



Patterns, Broken Symmetries & Computation

Emergent complexity in collective dynamics of spatially extended systems of oscillators



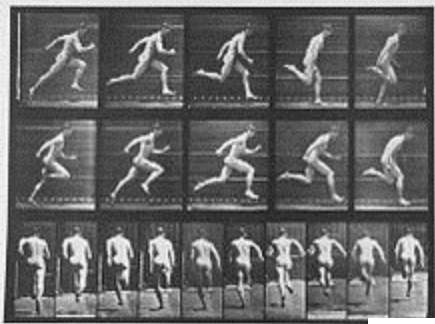
Sitabhra Sinha

in collaboration with

Rajeev Singh, Varsha Sreenivasan, R Janaki,
Parveena Shamim *and* Shakti N Menon

Oscillations in nature: span many space & time scales

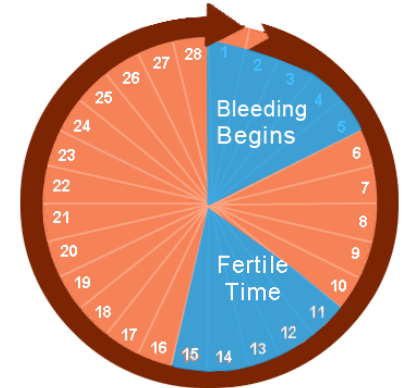
Locomotion gait patterns



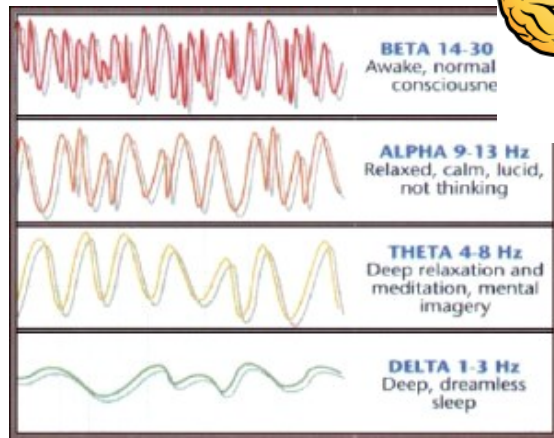
Circadian rhythm



The Menstrual Cycle



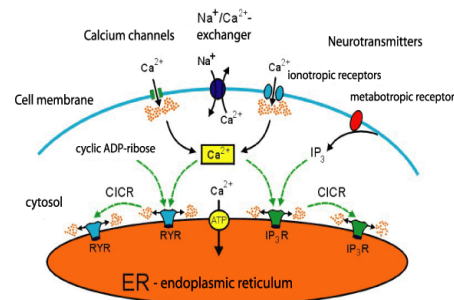
Spatial scale



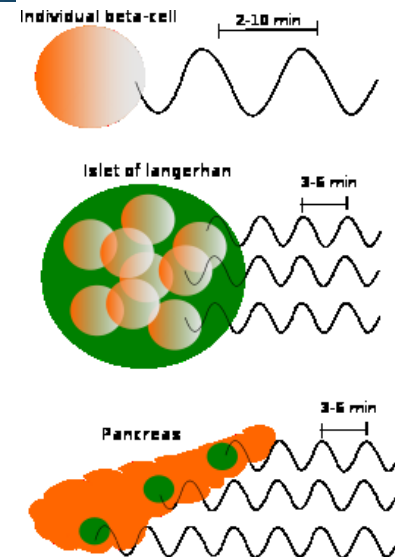
Brain activity oscillations



Intracellular oscillations



Insulin release oscillations



Time-scale

Collective ordering of spatially distributed oscillators is ubiquitous in nature

- Pacemaker cells in the heart
- β -cells in the pancreas
- Long-range synch across brain during perception
- Contractions in the pregnant uterus
- Rhythmic applause
- Pedestrians on a bridge falling in step with the swinging motion of bridge

Male Fireflies flashing in unison

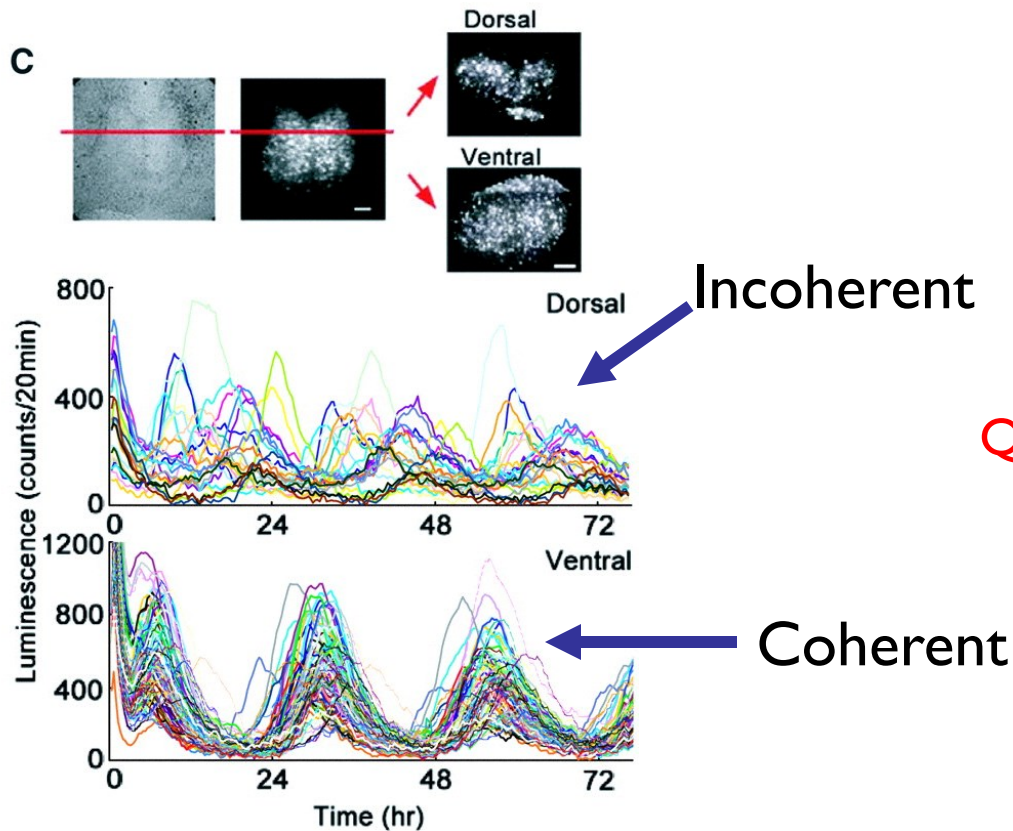
Each insect has its own rhythm – but the phase alters based on seeing its neighbors lights, bringing harmony



... and vital for the proper functioning of many biological systems

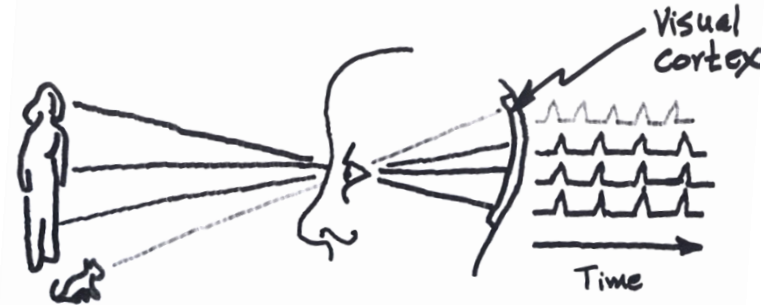
Examples:

Cellular clocks (day-night cycle)

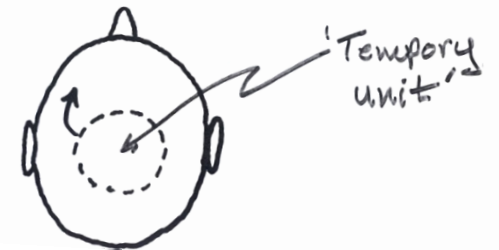


Yamaguchi et al., Science, **302**, 1408 (2003)

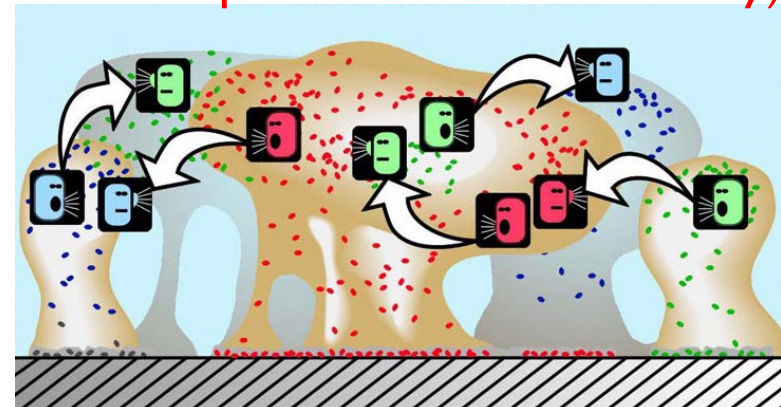
Synchrony in the Brain during



40 Hz oscillations



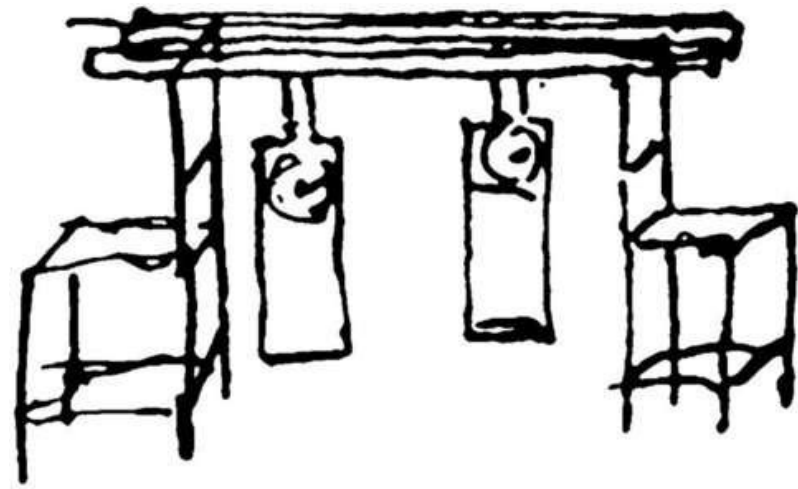
Quorum sensing (synchrony triggered by supra-threshold cell density)



<http://mpkb.org/>

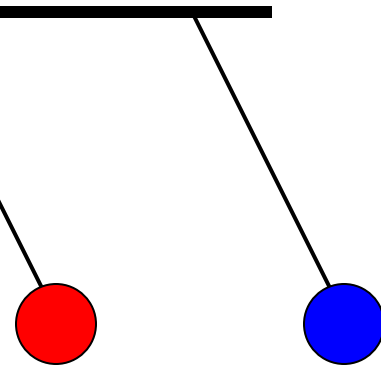
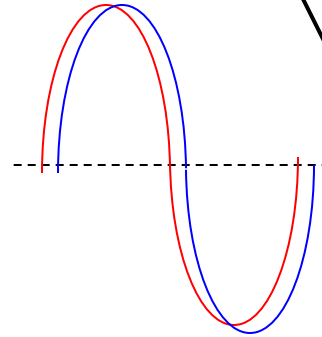
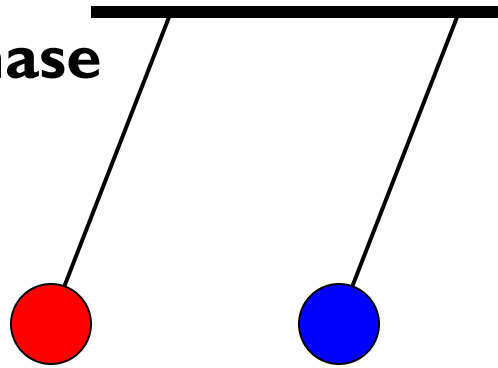
Synchronization of Coupled Oscillators

Feb 1665: Huygens observed phase-locking between two pendulum clocks hung side by side

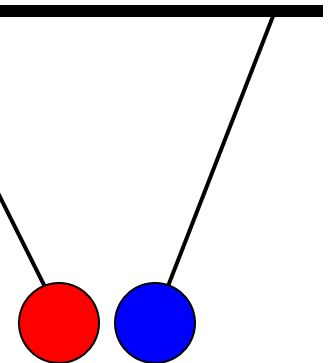
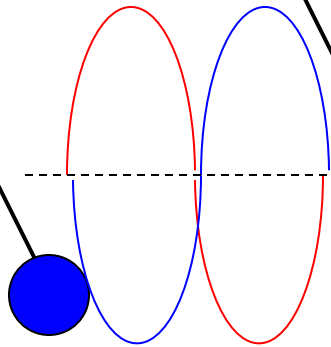
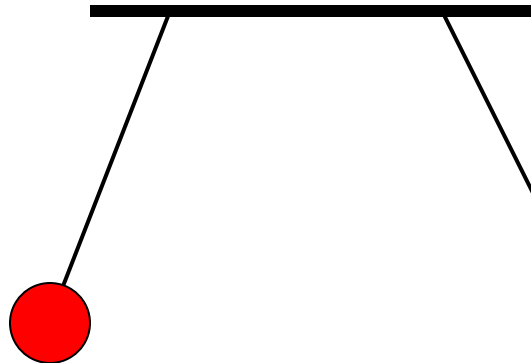


Christiaan Huygens
(1629-1695)

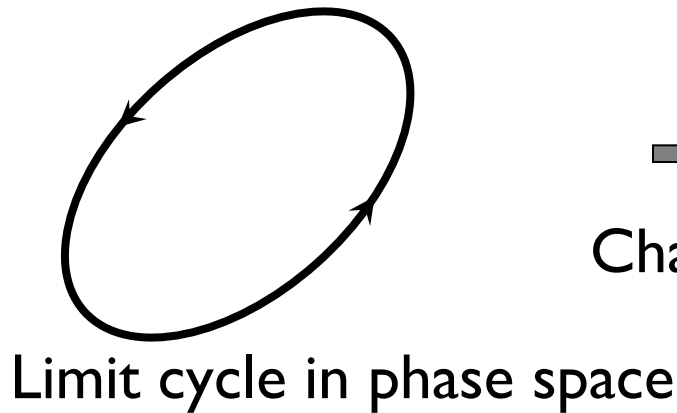
In-phase



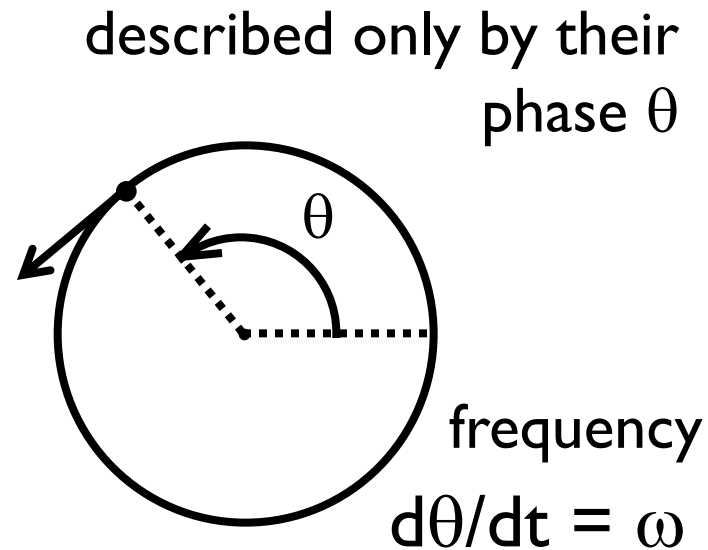
Anti-phase



Coupled Phase Oscillators



Changing variables



Consider many 'phase oscillators' : $d\theta_i / dt = \omega_i$ ($i=1,2,\dots,N \gg 1$)

The coupled system: $d\theta_i / dt = \omega_i + \sum_{j=1}^N k_{ij} (\theta_j - \theta_i)$

$$k_{ii} = 0, \quad k_{ij}(\phi) = k_{ij}(\phi \pm 2\pi)$$

Assumption: Rapid convergence to limit cycle attractor

Kuramoto model (1975): $k_{ij}(\phi) = k \sin \phi$

$$\frac{d\theta_n}{dt} = \omega_n + \frac{k}{N} \sum_{m=1}^N \sin(\theta_m - \theta_n)$$

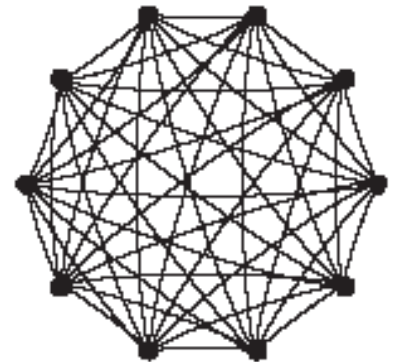
$n = 1, 2, \dots, N$ $k = (\text{coupling constant})$



Yoshiki Kuramoto
(1940 -)

- Assumes sinusoidal **all-to-all** coupling.
- Macroscopic coherence in the system is characterized by the order parameter:

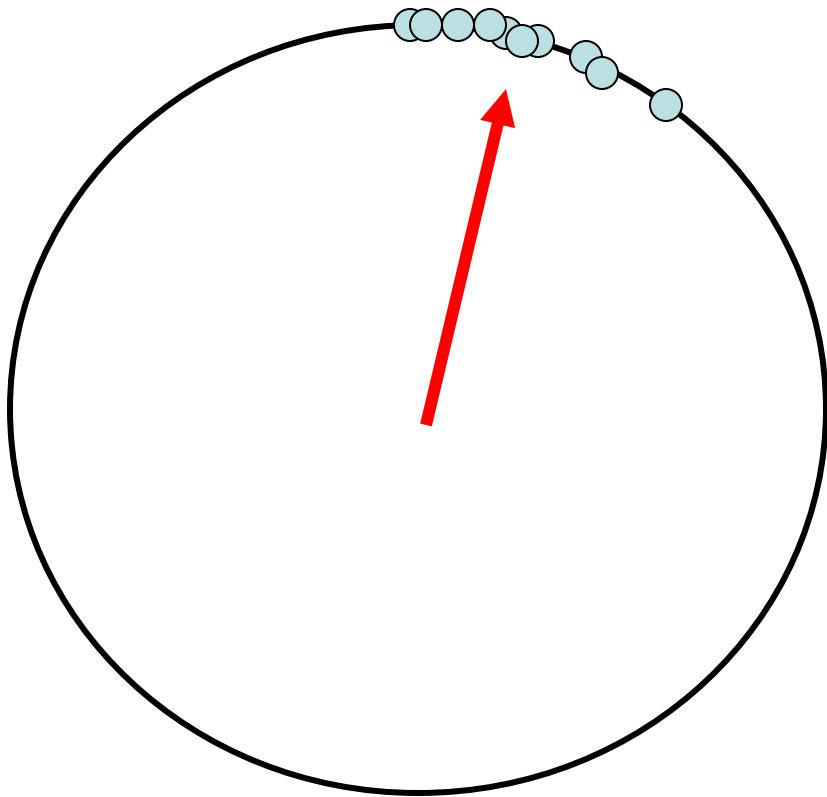
$$r = \left| \frac{1}{N} \sum_{m=1}^N \exp(i\theta_m) \right|$$



