

Systems Biology Across Scales: A Personal View

XXI. Temporal patterns: Discrete time models in biology

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Discrete-time dynamics

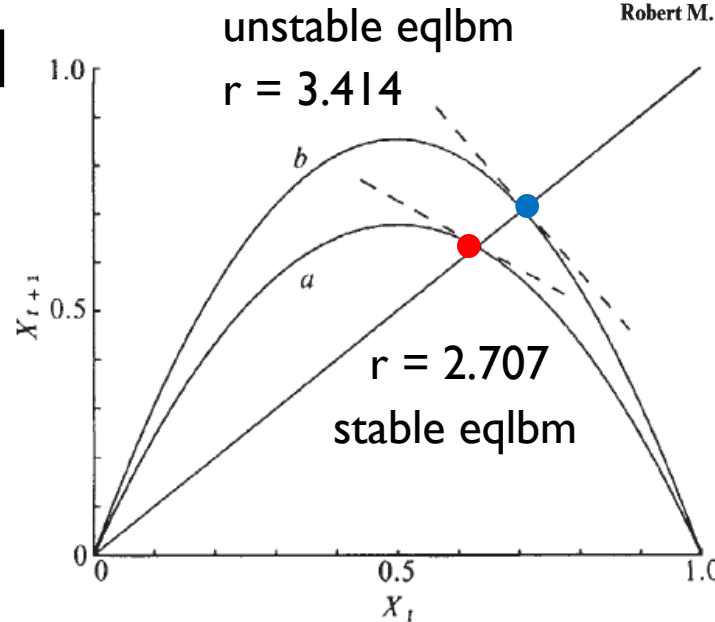
Nature Vol. 261 June 10 1976

Simple mathematical models with very complicated dynamics

Robert M. May*

$$X(t+1) = F(X(t))$$

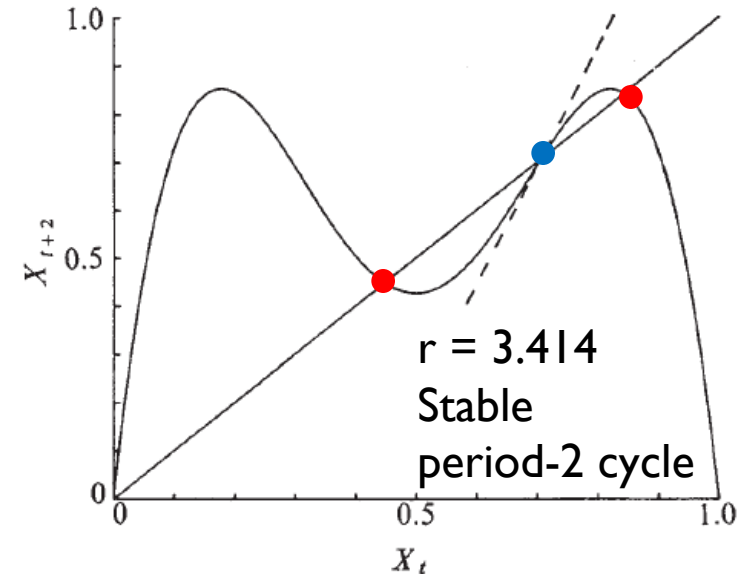
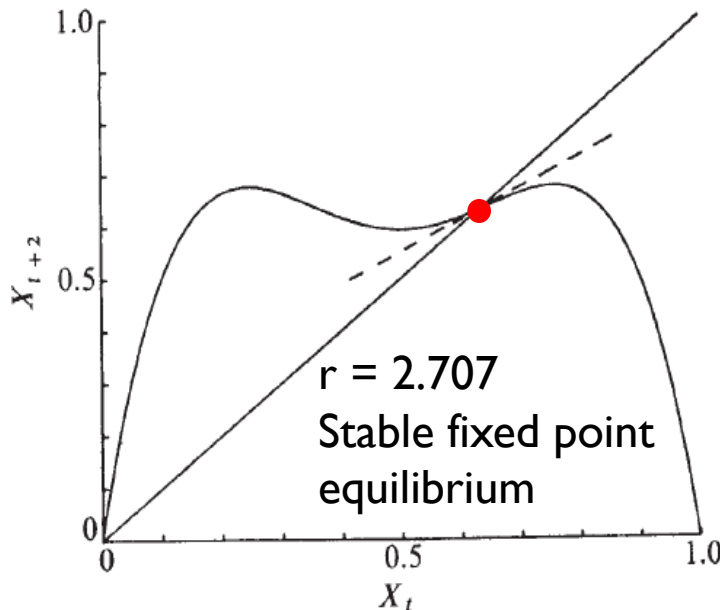
Used to describe population dynamics in seasonally breeding populations in which the generations do not overlap, e.g., insects such as crop pests in temperate zones



e.g., Logistic Map

$$X(t+1) = r X(t) [1 - X(t)]$$

Twice-iterated map
 $X(t+2) = F(F(X(t)))$
alerts us to creation of periodic cycles



