Systems Biology Across Scales: A Personal View XVII. Interactions in Ecology

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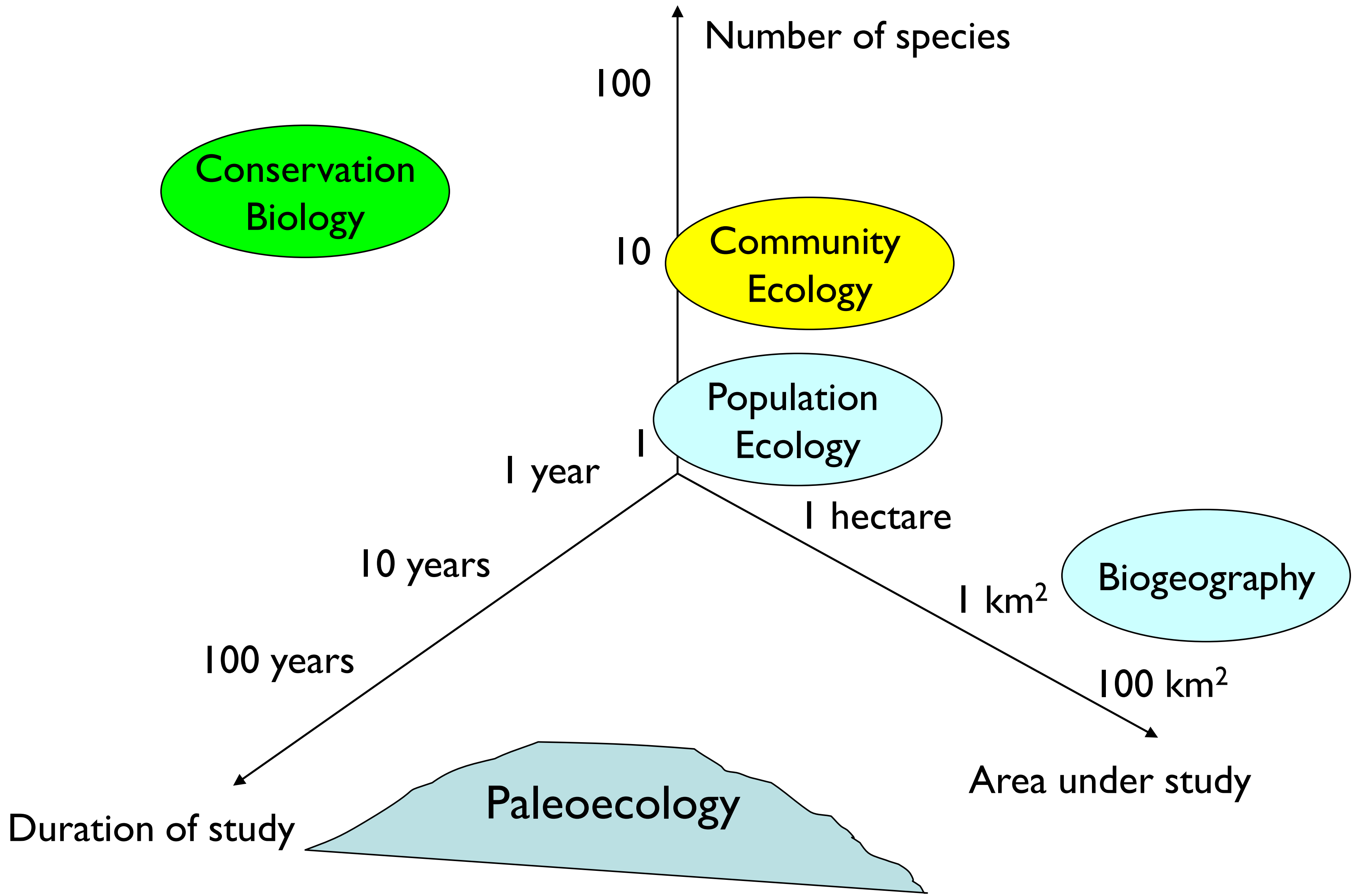
Spatial & Temporal Scales in Ecological Studies



