

**Systems Biology Across Scales:  
A Personal View  
XXII. Building elements for  
Biological circuits**

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# Elements of biological circuits

Figures and text from  
Tyson, Chen & Novak, “Sniffers, buzzers, toggles  
and blinkers,” *Curr. Opin. Cell Biol.* **15**:221  
(2003).

A molecular network looks strikingly similar to the wiring diagram of a modern electronic gadget. Instead of resistors, capacitors and transistors hooked together by wires, one sees genes, proteins and metabolites hooked together by chemical reactions and intermolecular interactions.

- Complex molecular networks, like electrical circuits, seem to be constructed from simpler modules: sets of interacting genes and proteins that carry out specific tasks and can be hooked together by standard linkages.
- Simple signalling pathways can be embedded in networks using positive and negative feedback to generate more complex behaviours — toggle switches and oscillators — which are the basic building blocks of the exotic, dynamic behaviour shown by nonlinear control systems.

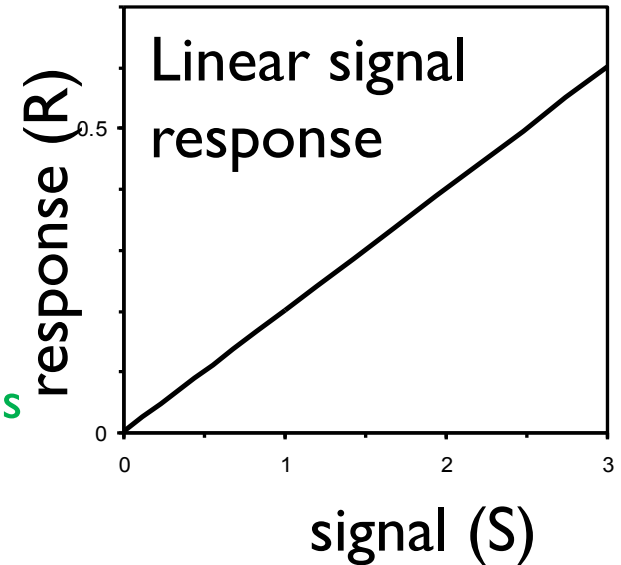
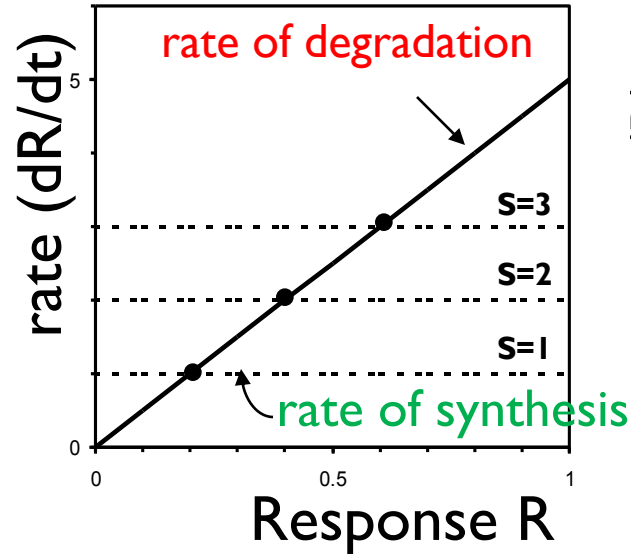
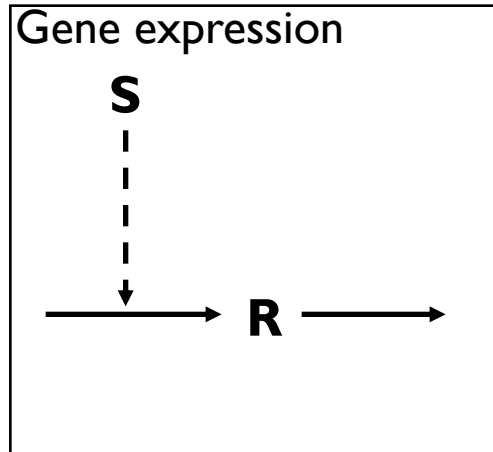
# Simple modules for building complex dynamic networks

- Linear and hyperbolic signal response : graded & reversible
- Sigmoidal response: reversible but abrupt (“buzzer”)
- Perfectly adapted response: transient response (“sniffer”)
- Positive feedback : discontinuous switch
  - hysteresis
  - mutual activation (“one way” switch)
  - mutual inhibition (“toggle” switch)
- Negative feedback
  - homeostasis
  - oscillations (“blinker”)

# Protein synthesis and degradation

S = signal concentration (e.g., concentration of mRNA)

R = response magnitude (e.g., concentration of protein)



