Systems Biology: A Personal View XII. Importance of Modularity in Networks

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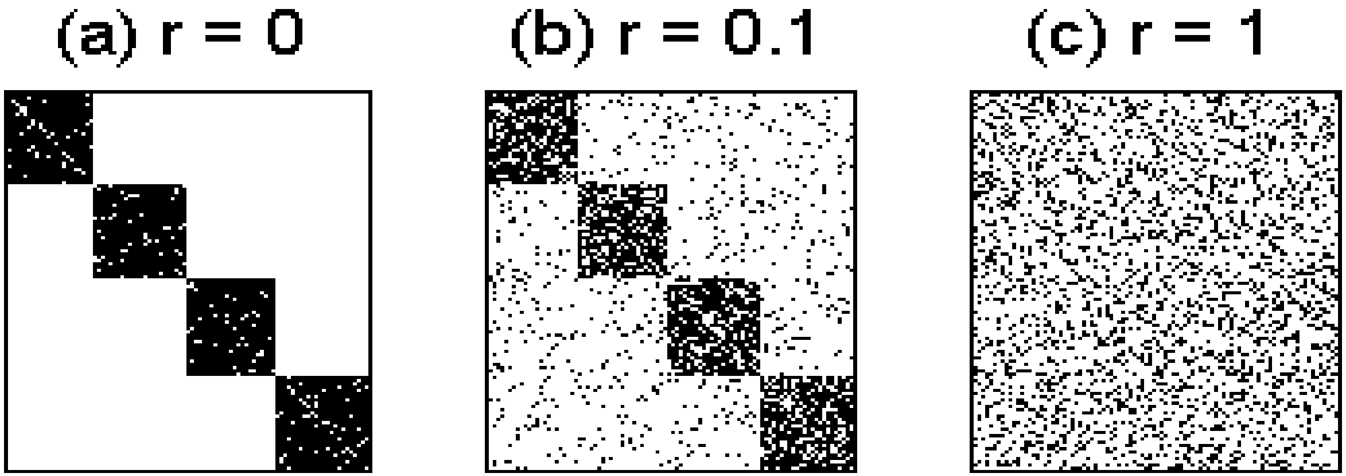
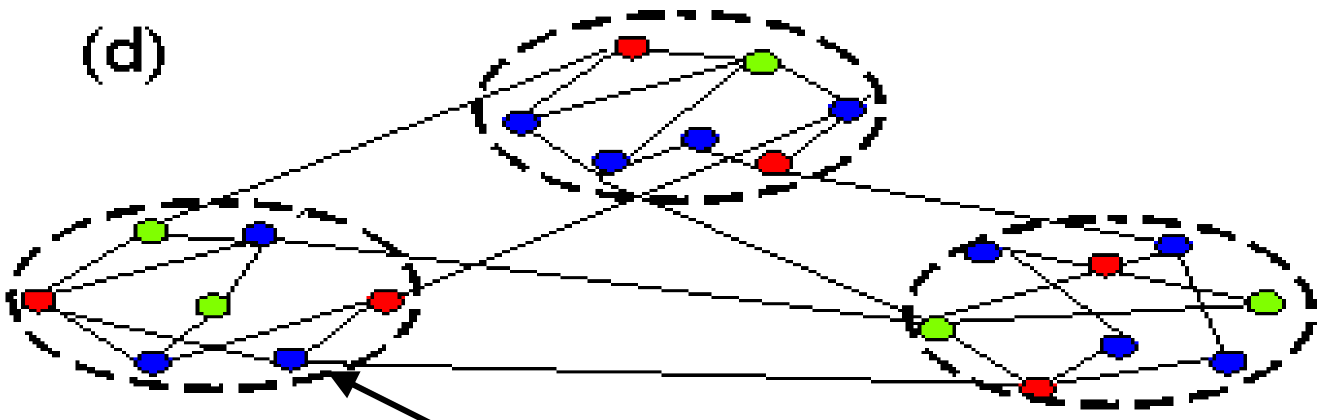
A simple model of modular networks

Model parameter *r* :

