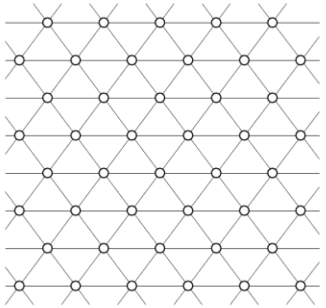
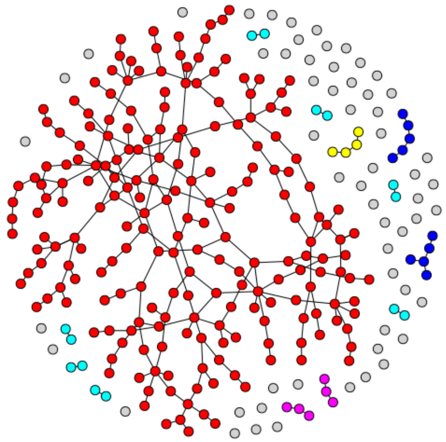
Systems Biology: A Personal View
V. Networks: Models I

Sitabhra Sinha
IMSc Chennai

Theoretical understanding of networks



- Regular lattice or grid (*Physics*)
  - average path length ~ N (no. of nodes)
  - clustering *high*
  - delta function distribution of degree (links/node)

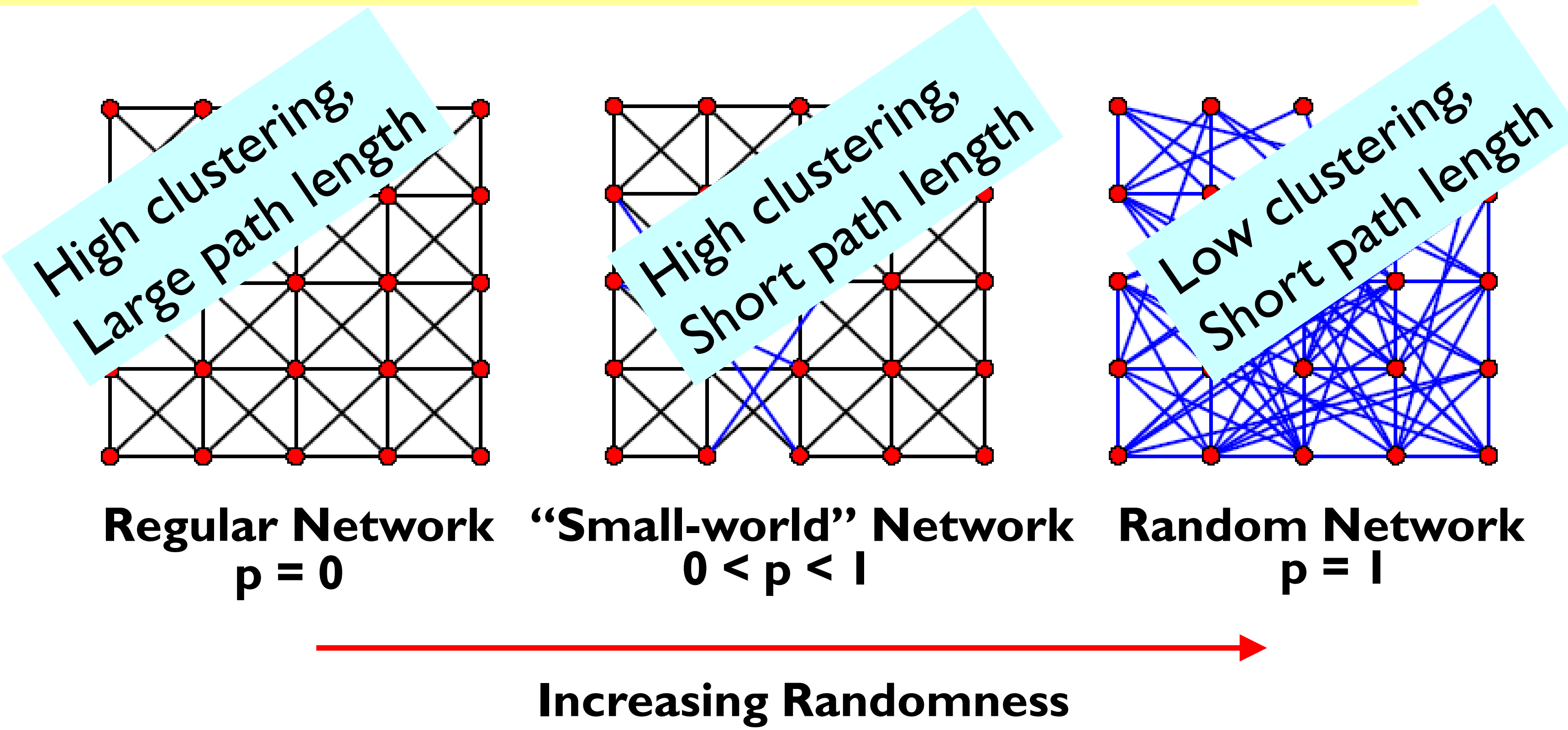


- Random networks (*Graph theory*)

