

Mathematical Methods I

Assignment 1

Due on August 16, 2012

1. Perform the following indefinite integrals

(a) $\int x \sin(2x) dx$

(b) $\int x \ln(x) dx$

(c) $\int x [\cos(3x) + 4 \sin(3x)] dx$

2. Evaluate

(a) $\int_0^\infty \exp(-\alpha x) dx$

(b) $\int_0^\infty x^n \exp(-\alpha x) dx$ (Use parametric differentiation, i.e., differentiating w.r.t. α within the integral)

(c) Using the above result show that $\Gamma(n) = (n-1)!$ when n is a positive integer

3. Use the definition of error function to evaluate $\int_0^x t^2 \exp(-t^2) dt$.

(Hint: introduce a parameter α in the integral definition of error function so that it has $\int_0^x \exp(-\alpha t^2) dt$ and differentiate it w.r.t. α . Next change variables in this integral from t to u such that $u^2 = \alpha t^2$ and differentiate this integral w.r.t. α . Use the fact that both integrals have the same value.)

4. Use similar technique as above to show $\Gamma(x+1) = x \Gamma(x)$.

5. Sketch the following functions by hand (do not use computers)

(a) $x/(x^2 + a^2)$

(b) $x/(x^3 + a^3)$

(c) $(x-a)\exp(-ax)$

(d) $(x-a)/[a^2 - (x-a)^2]$