

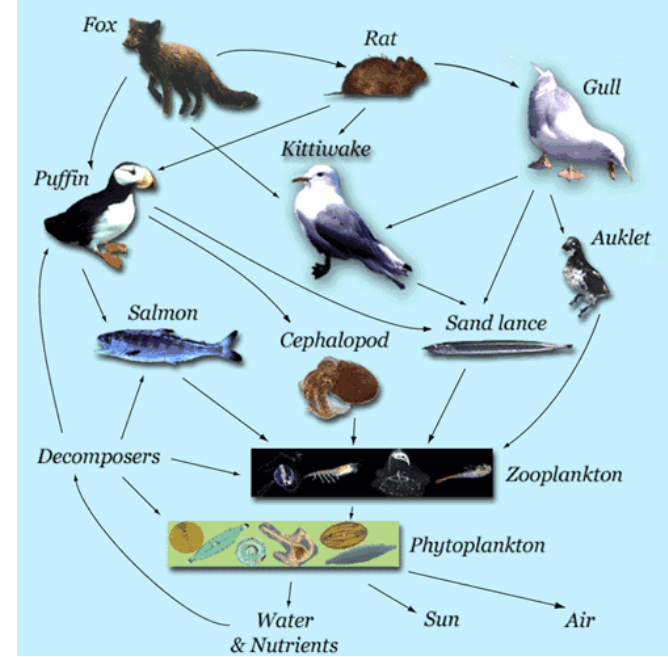
# Systems Biology: A Personal View

## XVII. Food Webs & Stability of Ecological Networks

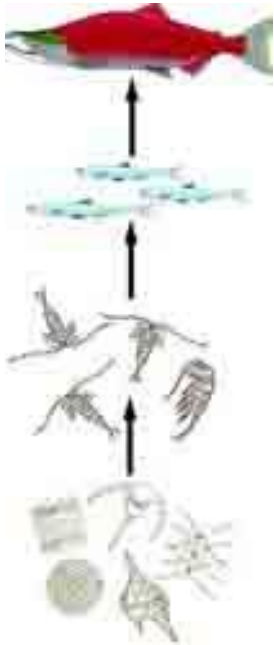
Sitabhra Sinha  
IMSc Chennai

# Network of Ecological Interactions

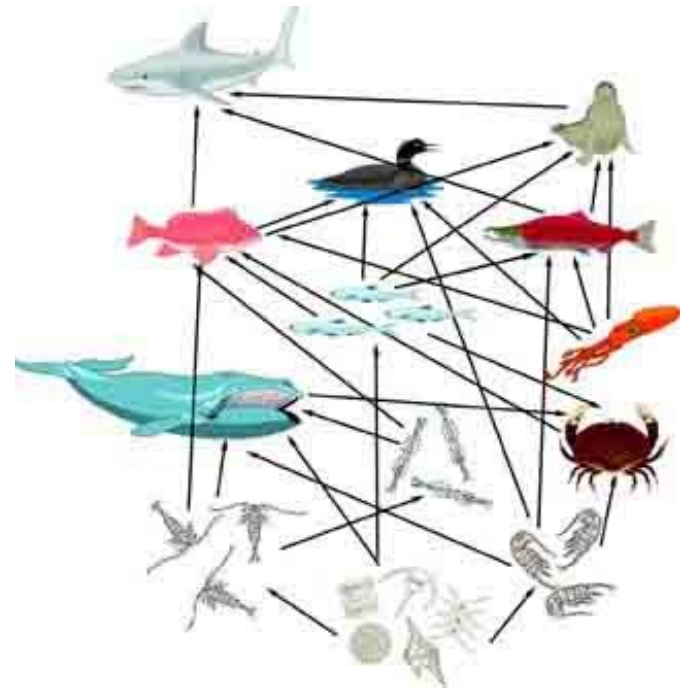
Alaskan food web



Simple food chains ...are embedded in more... Complex food webs



*Arrows indicate direction of energy flow*



# Early understanding of food webs



“Not a single plant, not even a lichen, grows on this island; yet it is inhabited by several insects and spiders”

Charles Darwin, 1839

“In February, 1832, Darwin described the food web of St. Paul's Rocks near the equator in the middle of the Atlantic Ocean, and remarked with surprise on the apparent absence of plants.”

J E Cohen (1994) in *Frontiers of Mathematical Biology* (ed S A Levin)

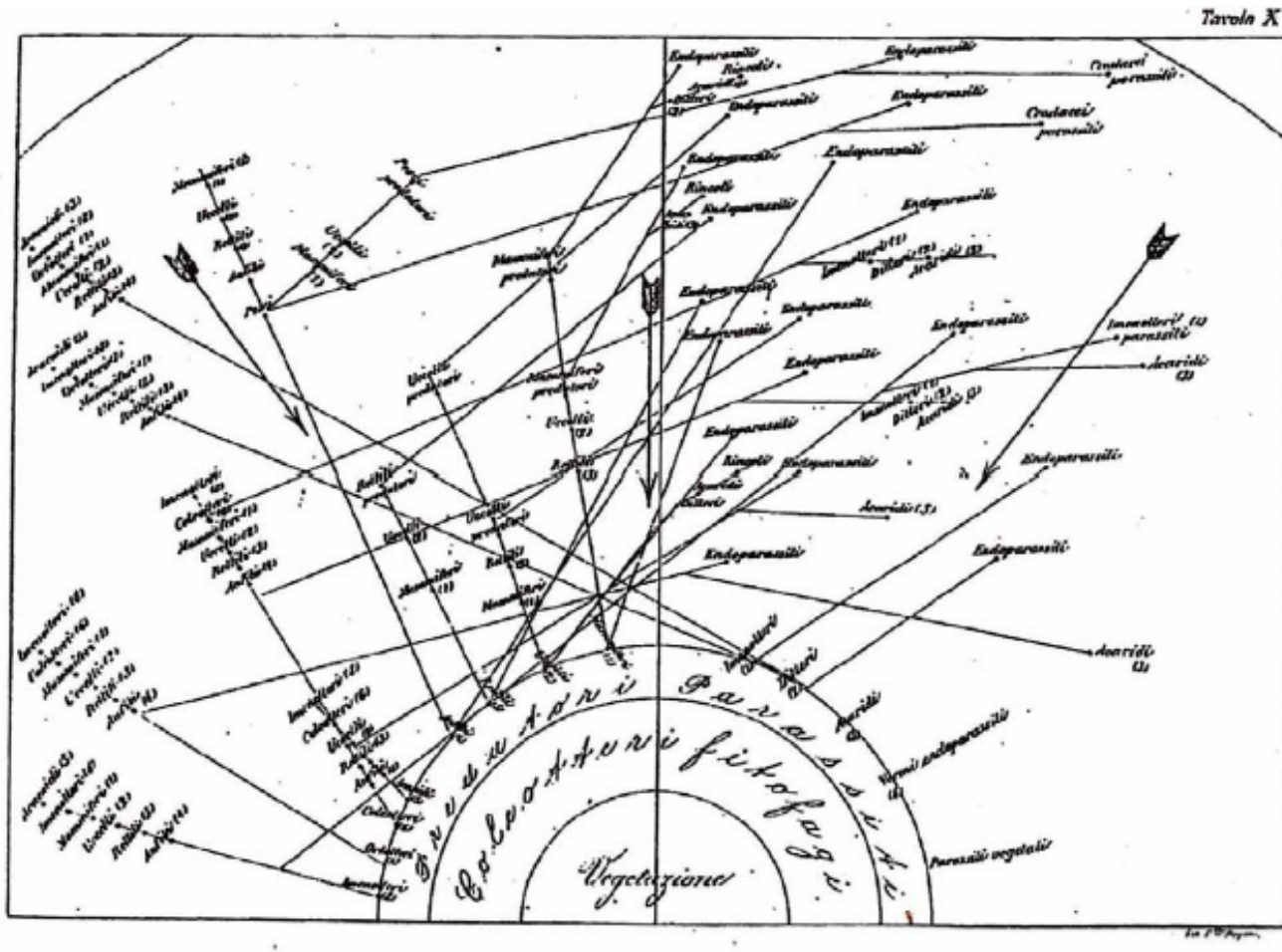
Abundance of each species maintained at a natural equilibrium:

“Möbius in 1877... recognized the importance of interspecific nutritive relationships while he was studying the organisms living on the oyster-beds of Schleswig-Holstein. To Möbius is due also the credit for noting that the effect of these interspecific relationships is to establish a state of equilibrium.”

U d'Ancona (1954) *The Struggle for Existence*

# First known network of trophic relations

Lorenzo Camerano (1880)



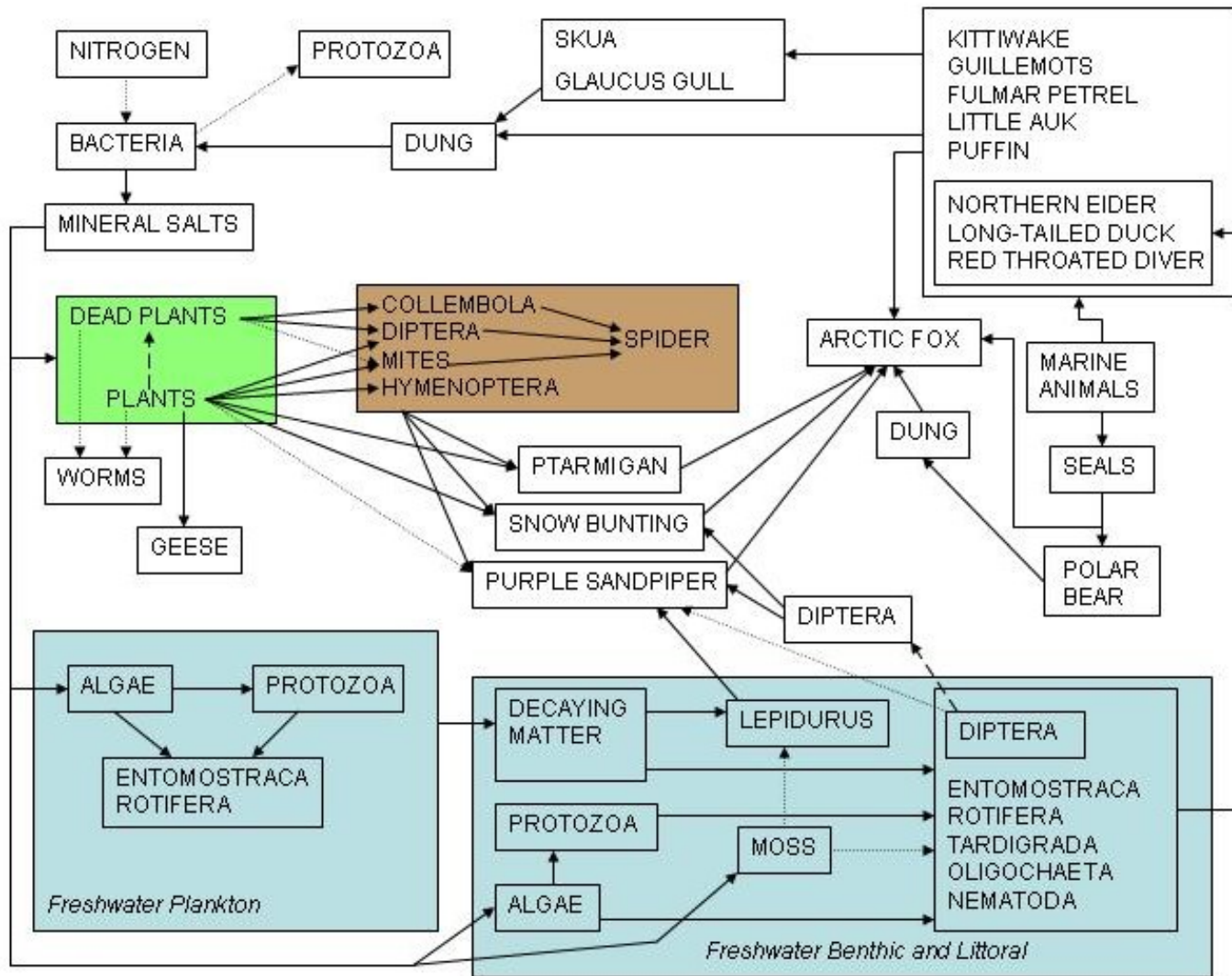
Network nodes  
classified into several  
taxa