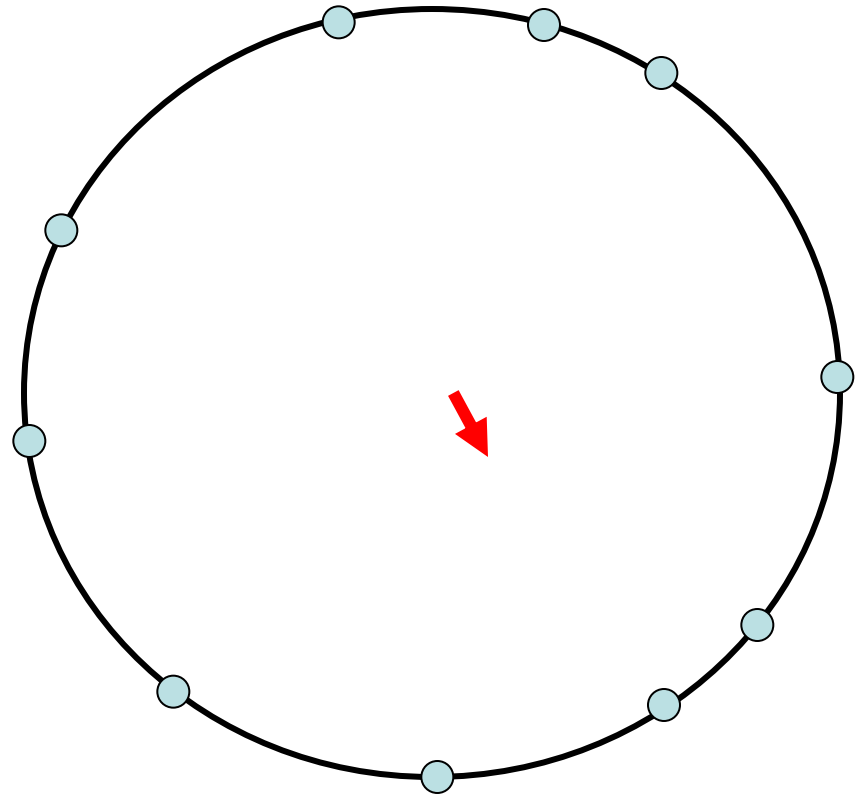
Global
coupling

Measuring coherence of oscillations in the system

$$r = \left| \frac{1}{N} \sum_{m=1}^N \exp(i\theta_m) \right|$$



$$r \approx 1$$

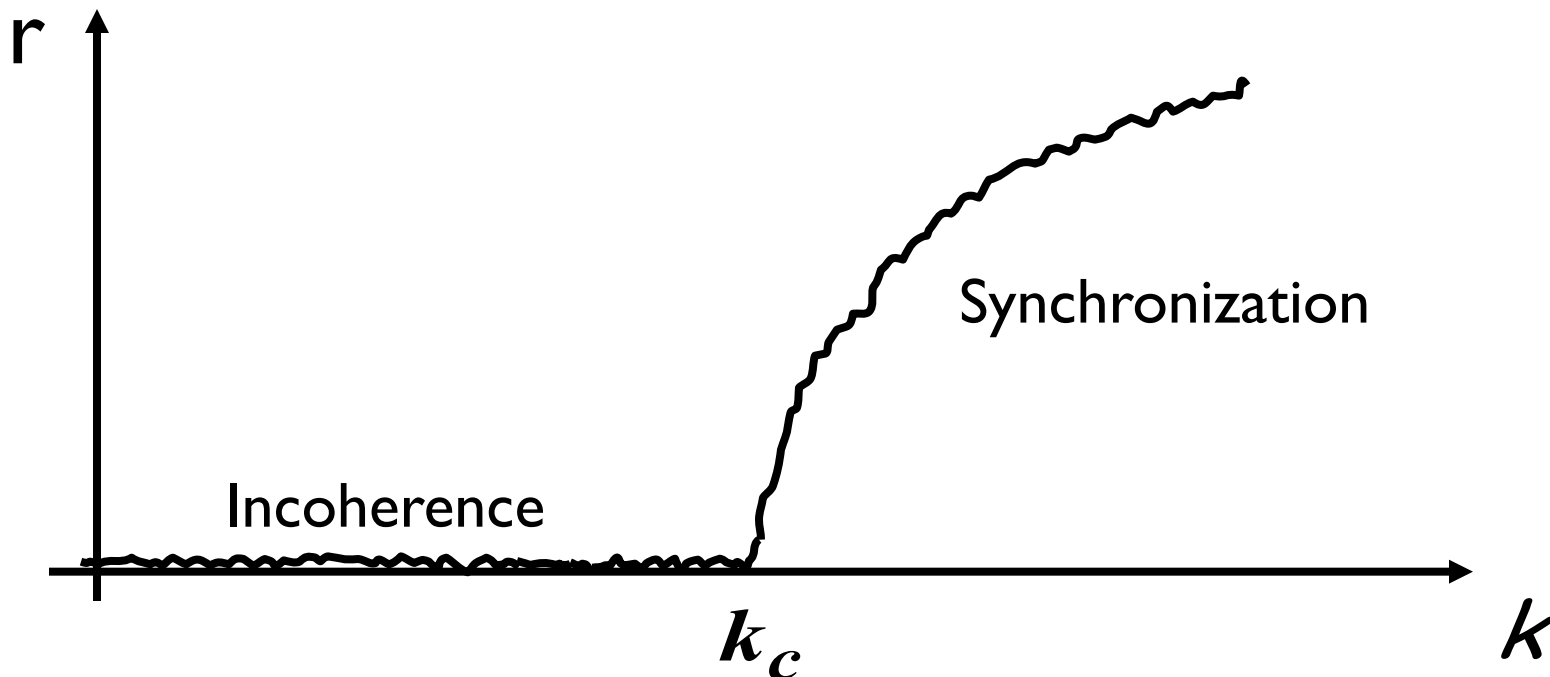
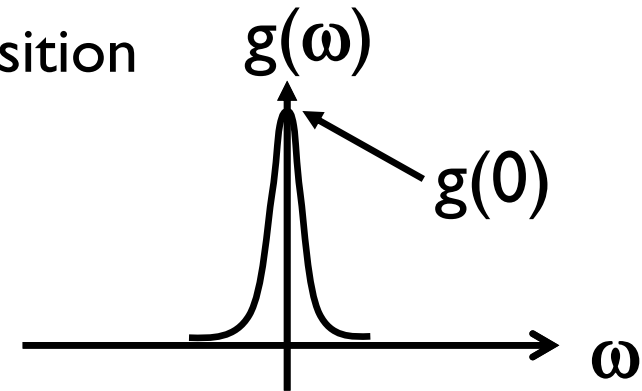


$$r \approx 0$$

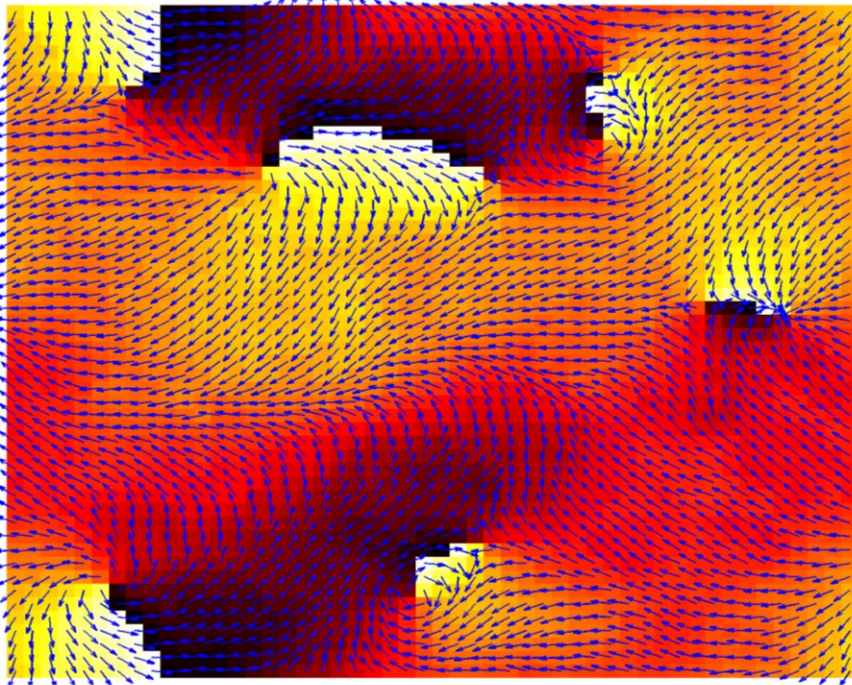
Synchronization-desynchronization transition

With increasing strength of coupling (k), a transition to coherence ($r > 0$) at a critical value of k

$$k_c = 2 / \pi g(0) N$$



Coupled Phase Oscillators on a Lattice

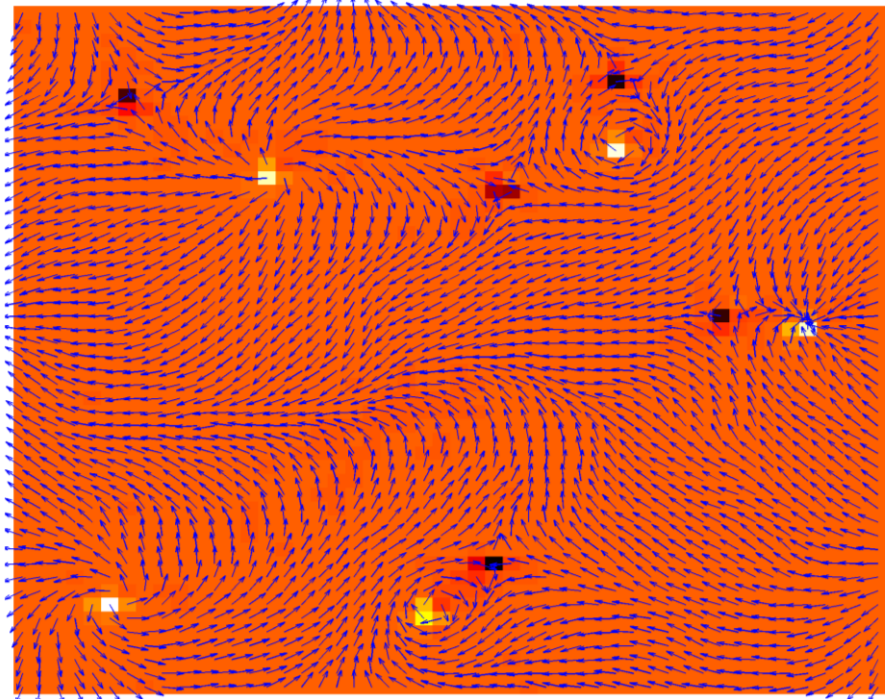


From the vector-field flow diagram of the phases it is clear that
Clockwise spirals: vortex
Anti-clockwise spirals: anti-vortex

Pairs of vortex-antivortex coalesce (annihilate) preserving net topological charge

In the presence of disorder, no global coherence – but long-range correlations still exist

Phase defects self-organize into clockwise and counter-clockwise rotating spiral waves



Pattern formation through interactions between oscillators

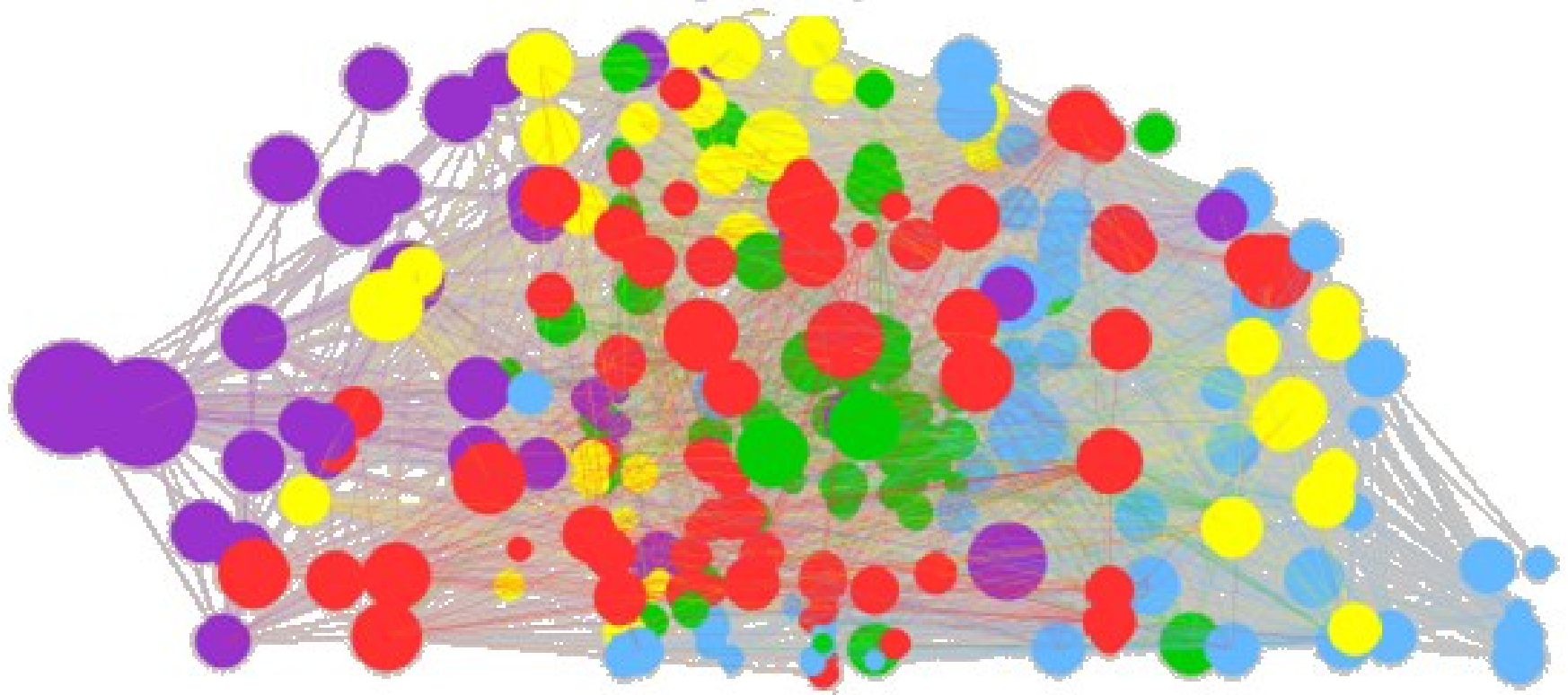
Can we use these ideas to understand synchronization-desynchronization transitions in chemical or biological complex systems where the individual elements are oscillating ?

Biologically motivated interactions may lead to surprising outcomes, such as, synchronization giving way to de-synchronization with increased coupling

Collective dynamics of a network of brain regions

The Macaque Brain

lu



CoCoMac :
links between areas
of macaque brain

(Revised from Modha & Singh, 2010)

Colors represent modules



Modeling Dynamics of a Local Brain Region

$$\tau_u \frac{du}{dt} = -u + (1 - u) S(w_{uu} u - w_{uv} v + I_u^{\text{ext}})$$

E.g., AMPA
or NMDA

Sub-population of
excitatory neurons

u

u: fraction of active excitatory
neurons

v: fraction of active inhibitory
neurons

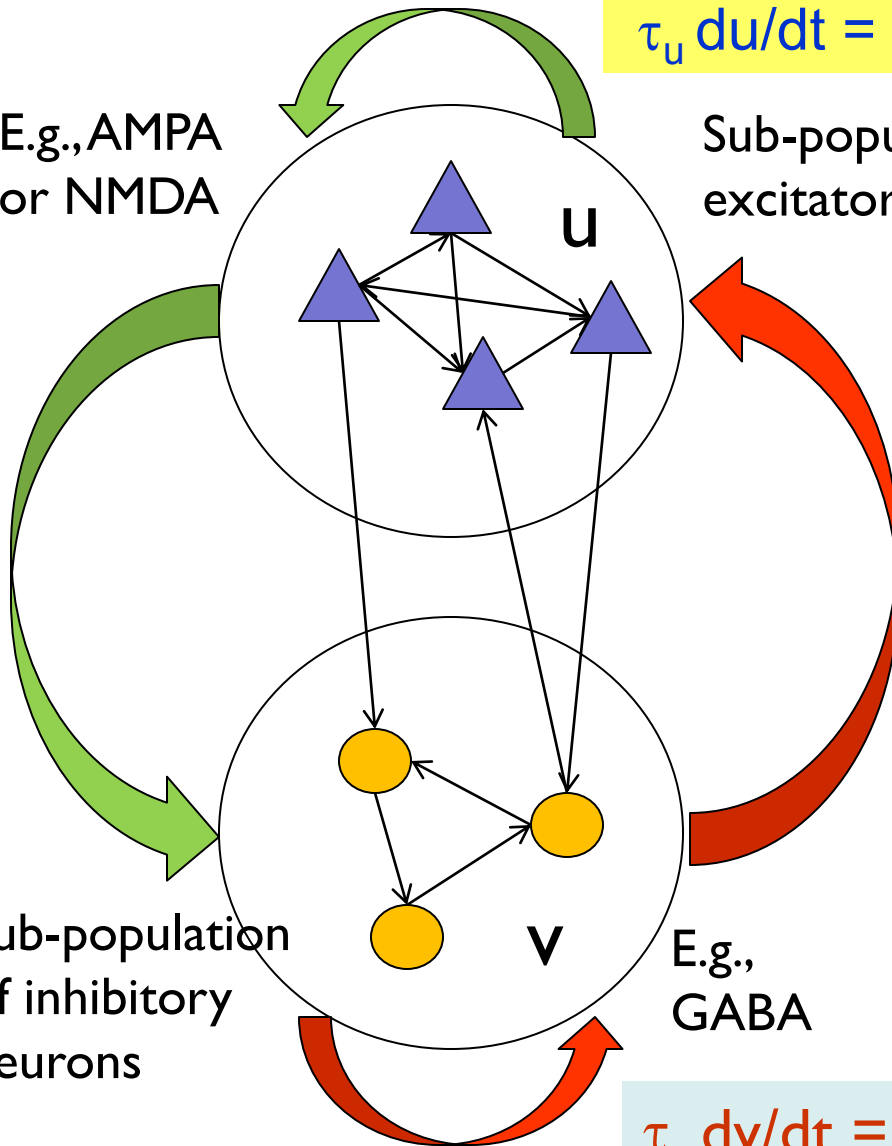
S: input-output function, usually of
sigmoidal nature