$$\frac{dR}{dt} = k_0 + k_1 S - k_2 R,$$

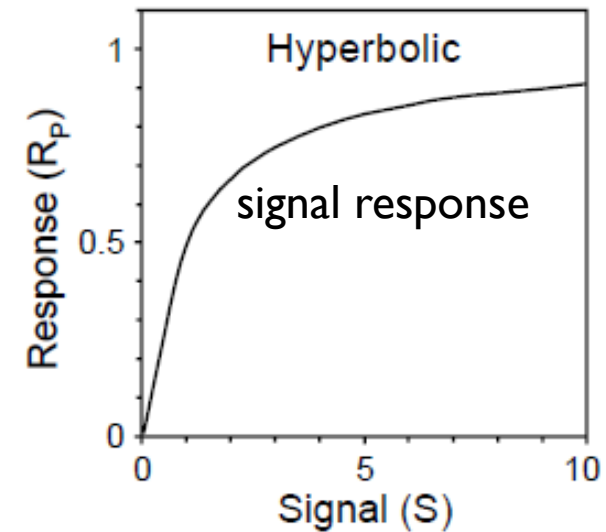
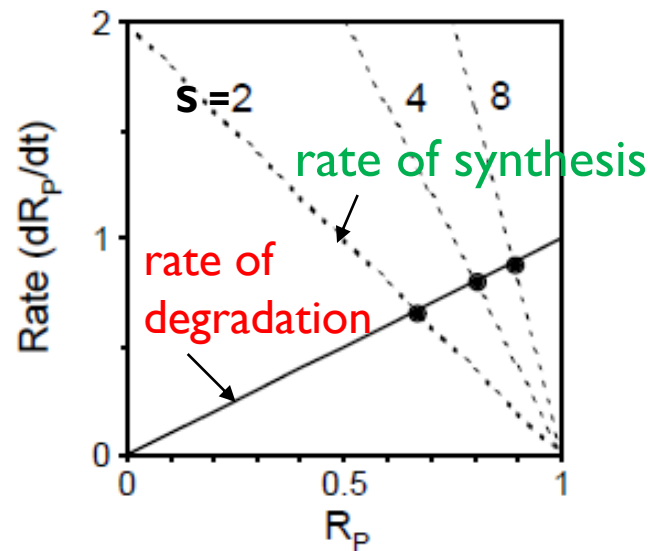
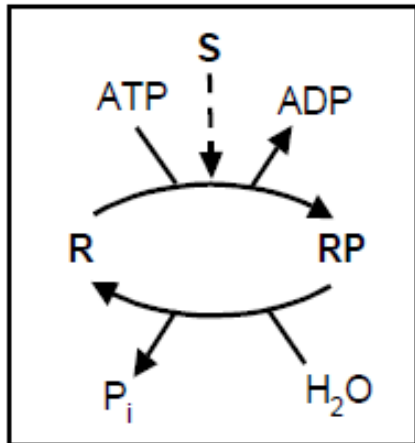
Steady-state solution  $R_{ss} = \frac{k_0 + k_1 S}{k_2}$

# Protein Phosphorylation/Dephosphorylation

R = unphosphorylated protein

R<sub>p</sub> = phosphorylated protein

R<sub>T</sub> = R + R<sub>p</sub> = total protein concentration



$$\frac{dR_P}{dt} = k_1 S (R_T - R_P) - k_2 R_P$$

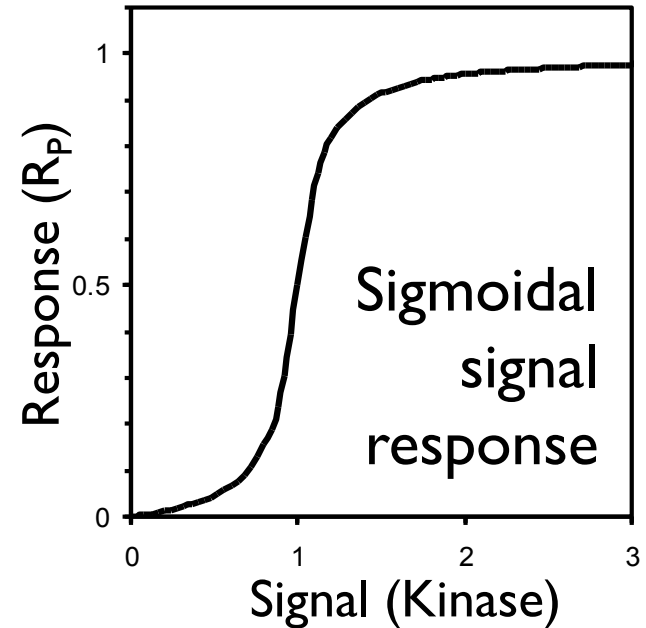
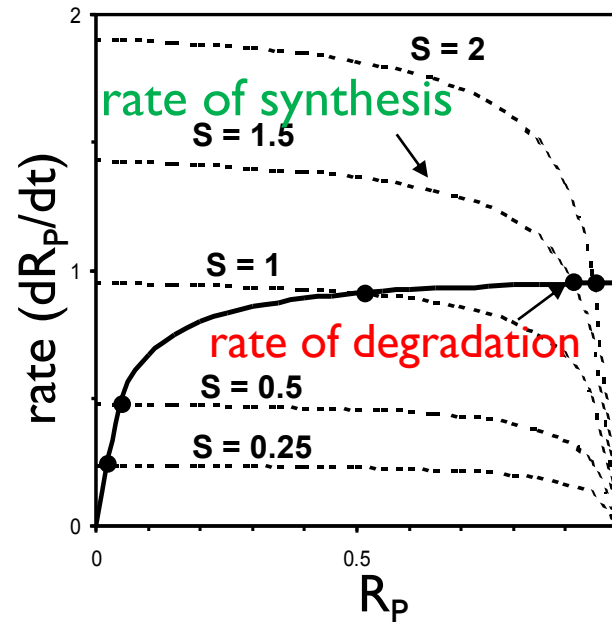
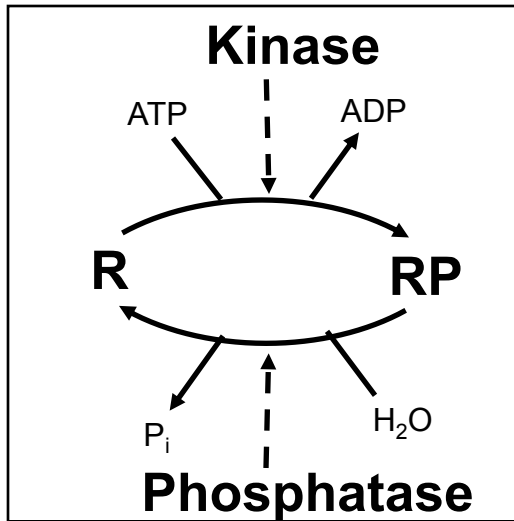
Steady-state solution  $R_{P,ss} = \frac{R_T S}{(k_2/k_1) + S}$

# Protein Phosphorylation/Dephosphorylation

## Michelis-Menten kinetics

Switch-like response or zero-order ultrasensitivity (“buzzer”)

Goldbeter & Koshland, PNAS 78, 6840 (1981)



$$\frac{dR_P}{dt} = \frac{k_1 S (R_T - R_P)}{K_{m1} + R_T - R_P} - \frac{k_2 R_P}{k_{m2} + R_P}$$