Plants  
Parasitic plants  
Insects  
Worms  
Spiders  
Crustaceans  
Fish  
Amphibians  
Reptiles  
Birds  
Mammals

First graphical representation of a food web as a network of groups of species  
linked by feeding relations

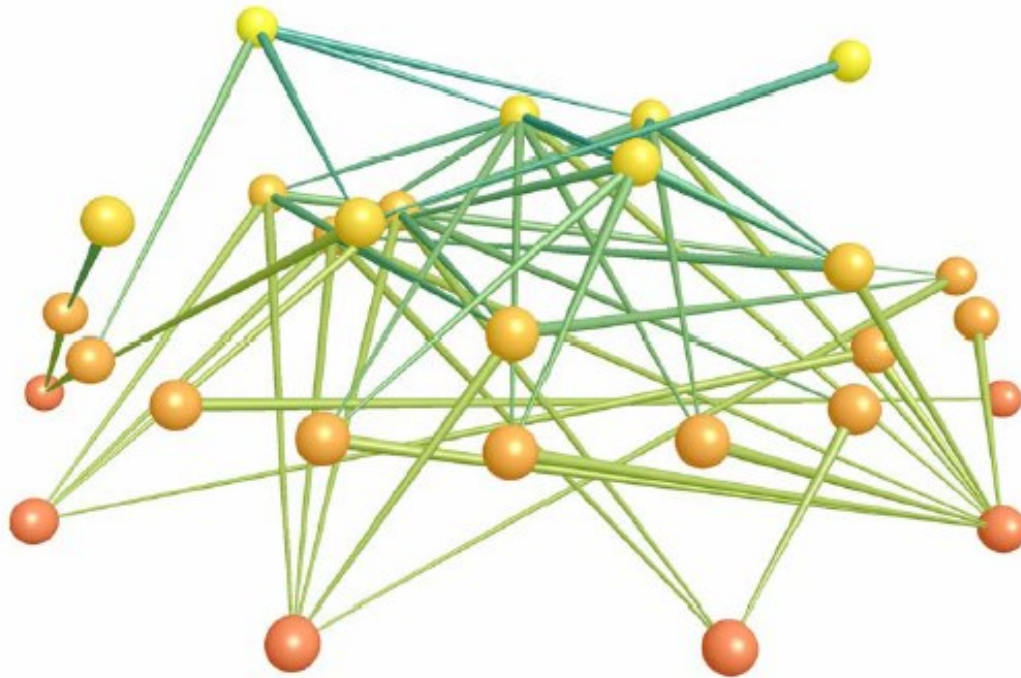
# Summerhayes and Elton (1923)

## Food web of Bear Island

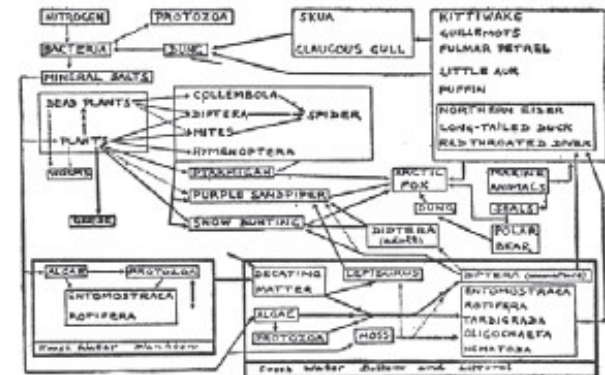




## Bear Island



1 bacteria, 4 autotrophs, 13 invertebrates, 6 birds, 4 mammals



$S$  (# taxa) = 28

$L$  (# links) = 59

$L/S$  (links/species) = 2.1

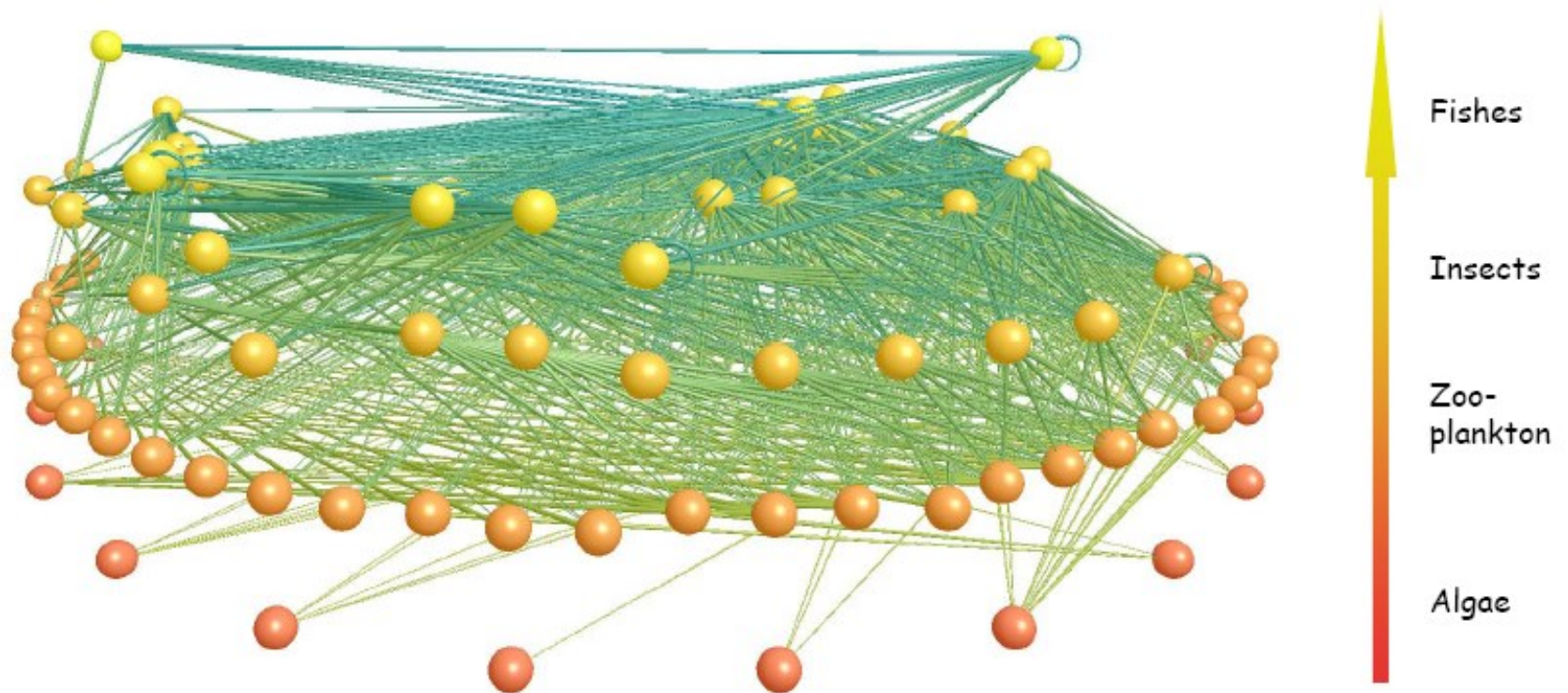
$C$  (connectance;  $L/S^2$ ) = 0.075

$TL$  (mean trophic level) = 2.07

**Directed Connectance ( $C$ ):** Proportion of possible links ( $S^2$ ) that are realized ( $L$ )

Source: Neo Martinez

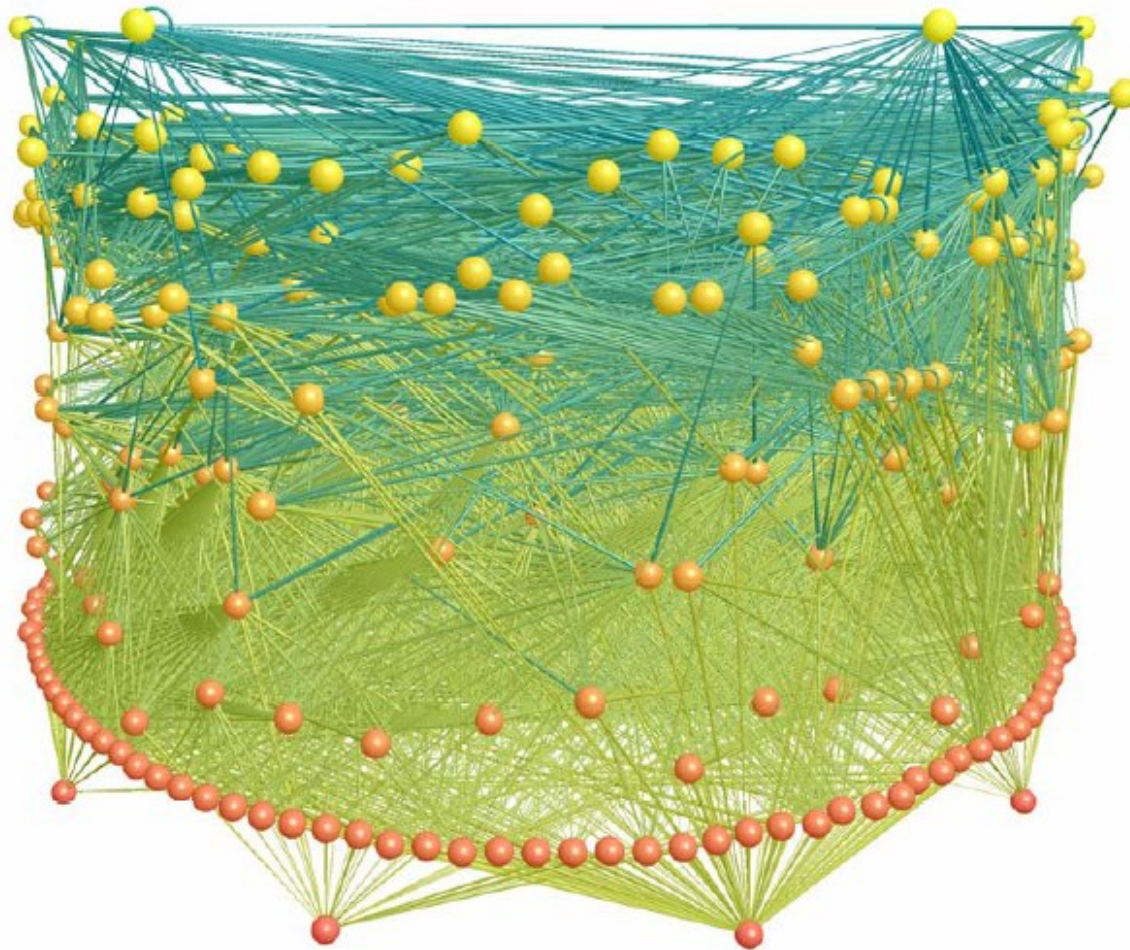
# Food web of Little Rock Lake, Wisconsin



