Systems Biology: A Personal View
XXI. Temporal patterns: Discrete time models in biology

Sitabhra Sinha

IMSc Chennai

Discrete-time dynamics

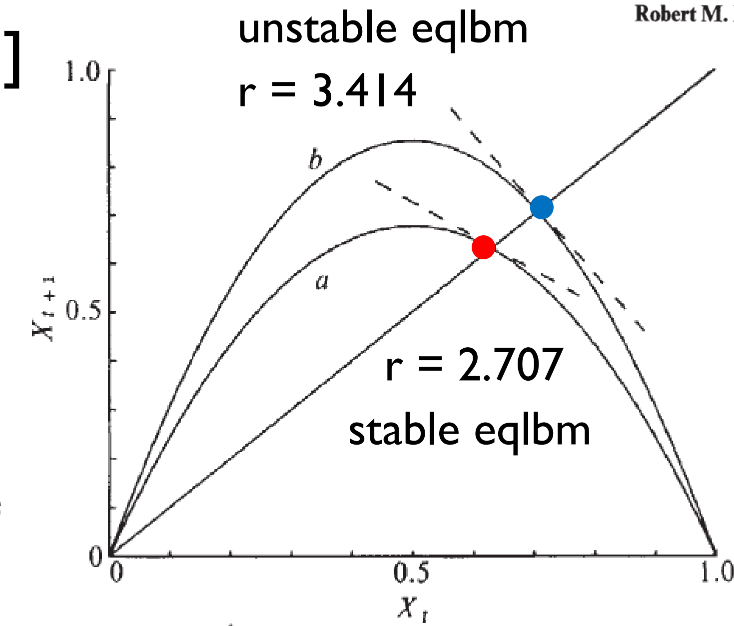
Nature Vol. 261 June 10 1976

Simple mathematical models with very complicated dynamics

Robert M. May*

$X(t+1) = F[(X(t)]$

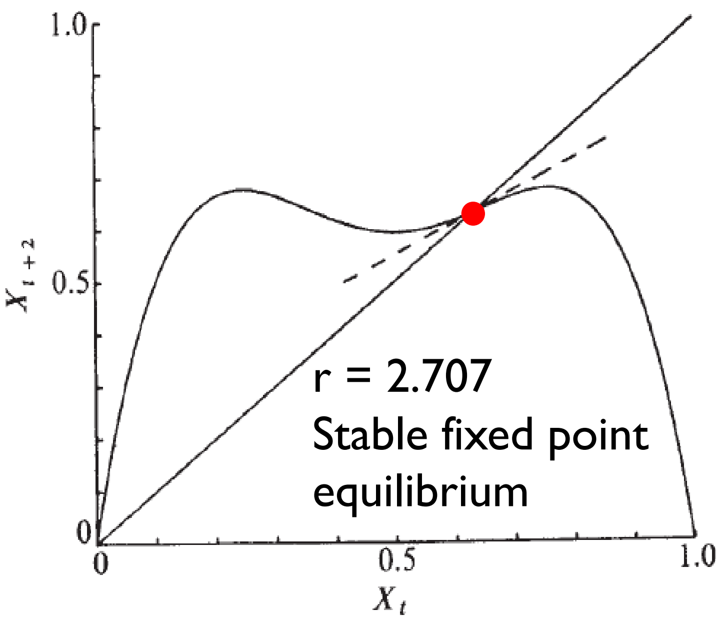
Used to describe population dynamics in seasonally breeding populations in which the generations do not overlap, e.g., insects such as crop pests in temperate zones