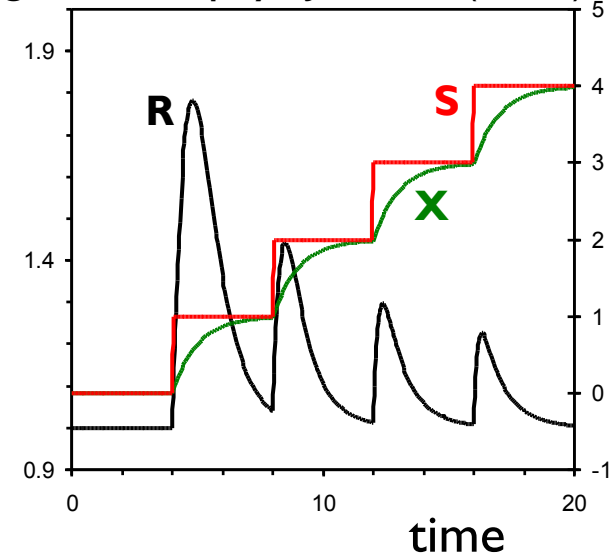
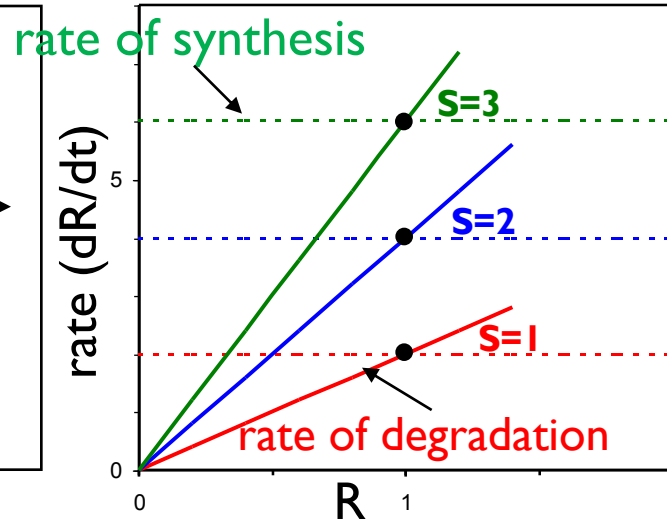
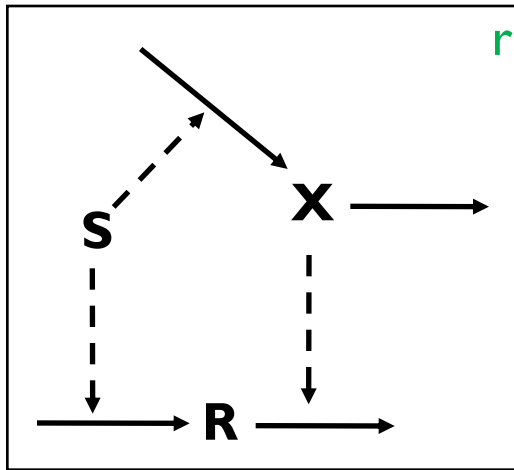
Steady-state concn  
is a solution of

$$k_1 S (R_T - R_P) (K_{m2} + R_P) = k_2 R_P (K_{m1} + R_T - R_P)$$

# Perfectly adapted signal response (“sniffer”)

by supplementing the simple linear response element with a second signaling pathway (via species X)

Levchenko & Iglesias, Biophys J 82, 50 (2002)



*Perfect adaptation:* Although the signaling pathway exhibits a transient response to changes in signal strength, its steady-state response

$$\frac{dR}{dt} = k_1 S - k_2 X \cdot R \quad R_{ss} = \frac{k_1 k_4}{k_2 k_3}$$

$$\frac{dX}{dt} = k_3 S - k_4 X \quad X_{ss} = \frac{k_3 S}{k_4}$$

$R_{ss}$  is independent of S

Typical of chemotactic systems responding to abrupt changes but insensitive to a constant signal

# Feedback

some component of a response pathway may feed back on the signal.

Feedback can be

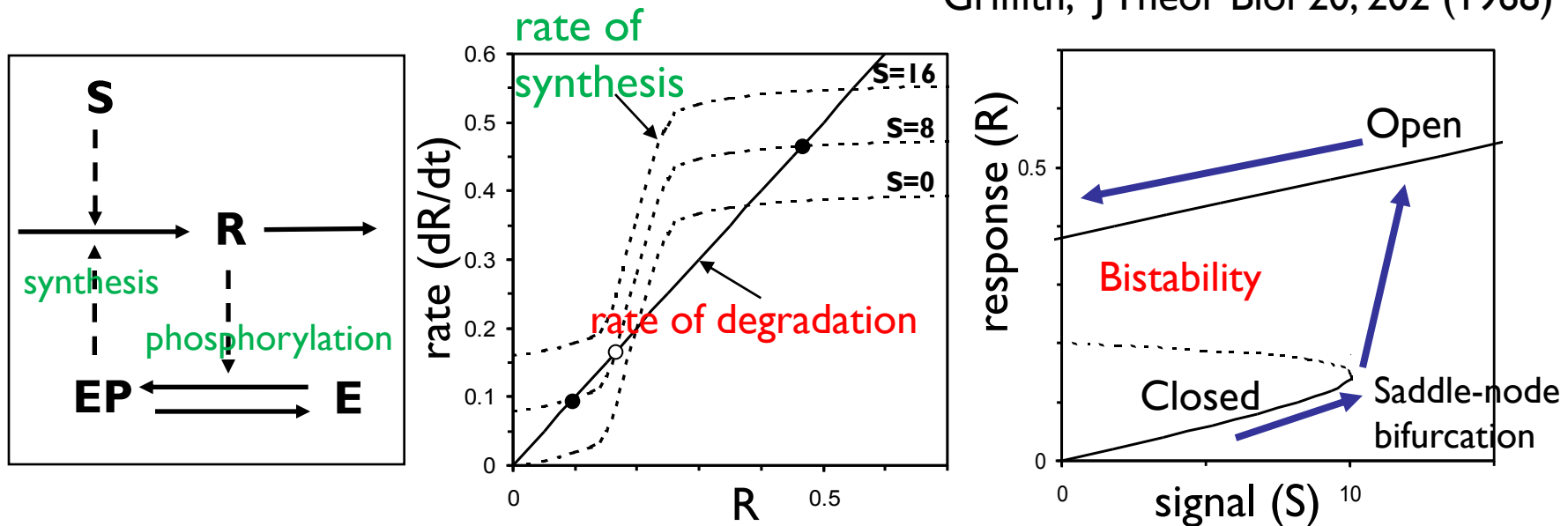
- positive,
- negative or
- mixed



# Positive Feedback: Mutual activation

- R activates protein E by phosphorylation
- EP enhances the synthesis of R

Griffith, J Theor Biol 20, 202 (1968)