$S = 92, L = 997, L/S = 11, C = 0.12, TL = 2.40$



# Antarctic Weddell Sea Food Web



Highly & Evenly Resolved

Original species = 492

62 autotrophs  
4 mixotrophs  
345 invertebrates  
48 ectotherm vertebrates  
29 endotherm vertebrates  
3 detritus  
1 bacteria

$S = 290$

$L = 7200$

$L/S = 24.8$

$C = 0.086$

Mean TL = 3.79

Data compiled by Ute Jacob

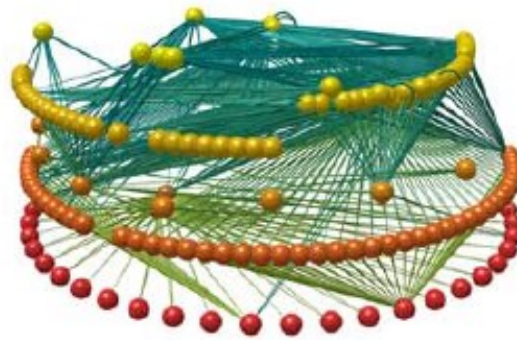
Source: Neo Martinez



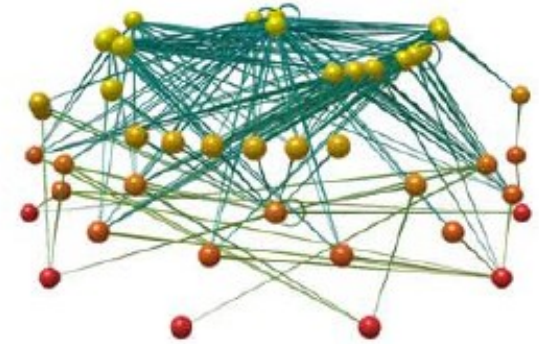
# Reconstructing Food Webs from the Cambrian Period

Dunne et al, PLoS Biology (2008)

## Burgess Shale

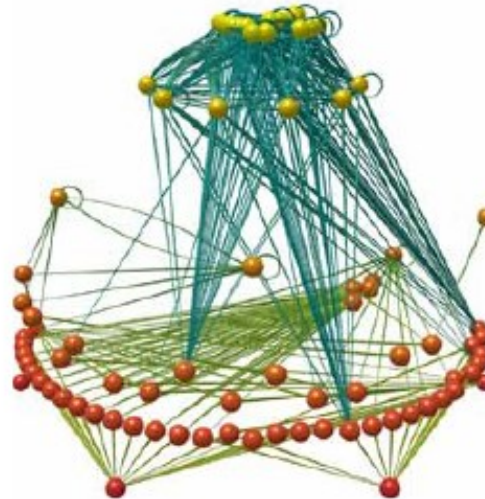


Original Species  
 $S = 142$ ,  $L = 771$ ,  $C = 0.038$   
 $TL = 2.42$ ,  $MaxTL = 3.67$

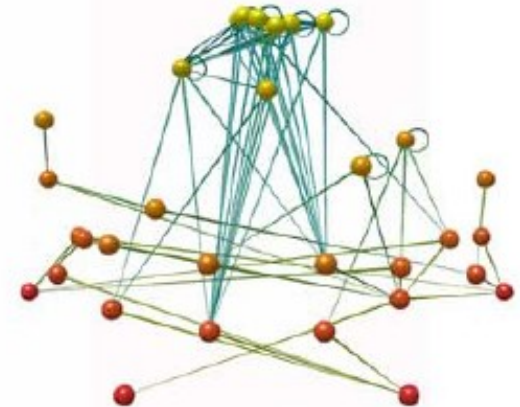


Trophic Species  
 $S = 48$ ,  $L = 249$ ,  $C = 0.108$   
 $TL = 2.72$ ,  $MaxTL = 3.78$

## Chengjiang Shale

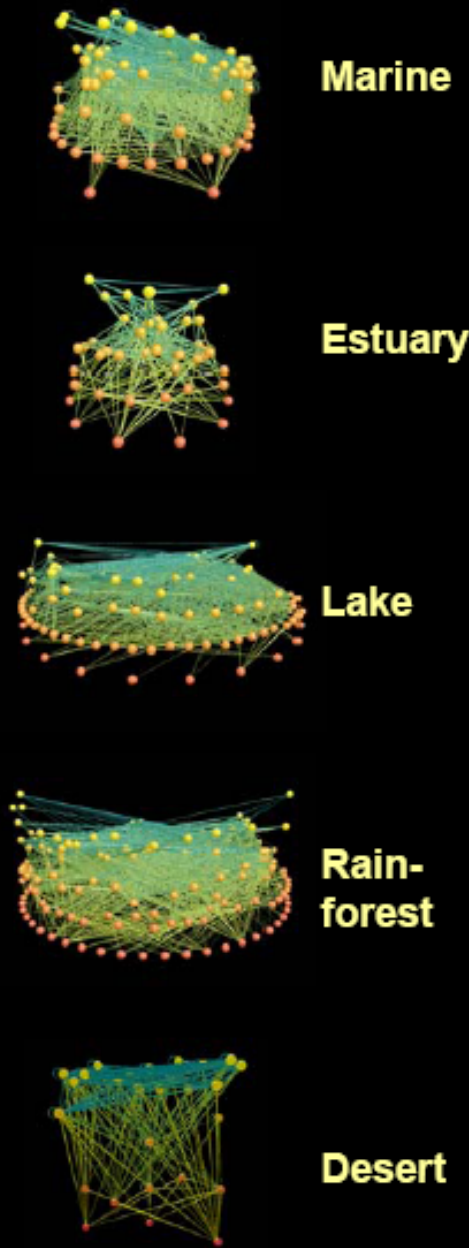


Original Species  
 $S = 85$ ,  $L = 559$ ,  $C = 0.077$   
 $TL = 2.99$ ,  $MaxTL = 5.15$



Trophic Species  
 $S = 33$ ,  $L = 99$ ,  $C = 0.091$   
 $TL = 2.84$ ,  $MaxTL = 4.36$

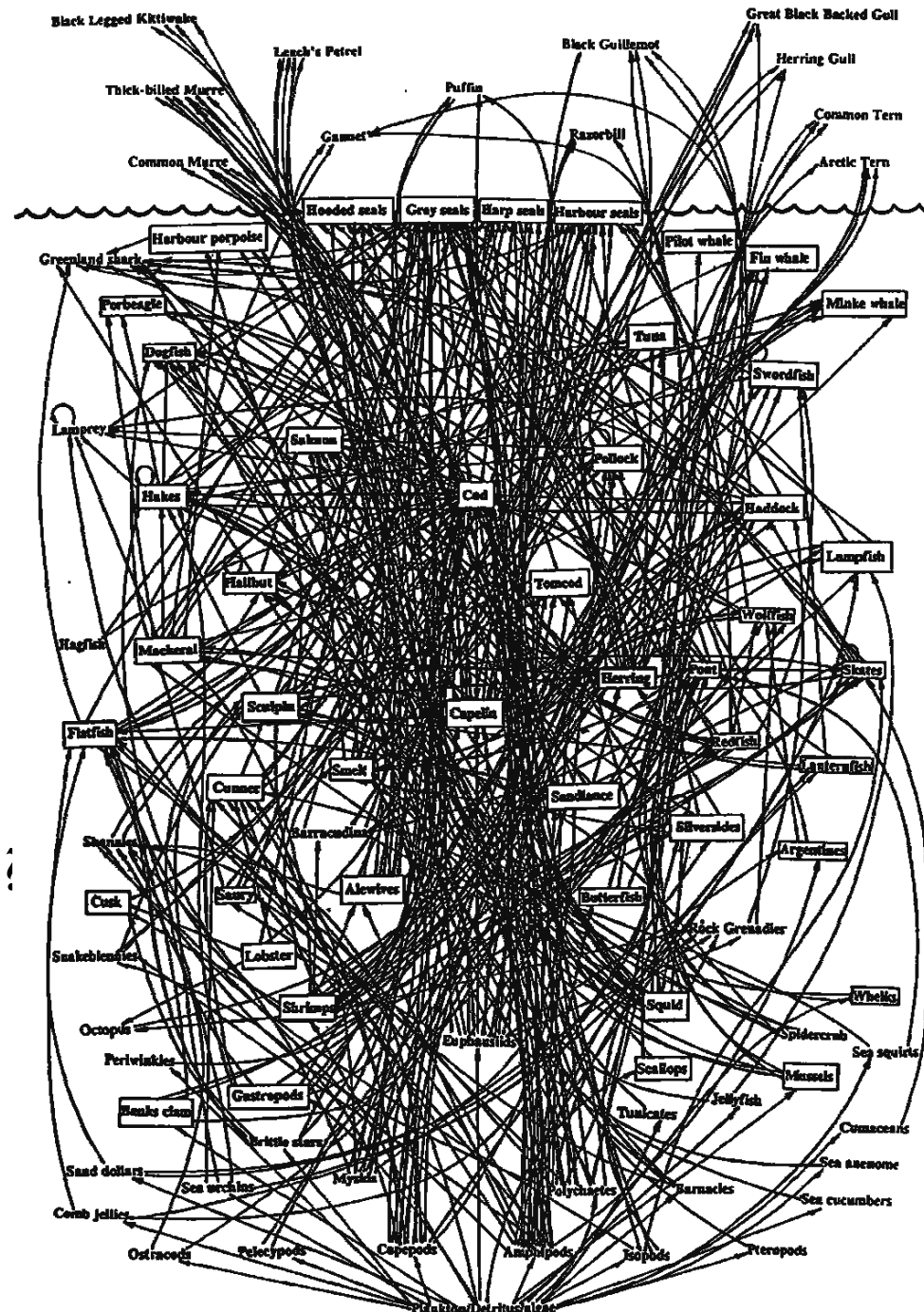
# A Systems-level Question



Instead of considering each food web in isolation as an unique case, is it possible to understand the general features of such networks ?  
To understand why and how such networks occur ?

# Robustness of Ecological Networks

- ❑ How do ecosystems collapse ?  
Cascades of extinction events triggered by small fluctuations
- ❑ Ecosystem management:  
Effect of human intervention ?
- ❑ Is higher diversity good or bad for the stability of the network ?
- ❑ How do robust networks emerge ?



The Scotian Shelf food web

# Are Complex Networks Unstable ?

Do complex networks become more susceptible to perturbations as:

- the number of nodes,
  - the density of connections, and,
  - the strength of interaction between the nodes,
- is increased ?

## Puzzle:

Theoretical results imply that complexity decreases stability, while observations (e.g., in ecology) sometimes show the opposite.

## But...

Most results were obtained assuming networks are random and at equilibrium (both at level of nodes as well as the network) !





# The Empiricists' View

Diversity is essential for maintaining network stability

**Charles Elton (1958)**

Simple ecosystems less stable than complex ones

Charles Elton (1900-1991 )

## Field observations:

- ☐ Violent fluctuations in population density more common in simpler communities.
- ☐ Simple communities more likely to experience species extinctions.
- ☐ Invasions more frequent in cultivated land.
- ☐ Insect outbreaks rare in diverse tropical forests – common in less diverse sub-tropical forests.