Ratio of inter- to intra-modular connection density





Module ≡ random network

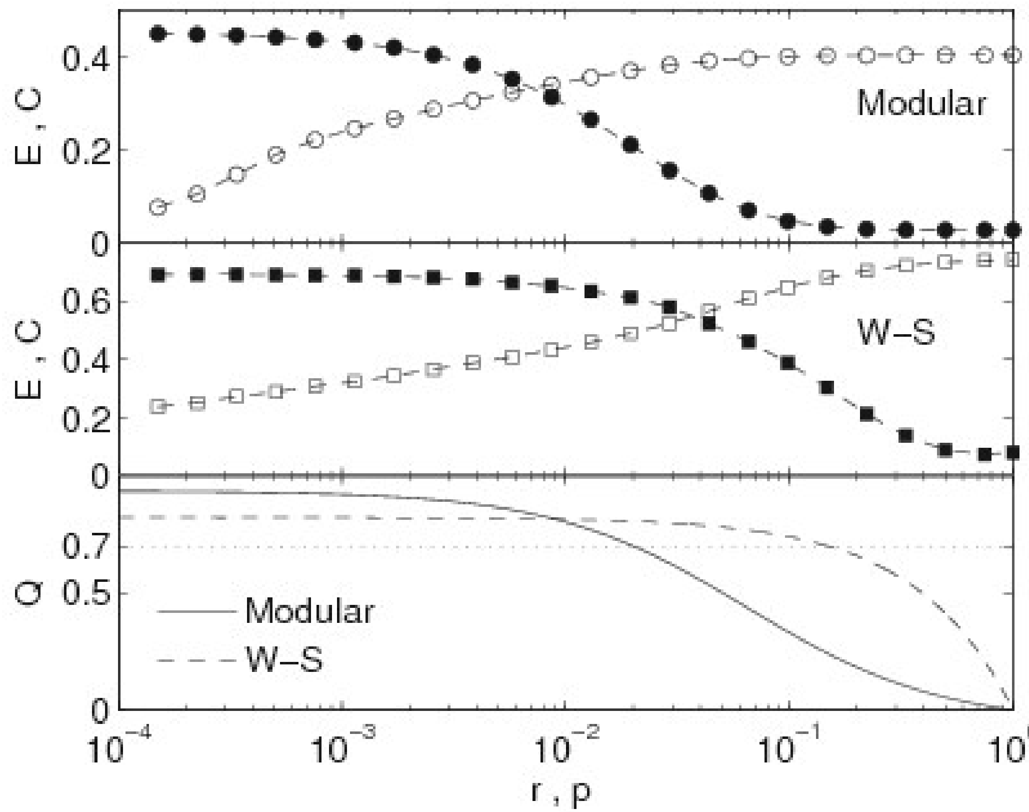
Comparison with Watts-Strogatz model

Structural measures used:

Communication efficiency: $E = [\text{avg path length}, \ell\,]^{-1} = 2/N(N-1) \sum_{i>j} d_{ij}$

Clustering coefficient: C = fraction of observed to potential triads $= (1/N) \sum_i 2n_i / k_i (k_i - 1)$

WS and Modular networks behave similarly as function of *p* or *r* (Also for between-ness centrality, edge clustering, etc)



In fact, for same N and <k>, we can find *p* and *r* such that the WS and Modular networks have the same "*modularity*" Q

Pan and Sinha, EPL (2009)

Pan and Sinha, EPL (2009)

How can you tell them apart ?