Adapted from Pimm, *The Balance of Nature ?*

Modeling population dynamics of species

A single species: population dynamics with intra-species competition represented by a logistic term
(like mean-field theory, we assume its environment to be fixed)

$$dx/dt = r\ x\,[1 - (x/K)]$$

Verhulst (19th century)

r: Growth rate, K: carrying capacity

Introducing interactions between species

Mutual competition between two species can be modeled as:

$$dx/dt = r\ x\,[1 - (x/K) - (\alpha\ y/K)]$$

α: interspecies competition

Lotka-Volterra competition model

Models ***interference competition*** among species involving direct contact or conflict
(Individuals of one species are assumed to be equivalent to a certain number of individuals of another species)

Dynamics of Exploitative Competition

Instead of interference competition, we can consider ***exploitative competition*** : each species diminishes the resources of the other (viz. reduces the capacity of the environment to support another species by acting on the carrying capacity of that species).

For 2 species, the populations evolve as:

r_1, r_2 : growth rates
d_1, d_2 : death rates
K, K' : competition intensity

$$dx_1/dt = [r_1\, x_1 \,/\, (x_1 + K\, x_2)] - d_1\, x_1$$

$$dx_2/dt = [r_2\, x_2 \,/\, (x_2 + K'\, x_1)] - d_2\, x_2$$

e.g., Schoener, Theo Popn Biol (1976)

For N species, can be written as a system of N coupled ODEs:

K : competition matrix

$$dx_i/dt = [r_i\, x_i \,/\, (x_i + K_{ij}\, x_j)] - d_i\, x_i$$

But interactions can be of different kind, viz., predator-prey !

2 species Resource-Consumer Dynamics

- Prey-Predator trophic relations
- Host-parasite
- Plant-Herbivore

In general can be of the form:

$$dx/dt = \phi(x) - g(x, y)\,y$$
$$dy/dt = n(x, y)\,y - d\,y$$

$\phi(x)$: growth of prey in absence of predators
g(x,y): capture rate of prey per predator
n(x,y): rate at which captured prey is converted into predator population increase – assumed to be ε g(x,y) where ε is efficiency
d: rate at which predators die in absence of prey

Different choices of the functional forms can give different models

Lotka-Volterra Model (1920-6)

Alfred Lotka

The first model of consecutive chemical reactions giving rise to oscillations in molecular concentrations

$$A + X \rightarrow 2X$$
$$X + Y \rightarrow 2Y$$
$$Y \rightarrow P$$

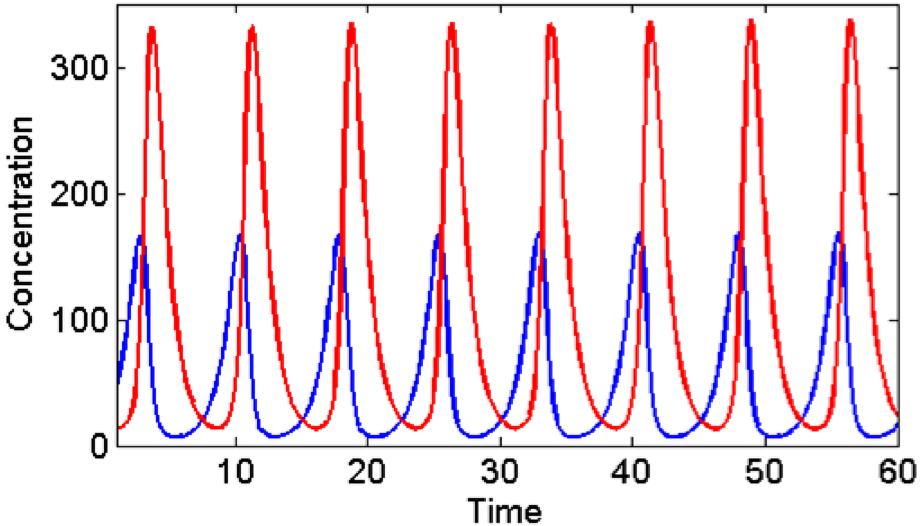
(*Autocatalysis:* +ve feedback)