Robert MacArthur: theoretical attempt at justification

Multiple links  $\equiv$  Insurance !

# But ...

This view was challenged by:

- Numerical experiments on the stability of random networks by Gardner & Ashby (1970).
- Theoretical analysis of randomly constructed ecological networks by May (1972).

*Observation:* Stability decreases as network size, connectivity and interaction strength increases.

## The Theorist's View

Increasing diversity leads to network instability



Robert M May (1936- )

Basis for the **Stability vs. Diversity** debate in ecology.



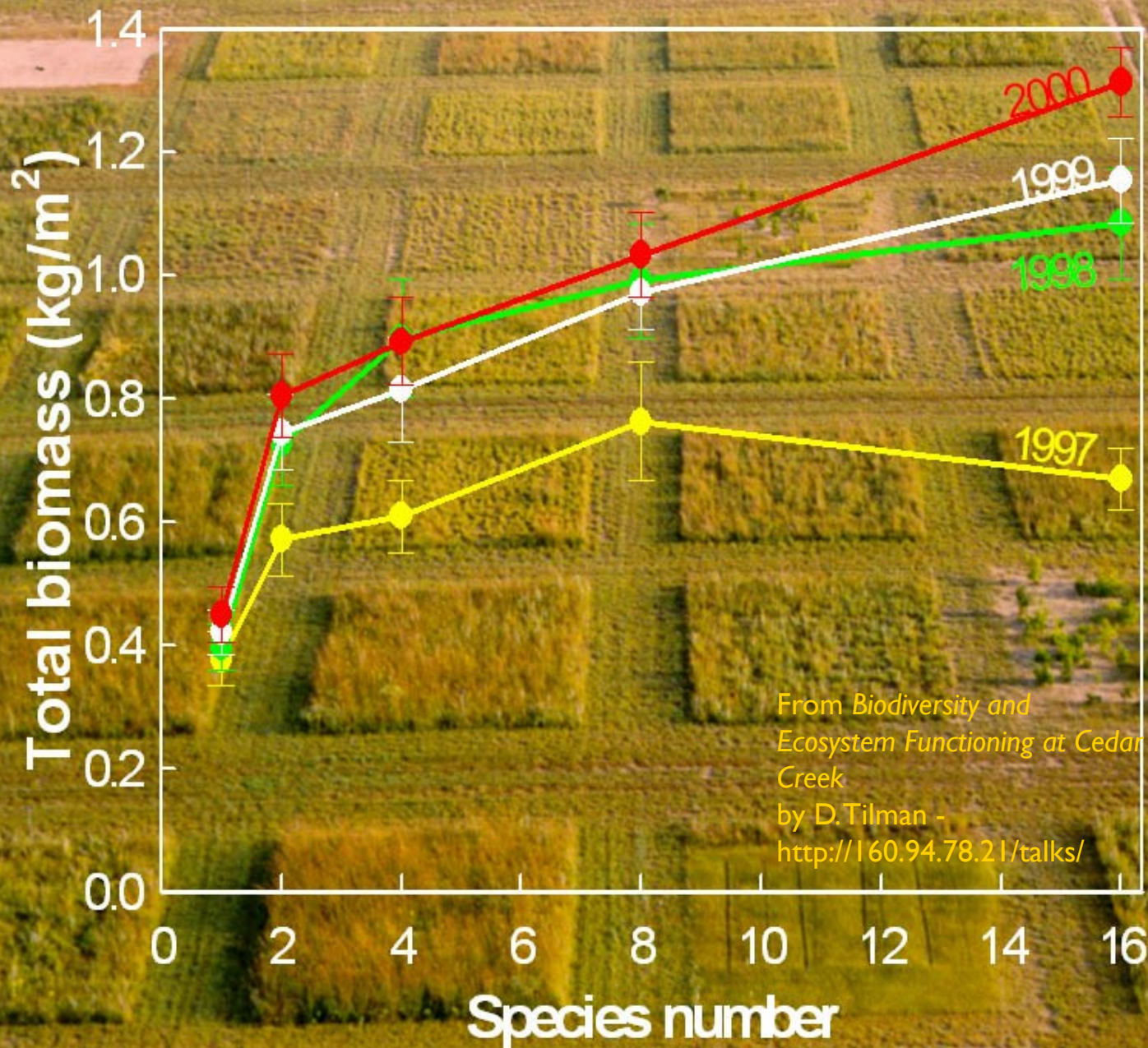


## Experimental evidence: *Common garden experiments (e.g. Cedar Creek)*

- ❑ diversity treatments divided over hundreds of experimental plots.
- ❑ examine response of population and community level biomass to environmental perturbation.

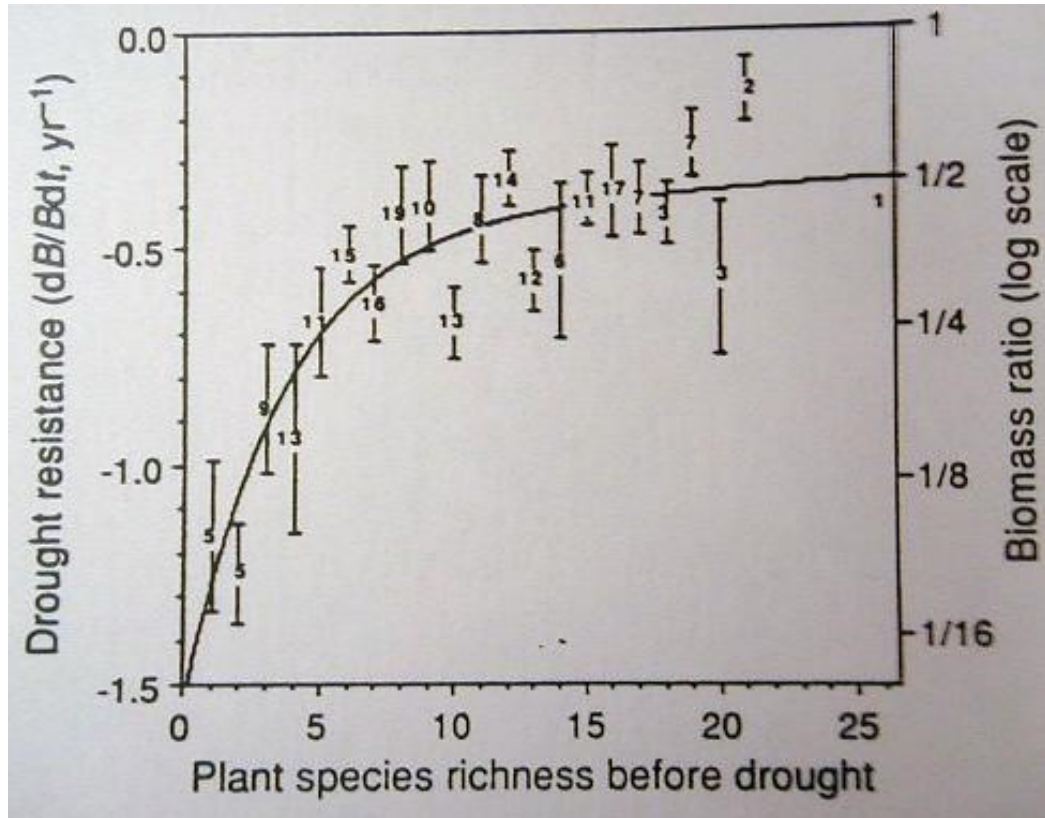


# Diverse Systems Are More Productive





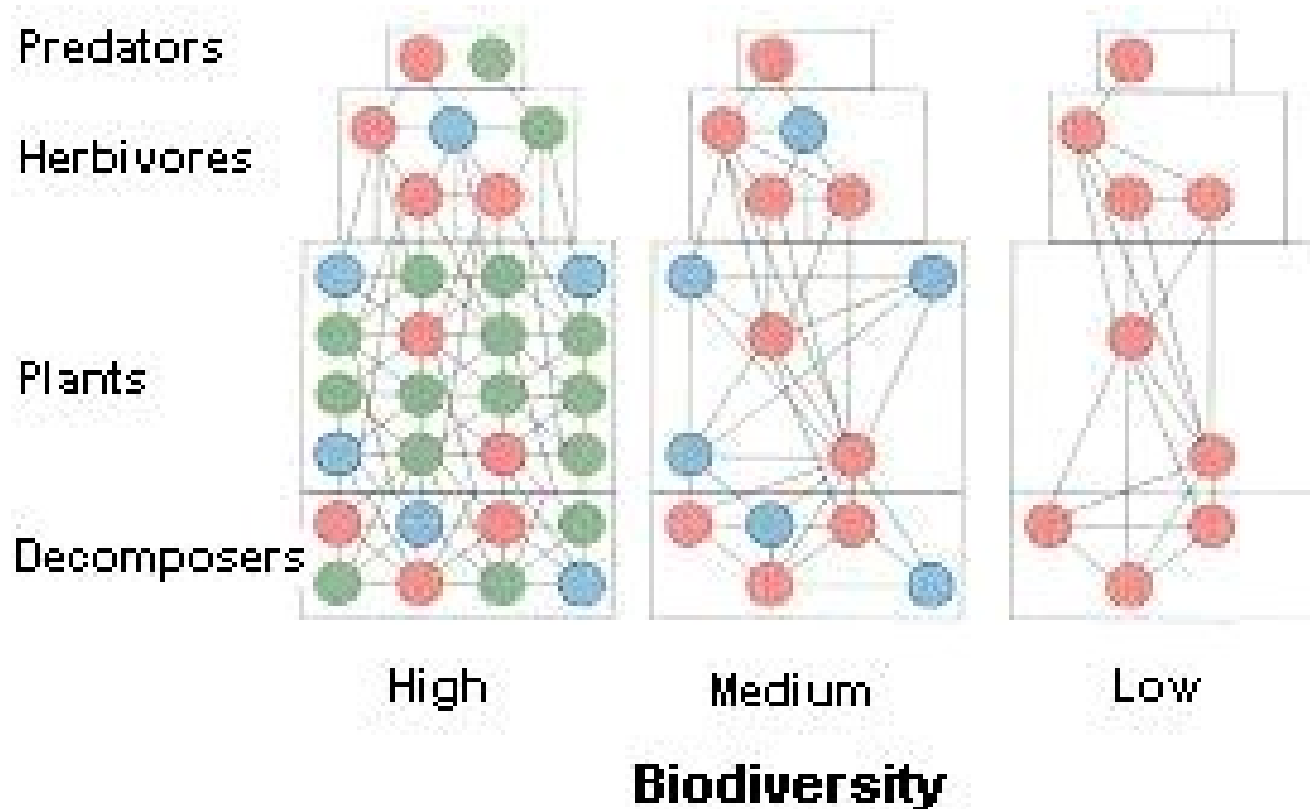
...and more resistant



Tilman et al. (1996)

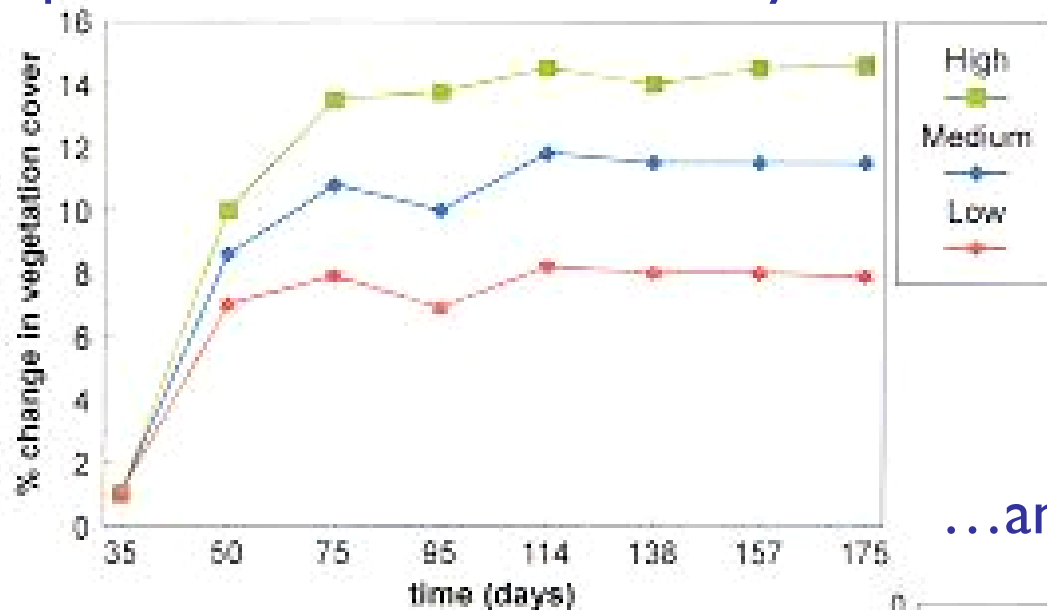
- ❑ but no effect on population variability.
- ❑ indicates averaging effect.