  Also known as Erdos-Renyi networks
  - average path length ~ log N
  - clustering low
  - Poisson distribution of degree

Empirical networks are not random – many have certain structural patterns

Example: *small-world* networks



p: fraction of random, long-range connections

Watts and Strogatz (1998): Many biological, technological and social networks have connection topologies that lie between the two extremes of completely regular and completely random.

"It's a *small world*": The Milgram Experiment

eweb.furman.edu

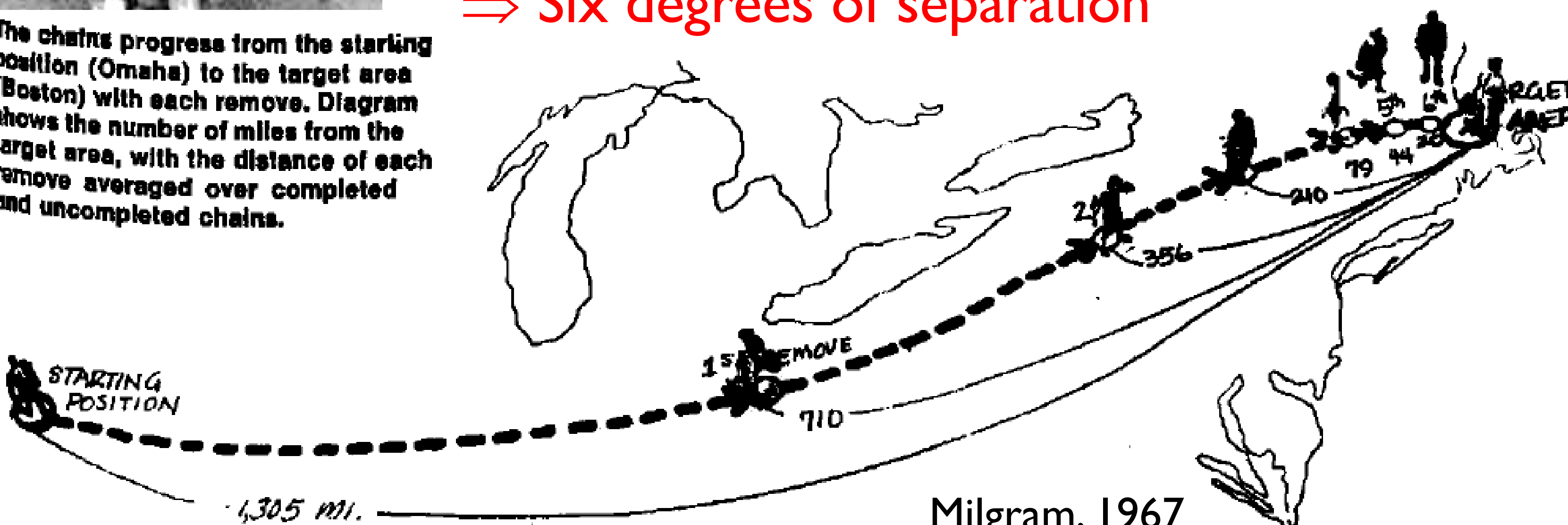
Stanley Milgram (1933-1984), US social psychologist

Arbitrarily selected individuals in Nebraska were asked to generate acquaintance chains (knowing on first name basis) connecting them to a target individual in Boston

In one experiment, 64 of the 296 chains initiated eventually reached the target – the mean number of intermediaries between source and target being slightly larger than 5