More reasonable in the context of ecology

Prey (x) – Predator (y) eqns

$$dx/dt = r\ x - k_{xy}\ x\ y$$
$$dy/dt = k_{yx}\ x\ y - d\ y$$

Volterra (1926)



Lotka-Volterra Model (1920-6)

Vito Volterra

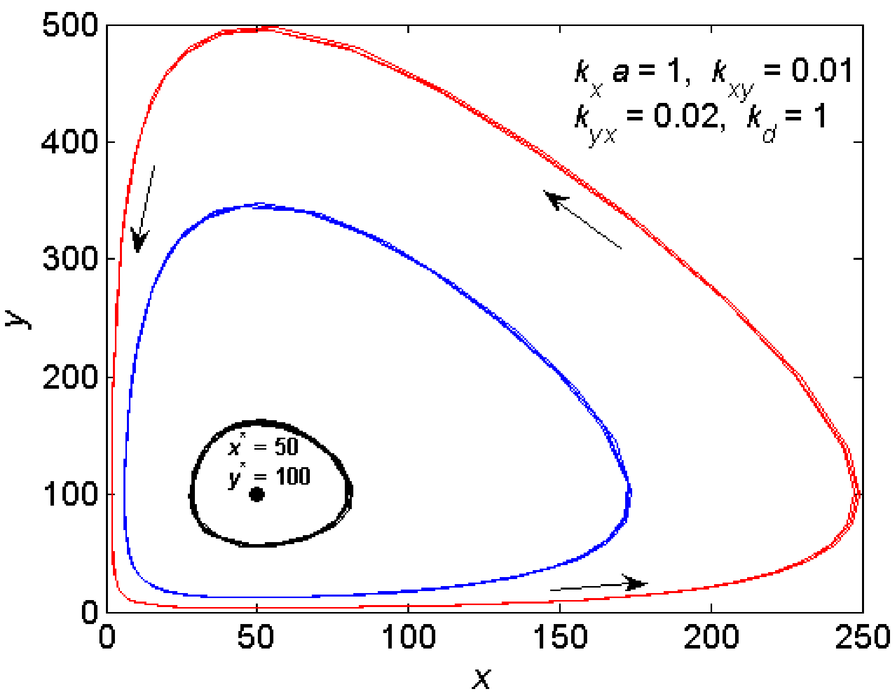
D'Ancona: The problem of high proportion of predator fishes in the Adriatic Sea during WWI (1914-1918)

Prey (x) – Predator (y) eqns + effect of fishing

$$dx/dt = (r - a)\, x - k_{xy}\, x\, y$$
$$dy/dt = k_{yx}\, x\, y - (d + b)\, y$$

Volterra (1926)