# Experimental evidence: *Bottle Experiments (e.g. Ecotron)*



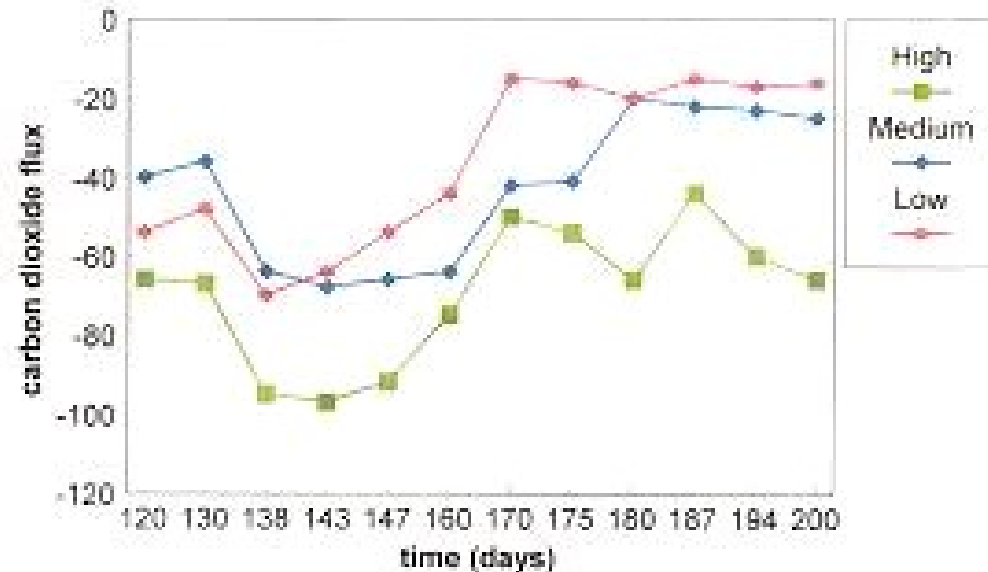
Setup allows manipulating diversity while maintaining food web structure.

High diversity communities are more productive than low diversity ones.



...and consume more CO<sub>2</sub>...

...but its unclear how these results scale to real communities.



# The Theorist's View

Increasing diversity leads to network instability

Consider a simple community of one predator and one prey

$$f_1 = \frac{dX}{dt} = X(a - bY)$$

$$f_2 = \frac{dY}{dt} = Y(-d + cX)$$

Taylor expansion around the equilibrium yields the Jacobian or “community matrix”

$$J = \begin{pmatrix} a & -bd/c \\ ac/b & -d \end{pmatrix}$$

The system is stable if the largest real component of the eigenvalues  $\text{Re}(\lambda_{\max}) < 0$ .



# The Theorist's View

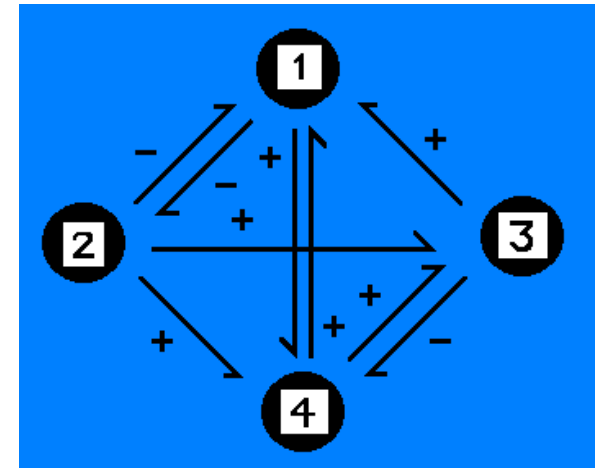
Increasing diversity leads to network instability

Will a Large Complex System be Stable?

ROBERT M. MAY

Robert May (1972) constructed *randomly* generated matrices representing interaction strengths in a network of  $N$  nodes whose isolated nodes are stable ( $J_{ii} = -1$ )

$$J = \begin{bmatrix} -1.15 & 0 & 0.33 \\ -1.66 & 0.17 & 2.18 \\ 0.12 & 0 & -0.14 \\ 0.29 & 0 & 0.75 \end{bmatrix}$$



obtained its eigenvalues  $\lambda \dots$

$\dots$ used the criterion that if  $\lambda_{\max} > 0$ , the system is unstable.

*Observation:* Stability decreases as network size, connectivity and interaction strength increases.

# Stability of large networks:

State of the network of  $N$  nodes:  $N$ -d vector  $x = (x_1, x_2, \dots, x_N)$ ,  $x_i$ : state of the  $i^{\text{th}}$  node.

Time evolution of  $x$  is given by a set of equations (e.g., Volterra-Lotka)

$$d x_i / d t = f_i (x) \quad (i = 1, 2, \dots, N)$$

Fixed point equilibrium of the dynamics :  $x^0 = (x^0_1, x^0_2, \dots, x^0_N)$  such that  $f(x^0) = 0$

Local stability of  $x^0$  : Linearizing about the eqblm:  $\delta x = x - x^0$

$$d \delta x / d t = A \delta x \text{ where Jacobian } A: A_{ij} = \partial f_i / \partial x_j |_{x=x^0}$$

Long time behavior of  $\delta x$  dominated by  $\lambda_{\max}$  (the largest real part of the eigenvalues of  $A$ )  
 $|\delta x| \sim \exp(\lambda_{\max} t)$

The equilibrium  $x = x^0$  is stable if  $\lambda_{\max} < 0$ .

What is the probability that for a network,  $\lambda_{\max} < 0$  ?

Each node is independently stable  $\Rightarrow$  diagonal elements of  $A < 0$  (choose  $A_{ii} = -1$ ).

Let  $A = B - I$  where  $B$  is a matrix with diagonal elements 0 and  $I$  is  $N \times N$  identity matrix.

For matrix  $B$ , the question: What is the probability that  $\lambda'_{\max} < 1$  ?

# Applying Random Matrix Theory:

Simplest approximation: **no particular structure** in the matrix  $B$ , i.e.,  $B$  is a random matrix.

$B$  has **connectance**  $C$ , i.e.,  $B_{ij} = 0$  with probability  $1 - C$ .

The non-zero elements are independent random variables from  $(0, \sigma^2)$  Normal distribution.

For large  $N$ , **Wigner's theorem** for random matrices apply.

Largest real part of the eigenvalues of  $B$  is  $\lambda'_{\max} = \sqrt{(N C \sigma^2)}$ .

For eigenvalues of  $A$ :  $\lambda_{\max} = \lambda'_{\max} - 1$

For large  $N$ , probability of stability  $\rightarrow 0$  if  $\sqrt{(N C \sigma^2)} > 1$ ,  
while, the system is **almost surely stable** if  $\sqrt{(N C \sigma^2)} < 1$ .

Large systems exhibit **sharp transition** from stable to unstable behavior when  $N$  or  $C$  or  $\sigma^2$  exceeds a critical value.

Numerical computations in good agreement with theory  
(Gardner & Ashby, 1970; May, 1973).

Early empirical data  
supporting May  
(McNaughton, 1978)

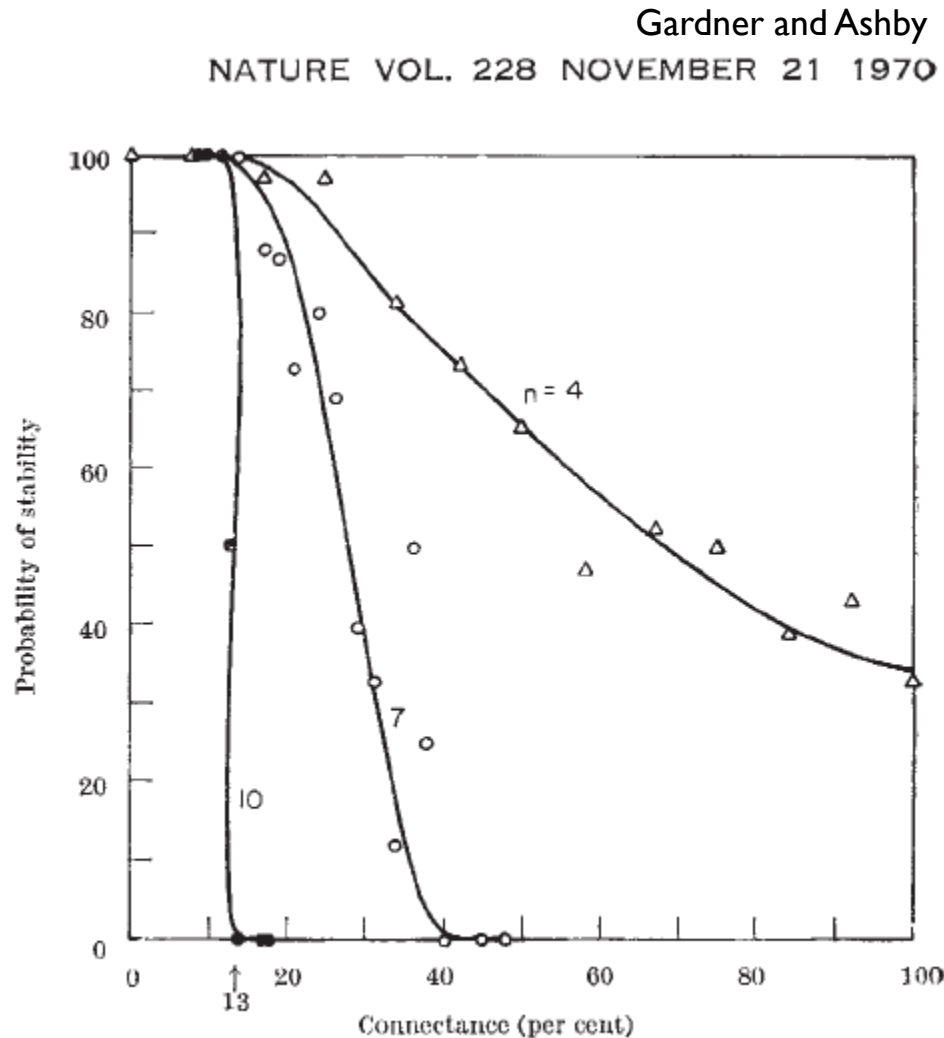
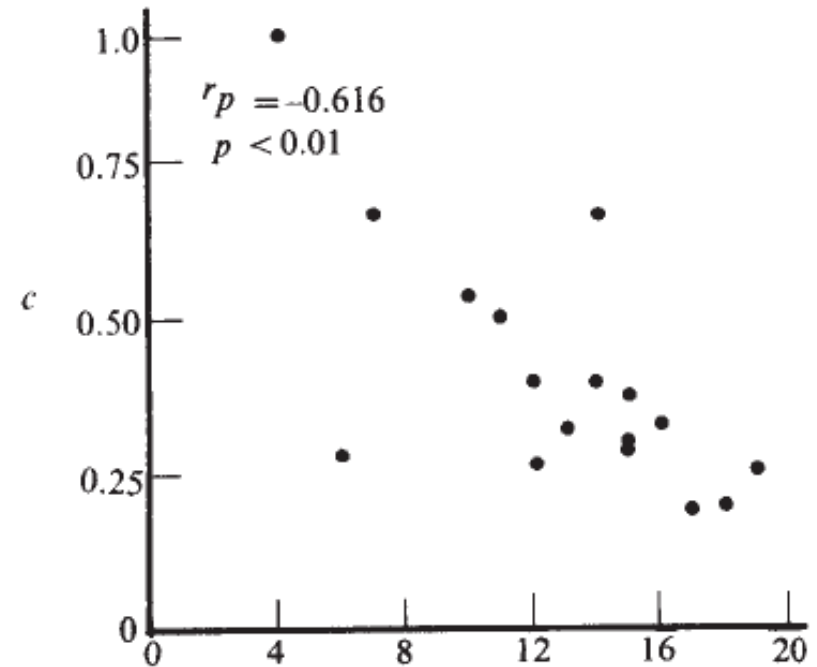