Sub-population
of inhibitory
neurons

E.g.,
GABA

v

$$\tau_v \frac{dv}{dt} = -v + (1 - v) S(w_{vu} u - w_{vv} v + I_v^{\text{ext}})$$

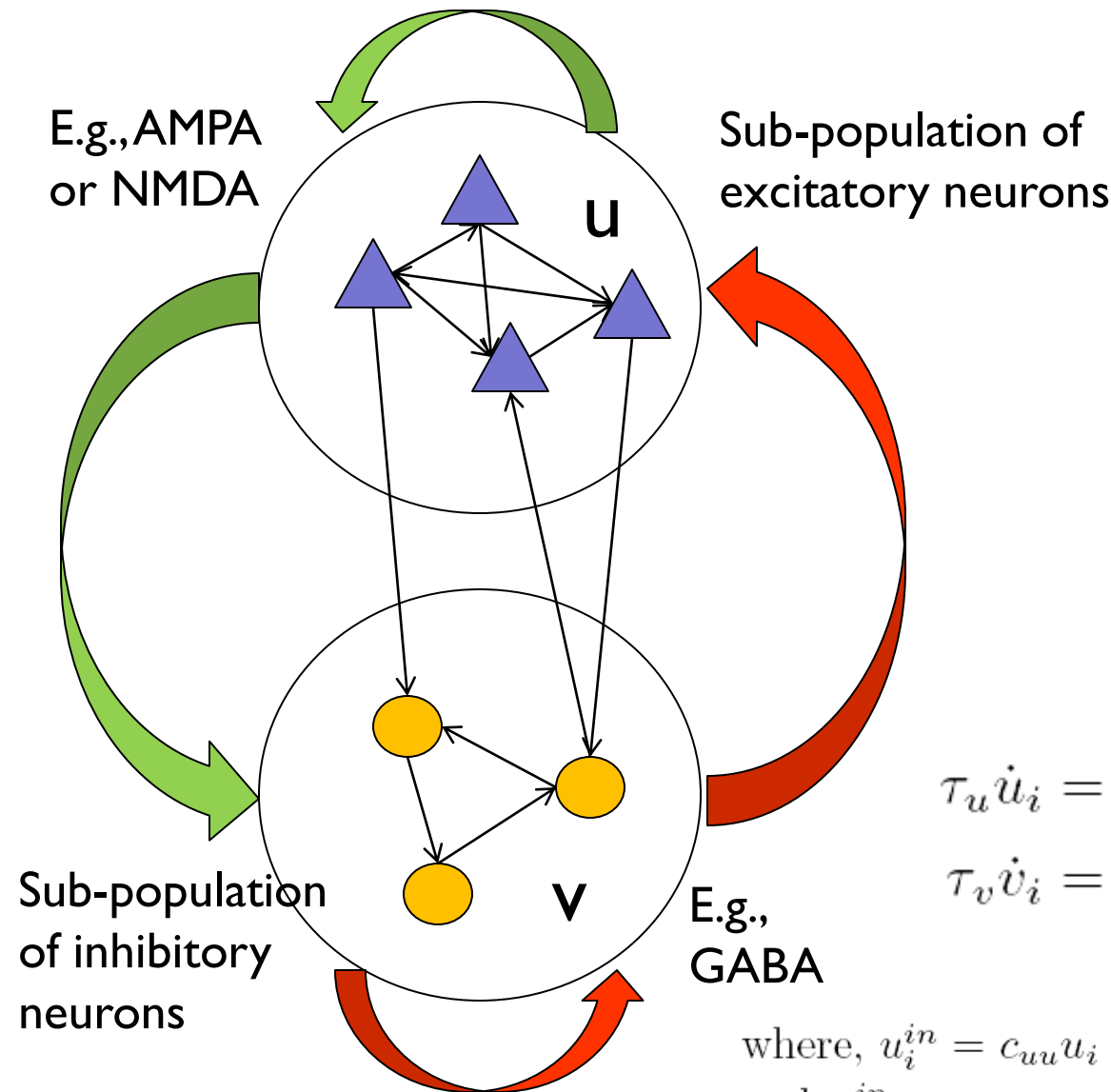


Modeling Dynamics of a Local Brain Region

Wilson-Cowan Model



Phenomenological description of activity in a system of coupled subpopulations of excitatory & inhibitory neurons



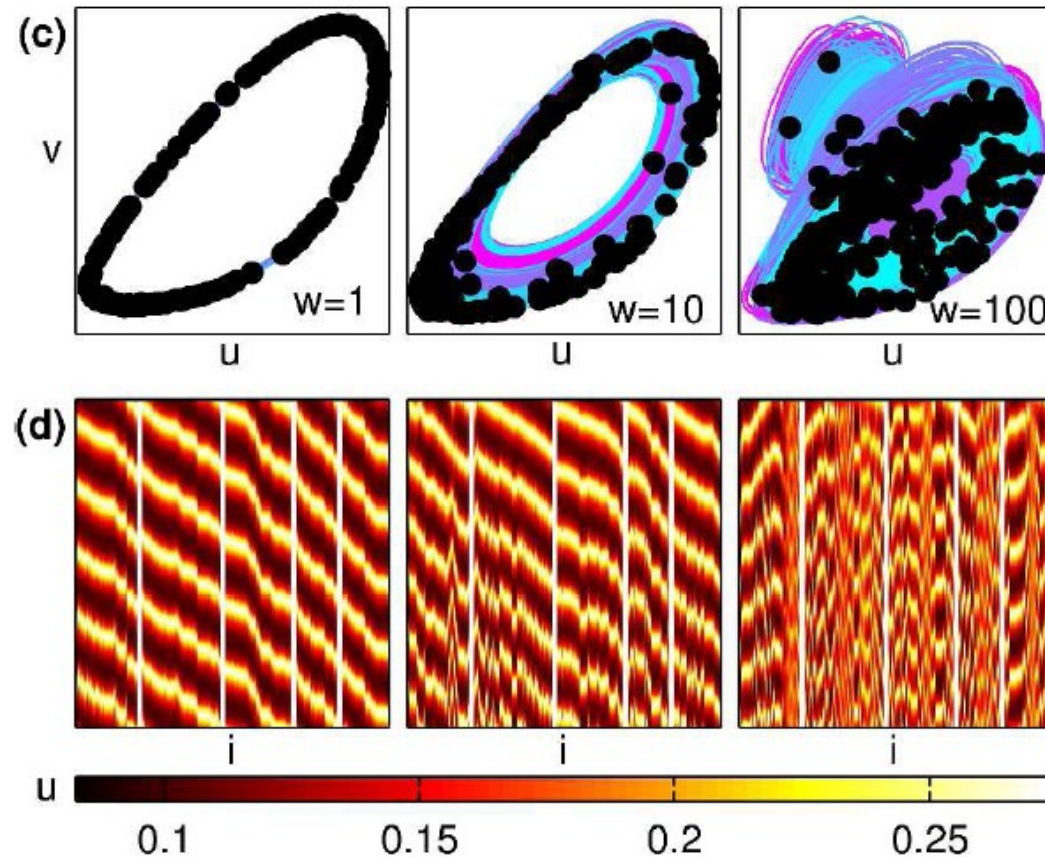
$$\tau_u \dot{u}_i = -u_i + (\kappa_u - r_u u_i) \mathcal{S}_u(u_i^{in}),$$

$$\tau_v \dot{v}_i = -v_i + (\kappa_v - r_v v_i) \mathcal{S}_v(v_i^{in}),$$

where, $u_i^{in} = c_{uu}u_i - c_{uv}v_i + \sum' (w_{ij}^{uu}u_j - w_{ij}^{uv}v_j) + I_u^{ext}$
 and $v_i^{in} = c_{vu}u_i - c_{vv}v_i + \sum' (w_{ij}^{vu}u_j - w_{ij}^{vv}v_j) + I_v^{ext}$

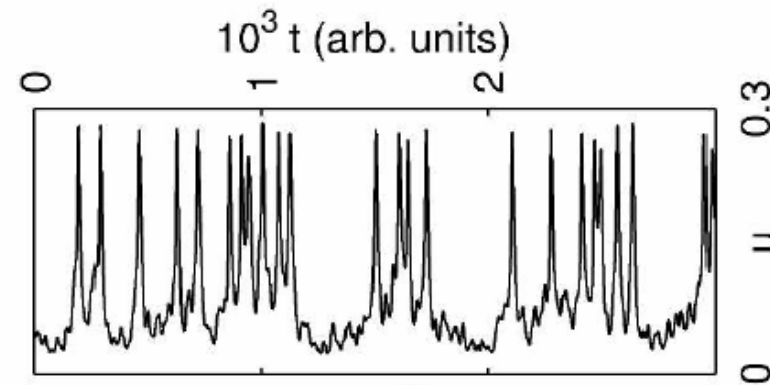
Collective Dynamics of Brain Networks

Increasing connection strength $\rightarrow$



Parameters of WC model
chosen so that each isolated
node has limit cycle dynamics:
Relaxation oscillations

Each of the N nodes of the
Macaque cortico-cortical network
is modeled by the Wilson-Cowan
oscillator

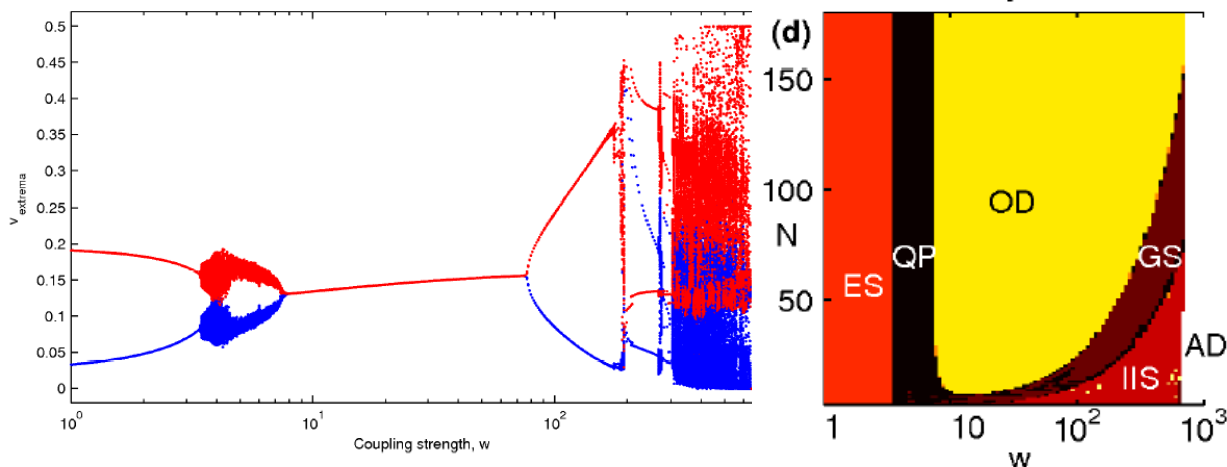
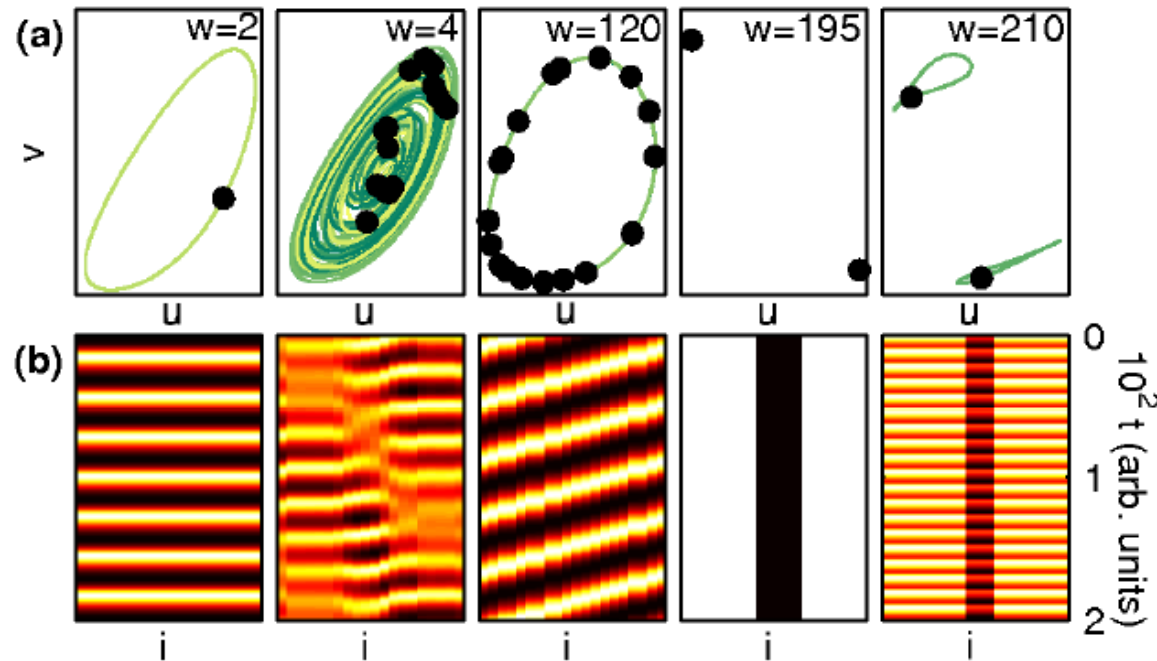


Resulting time-series of node activity are
qualitatively similar to experimentally
recorded activity in Macaque brain regions

Observed Patterns = Network + Dynamics

*Spontaneously
broken symmetry*

To understand better the genesis of the different spatio-temporal patterns in the network of Macaque brain regions, we can separately analyze the contributions of the
(a) brain-region dynamics and
(b) network organization



For (a), we study the dynamics of N globally coupled WC oscillators as a function of coupling

Analysis: Contribution of Dynamics

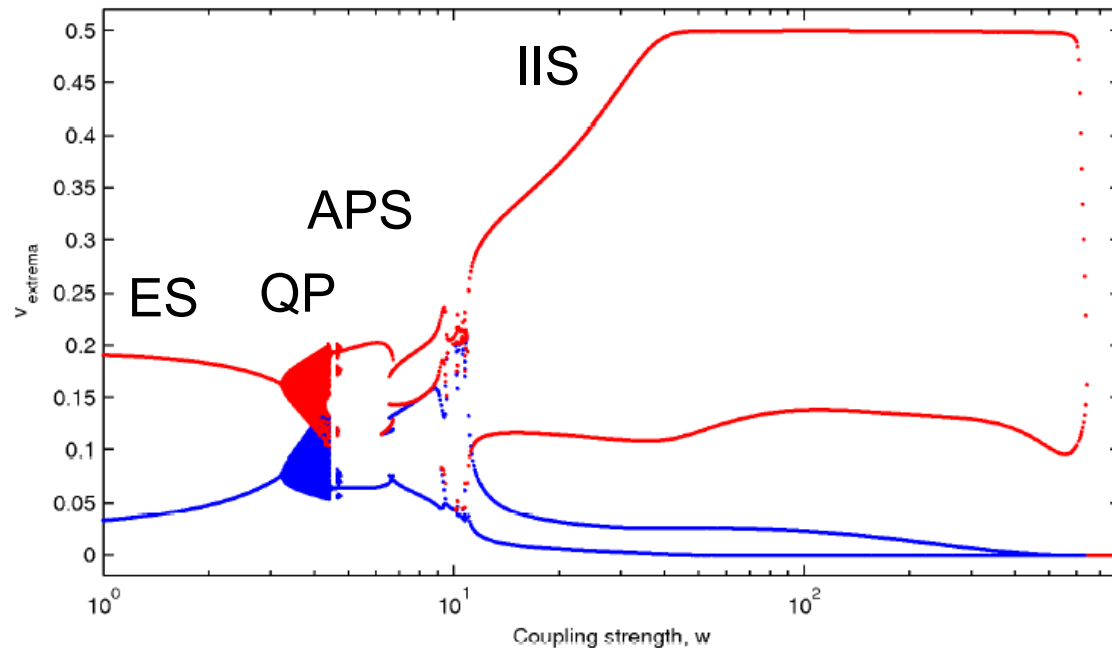
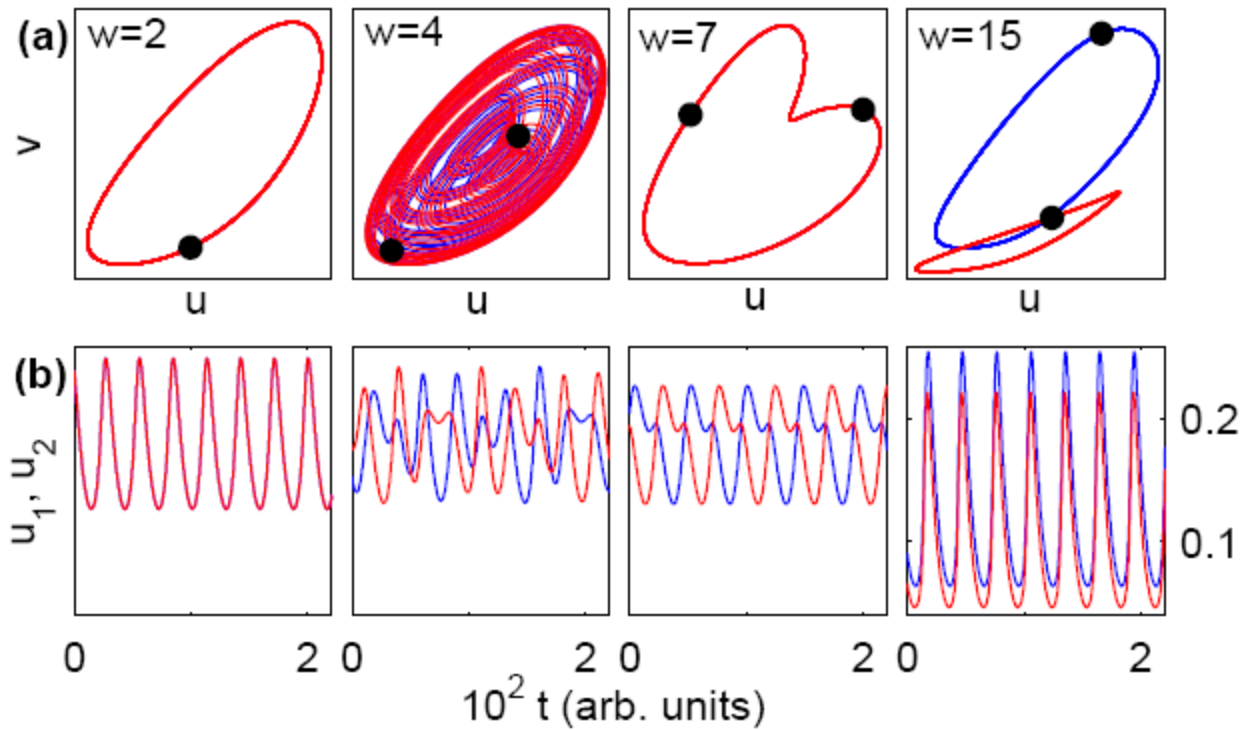
ES: exact synchronization

QP: quasiperiodicity

APS: anti-phase synch

IIS: Inhomogeneous

in-phase synchronization



Several observed dynamical patterns can be understood in terms of bifurcations in the global dynamics of a simple system of coupled “brain-region” oscillators

ES movie

APS movie

Many of the qualitative patterns seen in brain networks can be explained from the collective dynamics of coupled oscillators having activity like brain regions

In particular, suggests a relation between

- anaesthetic-induced loss of consciousness occurs through progressive disruption of communication between brain areas $\Rightarrow$ **weaker interactions**

L. D. Lewis et al., PNAS 109, E3377 (2012).

- functional connectivity networks reconstructed from EEG data become increasingly dense with the development of fatigue in sleep-deprived subjects $\Rightarrow$ **increased synchronization**

S. Kar et al., Clin. Neurophysiol. 122, 966 (2011)

Although it may appear counter-intuitive that decreased coupling strength results in increased synchronization, our findings show that these results are not incompatible.