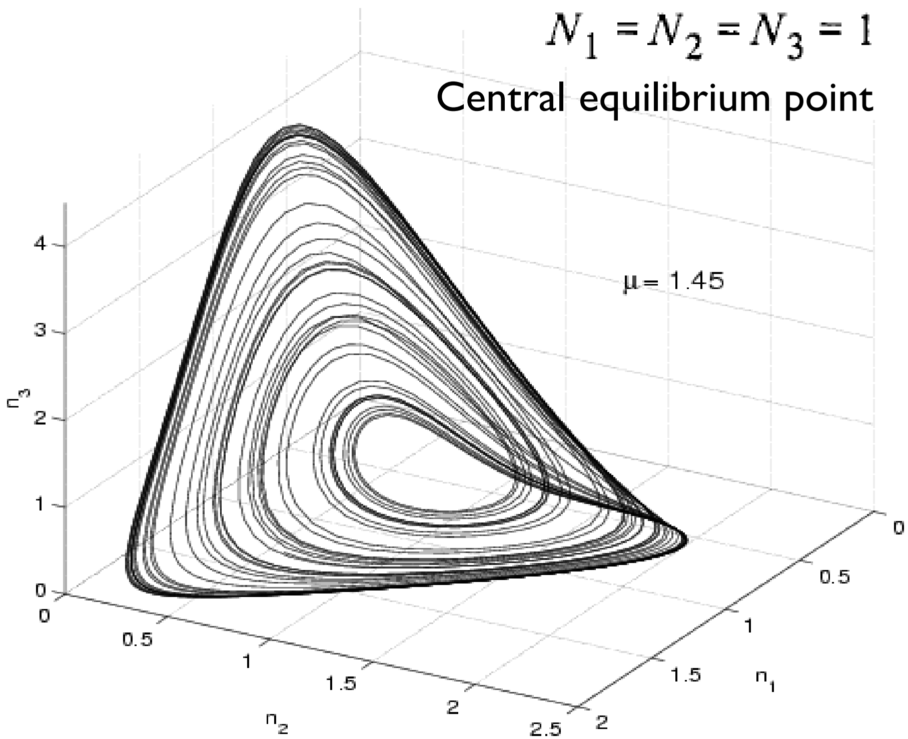
Equilibrium populations:
$x^* = (d+b)/k_{xy}$, $y^* = (r-a)/k_{xy}$



In absence of fishing during the war years (a=0,b=0), x* decreased while y* increased

In general, we observe oscillations because cycles around the equilibria are neutrally stable

Chaos in Lotka-Volterra System



$$\partial_t N_i = N_i \sum_{j=1}^{3} \alpha_{ij}(1 - N_j)$$

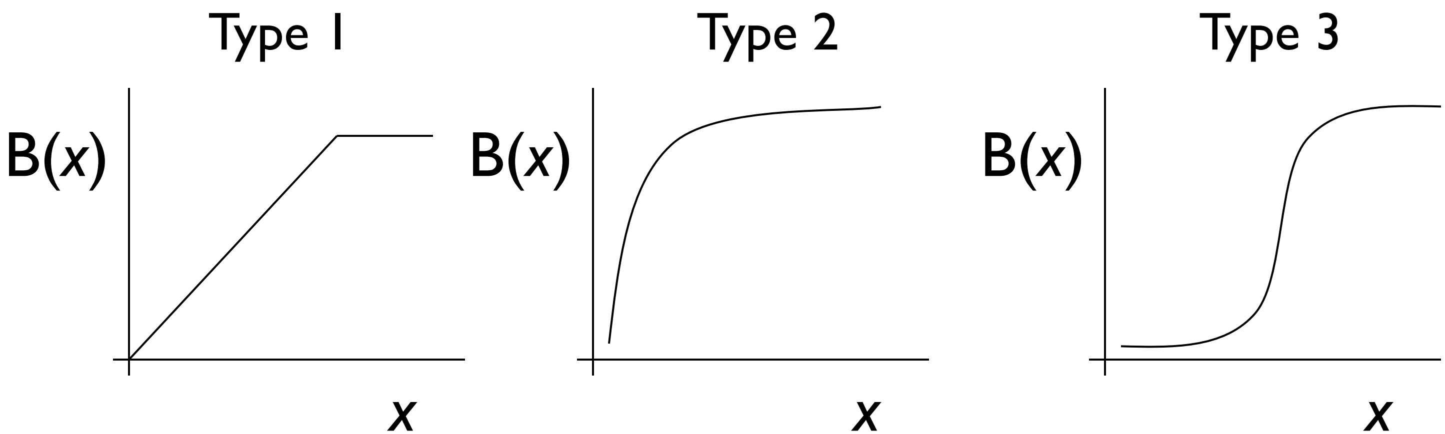
with $$(\alpha_{ij})_\mu = \begin{pmatrix} 0.5 & 0.5 & 0.1 \\ -0.5 & -0.1 & 0.1 \\ \mu & 0.1 & 0.1 \end{pmatrix}$$

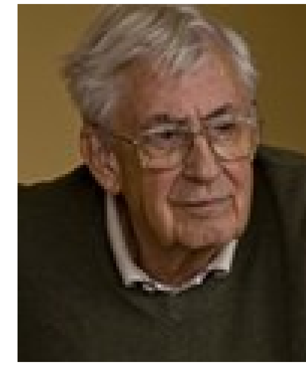
- Two-species Lotka-Volterra systems: periodic oscillations.
- Chaos in Lotka Volterra system with three species (Arneodo, Coullet and Tresser, 1980).
- In principle - possible for a species to persist in food web that is not dynamically stable by going through complex cycles.
- However, the cycles must be strictly bounded - otherwise populations will reach levels from which they cannot recover.