Discontinuous switch: cellular response changes abruptly and irreversibly as signal magnitude crosses a critical value.

$$\frac{dR}{dt} = k_0 E_P(R) + k_1 S - k_2 X \cdot R$$

$$E_P(R) = G(k_3 R, k_4, J_3, J_4)$$

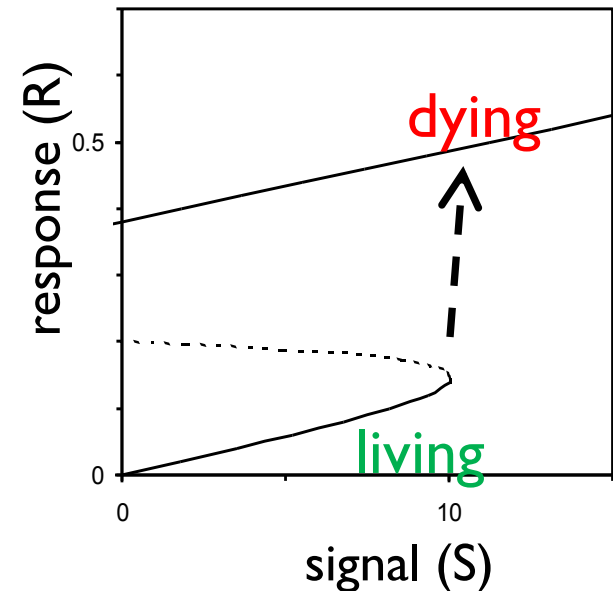
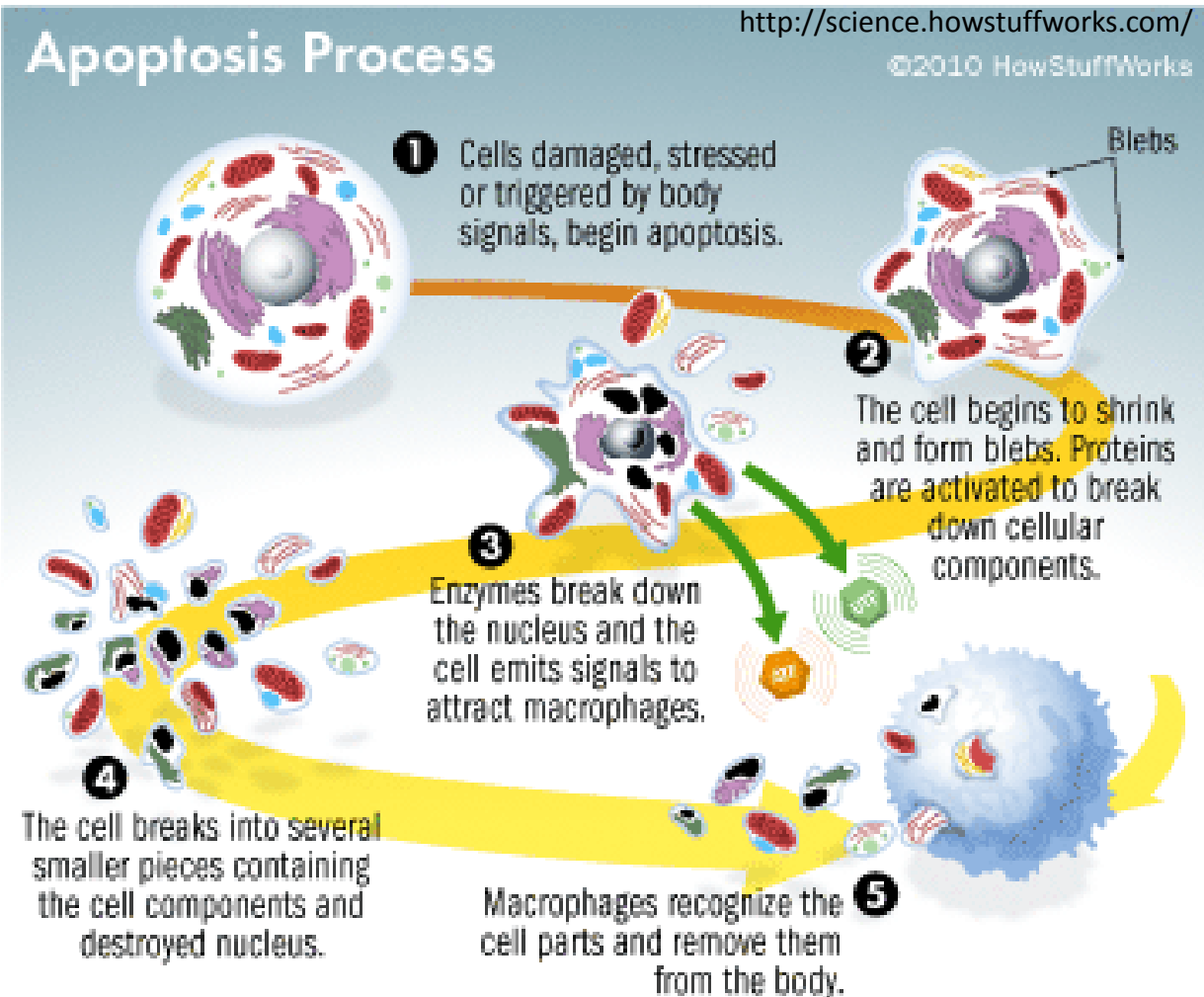
**Irreversible switch** (“fuse”): once response goes to high, it remains there even when signal becomes low

# One-way switch (“fuse”)

One-way switches presumably play major roles in developmental processes characterized by a “point of no return”