$w = 1$ (connectivity = 0.326)

Fatigue or anaesthetic induced effects → decrease in effective coupling strength w between cortical regions

Networks constructed from cross-correlations between activity in different brain regions essentially

measure this decreased communication between the regions brought about by weakened coupling

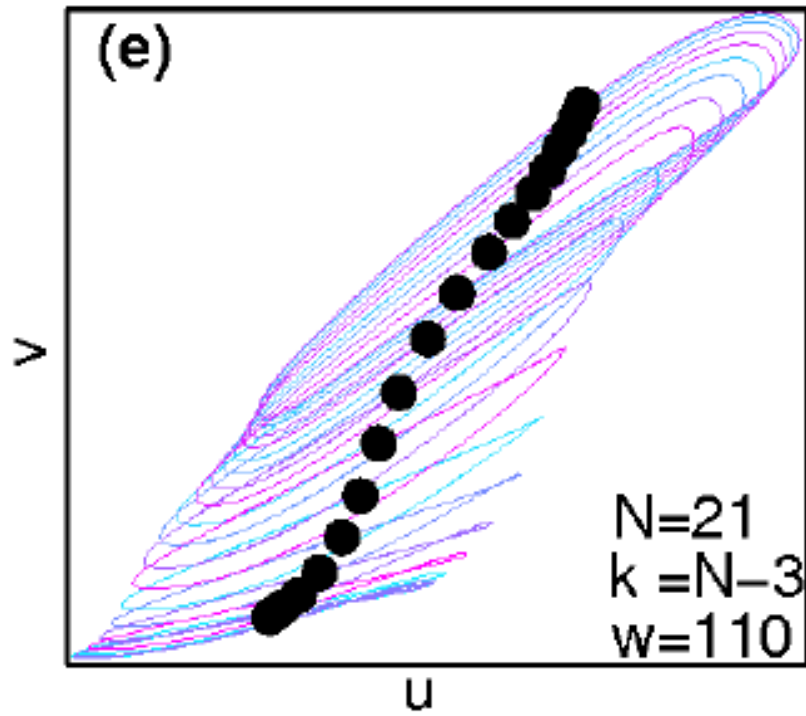
Network becomes increasingly sparse with increasing w as dynamics of different nodes become more desynchronized

Decreased coupling between different brain regions can be responsible for the higher connectivity observed in functional brain networks reconstructed from EEG recordings

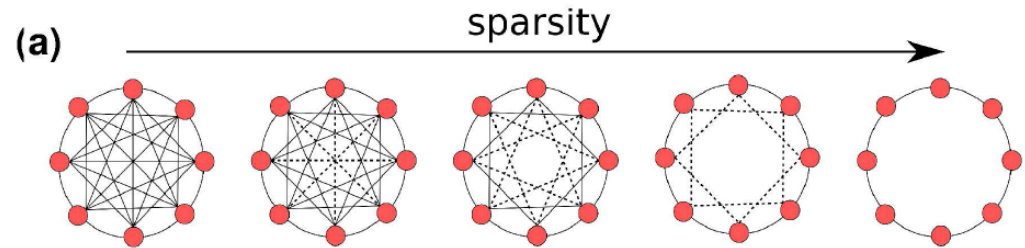
$w = 10$ (connectivity = 0.133)

$w = 100$ (connectivity = 0.053)

What is the contribution of the network structure ?



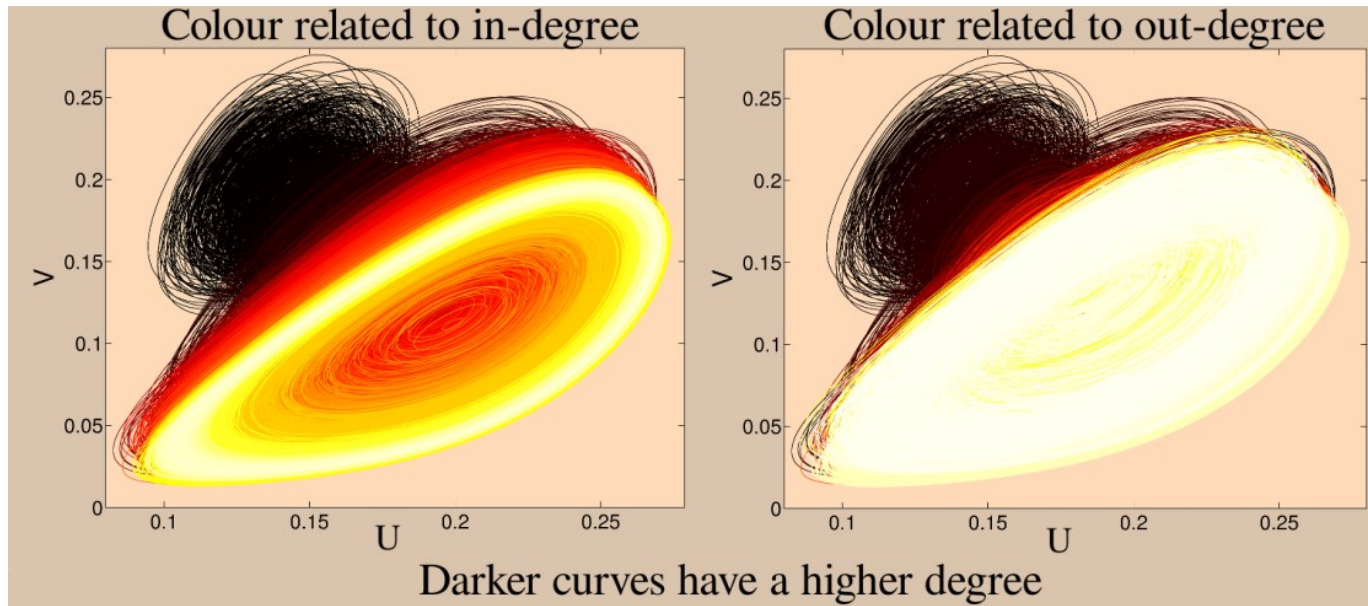
A reduction of just 2 links per node causes the trajectory in the IIS state to split into many more ($\sim N$) projections than seen for the fully connected case (~ 2).



Dramatic quantitative reduction in the area of parameter space corresponding to Oscillator Death even with the reduction of one link per node.

Surprising: one would expect that a marginal deviation from the global coupling limit in large systems will not result in a perceptible change from the mean-field behavior.

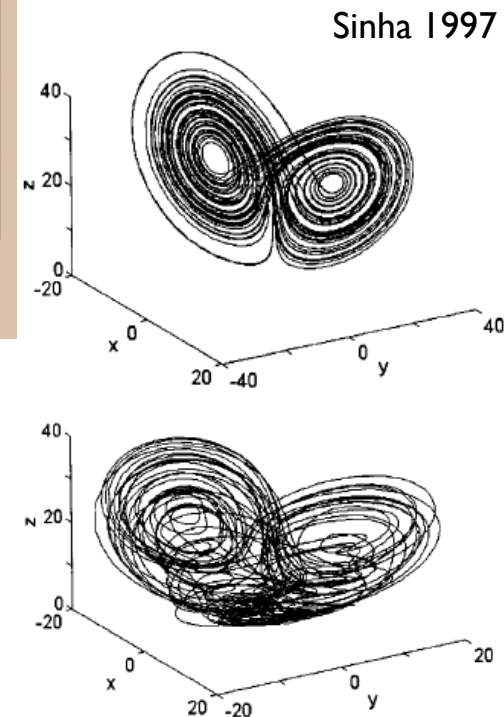
Analysis: Contribution of Nodal Degree



The divergence of phase space trajectories of different local regions of the Macaque cortical network at higher connection strengths is related to the density of incoming connections

“Frustrated synchronization”

Many systems trying to simultaneously synchronize another result in conflict, “frustrating” each other through competition



But

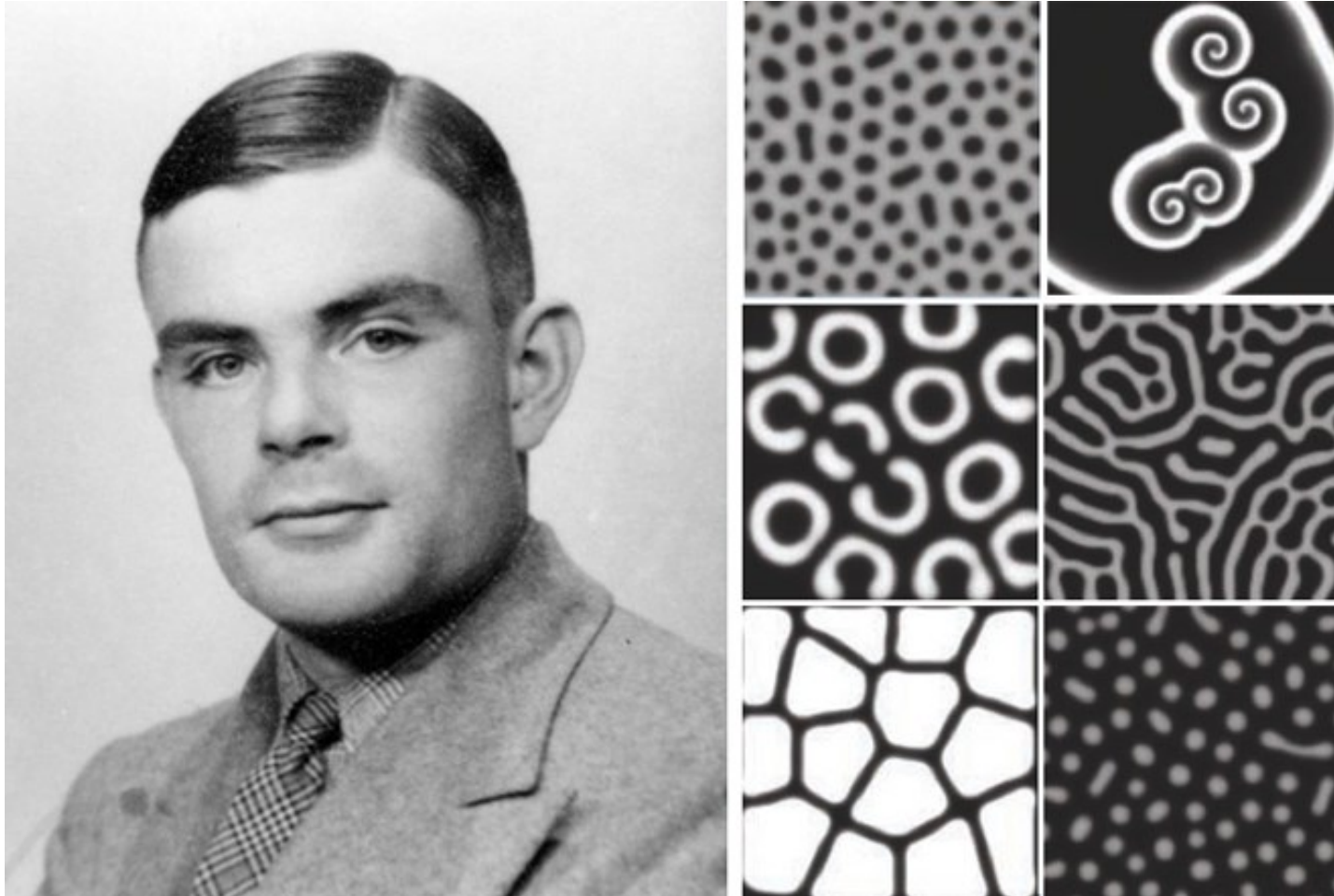
Could the different synchronization patterns observed be simply a product of the specific type of nonlinear interaction between WC oscillators ?

In general what kind of patterns can arise as a result of interactions between neighboring oscillating elements in a lattice ?

Arrays of coupled chemical relaxation oscillators with lateral inhibition provide a fascinating glimpse into the variety of collective ordering phenomena that are possible

The magnificent patterns of Alan Turing

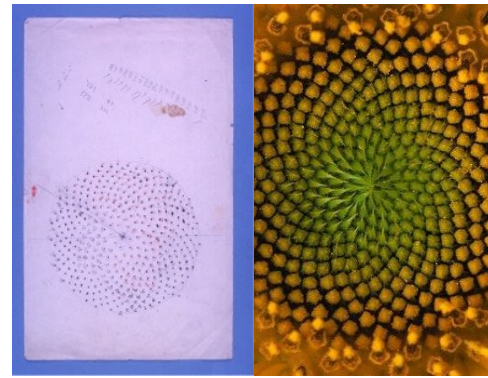
A. Turing
The Chemical Basis of Morphogenesis
Phil. Trans. Roy. Soc. Lond. B 237 (1952) 37



How to explain the development of patterns arising spontaneously in nature ?

Morphogenesis by lateral inhibition

Morphogenesis, development of shape or form in plants and animals explained using reaction-diffusion model systems of two substances with concentrations u_1, u_2



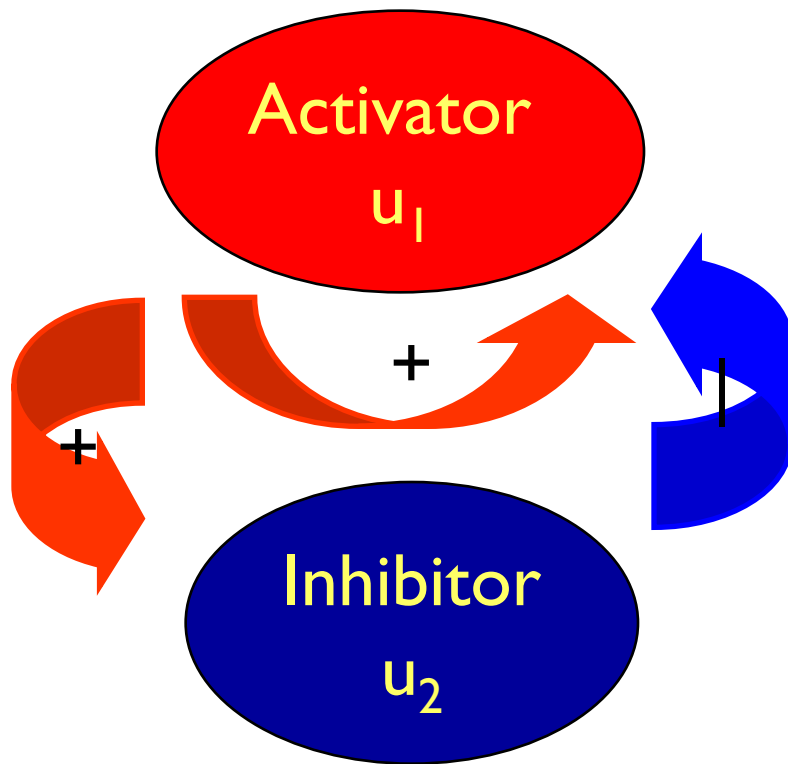
$$\partial_t u_1 = f_1(u_1, u_2) + D_1 \partial_x^2 u_1$$

$$\partial_t u_2 = f_2(u_1, u_2) + D_2 \partial_x^2 u_2$$

$\equiv$

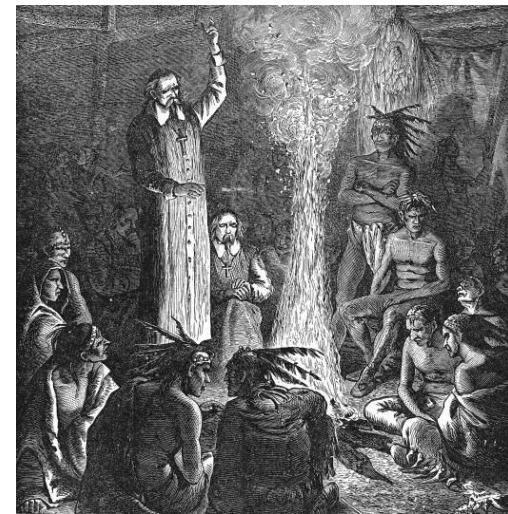
$$\partial_t \mathbf{u} = \mathbf{f}(\mathbf{u}) + \mathbf{D} \partial_x^2 \mathbf{u}$$

$$\mathbf{D} = \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix}$$



- ❑ Activator u_1 : stimulates increase in concentration of both chemicals
- ❑ Inhibitor u_2 : leads to a decrease in concentration of activator
- ❑ Turing: such a system can produce stationary pattern through **spontaneous symmetry-breaking** if inhibitor diffuses much faster than activator : **Local activation with lateral inhibition**

Turing's analogy: Missionaries vs. Cannibals



An island populated by
(i) cannibals & (ii) missionaries.

- **Missionaries** [inhibitors]
 - are all celibate
 - depend on recruiting to maintain their population as members gradually die.
- **Cannibals** [activators]
 - also die,
 - but can reproduce, increasing their population.
- **When two missionaries meet a cannibal, (s)he is converted to missionary status**

When both populations mixed together, stable balance reached between reproduction & conversion.

If disturbed by a small amount of noise, the system returns to balanced state.