Rosenzweig-MacArthur Model (1963)

Predator-Prey model with Holling Type 2 (hyperbolic) functional response

Functional response of predation to prey density (Holling, 1959)





C S Holling (1930-)

Prey (x) – Predator (y) eqns

$$dx/dt = r\; x\,[1-(x/K)] - q\,y\,[x/(b+x)]$$
$$dy/dt = -d\,y + \varepsilon\, q\, y\,[x/(b+x)]$$

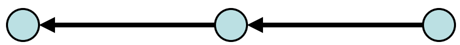
Gives rise to limit cycle oscillations

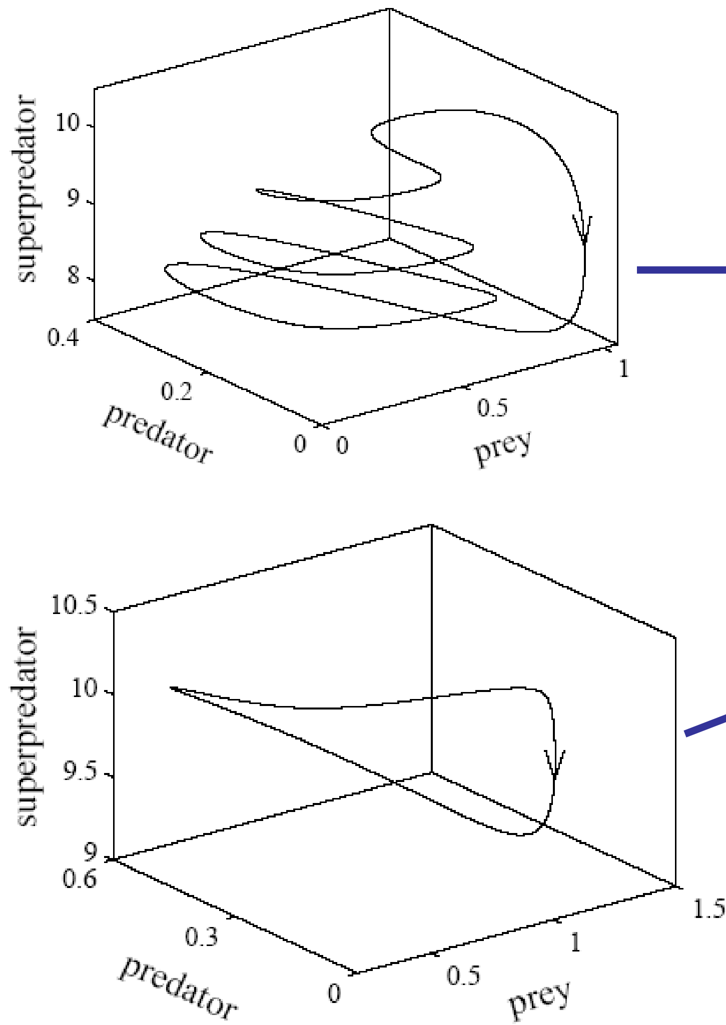
3 species Rosenzweig-MacArthur Model

$$\frac{dx_1}{dt} = x_1\left[r\left(1-\frac{x_1}{K}\right)-\frac{a_2x_2}{b_2+x_1}\right]$$