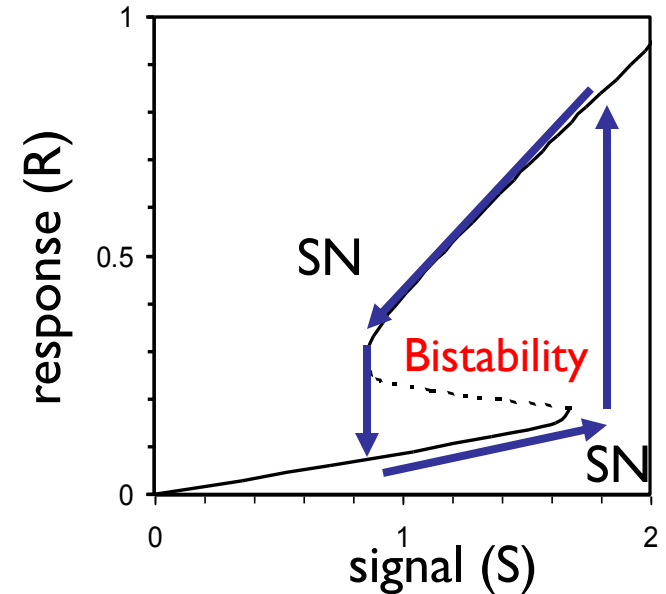
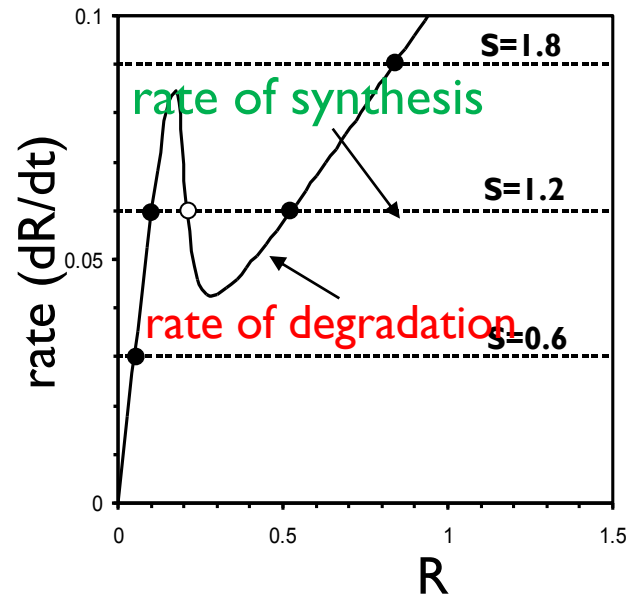
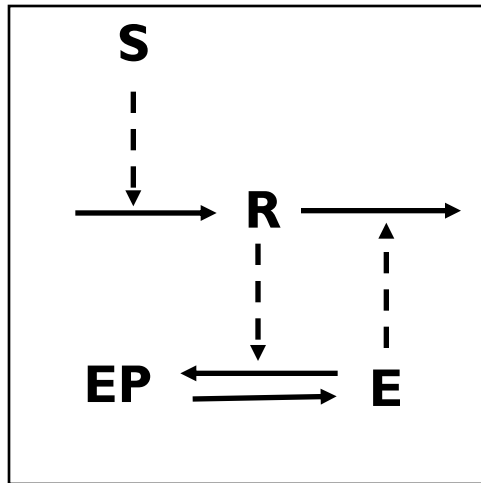
Laurent & Kellershohn, Trends Biochem Sci 24, 418 (1999)

## Example: Apoptosis (Programmed Cell Death)



# Positive Feedback: Mutual inhibition

- R inhibits E
- E promotes the degradation of R



**Toggle switch:** if the signal is increased beyond a critical value the system will switch to a high state and if signal decreases enough, the switch will go back to the low state but in bistable region will display hysteresis (state depends on how signal is changed – increased or decreased)

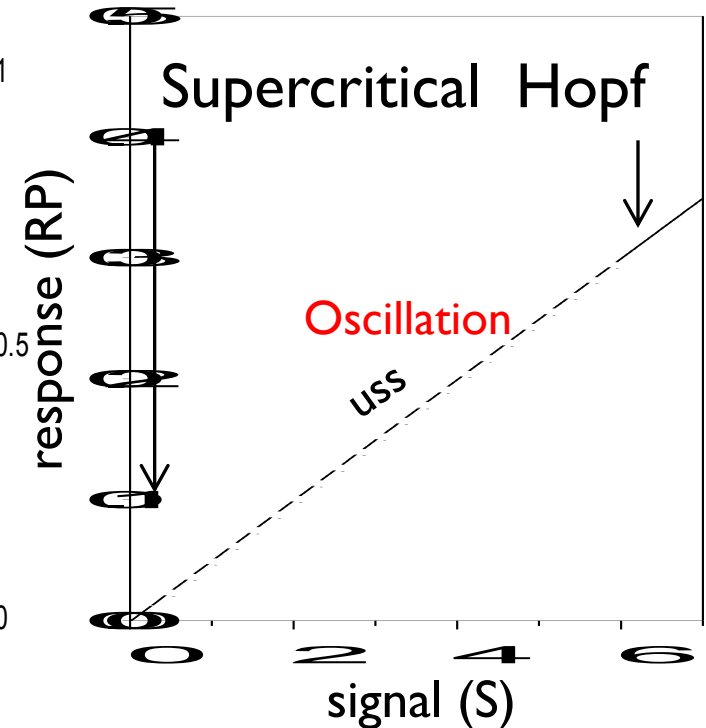
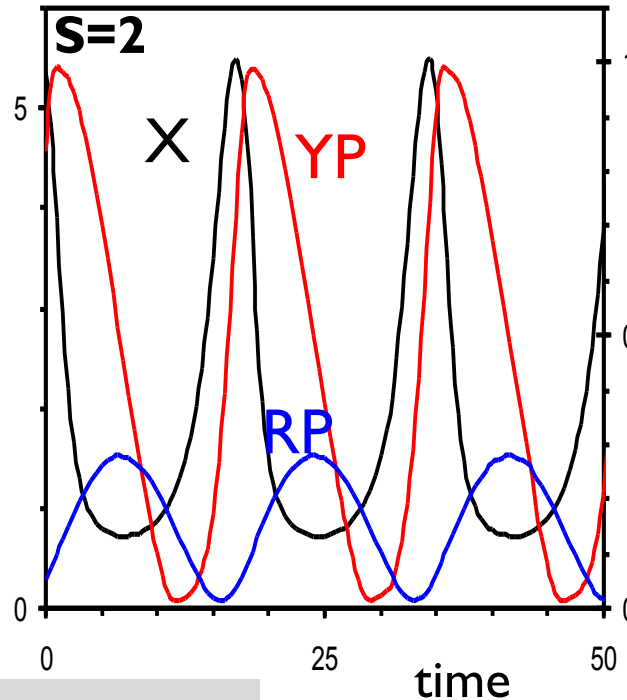
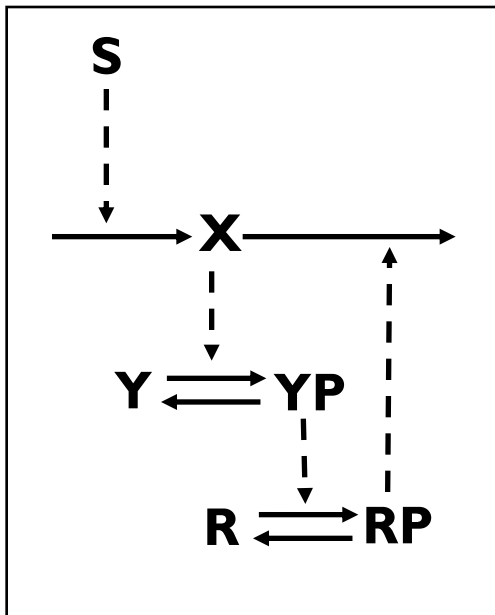
$$\frac{dR}{dt} = k_0 + k_1 S - k_2 R - k'_2 E(R) \cdot R$$