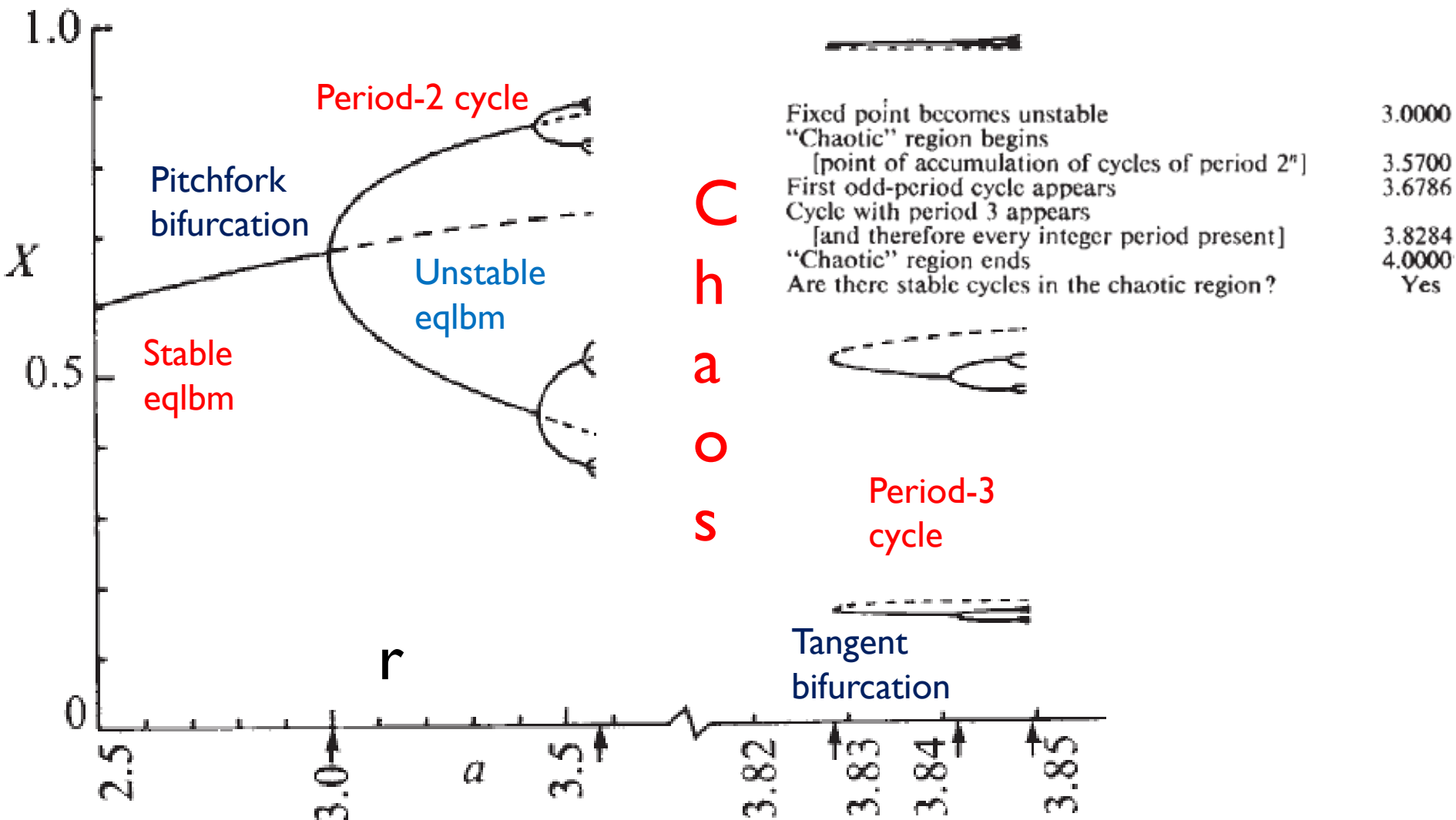
Bifurcation diagram

Period-doubling route to chaos

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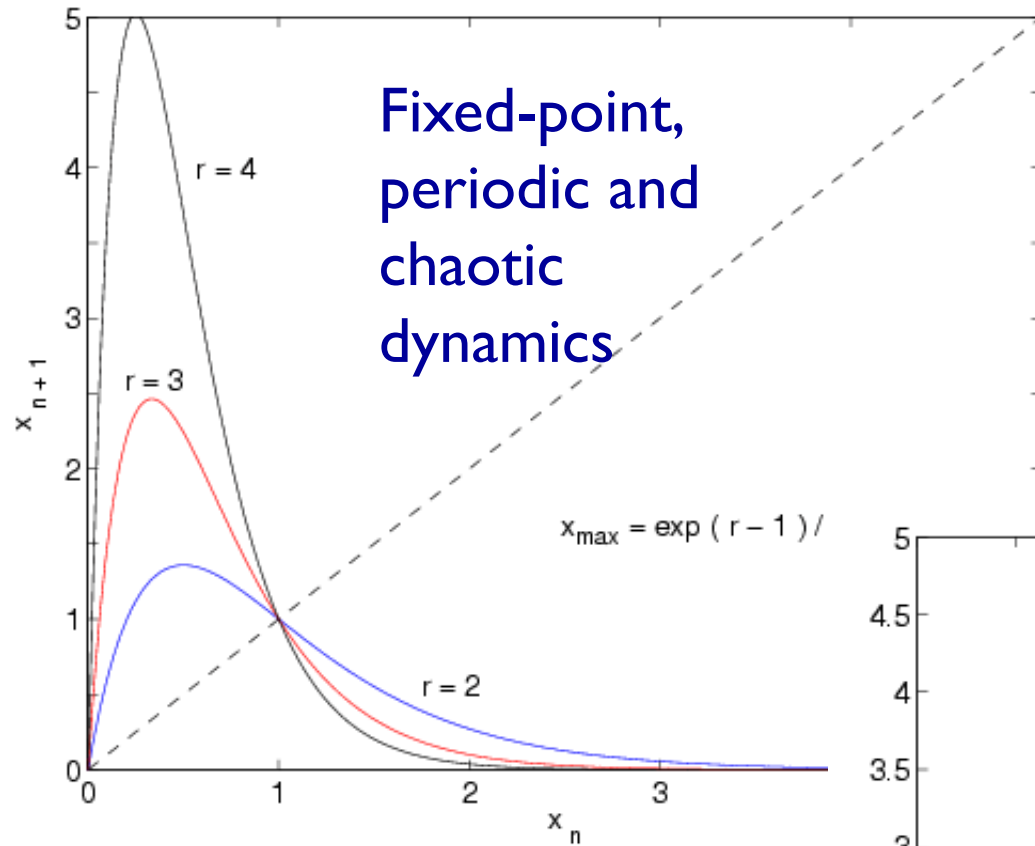