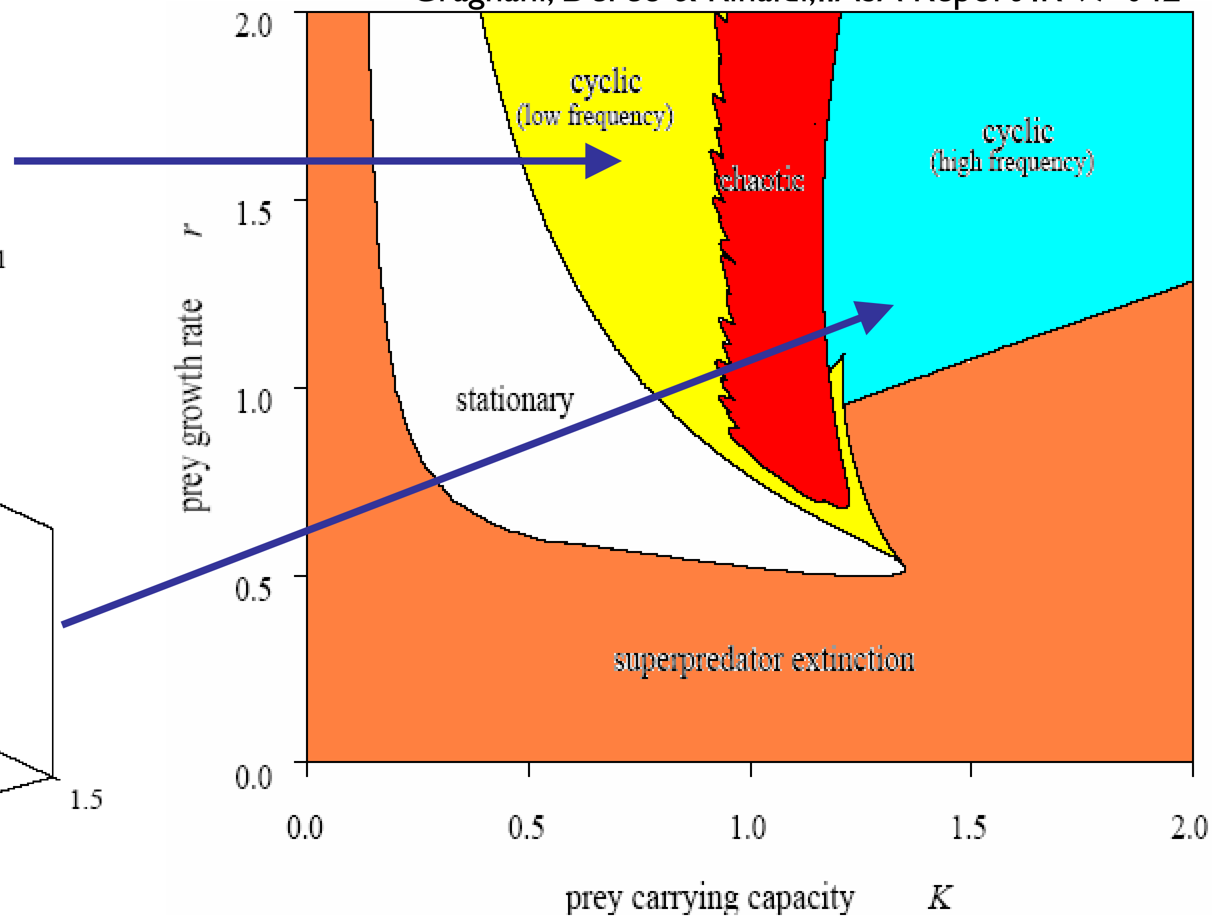
$$\frac{dx_2}{dt} = x_2\left[e_2\frac{a_2x_1}{b_2+x_1}-\frac{a_3x_3}{b_3+x_2}-d_2\right]$$

$$\frac{dx_3}{dt} = x_3\left[e_3\frac{a_3x_2}{b_3+x_2}-d_3\right]$$





Gragnani, DeFeo & Rinaldi, IIASA Report IR-97-042



Going beyond 3 species opens up an entire world of dynamical possibilities…

Ecological (Interaction) Networks

And specifically,

Food Webs

that comprise links representing trophic relations