$$E(R) = G(k_3, k_4 R, J_3, J_4)$$

# Negative Feedback Oscillator

RP activates the degradation of X

Goodwin, Nature 209, 479 (1966)



$$\frac{dX}{dt} = k_0 + k_1 S - k_2 X + k'_2 R_P \cdot X$$

$$\frac{dY_P}{dt} = \frac{k_3 X (Y_T - Y_P)}{K_{m3} + Y_T - Y_P} - \frac{k_4 Y_P}{K_{m4} + Y_P}$$

$$\frac{dR_P}{dt} = \frac{k_5 Y_P (R_T - R_P)}{K_{m5} + R_T - R_P} - \frac{k_6 R_P}{K_{m6} + R_P}$$

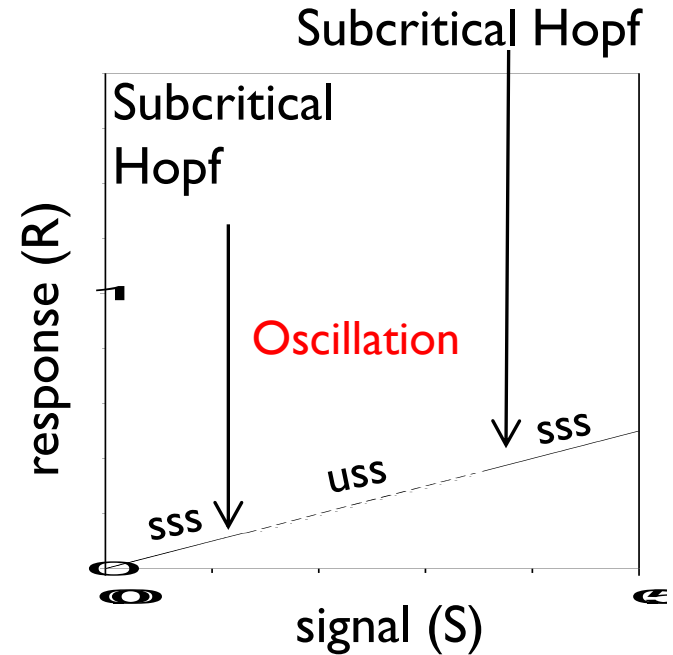
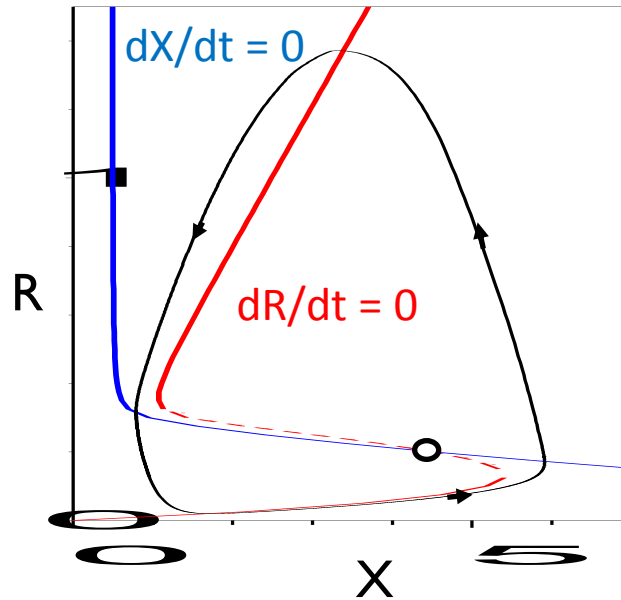
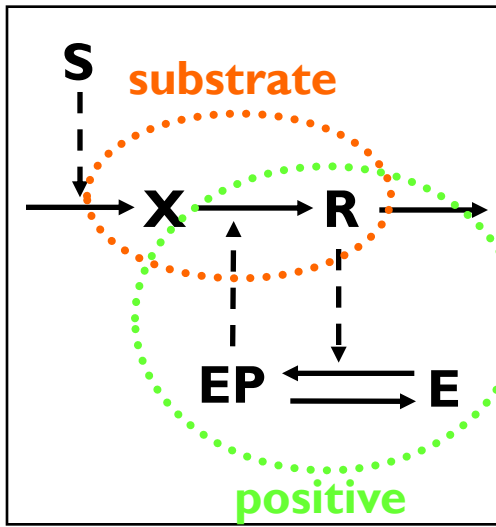
Negative feedback has been proposed as a basis for oscillations in protein synthesis

**Repressilator** (Elowitz & Leibler): artificial oscillating genetic network consisting of 3 operons that repress one another in a loop.