Equilibrium.

$$U, V > 0$$

$$-2.5U - 4V + 6.5 + 4 \frac{dU}{dx} = 0$$

$$2.5U + 3.5V - 6 + \frac{dV}{dx} = 0$$

Try a solution of form $U = e^{2x}$, $V = e^{2x}$

$$-2.5U - 4V + 4 \frac{dU}{dx} = 0$$

$$2.5U + 3.5V + \frac{dV}{dx} = 0$$

$$\frac{U}{V} = \frac{4}{4q^2 - 2.5} = - \frac{3.5 + q^2}{4q^2 - 2.5}$$

$$\frac{1}{q^2 - 0.625} + \frac{3.5 + q^2}{2.5} = 0$$

$$q^4 + 2.875q^2 + 0.3125 = 0$$

$$(q^2 + 2.0)(q^2 + 0.1) = 0$$

$$q_1 = 1.4142i, \quad q_2 = 0.31622i$$

$$\frac{1}{q_1^2 - 0.625} = \frac{1}{-0.5} = -2$$

$$\frac{1}{q_2^2 - 0.625} = \frac{1}{-0.2087} = -4.788$$

In mixed regions

$$A \cos q_1 x + B \cos q_2 x + D = U$$

$$A' \cos q_1 x + B' \cos q_2 x + D' = V$$

$$-2.5A - 4A' - 4q_1^2 A = 0$$

$$-2.5B - 4B' - 4q_2^2 B = 0$$

$$-2.5D - 4D' - 6.5 = 0$$

$$2.5A + 2.5A' - 4q_1^2 A = 0$$

$$2.5B + 2.5B' - 4q_2^2 B = 0$$

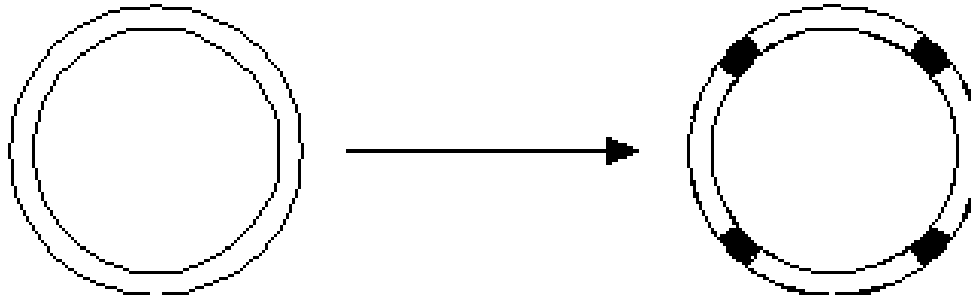
$$2.5D + 2.5D' - 6 = 0$$

Choose $D = D' = 1$

$$\frac{A}{A'} = -1.7526, \quad \frac{B}{B'} = -1.3548$$

Pattern formation via diffusive instability

Missionaries (on cycles) vs. Cannibals (on foot)



<http://www.swintons.net/deodands/>

Now introduce space in the model:

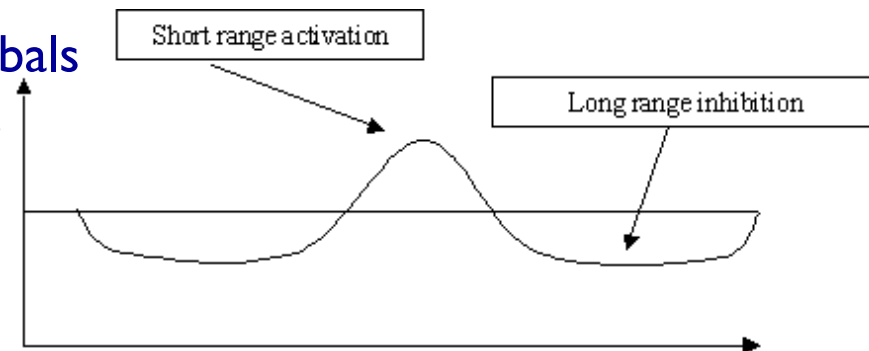
- consider the populations to be spread out in a thin ring around the narrow beach of the island,
- individuals interact only with their nearest neighbors
- while diffusing at random

But the missionaries have bicycles and move faster

⇒ **Instability in system:**

if there is at any point a small excess of cannibals

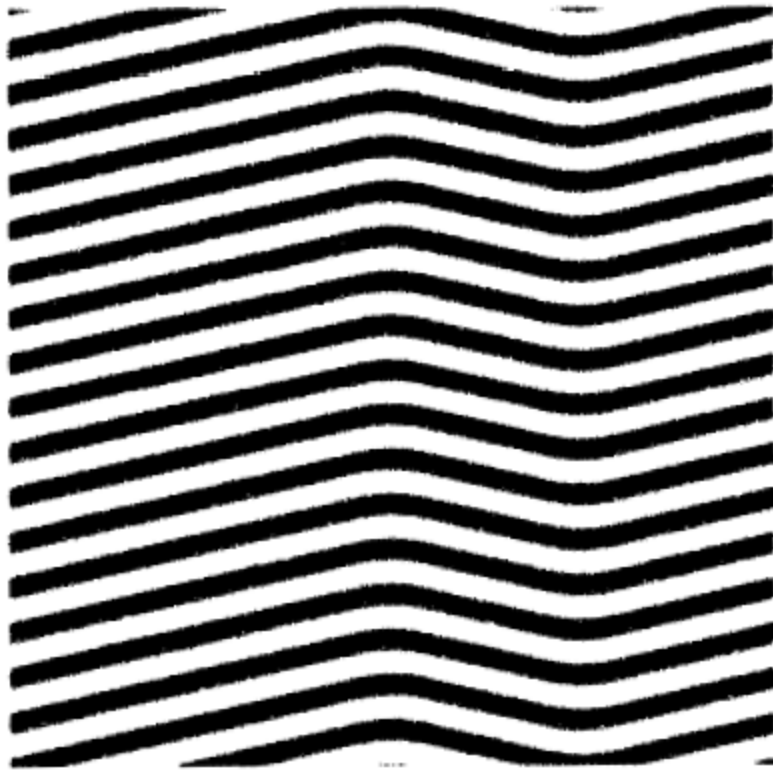
- excess 'production' of more cannibals
- more missionaries
(as more targets for conversion).



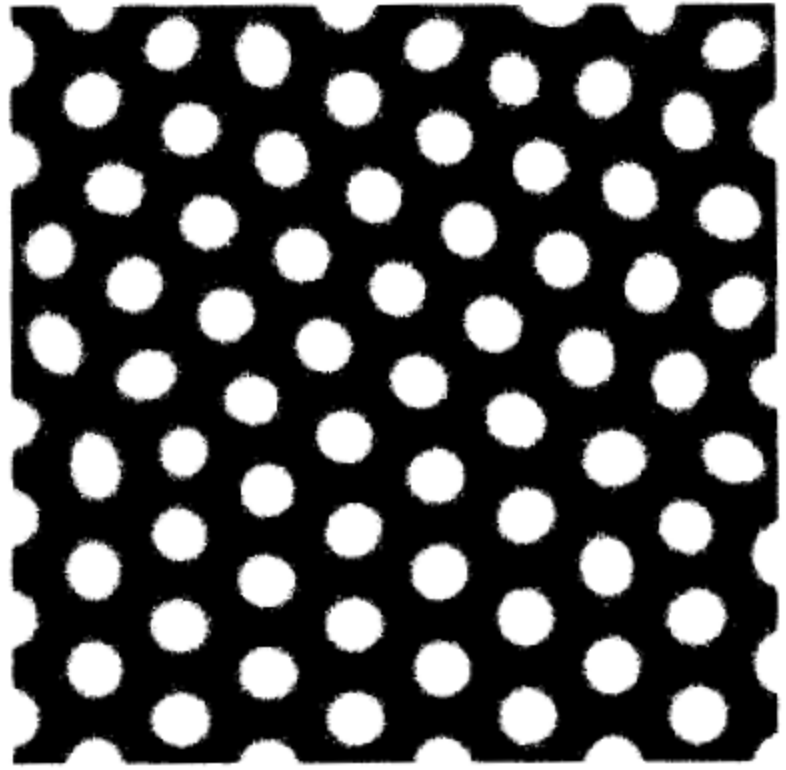
Without diffusion, extra missionaries reduce cannibal excess, system returns to balance.

But with diffusion, missionary excess transported away faster ⇒ a pattern develops with cannibal excess in center and excess missionaries at edge.

Typical patterns of 2-Dimensional Turing reaction-diffusion system



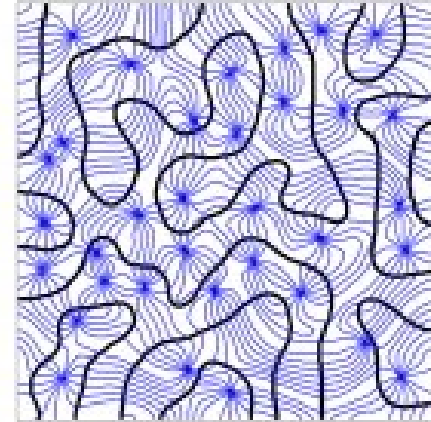
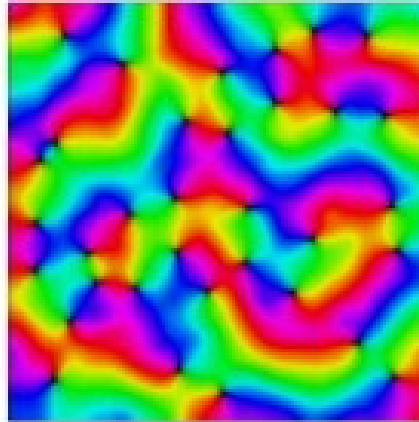
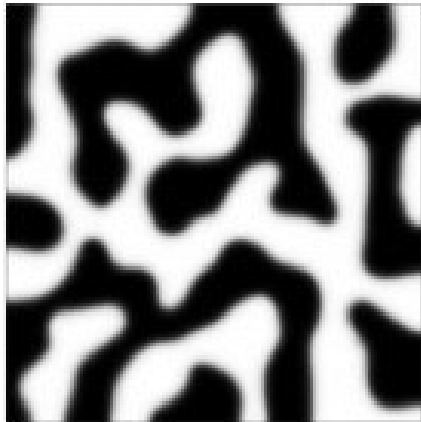
Stripes



Spots

In general,
the concept of lateral inhibition can give rise
to a variety of patterns...

E.g., Orientations columns in the visual cortex



G J Goodhill, Neuron (2007)

Brian C. Goodwin (1931–2009)

Lateral Inhibition with Oscillators

Goodwin: Development of patterns arising from coupled biochemical oscillators ?

In many biological systems, the individual entities undergo periodic oscillations instead of remaining in a constant state

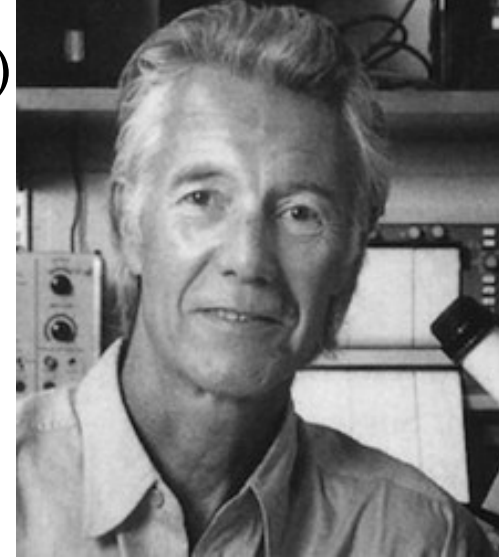
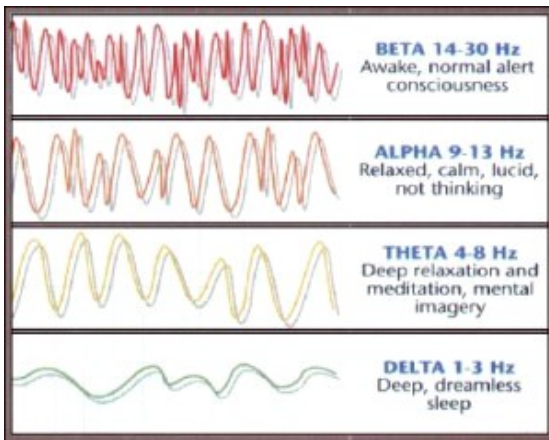
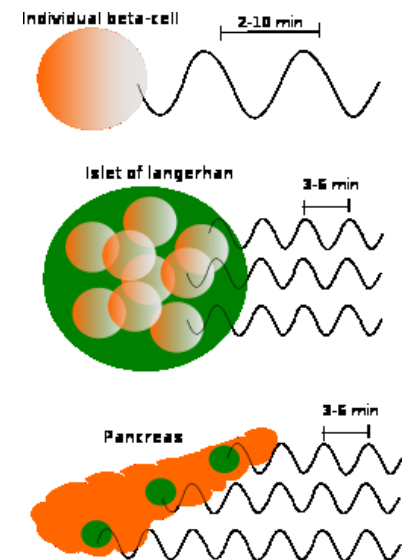
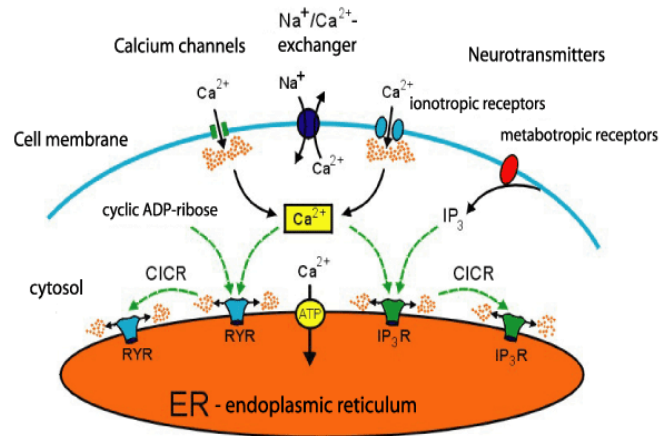


Image source: Wikipedia



Neuronal activity oscillations



Intracellular oscillations

Insulin release oscillations

What will happen if we allow these oscillators to have activating-inhibiting diffusive interactions with each other ?

Example: Chemical oscillators e.g. B-Z reaction

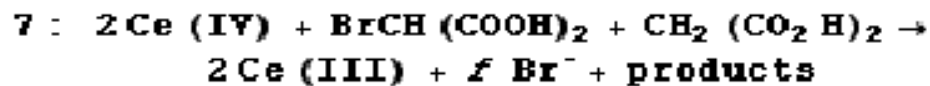
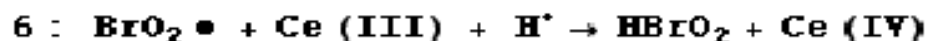
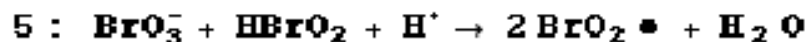
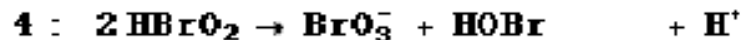
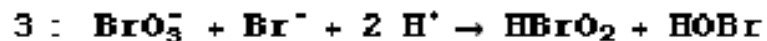
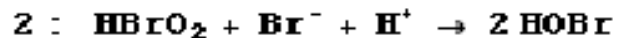
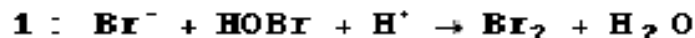
A family of oscillating homogeneous chemical reactions

B P Belousov (1951) Investigated a solution of bromate, citric acid (the reductant) and ceric ions (the catalyst)



Instead of monotonic conversion of yellow Ce^{4+} (reduced) to colorless Ce^{3+} (oxidized), saw **periodic oscillations** of the color

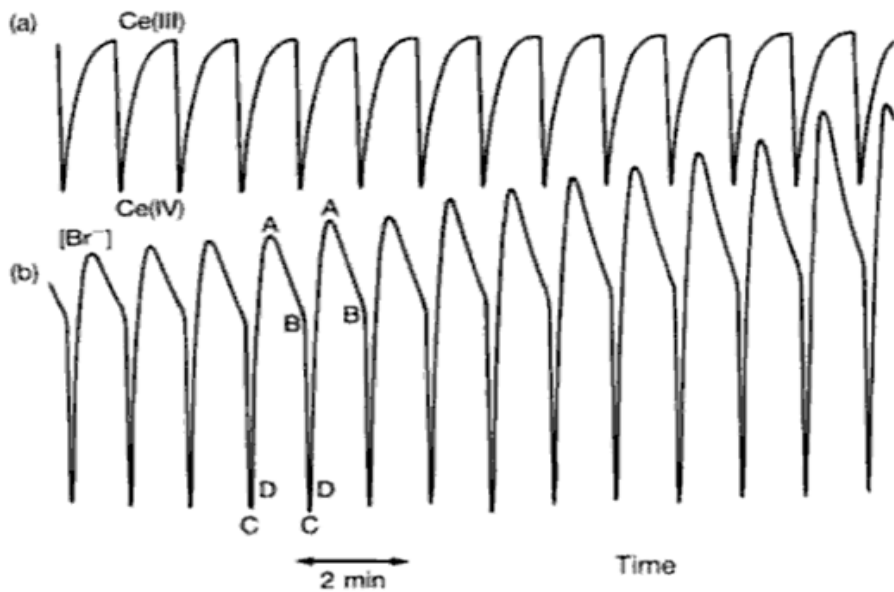
A M Zhabotinskii (1961+) Established the validity of Belousov's results – showed the phenomenon to be robust



Boris P. Belousov



A. M. Zhabotinskii



BZ oscillations

Typical experimental records from (a) Pt electrode and (b) Br ion sensitive electrode for BZ reaction

