Mathematical Methods I

Assignment 3

Due on November 16, 2012

1. Compute the sum of all the n-th roots of 1 for n=2, 3 and 4. Also indicate the general expression for arbitrary integer $n > 1$.
2. Can you solve for z in $e^z = 0$?
3. Simplify the following numbers:
   (a) $\frac{1+i}{1-i}$
   (b) $\frac{-1+i\sqrt{3}}{1+i\sqrt{3}}$
   (c) $(\frac{\sqrt{3}+i}{1+i})^2$
4. Plot the trajectories of the roots of the quadratic equation $z^2 + bz + c = 0$ in the complex plane as the parameter b is varied from $-\infty$ to ∞ for the two cases (i) $c = 1$ and (ii) $c = -1$.
5. By integrating the relation $\frac{1}{1+x^2} = \frac{i}{2}(\frac{1}{x+i} - \frac{1}{x-i})$, show that $\int_0^1 \frac{dx}{1+x^2} = \frac{\pi}{4}$
6. Do the following equations have any roots other than real ones ?
   (a) $\sin(z) = 0$
   (b) $\cos(z) = 0$
   (c) $\tan(z) = 0$
7. Write the Laurent series expansion about the singularity at $z = i$ of the function $1/(1 + z^2)$. [Hint: Note that $1 + z^2 = (z - i)(z + i) = (z - i)(2i + z - i)$ and use the binomial expansion.]
8. Evaluate $\int_0^i dz \frac{1}{1-z^2}$.
9. Obtain the Laurent series expansion about $z = 0$ with (i) $|z| < 1$ and (ii) $|z| > 1$ for the following functions. Also obtain the residue at $z = 0$ for each function for $|z| < 1$.
   (a) $\sin(z)/z^4$
   (b) $\frac{e^z}{z^2(1-z)}$
10. Evaluate $\int_C dz e^{-z} z^{-n}$ where C is a circle of radius R about the origin.
11. Evaluate $\int_{-\infty}^{\infty} dx \frac{\sin^2(bx)}{x(a^2+x^2)}$.