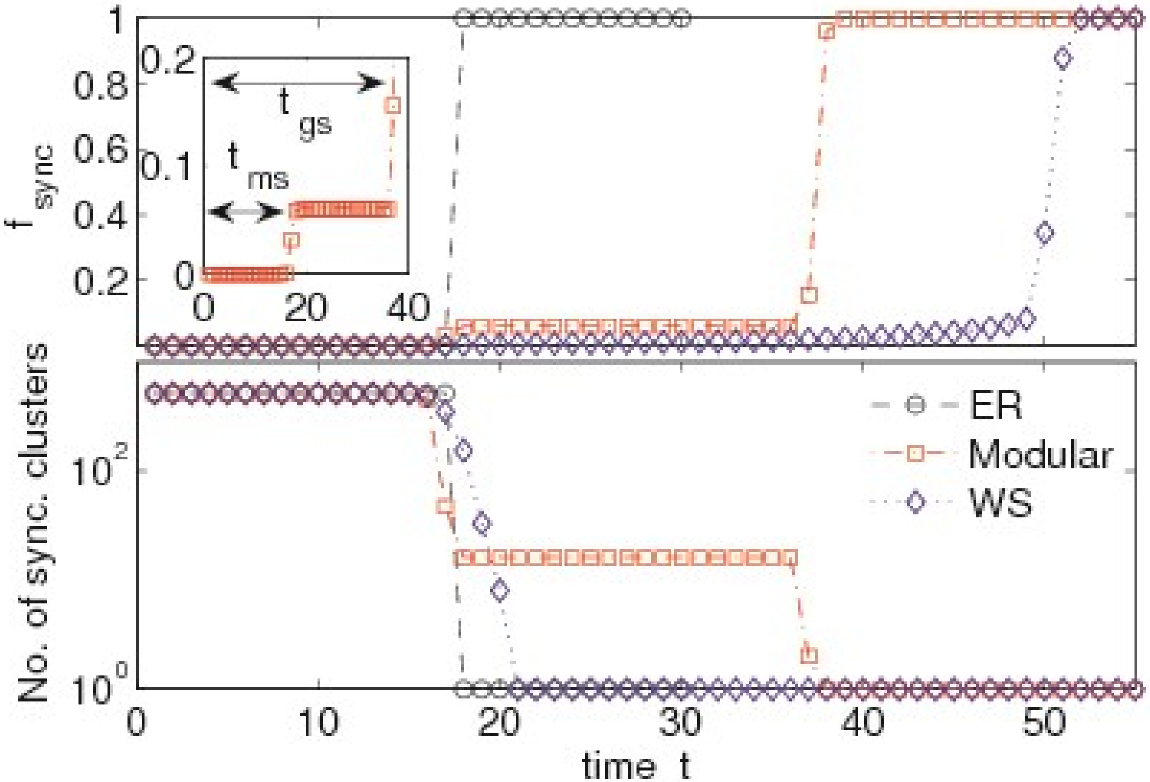
Dynamics on Modular networks different from that on Watts-Strogatz small-world networks

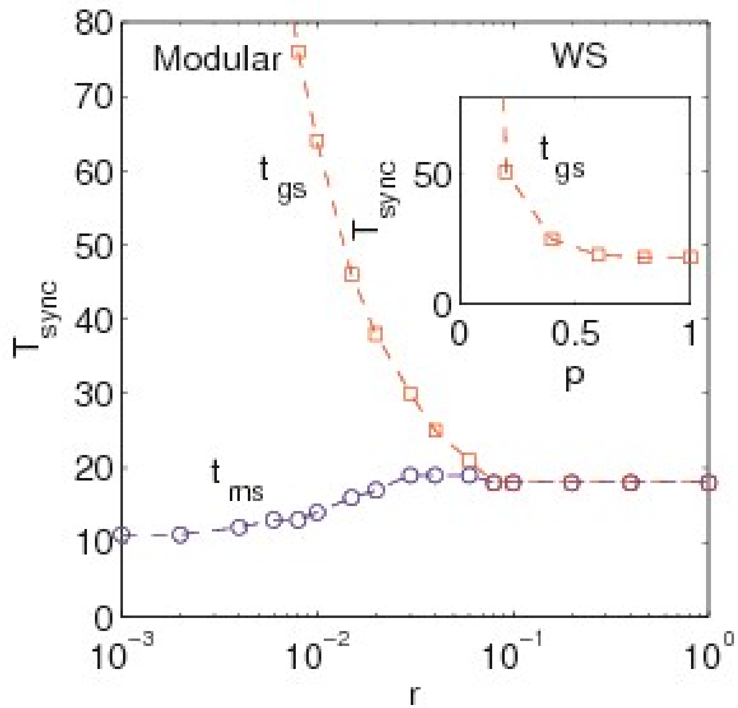
Consider synchronization on modular networks

Network topology

e.g., phase oscillators: $d\theta_i/dt = w + (1/k_i)\sum K_{ij} \sin(\theta_j - \theta_i)$

2 distinct time scales in Modular networks: $t_{modular}$ & t_{global}





Existence of distinct time-scales in Modular networks

Pan and Sinha, EPL (2009)