Spatial patterns in the BZ reaction

Target and Spiral waves

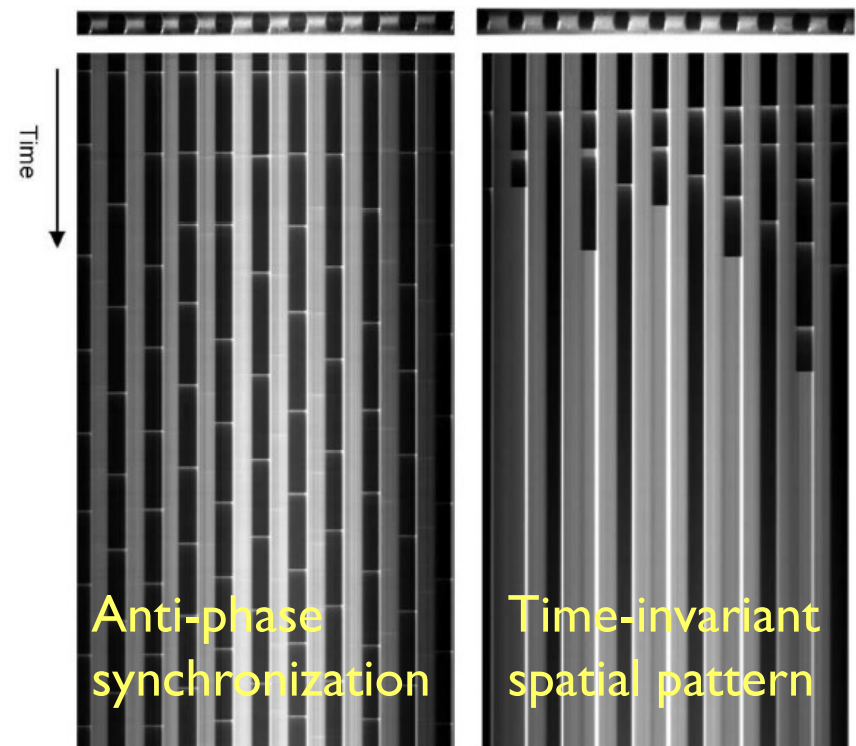
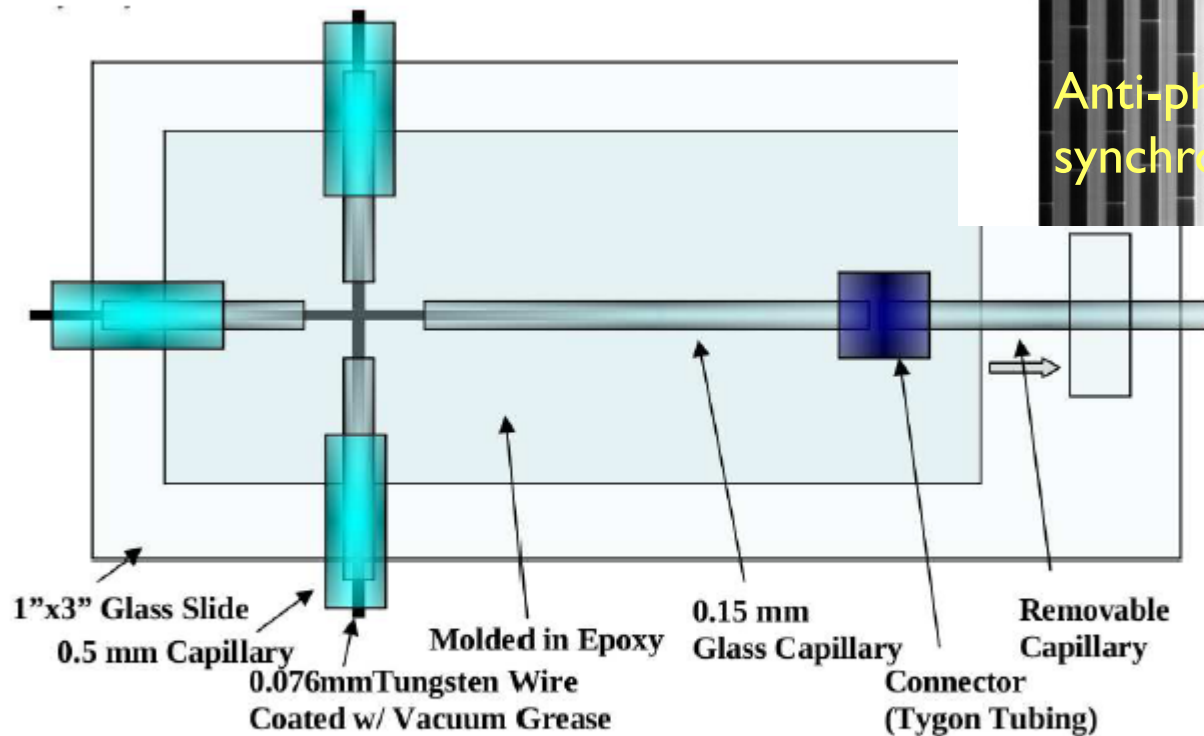
$$\begin{aligned}\frac{\partial x}{\partial t} &= F(x, y) + D_x \nabla^2 x \\ \frac{\partial y}{\partial t} &= G(x, y) + D_y \nabla^2 y\end{aligned}$$

Introducing spatial interactions through diffusion



Array of coupled chemical oscillators: Experiments

Recent experiments on microfluidic device show (i) anti-phase synch and (ii) spatially inhomogeneous time-invariant patterns



Beads containing BZ reactive solution suspended in a chemically inert medium that allows passage of only **inhibitory** chemical species

Figure S1. Schematic drawing showing the layout of the microfluidic device. Tungsten wires are removed after the epoxy cures. 0.5 mm capillaries provide inputs for the BZ solution and oil. Droplets produced at the junction are flowed into a removable capillary and put under a microscope for imaging. Toiya, Vanag, Epstein, *Angew. Chem. Int. Ed.* 2008, 47, 7753

The model

Array of coupled relaxation oscillators with passive elements at boundaries



Nodal dynamics

Individual oscillators described by a modified Van der Pol model

$$\dot{u} = f(u, v) = u(1 - u)(u - \alpha) - v,$$

$$\dot{v} = g(u, v) = \epsilon(ku - v - b),$$

$$\alpha = 0.139, k = 0.6, \epsilon = 0.001$$

b: measure of asymmetry of the oscillator (ratio of times spent at high-u branch to low-u branch)

Coupling

Oscillators diffusively coupled via **inactivation variable** v

$$\dot{u}_i = f(u_i, v_i),$$

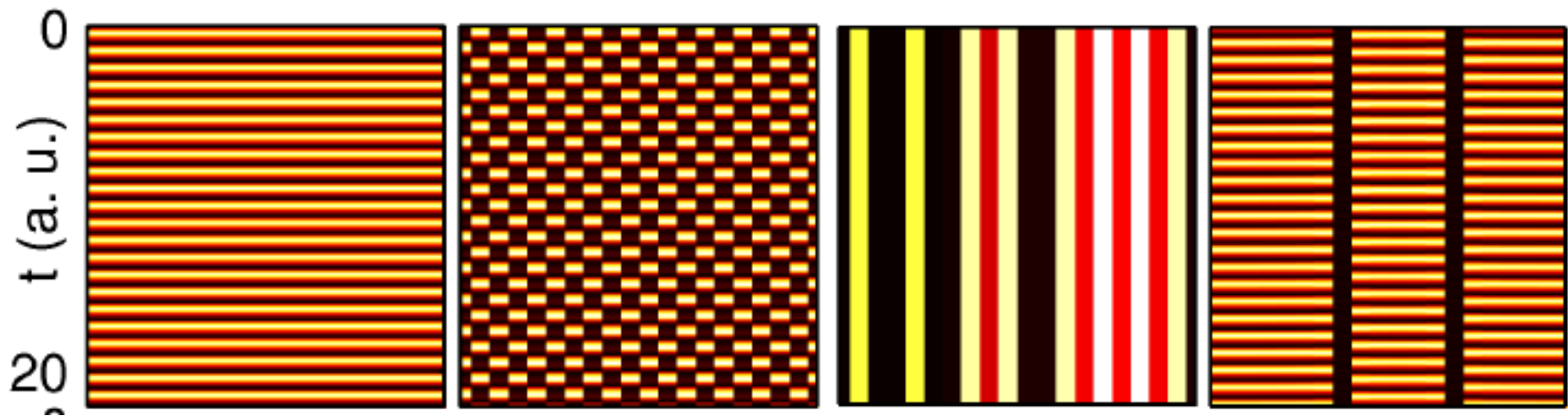
$$\dot{v}_i = g(u_i, v_i) + D_v(v_{i-1} + v_{i+1} - 2v_i),$$

D_v : strength of coupling betn neighboring oscillators

$$i = 1, 2, \dots, N$$

Boundry condn: periodic, 1 coupled to N to complete a ring

Variety of collective dynamical activity



ES

Synchronized
Oscillations

APS

Anti-Phase
Synchronization

SPOD

Spatially Patterned
Oscillation Death

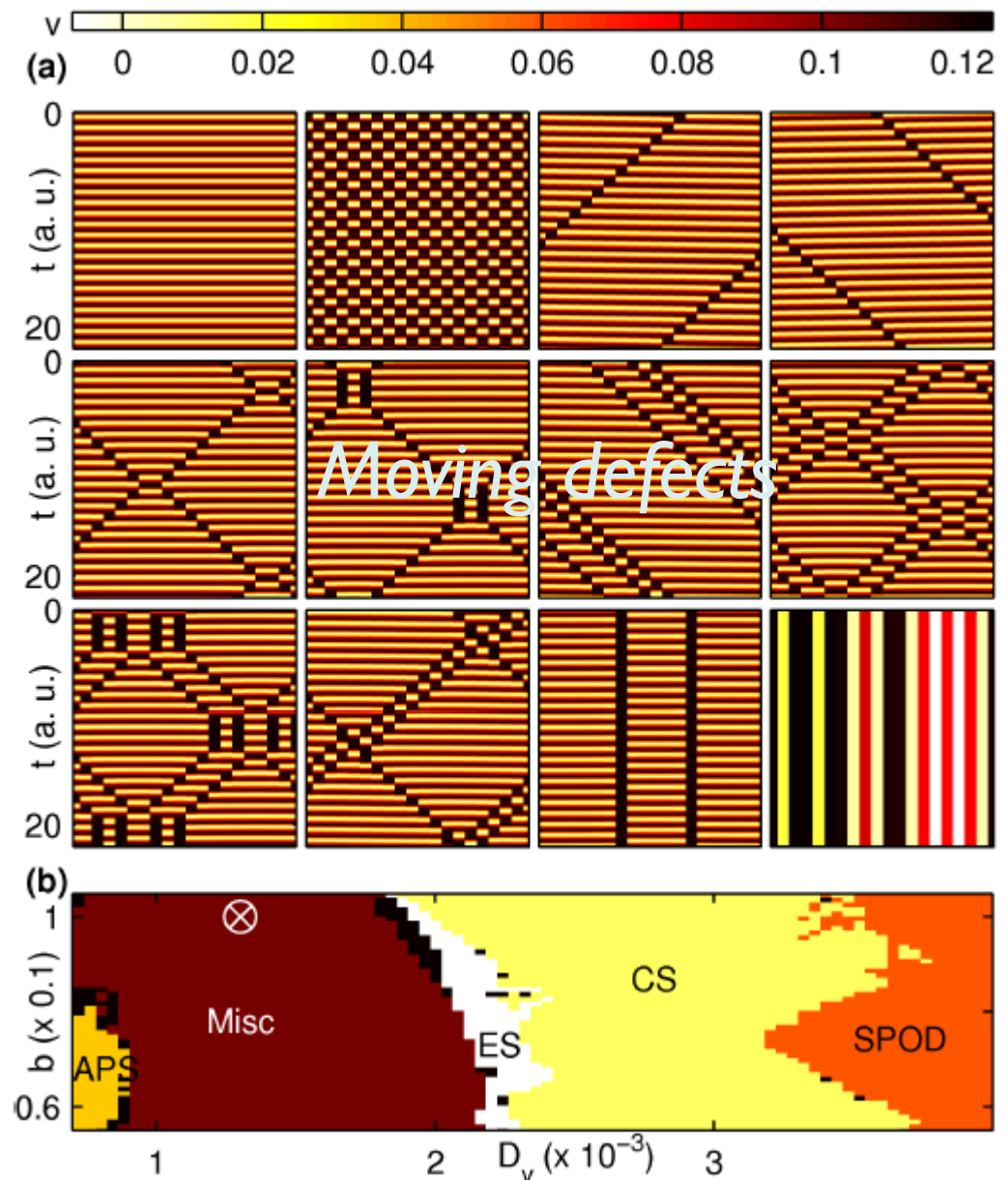
CS

Chimera
State

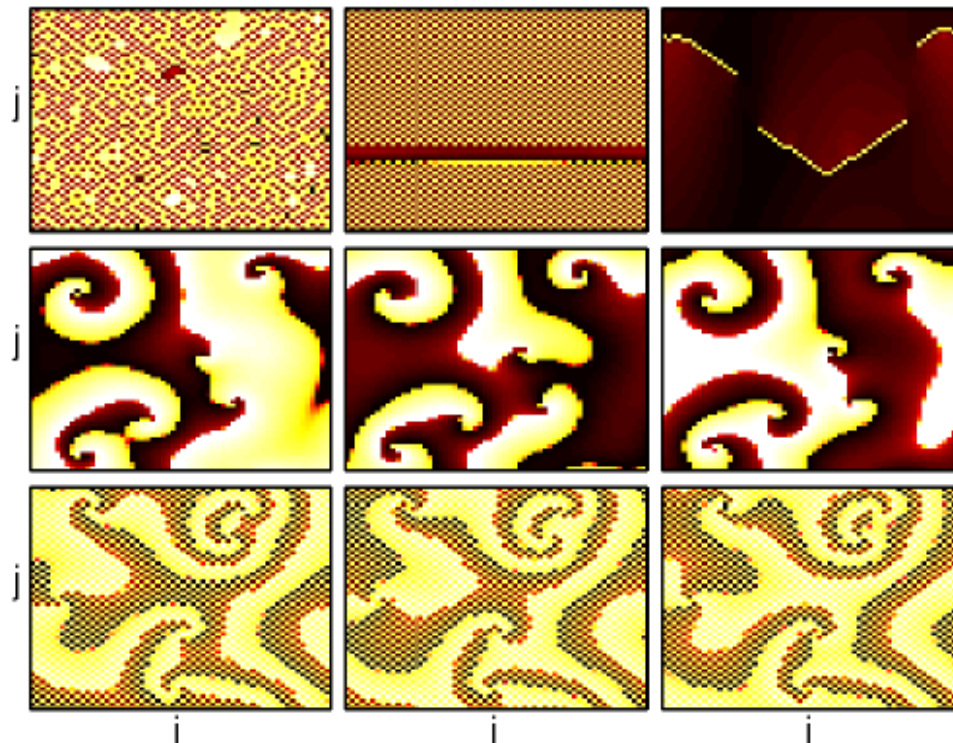
Basins of attraction of different spatio-temporal patterns in coupling-asymmetry space

Spatiotemporal evolution of a ring of $N = 20$ relaxation oscillators, each coupled diffusively to their nearest neighbors through the inactivation variable.

Dynamical regimes of a 1-D array of coupled relaxation oscillators ($N = 20$) showing regions where the majority (>50%) of initial conditions result in SO, APS, SPOD or CS



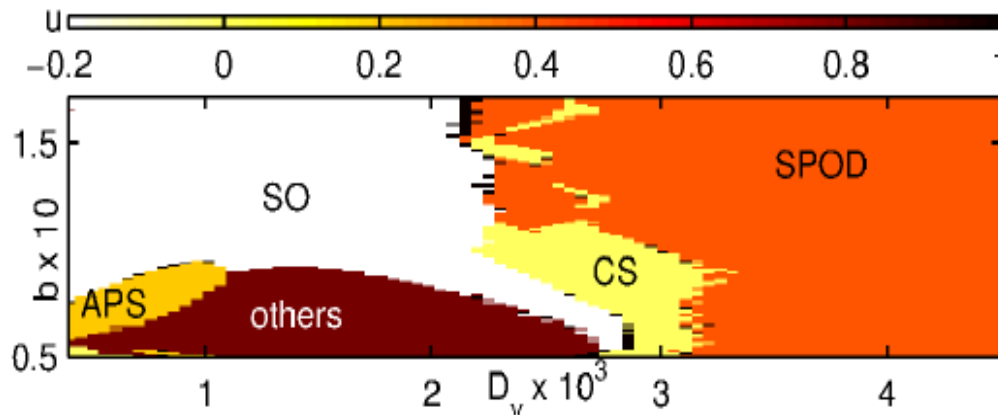
Patterns in 2 dimensions



Chimera state
Line dislocations,
“Glider-like” pattern

Spiral waves

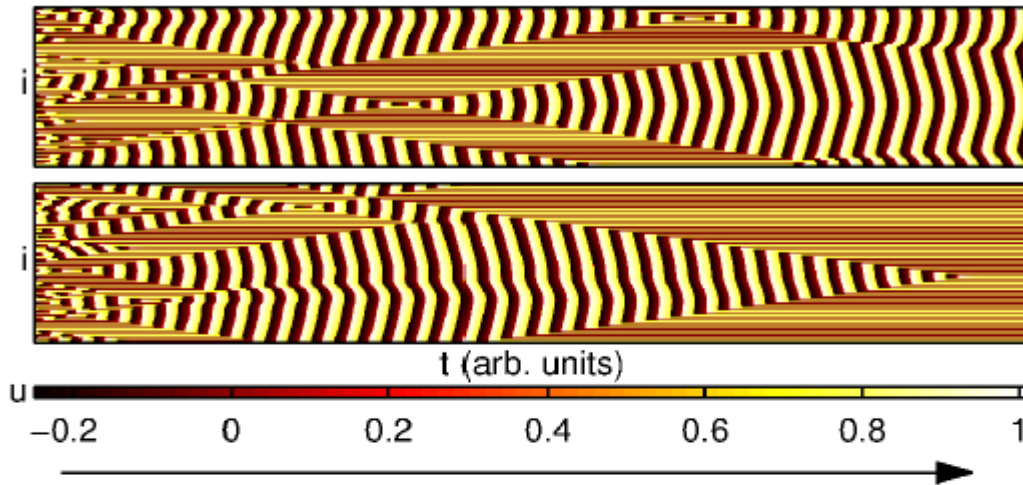
Spiral waves on an APS background



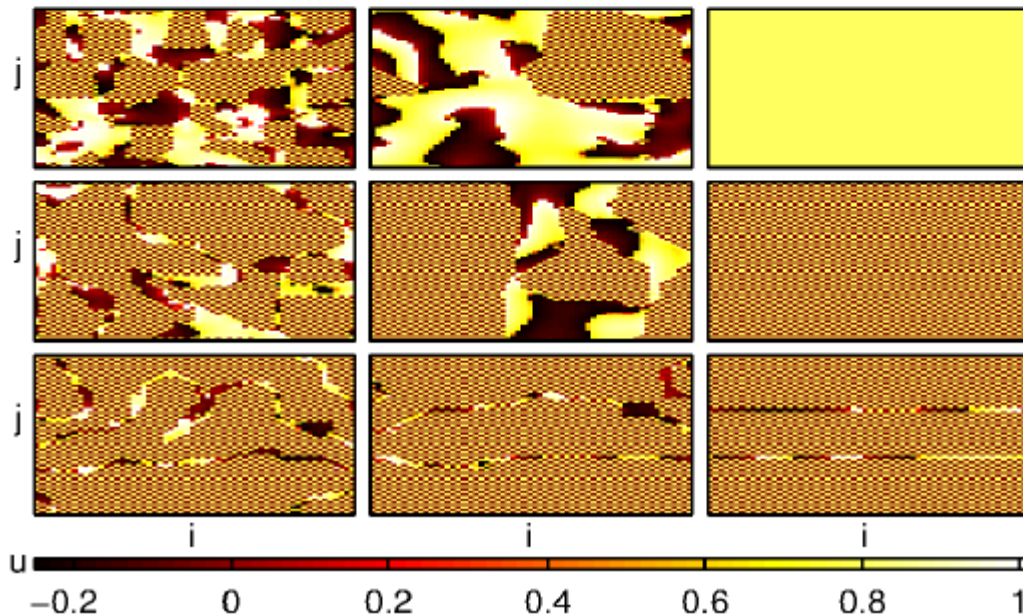
Dynamical regimes of a 2-D array of coupled relaxation oscillators showing regions where the majority (>50%) of initial conditions result in SO, APS, SPOD or CS

“Coarsening”

In both 1D and 2D, if the parameters lie on the border between the regimes corresponding to SO/CS and SPOD, we see behavior analogous to coarsening



1D ring: the system converges to either SO or SPOD depending on the initial condition.