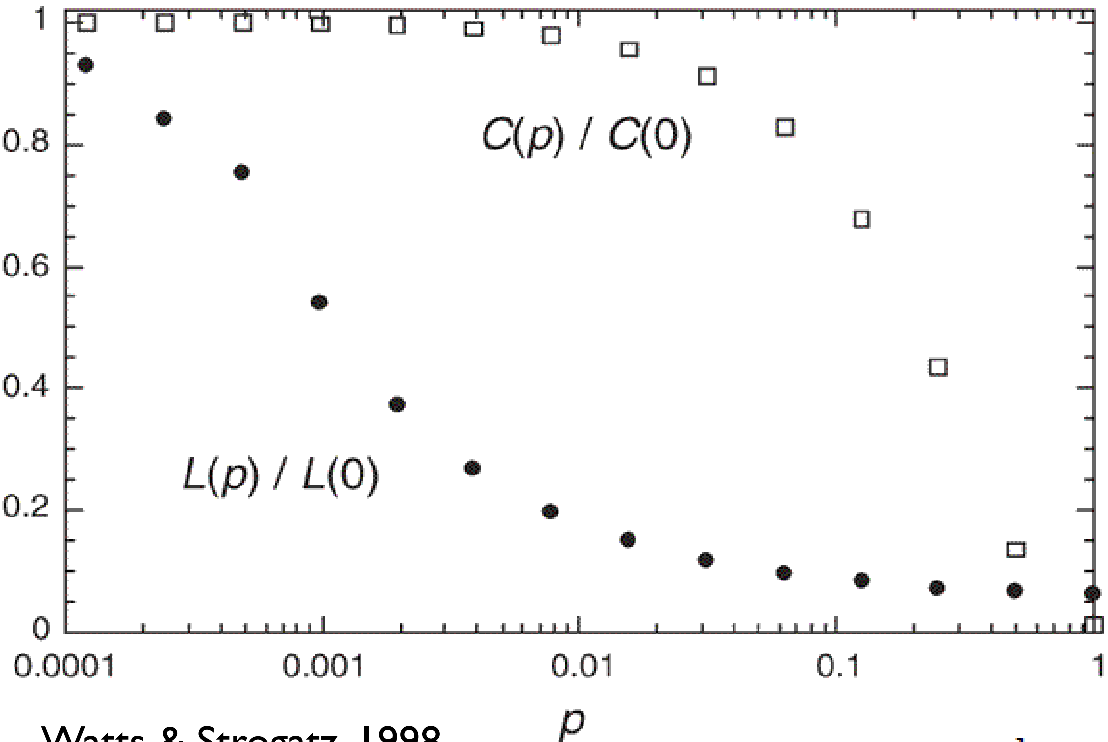
⇒ Six degrees of separation

The chains progress from the starting position (Omaha) to the target area (Boston) with each remove. Diagram shows the number of miles from the target area, with the distance of each remove averaged over completed and uncompleted chains.



Milgram, 1967

"Small world": Local properties of regular networks but global properties of random networks



Watts & Strogatz, 1998

Characteristic path length

$$L = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \geq j} d_{ij}$$

d_{ij} : shortest distance between nodes i and j

Alternatively $\ell^{-1} = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \geq j} d_{ij}^{-1}$

Clustering coefficient

$$C = \frac{1}{n} \sum_i C_i$$

where

$$C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}$$

Alternatively $C = \frac{3 \times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}}$

Small-world networks can be highly clustered (like regular networks), yet have small characteristic path lengths (as in random networks).

Epidemics on "Small world"