Consider linearized dynamics around synchronized state

$d\theta_i /dt = - (\kappa /k_i)\sum_j L_{ij}\, \theta_j$, $(i = 1,\ldots,N)$

L: Laplacian

κ: coupling strength of oscillators

Focus on the normal modes:

$\phi_i (t) = \sum_j B_{ij}\, \theta_j = \phi_i (0) \exp(-\lambda_i\, t)$, $(i = 1,\ldots,N)$

B: matrix of eigenvectors
λ_i :eigenvalues
} of $L' = D^{-1} L$,
D: diagonal matrix s.t. $D_{ii}=k_i$

$L' \rightarrow L=D^{1/2} L' D^{-1/2}$ is symmetric, normalized Laplacian $\Rightarrow \lambda_i$ real

Differences in time-scales of modes $\Rightarrow$ gap in spectrum of L

Mode for smallest λ_i : associated with global synchronization

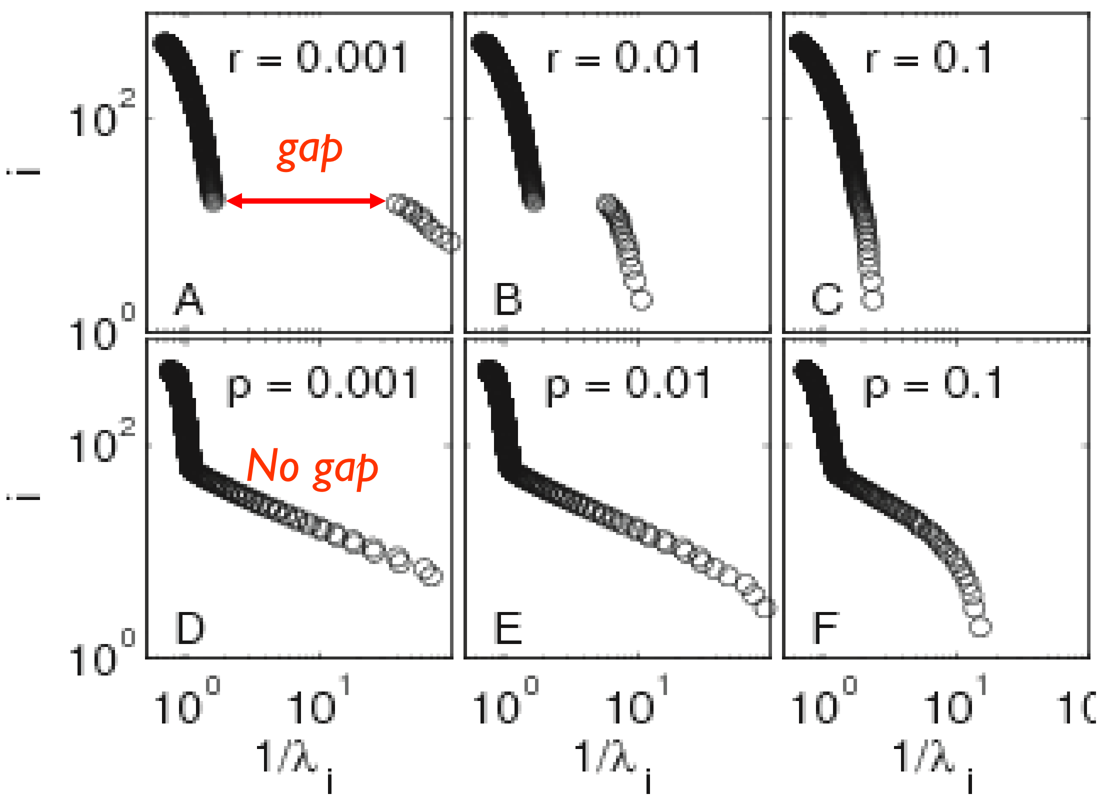
Other modes : synchronization within different groups of oscillators

Pan and Sinha, EPL (2009)