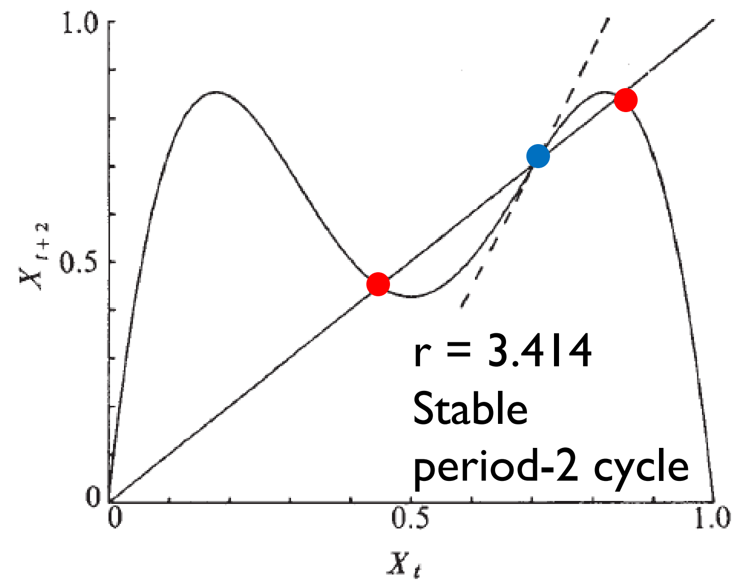


e.g., Logistic Map

$X(t+1) = r\,X(t)\,[1 - X(t)]$

Twice-iterated map
$X(t+2) = F(F(X(t)))$
alerts us to creation of periodic cycles