# Substrate Depletion Oscillator

Oscillations via positive & negative feedback

+ve feedback creates a bistable system and -ve feedback drives the system back and forth between the two stable steady states



$$\frac{dX}{dt} = k_1 S - [k'_0 + k_0 E_P(R)] \cdot X$$

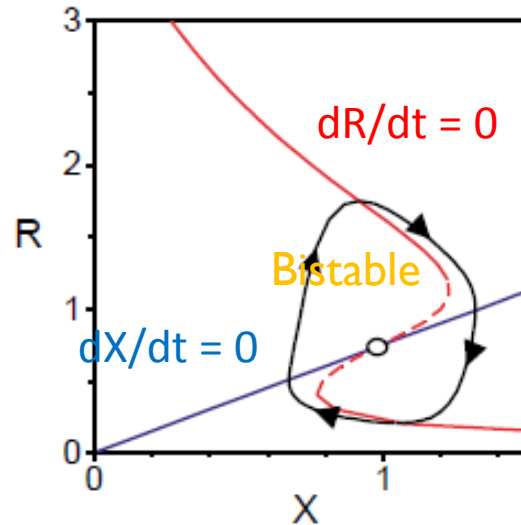
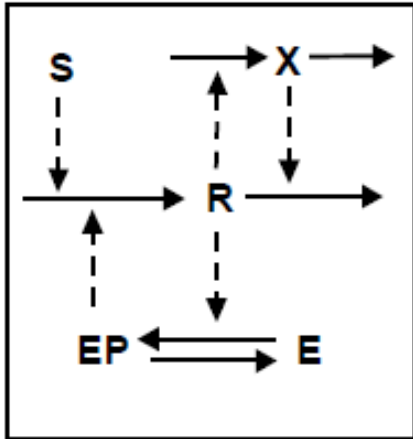
$$\frac{dR}{dt} = [k'_0 + k_0 E_P(R)] \cdot X - k_2 R$$

$$E_P(R) = G(k_3 R, k_4, J_3, J_4)$$

X is converted into R in an autocatalytic process. Initially, X is abundant and R is scarce. As R builds up, production of R accelerates until there is an explosive conversion of the entire pool of X into R. Then the autocatalytic reaction shuts off for lack of substrate, X. R is degraded, and X must build up again before another burst of R is produced.

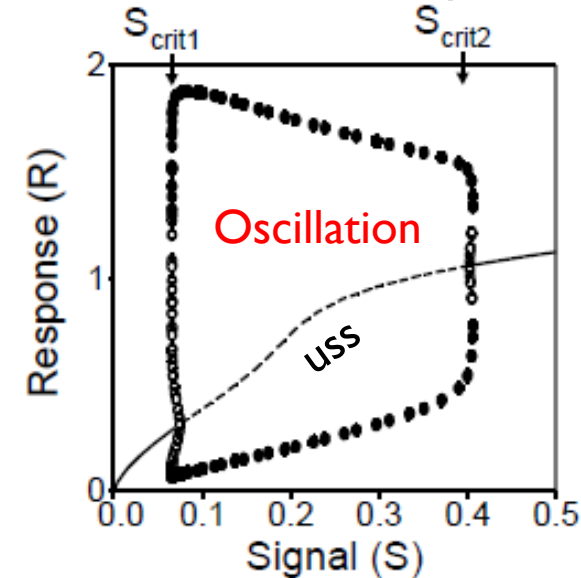
# Activator-Inhibitor Oscillator

R is created in an autocatalytic process, then it promotes the production of an inhibitor, X, which speeds up R removal. First, R builds up, then comes X to force R back down, then X disappears and R can rise again.



Goldbeter, *Biochemical oscillations and cellular rhythms* (1997)