Eigenvalue spectra of the Laplacian

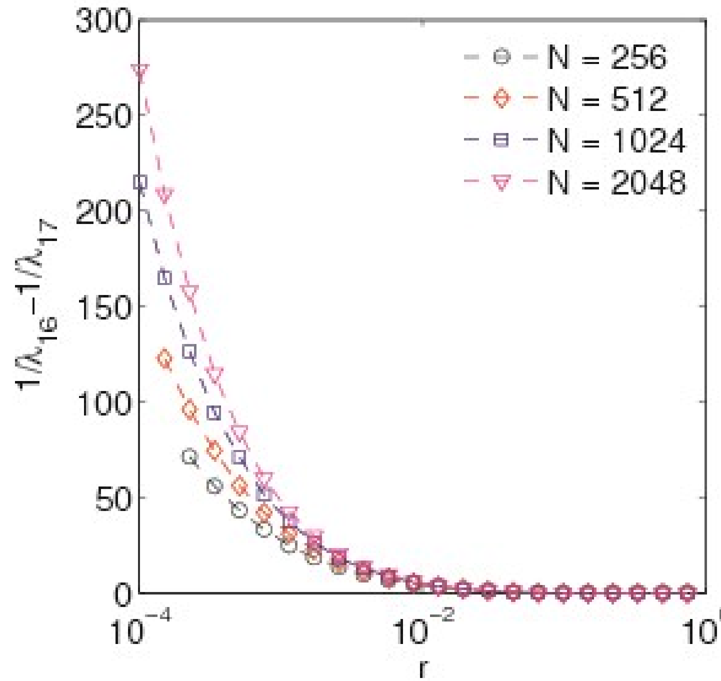
Shows the existence of spectral gap ⇒ distinct time scales

