Systems Biology: A Personal View
XVI. Interactions in Ecology

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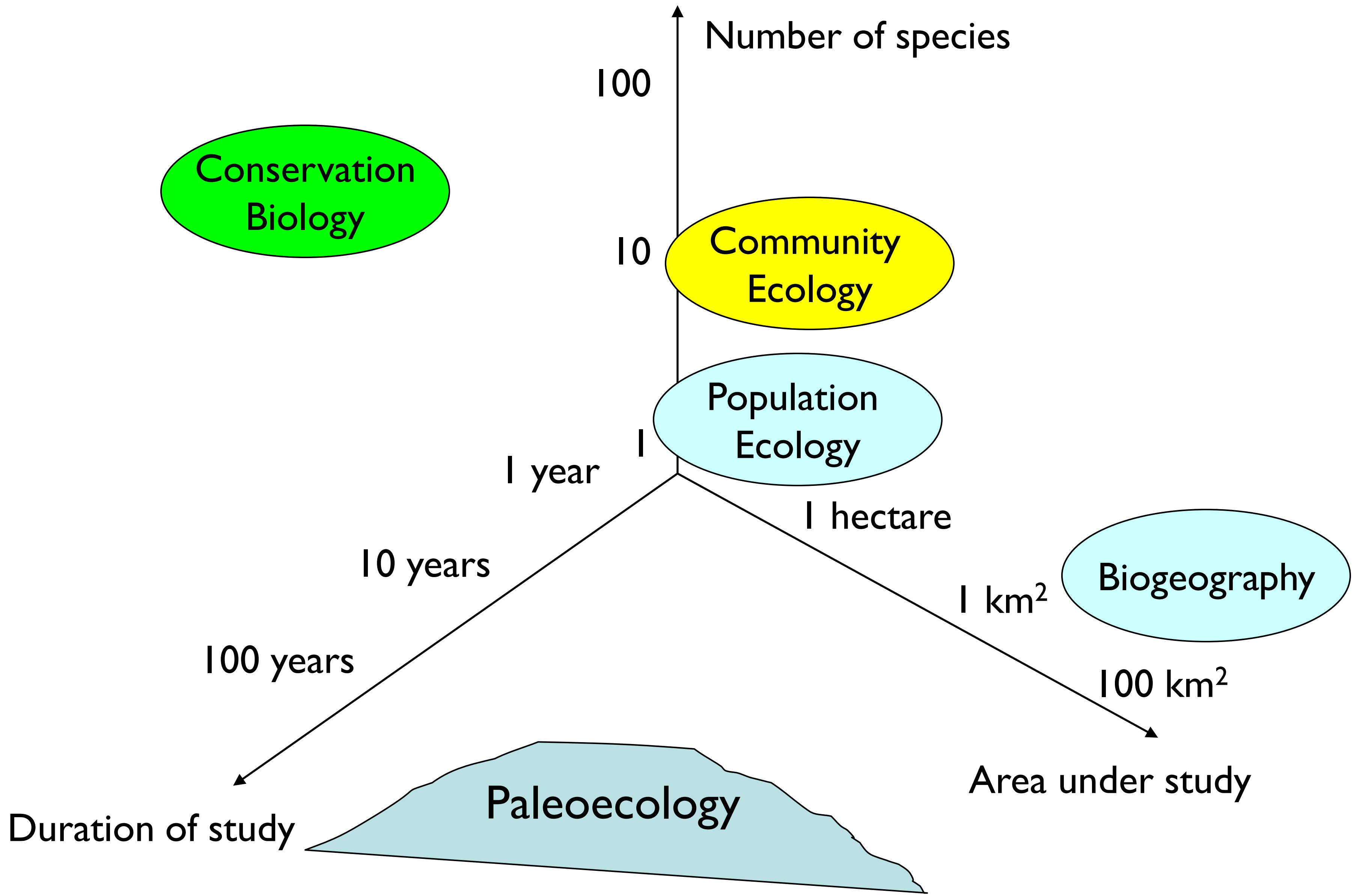
Spatial & Temporal Scales in Ecological Studies



Adapted from Pimm, *The Balance of Nature ?*

Modeling population dynamics of species

A single species: population dynamics with intra-species competition represented by a logistic term
(like mean-field theory, we assume its environment to be fixed)

$$dx/dt = r\ x\ [1 - (x/K)]$$

Verhulst (19th century)

r: Growth rate, K: carrying capacity

Introducing interactions between species

Mutual competition between two species using the same resources can be modeled as:

$$dx/dt = r\ x\ [1 - (x/K) - (x'/K)]$$

But interactions can be of different kind, viz., predator-prey !