Watts & Strogatz, 1998

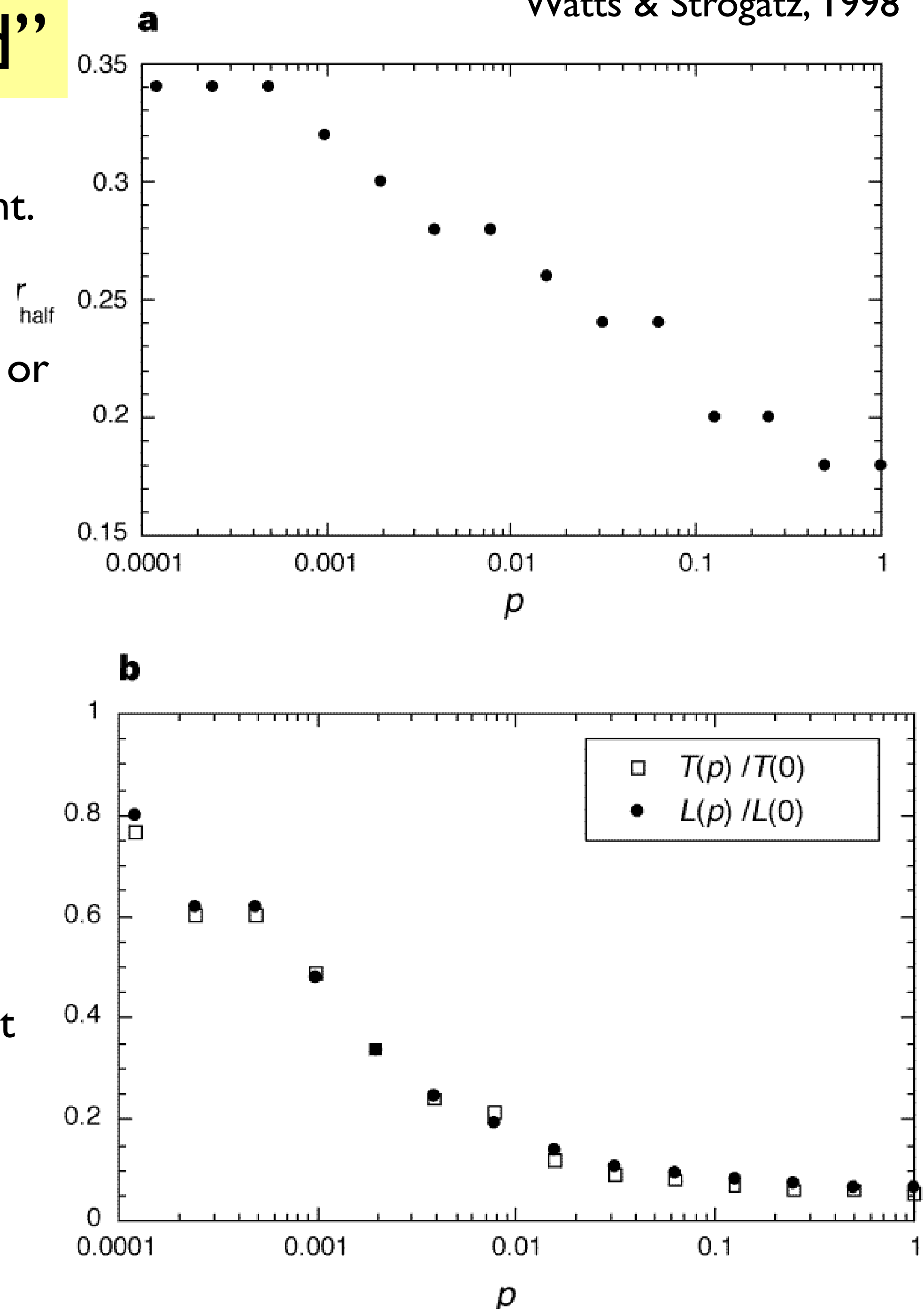
Dynamical process:

- Time $t = 0$: single infected individual present.
- Each infected agent can infect any of its neighbours with probability r.
- Infected individuals removed (by immunity or death) after unit period of sickness.

Key Results:

- Critical infectiousness r_{half}, at which the disease infects half the population, decreases with p
- Time required for a maximally infectious disease (r = 1) to spread throughout the entire population $T(p)$ has same form as characteristic path length $L(p)$

⇒ rewiring only a few links in the original lattice causes global infection to occur almost as fast as in random network



Implication:

"Control the truck-drivers"

Do *small-world* networks occur in real life ?

	# nodes	Avg degree	Avg path length		Clustering coefficient	
Network	Size	$\langle k \rangle$	ℓ	ℓ_{rand}	C	C_{rand}
WWW, site level, undir.	153 127	35.21	3.1	3.35	0.1078	0.00023
Internet, domain level	3015–6209	3.52–4.11	3.7–3.76	6.36–6.18	0.18–0.3	0.001
Movie actors	225 226	61	3.65	2.99	0.79	0.00027
LANL co-authorship	52 909	9.7	5.9	4.79	0.43	1.8×10^{-4}
MEDLINE co-authorship	1 520 251	18.1	4.6	4.91	0.066	1.1×10^{-5}
SPIRES co-authorship	56 627	173	4.0	2.12	0.726	0.003
NCSTRL co-authorship	11 994	3.59	9.7	7.34	0.496	3×10^{-4}
Math. co-authorship	70 975	3.9	9.5	8.2	0.59	5.4×10^{-5}
Neurosci. co-authorship	209 293	11.5	6	5.01	0.76	5.5×10^{-5}
E. coli, substrate graph	282	7.35	2.9	3.04	0.32	0.026
E. coli, reaction graph	315	28.3	2.62	1.98	0.59	0.09
Ythan estuary food web	134	8.7	2.43	2.26	0.22	0.06
Silwood Park food web	154	4.75	3.40	3.23	0.15	0.03
Words, co-occurrence	460.902	70.13	2.67	3.03	0.437	0.0001
Words, synonyms	22 311	13.48	4.5	3.84	0.7	0.0006
Power grid	4941	2.67	18.7	12.4	0.08	0.005
C. Elegans	282	14	2.65	2.25	0.28	0.05

Albert & Barabasi, 2003