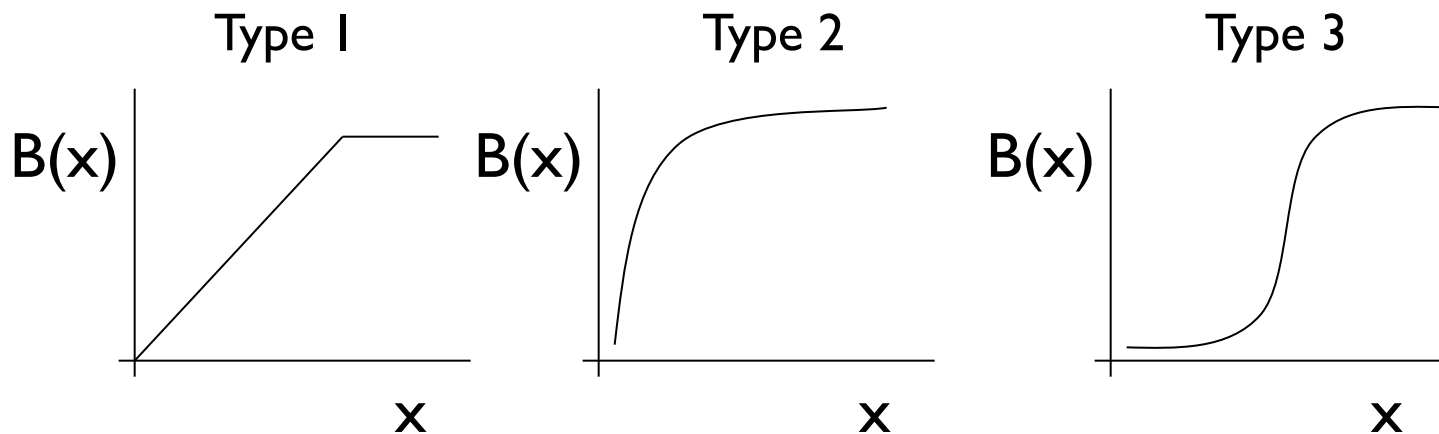
2D lattice: the system converges to SO, SPOD, or line defects depending on initial conditions

Similar spatiotemporal patterns can be seen in
Example: ecological oscillators

Rosenzweig-MacArthur Predator-Prey Model

Predator-Prey model with Holling Type 2 (hyperbolic) functional response

Functional response of predation to prey density (Holling, 1959)



Prey (x) – Predator (y) eqns

$$\frac{dx}{dt} = F(x,y) = r x \left[1 - \left(\frac{x}{K} \right) \right] - q y \left[\frac{x}{b+x} \right]$$

$$\frac{dy}{dt} = G(x,y) = -d y + \varepsilon q y \left[\frac{x}{b+x} \right]$$

Gives rise to limit cycle oscillations

Predator-Prey on a lattice:

Allowing diffusive spreading of only the predator species

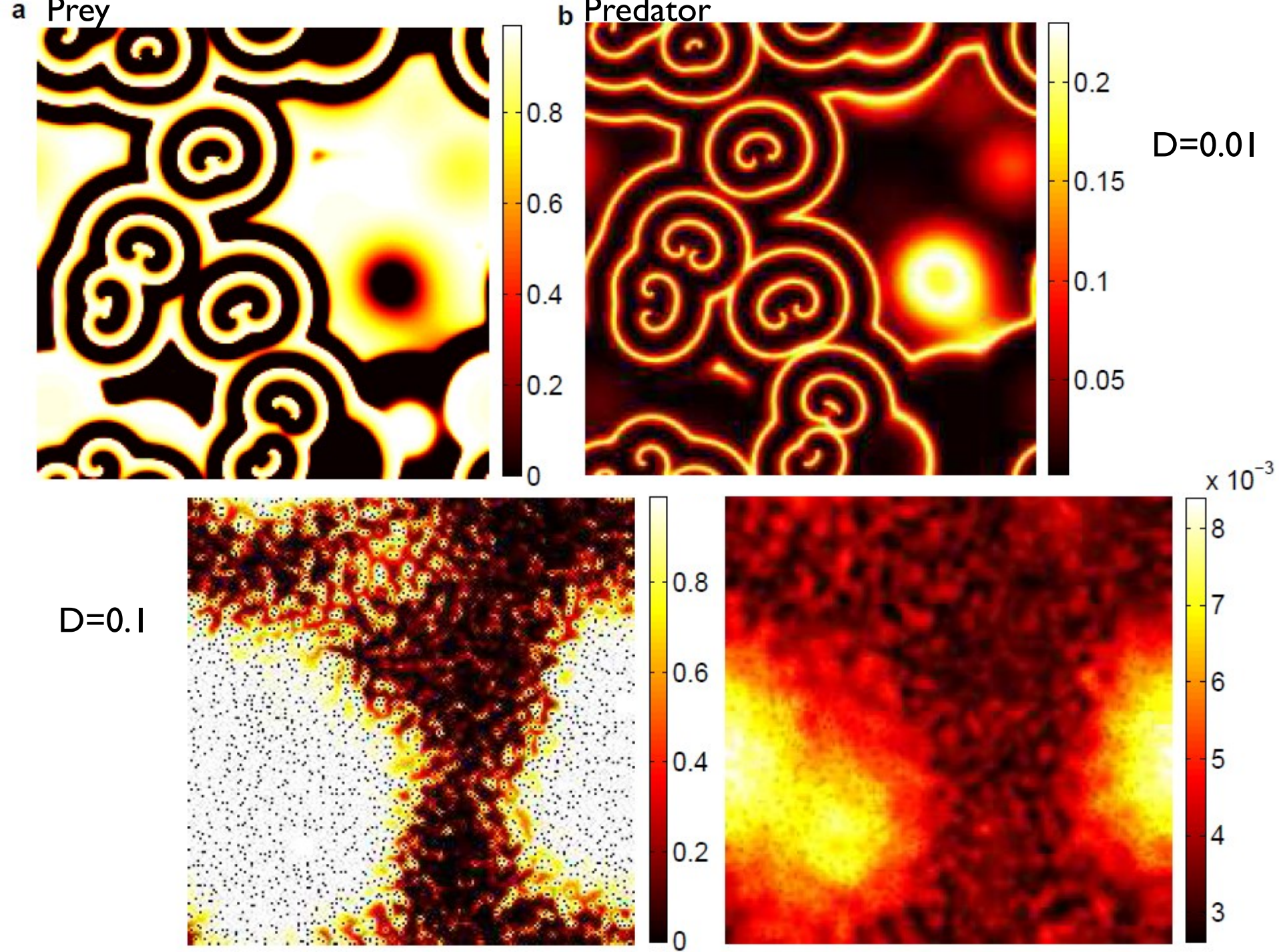
$$\begin{aligned}\partial x / \partial t &= r x [1 - (x / K)] - q y [x / (b + x)] + D_x \nabla^2 x \\ \partial y / \partial t &= -d y + \varepsilon q y [x / (b + x)] + D_y \nabla^2 y\end{aligned}$$

In many real predator-prey systems movement of only predator species is significant (e.g., phytoplankton-zooplankton)

Assuming only predators can move

$$D_x = 0, \quad D_y = D$$

Increasing D results in a variety of patterns including spiral waves and temporally invariant spatially heterogeneous structures

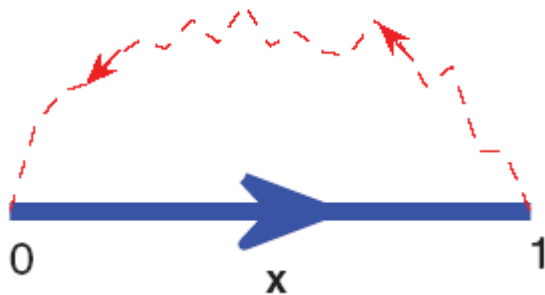


Explaining anti-phase oscillations

To understand the origin of antiphase oscillations, consider

- (a) the relaxation limit $\epsilon \rightarrow 0$
- (b) extreme asymmetry, i.e., system spends entire time in slow segment of limit cycle – the remaining segment of the cycle being traversed extremely fast

(a)



Limit cycle behavior for oscillators in the relaxation limit and extreme asymmetry

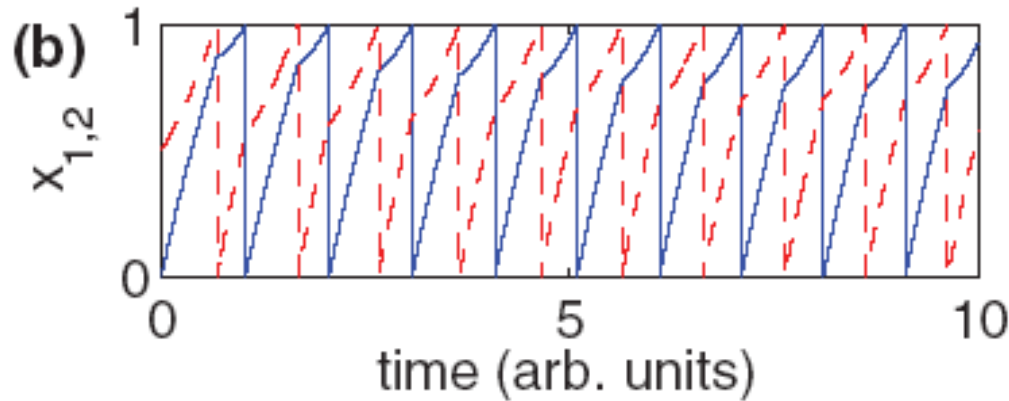
The system reduces to 1-D dynamical system $\dot{x} = \omega(x)$

x : parametrizes slow part of limit cycle, redefined to belong to $(0, 1)$

The model can be exactly solved if $\omega(x)$ is a constant
but geometrical argument valid for any arbitrary +ve definite function defined over $(0, 1)$.

By appropriate choice of time scale set the period $\omega^{-1} = 1$

Anti-phase behavior in pair of coupled oscillators



Time series of two coupled oscillators in relaxation limit and extreme asymmetry

A system of two such diffusively coupled oscillators can be described by

$$\dot{x}_1 = 1 + D(x_2 - x_1), \quad \dot{x}_2 = 1 + D(x_1 - x_2)$$

Given the values of x_1, x_2 at some arbitrary initial time t' , the solution at a later time t :

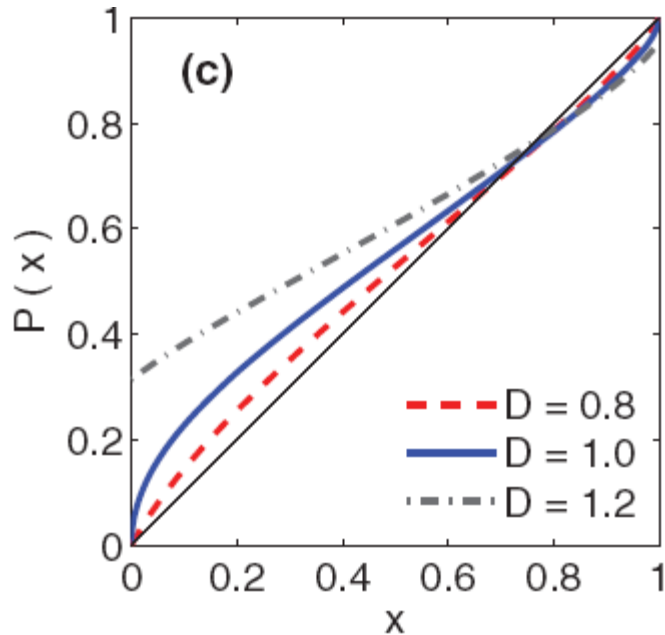
$$x_1(t) + x_2(t) = x_1(t') + x_2(t') + 2(t - t'),$$

$$x_1(t) - x_2(t) = [x_1(t') - x_2(t')] \exp[-2D(t - t')]$$

till time t'' when $\max(x_1, x_2)$ reaches $x = 1$. After this, the larger of (x_1, x_2) is mapped back to $x = 0$ (because of the instantaneous nature of the remaining segment of the limit cycle) and t' is replaced by t'' .

This allows a Poincare map $P(x)$ to be constructed.

Anti-phase oscillations: Poincare map



Poincare map for 2 coupled oscillators for different couplings D showing stable APS and unstable SO

Poincare map $P(x)$ is constructed in two steps

(a) If x_1 starts at 0 and x_2 starts at some point $0 < x < 1$, find location of $x_1 [=f(x)]$ at some time t when $x_2 = 1$ (which is then immediately mapped to $x_2 = 0$).

(b) Starting with $x_2=0$ and $x_1=f(x)$, when $x_1 = 1$ find the location of x_2 : $x = f[f(x)] = P(x)$.

Solving the system of 2 coupled oscillators with appropriate initial conditions,

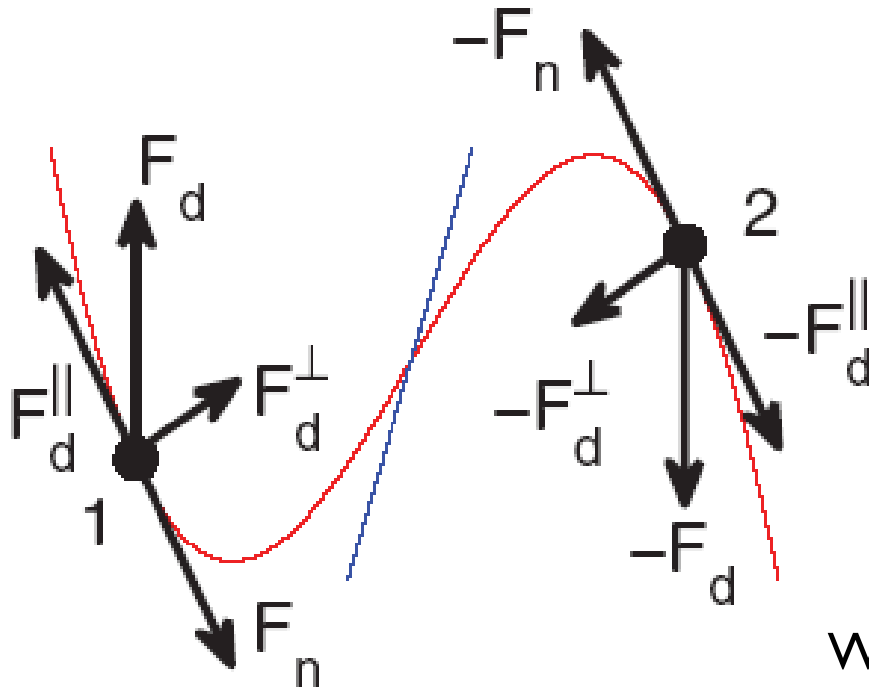
$$f(x) = 1 + D^{-1} W\{-Dx \exp[D(x-2)]\}$$

where $W(\)$ is the Lambert W function

The Poincare map has one stable (APS) and one unstable (SO) fixed point. Thus APS is the only stable state!

Relaxing the extremal conditions under which this was derived may allow a stable SO state to coexist with the stable APS state.

Oscillator death states at high values of coupling



Phase-plane diagram indicating general mechanism for oscillator death for 2 coupled oscillators.

To understand genesis of SPOD at strong coupling, focus on a pair of oscillators in relaxation limit

The parameter b chosen such that v nullcline is placed symmetrically between two branches of u -nullcline with oscillator taking equal time to traverse each branch

When the two oscillators (1 and 2) are in opposite branches, the two opposing forces acting on each oscillator,

- (i) the coupling [$F_d = Dv(v_2 - v_1)$] and
 - (ii) the intrinsic kinetics (F_n),
- can exactly cancel when coupling is strong
 $\Rightarrow$ **oscillator death.**

Symmetry ensures that the force due to the intrinsic kinetics for the two oscillators is identical in magnitude but oppositely directed in the steady state.

Chimera states arise through competition

Chimera: comprises regions with dynamically distinct behavior, as opposed to its recent usage referring to the co-occurrence of coherent and noncoherent domains



At intermediate values of coupling in large arrays, the competition of the oscillator death inducing mechanism with the intrinsic oscillatory dynamics dominant at low coupling, may give rise to *chimera states*.

Remarkable! system exhibits a heterogeneous dynamical state in spite of the bulk being homogeneous.

Not dependent on boundary conditions – as seen with periodic as well as no-flux boundary conditions.

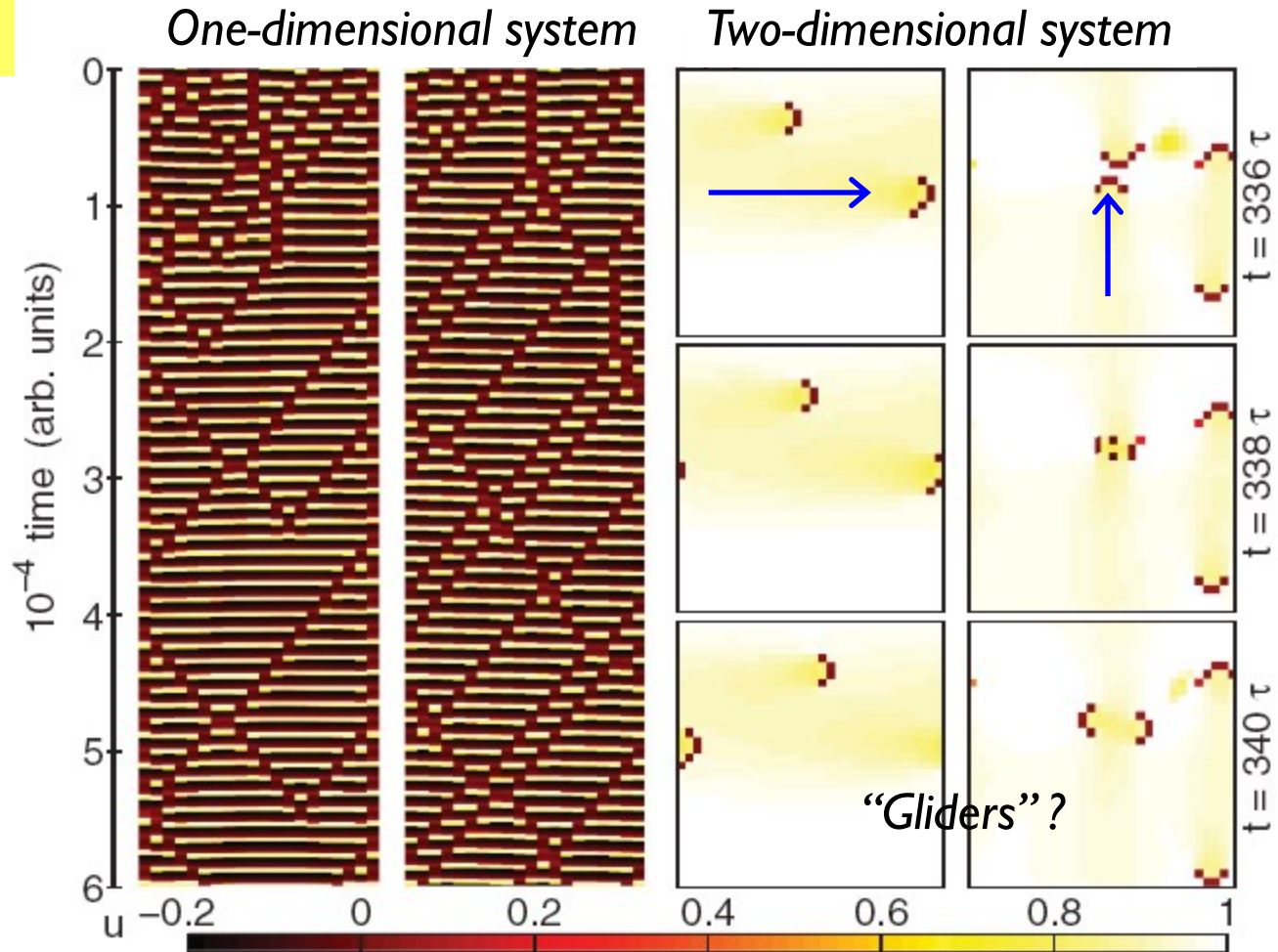
Propagating defects in 1D and 2D systems

Attractors having point-like “phase defects” (i.e., with a discontinuity of phase along the oscillator array at this point), moving in the background of system-wide oscillations.