2 species Resource-Consumer Dynamics

- Prey-Predator trophic relations
- Host-parasite
- Plant-Herbivore

In general can be of the form:

$$\frac{dx}{dt} = \phi(x) - g(x, y)\, y$$
$$\frac{dy}{dt} = n(x, y)\, y - d\, y$$

$\phi(x)$: growth of prey in absence of predators
$g(x,y)$: capture rate of prey per predator
$n(x,y)$: rate at which captured prey is converted into predator population increase – assumed to be $\varepsilon\, g(x,y)$ where ε is efficiency
d: rate at which predators die in absence of prey

Different choices of the functional forms can give different models

Lotka-Volterra Model (1920-6)

Alfred Lotka

The first model of consecutive chemical reactions giving rise to oscillations in molecular concentrations

$$A + X \rightarrow 2X$$
$$X + Y \rightarrow 2Y$$
$$Y \rightarrow P$$

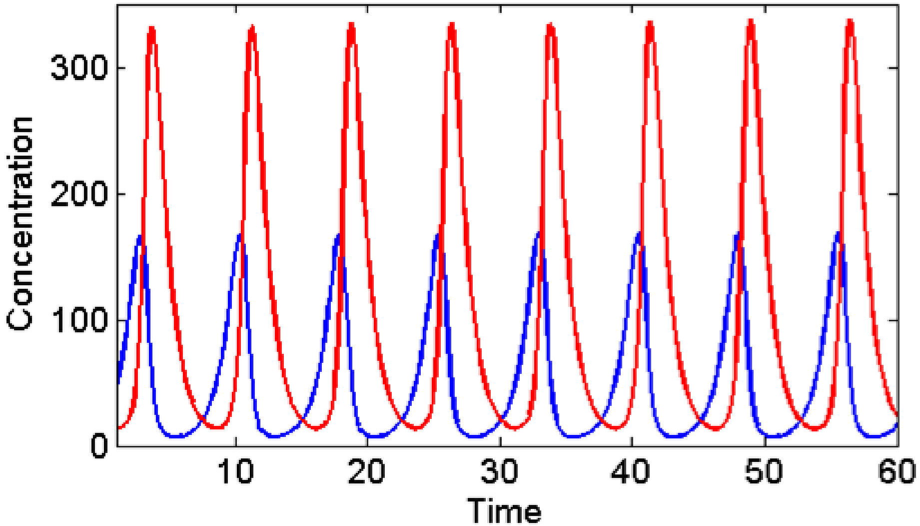
(*Autocatalysis:* +ve feedback)

More reasonable in the context of ecology

Prey (x) – Predator (y) eqns

$$dx/dt = r\ x - k_{xy}\ x\ y$$
$$dy/dt = k_{yx}\ x\ y\ - d\ y$$

Volterra (1926)



Lotka-Volterra Model (1920-6)

D'Ancona: The problem of high proportion of predator fishes in the Adriatic Sea during WWI (1914-1918)