Modular network Laplacian spectra



WS network Laplacian spectra

Spectral gap in modular networks diverges with decreasing r

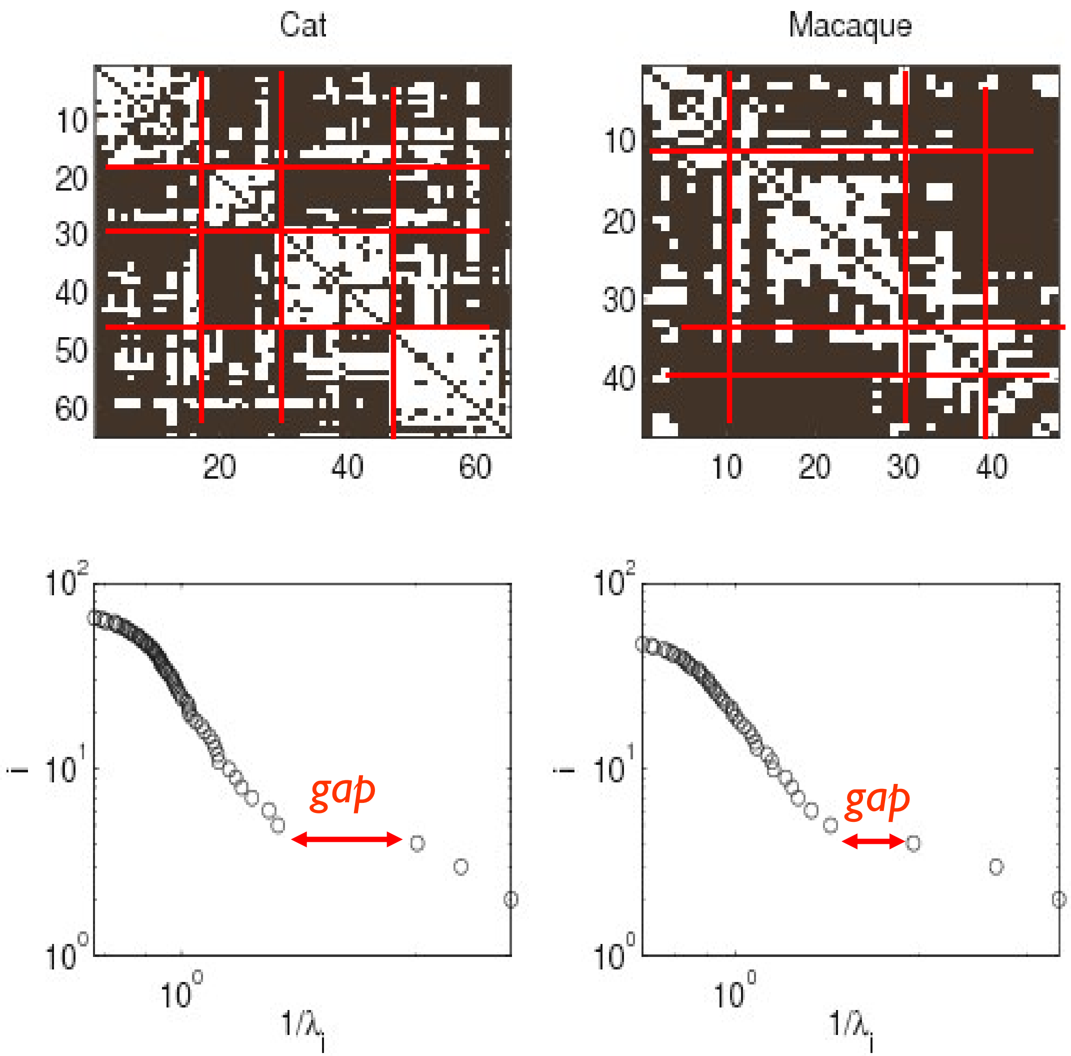


Existence of distinct time-scales in Modular networks

No such distinction in Watts-Strogatz small-world networks

How about "real" SW networks ?

Pan and Sinha, EPL (2009)



The networks of cortical connections in mammalian brain have been shown to have *small-world* structural properties

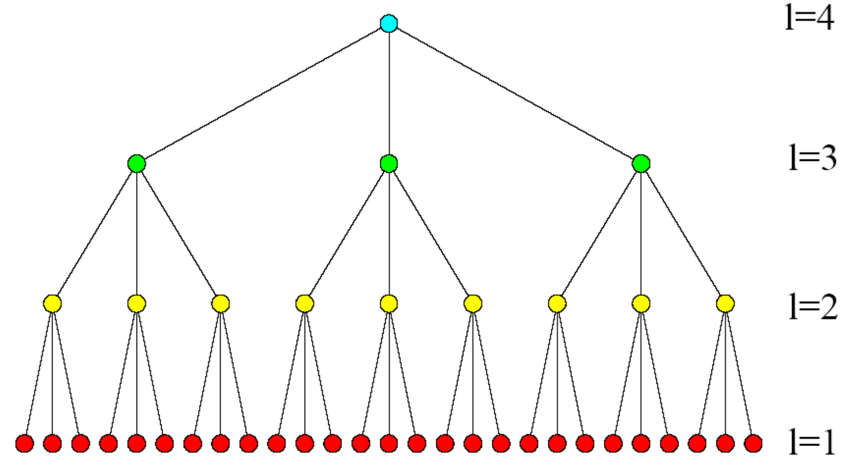
Our analysis reveals their dynamical properties to be consistent with modular "small-world" networks

Fast synchronization of neuronal activity within a module :

The mechanism for efficient neural information processing ?

How about other kinds mesoscopic structures ?

