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Perspective

How can statistical mechanics contribute to social science?

*Steven N. Durlauf**

Department of Economics, University of Wisconsin, 1180 Observatory Drive, Madison, WI 53706-1393

A model of interdependent decision making has been developed to understand group differences in socioeconomic behavior such as nonmarital fertility, school attendance, and drug use. The statistical mechanical structure of the model illustrates how the physical sciences contain useful tools for the study of socioeconomic phenomena.

Recent economic analyses have expanded the set of causal factors traditionally studied in understanding individual decision making for contexts ranging from the seemingly trivial cases such as choice of hairstyle to life determining choices such as out-of-wedlock births. This new research has been driven by an increased focus on group, as opposed to individual-specific, determinants of behavior. By group determinants, I refer to the role of peer groups, role models, and social networks in influencing an individual's beliefs, preferences, and opportunities. Group memberships occur in a wide variety of contexts, ranging across neighborhood residence, ethnicity, enrollment at a particular school, and employment at a given firm.

From the perspective of classical economic reasoning, in which individuals purposefully make choices that may be characterized as maximizing a utility function subject to constraints, two things are distinctive about this new approach. First, such group-level influences on individuals are not directly mediated by markets; rather, these influences affect the ways individuals evaluate the consequences of their choices. Second, this approach introduces an explicit sociological perspective on individual behavior. By moving away from market-mediated interrelationships to direct interdependencies in behavior, this new approach to analysis represents an "interactions-based" approach to socioeconomic behavior in which individual decisions are explicitly understood as determined by one's social context. A large class of these interactive decisions are binary: Staying in or dropping out of school, the decision to have an out-of-wedlock birth, the use of drugs, and entry into illegal activity all have this characteristic. These binary choices are linked by their effects on an individual's long-term socioeconomic status and help form the basis for understanding inequality over the individual's life cycle. Indeed, many forms of intergroup inequality, such as black–white differences in income, are related to differences group rates of these choices.

Interestingly, this shift in thinking about the causal determinants of individual behavior has led to a shift in the mathematical tools used to formalize socioeconomic environments. Specifically, the mathematics of statistical mechanics has proven useful in a range of contexts (examples include refs. 1–3). This utility may seem strange at first glance, in that the purposeful decisions of individuals are hardly similar to the characteristics of atoms. However, a powerful connection exists between formal individual choice models and the mathematics of statistical mechanics, which suggests that there are many useful tools which social scientists can borrow from physics. Just as statistical mechanics models explain how a collection of atoms can exhibit the correlated behavior necessary to produce a magnet, social science models wish to explain interdependent behaviors. The basic idea in statistical mechanics—that the behavior of one atom is influenced by the behavior of other atoms—is thus similar to the social science claim that one individual's decisions depend upon the decisions of others; therein lies the possibility of a common mathematical structure.

Here I outline a basic framework for the study of interactions-based socioeconomic models that William Brock and I have developed in recent work (4, 5). The framework can accommodate a number of standard models of interactions that have appeared in the economics literature (6). Mathematically, the model is a standard one from statistical mechanics. At the same time, it illustrates how the substantive reasoning behind social science models differs from that used in the physical sciences.

A Basic Model

Consider a model of binary decisions made by each of I individuals who form a common group. One can think of the exercise as trying to model the individual and collective decisions of white male teenagers in Madison, Wisconsin concerning cigarette smoking. (See ref. 7 for evidence of large differences in smoking rates among teenagers of differing ethnicities and genders as well as for discussion of the role of social interactions in smoking decisions.) In standard formulations, each individual i is viewed as making a choice w_i. The alternatives are coded as -1 (smoke) and 1 (not smoke). The choice is assumed to represent the solution to a utility maximization problem, which is a formalization of the notion that an agent has a preference ordering across the choices that depends on a range of factors,

$$max_{\omega_i \in \{-1,1\}} h_i \omega_i + \varepsilon_i(\omega_i). \qquad [1]$$

Here, h_i measures an observable (from the perspective of the modeler) difference in the payoff of the two choices whereas $\varepsilon_i(\omega_i)$ denotes an individual-specific random and unobservable (for the modeler) component to the payoffs associated with the two choices. So for the smoking example, h_i reflects the influence of observable factors such as parental education and the level of anti-smoking advertising on the way a teenager assesses the utility to smoking whereas $\varepsilon_i(\omega_i)$ reflects unobservable factors such as an individual's reaction to the taste of tobacco.