Exponential logistic growth model

$$X_{n+1} = X_n \exp [r(1 - X_n)]$$

X_n : population density at n-th generation

r: growth rate

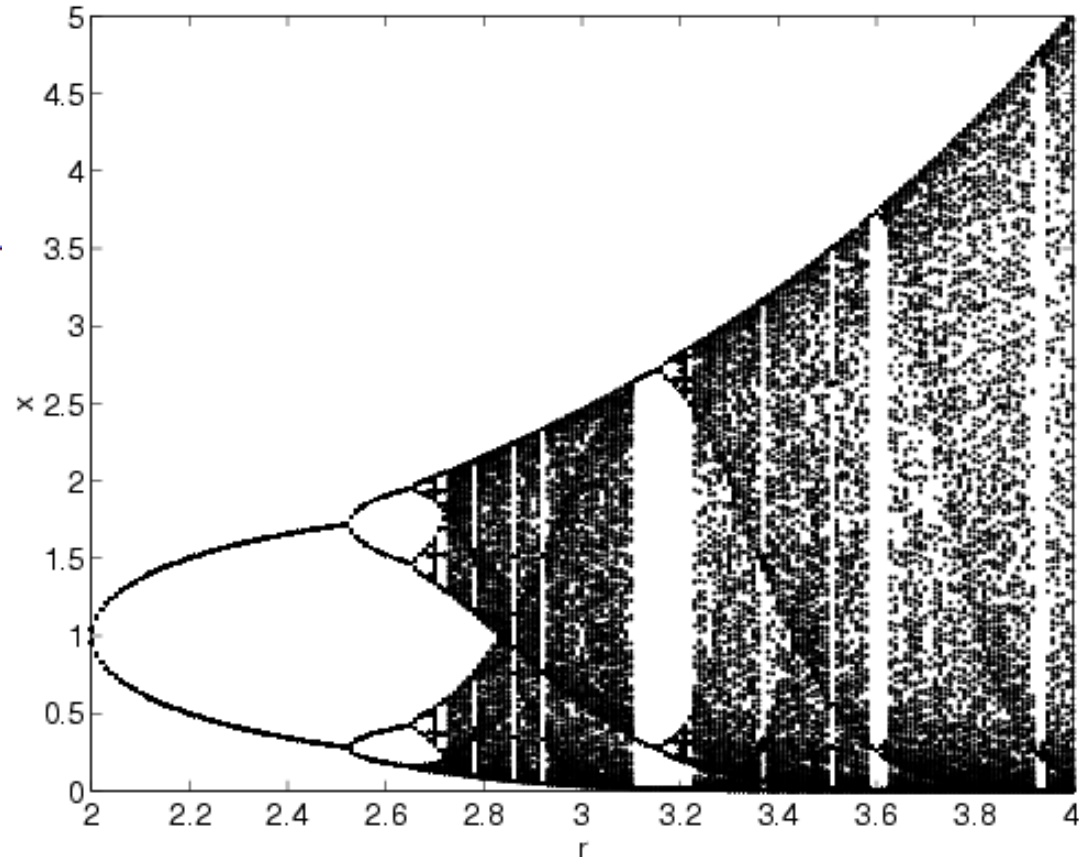


Fixed-point,
periodic and
chaotic
dynamics

Used as a model for fisheries dynamics (Ricker)

Fixed point becomes unstable
 "Chaotic" region begins
 [point of accumulation of cycles of period 2^n]
 First odd-period cycle appears
 Cycle with period 3 appears
 [and therefore every integer period present]
 "Chaotic" region ends