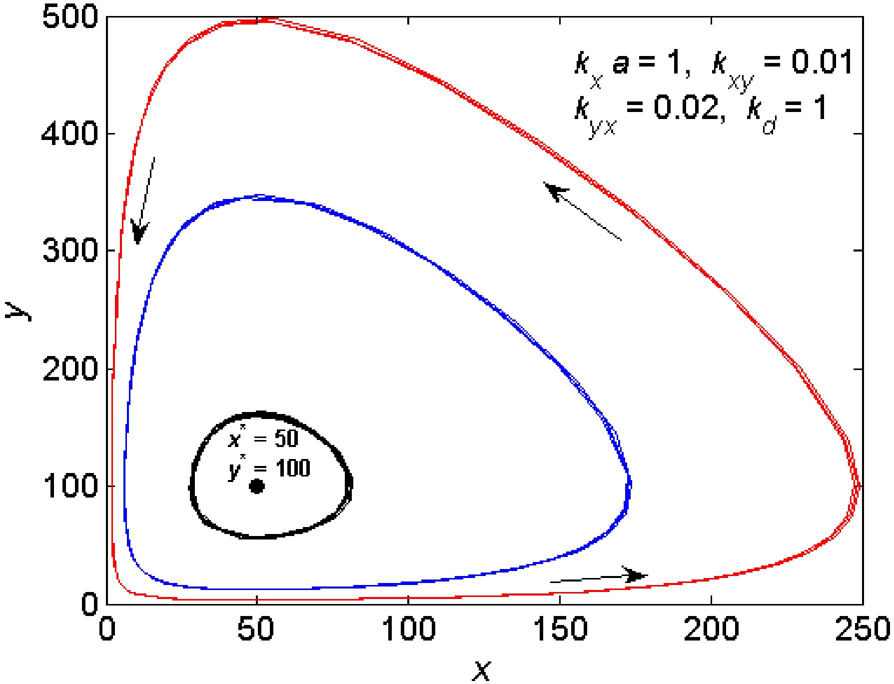
Vito Volterra

Prey (x) – Predator (y) eqns + effect of fishing

$$dx/dt = (r - a)\, x - k_{xy}\, x\, y$$
$$dy/dt = k_{yx}\, x\, y - (d + b)\, y$$

Volterra (1926)

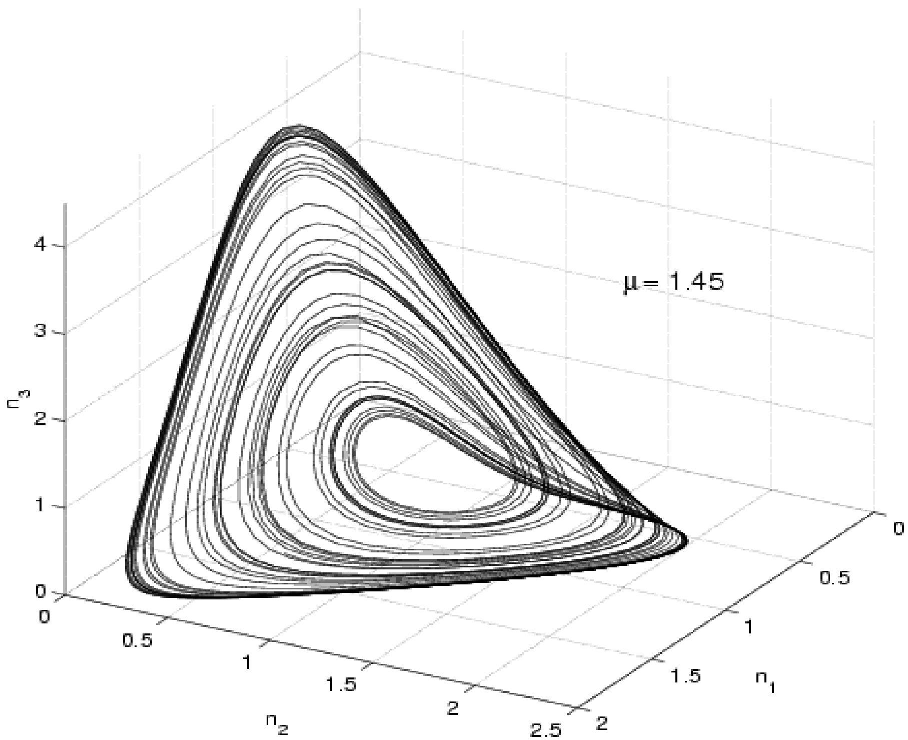
Equilibrium populations:
$x^* = (d+b)/k_{xy}$, $y^* = (r-a)/k_{xy}$



In absence of fishing during the war years (a=0,b=0), x^* decreased while y^* increased

In general, we observe oscillations because cycles around the equilibria are neutrally stable

Chaos in Lotka-Volterra System



• Two-species Lotka-Volterra systems: periodic oscillations.

• Chaos in Lotka Volterra system with three species
(Arneodo, Coullet and Tresser, 1980).

• In principle - possible for a species to persist in food web that is not dynamically stable by going through complex cycles.