Fig. 1. Variation of stability with connectance.



Inverse relation between  
connectance and number of  
species in grassland samples  
from Serengeti (Tanzania)  
May-June 1977

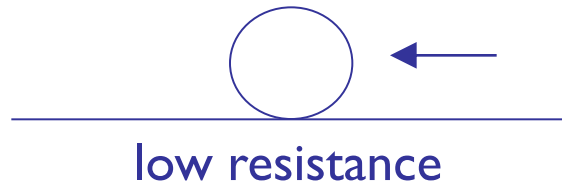
# Other “stabilities”

## The complexity and stability of ecosystems

Stuart L. Pimm

**Global stability** : system is stable if it returns to equilibrium after any perturbation (large or small) – size of basin of attraction

**Resistance** : the ability of a community to resist change in the face of a potentially perturbing force.



**Resilience** : the ability of a community to recover to normal levels of function after disturbance.



**Variability** : the variation in population or community densities over time. Usually measured as the coefficient of variation ( $CV = \text{mean} / \text{variance}$ )



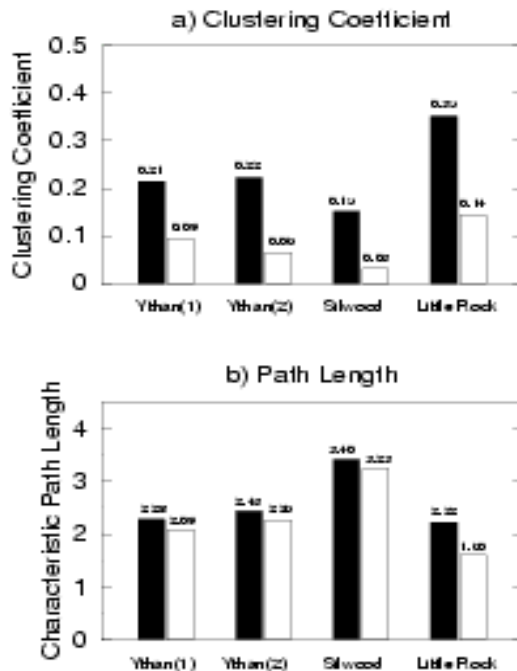
In nature, networks are not **random** –  
many have certain **structural patterns**

So...

How does network topological  
structure affect dynamical stability ?

# Small World Structure in Ecological Networks ?

Montoya and Sole (2001)



Network analysis of some food webs:

- Ythan estuary : freshwater-marine interface
- Silwood Park: field site
- Little Rock Lake: freshwater habitat

High clustering → small-world !

Challenged by Dunne et al (2002): Analysis of 16 food webs  
“Most food webs do not display typical small-world topology”

Does small-world topology affect the stability of a network ?

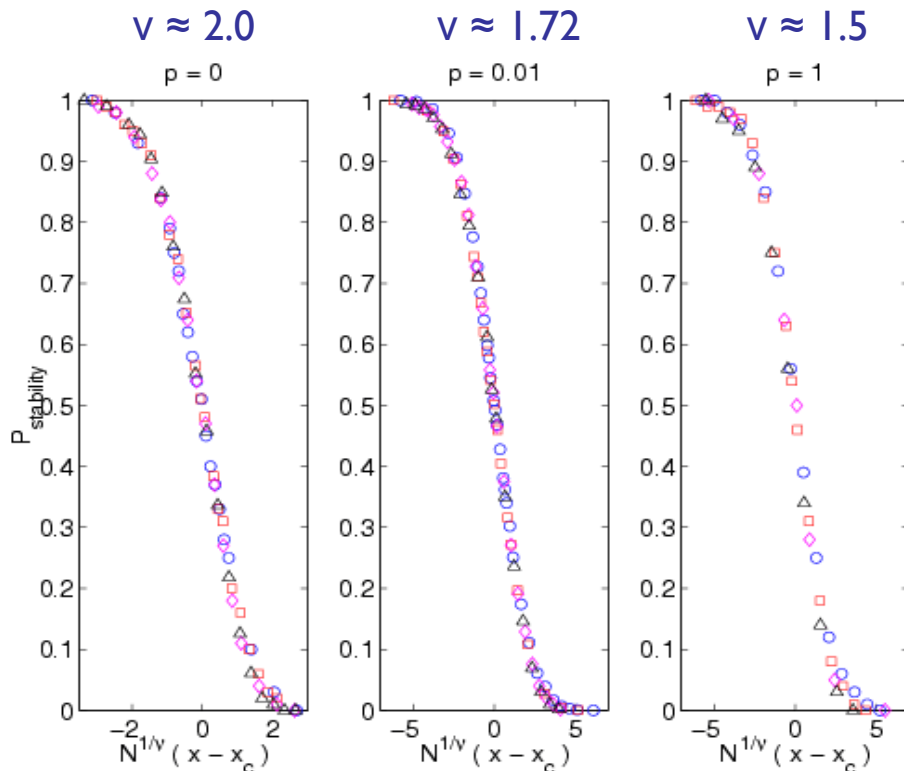
## Question:

Does small-world topology affect the stability of a network ?

**Answer: NO!** (Sinha, 2005)

Probability of stability in a network

Finite size scaling:  $N = 200, 400, 800$  and  $1000$ .

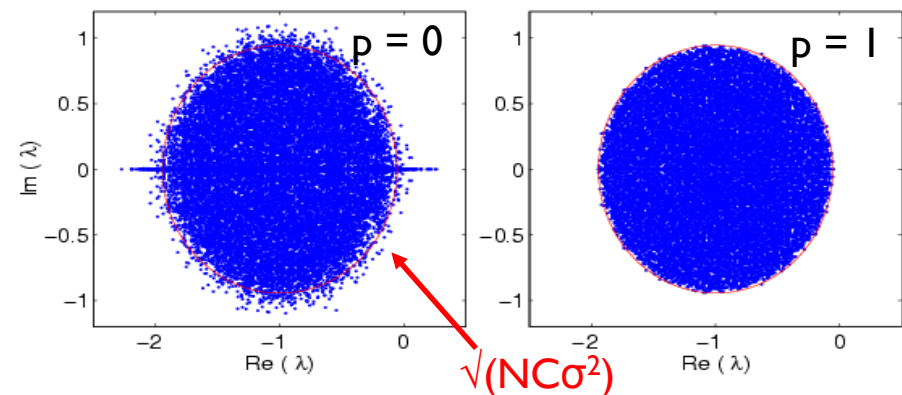


Regular vs Random Networks

The stability-instability transition occurs at the same critical value as random network ....

**but** transition gets sharper with randomness

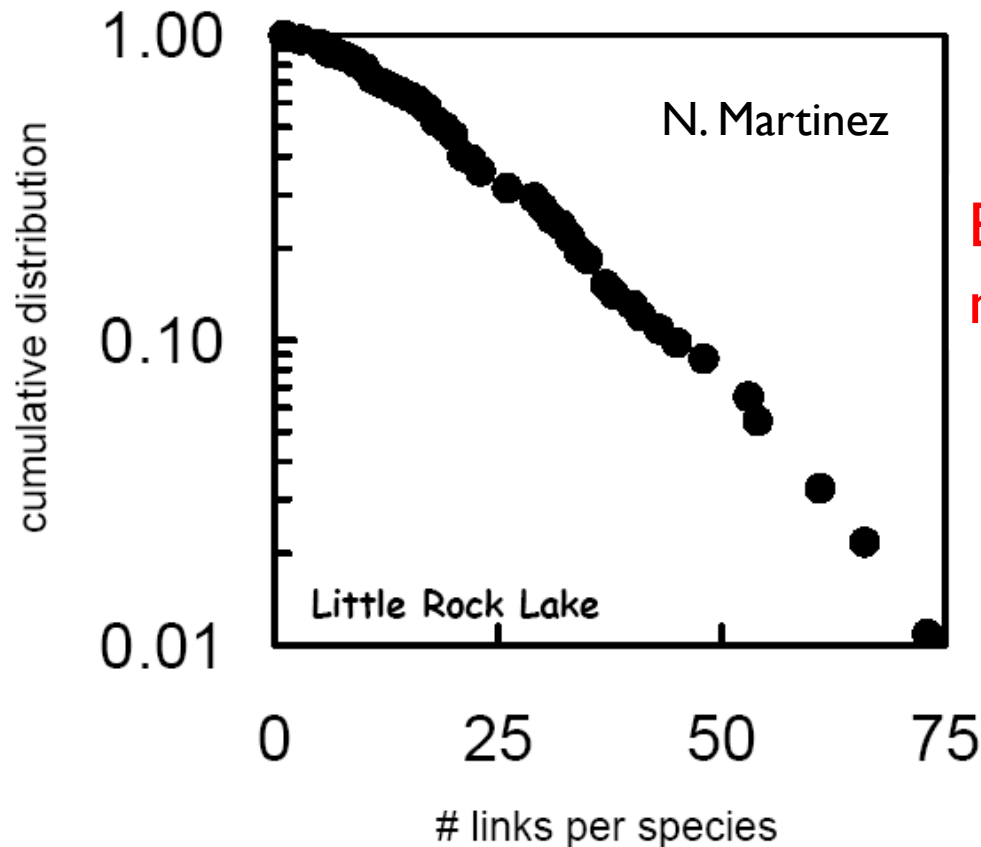
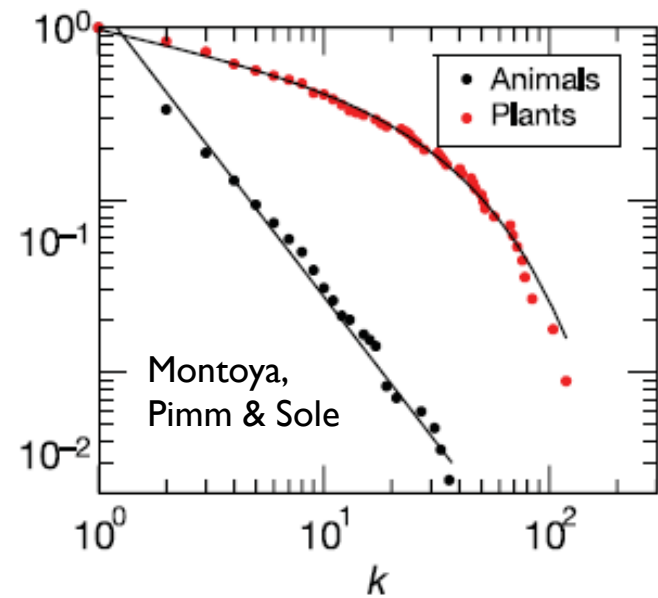
The eigenvalue plain