Movie: propagating gliders

After initial transients defects move in medium with interactions between two such entities resulting in (a) the two being deflected in opposite directions, or (b) either both or only one getting annihilated.

Movie: colliding gliders

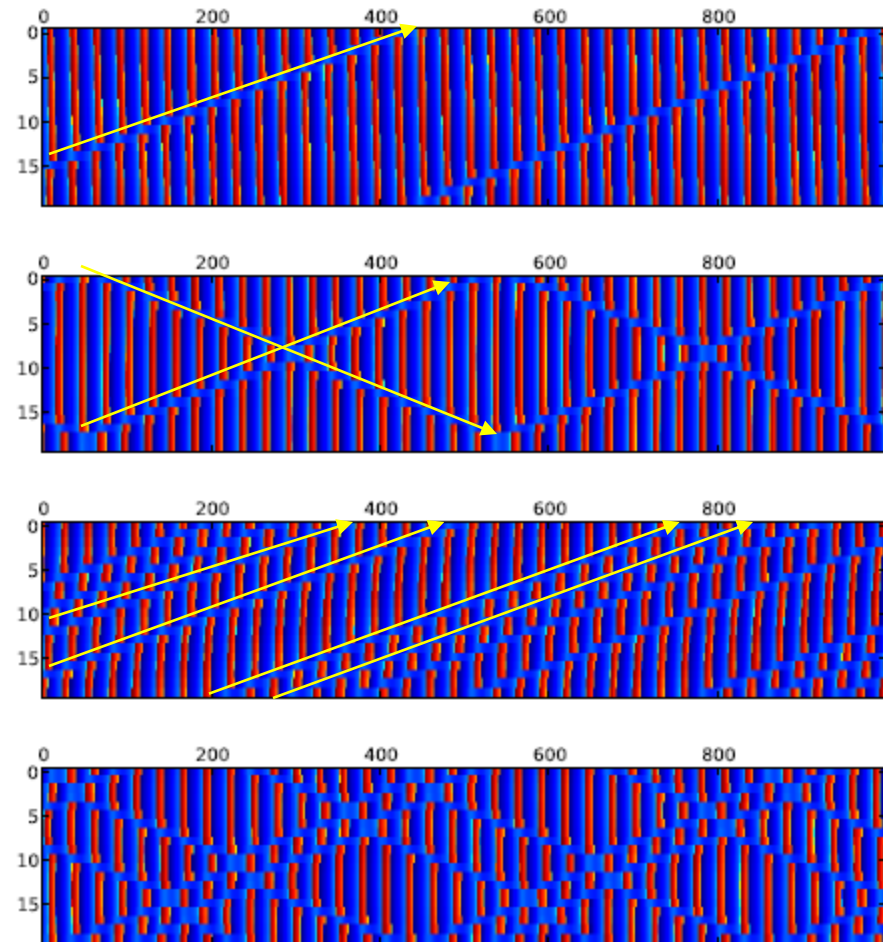
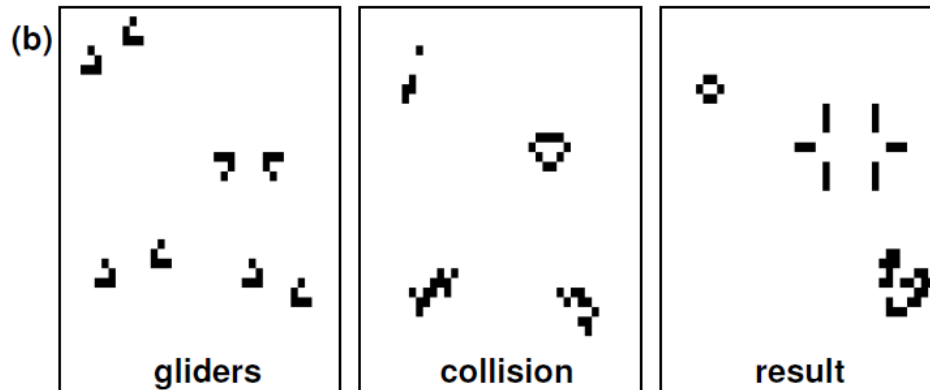
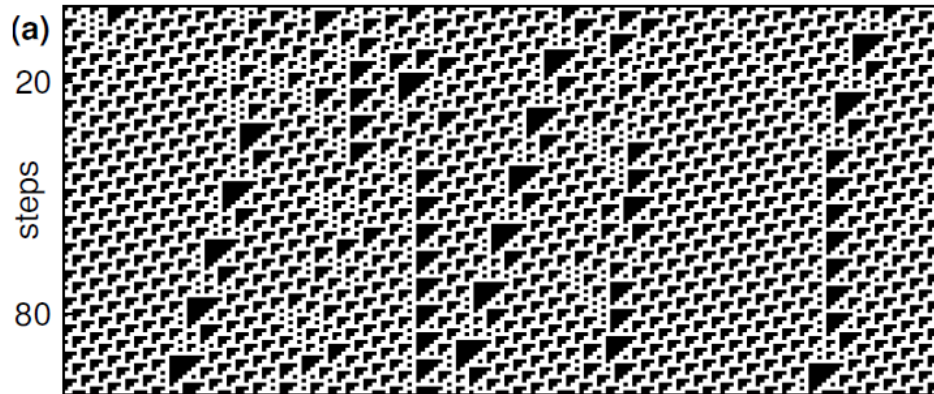


“Can Droplets and Bubbles Think?”

“Bubbles flowing through narrow channels can be encoded with information and made to perform logic operations like those in computer.” — J Epstein (2007)

Possible implementation of chemical computation ?

Moving defects resemble gliders in cellular automata



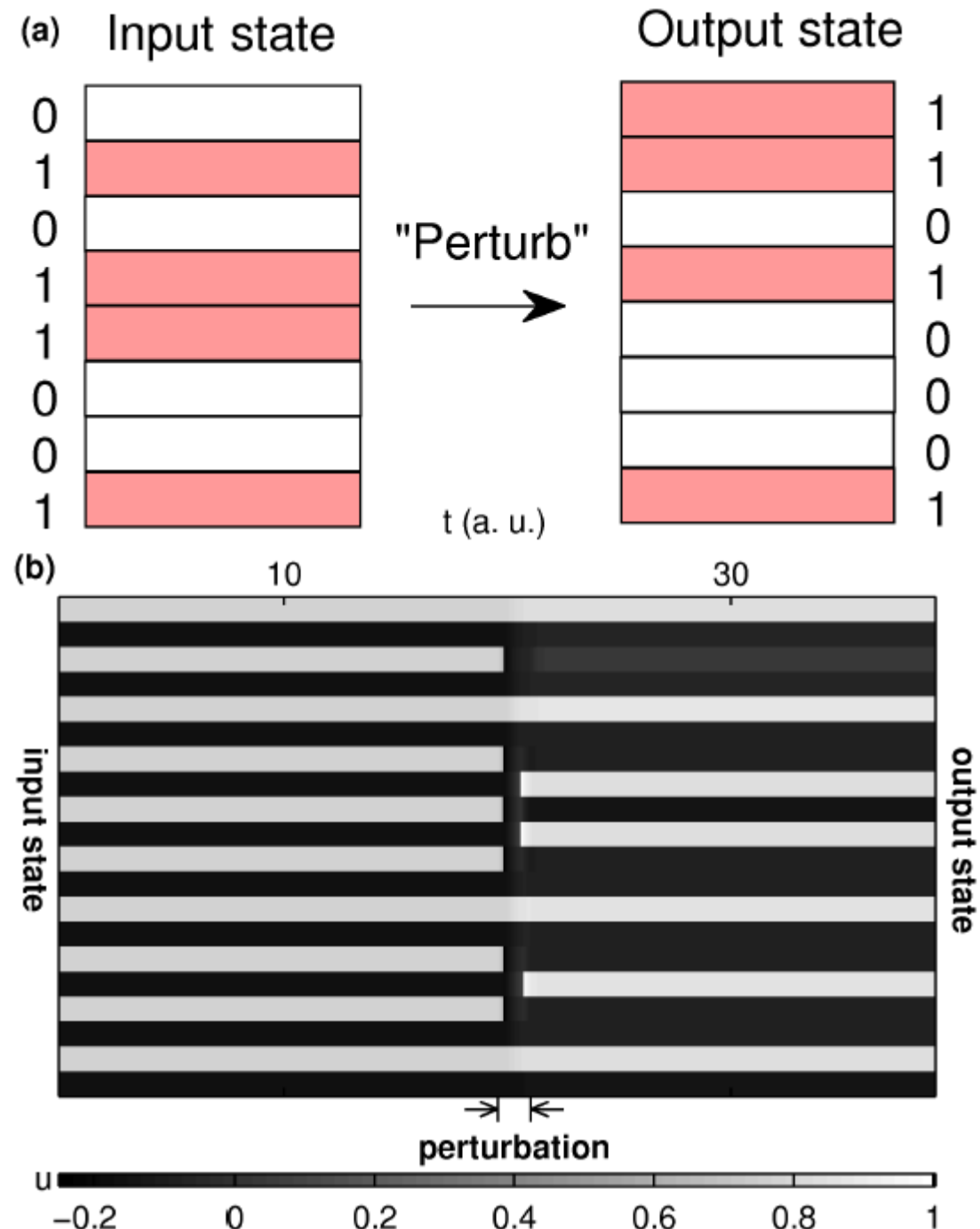
Computing in the SPOD regime

Applying a local perturbation to specific oscillators (rows) results in a different configuration of high and low values that can be interpreted as the output binary string.

Example: a computation that transforms the input sequence $(10)^{10}$ to $10(001)^2 01(001)^3 0$.

Perturbation: stimulating the inactivation component of 6 oscillators (rows 3, 7, 9, 11, 15 and 17, counting from top) for a short duration

Functionally similar to NOT gate

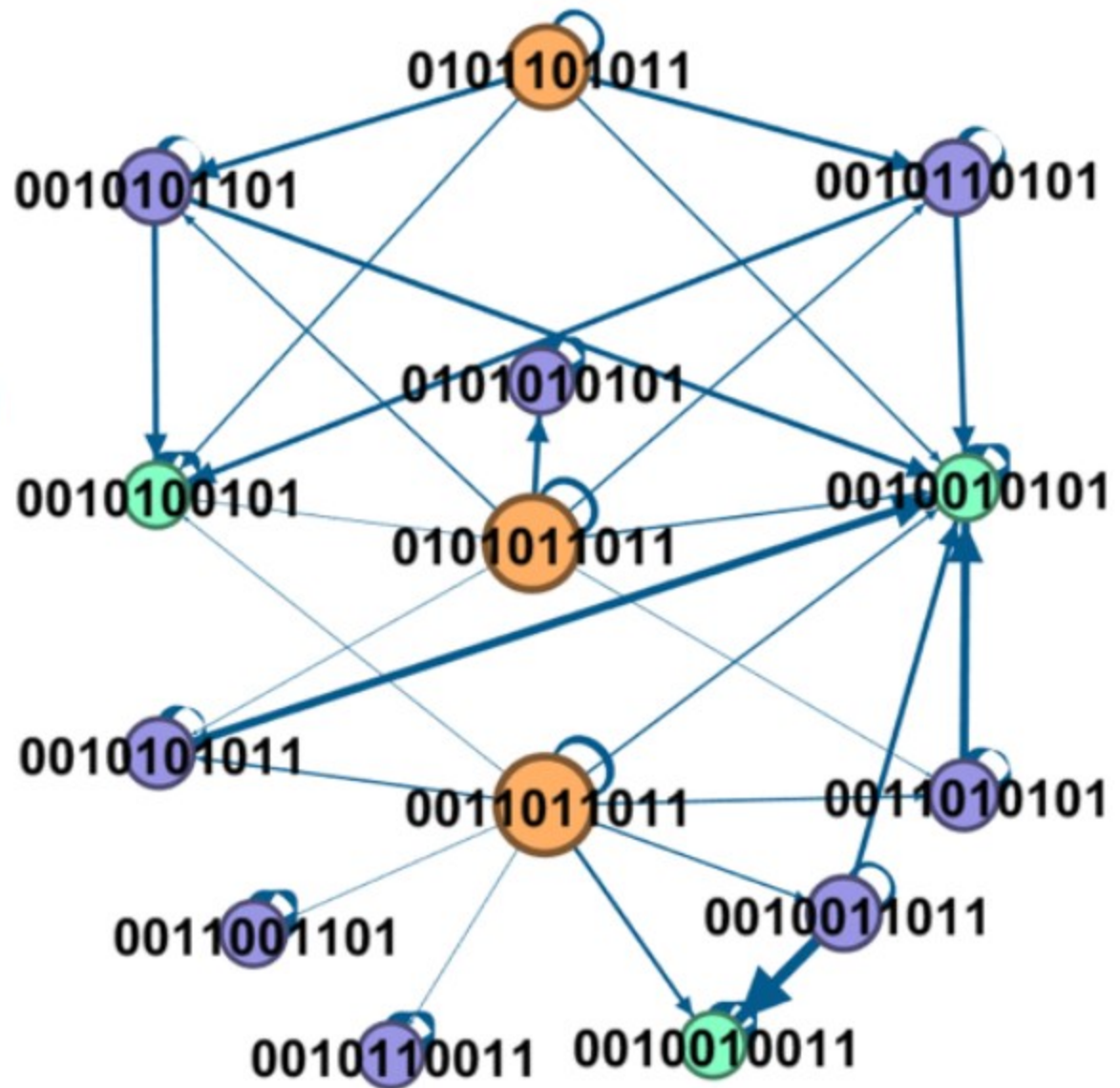


Computing as Transitions between Dynamical States

Not all possible 2^N states representing the distinct combinations of N elements arrested in high and low activity are stable in the SPOD regime

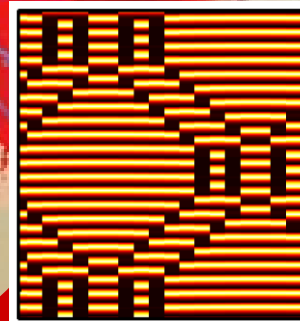
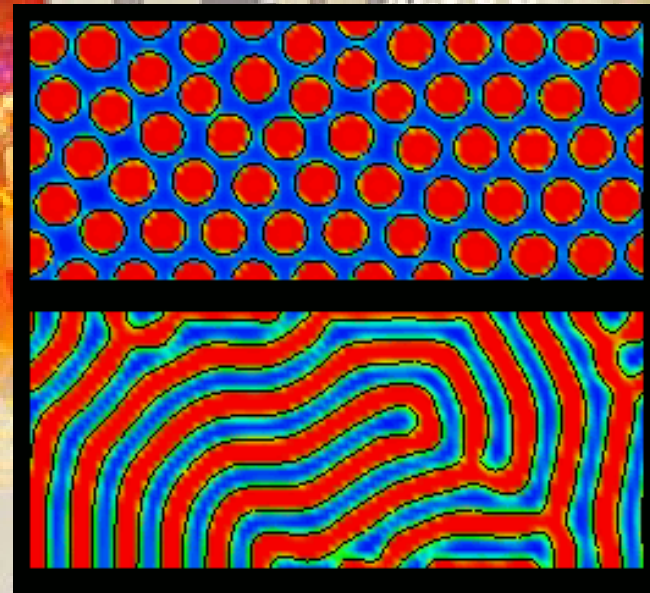
Only states having at most 2 consecutive 0s & 1s allowed
⇒ Fibonacci sequence

Mapping how perturbations (inputs) of one or more elements in a given state yields another state (output)
⇒ State transition diagram (computation)

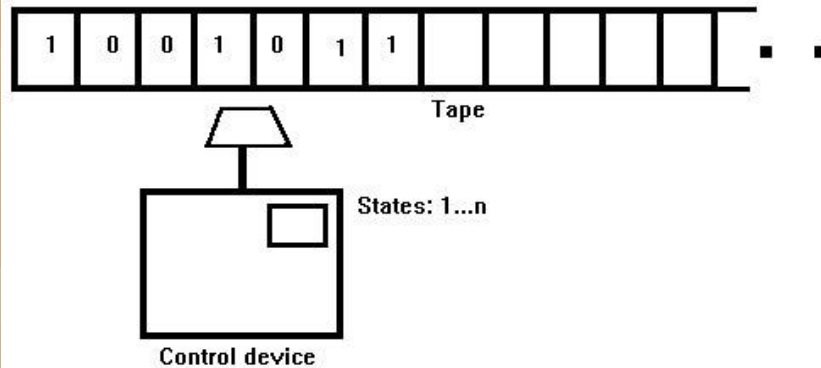


Linking the two enduring legacies of Turing

Pattern formation



Universal computation
& Turing Machines



Outlook

- Oscillations are not curiosities, but occur widely across nature – especially in the biological context
- Spatial and temporal patterns indicators of far-from-equilibrium condition
- Often characterized by spontaneously broken symmetries
- By understanding the physics of pattern formation in such systems, we can come up with
 - possibility of chemical computers and
 - successful methods of controlling such patterns in health & disease

Funding: Dept of Atomic Energy, IMSc Complex Systems Project

Thanks

R Singh, S N Menon & SS, Scientific Reports **6** (2016) 22074

V Sreenivasan, S N Menon & SS, Sci Rep **7** (2017) 1594

R Singh & SS, Phys Rev E **87** (2013) 012907

S N Menon & SS, arXiv:1405.2789 (2014)