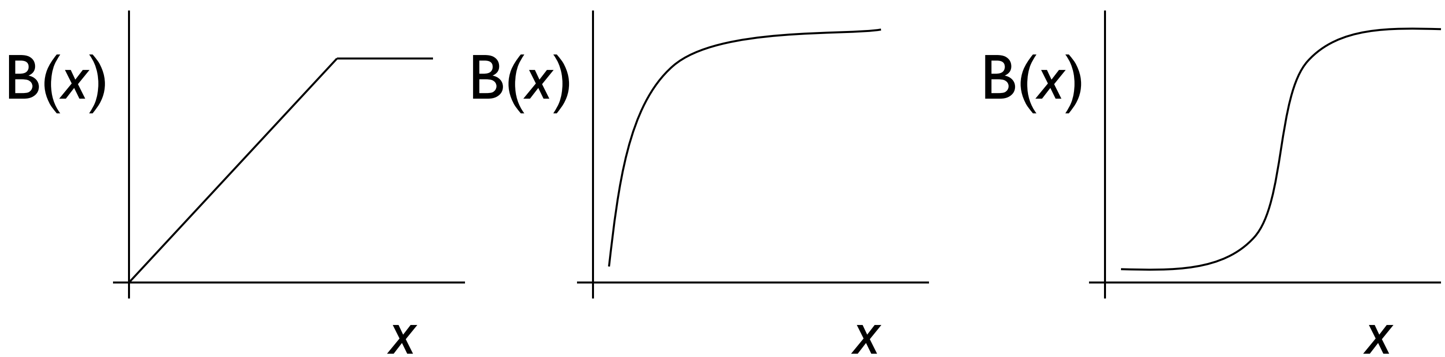
• However, the cycles must be strictly bounded - otherwise populations will reach levels from which they cannot recover.

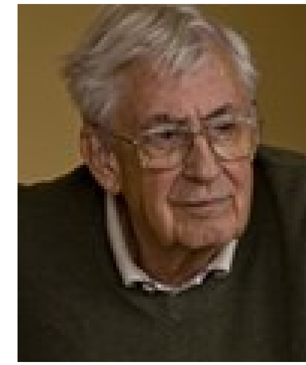
Rosenzweig-MacArthur Model (1963)

Predator-Prey model with Holling Type 2 (hyperbolic) functional response

Functional response of predation to prey density (Holling, 1959)

Type 1 Type 2 Type 3





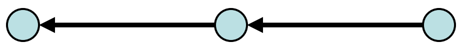
C S Holling
(1930-)

Prey (x) – Predator (y) eqns

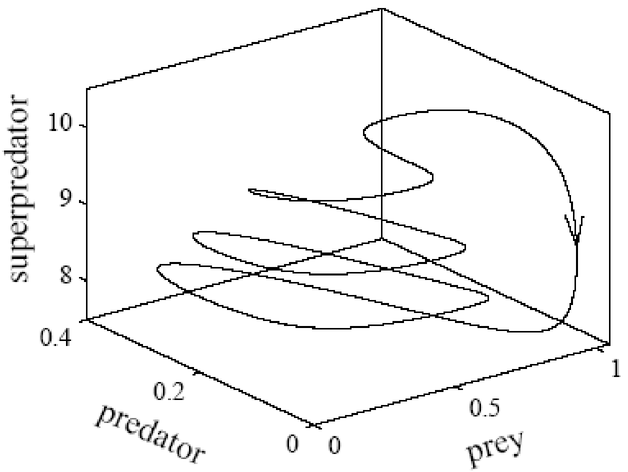
$$dx/dt = r\ x\,[1-(x/K)] - q\,y\,[x/(b+x)]$$
$$dy/dt = -d\,y + \varepsilon\,q\,y\,[x/(b+x)]$$