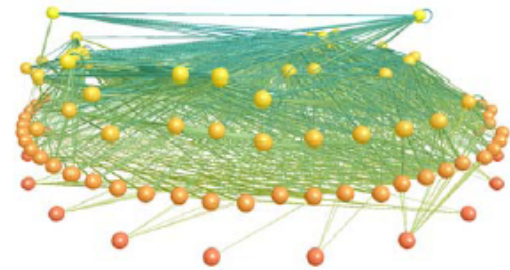
$N = 1000, C = 0.021, \sigma = 0.206$

# Scale-Free Degree Distribution in Ecological Networks ?

Montoya and Sole (2006): power-law distribution ! [Kyoto plant-pollinator web]  
Challenged by Martinez et al



Exponential distribution,  
not power law



N Martinez

So how can complex networks be robust at all ?

We have not yet considered the dynamics of networks!

## Possible solution: Network Evolution

Predator Adaptation or Prey Switching at short time scales

The trophic links between species may change depending on their relative densities

Community Assembly at long time scales

Networks do not occur fully formed but gradually evolve over time



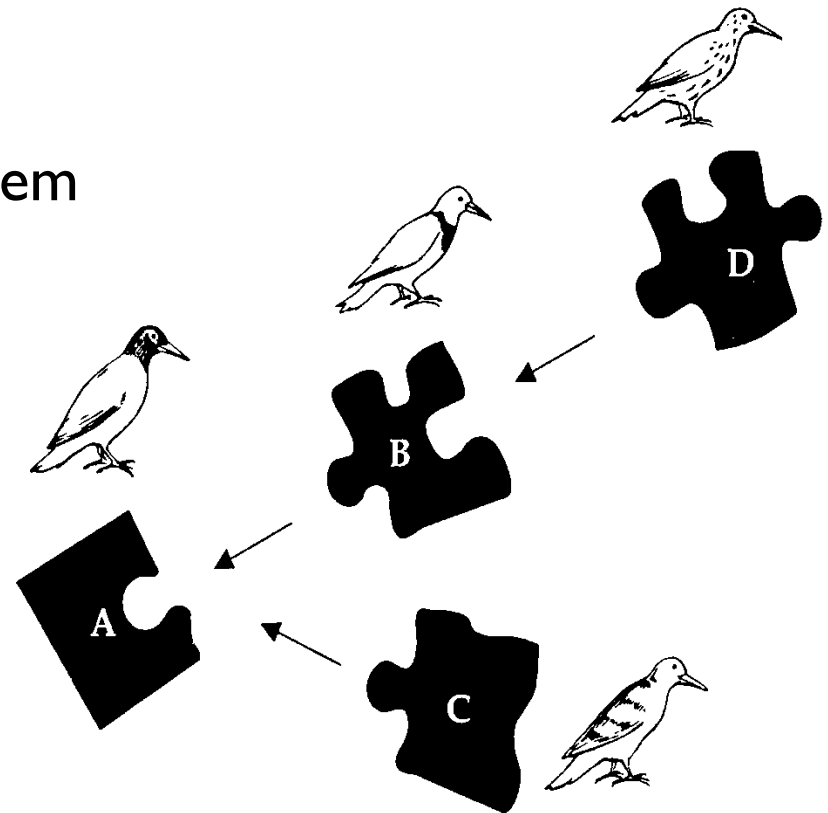
# Example:

## Assembling ecological communities

How are ecological networks gradually organized over time by species introduction and/or extinction ?

Community Assembly Rules decide

- ☐ which species can coexist in a system
- ☐ the sequence in which species are able to colonize a habitat



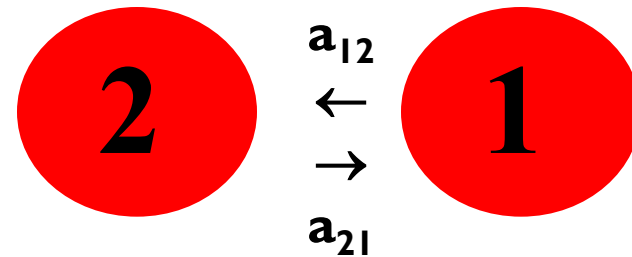
E. O. Wilson

# Network Evolution

## WSB Network Assembly Model

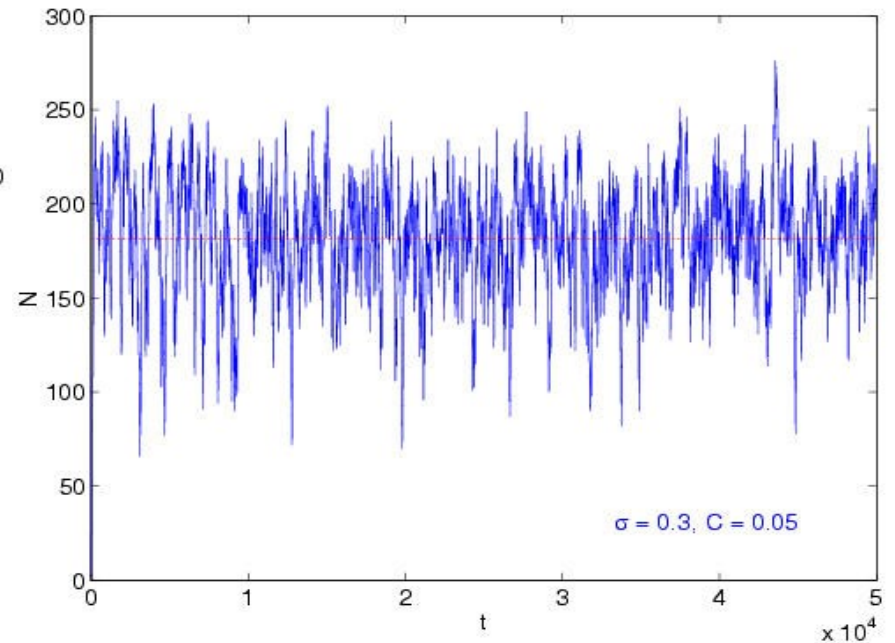
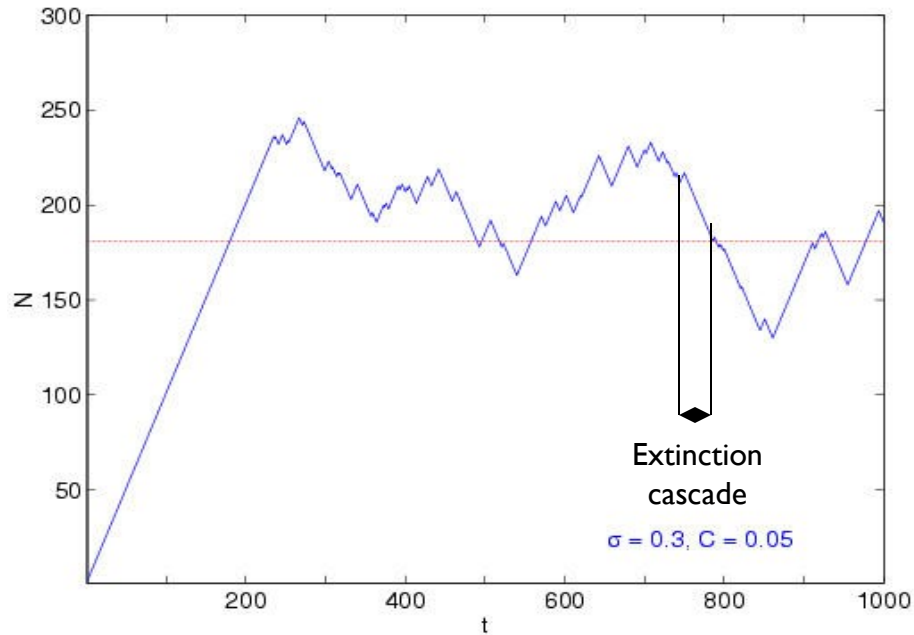
(Wilmers, Sinha & Brede, 2002)

- Start with one node.



- Add another node with random number of links, and randomly chosen interaction strengths  $a_{ij}$ .
- Check stability of the resultant network :
  - If unstable, remove a node at random and analyze the stability again.
  - If stable, add another node.

Network initially grows in size monotonically ...