This dichotomy between observable and unobservable components is useful in that it preserves a parallel between the theoretical model and its statistical implementation, where one must be careful to distinguish between those determinants of individual behavior that are and are not observable to an analyst. To be precise, in a statistical implementation of the model, h_i is assumed to be observable up to a set of parameters, which are estimated from a data set that contains information

*To whom reprint requests should be addressed. E-mail: sdurlauf@ssc.wisc.edu.

on the determinants of h_i. A common assumption is that the difference of the random terms $\varepsilon_i(-1) - \varepsilon_i(1)$ is logistically distributed,

$$Prob(\varepsilon_i(-1) - \varepsilon_i(1) \leq z) = \frac{1}{1 + exp(-\beta_i z)^i};\ \beta_i \geq 0, \quad [2]$$

and that these random components are independent across individuals.

To introduce interactions, one augments this basic problem by introducing an additional component to the payoff function. Letting $E_i(x)$ denote the subjective expected value assigned by i to x, this augmented decision problem is

$$max_{\omega_i \in \{-1,1\}} h_i\omega_i - E_i \sum_{j \neq 1} \frac{J_{i,j}}{2} (\omega_i - \omega_j)^2 + \varepsilon_i(\omega_i). \quad [3]$$

The middle term $-E_i\Sigma_{j\neq i}\ (J_{i,j}/2)(\omega_i - \omega_j)^2$ embodies the interactive aspect of decision making, in that it says that each agent has beliefs about the behavior of others in a reference group and that these beliefs influence the payoff associated with the agent's decision. When $J_{i,j}$ is positive, the specification means that agent i experiences some incentive to conform to the behavior of agent j. So, for our example, the larger the values of $J_{i,j}$, the stronger the incentives to smoke when one's peers are expected to do so.

This specification is sufficient to allow for calculations of the probabilities of various choices. For example, given the beliefs of an individual about behaviors within the group, the conditional probability of a particular choice is

$$Prob(\omega_i|h_i, E_i(\omega_j)\ \forall\ j) \sim exp(\beta_i h_i \omega_i + \sum_{j \neq i} \beta_i J_{i,j} \omega_i E_i(\omega_j)), \quad [4]$$

where "$\sim$" means "is proportional to." The joint probability measure over the vector of population choices $\underset{\sim}{\omega}$ therefore is

$$Prob(\underset{\sim}{\omega}|h_i E_i(\omega_j)\ \forall\ i,j) \sim \prod_i exp(\beta_i h_i \omega_i + \sum_{j \neq i} \beta_i J_{i,j} \omega_i E_i(\omega_j)). \quad [5]$$

The model is closed by assuming that each agent possesses rational expectations, which means that all subjective expectations $E_i(\omega_j)$ can be replaced with mathematical expectations $E(\omega_j)$ where these expectations are conditioned on the parameters h_i, β_i, and $J_{i,j}$, $\forall\ i, j$. Recalling that $tanh(x) = [exp(x) - exp(-x)]/[exp(x) + exp(-x)]$, this implies that the mathematical expectations of the individual choices are characterized by the set of I equations (one for each i) that describe the expected value of each choice as

$$E(\omega_i) = tanh(\beta_i h_i + \sum_{j \neq i} \beta_i J_{i,j} E(\omega_j)). \quad [6]$$

Any set of expected values $E(\omega_i)$ that solve these equations represents a possible solution to the decisions of the I actors. What is interesting about this model is the interplay of the observable individual incentives that influence choice, h_i, the degree of heterogeneity in individual unobservable characteristics, β_i, and the weights that describe interdependencies between choices, $J_{i,j}$.

Relation to Statistical Mechanics Models

Eq. **5**, which expresses the equilibrium probability measure for the population's decisions, has the same functional form, known as a Gibbs measure, as is found in many statistical mechanics models (ref. 8 is an accessible reference for social scientists). The similar structure between models of individual choice and models in statistical mechanics occurs because in each case one attempts to model the aggregate behavior of a population of binary random variables in which the elements are interdependent. Notice that this mathematical equivalence does not require any deviation from the economist's standard notion of rationality or the assumption of purposeful decision making.

The substantive foundations of an economist's model of interdependent decisions are, of course, fundamentally different from that employed in physical contexts. The statistical mechanics literature typically treats the conditional probability structure of a system as a primitive from which to reason about aggregate behavior. One moves from statements of the form $Prob(\omega_i|\omega_j, j \in N_i)$, where N_i denotes a neighborhood of i, to $Prob(\underset{\sim}{\omega})$, the description of the entire system. In socioeconomic contexts, the logic proceeds from a specification of individual objectives, beliefs, and possible actions. Conditional probabilities are determined from these primitives.