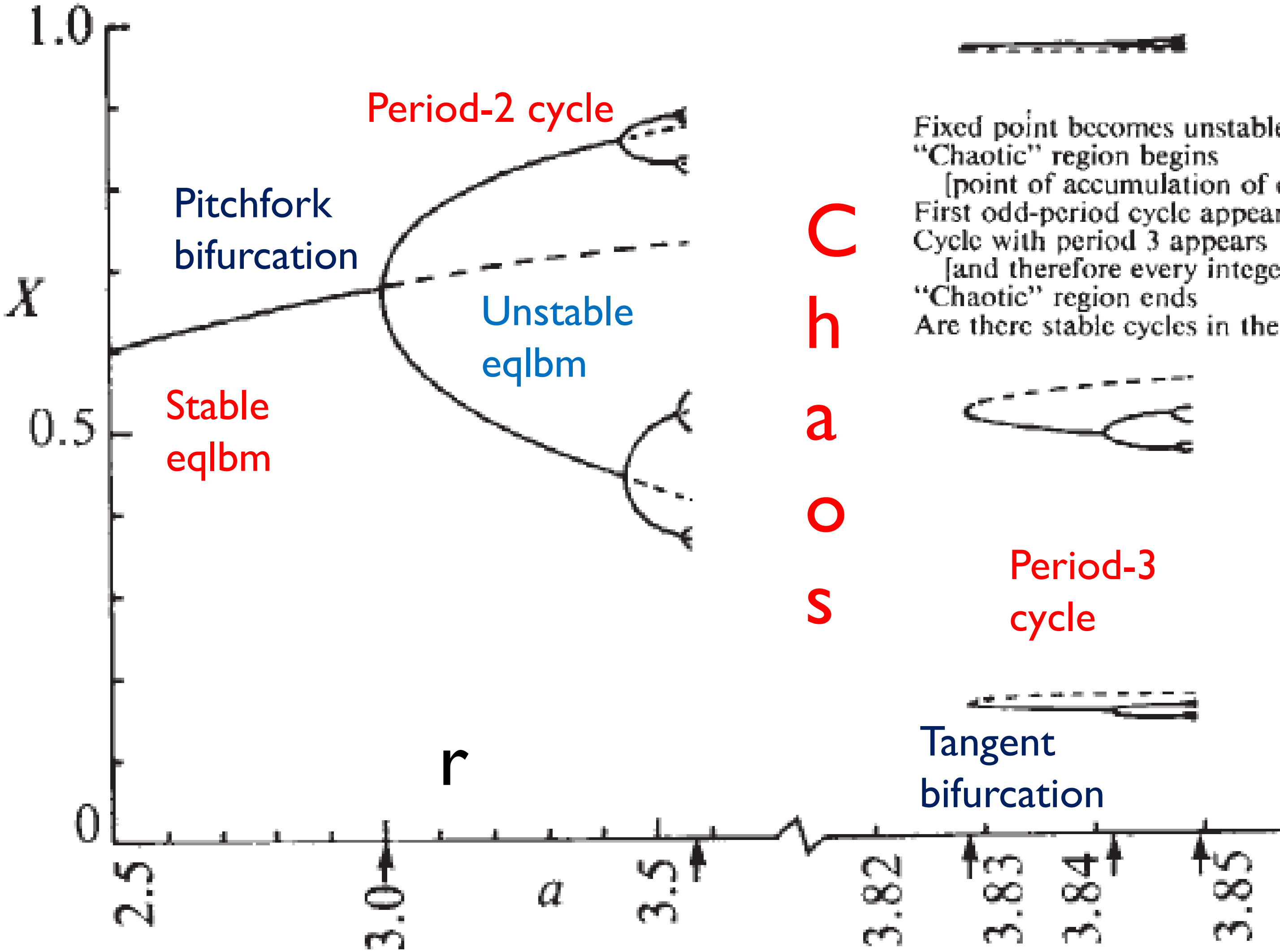
Bifurcation diagram

Period-doubling route to chaos

Nature Vol. 261 June 10 1976

Simple mathematical models with very complicated dynamics

Robert M. May*



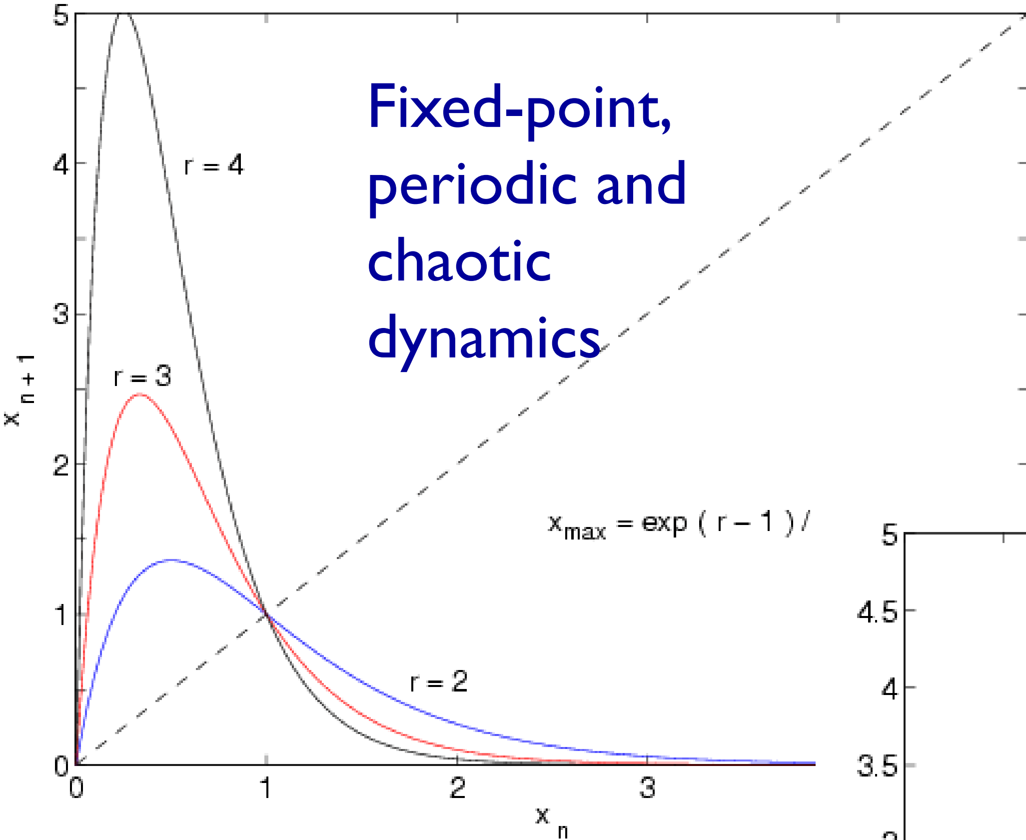
Fixed point becomes unstable	3.0000
"Chaotic" region begins [point of accumulation of cycles of period 2^n]	3.5700
First odd-period cycle appears	3.6786
Cycle with period 3 appears [and therefore every integer period present]	3.8284
"Chaotic" region ends	4.0000
Are there stable cycles in the chaotic region?	Yes