## Subcritical Hopf



$$\frac{dR}{dt} = k_0 E_P(R) + k_1 S - k_2 R - k'_2 X \cdot R$$

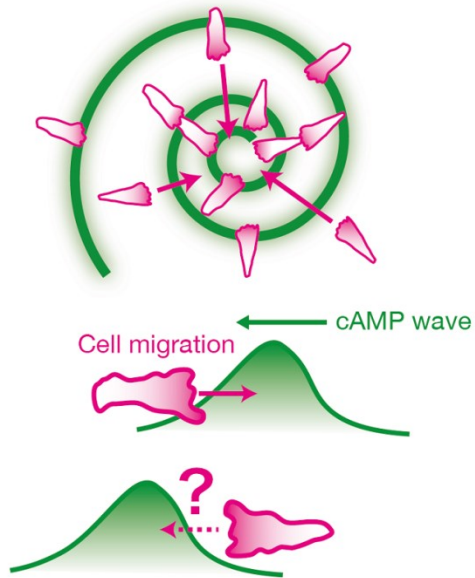
$$\frac{dX}{dt} = k_5 R - k_6 X$$

$$E_P(R) = G(k_3 R, k_4, J_3, J_4)$$

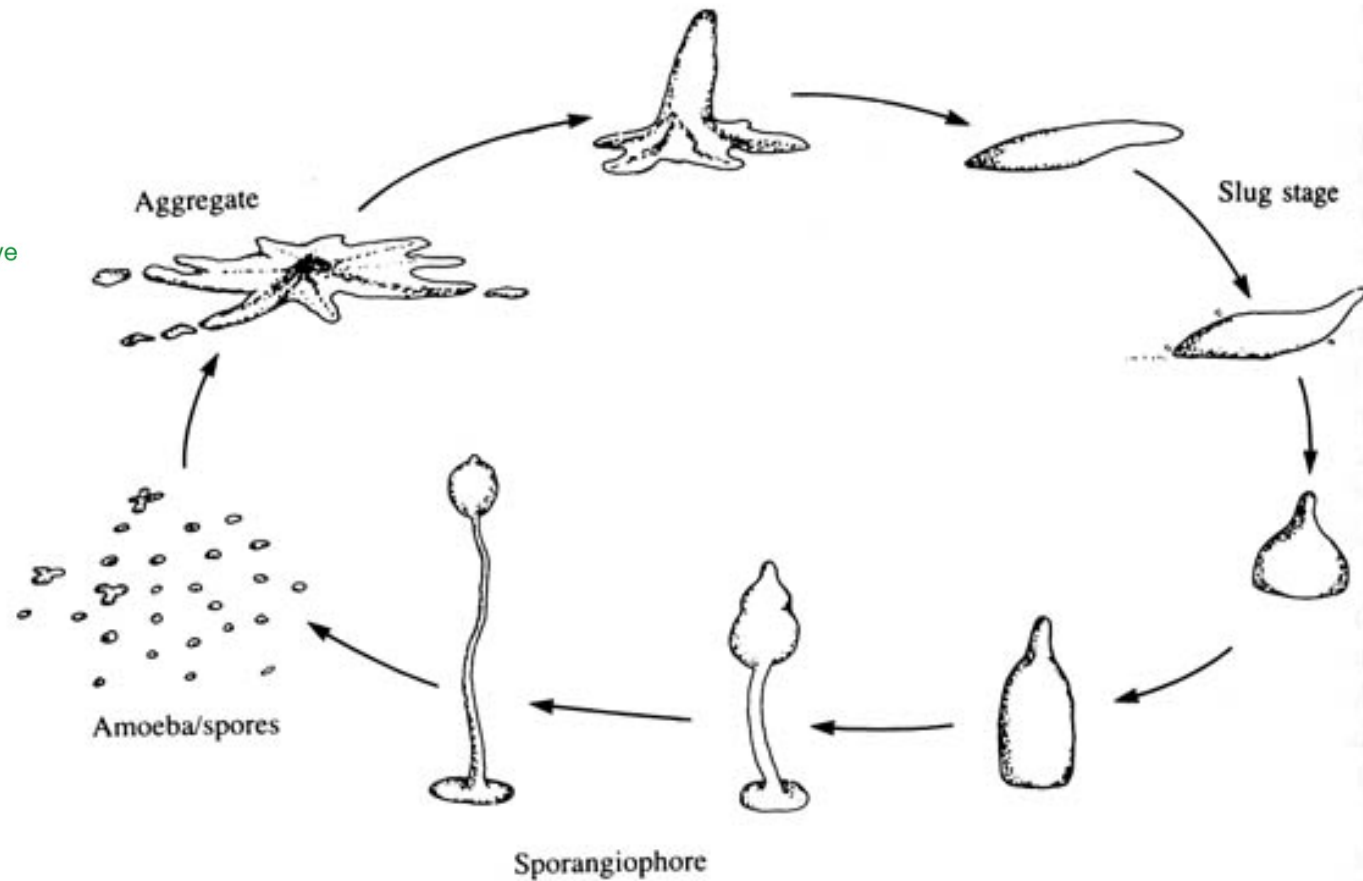
## Example: cyclicAMP production in slime mold

External cAMP binds to a surface receptor, stimulates adenylate cyclase to produce and excrete more cAMP. At the same time, cAMP-binding pushes the receptor into an inactive form. After cAMP falls off, the inactive form slowly recovers its ability to bind cAMP and stimulate adenylate cyclase again.

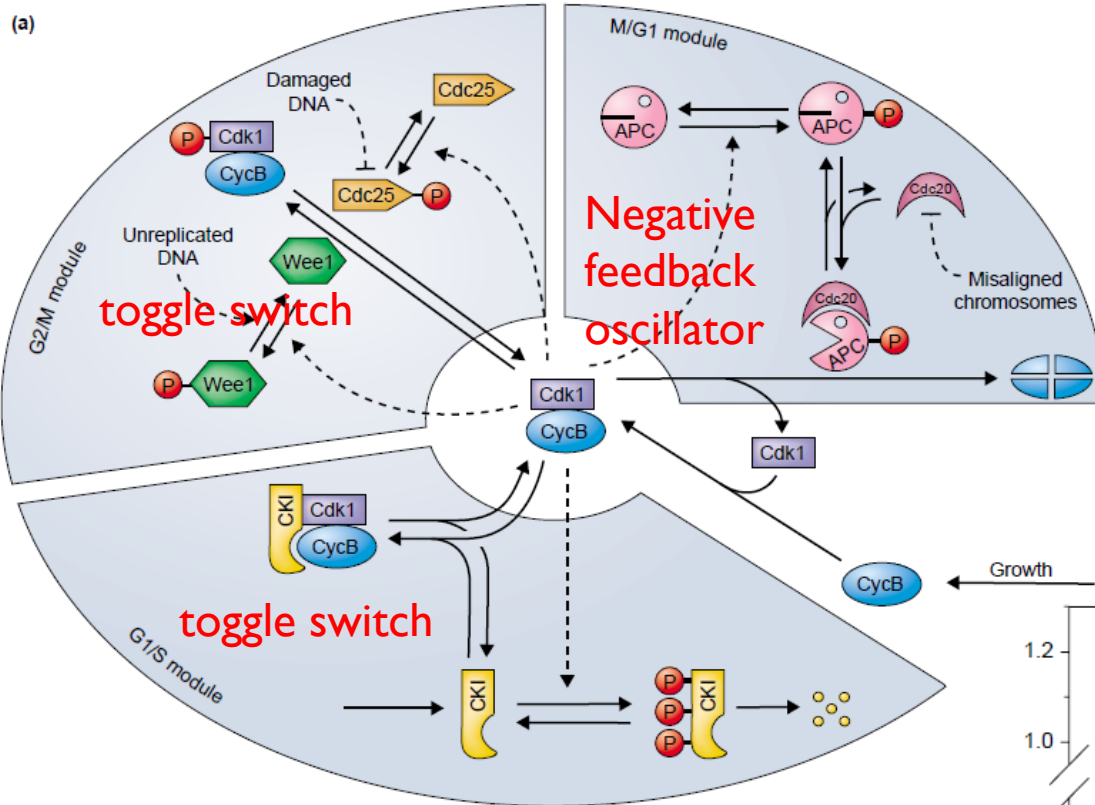
# cyclicAMP oscillations in slime mold



[www.u-tokyo.ac.jp](http://www.u-tokyo.ac.jp)



# Cell-cycle regulation in eukaryotes



Cyclin-dependent kinase (Cdk) network regulating DNA synthesis and mitosis