Does this additional step in reasoning matter? It does in several respects. First, without an explicit statement of the individual decision-making process, it is impossible to determine how changes in outside factors such as government policy will affect the equilibrium behavior of the system because we need to know how individuals will, given their objectives, react to the change. Second, the underlying microeconomic structure dictates the appropriateness of particular modeling assumptions. For example, the baseline model assumed that individuals react not to the realized behavior of others but, rather, to their beliefs about this behavior. Such an assumption is relatively appealing when interaction groups are large, as occurs for groups defined by ethnicity, gender, or age. Third, there is an important technical advantage. By focusing on expectations, there is no need to appeal to large population sizes in analyzing the model because the mathematical structure is greatly simplified.

What Do These New Models Add?

Interactions-based models of individual behavior have the potential to provide new understandings of socioeconomic phenomena. First, there are a number of abstract insights. Because characterization of population wide-behavior depends on the distribution of h_i's, β_i's, and $J_{i,j}$'s, I consider the special case where the deterministic private incentives and the distribution of random terms are identical across individuals ($h_i = h$ and $\beta_i = \beta\ \forall i$) and each person cares about the average choices of others ($J_{i,j} = J/(I-1)\ \forall i,j$). In this case, the system of equations (6) implies that the expected average choice in a population, m, is a solution to the equation

$$m = tanh(\beta h + \beta J m) \quad [7]$$

which is the solution to the mean field approximation of the Curie-Weiss model of ferromagnetism (cf. ref. 8). What is an approximation to a physicist is an exact solution to an economist because we have assumed that individuals make decisions based on their beliefs about the behaviors of others, not on actual behavior.

This equation has the key property that, when $\beta J > 1$, there exists a threshold H (dependent on βh) such that, if $\beta h < H$, then there exist three solutions to Eq. **7** whereas, if $\beta h > H$, there exists a single solution. Intuitively, this property means that, when private incentives are sufficiently weak (h is small enough), then there is a possibility for multiple, self-consistent levels of population-wide behaviors due to the desire for conformity, which is present in individual decisions, as measured by J. In the smoking example, strong conformity effects can lead to different average smoking levels for populations of white male teenagers from different cities, even if these populations possess common observable characteristics such as the income distributions of the respective parents. This multiplicity means that the microeconomic specification of a model may not uniquely determine its macroeconomic properties (see ref. 9 for related models).

In turn, this multiplicity of possible aggregate outcomes means that the relationship between individual incentives and aggregate outcomes can be highly nonlinear. For example, small changes in private incentives h_i can have the effect of changing the number of equilibria in the model. Even when an equilibrium is unique, there is the potential for a large social multiplier (10) in which the presence of interaction effects means that a small change in private incentives (measured by h_i) can lead to large changes in aggregate behavior.

Second, at least metaphorically, these models are suggestive in terms of how to think about specific social problems. Consider social pathologies such as out-of-wedlock birth, drug use, and high school dropout rates. Conventional liberal explanations of high rates of social pathologies such as drug use and crime in poor communities focus on the absence of alternative routes leading to economic success. Conventional conservative explanations focus on a "culture of poverty" in which socially undesirable behaviors are reinforced through social norms. This baseline model illustrates how these are in fact complementary explanations. It is only when the private incentives h_i are weak that multiple equilibria, and hence socially reinforced yet undesirable outcomes, can emerge due to social interactions.

A final advantage of this formulation is statistical. As initially recognized in refs. 1 and 2, the logistic distribution assumption (Eq. **2** above) means that the theoretical model under analysis corresponds to a standard econometric model of binary choice. Therefore, models of this type can be taken to data for estimation of the various model parameters. Interestingly, the nonlinearities that naturally arise in this model (because individual choices possess a bounded support whereas the observable private incentives, h_i, do not) facilitate identification of the model's parameters from nonexperimental data sets (see the contrast between the analyses in refs. 5 and 10).

Further, these models suggest new types of statistics that should be computed to better understand cross-group behavior. One example is multimodality in the cross-group distribution of percentages of out-of-wedlock births, and so on, due to multiple equilibria. Yet another is excess cross-group variance in aggregate outcomes once differences in population characteristics are removed (11).

Summary and Conclusions

Although the use of statistical mechanics methods in economic and social modeling is in its infancy, these techniques have already proven valuable in understanding the interplay of individual- and group-level influences in determining population-wide behaviors. In terms of theory, it is important to extend these models to account for the rules by which groups are formed: neighborhood residence, school enrollment, and employment are all contexts in which individual actors choose, subject to various constraints, which interaction environments they experience. In terms of econometrics, the development of statistical analyses that relax some of the assumptions necessary for development of the theory needs to be further explored. Robust measurement of the nature and strength of interaction effects will, in turn, shape further developments of the theory.

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