Exponential logistic growth model

$$X_{n+1} = X_n \exp [r(1 - X_n)]$$

X_n: population density at n-th generation

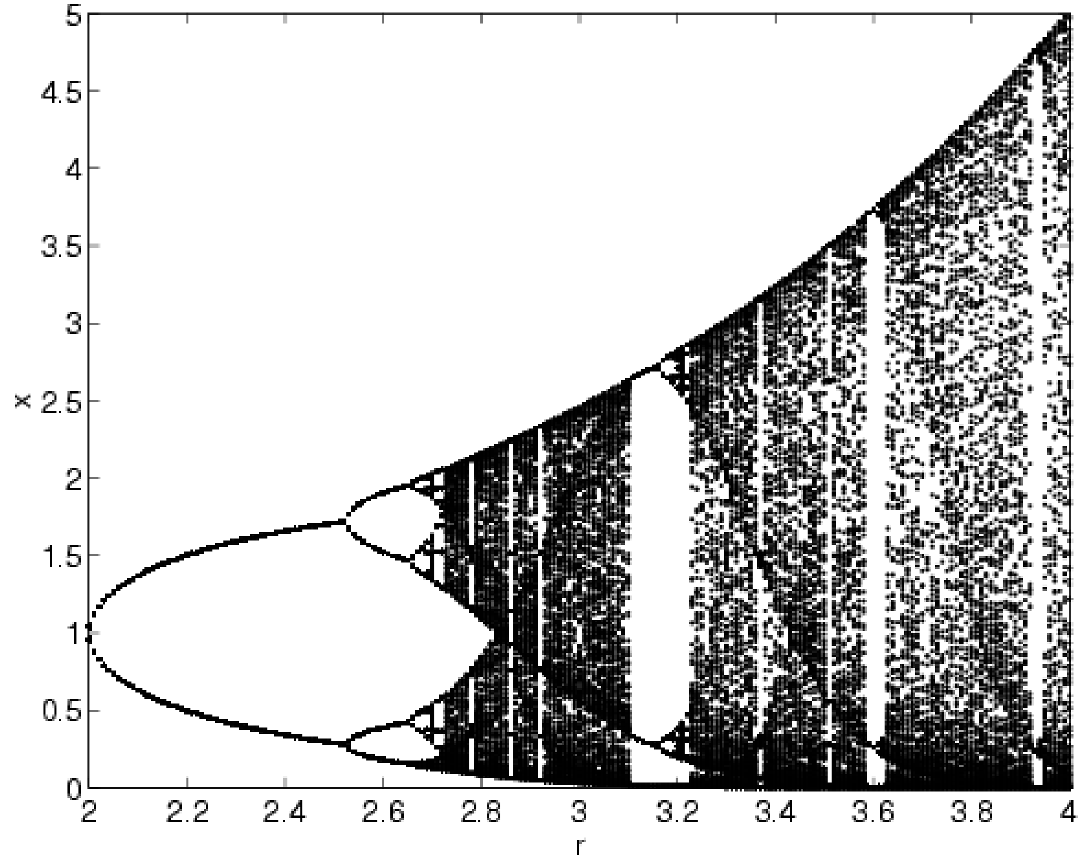
r: growth rate



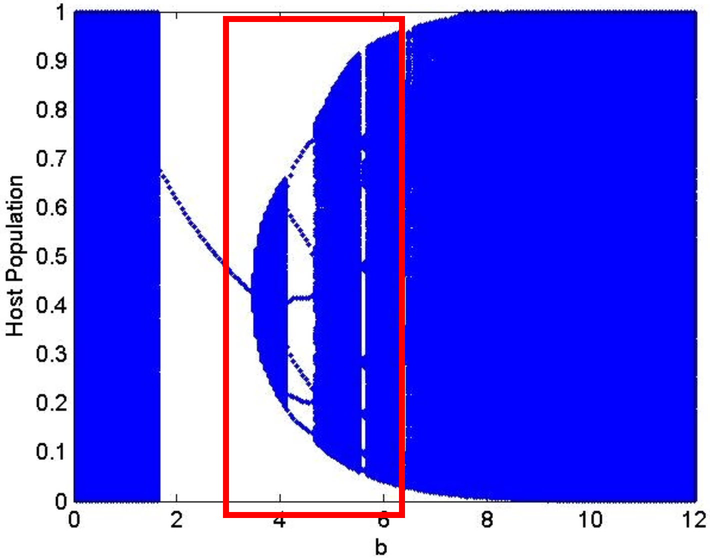
Fixed-point, periodic and chaotic dynamics

Used as a model for fisheries dynamics (Ricker)

Fixed point becomes unstable	2.0000
"Chaotic" region begins [point of accumulation of cycles of period 2^n]	2.6924
First odd-period cycle appears	2.8332
Cycle with period 3 appears [and therefore every integer period present]	3.1024