E.g., Hierarchy

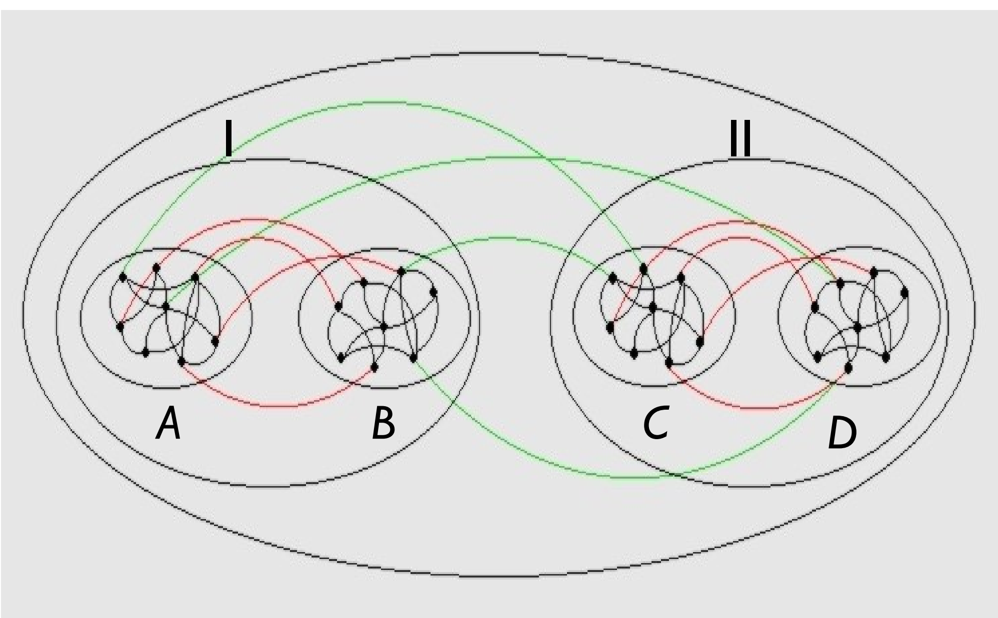


Hierarchical Modular networks

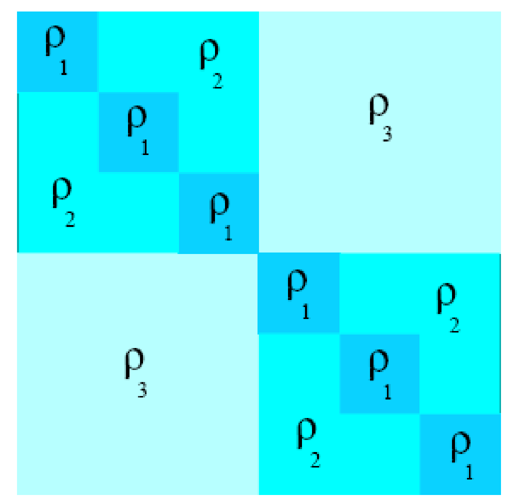
Modules may occur at different levels of hierarchy

Level 1: Modules *A*, *B*, *C*, *D*

Level 2: Meta-Modules I, II



- r = 1 : randomly coupled network.
- r = 0: isolated sub-networks (modules)
- 0 < r < 1 : hierarchically structured network.

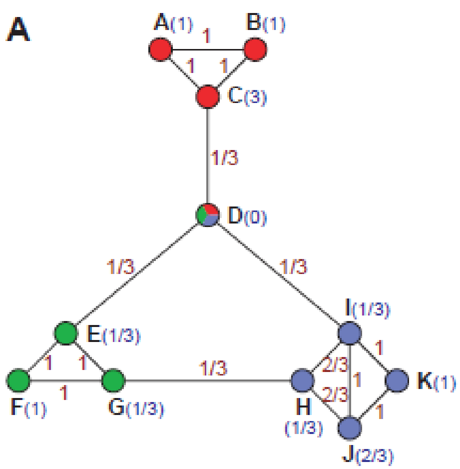


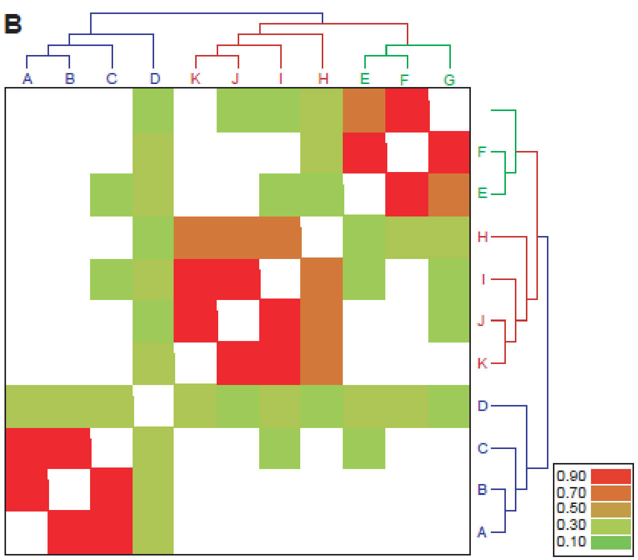
Hierarchical modularity in metabolic network