 Are there stable cycles in the chaotic region?

2.0000
 2.6924
 2.8332
 3.1024
 ∞
 Yes

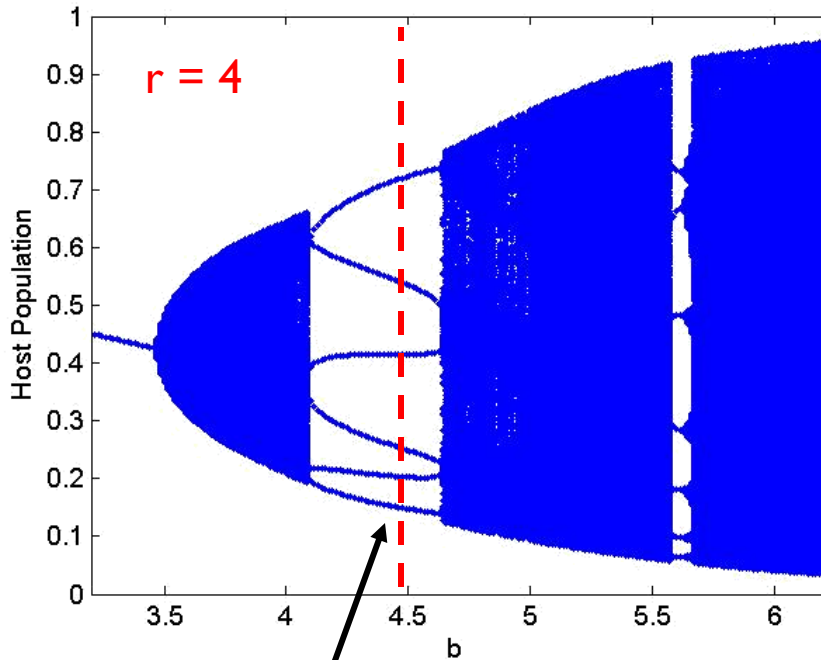
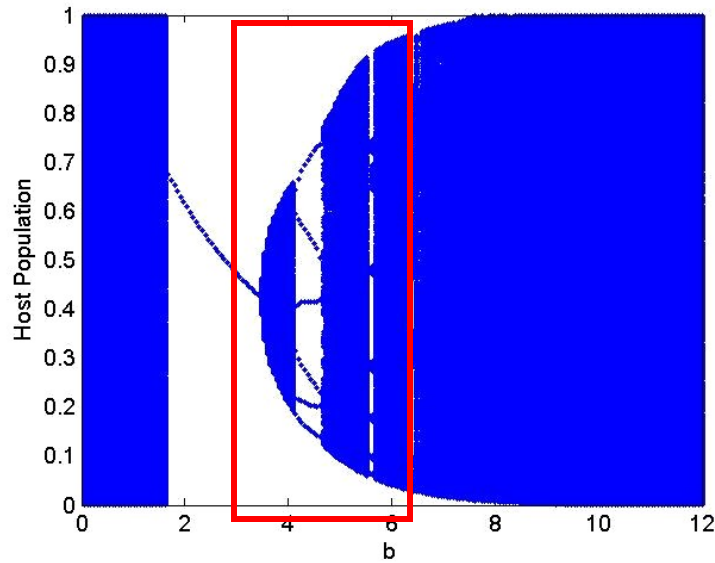


Host-Parasite Model

$$H_{n+1} = f(H_n, P_n) = r H_n (1 - H_n) \exp(-b P_n)$$

$$P_{n+1} = g(H_n, P_n) = c H_n [1 - \exp(-b P_n)]$$

Nicholson & Bailey, 1935



$b = 4.5$ Period 6 cycle

- r : rate of increase of host population
- c : parasite conversion efficiency (=1)
- b : parasite attack rate
- $\exp(-bP)$: host fraction escaping parasitism