"Chaotic" region ends	∞‡
Are there stable cycles in the chaotic region?	Yes



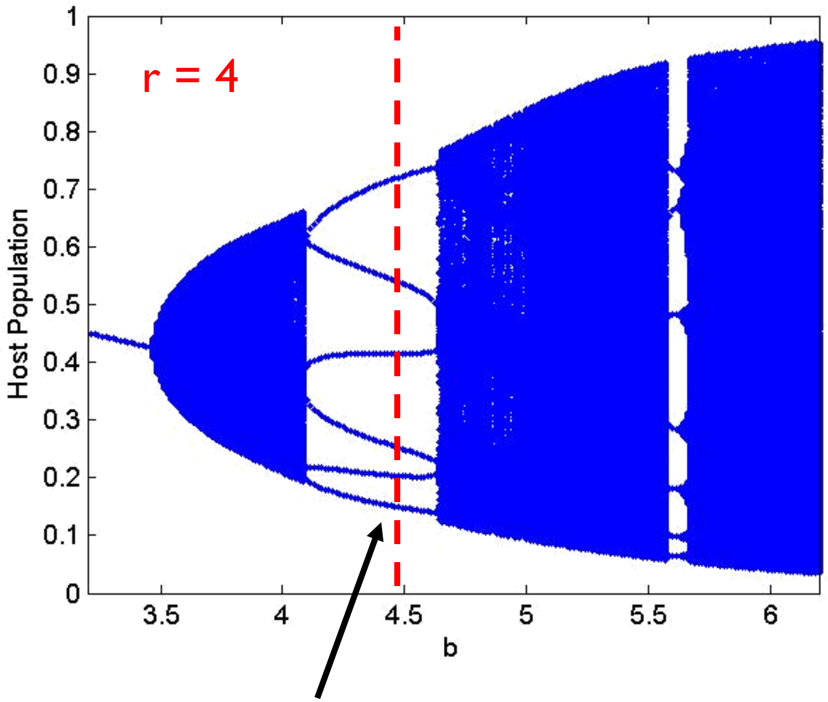
Host-Parasite Model



$$H_{n+1} = f(H_n,P_n) = r\, H_n\, (1 - H_n)\, \exp(-\, b\, P_n)$$

$$P_{n+1} = g(H_n,P_n) = c\, H_n\, [1 - \exp(-\, b\, P_n)]$$

Nicholson & Bailey, 1935



r : rate of increase of host population
c: parasite conversion efficiency (=1)
b: parasite attack rate
exp (- bP): host fraction escaping parasitism

b = 4.5 Period 6 cycle