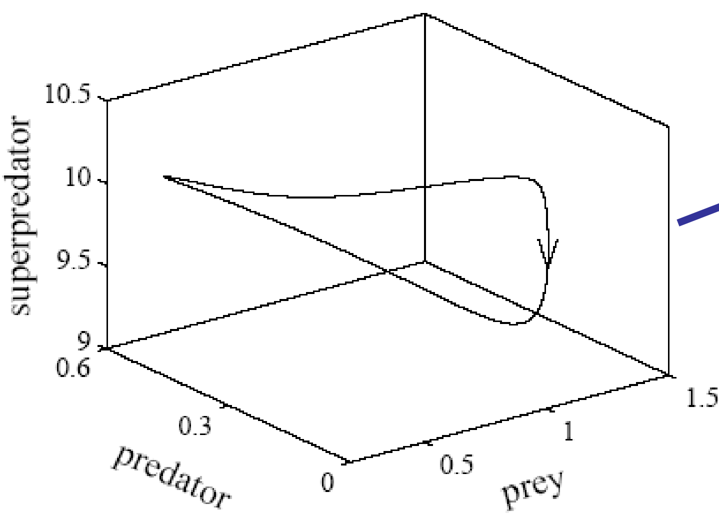
Gives rise to limit cycle oscillations

3 species Rosenzweig-MacArthur Model

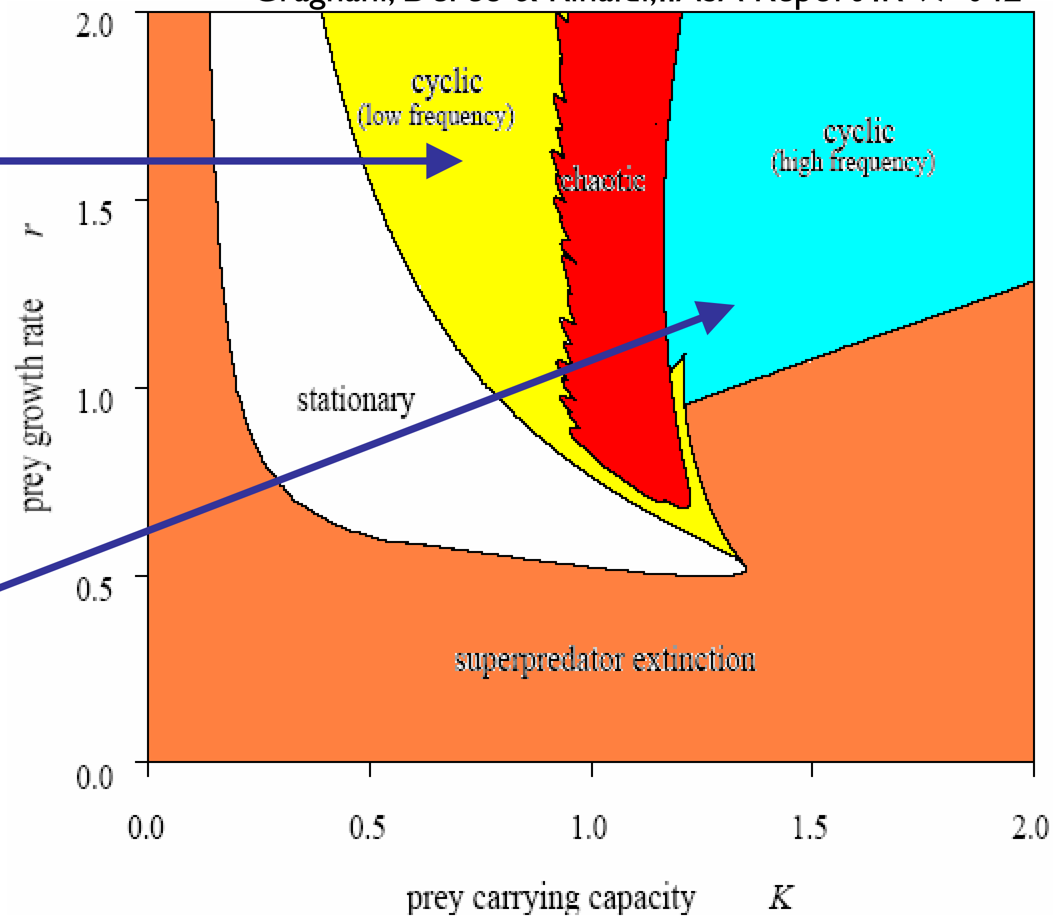


$$\frac{dx_1}{dt} = x_1\left[r\left(1-\frac{x_1}{K}\right)-\frac{a_2x_2}{b_2+x_1}\right]$$
$$\frac{dx_2}{dt} = x_2\left[e_2\frac{a_2x_1}{b_2+x_1}-\frac{a_3x_3}{b_3+x_2}-d_2\right]$$
$$\frac{dx_3}{dt} = x_3\left[e_3\frac{a_3x_2}{b_3+x_2}-d_3\right]$$





Gragnani, DeFeo & Rinaldi,IIASA Report IR-97-042



Going beyond 3 species opens up an entire world of dynamical possibilities…

Ecological (Interaction) Networks

And specifically,

Food Webs

that comprise links representing trophic relations