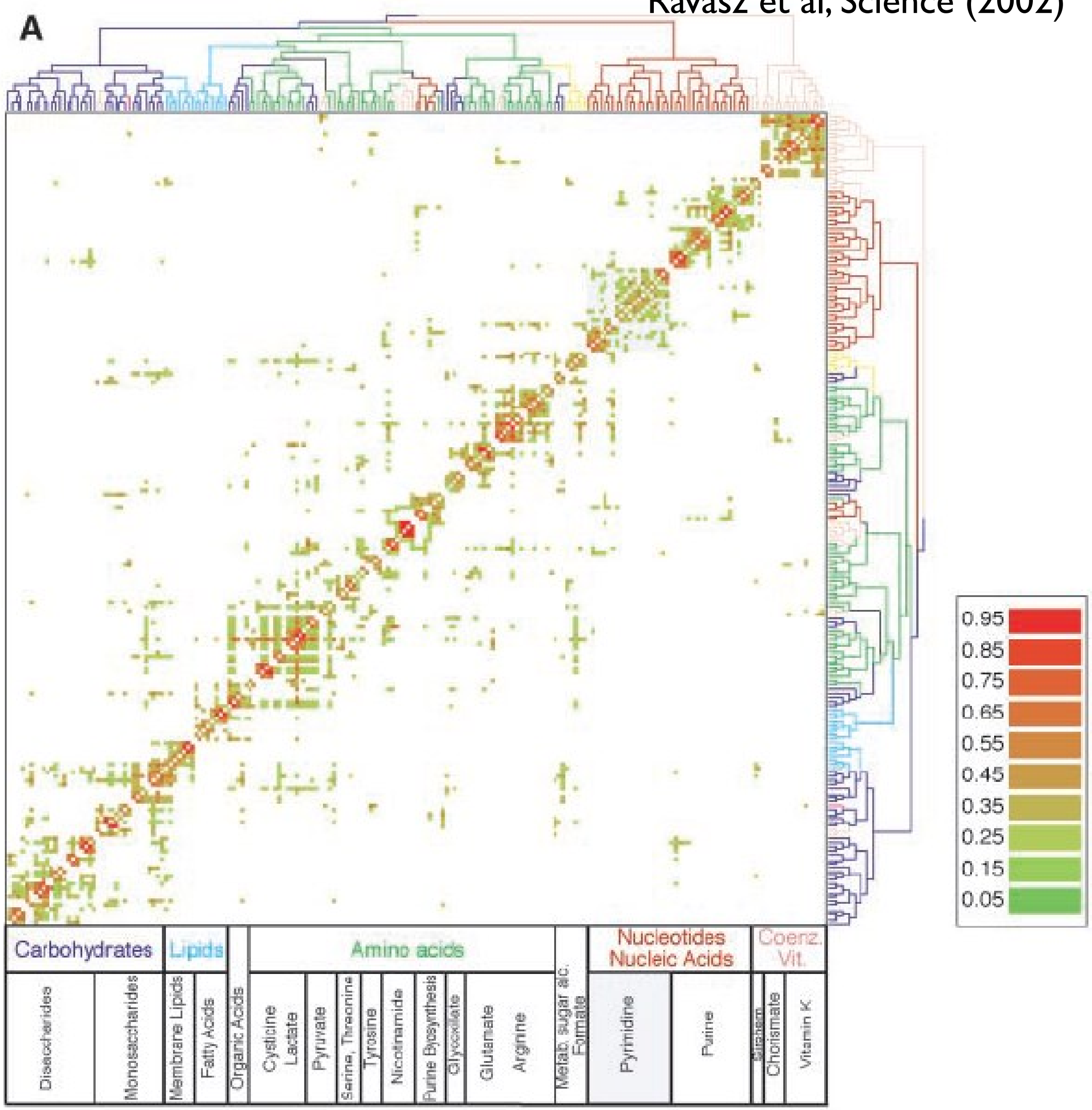
Topological overlap:

Ravasz et al, Science (2002)

$$O_T(i,j) = \frac{\sum_{l=1}^{N}{}' \, l_{i,l} \cdot l_{j,l} + l_{i,j}}{\min(k_i, k_j) + 1 - l_{i,j}}$$







Hierarchical Modular Networks exhibit several distinct time-scales – equal to the number of hierarchical levels (Sinha & Poria, 2011)

Synchronization of phase oscillators in hierarchical modular network show as many distinct time-scales as number of hierarchical levels …
Reflected in the eigenvalue spectra