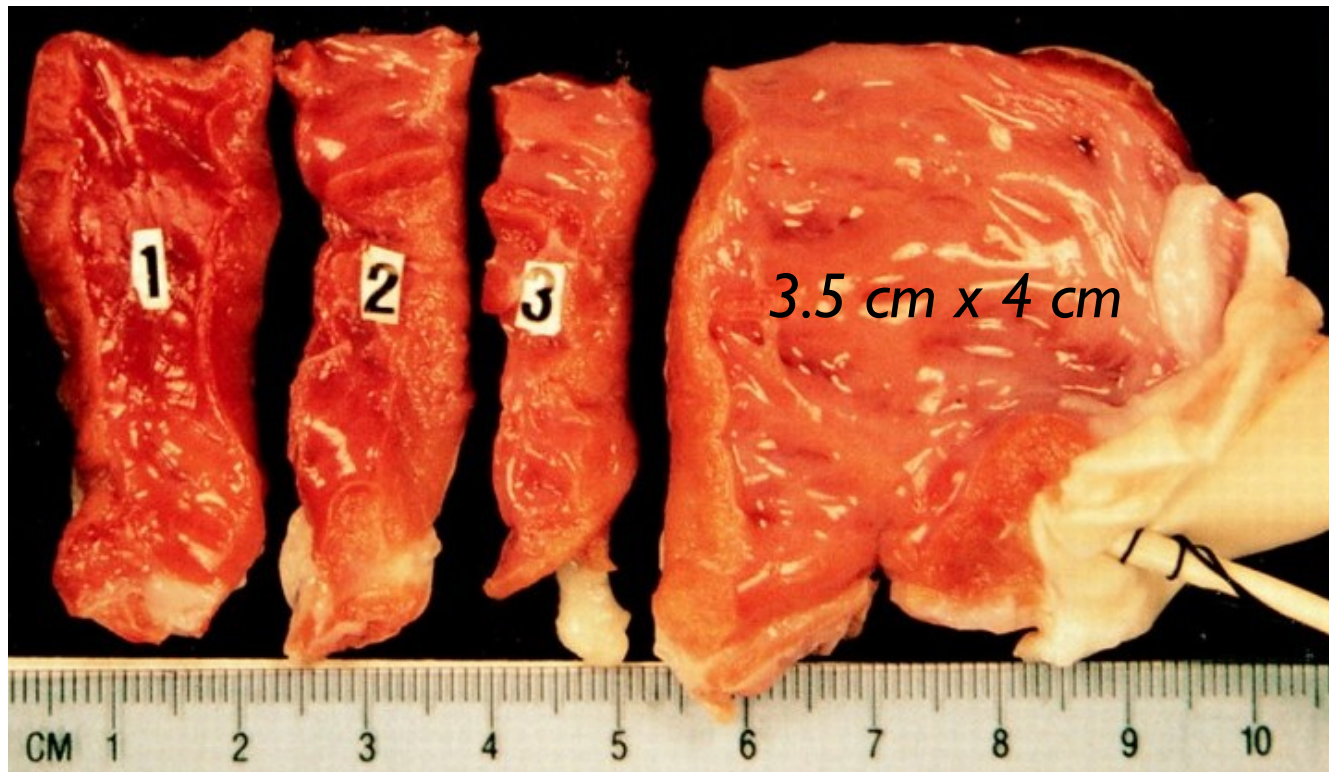
The lifetime of the chaotic transient increases with size  $L$ .