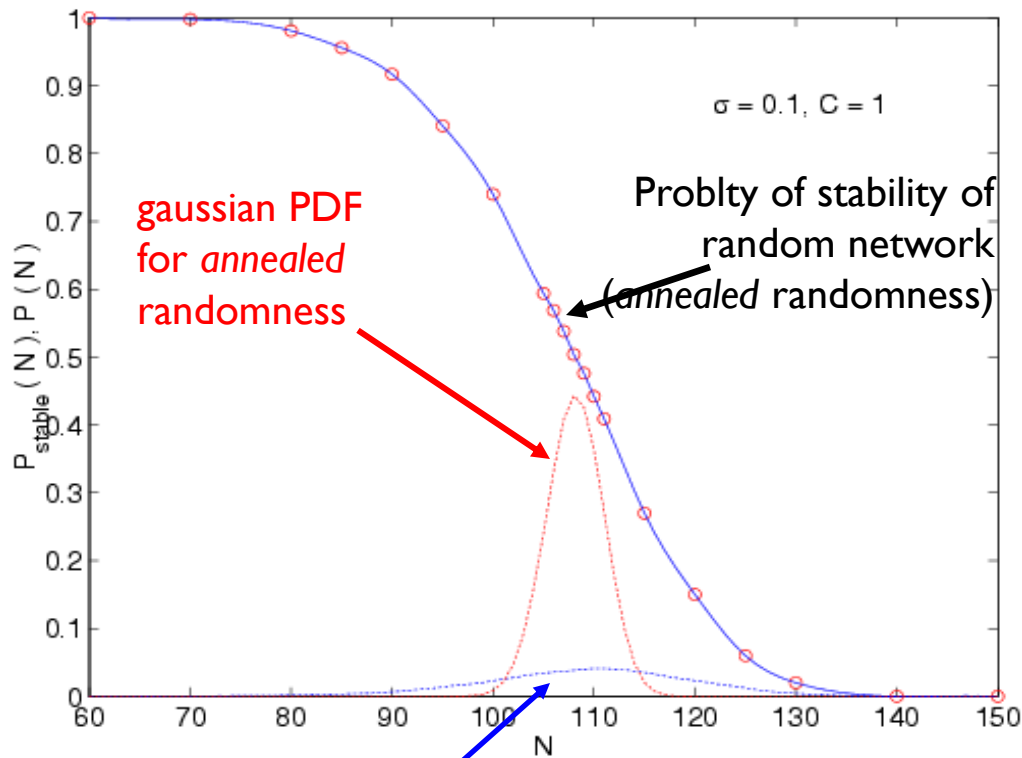


...and then settles down to a pattern of growth spurts & collapses

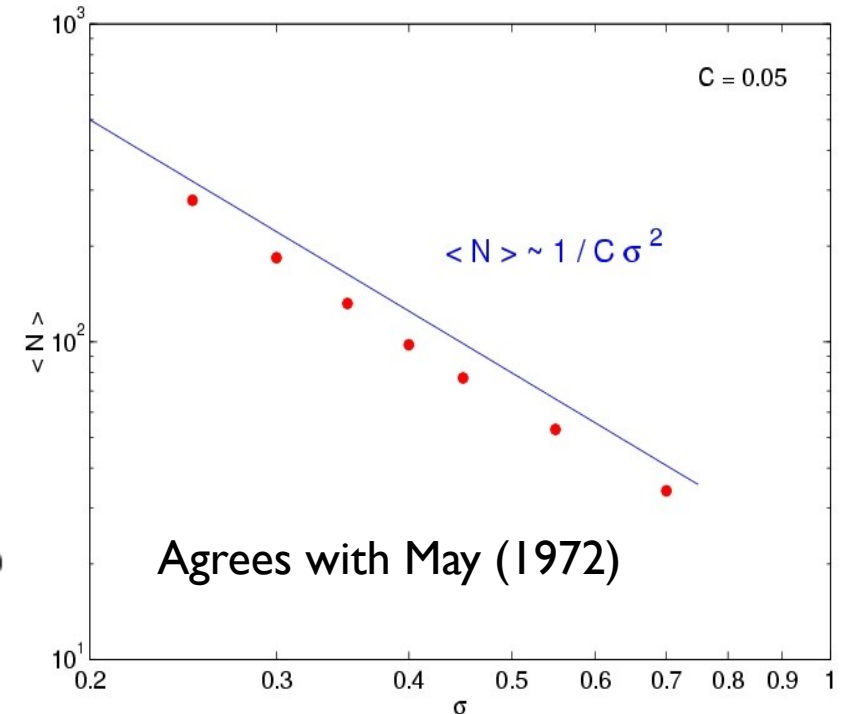
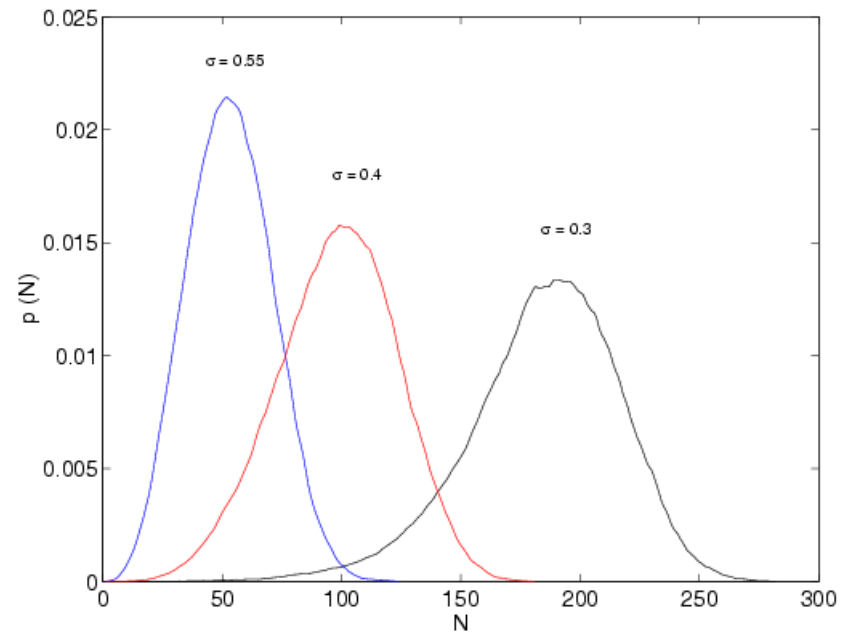
Communities with overall weaker interactions support a larger mean number of species

→ weak links are stabilizing (R. May).

The randomness in network connectivity is *quenched* → long-range memory!

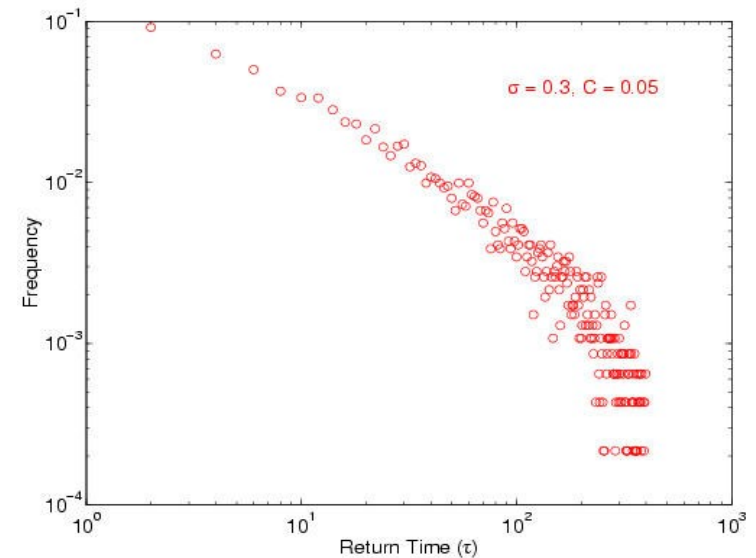
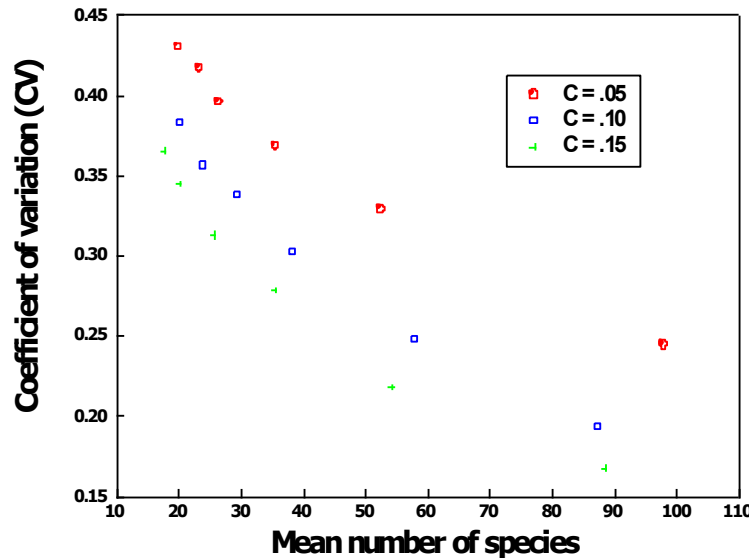


observed PDF for *quenched* randomness



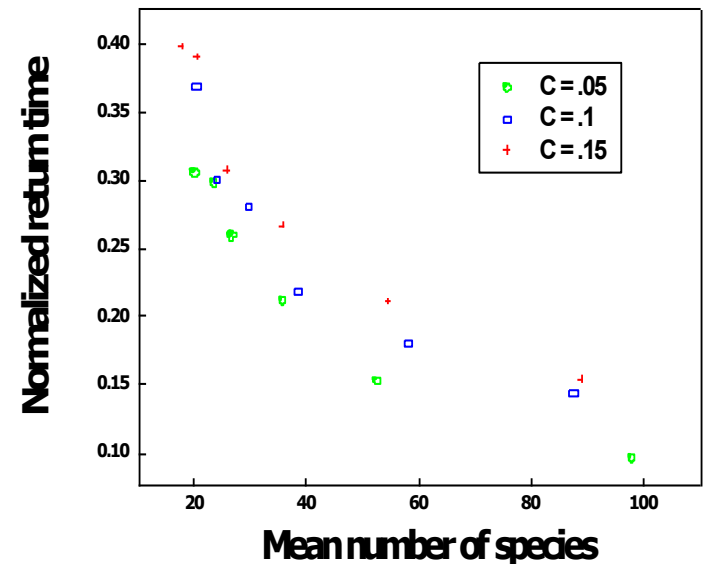
# Surprise!

For the evolved networks : complexity  $\rightarrow$  robustness



Larger networks are

- less variable (i.e., more robust)
- more resilient  
(resilience = normalized mean return time to average network size)

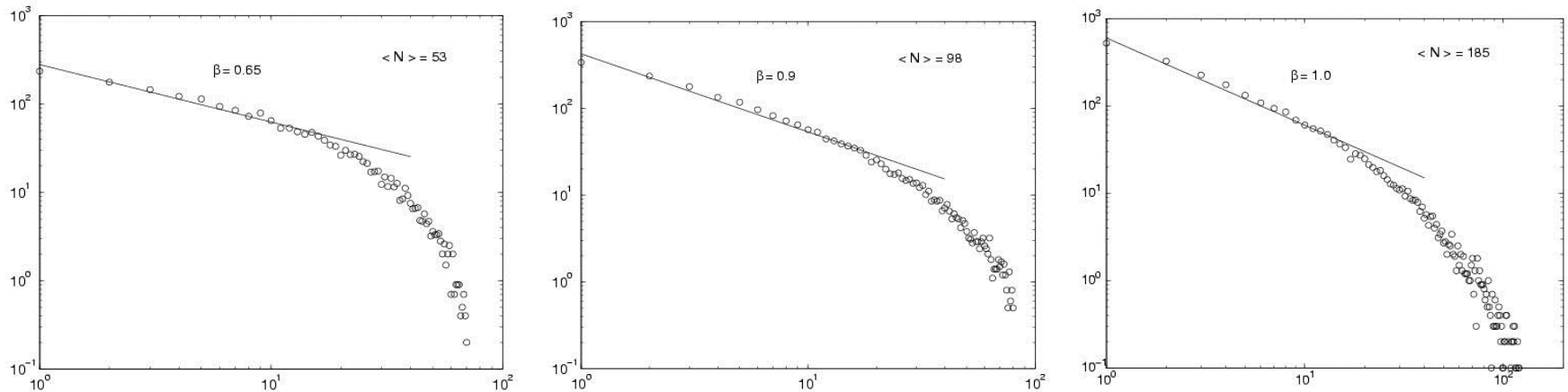




# Surprise!

For the evolved networks : complexity  $\rightarrow$  robustness

Frequency Distribution of Extinction Cascades:



Larger networks have smaller chance of a large magnitude collapse  $\rightarrow$  **increased resistance**

# Implications

❑ Introducing explicit dynamics and/or complex structure into networks: does not change the likelihood of a network to become unstable at increased complexity

❑ Introducing network evolution → Complex yet stable networks can evolve !

❑ The results imply that the traditional approach of taking snapshot views of networks may be inadequate to build an understanding of their stability.