## Size does matter!

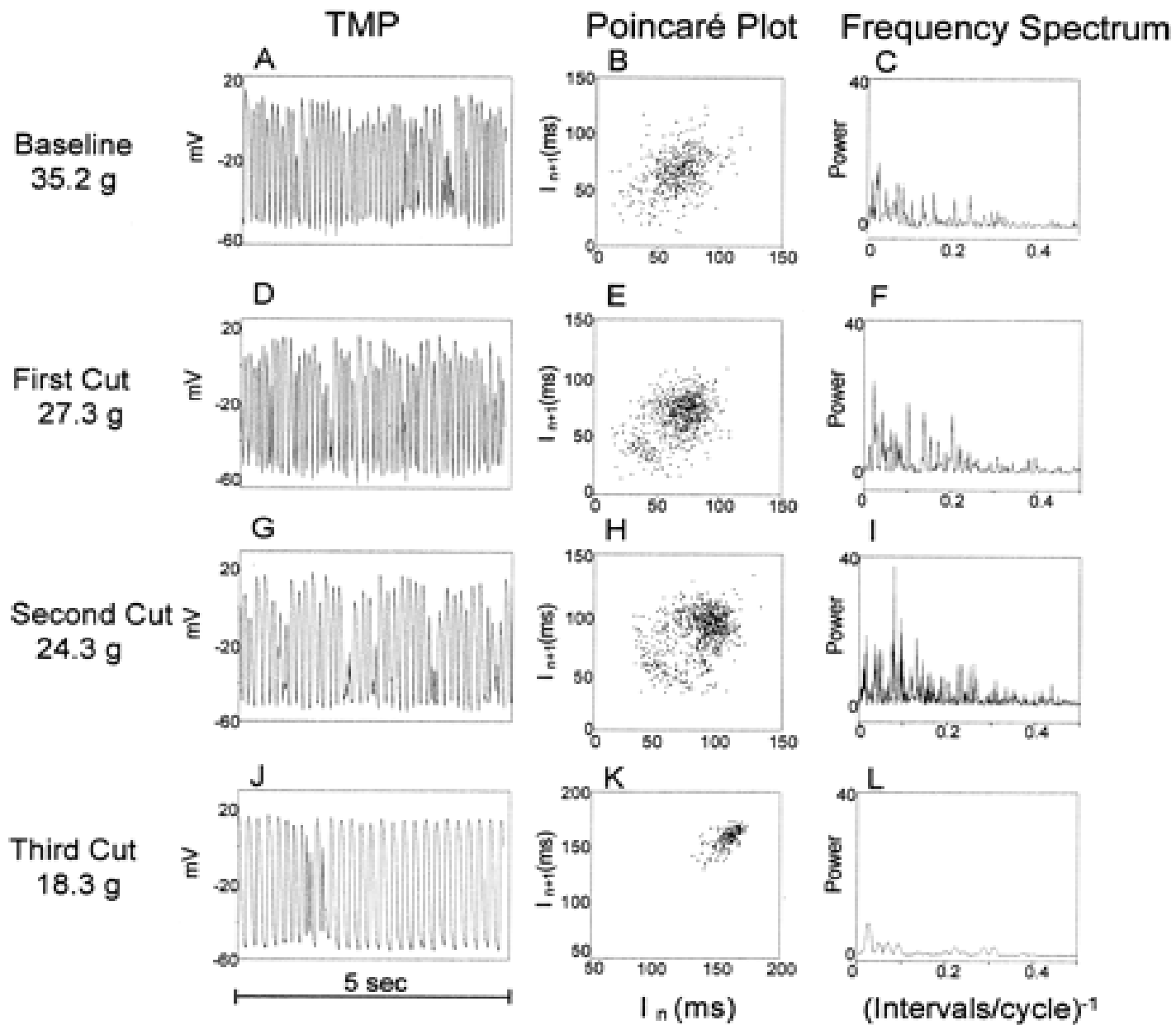
Decreasing the size of heart drastically reduces the duration of the chaotic transient.

*Expts:* Hearts of smaller mammals less likely to fibrillate.



Y-H Kim et al, *J Clin Invest* 100 (1997) 2486

Tissue mass reduction of swine ventricle by sequential cutting



Y-H Kim et al, *J Clin Invest* 100 (1997) 2486

Transition from VF to periodicity with reduction of heart tissue

**But...**

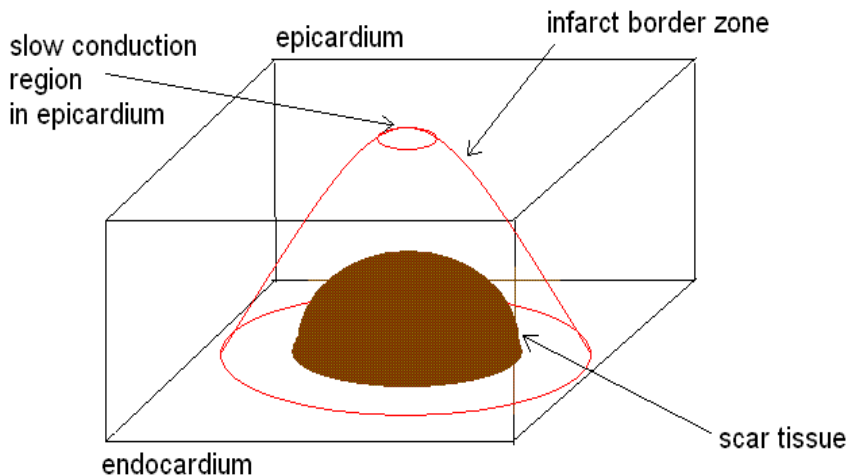
**Life gets even more complicated**

# Enter disorder (inhomogeneity)

## **Example:**

Cardiac tissue damaged by myocardial infarction (heart attack)

*Heterogeneities:* scar tissue through cell death due to lack of oxygenated blood (structural disorder)



Normal (healthy) tissue: *excitable*

Scar tissue: *Inexcitable*

Recovered tissue: *partially excitable*

In theoretical models, heterogeneity in

- diffusion coefficients (conductivity)
- excitation parameters